关于局部对称空间中具有平行平均曲率向量子流形的 Pinching 定理

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摘要 : 设 M 为单位球面 $S^{n+p}(1)$ 中的一个紧致子流形. $\cup M = \bigcup_{x\in M} \cup M_x$ 是 M 的单位切丛. 陈卿引入函数 $f(x) = \max_{u,v\in\cup M_x} \|B(u,u) - B(v,v)\|^2$,其中 B 是 M 的第二基本形式. 当 M 具有平行平均曲率向量时 ,陈卿通过研究函数 $f(x)$,得到一个 Pinching 定理. 当考虑外围流形为局部对称空间时 ,我们应用 Gauss 方程 ,Ricci 方程和外围空间的局部对称性质等方法得到 :若 $f(x)$ 满足一个 Pinching 条件 ,则 M 或是全脐的或是一个 Veronese 曲面. 当 $p \geq 2$ 时 ,所得的结果改进了陈卿研究的相应结果.

关键词 : 平均曲率 ;局部对称空间 ;第二基本形式
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1 介 绍

设 M^n 是 S^{n+p} 的一个紧致浸入子流形, B 是 M 的第二基本形式. $\cup M = \bigcup_{x\in M} \cup M_x$ 是 M 的单位切丛, $\cup M_x = \{u \in T_xM, |u|^2 = 1\}$,设 $f(x) = \max_{u,v\in\cup M_x} \|B(u,u) - B(v,v)\|^2$.

陈卿在文献[1] 中证得如下定理 :

定理 A 设 M^n 是 S^{n+p} 的一个具有平行平均曲率向量的紧致子流形

(i) $P = 1$. 若 $f(x) \leq 4$,则 M 是全脐超曲面或是 $S^l(\frac{1}{\sqrt{2}}) \times S^m(\frac{1}{\sqrt{2}})$, $(l + m = n)$,

(ii) $P \geq 2$. 若 $f(x) \leq \frac{2n}{2n-1}$,则 M 或是全脐的或是一个 Veronese 曲面.

对于 $P \geq 2$ 的情况 ,本文把上述结果改进为如下 :

定理 1 设 M^n 是 S^{n+p} 的一个具有平行平均曲率向量的紧致子流形, $P \geq 2$,若 $f(x) \leq \frac{8n}{7n-2}$,则 M 或是全脐的或是一个 Veronese 曲面.

对于外围空间为局部对称空间 N^{n+p} 的情况 ,有下面结论 :

定理 2 设 M^n 是局部对称空间 N^{n+p} 中一个具有平行平均曲率向量的紧致子流形, N^{n+p} 的截曲率 K^N 满足 : $\frac{1}{2} < \delta \leq K^N \leq 1$.

(i) $P = 1$,若 $c_1 \leq f(x) \leq c_2$,则 M 是全脐超曲面或是 $S^l(\frac{1}{\sqrt{2}}) \times S^m(\frac{1}{\sqrt{2}})$, $(l + m = n)$;

(ii) $P \geq 2$,若 $c_3 \leq f(x) \leq c_4$,则 M 或是全脐的或是一个 Veronese 曲面.

其中 c_1, c_2 为关于 x 的方程

$$-\frac{1}{4}x^2 + \left[(2\delta - 1)n - \frac{1}{2}(1-\delta)\right]x - \frac{1}{2}(1-\delta)n^2H^2 = 0$$

的两根, c_3, c_4 为关于 x 的方程

$$-\frac{m-2}{8}x^2 + [(2\delta - 1)n - \frac{13}{3}(1-\delta)(n-1)\sqrt{P}]x - (1-\delta)\frac{n^2}{n-1}\sqrt{P}H^2 = 0$$

的两根.

注 $\delta = 1$ 时 ,定理 2 就是定理 1.

2 准 备

本文约定 : $1 \leq A, B \cdots \leq n + P$, $1 \leq i, j \cdots \leq n$, $n + 1 \leq \alpha, \beta \cdots \leq n + P$.

设 $\cup M = \bigcup_{x\in M} \cup M_x$ 是 M 的单位切丛, $\cup M_x = \{u \in T_xM : \|u\| = 1\}$.

