

# A New Forecasting Model for USD/CNY Exchange Rate

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This paper models the return series of USD/CNY exchange rate by considering the conditional mean and conditional volatility simultaneously. An index type functional-coefficient model is adopted to model the conditional mean part and a GARCH type model with a policy dummy variable is applied to the conditional volatility model. We show that the government policy indeed has an impact on the exchange rate dynamic. To evaluate the out-of-sample forecasting ability, a prediction interval is computed by employing nonparametric conditional quantile regression. Our method outperforms other popular models in terms of various criteria.

Keywords: Nonlinearity; Functional-coefficient regression model; GARCH model; Index model; Quantile regression.

# 1 Introduction

It is commonly assumed that the exchange rate follows a martingale difference sequence (MDS) process, which implies that the future returns are unpredictable using public available information. As a result, many empirical studies in 1990s modeled exchange rates by focusing only on volatility forecasts. The most popular specification to model volatility is the GARCH type model due to Bollerslev (1986, 1987). By incorporating the MDS hypothesis and using GARCH type models or their variants, most studies found evidence of nonlinearity in volatilities of exchange rates; see for example, Bollerslev (1990), Brock, Hsieh and Lebaron (1991), Engle, Ito and Lin (1990), West and Cho (1995), among others.

However, by applying a generalized spectral test, Hong and Lee (2003) examined some major exchange rates in the world and found that for some exchange rates, there exist strong non-linearities in the conditional mean of exchange rates in addition to the nonlinearity in conditional volatility. This finding was advocated by Fan, Yao and Cai (2003) by using a nonparametric regression technique. Therefore, during the recent years there have been increasing interests in predicting the changes of exchange rates using nonlinear time series models. For example, Michael, Nobay and Peel (1997) employed a smooth transition autoregressive (STAR) model to analyze nonlinearities for three exchange rates. For every exchange rate examined, they rejected linearity hypothesis and found strong support for exponential STAR (ESTAR) model. Sarantis (1999) adopted a STAR model to test the nonlinearities of the real effective exchange rates for the 10 major industrial countries (the G-10). Their tests rejected the linearity hypothesis for eight out of ten industrial countries during the 1980s and 1990s. Moreover, they demonstrated some evidence that indeed, the STAR model can improve the forecasts compared to the simple random walk model, although the degree of improvement is not always unambiguous. However, the empirical evidence supporting parametric nonlinear time series models seems to be mixed. For example, Meese and Rose (1991) found that incorporation of nonlinearities into the conditional mean models does not help to improve the forecasts of the changes of exchange rates.

The advantage of modeling nonlinearities flexibly makes the nonparametric method popular in the literature. Kuan and Liu (1995) used a feed-forward and recurrent neural network model to forecast exchange rates and found a lower mean squared forecasting error (MSFE) than that in the martingale model, whereas Diebold and Nason (1990) applied nonparametric kernel regressions to analyze the nonlinearities of 10 major dollar spot rates in the post-1973 float period. Gencay (1999) investigated the predictability of spot foreign exchange rate returns from the past buy-sell signals of the simple technical trading rules by using the nearest neighbors and the feed-forward network regressions. The results indicated that the simple technical trading rules provided significant forecast improvements for the current returns over the random walk model.

Chinese foreign trade and investment, which have been started from 1978, are of capital importance in the world economy. It is well known that the USD/CNY exchange rate has been one of the most important economy indexes in the world during the last decade. Due to the specialities of Chinese economy, the whole mechanism of the CNY/USD exchange rate is different from that

for the major exchange rates in the world. The study of Chinese exchange rate has been of independent research interest in the recent years due to economic and political reasons. Moreover, the USD/CNY exchange rate is changed wildly different from one period to another according to the economic reforms and policies.

In this paper, we model the dynamic of the daily USD/CNY exchange rate by considering conditional mean and conditional volatility simultaneously. Concretely, we apply the index functional-coefficient regression method proposed in Fan, Yao and Cai (2003) to model the conditional mean model of the changes of the exchange rate and a GARCH model with policy dummy variables is adopted to describe the conditional volatility. The reason of why a policy dummy variable is considered in the model is that the Chinese government made several reforms on the daily USD/CNY exchange rate, so that the policy changes should have an impact on the dynamic of the exchange rate. The functional-coefficient regression method allows more flexibility of the dynamic and can avoid the curse of dimensionality, and furthermore the nonparametric natural modeling can avoid a possible misspecification and improve the forecasting performance.

