To give and get: poverty alleviation as a local public good

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11 December, 2010

Abstract

The paper theoretically analyzes the public choice of transfer payments to the poor (welfare spending) by modeling poverty alleviation as a public good provided by local governments. Voters that are not welfare recipients support welfare spending out of self-interest, rather than altruism, due to the public good property of poverty alleviation. Equilibrium policies are then analyzed according to characteristics of localities, such as population density and income inequality. More generally, our paper provides a technique to solve certain multiple peak problems that arise when a public goods policy has an explicitly redistributive component. To provide empirical support for our model, we use county-level demographic and government expenditure data from the United States Census.

Keywords: collective choice, poverty policy, public goods JEL classification: D72, H75

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1 Introduction

A fundamental service provided by governments is to assist the poor, however they are defined, by providing income transfers or goods in kind. Government spending on poverty alleviation provides direct benefit (positive net transfer) only to the poor, who are a minority of the population, at the cost of the non-poor majority. If poverty alleviation policy is the result of a public choice process and if voters are self-interested egoists, then we should not observe such targeted spending. Majority rule would block income transfers to the poor, since the majority receives a negative net transfer. Yet, we observe poverty alleviation policies being pursued by all levels of government. This paper rationalizes, within a public choice context, the existence of income transfers that are not received by the majority of voters.

In the standard theory of public finance, income redistribution is achieved by linear income taxation and lump-sum transfers.¹ In this case, all are subject to the same income tax rate and all receive the same transfer, which is the tax rate times the average income in the jurisdiction. The net transfer received by an individual is decreasing in individual income, and is negative for anyone with an income greater than the mean income. All individuals who receive a positive net transfer support redistribution to some degree, while all those with income greater than the mean do not support redistribution of any degree in the standard theory. Since income distributions are right-skewed, the median is less than the mean in any income distribution, which implies that at least half the voters will receive a positive net transfer from a policy of redistribution. Therefore, the existence of redistributive policies is not surprising within the standard theory since a majority receives a positive net transfer.

In practice, however, governments do not pursue redistribution through the policy tools described by the standard theory. Rather, governments target redistribution to those in the very bottom of the income distribution (below the poverty line), often, in the form of cash transfers. What percentage of a polity receives welfare transfers varies across jurisdictions of course, but it is certainly always less than half. The existence of targeted welfare spending, then, cannot be rationalized within the standard theory, since the majority don't receive a transfer and have the political puissance to vote welfare spending to zero.

One way to rationalize the existence of welfare spending is to consider pro-social, or other-regarding, motivations. When individual utility is affected by the state of

¹See Persson and Tabellini [2000] for example.

others, we can understand political support for poverty alleviation within the context of rational public choice. Formally, we shall say that the poverty level one observes is an argument of the utility function.² We characterize the social problem of poverty as a pure public bad, implying that poverty observation affects all in society equally. The government's role in our model is to provide the public good of poverty alleviation, the degree of which is the subject of public choice.

We positively analyze the public choice of poverty alleviation via income transfers to the poor (welfare spending). Public policy (hence, a candidate's policy platform) is one-dimensional in our model; elected governments establish and publicly finance a minimum consumption threshold, which has pure public value. The one-dimensional policy is determined by majority rule, and we identify a *Condorcet* winner typical to models of spatial political competition.

We prove the existence of an equilibrium that features positive poverty alleviation, despite transfer recipients being in minority.

More generally, our paper provides a technique to solve certain multiple peak problems that arise when a public goods policy has an explicitly redistributive component. We isolate preferences for the public good property of the minimum income threshold policy, and characterize these preferences before considering any transfers that are associated with the policy. The same technique could be used to address puzzles concerning the existence of targeted policies that could have some tangential public value.³

In our model, individuals are affected by *observing* poverty. The notion that utility interdependence is conditional on the observation of another's income (hence, utility level) gives a *local* element to considerations of poverty that follows Pauly [1973].⁴ Thus,

²It is clear that individual utility should be decreasing in the rate of poverty one observes, but economists would argue over why exactly that should be so. A popular explanation is that people are altruistic, and care about poverty because it means that others are in a bad state. See Andreoni [2006] for a review of altruism as it relates to incentives for philanthropic behavior. Perhaps observing poverty gives the altruist a sense of unfairness and raises unpleasant questions about the failure of society, which gives dis-utility. Alternatively, poverty may enter an individual's utility function for purely selfish reasons. Perhaps economic man does not care about the utility of others altruistically, but rather realizes that there are negative externalities associated with poverty, and views poverty as a public bad due to its associated externalities. Crime for example, may be associated with poverty, and an individual who wishes to reduce crime out of egoistic concern may instrumentally wish to reduce poverty. Whatever the motivation, we assume that poverty is a public bad, and that individual utility is decreasing in the observed poverty rate.

 $^{^{3}}$ We can imagine some sorts of strategic trade policies or financial bailouts that are sold to the policy as a public good, which is accomplished by direct transfers in the form of export subsidy or re-capitalization.

⁴Indeed, Pauly [1973] proves that heterogeneity of districts can imply that redistributing income locally is Pareto-superior to redistribution by a federal government.

we treat welfare spending as a local public policy, and examine how variation in the degree of welfare spending can be explained by heterogeneity of localities, for example, in population density.⁵

Modeling poverty policy at the local level permits examination of a large crosssection of jurisdictions. In the United States, there is wide variety across counties as to the level of per capita expenditure on local welfare policies, so the jurisdictional units of our analysis are counties. Consideration of county-level policies allows for a rich cross-section (there are over 3000 counties in the US) for an empirical investigation of our model's comparative static predictions. We conjecture that the equilibrium level of per capita welfare spending is increasing in population density, after controlling for the poverty rate and other characteristics of the county. Our empirical analysis supports this conjecture.

The paper is organized as follows. The next section sets out the basic model and derives our main theoretical results. The third section briefly considers some empirical literature on the economics of charitable giving to justify a utility configuration assumption that we make. We present our data set, describe our empirical methodology and present some empirical support for our model in the fourth section. The final section concludes briefly.

2 The model

In the model society, income, y, is distributed according to a known cdf F and associated pdf, f. The median income is denoted by y^m and the mean income is denoted by \bar{y} . There is a well-defined poverty line, c^P , so the proportion of people living in poverty is $F(c^P)$.

2.1 Individuals

People receive utility from consuming goods and dis-utility from observing poverty, which we model as a public bad. Poverty observation is determined by the poverty rate, scaled by a jurisdiction's population density. When there is no poverty alleviation,

⁵Quite simply, if it is the observation of poverty that gives dis-utility, then the same poverty rate will give more dis-utility in a more densely populated area, since more poor are observed in the higher density area. Pauly [1973] notes that "the desire to do good is conditional on the perception of bad circumstances, and bad circumstances close at hand are more likely to be perceived than those at a distance."

hence no taxation, an individual with consumption level $c^i = y^i$ has utility

$$u\left(c^{i}\right) - \delta \int_{0}^{c^{P}} h\left(c\right) dF\left(y\right),$$

where δ is the population density of a jurisdiction. We make the following standard assumptions on consumption: u'(c) > 0, u''(c) < 0. For poverty dis-utility we assume h is differentiable everywhere and that h(c) > 0, h'(c) < 0 and h''(c) > 0. The consumption of individuals above the poverty line has no public value.⁶

2.2 Government

The sole purpose of the local government in this society is to reduce poverty through welfare transfers. We model a government which only has the power to establish a minimum consumption threshold, denoted by \underline{c} . The transfers associated with such a policy must be fully-funded by the proportional income tax rate, t, which balances the budget. We pose that the minimum consumption threshold policy is determined by one-dimensional political competition.

