RAIRO Operations Research
RAIRO Oper. Res. **36** (2002) 191-208
DOI: 10.1051/ro:2003002

OPTIMAL QOS CONTROL OF INTERACTING SERVICE STATIONS

Abdelkrim Haqiq[1], I. Lambadaris[2], N. Mikou[3] and L. Orozco–Barbosa[4]

Communicated by Naoto Kaio

Abstract. We consider a system of three queues and two types of packets. Each packet arriving at this system finds in front of it a controller who either sends it in the first queue or rejects it according to a QoS criterion. When the packet finishes its service in the first queue, it is probabilistically routed to one of two other parallel queues. The objective is to minimize a QoS discounted cost over an infinite horizon. The cost function is composed of a waiting cost per packet in each queue and a rejection cost in the first queue. Subsequently, we generalize this problem by considering a system of $(m+1)$ queues and n types of packets. We show that an optimal policy is monotonic.

Keywords. Queues, flow control, dynamic programming, Policies, IP network.

Received May, 2001.

[1] Faculté des Sciences et Techniques, Département de Mathématiques et Informatique, route de Casablanca, BP. 577, Settat, Maroc; e-mail: haqiq@netscape.net

[2] Carleton University, Department of Systems and Computer Engineering, 1125 Colonel By Drive, Ottawa, Ontario, K1 S5 B6, Canada

[3] Faculté des Sciences Mirande, Laboratoire Informatique de Recherche sur les Systèmes Intelligents et leurs Applications, BP. 47870, 21078 Dijon Cedex, France

[4] University of Ottawa, School of Information, Technology and Engineering, 161 Louis Pasteur, Ottawa, Ontario, K1 N6 N5, Canada

1. Introduction

In this paper we first study a queueing system composed of three exponential servers. The input to the system comes from two different Poisson processes and can be controlled by accepting or rejecting arriving packets. A packet entering queue 1 is then routed to queue 2 or queue 3 with given probabilities. There is a waiting cost per packet in each queue and a reject cost in the first queue. Our goal is to prove that there exits an optimal admission control that minimizes a waiting/rejection cost. We furthermore derive monotonicity properties of the control policies. We use induction on a sequence of finite-horizon problems to establish this and other interesting properties of optimal policies, for both finite and infinite horizon problems, with discounting. Furthermore, we generalize this problem by considering a system of $(m+1)$ exponential servers. The input to the system comes from n different Poisson processes and can be controlled by accepting or rejecting arriving packets, each arrival enters queue 1 and then routed to one of the m other queues with given probabilities.

The routing probabilities are chosen according to the service rates and the coefficients of the waiting costs in queues 2 and 3 for the first case, and in queues 2, 3, ..., and $m+1$ in the second case.

Admission control and routing are key issues arising in the design and operation of communication and computer networks, and have received considerable attention in the last years. The admission control problem implies a determination of efficient policies for allowing incoming packets to gain access to network facilities. The routing problem involves selecting paths from several alternatives in the network along which accepted packets can be efficiently forwarded to their destinations. Admission control and routing play a key role for Quality of Service provisioning in modern broadband packet networks (such as IP, ATM etc.).

Numerous studies of admission control and routing problems at an entrance node or at several intermediate nodes of a network can be found in the literature. We cite below some of the studies relevant to our work; this list is by no means exhaustive.

Stidham [22] has considered admission control policies for several simple queuing models. The optimal admission control policies for all these models share the characteristic that they can be expressed in terms of a "switching curve". Viniotis and Ephremides [25] have demonstrated a similar characterization of the optimal admission strategy at a simple node in an Integrated Services Digital Network. Results in the same vein have been obtained by Christidou *et al.* [2] for a cyclic interconnection of two queues, and by Lambadaris *et al.* [15] for a circuit-switched node. Farrel [5] considered the problem of routing packets to one of two queues, but without the option of rejecting an arriving packet. Davis [3] considered the routing problem for two parallel queues with the rejection option. He showed that the optimal policy is monotonic (*i.e.*, the optimal rejection region is an increasing set) and that it is optimal to send an accepted packet to the shorter of the two queues. (The latter property has also been proved by Winston [28], Weber [27] and Ephremides *et al.* [4].) Davis also demonstrated that the optimal rejection has an additional property: moving a packet from the longer to the shorter of the two queues makes it less advantageous to accept an arriving packet. In addition to being of interest in its own right, this property was crucial to the proof that the optimal rejection region is an increasing set. Ghoneim [7] and Ghoneim and Stidham [8] studied a system with two exponential servers in series and Poisson arrivals to each server, with the rejection option. Stidham and Weber [23] have extended some of the results of Ghoneim [7] for two queues in series to an arbitrary finite number of queues in series.

Hajek [9] studied a class of control problems for a general two-node network, in which it is possible to control routing of incoming packets, transfers from one node to another, and service priorities. He showed that an optimal control policy is characterized by a monotonic switching curve, with the property that one should switch attention from node 1 to node 2 when the number of packets in node 2 decreases or the number in node 1 increases. Hajek's model includes as a special case optimal routing of arrivals between two parallel nodes (the problem considered by Farrel [5], Winston [28], Weber [27], and Ephremides *et al.* [4]), but it does not include Davis's, Ghoneim's and Stidham's models and the one in the present paper. Haqiq and Mikou [10] studied a network consisting of two different parallel exponential servers with two types of packets, in which the first server is subject to intermittent breakdowns and it is possible to control routing of incoming packets of type one. Haqiq [11] has extended the latter problem by adding the rejection option and by limiting the capacity of the second queue. Lazar [16] examined control of arrivals to an arbitrary network of queues, with the objective of maximizing throughput subject to a constraint on average response time. As a consequence, Lazar necessarily restricted attention to policies based on the total number of packets in the network. Among such policies, he showed that it is optimal to use "end-to-end" control, that is, to reject arrivals when the total number of packets in the network exceeds a critical level. Lambadaris and Narayan [14] studied the problem of jointly optimal admission and routing controls at a network node, they proved that optimal admission and routing strategies are characterized by means of "switching curves".