定义函数 $f(x)$ 如下

$$f(x) = \max_{u,v\in\cup M_x} \|B(u,u) - B(v,v)\|^2,$$

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设 $x_0 \in M$,使得 $f(x_0) \neq 0$,因而存在 $u_0, v_0 \in \cup M_{x_0}$ 满足:$f(x_0) = \|B(u_0,u_0) - B(v_0,v_0)\|^2$.

选取 x_0 的局部标架场 $\{e_A\}$,使得:

$$e_{n+1} = \frac{B(u_0,u_0) - B(v_0,v_0)}{\|B(u_0,u_0) - B(v_0,v_0)\|^2},$$

则有[1]

$$f(x_0) = (h_{11}^{n+1} - h_{nn}^{n+1})^2 \tag{1}$$

$$h_{11}^{\alpha} = h_{nn}^{\alpha}, (\alpha \neq n+1) \tag{2}$$

$$\sum_{\alpha}(h_{ij}^{\alpha})^2 \leq \frac{1}{4} f(x), \ (\forall i \neq j) \tag{3}$$

$$h_{11}^{n+1} \geq h_{22}^{n+1} \geq \cdots \geq h_{nn}^{n+1}, h_{ij}^{n+1} = 0, (i \neq j) \tag{4}$$

在 M 上定义一个张量场 $H = (H_{ijkl})$ 如下

$$H_{ijkl} = \sum_{\alpha} h_{ij}^{\alpha} h_{kl}^{\alpha},$$

则在上述标架下有

$$f(x_0) = H_{1111} + H_{nnnn} - 2H_{11nn},$$

记

$$\Delta(1,n) = (\Delta H)_{1111} + (\Delta H)_{nnnn} - 2(\Delta H)_{11nn},$$

则有[1]

$$\frac{1}{2}\Delta(1,n) \geq \sum_{i}(h_{11}^{n+1} - h_{nn}^{n+1})(h_{11ii}^{n+1} - h_{nnii}^{n+1}) \tag{5}$$

又

$$h_{ijkl}^{\alpha} - h_{ijlk}^{\alpha} = \sum_{m} h_{im}^{\alpha} R_{mjkl} + \sum_{m} h_{mj}^{\alpha} R_{mikl} - \sum_{\beta} h_{ij}^{\beta} R_{\alpha\beta kl} \tag{6}$$

$$R_{ijkl} = (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jl}) + \sum_{\alpha}(h_{ik}^{\alpha} h_{jl}^{\alpha} - h_{il}^{\alpha} h_{jl}^{\alpha}) \tag{7}$$

$$R_{\alpha\beta ij} = \sum_{k}(h_{ik}^{\alpha} h_{jk}^{\beta} - h_{jk}^{\alpha} h_{ik}^{\beta}) \tag{8}$$

$$h_{ijk}^{\alpha} = h_{ikj}^{\alpha} \tag{9}$$

由式(5) ~ (9) 得

$$\frac{1}{2}\Delta(1,n) \geq A + B \tag{10}$$

其中

$$A = nf(x) - 2(h_{11}^{n+1} - h_{nn}^{n+1}) \cdot \sum_{\alpha,i}[(h_{11}^{n+1} - h_{ii}^{n+1})(h_{1i}^{\alpha})^2 + (h_{ii}^{n+1} - h_{nn}^{n+1})(h_{ni}^{\alpha})^2],$$

$$B = f(x)\{-\sum_{i}(h_{ii}^{n+1})^2 + \sum_{i,\alpha\neq n+1} h_{11}^{\alpha} h_{ii}^{\alpha} + \sum_{i}(h_{11}^{n+1} + h_{ii}^{n+1}) h_{ii}^{n+1}\}.$$

引理 1　$A \geq f(x)\{n - \frac{n}{2} f(x)\}$.

证明　由式(3)、(4) 得

$$A \geq nf(x) - 2(h_{11}^{n+1} - h_{nn}^{n+1}) \cdot \sum_{i}\{(h_{11}^{n+1} - h_{ii}^{n+1})\frac{f(x)}{4} + (h_{ii}^{n+1} - h_{nn}^{n+1})\frac{f(x)}{4}\} = f(x)\{n - \frac{n}{2} f(x)\}.$$

引理 2　$\sum_{\alpha\neq n+1, i\neq 1,n} h_{11}^{\alpha} h_{ii}^{\alpha} + \frac{1}{4}\sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1}) \cdot (h_{ii}^{n+1} - h_{nn}^{n+1}) \geq -\frac{n-2}{8} f(x)$.