The expenditures, hence revenues, of the government depend on who is eligible to receive a transfer. For a policy of \underline{c} , individuals receive the following transfer:

$$\text{transfer} = \begin{cases} \underline{c} - (1-t)y^{i} & \text{if} \quad (1-t)y^{i} \leq \underline{c} \\ 0 & \text{if} \quad \underline{c} < (1-t)y^{i} < c^{P} \\ 0 & \text{if} \quad (1-t)y^{i} > c^{P} \end{cases}$$

With a minimum consumption threshold of \underline{c} , the government's budget constraint is

$$\Gamma(\underline{c},t) \equiv t\overline{y} - \int_0^{\frac{c}{(1-t)}} \left[\underline{c} - (1-t)y\right] f(y) \, dy = 0 \tag{1}$$

where we integrate the transfer expenditures up to the income level that is eligible when the policy is \underline{c} . Let $t(\underline{c})$ be the proportional tax that solves (1). Such a function always

⁶The structure of poverty dis-utility implies that people care about both the number of individuals consuming below the poverty line and their absolute consumption level. Hence, if the income distribution F first-order stochastically dominates G then G gives higher dis-utility than F.

exists since for any $\underline{c} > 0$, we have that $\Gamma(\underline{c}, 0) < 0, \Gamma(\underline{c}, 1) > 0$ and

$$\Gamma_t(\underline{c},t) = \overline{y} - \int_0^{\frac{c}{1-t}} y dF(y) > 0.$$

2.3 Analysis

An individual with income y^i consumes his entire net income if it is above the minimum consumption threshold, \underline{c} . For an individual with income y^i , consumption is a function $c(\underline{c}, y^i)$, where

$$c\left(\underline{c}, y^{i}\right) = \begin{cases} \underline{c} & \text{if} & \left[1 - t\left(\underline{c}\right)\right] y^{i} \leq \underline{c} \\ \left[1 - t\left(\underline{c}\right)\right] y^{i} & \text{if} & \left[1 - t\left(\underline{c}\right)\right] y^{i} > \underline{c} \end{cases}$$

Given a balanced budget, induced preferences over \underline{c} for an individual with income y^i are composed of utility from individual consumption level and dis-utility from observing the poverty that remains after raising the consumption of the (eligible) poor up to \underline{c} .

$$V\left(\underline{c}; y^{i}\right) = u\left[c(\underline{c}; y^{i})\right] - \delta\left[h\left(\underline{c}\right) F\left(\frac{\underline{c}}{1 - t\left(\underline{c}\right)}\right) + \int_{\frac{\underline{c}}{1 - t(\underline{c})}}^{\frac{\underline{c}^{P}}{1 - t(\underline{c})}} h\left(\left[1 - t\left(\underline{c}\right)\right]y\right) dF\left(y\right)\right]$$
(2)

In our model, the policy \underline{c} has public value, but this is not a standard public goods problem due to the fact that for a given provision level \underline{c} , a mass of voters $F(\underline{c})$, receives a positive net income (consumption) transfer. These voters do not view candidate policies as simply public good provision levels, as the voters not in poverty do. We begin as if this were a standard public good problem, and proceed as if there were no transfers, i.e., as if $c(\underline{c}; y^i) = [1 - t(\underline{c})] y^i$. By ignoring the redistributive implications of a consumption threshold policy, we can isolate the demand for the public good. Consider preferences over \underline{c} for an alternative indirect utility function that ignores income transfers, $\widetilde{V}(\underline{c}; y^i)$:

$$\widetilde{V}\left(\underline{c}; y^{i}\right) = u\left(\left[1 - t\left(\underline{c}\right)\right] y^{i}\right) - \delta\left[h\left(\underline{c}\right) F\left(\frac{\underline{c}}{1 - t\left(\underline{c}\right)}\right) + \int_{\frac{\underline{c}}{1 - t\left(\underline{c}\right)}}^{\frac{c^{P}}{1 - t\left(\underline{c}\right)}} h\left(\left[1 - t\left(\underline{c}\right)\right] y\right) dF\left(y\right)\right]$$

$$(3)$$

Note that (3) only differs from (2) by the consumption argument of $u(\cdot)$; there is no consideration of the transfer associated with the public good in $\widetilde{V}(\underline{c}; y^i)$. We characterize (3) as single-peaked in \underline{c} , and establish that peak-points are monotonic in income

under certain preference configurations. We then analyze the majoritarian political dynamics when the poor do consider the transfer associated with the public good provision, i.e. we then analyze $V(\underline{c}; y^i)$. While $\widetilde{V}(\underline{c}; y^i)$ is shown to be single-peaked for all voters, $V(\underline{c}; y^i)$ may not be. We identify multiple peak points for poor voters who consider the transfer: there is always a second peak point at full poverty alleviation. We then identify the unique *Condorcet* winner for given preference configurations. To begin the analysis, we rely on two claims, which we prove in the the appendix.

Claim 1. The first-order derivative of $\widetilde{V}(\underline{c}; y^i)$ with respect to \underline{c} can be written

$$\frac{\partial \widetilde{V}\left(\underline{c};y^{i}\right)}{\partial \underline{c}}=t'\left(\underline{c}\right)\Delta\left(\underline{c},y^{i}\right),$$

where $t'(\underline{c}) > 0$ and

$$\Delta\left(\underline{c}, y^{i}\right) \equiv \delta\left[\int_{\frac{\underline{c}}{1-t(\underline{c})}}^{\frac{\underline{c}^{P}}{1-t(\underline{c})}} h'\left(\left[1-t\left(\underline{c}\right)\right]y\right)ydF\left(y\right) - h'\left(\underline{c}\right)\Gamma_{t}(\underline{c}, t\left(\underline{c}\right)\right)\right] - u'\left(\left[1-t\left(\underline{c}\right)\right]y^{i}\right)y^{i}$$

$$\tag{4}$$

Proof. See appendix.

Claim 2. The first-order derivative of $\Delta(\underline{c}, y^i)$ with respect to \underline{c} is

$$\frac{\partial \Delta\left(\underline{c},y^{i}\right)}{\partial \underline{c}} \equiv \Delta_{\underline{c}}\left(\underline{c},y^{i}\right) < 0.$$

Proof. See appendix.

Lemma 1. An individual with indirect utility function $\widetilde{V}(\underline{c}; y^i)$ has single-peaked preferences for \underline{c} over the possible policy space $\underline{c} \in [0, c^P]$.