The paper contains two parts, in the first we study a system with three interacting queues and two types of packets and it is organised as follows: we first informally describe the system, and give a mathematical formulation, after that, the continuous-time problem is transformed into another one with discrete time by looking at an embedded Markov chain. In Section 3.3, the structural properties for the optimal policies are given for the finite and infinite horizon and discounted cost problems. Section 3.4 gives a characterization of the optimal policies. Section 3.5 gives the proof of the structural properties and we finish this part by a conclusion. In the second part, we generalize the first study by choosing a system with $m+1$ interacting queues and n-types of packets. The next section is a motivation of our problem.

2. Motivation

The Internet Protocol (IP) architecture is based on a connectionless packet service using the IP protocol. The advantages of its connectionless design, flexibility and robustness, have been amply demonstrated. However, these advantages are not without cost: careful design is required to provide good service under heavy load. In fact, lack of attention to the dynamics of packet forwarding can result in severe service degradation. This phenomenon was first observed during the early growth phase of the Internet of the mid by Nagle [19], and is technically called "congestion collapse".

Beginning in 1986, Jacobson [12] developed the congestion avoidance mechanisms that are now required in TCP implementations [1, 12]. These mechanisms operate in the hosts to cause TCP connections to "back off" during congestion. It is primarily these TCP congestion avoidance algorithms that prevent the congestion collapse of today's Internet.

Considerable research has been done on Internet dynamics since 1988, and the Internet has grown. It has become clear that TCP congestion avoidance mechanisms [21], while necessary and powerful, are not sufficient to provide good service in all circumstances. Basically, there is a limit to how much control can be accomplished from the edges of the network. Some mechanisms are needed in the routers to complement the endpoint congestion avoidance mechanisms.

It is useful to distinguish between two classes of router algorithms related to congestion control: "queue management" *versus* "scheduling" algorithms. Queue management algorithms manage the length of packet queues by dropping packets when necessary or appropriate, while scheduling algorithms determine which packet to send next and are used primarily to manage the allocation of bandwidth among flows.

In this paper, we propose to control dynamically the packets before entering into the router by accepting or rejecting them in order to minimize the waiting and the rejection discounted costs. The router is modelled by the first queue. Our control prevents the congestion in the network by rejecting packets arriving at the router, when the load of the system exceeds a certain threshold. This is another method which is in agreement with that of Random Early Detection (RED) gateways presented in [6], where the gateway detects incipient congestion by computing the average queue size. When the average queue size exceeds a certain threshold, the gateway drops or marks each arriving packet with a certain probability, where the exact probability is a function of the average queue size.

3. Optimal Control of three interacting queues with two types of arrivals

The main model considered in this part is pictured in Figure 1 and described below.

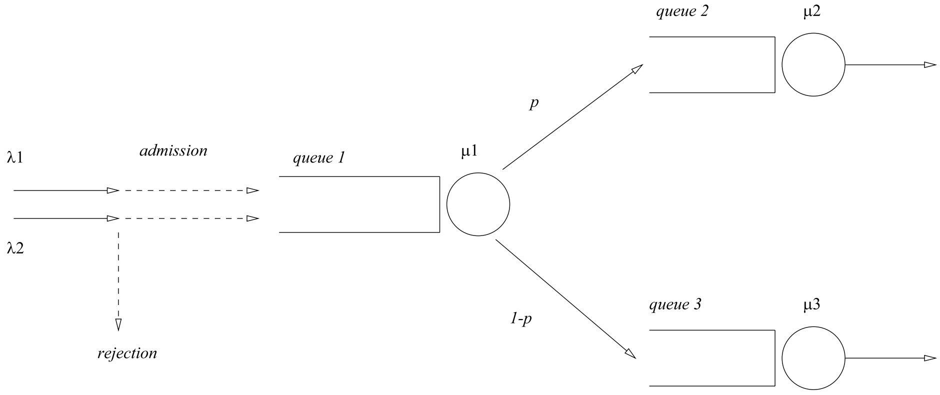

FIGURE 1. Illustration of basic system.

3.1. DESCRIPTION AND FORMULATION OF PROBLEM

The evolution of the system is influenced by a possible time-varying state-dependent control with values $z = (z^1, z^2) \in \{0,1\}^2$.

Two types of packets arrive to queue 1 in two Poisson streams of rates λ_1 and λ_2 (Fig. 1). Packets (of variable length) are served at station 1 by an exponential server at rate μ_1 and they routed to station 2 with probability p and to station 3 with probability $1-p$. Packets are served at station 2 and 3 by two exponential servers at rates μ_2 and μ_3 respectively. After being served by one of these servers a packet departs from the system.

An input controller at queue 1 has the option of refusing entry to each packet, based on the size of the three queues at the arrival time of each packet.

However, a cost ξ_1 (respectively, ξ_2) must be paid for each rejection of packet of type 1 (respectively type 2), such that $\xi_2 < \xi_1$. Rejected packets are assumed lost.

The state of the system at time t, $t \geq 0$, is defined by a stochastic process $(x_t, t \geq 0)$, describing the evolution of the total load of the system, where $x_t = (x_t^1, x_t^2, x_t^3)$ takes values in the state space $S = \mathbb{N}^3$ and x_t^i is the total number of packets in queue i (including the packet in the server).

With each state x in S we associate a set of admissible actions $D = \{0,1\}^2$. Thus, an admissible action $z_t(x)$ in state x at time t, with values in D will have the form

$$z_t(x) = (z_t^1(x), z_t^2(x))$$

where $z_t^i = 1$ or 0 according to whether an arriving packet of type i is accepted into the first queue or is rejected (and lost).

Defining the action space to be the set $A = D^S$, we can now represent an admissible Control Strategy (CS) as an A-valued stochastic process $(z_t, t \geq 0)$, where $z_t = (z_t(x), x \in S)$.

Hereafter, we shall use the abbreviated notation z for the CS $(z_t, t \geq 0)$. Let P denote the set of all admissible control strategies. A law of motion corresponding to a CS z is specified by a transition probability $P(y|x, z_t)$, $x, y \in S$, $t \geq 0$, denoting the conditional probability that the system moves to state y at time t^+ when the action $z_t(x)$ is applied to it at time t while in state x.