证明

$$\sum_{\alpha\neq n+1, i\neq 1,n} h_{11}^{\alpha} h_{ii}^{\alpha} \geq -\frac{1}{4}\sum_{\alpha\neq n+1, i\neq 1,n}(h_{11}^{\alpha} - h_{ii}^{\alpha})^2 = -\frac{1}{4}\sum_{\alpha, i\neq 1,n}(h_{11}^{\alpha} - h_{ii}^{\alpha})^2 + \frac{1}{4}\sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})^2 \geq -\frac{n-2}{4} f(x) + \frac{1}{4}\sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})^2 \tag{11}$$

$$\sum_{\alpha\neq n+1, i\neq 1,n} h_{nn}^{\alpha} h_{ii}^{\alpha} \geq -\frac{1}{4}\sum_{\alpha\neq n+1, i\neq 1,n}(h_{nn}^{\alpha} - h_{ii}^{\alpha})^2 = -\frac{1}{4}\sum_{\alpha, i\neq 1,n}(h_{nn}^{\alpha} - h_{ii}^{\alpha})^2 + \frac{1}{4}\sum_{i\neq 1,n}(h_{nn}^{n+1} - h_{ii}^{n+1})^2 \tag{12}$$

$$\sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})^2 + \sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})(h_{ii}^{n+1} - h_{nn}^{n+1}) = \sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})\{(h_{11}^{n+1} - h_{ii}^{n+1}) + (h_{ii}^{n+1} - h_{nn}^{n+1})\} = \sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})(h_{11}^{n+1} - h_{nn}^{n+1}) \tag{13}$$

$$\sum_{i\neq 1,n}(h_{nn}^{n+1} - h_{ii}^{n+1})^2 + \sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})(h_{ii}^{n+1} - h_{nn}^{n+1}) = \sum_{i\neq 1,n}(h_{ii}^{n+1} - h_{nn}^{n+1})\{(h_{ii}^{n+1} - h_{nn}^{n+1}) + (h_{11}^{n+1} - h_{ii}^{n+1})\} = \sum_{i\neq 1,n}(h_{ii}^{n+1} - h_{nn}^{n+1})(h_{11}^{n+1} - h_{nn}^{n+1}) \tag{14}$$

由式(2),(11) ~ (14) 得:

$$\sum_{\alpha\neq n+1, i\neq 1,n} h_{11}^{\alpha} h_{ii}^{\alpha} + \frac{1}{4}\sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})(h_{ii}^{n+1} - h_{nn}^{n+1}) = \frac{1}{2}\sum_{\alpha\neq n+1, i\neq 1,n} h_{11}^{\alpha} h_{ii}^{\alpha} + \frac{1}{2}\sum_{\alpha\neq n+1, i\neq 1,n} h_{nn}^{\alpha} h_{ii}^{\alpha} + \frac{1}{4}\sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1})(h_{ii}^{n+1} - h_{nn}^{n+1}) \geq -\frac{n-2}{4} f(x) + \frac{1}{8}\{\sum_{i\neq 1,n}(h_{11}^{n+1} - h_{ii}^{n+1}) \cdot (h_{11}^{n+1} - h_{nn}^{n+1}) + \sum_{i\neq 1,n}(h_{ii}^{n+1} - h_{nn}^{n+1})(h_{11}^{n+1} - h_{nn}^{n+1})\} = -\frac{n-2}{4} f(x) + \frac{n-2}{8} f(x) = -\frac{n-2}{8} f(x).$$

引理 3　$B \geq -\frac{3n-2}{8} f^2(x)$.

证明

$$B = f(x)\{\sum_{i}(h_{11}^{n+1} - h_{ii}^{n+1})(h_{ii}^{n+1} - h_{nn}^{n+1}) + nh_{11}^{n+1} h_{nn}^{n+1} + \sum_{i,\alpha\neq n+1} h_{11}^{\alpha} h_{ii}^{\alpha}\} \geq f(x)\{\sum_{i}(h_{11}^{n+1} - h_{ii}^{n+1})(h_{ii}^{n+1} - h_{nn}^{n+1}) - \frac{n}{4}(h_{11}^{n+1} - h_{nn}^{n+1})^2 + \sum_{i\neq 1,n,\alpha\neq n+1} h_{11}^{\alpha} h_{ii}^{\alpha}\} = f(x)\{\sum_{i}(h_{11}^{n+1} - h_{ii}^{n+1})(h_{ii}^{n+1} - h_{nn}^{n+1}) - \frac{n}{4} f(x) + \sum_{i\neq 1,n,\alpha\neq n+1} h_{11}^{\alpha} h_{ii}^{\alpha}\} \tag{15}$$