The rest of the paper is organized as follows. Section 2 introduces some backgrounds of China's foreign exchange reform. Section 3 introduces the model and the estimation approach as well as inference methods in details. Data description and some characteristics of the daily CNY/USD exchange rate series are briefly discussed at the beginning of Section 3. Section 4 presents empirical results. Section 5 compares our methods to other popular models in the literature. Section 6 concludes.

## **2 China's Foreign Exchange Reforms and Its Characteristics**

Since the whole mechanism of the CNY/USD exchange rate is different from that for the major exchange rates in the world such as EUR/USD, JPY/USD and KRW/USD, this section is devoted to introducing briefly the Chinese foreign exchange's characteristics and its reforms in the recent years. Prior to the economic reform launched in 1978, Chinese trade took place within the context of the so called import substitution policy. The regime maintained an overvalued exchange rate to subsidize the import of capital goods in heavy and chemical industries. In order to maintain the overvaluation, a rigid exchange control was implemented. As described by Branstetter and Lardy (2008), key elements of the control system included a 100 percent foreign exchange surrender requirement, tight limitations on individuals to hold foreign currency, and strict controls on the outflow of foreign capital. Beginning from the early of 1980s, the government relaxed almost all of the above restrictions progressively. The USD/CNY exchange rate was 1.5 yuans (Chinese currency unit) to one dollar in 1981, and was devalued to 8.7 yuans in 1994. After a modest appreciation, the authorities fixed the exchange rate around 8.3 yuans in 1995 and kept this exchange rate until the summer of 2005. After that, the market oriented reform of foreign exchange was accelerated.

On July 21, 2005, the Chinese authority announced that the exchange rate regime would move immediately into a managed floating exchange rate regime based on market supply and demand, and furthermore the authority scrapped renminbi (RMB, Chinese currency)'s peg to the US dollar,

shifting to referring to a basket of main currencies to determine the value of its currency. At the same time, the USD/CNY exchange rate was raised to 8.11 from around 8.28 on that day, that is, the value of Chinese yuan was increased about 2 percent. Moreover, the daily trading price of the USD/CNY exchange rate in the inter-bank foreign exchange trading market would be allowed to float within a band of 0.3 percent around the central parity published by the central bank. The float range of RMB exchange against the dollar was raised again from 0.3 percent to 0.5 percent on May 21, 2007.

In order to further promote the flexibility of the foreign exchange, the People's Bank of China (Chinese Central bank) decided to introduce the over-the-counter (OTC) transactions in the inter-bank spot foreign exchange market on January 4, 2006. Before the introduction of OTC transactions, the central parity of exchange rate was determined based on the closing quotation in the inter-bank foreign exchange market. Under the system of OTC transactions, the formation mechanism of the central parity was changed. The China Foreign Exchange Trade System makes offers to all market makers before the opening of the inter-bank foreign exchange market, and the quotations of all market makers are taken except the highest and the lowest quotations. The central parity of exchange rate of RMB against US dollar for the current day is confirmed by the weighted average of all remaining quotations. The weight is determined by the China Foreign Exchange Trade System in the light of transaction volumes and the quotation conditions and other indexes.

### **3 The Econometric Modeling**

#### **3.1 The Data**

We concern the daily USD/CNY exchange rate series from January 4, 2006 to July 18, 2008, which forms a series of 619 observations. The data are available from the Chinese State Administration of Foreign Exchange (<http://www.safe.gov.cn/>). Let  $X_t$  be the exchange rate on the  $t$ th day. Figure 1(a) shows that the price series has an obvious decreasing time trend, which reflects the fact of a gradual appreciation of RMB since 2006. Denote the return series by  $r_t = 100 \log(X_t/X_{t-1})$ , the so-called scaled logarithmic difference. Figure 1(b) presents the time series graph of the return of exchange rate and it clearly shows a structural change on May 21, 2007. On that day, the authorities raised the float range of USD/CNY exchange rate from 0.3 percent to 0.5 percent.