Our objective is to find a CS z in P minimizing the following discounted cost:

$$(P1) \qquad \limsup_{T\to\infty} E_x^z \left[\int_0^T \mathrm{e}^{-\alpha t} \left(\sum_{i=1}^{3} c_i x_t^i + \sum_{i=1}^{2} \lambda_i \xi_i 1\!\!1_{\{z_t^i = 0\}} \right) \mathrm{d}t \right]$$

where $\alpha > 0$ and $c_i \in \mathbb{R}_+^*$, $i = 1, 2, 3,$ such that $c_2 < c_1$ and $c_3 < c_1$.

If such a minimizing CS exists, it will be called the "optimal strategy" for the discounted cost problem $(P1)$.

Now, we introduce two special classes of relevant CS's.

An admissible CS which is an independent identically distributed (i.i.d) stochastic process will be called a Stationary Randomized Strategy (SRS). Furthermore, if the common distribution of SRS z has all its mass concentrated at some point in A, we shall refer to it as a Stationary Strategy (SS). Let $P_s \subset P$ denote the set of all SS's.

For our problem, we assert from Lippman [18] (p. 1228) and Walrand [26] (p. 275) that an optimal CS exists which, furthermore, is stationary.

The inter-arrival and inter-departure times of the packets are exponentially distributed. Furthermore, the action set D is finite. The assumptions of [18] (Th. 1, p. 1229) and [26] (Prop. 8.5.3, p. 275) are therby satisfied, leading to our assertion above.

Hereafter, we replace $z_t(x_t)$ by z_t for notational convenience. Furthermore, in view of our previous assertion, we restrict attention to stationary CS's and define the α-discounted cost starting with initial state x associated with the problem (P_1) by:

$$J^\alpha(x) = \min_{z\in P_s} E_x^z \left[\int_0^\infty \mathrm{e}^{-\alpha t} \left(\sum_{i=1}^{3} c_i x_t^i + \sum_{i=1}^{2} \lambda_i \xi_i 1\!\!1_{\{z_t^i = 0\}} \right) \mathrm{d}t \right] \cdot \tag{1}$$

The minimum cost in (1) can be expressed in an alternative form which facilitates further analysis. Let $0 = t_0 < t_1 < ... < t_n...$ be the (random) instants in time denoting transition epochs of the system state $(x_t, t \geq 0)$, where each transition epoch represents an arrival of a packet into the system, or a transfer of a packet from queue 1 to queue 2, or a transfer of a packet from queue 1 to queue 3, or a departure of a packet from the system.

It is convenient to introduce at this point the α-discounted expected cost over the time-horizon $[0, t_n)$, with initial state x, and following a control strategy z

in P_s, namely,

$$V_n^\alpha(x,z) = E_x^z\left[\int_0^{t_n} \mathrm{e}^{-\alpha t}\left(\sum_{i=1}^{3} c_i x_t^i + \sum_{i=1}^{2}\lambda_i\xi_i 1\!\!1_{\{z_t^i=0\}}\right)\mathrm{d}t\right]. \qquad (2)$$

Let

$$J_n^\alpha(x) = \min_{z\in P_s} V_n^\alpha(x,z), \qquad n=0,1,...$$
$$J_\infty^\alpha(x) = \lim_{n\to\infty} J_n^\alpha(x).$$

In the same way as [14], we can prove that the minimum cost in (1) has the alternative expression:

$$J^\alpha(x) = J_\infty^\alpha(x) \qquad (3)$$

for every initial state x.

3.2. The equivalent discrete-time problem

In this section we convert the original continuous-time problem (P_1) into its discrete-time equivalent by the standard procedure of "uniformization" [9, 13, 20].

As previously, $0 = t_0 < t_1 < ... < t_n...$ are the (random) instants in time denoting transition epochs of the system state. By suitably introducing dummy departures as in [17, 20], the inter-epoch intervals are seen to be i.i.d. random variables with distribution:

$$Pr[t_{k+1} - t_k > t] = \mathrm{e}^{-t(\lambda_1+\lambda_2+\mu_1+\mu_2+\mu_3)}$$

for $k = 0, 1...$

Consider the discrete time system obtained as in [13,20] by sampling the original continuous-time system at its transition epochs. To this end, we introduce the notation $x_k \triangleq x_{t_k}$ and $z_k \triangleq z(x_{t_k})$ and define:

$$\beta = \frac{\lambda_1+\lambda_2+\mu_1+\mu_2+\mu_3}{\lambda_1+\lambda_2+\mu_1+\mu_2+\mu_3+\alpha}$$

hence $0 < \beta < 1$. We can then conveniently convert the continous time optimization problem into an equivalent discrete time problem.

The β-discounted cost incurred by the n-step discrete time system for the CS z is defined [13, 20] as:

$$\widetilde{V}_n^\beta(x,z) \triangleq E_x^z\left[\sum_{k=0}^{n-1}\beta^k\left(\sum_{i=1}^{3} c_i x_k^i + \sum_{i=1}^{2}\lambda_i\xi_i 1\!\!1_{\{z_k^i=0\}}\right)\right].$$

It then follows as in [13, 20] that

$$V_n^\alpha(x,z) = \frac{1-\beta}{\alpha}\widetilde{V}_n^\beta(x,z). \qquad (4)$$

Let

$$\widetilde{V}^{\beta}(x,z) \triangleq \lim_{n\to\infty} \widetilde{V}_n^{\beta}(x,z).$$

We can now state the minimization problem (P1) in terms of a discrete-time problem of equivalent cost as follows. Define the minimum β-discounted cost for the n-step and infinite horizon discrete-time systems, respectively, by:

$$\widetilde{J}_n^{\beta}(x) \triangleq \min_{z\in P} \widetilde{V}_n^{\beta}(x,z) \tag{5}$$

and

$$\widetilde{J}^{\beta}(x) \triangleq \min_{z\in P} \widetilde{V}^{\beta}(x,z). \tag{6}$$

Letting

$$\widetilde{J}_{\infty}^{\beta}(x) \triangleq \lim_{n\to\infty} \widetilde{J}_n^{\beta}(x). \tag{7}$$

As in previous section, we have

$$\widetilde{J}_{\infty}^{\beta}(x) = \widetilde{J}^{\beta}(x)$$

for every initial state x.