由引理 2,式(15) 得:

$$B \geq f(x)\{\frac{3}{4}\sum_i (h_{11}^{n+1}-h_{ii}^{n+1})(h_{ii}^{n+1}-h_{nn}^{n+1}) - \frac{n}{4}f(x) - \frac{n-2}{8}f(x)\} = -\frac{3n-2}{8}f^2(x).$$

引理 4 $f\equiv 0$,当且仅当 M 是全脐点子流形.

3 定理 1 的证明

由式(10),引理 1 及引理 3 得

$$\frac{1}{2}\Delta(1,n) \geq f(x)\{n-\frac{7n-2}{8}f(x)\}.$$

若 $f\equiv 0 \Rightarrow M$ 是全脐点子流形.

若 $f\equiv \frac{8n}{7n-2}$,由式(15) 取等号得:

$$\forall \alpha \neq n+1, h_{11}^{\alpha} = h_{nn}^{\alpha} = 0, n = 2 \qquad (16)$$

由 $nh_{11}^{n+1}h_{nn}^{n+1} = -\frac{1}{4}(h_{11}^{n+1}-h_{nn}^{n+1})^2$ 和式(16) 得

$h_{11}^{n+1}+h_{nn}^{n+1} = 0.$

所以,M 是极小的,因此 M 是一个 Veronese 曲面[2].

4 定理 2 的证明

$$h_{11ii}^{\alpha} = h_{1i1i}^{\alpha} - K_{\alpha 11ii}^{N} = h_{i11i}^{\alpha} - K_{\alpha 11ii}^{N} = h_{i1i1}^{\alpha} + h_{im}^{\alpha}R_{m11i} + h_{m1}^{\alpha}R_{mi1i} - h_{i1}^{\beta}R_{\alpha\beta 1i} - K_{\alpha 11ii}^{N} = h_{im}^{\alpha}R_{m11i} + h_{m1}^{\alpha}R_{mi1i} - h_{i1}^{\beta}R_{\alpha\beta 1i} - K_{\alpha i1i1}^{N} - K_{\alpha 11ii}^{N} \qquad (17)$$

由式(5),(17) 以及 Gduss 方程,Ricci 方程以及 $0<\delta\leq K_{ijij}^{N}\leq 1$ 得

$$\frac{1}{2}\Delta(1,n) \geq \delta n f(x) + A + B + C \qquad (18)$$

其中

$$A = -2(h_{11}^{n+1}-h_{nn}^{n+1})\sum_{\alpha,i}[(h_{11}^{n+1'}-h_{ii}^{n+1})(h_{1i}^{\alpha})^2 + (h_{ii}^{n+1}-h_{n+1}^{n+1})(h_{ni}^{\alpha})^2],$$

$$B = f(x)\{-\sum_i (h_{ii}^{n+1})^2 + \sum_{i,\alpha\neq n+1} h_{11}^{\alpha}h_{ii}^{\alpha} + \sum_i (h_{11}^{n+1}+h_{ii}^{n+1})h_{ii}^{n+1}\},$$

$$C = -(h_{11}^{n+1}-h_{nn}^{n+1})\{\sum_i[K_{n+1,i1i1}^{N} + K_{n+1,11ii}^{N} - K_{n+1,inin}^{N} - K_{n+1,nnii}^{N}] + \sum_{\beta,i}[h_{i1}^{\beta}K_{n+1,\beta 1i}^{N} - h_{in}^{\beta}K_{n+1,\beta ni}^{N}]\}.$$