#### **3.2 The Model**

Figure 1(b) shows that the return series is nonlinear but hard to be modeled by an existing parametric nonlinear model. However, any nonlinear model can be approximated by a nonparametric time-varying parameter (TVP) linear model; see Granger (2008). Following this spirit, we propose to using a flexible functional-coefficient method of Cai, Fan and Yao (2000) to model the conditional mean part and to employing the GARCH model with policy dummies to consider the conditional volatility part.

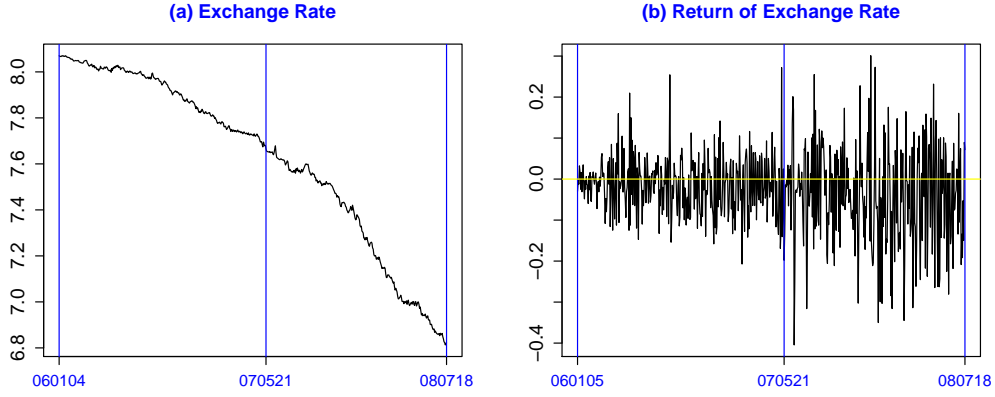


Figure 1: Time series plot of USD/CNY exchange rate

In order to forecast the return series of exchange rate, we are interested in estimating  $E(r_t|I_{t-1})$ , where  $I_{t-1}$  is the information set available at time  $t-1$ , that is,  $I_{t-1} \equiv \{r_{t-1}, \dots, r_1, X_{t-1}, \dots, X_1\}$ , where  $X_{t-1}$  denotes other available explanatory variables. The models can be summarized as follows:

$$r_t = \mu_t + \eta_t, \quad \text{where } \eta_t = \sigma_t \varepsilon_t. \quad (3.1)$$

For the conditional mean part  $\mu_t$ , it is ideal to model  $\mu_t$  by a nonparametric form as  $\mu_t = \mu(I_t)$  but it suffers from the so-called “curse of dimensionality” due to the high dimensional modeling. To overcome this problem, we employ a functional-coefficient regression model, in which the coefficients depend on some smoothing variables  $U_t$ . The functional-coefficient regression model imposes very little model assumptions but allows appreciable flexibility on the model structure, and furthermore, the additive structure can effectively avoid the “curse of dimensionality” in nonparametric regressions. A functional-coefficient regression model can be defined by

$$m(\mathbf{u}, \mathbf{x}) = E(Y_t | \mathbf{U}_t = \mathbf{u}, \mathbf{X}_t = \mathbf{x}) = \sum_{j=1}^d a_j(\mathbf{u}) \mathbf{x}_j \quad (3.2)$$

where,  $Y_t \in \mathbb{R}^1$  is a dependent variable,  $\mathbf{X}_t \in \mathbb{R}^p$  are explanatory variables and  $\mathbf{U}_t \in \mathbb{R}^k$  are smoothing variables. We assume that  $\{Y_t, \mathbf{X}_t, \mathbf{U}_t\}_{t=-\infty}^{\infty}$  are strictly stationary and  $\{a_j(\cdot)\}_{j=1}^d$  are measurable functions mapping from  $\mathbb{R}^k$  to  $\mathbb{R}^1$ ; see Cai, Fan and Yao (2000) for details.