Finally, the equivalence, in the sense of optimal discounted cost, between (P1) and the discrete-time formulation above follows readily from (4) and (7) by noting that:

$$J^{\alpha}(x) = \frac{1-\beta}{\alpha}\widetilde{J}^{\beta}(x).$$

Thus, we may restrict our attention hereafter to the discrete-time β-discounted cost problem defined by (6).

We can now proceed to develop the dynamic programming equation for the problem in (6).

Walrand [26] (Prop. 8.5.3, p. 275), proves that $\widetilde{J}^{\beta}(x)$ is the unique bounded solution of the following dynamic programming equation:

$$\widetilde{J}^{\beta}(x) = \min_{z\in P}\left\{ c(x,z) + \beta \sum_y P(y|x,z)\widetilde{J}^{\beta}(y) \right\}. \tag{8}$$

Where $c(x,z)$ is the instantaneous cost and $P(y|x,z)$ is the conditional probability (for the discrete-time problem) that the system moves to state y at time $n+1$ when the action $z(x)$ is applied to it at time n while in state x.

Then $\widetilde{J}^{\beta}$ is characterized by the dynamic programming optimality equation $\widetilde{J}^{\beta} = T\widetilde{J}^{\beta}$, where T is the operator defined on real-valued function on S by:

$$\begin{aligned} &Tf(x) = cx + (\alpha+\gamma)^{-1}.\beta\{\mu_2 f(D_2x) + \mu_3 f(D_3x) + p\mu_1 f(R_{12}x) \\ &+ (1-p)\mu_1 f(R_{13}x) + \lambda_1 \min\{f(A_1x), f(x)+\xi_1\} + \lambda_2 \min\{f(A_1x), f(x)+\xi_2\}\} \end{aligned}$$

where $\gamma = \lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3$, $x = (x_1, x_2, x_3) \in S$, $cx = \sum_{i=1}^{3} c_i x_i$, $c_i \in \mathbb{R}_+^*$, $D_2 x = (x_1, (x_2 - 1)^+, x_3)$, $D_3 x = (x_1, x_2, (x_3 - 1)^+)$, $A_1 x = (x_1 + 1, x_2, x_3)$, if $x_1 \geq 1$ then $R_{12} x = (x_1 - 1, x_2 + 1, x_3)$ and $R_{13} x = (x_1 - 1, x_2, x_3 + 1)$ else $R_{12} x = R_{13} x = x$, $x^+ = \max(0, x)$,

For convenience we assume: $\alpha + \gamma = 1$.

3.3. Structural properties for the optimal policies

In the same proposition that cited above, Walrand [26] provides a method for deriving structural properties of optimal policies.

Assume that the hypothesis of Theorem 1 (p. 1229) in [18] hold and that H is a set of functions from S into $\mathbb{R}$ that contains the function that is identically zero and is such that:

$$f \in H \Longrightarrow g(x) := \min_{z \in P} \left\{ c(x,z) + \beta \sum_{y \in S} P(y|x,z) f(y) \right\} \in H$$

and H is closed under pointwise limits (*i.e.*, if $f_n \in H$ for all n and $f_n(x) \longrightarrow f(x)$ for all $x \in S$, then $f \in H$). Then $\widetilde{J}^\beta \in H$. We are using this result to find properties of the optimal policies for our problem. In this way properties of $\widetilde{J}_n^\beta$ carry to $\widetilde{J}^\beta$.

We have the following properties:

Property 1. *If it is optimal to reject a packet (of type 1 or type 2) in state* $x = (x_1, x_2, x_3)$ *then it is also optimal to reject it in states* (x_1+1, x_2, x_3), (x_1, x_2+1, x_3) *and* $(x_1, x_2, x_3 + 1)$.

i.e. if $f(x) - f(A_1 x) < -\xi_i$ then $f(A_j x) - f(A_1 A_j x) < -\xi_i$ for $i = 1, 2$ and $j = 1, 2, 3$

i.e.

$$f(A_j x) - f(A_1 A_j x) \leq f(x) - f(A_1 x) \quad \text{for } j = 1, 2, 3 \tag{9}$$

where A_j corresponds to an arrival at station j (j = 1,2,3).

Property 2. *If it is optimal to reject a packet (of type 1 or type 2) in state* $x = (x_1, x_2 + 1, x_3)$ *or in state* $x = (x_1, x_2, x_3 + 1)$ *then it is also optimal to reject it in state* $x = (x_1 + 1, x_2, x_3)$.

i.e. if $f(A_2 x) - f(A_1 A_2 x) < -\xi_i$ or $f(A_3 x) - f(A_1 A_3 x) < -\xi_i$ then $f(A_1 x) - f(A_1 A_1 x) \leq -\xi_i$

i.e.

$$f(A_1 x) - f(A_1 A_1 x) \leq f(A_2 x) - f(A_1 A_2 x)$$

$$f(A_1 x) - f(A_1 A_1 x) \leq f(A_3 x) - f(A_1 A_3 x)$$

i.e.

$$f(x) - f(A_1 x) \leq f(R_{12} x) - f(A_1 R_{12} x) \tag{10}$$

$$f(x) - f(A_1x) \leq f(R_{13}x) - f(A_1R_{13}x) \tag{11}$$

for all $x \in \mathbb{N}^3$, with $x_1 > 0$.

Property 1 is equivalent to saying that the rejection region R for each type of packet is an increasing set, that is (x_1+1, x_2, x_3), (x_1, x_2+1, x_3) and (x_1, x_2, x_3+1) belong to R whenever (x_1, x_2, x_3) does. This is a generalization to three dimension of the monotonicity of optimal policies for optimal control of admission to a single queueing facility and two dimensions.

Property 2 is a generalization to three dimension of two-dimensional control problems, it says that moving one of the existing packets from queue 1 to queue 2 or queue 3 makes it more likely that a new packet (of type 1 or type 2) will be admitted.

For the sake of notation, we will denote $\widetilde{J}_n^\beta$ and $\widetilde{J}^\beta$ respectively by J_n and J.