由 N 的局部对称性,得:

$$C = -(h_{11}^{n+1}-h_{nn}^{n+1})\{3K_{n+1,\beta 1i}^{N}h_{1i}^{\beta} + 3K_{n+1,\beta ni}^{N}h_{ni}^{\beta} + K_{n+1,\beta i}^{N}(h_{11}^{\beta}-h_{nn}^{\beta}) + \sum_{i\neq 1}K_{n+1,i1\beta}^{N}h_{i1}^{\beta} + \sum_{i\neq 1}K_{n+1,1\beta i}^{N}h_{1i}^{\beta} + \sum_{i\neq n}K_{n+1\,in\beta}^{N}h_{in}^{\beta} + \sum_{i\neq n}K_{n+1,n\beta i}^{N}h_{ni}^{\beta} - (h_{11}^{n+1}-h_{ii}^{n+1})K_{i1i1}^{N} - (h_{ii}^{n+1}-h_{nn}^{n+1})K_{inin}^{N} + K_{n+1,11\beta}^{N}h_{ii}^{\beta} - K_{n+1,nn\beta}^{N}h_{ii}^{\beta}\} \qquad (19)$$

(i) 先讨论 $p\geq 2$ 的情况.

引理 5 $C \geq -\frac{13}{3}(1-\delta)(n-1)\sqrt{p}f(x) - (1-\delta)nf(x) - (1-\delta)\frac{\sqrt{p}}{n-1}n^2H^2$

证明

$$3|K_{n+\beta,1i}^{N}h_{1i}^{\beta}(h_{11}^{n+1}-h_{nn}^{n+1})| \leq (1-\delta)\{\sum_{i\neq 1,\beta}\varepsilon(h_{1i}^{\beta})^2 + \frac{1}{\varepsilon}(h_{11}^{n+1}-h_{nn}^{n+1})^2\} \leq (1-\delta)\{\frac{\varepsilon}{4}(n-1)f(x) + \frac{n-1}{\varepsilon}pf(x)\} = (1-\delta)(n-1)f(x)\{\frac{\varepsilon}{4}+\frac{p}{\varepsilon}\} = (1-\delta)(n-1)\sqrt{p}f(x) \quad (\text{取 } \varepsilon = 2\sqrt{p}) \qquad (20)$$

同理

$$3|K_{n+1,\beta ni}^{N}h_{ni}^{\beta}(h_{11}^{n+1}-h_{nn}^{n+1})| \leq (1-\delta)(n-1)\sqrt{p}f(x) \qquad (21)$$

$$|\sum_{i\neq 1}K_{n+1,i1\beta}^{N}h_{i1}^{\beta}(h_{11}^{n+1}-h_{nn}^{n+1})| \leq \frac{1}{3}(1-\delta)(n-1)\sqrt{p}f(x) \qquad (22)$$

$$|\sum_{i\neq 1}K_{n+1,1\beta i}^{N}h_{i1}^{\beta}(h_{11}^{n+1}-h_{nn}^{n+1})| \leq \frac{1}{3}(1-\delta)(n-1)\sqrt{p}f(x) \qquad (23)$$

$$|\sum_{i\neq 1}K_{n+1,in\beta}^{N}h_{in}^{\beta}(h_{11}^{n+1}-h_{nn}^{n+1})| \leq \frac{1}{3}(1-\delta)(n-1)\sqrt{p}f(x) \qquad (24)$$

$$|\sum_{i\neq 1}K_{n+1,n\beta i}^{N}h_{in}^{\beta}(h_{11}^{n+1}-h_{nn}^{n+1})| \leq \frac{1}{3}(1-\delta)(n-1)\sqrt{p}f(x) \qquad (25)$$

$$-K_{n+1,\beta i}^{N}(h_{11}^{\beta}-h_{nn}^{\beta})(h_{11}^{n+1}-h_{nn}^{n+1}) = -K_{n+1,i,n+1,i}^{N}(h_{11}^{n+1}-h_{nn}^{n+1})^2 \geq -nf(x) \qquad (26)$$

$$(h_{11}^{n+1}-h_{nn}^{n+1})\{(h_{11}^{n+1}-h_{ii}^{n+1})K_{i1i1}^{N} + (h_{ii}^{n+1}-h_{nn}^{n+1})K_{inin}^{N}\} \geq \delta(h_{11}^{n+1}-h_{nn}^{n+1})\{(h_{11}^{n+1}-h_{ii}^{n+1}) + (h_{ii}^{n+1}-h_{nn}^{n+1})\} = n\delta f(x) \qquad (27)$$

$$|(K_{n+1,11\beta}^{N} - K_{n+1,nn\beta}^{N})h_{ii}^{\beta}(h_{11}^{n+1}-h_{nn}^{n+1})| \leq (1-\delta)\sum_{\beta}\frac{1}{\varepsilon}(h_{11}^{n+1}-h_{nn}^{n+1})^2 + \varepsilon(\sum_i h_{ii}^{\beta})^2 = (1-\delta)\{\frac{p}{\varepsilon}f(x) + \varepsilon n^2H^2\} = (1-\delta)(n-1)\sqrt{p}f(x) + (1-\delta)\frac{\sqrt{p}}{n-1}n^2H^2 \qquad (28)$$

由式(20)~(28) 得引理 5.