Proof. The first-order condition for optimization is written

$$\frac{\partial \widetilde{V}(\underline{c}; y^{i})}{\partial c} = t'(\underline{c}) \Delta\left(\underline{c}, y^{i}\right) \equiv 0.$$
(5)

We have shown previously that $t'(\underline{c}) > 0$, so it must be that $\Delta(\underline{c}, y^i) = 0$ at any optimum. To prove that $\widetilde{V}(\underline{c}; y^i)$ is single-peaked, it is sufficient to show that indirect utility is concave in \underline{c} , and that we have found a maximum. Taking the second-order

derivative of $\widetilde{V}(\underline{c}; y^{i})$ with respect to \underline{c} , we have that

$$\begin{array}{ll} \displaystyle \frac{\partial^2 \widetilde{V}\left(\underline{c};y^i\right)}{\partial \underline{c}^2} & = & t''\left(\underline{c}\right) \Delta\left(\underline{c},y^i\right) + t'\left(\underline{c}\right) \Delta_{\underline{c}}\left(\underline{c},y^i\right) \\ & = & t'\left(\underline{c}\right) \Delta_{\underline{c}}\left(\underline{c},y^i\right) < 0 \end{array}$$

where the equality follows from the observation that $\Delta(\underline{c}, y^i) = 0$ and the inequality follows from the previous claims that $t'(\underline{c}) > 0$ and $\Delta_{\underline{c}}(\underline{c}, y^i) < 0$.

Lemma 1 implies that each individual, without regard to possible transfer, has a unique most-preferred public goods policy. We solve the first-order condition (5) for an individual's most-preferred policy as an implicit function of income, $\tilde{\underline{c}}(y^i)$, where the tilde indicates that the individual does not consider the income transfers associated with the public good.

Lemma 2. $\underline{\widetilde{c}}(y^i)$ is monotone in income for certain preference configurations. The sign of $\underline{\widetilde{c}}'(y)$ depends on a preference configuration:

- $u_c(c^i) + u_{cc}(c^i)c^i > 0 \Rightarrow \underline{\tilde{c}}'(y) < 0$: SDI (Slope Decreasing in Income)
- $u_c(c^i) + u_{cc}(c^i)c^i < 0 \Rightarrow \underline{\tilde{c}}'(y) > 0$: SRI (Slope Rising in Income)

Proof. An individual's most-preferred policy is defined implicitly by the first-order condition in (5).

$$t'(\underline{c})\Delta\left[\underline{\widetilde{c}}\left(y^{i}\right),y^{i}\right]=0,$$

which implies

$$\Delta\left[\underline{\widetilde{c}}\left(y^{i}\right),y^{i}\right]=0,$$

which we totally differentiate to get

$$dy^{i}\left\{\Delta_{\underline{c}}\left[\underline{\widetilde{c}}\left(y^{i}\right),y^{i}\right]\underline{\widetilde{c}}'\left(y\right)+\Delta_{y^{i}}\left[\underline{\widetilde{c}}\left(y^{i}\right),y^{i}\right]\right\}=0,$$

which gives

$$\underline{\tilde{c}}'(y) = \frac{-\Delta_{y^i}(\underline{c}, y^i)}{\Delta_c(\underline{c}, y^i)}.$$
(6)

Since $\Delta_{\underline{c}}(\underline{c}, y^i) < 0$ from Claim 2 above, we know that $\underline{\tilde{c}}'(y)$ will have the same sign as $\Delta_{y^i}(\underline{c}, y^i)$. Individual income y^i does not appear in the public bad part of the utility

function, so $\Delta_{y^i}(\underline{c}, y^i)$ is easy to find.

$$\Delta_{y^{i}}\left(\underline{c}, y^{i}\right) = -\left\{u'\left(\left[1-t\left(\underline{c}\right)\right]y^{i}\right)+u''\left(\left[1-t\left(\underline{c}\right)\right]y^{i}\right)\left[1-t\left(\underline{c}\right)\right]y^{i}\right\}$$
$$= -\left[u_{c}\left(c^{i}\right)+u_{cc}\left(c^{i}\right)c^{i}\right].$$

These preference configurations are similar to those investigated by Epple and Romano [1996] in their paper about public provision of private goods: SRI (slope rising with income) and *SDI* (slope decreasing in income). The restrictions describe how individuals with different incomes trade off consumption for the public good on the margin. We focus on the case of SDI, which implies that the relatively poor are, on the margin, more willing to trade consumption for the public good than the relatively rich. Since poverty alleviation is a public good, the marginal benefit from a higher consumption threshold is the same for everyone in society. The marginal cost is comprised of two multiplicative effects. The first effect is a reduction in net income (consumption) due to the required higher level of taxes. Other things equal, this cost is larger for people with higher incomes since taxes are proportional. The second effect is the reduction in utility associated with this reduction in net income. Other things equal this cost is smaller for people with higher incomes since marginal utility is decreasing in income. The SDI preference restriction ensures that, on net, the marginal cost of poverty alleviation is increasing in y^i . A discussion of the empirical rationale for considering SDI as it relates to charitable giving follows in section 3.

To keep the discussion interesting (and empirically plausible), we assume that the median-income voter is never eligible for the transfer.

Assumption 1. $y^m [1 - t(c^P)] > c^P$.

To this point, we have been isolating the public good property of \underline{c} , but the policy platform that wins the election will have associated transfers. So, we turn the discussion to how policy affects $V(\underline{c}; y^i)$. We denote peak points to $V(\underline{c}; y^i)$ as a function of income by $\underline{c}(y^i)$. For all voters who are never eligible for an income transfer, we have that $c(\underline{c}, y^i) = [1 - t(\underline{c})] y^i$ for any policy, so $V(\underline{c}; y^i) = \widetilde{V}(\underline{c}; y^i)$, i.e., $[1 - t(c^P)] y^i > c^P \Rightarrow$ $\underline{\tilde{c}}(y^i) = \underline{c}(y^i)$. Since $\widetilde{V}(\underline{c}; y^i)$ is single-peaked, so too is $V(\underline{c}; y^i)$. For potential transfer recipients, however, this need not be. For voters with net incomes $[1 - t(c^P)] y^i < c^P$, $V(\underline{c}; y^i)$ is strictly increasing in the neighborhood of c^P because c^P is the maximum transfer. These voters have multiple peak points to $V(\underline{c}; y^i)$ at $\underline{\tilde{c}}(y^i)$ and at c^P , so $\underline{c}(y^i)$

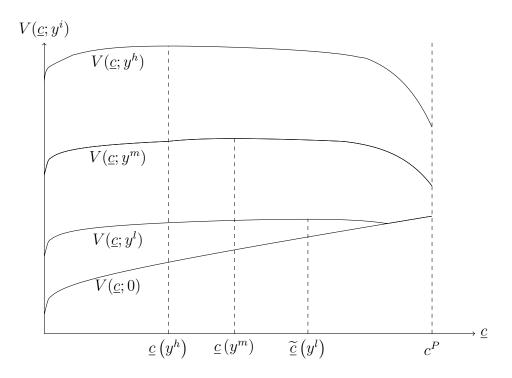


Figure 1: The indirect utility function $V(\underline{c}; y^i)$ for 3 different income levels, $y^l < [1-t(c^P)] y^i < y^m < y^h$ over the range $[0, c^P]$, with peak points $\underline{c}(y^h) < \underline{c}(y^m) < \underline{\tilde{c}}(y^l) < c^P$ due to the *SDI* preference restriction. Note that for a given \underline{c} , all those income levels $0 < [1-t(\underline{c})] y^i < \underline{c}$ have the same utility $V(\underline{c}; 0)$. Despite the non-single-peakedness of indirect utility at low income levels, the first proposition shows that \underline{c}^m is the Condorcet winner.

is always at least as big as $\underline{\widetilde{c}}(y^i)$ under both preference configurations. Figure 3.1 shows preferences for three different income levels with the *SDI* preference ordering.