Let H be the set of functions J_n defined by (5) such that:

1) J_n is increasing in x_1, x_2 and x_3;
2) J_n obeys the inequalities (9, 10) and (11) above.

Then the function g_n defined by:

$$g_n(x) = TJ_n$$

belongs to H whenever J_n does.

$$\begin{aligned} TJ_n(x) = {} & cx + \beta\{\mu_2 J_n(D_2x) + \mu_3 J_n(D_3x) + p\mu_1 J_n(R_{12}x) + (1-p)\mu_1 J_n(R_{13}x) \\ & + \lambda_1 \min\{J_n(A_1x), J_n(x) + \xi_1\} + \lambda_2 \min\{J_n(A_1x), J_n(x) + \xi_2\}\}. \end{aligned}$$

We assume that: $J_0(.) = 0$.

We shall prove this result in the Section 3.5.

3.4. Characterization of the optimal policies

In [26], the Proposition 8.5.3 (p. 275) assures the existence of the limit in (7) such that J is the unique bounded solution of the equation $TJ(x) = J(x)$ and $J(x) \in H$ for all $x \in S$.

The optimal policy is given by part (d) of the same proposition. It is the function $z = (z^1, z^2)$ such that each z^i achieves the minimum in the equation (8).

That is,

$$z^i(x) = \begin{cases} 1 & \text{if } J(A_1x) < J(x) + \xi_i \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

for $i = 1, 2$.

According to the properties of the optimal policies, there are two switching surfaces one for each type of arrival. These surfaces divide the state space into two regions, one of which relates to packet rejection and the other to packet admission.

The structure of these surfaces is illustrated in the Figure 2 in which z represents z^i $(i = 1, 2)$ and the rejection region is defined by the following set:

$$\{x \in S \ / \ z(x) = 0\}\cdot$$

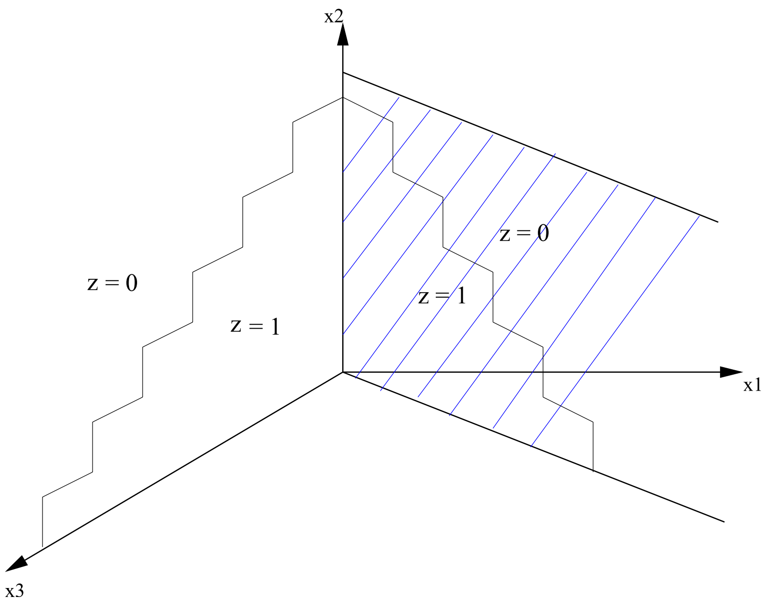

FIGURE 2. Illustration of the optimal policy.

3.5. PROOF OF STRUCTURAL PROPERTIES

Remark 1. 1) It is easy to see that the function:

$$TJ_0 : x \longmapsto cx$$

belongs to H.

2) All products of a function that belongs to H with a positive number is also in H, as well as all sum of functions of H.

Now, we are going to prove the assertions 1 and 2 given in the Section 2.3.

Assume that $J_n \in H$.

In the expression of $TJ_n(x) - TJ_n(A_1x)$, the terms which the coefficients are μ_2, μ_3, $p\mu_1$ and $(1-p)\mu_1$ belong in H (by using the above remark and hypothesis).

It is easy to see that the function TJ_n is increasing in x_1, x_2 and x_3 whenever J_n does.

Prove that TJ_n satisfies the inequality (9) for $j=1$ whenever J_n verifies this inequality for $j=1$.

We need only to prove that:

$$\min\{J_n(A_1x)+\xi_i, J_n(A_1^2x)\} - \min\{J_n(x)+\xi_i, J_n(A_1x)\} \\ \leq \min\{J_n(A_1^2x)+\xi_i, J_n(A_1^3x)\} - \min\{J_n(A_1x)+\xi_i, J_n(A_1^2x)\}$$

for $i=1,2$, *i.e.*, we need to prove an inequality like:

$$\min\{p_1,p_2\} - \min\{q_1,q_2\} \leq \min\{r_1,r_2\} - \min\{s_1,s_2\}\cdot \tag{13}$$

To prove this inequality we must distinguish sixteen cases, but by the following remark, we can only study four cases.

If we prove that:

$$p_i - \min\{q_1,q_2\} \leq \min\{r_1,r_2\} - s_j$$

for one i and one j, then we will have the inequality (13), because:

$$p_i \geq \min\{p_1,p_2\} \quad \text{and} \quad s_j \geq \min\{s_1,s_2\}\cdot$$

Put: $p_1 = J_n(A_1x)+\xi_i$, $p_2 = J_n(A_1^2x)$, $q_1 = J_n(x)+\xi_i$, $q_2 = J_n(A_1x)$, $r_1 = J_n(A_1^2x)+\xi_i$, $r_2 = J_n(A_1^3x)$, $s_1 = J_n(A_1x)+\xi_i$, $s_2 = J_n(A_1^2x)$.

For the case q_1, r_1 we choose p_1 and s_1 and for the case q_2, r_2 we choose p_2 and s_2, because the inequalities:

$$p_1 - q_1 \leq r_1 - s_1 \quad \text{and} \quad p_2 - q_2 \leq r_2 - s_2$$

are satisfied by hypothesis.