For the conditional volatility part  $\sigma_t$ , a GARCH type model is used with policy dummy variable. To address the structural change occurred on May 21, 2007, a policy dummy variable  $C_t$  is considered, which takes a value of zero for observations before that day and one for the remaining observations. That is,

$$C_t = \begin{cases} 0 & , \text{when } t < 20070521 \\ 1 & , \text{when } t \geq 20070521 \end{cases}$$

We also add the same policy dummy variable in the conditional mean part. By combining (3.1) and (3.2), the forecasting model takes the following form:

$$Y_t = a_C(U_t)C_t + \sum_{j=1}^p a_j(U_t)Y_{t-j} + \eta_t = \sum_{j=1}^{p+1} a_j(U_t)X_{tj} + \sigma_t \varepsilon_t, \quad (3.3)$$

$$\sigma_t^2 = \gamma_0 + \gamma_1 \eta_{t-1}^2 + \delta_1 \sigma_{t-1}^2 + \alpha_\sigma C_t \quad \text{with } \gamma_0 > 0, \gamma_1 \geq 0, \delta_1 \geq 0, \text{ and } \gamma_1 + \delta_1 < 1, \quad (3.4)$$

where  $X_{tj} = Y_{t-j}$ ,  $X_{t,p+1} = C_t$ ,  $U_t$  is a smoothing variable determined later and  $\{a_j(\cdot)\}$ ,  $0 \leq j \leq p+1$ , are continuous functions.

### 3.3 The Nonparametric Estimation

There are several nonparametric estimation techniques available to estimate the functional coefficient  $\{a_j(\cdot)\}$ . Here we employ the local linear regression method due to its attractive properties such as the boundary correction and minimax efficiency; see Fan (1993) and Fan and Gijbels (1996). We assume throughout that  $a_j(\cdot)$  has a continuous second derivative. Then, for any given grid point  $u_0 \in \mathbb{R}^k$ , when  $U_t$  is in a neighborhood of  $u_0$ ,  $a_j(U_t)$  is approximated locally at  $u_0$  by the Taylor expansion. That is,  $a_j(U_t) \approx a_j(u_0) + \dot{a}_j(u_0)^\top (U_t - u_0)$ . The local linear estimate is defined as  $\hat{a}_j(u_0) = \hat{a}_j$ ,  $\hat{a}_j(u_0) = \hat{b}_j$ , where  $(\hat{a}_j, \hat{b}_j)$  minimizes the sum of weighted squares:

$$\sum_{t=1}^n \left[ Y_t - \sum_{j=1}^{p+1} (a_j + b_j^\top (U_t - u_0)) X_{tj} \right]^2 K_h(U_t - u_0), \quad (3.5)$$

where  $K_h(\cdot) = h^{-k} K(\cdot/h)$  is a kernel function on  $\mathbb{R}^k$ ,  $h > 0$  is a bandwidth, and  $h \rightarrow 0$  as  $n \rightarrow \infty$ . By moving  $u_0$  along the whole domain of  $U_t$ , the entire estimated surface of  $a_j(u_0)$  is obtained.

The data-driven fashion and optimal bandwidth is chosen by the nonparametric AIC developed by Hurvich, Simonoff and Tsai (1998) and Cai and Tiwari (2000), which is specifically designed as an approximately unbiased estimator of the expected Kullback-Leiber information criterion in nonparametric regression settings. Also, the nonparametric AIC can be used for choosing the ‘‘optimal’’ order  $p$ ; see Cai and Tiwari (2000) for details.

### 3.4 The Selection of Smoothing Variable

It is important to choose an appropriate smoothing variable  $U_t$  when applying the functional-coefficient regression model. Knowledge on physical background of the data or economic theory and events may be very helpful. Some data-driven methods to choose the smoothing variables are also available, such as the Akaike information criterion, cross-validation, and other criteria; see Cai, Fan and Yao (2000) and Fan, Yao and Cai (2003). We benefit from the fact that the value of RMB pegs to a basket of main currencies. The Governor of the People’s Bank of China, Xiaochuan Zhou, stated on August 9, 2005 that the major currencies in the basket include US dollar(USD), euro(EUR), Japanese yen(JPY), and Korean won(KRW). Therefore, we choose the smoothing variable as a linear combination of the return series of three major exchange rates: EUR/USD, KRW/USD and JPY/USD and

the weights of each exchange rate is automatically determined by the data.

$$U_t = \beta_{EUR} \cdot r_{EUR,t} + \beta_{KRW} \cdot r_{KRW,t} + \beta_{JPY} \cdot r_{JPY,t}, \quad (3.6)$$

where  $\beta = (\beta_{EUR}, \beta_{KRW}, \beta_{JPY})$  are weights satisfying the identification condition  $\beta_{EUR} + \beta_{KRW} + \beta_{JPY} = 1$ , and  $r_{EUR,t}$ ,  $r_{KRW,t}$  and  $r_{JPY,t}$  are the return series of EUR/USD, KRW/USD and JPY/USD respectively.