Proposition 1. When the minimum consumption threshold is determined by majority voting and preferences are restricted by SDI, the most-preferred policy of the median-income voter is the Condorcet winner.

Proof. Consider the minimum consumption threshold that maximizes $V(\underline{c}; y^i)$ for the median-income voter. Since the median receives no transfer by assumption 1, maximizing $\widetilde{V}(\underline{c}, y^m)$ solves the same problem as maximizing $V(\underline{c}, y^m)$, so $\underline{\widetilde{c}}(y^m) = \underline{c}(y^m) \equiv \underline{c}^m$. Ignoring income transfers, most-preferred policies are decreasing in income for the case of SDI. For all individuals with income $y^i > y^m$, we know that $\underline{\widetilde{c}}(y^i) = \underline{c}(y^i) < \underline{c}^m$ by lemma 2. Hence, for all individuals with income $y^i > y^m$ utility is decreasing over the range $\underline{c} \in [\underline{c}^m, c^P]$, which implies that all such individuals prefer \underline{c}^m over $\underline{c}' > \underline{c}^m$.

Similarly, for all individuals with net income below the median, but above the poverty line, $c^P < [1-t(c^P)] y^i < y^m$, we know that $\underline{\tilde{c}}(y^i) = \underline{c}(y^i) > \underline{c}^m$, again by lemma 2. For these voters, utility is decreasing over the range $\underline{c} \in [0, \underline{c}^m]$ which implies that all such individuals prefer \underline{c}^m over $\underline{c}' < \underline{c}^m$. Finally, for potential transfer recipients, we know that $\underline{\tilde{c}}(y^i) > \underline{c}^m$ by lemma 2, so $\underline{c}(y^i) > \underline{c}^m$ by transitivity. In other words, both peak points to $V(\underline{c}; y^i)$ are greater than \underline{c}^m , so we know that \underline{c}^m is on the (first) upward-sloping segment of of $V(\underline{c}; y^i)$ for potential transfer recipients. Therefore, all with net incomes $[1 - t(c^P)] y^i$ prefer \underline{c}^m over any $\underline{c}' < \underline{c}^m$. In sum, no policy besides \underline{c}^m can win the support of at least half the population.

Note that we cannot simply invoke the Median Voter Theorem due to the multiple peak points in the preferences of potential welfare recipients. However, in the case of SDI, the income transfer does not affect the political outcome in any meaningful way. In the SDI environment, the relatively poor are more willing to trade off consumption for the public good on the margin. Thus, both peak points for the poor are at higher consumption thresholds than \underline{c}^m and there is no popular support for anything greater than \underline{c}^m . Since for any potential welfare recipient, $\underline{c}^m < \underline{\widetilde{c}}(y^i) < \underline{c}(y^i)$, the second peak point at c^P never comes into play. The *Condorcet* result is clear in this case.

Note that $\underline{c}^m > 0$ despite the fact that the median's net transfer is negative. Proposition 1 rationalizes the existence of a transfer policy targeted to a minority segment of the population. Note further that the median-income voter is decisive here due to the assumption that preferences are ordered according to SDI. Section 3 justifies the SDI assumption by looking at some empirical results on personal charitable contributions.⁷

2.4 Comparative statics

We now examine how the equilibrium minimal consumption threshold changes as the parameters of the model change by analyzing how the median voter's preferences change. For what follows we assume that equation (4) characterizes the most preferred minimum consumption threshold for the median-income individual. Where relevant we use the implicit function theorem and the fact that $\Delta_{\underline{c}}(\underline{c}, y^i) < 0$. See the appendix for detailed proofs of the comparative static results.

⁷An appendix is available from the authors which also considers when preferences are ordered according to the SRI assumption, and finds an ends-against-middle equilibrium in which the median-income voter can never be decisive.

• Population Density, δ ...

$$\frac{\partial \underline{c}^{m}}{\partial \delta} = -\frac{\Delta_{\delta}\left(\underline{c}, y^{m}\right)}{\delta \Delta_{c}\left(\underline{c}, y^{m}\right)} > 0$$

A decrease in the population density parameter, δ , has the effect of making poverty less noticeable. This reduces the marginal benefit associated with each level of poverty alleviation, shifting the $MB(\underline{c})$ schedule down. All voters demand less poverty alleviation, including the median voter.

• Median income, y^m ...

$$\frac{\partial \underline{c}^m}{\partial y^m} < 0$$

Since demand for poverty alleviation is decreasing in income, a higher median income results in lower demand for poverty alleviation. This follows immediately from lemma 2 under *SDI*.

• Average Income \bar{y} ...

$$\frac{\partial \underline{c}^{m}}{\partial \bar{y}} = \frac{\Delta_{\bar{y}}\left(\underline{c}, y^{m}\right)}{\Delta_{\underline{c}}\left(\underline{c}, y^{m}\right)} > 0$$

Increasing average income reduces the marginal cost of providing income assistance for the median voter since his share of the tax burden for an additional increase in the minimum income threshold is smaller. A higher mean income also reduces the tax required to balance the public budget. This further reduces the marginal cost of additional poverty alleviation. As a result, the median voter demands more poverty alleviation.

3 On the relatively rich preferring lower tax rates

An implication of the *SDI* assumption is that the relatively rich support lower levels of poverty alleviation. In an *SDI* environment, the relatively rich have a lower level of poverty alleviation that solves their optimization problem, as shown in lemma 2. The rich, therefore, prefer lower tax rates, which is common when public goods are financed with proportional taxation. This may seem odd at first glance, if we think that poverty alleviation is a normal good. The relevant issue, however, is not how income affects the optimal *level* of poverty alleviation, but rather how income affects the optimal *percentage* of income one would give up to fight poverty. The statement that the rich prefer lower tax rates does not contradict the statement that poverty alleviation is a normal good.