We need only to prove the inequality when $\min\{q_1,q_2\}=q_1$ and $\min\{r_1,r_2\}=r_2$ or when $\min\{q_1,q_2\}=q_2$ and $\min\{r_1,r_2\}=r_1$.

Case q_1, r_2

We choose p_2 and s_1.

Prove that: $p_2 - q_1 \leq r_2 - s_1$

i.e. $J_n(A_1^2x) - (J_n(x)+\xi_i) \leq J_n(A_1^3x) - (J_n(A_1x)+\xi_i)$ for $i=1,2$

i.e. $J_n(A_1^2x) - J_n(x) \leq J_n(A_1^3x) - J_n(A_1x)$

i.e. $J_n(A_1x) - J_n(x) \leq J_n(A_1^3x) - J_n(A_1^2x)$

which is true by hypothesis.

Case q_2, r_1

We choose p_1 and s_2.

Prove that: $p_1 - q_2 \leq r_1 - s_2$

i.e. $(J_n(A_1x)+\xi_i) - J_n(A_1x) \leq (J_n(A_1^2x)+\xi_i) - J_n(A_1^2x)$ for $i=1,2$

i.e. $\xi_i \leq \xi_i$ for $i=1,2$

which is true.

Conclusion. TJ_n satisfies the inequalitiy (9) for $j = 1$.

Prove that TJ_n satisfies the inequality (9) for $j = 2$ whenever J_n verifies this inequality for $j = 2$.

We need only to prove that:

$$\min\{J_n(A_1x) + \xi_i, J_n(A_1^2x)\} - \min\{J_n(x) + \xi_i, J_n(A_1x)\} \\ \leq \min\{J_n(A_1A_2x) + \xi_i, J_n(A_1^2A_2x)\} - \min\{J_n(A_2x) + \xi_i, J_n(A_1A_2x)\}$$

for $i = 1, 2$.

Put: $p_1 = J_n(A_1x) + \xi_i$, $p_2 = J_n(A_1^2x)$, $q_1 = J_n(x) + \xi_i$, $q_2 = J_n(A_1x)$, $r_1 = J_n(A_1A_2x) + \xi_i$, $r_2 = J_n(A_1^2A_2x)$, $s_1 = J_n(A_2x) + \xi_i$, $s_2 = J_n(A_1A_2x)$.

For the case q_1, r_1 we choose p_1 and s_1 and for the case q_2, r_2 we choose p_2 and s_2, because the inequalities:

$$p_1 - q_1 \leq r_1 - s_1 \quad \text{and} \quad p_2 - q_2 \leq r_2 - s_2$$

are satisfied by hypothesis.

Case q_1, r_2

We choose p_1 and s_2.

Prove that: $p_1 - q_1 \leq r_2 - s_2$

i.e. $(J_n(A_1x) + \xi_i) - (J_n(x) + \xi_i) \leq J_n(A_1^2A_2x) - J_n(A_1A_2x)$ for $i = 1, 2$

i.e. $J_n(A_1x) - J_n(x) \leq J_n(A_1^2A_2x) - J_n(A_1A_2x)$

which is true because $J_n(A_1x) - J_n(x)$ is increasing in x_1 and in x_2.

Case q_2, r_1

We choose p_1 and s_2.

Prove that: $p_1 - q_2 \leq r_1 - s_2$

i.e. $(J_n(A_1x) + \xi_i) - J_n(A_1x) \leq (J_n(A_1A_2x) + \xi_i) - J_n(A_1A_2x)$ for $i = 1, 2$

i.e. $\xi_i \leq \xi_i$ for $i = 1, 2$

which is true.

Conclusion. TJ_n satisfies the inequalitiy (9) for $j = 2$.

We can prove that TJ_n satisfies the inequality (9) for $j = 3$ whenever J_n verifies this inequality for $j = 3$ in the same way that for $j = 2$.

Prove that TJ_n satisfies the inequality (10) whenever J_n verifies this inequality, for all $x \in \mathbb{N}^3$, with $x_1 > 0$.

We prove that:

$$\min\{J_n(A_1R_{12}x) + \xi_i, J_n(A_1^2R_{12}x)\} - \min\{J_n(R_{12}x) + \xi_i, J_n(A_1R_{12}x)\} \\ \leq \min\{J_n(A_1x) + \xi_i, J_n(A_1^2x)\} - \min\{J_n(x) + \xi_i, J_n(A_1x)\}$$

for $i = 1, 2$.

Put: $p_1 = J_n(A_1R_{12}x) + \xi_i$, $p_2 = J_n(A_1^2R_{12}x)$, $q_1 = J_n(R_{12}x) + \xi_i$, $q_2 = J_n(A_1R_{12}x)$, $r_1 = J_n(A_1x) + \xi_i$, $r_2 = J_n(A_1^2x)$, $s_1 = J_n(x) + \xi_i$, $s_2 = J_n(A_1x)$.

For the case q_1, r_1 we choose p_1 and s_1 and for the case q_2, r_2 we choose p_2 and s_2, because the inequalities:

$$p_1 - q_1 \leq r_1 - s_1 \quad \text{and} \quad p_2 - q_2 \leq r_2 - s_2$$

are satisfied by hypothesis.

Case q_1, r_2
We choose p_1 and s_2.
Prove that: $p_1 - q_1 \leq r_2 - s_2$
i.e. $(J_n(A_1R_{12}x) + \xi_i) - (J_n(R_{12}x) + \xi_i) \leq J_n(A_1^2x) - J_n(A_1x)$ for $i = 1, 2$
i.e. $J_n(A_1R_{12}x) - J_n(R_{12}x) \leq J_n(A_1^2x) - J_n(A_1x)$
which is true because:

$$\begin{aligned} J_n(A_1R_{12}x) - J_n(R_{12}x) &\leq J_n(A_1x) - J_n(x) \\ &\leq J_n(A_1^2x) - J_n(A_1x) \end{aligned}$$

these inequalities are true by hypothesis.

Case q_2, r_1
We choose p_1 and s_2.
Prove that: $p_1 - q_2 \leq r_1 - s_2$
i.e. $(J_n(A_1R_{12}x) + \xi_i) - J_n(A_1R_{12}x) \leq (J_n(A_1x) + \xi_i) - J_n(A_1x)$ for $i = 1, 2$
i.e. $\xi_i \leq \xi_i$ for $i = 1, 2$
which is true.