由式(18),引理 1,3,5 得

$$\frac{1}{2}\Delta(1-n) \ge -\frac{7n-2}{8}f^2(x) + \{-(1-2\delta)n - \frac{13}{3}(1-\delta)(n-1)\sqrt{p}\}f(x) - (1-\delta)\frac{n^2\sqrt{p}}{n-1}H^2 \quad (29)$$

由题设和式(29) 知:式(26) ～ (28) 取等号.

式(26) 取等号得

$$K^N_{n+1,i,n+1,i} = 1, \forall i \quad (30)$$

式(27) 取等号得

$$K^N_{i1i1} = K^N_{inin} = \delta, \forall i \ne 1, n \quad (31)$$

式(28) 取等号得

$$K^N_{n+1,1,n+1,1} = \delta \text{或} K^N_{n+1,n,n+1,n} = \delta \quad (32)$$

由式(30) , (32) 知 $\delta = 1$.

所以由定理 1 立即得定理 2.

(ii) $p = 1$ 的情况.

引理6　$C \ge -(1-\delta)nf(x) - \frac{1}{2}(1-\delta)f(x) - \frac{1}{2}(1-\delta)n^2H^2$.

证明　由式(19) 知

$$C = -K^N_{n+1,i,n+1,i}(h_{11}^{n+1} - h_{nn}^{n+1})^2 + (h_{11}^{n+1} - h_{nn}^{n+1})\{(h_{11}^{n+1} - h_{ii}^{n+1})K^N_{i1i1} + (h_{ii}^{n+1} - h_{nn}^{n+1})K^N_{inin}\} + (h_{11}^{n+1} - h_{nn}^{n+1})\cdot\{(K^N_{n+1,1,n+1,1} - K^N_{n+1,n,n+1,n})h_{ii}^{n+1}\} \ge -nf(x) + n\delta f(x) + (h_{11}^{n+1} - h_{nn}^{n+1})\cdot(K^N_{n+1,1,n+1,1} - K^N_{n+1,n,n+1,n})h_{ii}^{n+1} \quad (33)$$

$$|(h_{11}^{n+1} - h_{nn}^{n+1})(K^N_{n+1,1,n+1,1} - K^N_{n+1,n,n+1,n})h_{ii}^{n+1}| \le \frac{1}{2}(1-\delta)\{(h_{11}^{n+1} - h_{nn}^{n+1})^2 + (\sum h_{ii}^{n+1})^2\} = \frac{1}{2}(1-\delta)\{f(x) + n^2H^2\} \quad (34)$$

由式(33) , (34) 得引理 6.

由引理 6 ,式(18) 得

$$\frac{1}{2}\Delta(1,n) \ge -\frac{n}{4}f^2(x) + [(2\delta-1)n - \frac{1}{2}(1-\delta)]f(x) - \frac{1}{2}(1-\delta)n^2H^2 \quad (35)$$

由题设,式(35) 得(33) 、(34) 取等号,所以 $\delta = 1$.

由定理 A ,得定理 2 成立.

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A Pinching Theorem for Submanifolds of Locally Symmetric Space with Parallel Mean Curvature

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Abstract: Let M^n be a compact submanifold of unit sphere $S^{n+p}(1)$ $\cup M = \bigcup_{x\in M} \cup M_x$ is the unit tangent bundle on M. Chen Qing constructed a function $f(x) = \max_{u,v\in\cup M_x} \|B(u,u) - B(v,v)\|^2$, where B is the second fundamental form of M. When M has parallel mean curvature vector, Chen Qing obtained a Pinching theorem through studying function $f(x)$. When considering outer space which is locally symmetry, we apply Gauss equation, Ricci equation and the property of locally symmetry of the outer space, then we obtain the following theorem: if M satisfys a Pinching condition, M is a totally umbilical submanifold or a Veronese surface. When $p \ge 2$, What we obtain improves the corresponding theorem of Chen qin's.

Key words: mean curvature; locally symmetric space; second fundamental form