The weights  $\beta$  are estimated by a hybrid backfitting algorithm method proposed by Fan, Yao and Cai (2003). Basically, the estimation method is an alternating iteration between estimating the linear index through a one-step scheme proposed by Bickel (1975) and estimating the functional coefficients through an one-dimensional local linear smoothing method. To minimize the  $E(Y - \hat{Y})^2$ , one should search for  $\beta$  to minimize

$$R(\beta) = \frac{1}{n} \sum_{t=1}^n \left\{ Y_t - \sum_{j=1}^{p+1} a_j(U_t) r_{t-j} \right\}^2 w(U_t), \quad (3.7)$$

where  $w(\cdot)$  is a known weighting function. Suppose that  $\hat{\beta}$  is the minimizer of the above equation, then  $\dot{R}(\hat{\beta}) = 0$ , where  $\dot{R}(\cdot)$  denotes the derivative of  $R(\cdot)$ . For any  $\beta^{(0)}$  close to  $\hat{\beta}$ , we have the Taylor approximation  $0 = \dot{R}(\hat{\beta}) \approx \dot{R}(\beta^{(0)}) + \ddot{R}(\beta^{(0)})(\hat{\beta} - \beta^{(0)})$ , where  $\ddot{R}(\cdot)$  is the Hessian matrix of  $R(\cdot)$ . This leads to the one-step iterative estimate similar to the Newton-Raphson procedure:

$$\beta^{(1)} = \beta^{(0)} - \ddot{R}(\beta^{(0)})^{-1} \dot{R}(\beta^{(0)}), \quad (3.8)$$

where  $\beta^{(0)}$  is the initial value. At each iteration, we re-scale  $\beta^{(1)}$  such that it has unit sum. We refer the reader to the paper by Fan, Yao and Cai (2003) for details.

Alternatively, one might consider other smoothing variables used in the literature, such as the moving average technique trading rule (MATTR)

$$U_{t,MATTR} = \frac{Y_{t-1}}{\sum_{j=1}^L Y_{t-j}/L} - 1$$

for certain  $L$  (say,  $L = 21$ ), as in Brock, Lakonishock and Lebaron (1992) and Hong and Lee (2003). Indeed,  $U_{t,MATTR}$  has a nice economic interpretation; see the aforementioned papers for details. Based on our empirical study, we find that the linear combination of several main currencies versus US dollar outperforms MATTR in terms of the nonparametric AIC criterion.

### 3.5 Goodness-of-Fit Tests

It is important to consider the goodness-of-fit of the nonparametric model proposed above. For example, it is interesting to test whether the policy dummy variable is significant.

Firstly, we test for the linear regression model against the nonparametric functional-coefficient regression model by employing the method proposed by Cai, Fan and Yao (2000). The null hypothesis is defined as:

$$H_0 : a_j(U_t) = \alpha_j \quad \text{for } 0 \leq j \leq p + 1, \quad (3.9)$$

so that model (3.3) becomes a  $p$ th order autoregressive model with an exogenous variable  $C_t$ , where  $\{\alpha_j\}$  are constant parameters in the AR( $p$ ). In particular, it is interesting to see if the policy dummy variable is significant. That is to test the null hypothesis defined as

$$H_0 : a_C(U_t) = 0. \quad (3.10)$$

To consider the test in (3.9) and (3.10), one can apply a generalized likelihood ratio (or generalized F-type) test proposed by Cai, Fan and Yao (2000) and studied by Fan, Zhang and Zhang (2001), which can be constructed as

$$F = \frac{RSS_0 - RSS_1}{RSS_1} = \frac{RSS_0}{RSS_1} - 1, \quad (3.11)$$

where  $RSS_0$  and  $RSS_1$  are the residual sum of squares under the null and alternative hypotheses respectively. The null hypothesis is rejected for large value of the test statistic. To calculate the  $p$ -value, a nonparametric wild bootstrap approach can be employed; see Cai, Fan and Yao (2000) for details.