Conclusion. TJ_n satisfies the inequalitiy (10).

We can prove that TJ_n satisfies the inequality (11) whenever J_n verifies this inequality in the same way that we proved the inequality (10).

Thus $TJ_n \in H$ for all $n \in \mathbb{N}$.

3.6. Conclusion

We proved that there are two optimal admission control policies which are "switching surfaces", one for each type of packets. The switching surfaces divide the state space into two regions, one of which relates to packet rejection and the other to packet admission. This result generalizes the result found for a system consisting of two queues in series with the rejection option and for which the optimal policy is also expressed in terms of a "switching curve".

We also proved that if the rejection costs decrease or the load of the system increases, the rejection regions increase. Consequently, the rejection region for packets of type one is included in the rejection region for packets of type two because $\xi_2 < \xi_1$, which means that we accept more packets of type one in the network than those of type two.

4. Optimal control of $m+1$ interacting queues with n-types of arrivals

In this section, we generalize the previous problem by considering n-types of packets and $m+1$ queues.

The packets of type i arrive in queue 1 in a Poisson stream with constant rate λ_i $(i = 1, ..., n)$. The packets are served at this queue by an exponential server at rate μ_1 and they routed to queue j with probability p_{j-1} $(j = 2, ..., m+1)$, the queues $2, 3, ..., m+1$ are parallel and they have exponential servers at rates $\mu_2, \mu_3, ..., \mu_{m+1}$ respectively.

As before, denote by ξ_i the rejection cost of packets of type i.

We have: $\displaystyle\sum_{j=1}^{m} p_j = 1$ and $\xi_i < \xi_j$ for $1 \leq i < j \leq n$.

The state space is: $S = \mathbb{N}^{m+1}$, the admissible action space is: $D = \{0,1\}^n$, and the instantaneous cost function is defined by:

$$c(x_t, z_t) = \sum_{i=1}^{m+1} c_i x_t^i + \sum_{i=1}^{n} \lambda_i \xi_i \mathbb{1}_{\{z_t^i = 0\}} \tag{14}$$

where $x_t = (x_t^1, x_t^2, ..., x_t^{m+1})$, $z_t = (z_t^1, z_t^2, ..., z_t^n)$, and $c_1, c_2, ..., c_{m+1} \in \mathbb{R}_+^*$ (such that $c_j < c_1$, for $j = 2, ..., m+1$) with

$$z_t^i = \begin{cases} 1 & \text{if at time } t \text{ a packet of type } i \text{ is accepted into the first queue} \\ 0 & \text{otherwise} \end{cases}$$

and $x_t^i =$ is the total number of packets in the queue i (including the packet in the server).

The objective is to find an optimal control z minimizing the following discounted cost:

$$(P2) \qquad \limsup_{T\to\infty} E_x^z \left[\int_0^T \mathrm{e}^{-\alpha t} c(x_t, z_t) \mathrm{d}t \right]$$

where $\alpha \in \mathbb{R}_+^*$.

Let A_1 corresponds to an arrival at station 1, let R_{1j} corresponds to a potential transfer from queue 1 to queue j and let D_j corresponds to a potential departure at station j, with $j = 2, 3, ...m+1$.

We use the same notations that the first case and we only replace in the previous formulas the cost function by the cost defined in (14).

Let H be the set of functions $\widetilde{J}_n^\beta$ (or J_n if there is no ambiguity) defined as (5) and verify the following properties:

1) J_n is increasing in x_1, x_2,..., x_{m+1};
2) $J_n(A_1 x) - J_n(x) \leq J_n(A_1 A_j x) - J_n(A_j x)$, for $j = 1, 2, ..., m+1$;
3) $J_n(A_1 R_{1j} x) - J_n(R_{1j} x) \leq J_n(A_1 x) - J_n(x)$, for $j = 2, ..., m+1$, and for all $x \in \mathbb{N}^{m+1}$, with $x_1 > 0$.

We assume: $J_0(.) = 0$.

Property 1 says that the rejection region for each type of packets is an increasing set.

Property 2 says that moving a packet from queue 1 to queue j ($j = 2, 3, ..., m+1$) makes it more likely that a new packet will be admitted.

As the previous problem, we can prove that the function TJ_n belongs to H whenever J_n does, where

$$TJ_n = \sum_{i=1}^{m+1} c_i x_i + \beta(\alpha+\gamma)^{-1} \left\{ \sum_{j=2}^{m+1} \mu_j J_n(D_j x) \right.$$
$$\left. + \mu_1 \sum_{j=2}^{m+1} p_{j-1} J_n(R_{1j} x) + \sum_{i=1}^{n} \lambda_i . \min\{J_n(x) + \xi_i, J_n(A_1 x)\} \right\}$$

with

$$\beta = \frac{\sum_{i=1}^{n} \lambda_i + \sum_{j=1}^{m+1} \mu_j}{\alpha + \sum_{i=1}^{n} \lambda_i + \sum_{j=1}^{m+1} \mu_j}$$

and

$$\gamma = \sum_{i=1}^{n} \lambda_i + \sum_{j=1}^{m+1} \mu_j .$$

By using the Proposition 8.5.3 [26], we have also $J(x) \in H$, for all $x \in S$, and the optimal policy is the function $z = (z^1, z^2, ..., z^n)$ defined in a manner same as (12) for $i = 1, 2, ..., n$.

For all $x_2, ..., x_{m+1} \in \mathbb{N}$ (number of packets respectively in the queues $2, 3, ..., m+1$) and for all type of packets, the arrive packet will be rejected when the first queue reaches some threshold. This threshold is defined by:

$$w^i(x_2, x_3, ..., x_{m+1}) = \max\{x_1 \in \mathbb{N} / J(x_1+1, x_2, ..., x_{m+1}) < J(x_1, x_2, ..., x_{m+1}) + \xi_i\}$$

for $i = 1, 2, ..., n$.

It is easy to see that w^i, $i = 1, 2, ..., n$, is decreasing in $x_2, ..., x_{m+1}$.