To get an idea of whether imposing SDI is reasonable, we consider some empirical work from the literature on philanthropy and charitable giving.⁸ Charitable giving, which can be interpreted as private provision of the public good of poverty alleviation, has been shown empirically to increase with income levels, so it is a normal good in this sense. There seems to be a widely-held consensus among empirical researchers in this area, however, that the income elasticity of charitable giving is less than one. In a survey of empirical studies on charitable giving from 1985 to 1990, Steinberg [1990] reports that 20 of 22 studies he reviews find an income elasticity less than one. A recent and thorough analysis by Auten et al. [2002] isolates the effects of permanent and transitory income changes on charitable giving. The authors find that the elasticity of charitable giving to permanent and transitory income changes is significantly less than one. McClelland and Brooks [2004] is another recent study that finds a permanent income elasticity of less than one, and find further that elasticities decrease as income levels increase. In other words, as income rises, individuals make larger gross donations to charity, but the donations become a smaller fraction of income as incomes rise.⁹ A number of explanations have been given for the (somewhat) counter-intuitive result that the poor are relatively more generous. Wiepking [2007], for example, argues that there may be social norms for the *level* of giving (a giving standard) that are common across income levels, leading the relatively rich to donate smaller proportions of their incomes.¹⁰ Thus, we turn to an empirical test of the equilibrium predictions, which rely

⁸Andreoni [2006] provides an excellent survey of the economics literature, while a survey across the social sciences is given by Bekkers and Wiepking [2007].

⁹The result from McClelland and Brooks [2004] that elasticities decrease with income levels is contrary to an earlier result from Lankford and Wyckoff [1991], who find that income elasticities are not monotonic with income levels. Lankford and Wyckoff [1991] do find, however, that income elasticities are less than one at all income levels.

¹⁰The finding that charitable giving as a proportion of income is decreasing in income is not uniformly accepted in the literature. Other results posit that the relationship is not strictly monotone, but forms a U-shaped curve. The wealthy in society often make enormous philanthropic gifts, which skews the average relationship between giving and income, causing the upward slope of the U-shaped curve at high income levels. Wiepking [2007] argues that the reason many have found the U-shaped curve is because they have not properly taken into account the tax-incentives for giving. Correcting for tax-incentives for giving, or the price of giving, eliminates the U-shape of the curve and gives a decreasing relationship in her study. Andreoni [2006] notes the greater variability of donations of the extremely wealthy, such as academic buildings, or other prestigious gifts. When looking at the median gift as

on the SDI assumption, using data that describes local policy outcomes.

4 Empirical analysis

4.1 Data and methodology

To test the comparative static predictions of our model, we employ county-level data from the United States. The variables that we consider are presented in Table 1. The welfare expenditure data was taken from the 2002 round of the Census of Governments, and the demographic data was taken from the 2000 round of the Census of the Population. In addition to the variables that are required to test our comparative static predictions, we include a variable to account for racial heterogeneity across counties.¹¹ We also include state dummies to control for institutional differences across the states, such as differences in intra-state fiscal transfers. Summary statistics of the full sample are presented in Table 2.

The regression analysis is complicated by the presence of censoring in the dependent variable, since some of the counties have welfare expenditures equal to zero. The Tobit model (Tobin [1958]) accounts for censoring by assuming

$$y_i = \begin{cases} y_i^* & \text{if } y_i^* \ge 0\\ 0 & \text{if } y_i^* < 0 \end{cases}$$

where the latent variable $y_i^* = X_i\beta + \epsilon_i, \epsilon_i \sim N(0, \sigma^2)$.

The Tobit model implicitly restricts the relationship between the variables to be linear. We allow for a less restrictive relationship between the variables by undertaking a Box-Cox (Box and Cox [1964]) power transformation of the dependent variable, since the relation between the variables does not appear linear. The transformed variable is

$$y_i^{(\lambda)} = \begin{cases} (\widetilde{y}_i^{\lambda} - 1)/\lambda & \text{if } \lambda \neq 0\\ \log \widetilde{y}_i & \text{if } \lambda = 0 \end{cases}$$

a fraction of income over kernels of the income distribution, rather than the average gift, we see the negative relationship between income and donations as a fraction of income. See Andreoni [2006] and Bekkers and Wiepking [2007] for reviews of this literature.

¹¹We are motivated to include a measure of racial heterogeneity by the work on public goods provision and racial fragmentation of Alesina et al. [1999], as well as the work of Lee and Roemer [2006] on the role of racism in the politics of redistribution in the United States.

	Table 1: Description of Data
Variable	Definition
WelfarePC	Welfare Expenditure per Capita
MedianInc	Median Household Income
MeanInc	Mean Household Income
Poverty	Percentage of Individuals Below Poverty Line
Density	Population Density per Square Mile of Land
Color	Percent of Population Not White

Table 2: Summary Statistics for Full Sample

Table 2. Summary Statistics for Full Sample					
Variable	Obs	Mean	Std. Dev.	Min	Max
WelfarePC	3031	73.386	130.165	0	1,201.766
MedianInc	3031	$35,\!186.89$	8,746.106	$12,\!692$	$82,\!929$
MeanInc	3031	$44,\!638.67$	$10,\!159.51$	$19,\!395$	108,756
Density	3031	137.524	450.944	0.1	13043.6
Poverty	3031	14.167	6.534	2.1	56.9
Color	3031	15.009	16.025	0.3	95.5

 Table 3: Summary Statistics for Truncated Sample

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Table 5. Summary Statistics for Truncated Sample					
Variable	Obs	Mean	Std. Dev.	Min	Max
WelfarePC	2581	86.181	137.095	0.00176	$1,\!201.766$
MedianInc	2581	$35,\!898.11$	8860.455	$12,\!692$	$82,\!929$
MeanInc	2581	$45,\!439.3$	$10,\!357.57$	$19,\!395$	108,756
Density	2581	155.865	486.014	0.1	$13,\!043.6$
Poverty	2581	13.756	6.330	2.1	56.9
Color	2581	15.173	15.958	0.3	88.4

where $\tilde{y}_i = y_i + 1$ to accommodate the fact that some values of the dependent variable are zero and would be undefined for $\lambda = 0$. Allowing a more general functional form mitigates the restrictiveness of the Tobit model assumptions. In a likelihood maximizing procedure, we have found that the value for λ that makes the estimated linear coefficients for the full sample most likely is approximately $\lambda = 0.1$. We use the Box-Cox power transformation with $\lambda = 0.1$ below in the Heckman Two-Step procedure. We also provide the results for $\lambda = 0$, which reduces down to the log-linear specification that is common in the literature on charitable giving.

4.2 Heckman two-step procedure

We employ a Heckman two-step procedure to correct our estimates for the bias caused by the truncation of the dependent variable at zero. In the first stage of the Heckman procedure, we estimate a binary choice model by probit, which estimates the probability that welfare spending is positive, given the characteristics of a county and state dummies. We then construct the Heckman correction term to be used in the second stage.¹² In the second stage of the Heckman procedure, we reestimate the original model using only the observations for which the dependent variable is not zero, but including the estimated Heckman correction term. 450 observations were dropped from the full sample to form the limited sample, the summary statistics of which are presented in Table 3.