### 3.6 Prediction Intervals

It is easy to see from (3.3) and (3.4) that the  $\tau$ th conditional quantile of  $Y_t$  given  $U_t$  and  $\mathbf{X}_t$  is

$$q_\tau(U_t, \mathbf{X}_t) = \sum_{j=1}^{p+1} a_j(U_t) X_{tj} + \sigma_t F_\varepsilon^{-1}(\tau), \quad (3.12)$$

where  $F_\varepsilon(\cdot)$  is the distribution of  $\varepsilon$ . Therefore, a naive  $(1 - \alpha)100\%$  prediction interval can be constructed as

$$\left( \sum_{j=1}^{p+1} \hat{a}_j(U_t) X_{tj} - \hat{\sigma}_t F_\varepsilon^{-1}(\alpha/2), \sum_{j=1}^{p+1} \hat{a}_j(U_t) X_{tj} + \hat{\sigma}_t F_\varepsilon^{-1}(1 - \alpha/2) \right).$$

To make prediction interval in a nonparametric nature, the above quantile regression function in (3.12) is generalized to be more a general form as

$$q_\tau(U_t, \mathbf{X}_t) = a_{C,\tau}(U_t) C_t + \sum_{j=1}^p a_{j,\tau}(U_t) X_{tj}, \quad (3.13)$$

which was proposed by Cai and Xu (2008), where  $\{a_{j,\tau}(\cdot)\}$  might depend on  $\tau$ . To estimate  $\{a_{j,\tau}(\cdot)\}$  nonparametrically, one can use a nonparametric quantile regression estimation procedure as in Cai and Xu (2008). Use the Taylor expansion,

$$q_\tau(U_t, \mathbf{X}_t) \approx \beta_{0,\tau}^\top \mathbf{X}_t + \beta_{1,\tau}^\top \mathbf{X}_t (U_t - u_0),$$

and then find  $(\hat{\beta}_{0,\tau}, \hat{\beta}_{1,\tau})$  to minimize the following

$$\sum_{t=1}^n \rho_\tau(Y_t - \beta_{0,\tau}^\top \mathbf{X}_t - \beta_{1,\tau}^\top \mathbf{X}_t (U_t - u_0)) K_h(U_t - u_0),$$



where  $\rho_\tau(z) = z(\tau - I_{\{z < 0\}})$  is the loss function and  $I_A$  denotes the indicator function of any set  $A$ . Then, the nonparametric estimation of  $a_{j,\tau}(u_0)$  is the  $j$ th element of  $\hat{\beta}_{0,\tau}$ , so that the nonparametric estimation of  $q_\tau(u_0, \mathbf{X}_t)$  is  $\hat{\beta}_{0,\tau}^\top \mathbf{X}_t$ . By changing the value of  $\tau$  from 0 to 1, a set of quantile regressions are obtained. Particularly, when letting  $\tau_1 = 0.025$  and  $\tau_2 = 0.975$ , then a 95% prediction interval is obtained and it is  $(\hat{q}_{\tau_1}(U_t, \mathbf{X}_t), \hat{q}_{\tau_2}(U_t, \mathbf{X}_t))$ .

## 4 Empirical Results

For the conditional mean part, we use an index functional-coefficient regression model with the coefficients depending on the smoothing variable  $U_t$  given in (3.6). By using the nonparametric AIC method and the hybrid backfitting algorithm, the selected optimal bandwidth is  $h = 1.2$ , the selected optimal lag term is  $p = 2$  and the selected optimal weights are  $\hat{\beta} = (0.2966525, 0.3842938, 0.3190537)$ . The smoothing variable is constructed by a linear combination of the return series of EUR/USD, KRW/USD and JPY/USD with the estimated weights  $\hat{\beta}$  and its time series plot is presented in Figure 2. The estimated functional coefficient curves are displayed in Figure 3. Finally, both goodness-