For all $x_2, ..., x_{m+1} \in \mathbb{N}$, let

$$R_i(x_2, ..., x_{m+1}) = \{M(x_1, x_2, ..., x_{m+1}) \in P_n / x_1 > w^i(x_2, ..., x_{m+1})\}, \text{ for } i = 1, ..., n$$

where P_n is the affine space of n-dimentional and $M(x_1, x_2, ..., x_{m+1})$ is the point of P_n having $(x_1, ..., x_{m+1})$ as components.

The rejection region for packets of type i ($i = 1, 2, ..., n$) is defined as follows:

$$R_i = \bigcup_{x_2, x_3, ..., x_{m+1}} R_i(x_2, ..., x_{m+1})$$

and $R_n \subseteq R_{n-1} \subseteq ... \subseteq R_1$, since $\xi_i < \xi_j$ for $i < j$, which means that the rejection region increases when the rejection cost decreases.

4.1. Conclusion

We showed that there are n optimal admission control policies, one for each type of packets. We also established that these optimal policies are monotonic, *i.e.*, when the rejection costs decrease or the load of the system increases, the optimal rejection regions increase. Thus the rejection region for packets of type j is included in the rejection region for packets of type i if $1 \leq i < j \leq n$, because $\xi_i < \xi_j$, for $1 \leq i < j \leq n$, which means that we accept more packets of type j in the network than those of type i.

The mechanism of control studied in this paper may be considered for implementation in Internet routers in order to avoid packet congestion and achieve acceptable QoS.

References

[1] R. Braden, *Requirements for Internet Hosts Communication Layers*. STD 3, RFC 1122 (1989).

[2] I. Christidou, I. Lambadaris and R. Mazumdar, Optimal Control of Arrivals to a Feedback Queueing System, in *Proc. of the 27th CDC*. Austin, Texas (1988) 663-667.

[3] E. Davis, *Optimal Control of Arrivals to a Two-server Queueing System with Separate Queues*, Ph.D. Dissertation, Program in Operations Research. N.Y. State University, Raleigh (1977).

[4] A. Ephremides, P. Varaiya and J. Walrand, A Simple Dynamic Routing Problem. *IEEE Trans. Automat. Control* **25** (1980) 690-693.

[5] W. Farrell, *Optimal Switching Policies in a Nonhomogeneous Exponential Queueing System*, Ph.D. Dissertation. Graduate School of Management, University of California, Los Angeles (1976).

[6] S. Floyd and V. Jacobson, Random Early Detection Gateways for Congestion Avoidance. *IEEE/ACM Trans. Networking* (1993).

[7] H. Ghoneim, *Optimal Control of Arrivals to a Network of Two Queues in Series*, Ph.D. Dissertation, Program in Operations Research. North Carolina State University, Raleigh (1980).

[8] H. Ghoneim and S. Stidham, Optimal Control of Arrivals to Two Queues in Series. *Eur. J. Oper. Res.* **21** (1985) 399-409.

[9] B. Hajek, Optimal Control of Two Interacting Service Stations. *IEEE Trans. Automat. Control* **29** (1984) 491-499.

[10] A. Haqiq and N. Mikou, Contrôle Dynamique de Flux dans un Système d'Attente avec Panne. *RAIRO: Oper. Res.* **33** (1999) 69-86.

[11] A. Haqiq, Admission and routing dynamic control in a queueing system with breakdown, in *Troisième Conférence Internationale en Recheche Opérationnelle*. Marrakech, Maroc (2002).

[12] V. Jacobson, *Congestion Avoidance and Control*. ACM SIGCOMM (1988).

[13] P.R. Kumar and P. Varaiya, *Stochastic Systems: Estimation, Identification and Adaptive Control*. Prentice Hall (1986).

[14] I. Lambadaris and P. Narayan, Jointly Optimal Admission and Routing Controls at a Network Node. *Commun. Statist. Stochastic Models* **10** (1994) 223-252.

[15] I. Lambadaris, P. Narayan and I. Viniotis, Optimal Service Allocation among Two Heterogeneous Traffic Types at a Network Node with no Queueing, in *Proc. of the 26th CDC*. Los Angeles, Vol. CA (1987) 1496-1498.
[16] A. Lazar, Optimal Flow Control of a Class of Queueing Networks in Equilibrium. *IEEE Trans. Automat. Control* **28** (1983) 1001-1007.
[17] S. Lippman, Applying a New Device in the Optimization of Exponential Queueing Systems. *Oper. Res.* **23** (1975) 687-710.
[18] S. Lippman, On Dynamic Programming with Unbounded Rewards. *Management Sci.* **21** (1975) 1225-1233.
[19] J. Nagle, *Congestion Control in IP/TCP*. RFC 896 (1984).
[20] Z. Rosberg, P. Varaiya and J. Walrand, Optimal Control of Service in Tandem Queues. *IEEE Trans. Automat. Control* **AC-27** (1982) 600-610.
[21] W. Stevens, *TCP Slow Start, Congestion Avoidance, Fast Retransmit, and Fast Recovery Algorithms*. RFC 2001 (1997).
[22] S. Stidham, Optimal Control of Admission to a Queueing System. *IEEE Trans. Automat. Control* **AC-30** (1985) 705-713.
[23] S. Stidham and R. Weber, *Control of Service Rates in Cycles and Series of Queues*. Stat. Lab., Univ. Cambridge, Cambridge (1983).
[24] P. Varaiya, *Note on Optimization*. Van Nostrand Reinhold, New York (1972).
[25] I. Viniotis and A. Ephremides, Optimal Switching of Voice and Data at a Network Node, in *Proc. of the 26th CDC*, Vol. CA. Los Angeles (1987) 1504-1507.
[26] J. Walrand, *An Introduction to Queueing Networks*. Prentice Hall International Editions, Englwood Cliffs, New Jersey (1988).
[27] R. Weber, On the Optimal Assignment of Customers to Parallel Servers. *J. Appl. Probab.* **15** (1978) 406-413.
[28] W. Winston, Optimalily of the Shortest-Processing-Time Discipline. *J. Appl. Probab.* **14** (1977) 181-189.

To access this journal online:
www.edpsciences.org