Table 4 reports the results for specifications with $\lambda = 0.1$ and $\lambda = 0$, as well as for the one-stage log-linear regression that does not follow the Heckman procedure. Standard errors are in parentheses below the estimated coefficients. The first (second) column of Table 4 reports the estimated coefficients from the second-stage of the Heckman two-stage procedure for $\lambda = 0.1$ ($\lambda = 0$). The results are, in general, supportive of our comparative static predictions. The coefficients on Density, MedianInc, and MeanInc all have the sign predicted by our model, and are all statistically significant.¹³ The third column of results in Table 4 reports the estimated coefficients for the bias caused by the censoring of the dependent variable.

¹²See Heckman [1979] for details of the correction term.

¹³Note, however, that the standard errors reported below do not take into account the uncertainty associated with the estimated value of the Heckman correction term (estimated in the first stage). This is a known shortcoming of the Heckman Two-Step procedure, which results in downwardly-biased standerd errors in the second stage estimations.

100	(1)	$\frac{\text{cond-Stage Regression}}{(2)}$	(3)
	(1)	(2)	(0)
	$\frac{(\tilde{y}_i - 1)^{0.1}}{0.1}$	$\log\left(ilde{y}_{i} ight)$	$\log\left(\tilde{y}_{i} ight)$
	second-stage	second-stage	one-stage
Density	0.000182***	0.000125**	0.000143***
, , , , , , , , , , , , , , , , , , ,	(0.00007)	(0.000051)	(0.000057)
Ln(MedianInc)	-1.954^{***}	-1.331^{***}	-0.587
· /	(0.601)	(0.4383)	(0.439)
Ln(MeanInc)	0.993*	0.755^{*}	0.576
	(0.565)	(0.412)	(0.419)
Poverty	-0.0238^{*}	-0.0150^{*}	-0.0148^{*}
	(0.0123)	(0.00898)	(0.00886)
Color	0.00594^{*}	0.00409*	0.00579**
	(0.0032)	(0.00233)	(0.00238)
Constant	16.131***	10.728***	2.954
	(3.394)	(2.474)	(2.455)
Heckman	-0.0314	-0.0841	_
	(0.284)	(0.207)	_
Adjusted R^2	0.663	0.651	0.646
Censored Obs.	450	450	0
Uncensored Obs.	2581	2581	3031

5 Conclusion

This paper has positively examined public choice over a specific poverty alleviation tool, that of a minimum consumption threshold as a policy tool for poverty alleviation. The policy transfers income to a minority (the poor) from the majority, so the existence of such policies seems anomalous at first glance. The transfers can be rationalized, within a positive public choice perspective, as majority-supportable due to the public value of poverty alleviation. In short, the non-poor are willing to pay for a public good, which is implemented by making transfer payments. We isolate demand for the public good from its associated transfers when determining policy preferences, which allows us to overcome a multiple peak problem and identify a unique policy that is the *Condorcet* winner.

The technique developed in our paper could be used more generally to solve certain multiple peak problems that arise when a public goods policy has an explicitly redistributive component. The method could be used to address anomalies concerning the existence of targeted policies that have (perceived) public value, such as affirmative action, strategic protectionism, or banking bailouts, to name a few.

Our main result establishes an equilibrium policy, whose comparative static properties we investigate empirically using a Heckman two-step procedure. In general, our empirical results confirm the comparative static predictions of the model, namely that welfare spending is increasing in the population density of a jurisdiction.

A Proofs

Derivatives of $\Gamma(\underline{c}, t)$

Proof.

$$\begin{split} \Gamma(\underline{c},t) &= t\bar{y} - \int_{0}^{\frac{c}{(1-t)}} (\underline{c} - y(1-t)) f(y) \, dy \\ \Gamma_{\underline{c}}(\underline{c},t) &= -\left\{ \left[\underline{c} - \frac{\underline{c}(1-t)}{1-t} \right] f\left(\frac{\underline{c}}{1-t} \right) \left(\frac{1}{1-t} \right) - (\underline{c}) f(0) \right\} \\ &= -F\left(\frac{\underline{c}}{1-t} \right) < 0. \\ \Gamma_{t}(\underline{c},t) &= \bar{y} - \left\{ \left[\underline{c} - \frac{\underline{c}(1-t)}{1-t} \right] f\left(\frac{\underline{c}}{1-t} \right) \left(\frac{\underline{c}}{(1-t)^{2}} \right) - (\underline{c}) f(0) \right\} - \int_{0}^{\frac{c}{1-t}} y dF(y) \\ &= \bar{y} - \int_{0}^{\frac{c}{1-t}} y dF(y) > 0. \\ \Gamma_{\underline{c},\underline{t}}(\underline{c},t) &= -f\left(\frac{\underline{c}}{1-t} \right) \frac{\underline{c}}{(1-t)^{2}} < 0. \\ \Gamma_{\underline{c},\underline{c}}(\underline{c},t) &= -f\left(\frac{\underline{c}}{1-t} \right) \frac{1}{1-t} < 0. \\ \Gamma_{\underline{t},\underline{c}}(\underline{c},t) &= -\left[\frac{\underline{c}}{1-t} f\left(\frac{\underline{c}}{1-t} \right) \frac{1}{1-t} - 0f(0) \right] \\ &= -f\left(\frac{\underline{c}}{1-t} \right) \frac{\underline{c}}{(1-t)^{2}} < 0. \\ \Gamma_{\underline{t},\underline{t}}(\underline{c},t) &= -f\left(\frac{\underline{c}}{1-t} \right) \frac{\underline{c}^{2}}{(1-t)^{2}} < 0. \end{split}$$

So $\Gamma(\underline{c}, t)$ is convex. The following will also be used.

$$\begin{split} \Gamma_{t,\bar{y}}(\underline{c},t) &= 1 - \left(\frac{\underline{c}}{1-t}\right) f\left(\frac{\underline{c}}{1-t}\right) \left(\frac{\underline{c}}{\left(1-t\right)^2}\right) \frac{\partial t\left(\underline{c}\right)}{\partial \bar{y}} \\ &= 1 - f\left(\frac{\underline{c}}{1-t}\right) \left(\frac{\underline{c}^2}{\left(1-t\right)^3}\right) \frac{\partial t\left(\underline{c}\right)}{\partial \bar{y}} \end{split}$$