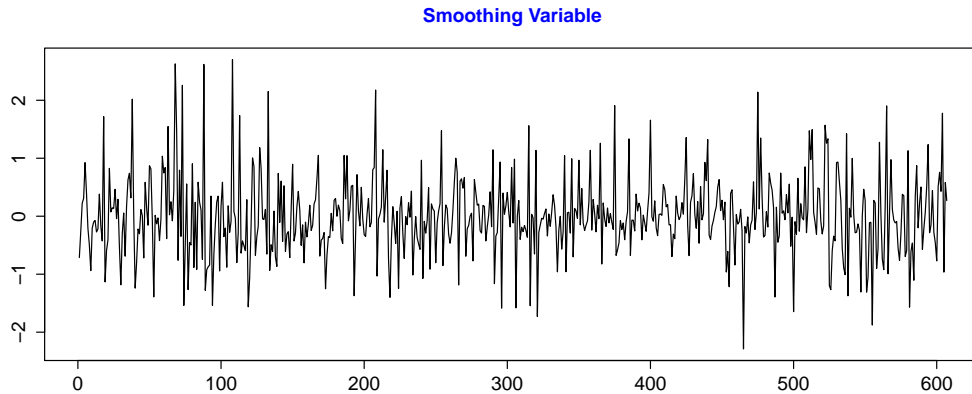


Figure 2: Time series plot of smoothing variable series.

of-fit tests for testing constant coefficients and testing significance of policy variable are considered. The p-values are 0.048 and 0.001, respectively, so that the coefficients are nonconstant and the policy variable is significant. Therefore, there exists a nonlinearity in the dynamic of USD/CNY exchange rate series and the government policy has an impact on the currency exchange.

For the conditional volatility part, we employ a GARCH model with a policy dummy variable  $C_t$ . The estimation results are reported as follows:

$$\begin{array}{rcl}
 r_t & = & \mu_t + \eta_t \quad , \quad \text{where } \eta_t = \sigma_t \varepsilon_t, \\
 \sigma_t^2 & = & 0.000959 + 0.188030\eta_{t-1}^2 + 0.735885\sigma_{t-1}^2 + 0.002045C_t \\
 p\text{-value} & & (0.0000) \quad (0.0000) \quad (0.0000) \quad (0.0000).
 \end{array}$$

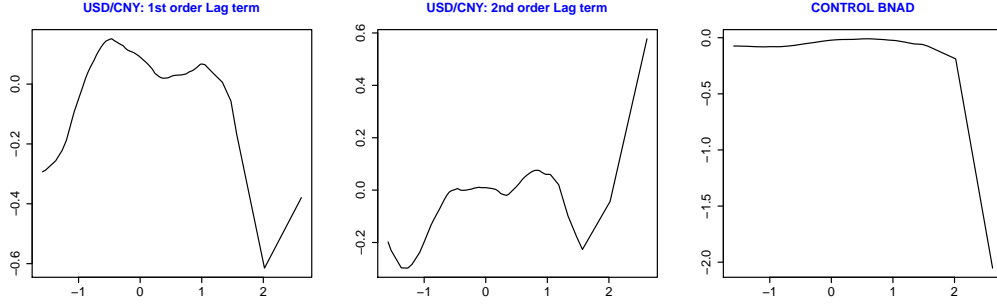


Figure 3: Nonparametric estimate of functional coefficients.

The coefficient of the policy dummy variable  $C_t$  is 0.002045 with associated p-value 0.0000, which is highly significant. This result coincides with our expectation that the increase of floating range has a significant impact on the mechanism of generating volatilities.

## 5 Comparisons with Other Models

There are several models commonly used in the forecast of exchange rates in the literature. For example, an incomplete list of these models includes the random walk model (RW), the autoregressive model, the threshold autoregressive model (TAR), the smooth transition autoregressive model (STAR), the artificial neural network model (ANN) and the autoregressive polynomial model. The forecasting performances of these models seem to be mixed. Meese and Rogoff (1983) compared the out-of-sample forecasting accuracy of various structural and time series exchange rate models. They found that a random walk model performs as well as any estimated model due to sampling error, while Michael, Nobay and Peel (1997) and Sarantis (1999) used parametric STAR model to analyze the nonlinearities for exchange rates and rejected the linearity hypothesis for the real effective exchange rates.