Proofs of claims

Claim 1

Proof. The first-order derivative of (3) with respect to \underline{c} is

$$\begin{split} \frac{\partial \widetilde{V}\left(\underline{c};y^{i}\right)}{\partial \underline{c}} &= -u'\left(\left[1-t\left(\underline{c}\right)\right]y^{i}\right)y^{i}t'\left(\underline{c}\right) - \delta\left\{h'(\underline{c})F\left(\frac{\underline{c}}{1-t\left(\underline{c}\right)}\right)\right.\\ &+h(\underline{c})f\left(\frac{\underline{c}}{1-t\left(\underline{c}\right)}\right)\left(\frac{1}{1-t\left(\underline{c}\right)} + \frac{\underline{c}t'\left(\underline{c}\right)}{\left(1-t\left(\underline{c}\right)\right)^{2}}\right)\right.\\ &+h\left(\frac{c^{P}\left[1-t\left(\underline{c}\right)\right]}{1-t\left(\underline{c}\right)}\right)f\left(\frac{c^{P}}{1-t\left(\underline{c}\right)}\right)\\ &-h\left(\frac{\underline{c}\left[1-t\left(\underline{c}\right)\right]}{1-t\left(\underline{c}\right)}\right)f\left(\frac{\underline{c}}{1-t\left(\underline{c}\right)}\right)\left(\frac{1}{1-t\left(\underline{c}\right)} + \frac{\underline{c}t'\left(\underline{c}\right)}{\left(1-t\left(\underline{c}\right)\right)^{2}}\right)\\ &-t'\left(\underline{c}\right)\left[\int_{\frac{\underline{c}}{1-t\left(\underline{c}\right)}}^{\frac{c^{P}}{1-t\left(\underline{c}\right)}}h'\left(\left[1-t\left(\underline{c}\right)\right]y\right)t'\left(\underline{c}\right)ydF\left(y\right)\right]\right] \end{split}$$

which is reduced to

$$t'\left(\underline{c}\right)\left\{\delta\left[\int_{\frac{\underline{c}}{1-t(\underline{c})}}^{\frac{\underline{c}^{P}}{1-t(\underline{c})}}h'\left(\left[1-t\left(\underline{c}\right)\right]y\right)ydF\left(y\right)-h'\left(\underline{c}\right)\frac{F\left(\frac{\underline{c}}{1-t(\underline{c})}\right)}{t'\left(\underline{c}\right)}\right]-u'\left(\left[1-t\left(\underline{c}\right)\right]y^{i}\right)y^{i}\right\}\right\}$$

$$(7)$$

by factoring out $t'(\underline{c})$ and noting that $h(c^P) = 0$. We would like to completely factor out the $t'(\underline{c})$. Partial differentiation of the definition of $\Gamma(\underline{c}, t(\underline{c}))$ with respect to \underline{c} yields

$$\frac{\partial \Gamma\left(\underline{c},t\left(\underline{c}\right)\right)}{\partial \underline{c}} + \frac{\partial \Gamma\left(\underline{c},t\left(\underline{c}\right)\right)}{\partial t}t'\left(\underline{c}\right) = 0$$

which implies that

$$t'(\underline{c}) = \frac{-\Gamma_{\underline{c}}(\underline{c},t)}{\Gamma_t(\underline{c},t)}.$$

Derivations of the partial derivatives of $\Gamma(\underline{c}, t)$ were previously shown. Note that $t'(\underline{c}) > 0$ since $\Gamma_{\underline{c}}(\underline{c}, t) < 0$ and $\Gamma_t(\underline{c}, t) > 0$. Further, note that

$$\frac{F\left(\frac{\underline{c}}{1-t(\underline{c})}\right)}{t'(\underline{c})} = \frac{-\Gamma_{\underline{c}}\left(\underline{c},t\left(\underline{c}\right)\right)}{t'(\underline{c})} = \Gamma_t\left(\underline{c},t\left(\underline{c}\right)\right)$$

so that the first-order derivative in (7) can be expressed as first order condition

$$t'(\underline{c})\Delta(\underline{c},y^i)=0$$

where

$$\Delta\left(\underline{c}, y^{i}\right) \equiv \delta\left[\int_{\frac{\underline{c}}{1-t(\underline{c})}}^{\frac{\underline{c}P}{1-t(\underline{c})}} h'\left(\left[1-t\left(\underline{c}\right)\right]y\right)ydF\left(y\right) - h'\left(\underline{c}\right)\Gamma_{t}(\underline{c}, t\left(\underline{c}\right))\right] - u'\left(\left[1-t\left(\underline{c}\right)\right]y^{i}\right)y^{i}.$$

Claim 2

Proof.

$$\begin{split} \frac{\partial\Delta\left(\underline{c},y^{i}\right)}{\partial\underline{c}} &= \delta\left\{h'\left(\frac{c^{P}\left[1-t\left(\underline{c}\right)\right]}{1-t\left(\underline{c}\right)}\right)\left(\frac{c^{P}}{1-t\left(\underline{c}\right)}\right)f\left(\frac{c^{P}}{1-t\left(\underline{c}\right)}\right)\right.\\ &\quad -h'\left(\frac{\underline{c}\left[1-t\left(\underline{c}\right)\right]}{1-t\left(\underline{c}\right)}\right)\left(\frac{\underline{c}}{1-t\left(\underline{c}\right)}\right)f\left(\frac{\underline{c}}{1-t\left(\underline{c}\right)}\right)\left(\frac{1}{1-t\left(\underline{c}\right)}+\frac{\underline{c}t'\left(\underline{c}\right)}{\left(1-t\left(\underline{c}\right)\right)^{2}}\right)\right.\\ &\quad -t'\left(\underline{c}\right)\int_{\frac{\underline{c}}{1-t(\underline{c})}}^{\frac{c^{P}}{1-t(\underline{c})}}h''\left(\left[1-t\left(\underline{c}\right)\right]y\right)y^{2}dF(y)-h''\left(\underline{c}\right)\Gamma_{t}\left(\underline{c},t\left(\underline{c}\right)\right)\\ &\quad -h'\left(\underline{c}\right)\left[\Gamma_{t\underline{c}}\left(\underline{c},t\left(\underline{c}\right)\right)+\Gamma_{tt}\left(\underline{c},t\left(\underline{c}\right)\right)t'\left(\underline{c}\right)\right]\right\}+u''\left(\left[1-t\left(\underline{c}\right)\right]y^{i}\right)y^{i^{2}}t'\left(\underline{c}\right)\end{split}$$

Note that $h'(c^P) = 0$ and that

$$\Gamma_{t\underline{c}}(\underline{c},t(\underline{c})) + \Gamma_{tt}(\underline{c},t(\underline{c}))t'(\underline{c}) = \left(\frac{-\underline{c}}{1-t(\underline{c})}\right)f\left(\frac{\underline{c}}{1-t(\underline{c})}\right)\left(\frac{1}{1-t(\underline{c})} + \frac{\underline{c}t'(\underline{c})}{(1-t(\underline{c}))^2}\right)$$

so $\Delta_{\underline{c}}(\underline{c}, y^i)$ reduces to

$$u''\left(\left[1-t\left(\underline{c}\right)\right]y^{i}\right)y^{i^{2}}t'\left(\underline{c}\right)-\delta\left[\int_{\frac{\underline{c}}{1-t(\underline{c})}}^{\frac{\underline{c}^{P}}{1-t(\underline{c})}}h''\left(\left[1-t\left(\underline{c}\right)\right]y\right)y^{2}dF(y)t'\left(\underline{c}\right)+h''(\underline{c})\Gamma_{t}(\underline{c},t\left(\underline{c}\right))\right]$$

which is negative by the concavity of $u(\cdot)$, the convexity of $h(\cdot)$, and the observations that $t'(\underline{c})$ is positive and that $\Gamma_t(\underline{c}, t(\underline{c}))$ is negative.

Comparative Statics Proofs

Population density

Proof. Expressing \underline{c}^m as an implicit function of δ , equation (4), evaluated at y^m , is written as

$$\Delta\left[\underline{c}^{m}\left(\delta\right), y^{m}\right] \equiv 0 \tag{8}$$

Partial differentiation of (8) gives

$$\frac{\partial \Delta \left[\underline{c}^{m}\left(\delta\right),y^{m}\right]}{\partial \delta}+\frac{\partial \Delta \left[\underline{c}^{m}\left(\delta\right),y^{m}\right]}{\partial \underline{c}^{m}}\cdot\frac{\partial \underline{c}^{m}}{\partial \delta}=0$$

which is rearranged as

$$\frac{\partial \underline{c}^{m}}{\partial \delta} = \frac{-\Delta_{\delta} \left[\underline{c}^{m} \left(\delta \right), y^{m} \right]}{\Delta_{\underline{c}} \left[\underline{c}^{m} \left(\delta \right), y^{m} \right]} \tag{9}$$

Note that we can rearrange (4), evaluated at y^m , as

$$\int_{\frac{\underline{c}}{1-t(\underline{c})}}^{\frac{\underline{c}^{P}}{1-t(\underline{c})}} h'\left(\left[1-t\left(\underline{c}\right)\right]y\right)ydF\left(y\right) - h'\left(\underline{c}\right)\Gamma_{t}\left(\underline{c},t\left(\underline{c}\right)\right) = \frac{u'\left(\left[1-t\left(y^{m}\right)\right]y^{m}\right)y^{m}}{\delta}$$

and that

$$\Delta_{\delta}\left[\underline{c}^{m}\left(\delta\right), y^{m}\right] = \int_{\frac{\underline{c}}{1-t(\underline{c})}}^{\frac{\underline{c}^{P}}{1-t(\underline{c})}} h'\left(\left[1-t\left(\underline{c}\right)\right]y\right) y dF\left(y\right) - h'\left(\underline{c}\right)\Gamma_{t}\left(\underline{c}, t\left(\underline{c}\right)\right)$$

 \mathbf{SO}

$$\Delta_{\delta}\left[\underline{c}^{m}\left(\delta\right), y^{m}\right] = \frac{u'\left(\left[1-t\left(y^{m}\right)\right]y^{m}\right)y^{m}}{\delta} > 0.$$

Since $\Delta_{\underline{c}}(\underline{c}, y^i) < 0$ by Claim 2, (9) must be positive.

Median income

Proof. The most preferred policy of the median-income voter is decreasing in income by lemma 2 in the SDI environment, so the sign of the derivative follows immediately. \Box

Mean income

Proof. Expressing \underline{c}^m as an implicit function of \overline{y} , (4) is written as

$$\Delta\left[\underline{c}^{m}\left(\bar{y}\right), y^{m}\right] \equiv 0 \tag{10}$$

Partial differentiation of (10) gives

$$\frac{\partial\Delta\left[\underline{c}^{m}\left(\bar{y}\right),y^{m}\right]}{\partial\bar{y}} + \frac{\partial\Delta\left[\underline{c}^{m}\left(\bar{y}\right),y^{m}\right]}{\partial\underline{c}^{m}} \cdot \frac{\partial\underline{c}^{m}}{\partial\bar{y}} = 0$$

which is rearranged as

$$\frac{\partial \underline{c}^{m}\left(y^{m}\right)}{\partial \bar{y}} = \frac{-\Delta_{\bar{y}}\left[\underline{c}^{m}\left(\bar{y}\right), y^{m}\right]}{\Delta_{\underline{c}}\left[\underline{c}^{m}\left(\bar{y}\right), y^{m}\right]}.$$
(11)

From (4), we know that

$$\begin{split} \Delta_{\bar{y}}\left[\underline{c}^{m}\left(\bar{y}\right),y^{m}\right] &= \delta \left\{ h'\left(\frac{c^{P}\left[1-t\left(\underline{c}\right)\right]}{1-t\left(\underline{c}\right)}\right) f\left(\frac{c^{P}}{1-t\left(\underline{c}\right)}\right) \left(\frac{c^{P}}{\left(1-t\left(\underline{c}\right)\right)^{2}}\right) t_{\bar{y}}(\underline{c}) \right. \\ &- h'\left(\frac{\underline{c}\left[1-t\left(\underline{c}\right)\right]}{1-t\left(\underline{c}\right)}\right) f\left(\frac{\underline{c}}{1-t\left(\underline{c}\right)}\right) \left(\frac{\underline{c}^{2}}{\left(1-t\left(\underline{c}\right)\right)^{3}}\right) t_{\bar{y}}(\underline{c}) \\ &- t_{\bar{y}}(\underline{c}) \int_{\frac{\underline{c}}{1-t(\underline{c})}}^{\frac{c^{P}}{1-t(\underline{c})}} h''\left(\left[1-t\left(\underline{c}\right)\right]y\right) y^{2} dF(y) \\ &- h'(\underline{c})\Gamma_{t\bar{y}}\left(\underline{c},t\left(\underline{c}\right)\right) t_{\bar{y}}(\underline{c}) \} + u''\left(\left[1-t\left(y^{m}\right)\right]y^{m}\right) \left(y^{m}\right)^{2} t_{\bar{y}}(\underline{c}) \end{split}$$

which reduces to

$$-\left\{\delta\left[h'(\underline{c})+\int_{\frac{c}{1-t(\underline{c})}}^{\frac{c}{1-t(\underline{c})}}h''([1-t(\underline{c})]y)y^2dF(y)t_{\bar{y}}(\underline{c})\right]-u''(\cdot)(y^m)^2t_{\bar{y}}(\underline{c})\right\}>0$$

since $h'(c^P) = 0$ and $\Gamma_{t,\bar{y}}(\underline{c},t) = 1 - f\left(\frac{\underline{c}}{1-t}\right) \left(\frac{\underline{c}^2}{(1-t)^3}\right) t_{\bar{y}}(\underline{c})$. That $\Delta_{\bar{y}}[\underline{c}^m(\bar{y}), y^m]$ is positive follows from the convexity of $h(\cdot)$ the concavity of $u(\cdot)$ and the fact that $t_{\bar{y}}(\underline{c})$ is negative. To see that $t_{\bar{y}}(\underline{c})$ is negative, partially differentiate $\Gamma(\underline{c}, t(\underline{c}))$ with respect to \bar{y} to find

$$\frac{\partial\Gamma\left(\underline{c},t\left(\underline{c}\right)\right)}{\partial\bar{y}} + \frac{\partial\Gamma\left(\underline{c},t\left(\underline{c}\right)\right)}{t\left(\underline{c}\right)} \cdot \frac{\partial t\left(\underline{c}\right)}{\partial\bar{y}} = 0$$

which is rearranged to show

$$t_{\bar{y}}(\underline{c}) = \frac{-t(\underline{c})}{\Gamma_t(\underline{c}, t(\underline{c}))} < 0$$

since $\Gamma_t(\underline{c}, t(\underline{c}))$ is positive. Since $\Delta_{\underline{c}}(\underline{c}, y^i) < 0$ by claim 2, (11) must be positive. \Box

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