Furthermore, we compute the out-of-sample one-step-ahead forecasts to compare our model with other models in terms of MSFE and MAFE:

$$\text{MSFE} = m^{-1} \sum_{i=1}^m (r_{T+i} - \hat{r}_{T+i})^2 \quad (5.1)$$

$$\text{MAFE} = m^{-1} \sum_{i=1}^m |r_{T+i} - \hat{r}_{T+i}| \quad (5.2)$$

where  $m$  is the forecasting period and  $m = 50$ . It has been well documented that the random walk model often outperforms complicated structural time series models in forecasting the conditional mean of exchange rate changes. The MSFE and MAFE for our model and other models are listed in Table 1.

Table 1: A comparison of MSFE values for all models

Model	FC	RW	TAR	STAR	ANN
MSFE	0.0141	0.0153	0.0176	0.0221	0.0155
MAFE	0.1016	0.1033	0.1091	0.1167	0.1031

Table 2: P-value of Superior Predictive Ability test

	$SPA_l$	$SPA_c$	$SPA_u$
SFE	0.1699	0.7321	0.8091
AFE	0.4818	0.7933	0.7933

Table 1 demonstrates that both MSFE and MAFE for our model is smaller than those of alternative models. Since the difference of MSFE and MAFE between our model and the random walk (RW) model is small, to check whether the difference is significant, the superior predictive ability tests (White, 2000; Hansen, 2005) are employed. The null hypothesis is that the proposed model is not inferior to all alternative models which include the random walk (RW) model, the threshold autoregressive (TAR) model, the smooth transition autoregressive (STAR) model, and the artificial neural network (ANN) model. We adopt the distance of squared forecasting errors (SFE) and the distance of absolute forecasting errors (AFE) respectively as the loss function to evaluate model performance.  $SPA_u$  is the reality check test proposed by White (2000), and  $SPA_l$  and  $SPA_c$  denote the superior predictive ability tests proposed by Hansen (2005). The  $SPA_l$  test simply deletes all poor alternatives in the test statistic, while the  $SPA_c$  test keeps alternative models with moderately poor performance. The latter test usually has better finite sample performance than the former because the  $SPA_c$  test accounts for the fact that poor alternatives may have an impact on the critical value. The p-values of all tests are presented in Table 2. All tests cannot reject the null hypothesis that our model is not inferior to any of the alternatives.

Finally, to evaluate the performance of our model, we compute a 95% prediction interval using nonparametric quantile estimation. The results are presented in Figure 4, which demonstrates the out-of-sample forecasting results for the last 50 days. From Figure 4, there are only two sample points outside the 95% prediction interval. This implies that the out-of-sample forecasting performance of our model performs reasonably well.

## 6 Conclusion

We model the conditional mean and conditional volatility of the return series of USD/CNY exchange rate simultaneously. A functional-coefficient index model is adopted to model the conditional mean part and a GARCH (1,1) model with a policy dummy is applied to the conditional volatility model. A prediction interval is computed by employing nonparametric conditional quantile regressions. Our method outperforms other popular models in terms of mean squared forecasting errors and absolute

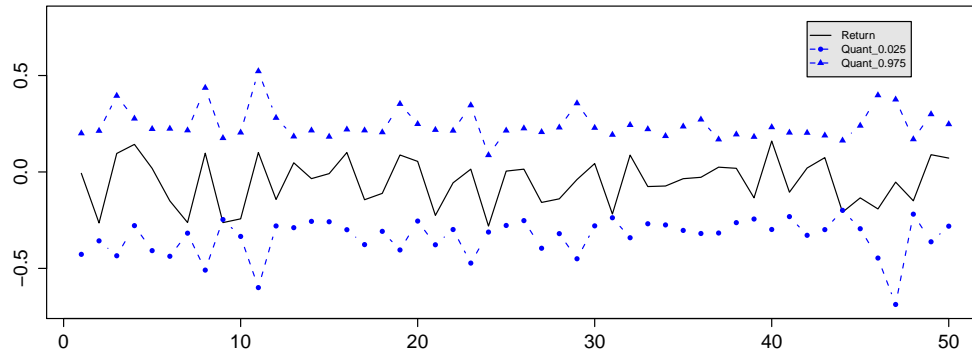


Figure 4: Quantile interval forecasting.

forecasting errors .

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