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Global Homeomorphism Theorem for Manifolds and Polyhedra (*).

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ABSTRACT - We improved a version of Global Homeomorphism Theorem due to Katriel such that it can be applied to more general geometric objects: Hilbert Riemannian manifolds, graphs, and polyhedra.

1. Introduction.

Hadamard's Global Homeomorphism Theorem (GHT) is concerned with a differential mapping F between Banach spaces X and Y, in which, the speed of decay of the function $\|F'(x)^{-1}\|^{-1}$ plays an important role. In fact, he (1906) only studied finite dimensional case. Extensions were made by many authors, e.g., P. Levy (1920), Caccioppoli (1932), Banach-Mazur (1934), etc. It would be natural to ask whether this may be extended to other geometric objects, or the differentiability of the map F may be removed. We should mention the pioneer works of F. Browder [Br], Prodi-Ambrosetti [PA], R. Plastock [Pl], and A. D. Ioffe [Io]. In [Ka], Katriel generalized the result to certain continuous mappings between metric spaces. The main point in his approach is to use the surjection constant due to Ioffe in [Io]. However, in [Ka], the metric space Y is assumed to be «nice». Although this notion includes Banach spaces, and the unit sphere of a Hilbert space as special examples, it is very restrictive. The purpose of this paper is to improve the version of GHT in [Ka], so that it can be applied to more general geometric objects. Following

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Browder and Plastock, our idea returns to show that F is a covering map. The crucial point is to verify a kind of weak properness of F. The new ingredient in the verification relies on a recent deformation lemma for continuous functions on metric spaces due to Corvellec [Co]. Accordingly, the restriction on Y is considerably reduced. Not only all Hilbert-Riemannian manifolds but also infinite graphs as well as the abstract polyhedra are included.

Before going to state our main result, we introduce the necessary notations and terminologies. Let (X, ϱ) be a metric space, $g : X \to R^1$ be a continuous function. $\forall x \in X$, $\forall \delta > 0$, a continuous map
$H : B_\delta(x) \times [0, \delta) \to X$ is called admissible, if it satisfies

$$\varrho(H(y, t), y) \leqslant t, \quad \forall (y, t) \in B_\delta(x) \times [0, \delta).$$

One (cf. J. N. Corvellec, M. DeGiovanni, and M. Marzocchi [CDM]) defines the weak slope of g at x to be

$$|dg|(x) = \sup \{\sigma \in [0, +\infty) \mid \exists \delta > 0, \exists H : B_\delta(x) \times [0, \delta) \to X, \text{ s.t.}$$
$$g(H(y, t)) \leqslant g(y) - \sigma t \ \forall (y, t) \in B_\delta(x) \times [0, \delta)\},$$

where H is admissible.

A point x_0 is called critical if $|dg|(x_0) = 0$, otherwise, it is called regular. For a map $F : X \to Y$ between two metric spaces, we set

$$Sur(F, x)(t) = \sup \{r \geqslant 0 \mid B_r(F(x)) \subset F(B_t(x))\} \quad \forall x \in X, \quad \forall t > 0,$$

and set

$$sur(F, x) = \liminf_{t \to +0} \frac{1}{t} Sur(F, x)(t), \quad \forall x \in X.$$

The later is called the surjection constant of F at x, see [Io]. Our main results, which includes the versions of GHT for manifolds and polyhedra as special cases reads as follow:

THEOREM 1.1. *Let $(X, \varrho), (Y, d)$ be two complete and path connected metric spaces. Assume*

$(H1)$ $\forall y_0 \in Y$, *there is a neighborhood $U = U(y_0)$, there is a continuous function $g = g_{y_0} : U \to R^1$, and there is a constant $\beta > 0$ such that*

$$g(y_0) = 0, \ g(y) > 0, \ |dg|(y) > \beta, \quad \forall y \in U \backslash \{y_0\}.$$

And assume that $F : X \to Y$ is a local homeomorphism satisfying

$(H2)$ $\forall y \in Y$, $\exists$ *a neighborhood* $U = U(y)$ *and* $\exists$ *a constant* $k > 0$ *such that*

$$sur(F, x) \geqslant k, \quad \forall x \in F^{-1}(U).$$

Then F *is a covering map. Furthermore, if* $\pi_1(Y) \subset F^* \pi_1(X)$, *then* F *is a global homeomorphism.*

Let us recall the weak properness of a map F between path connected topological spaces X and Y:

$(H_3)\forall$ path $\sigma : [0, 1] \to Y$ with $\sigma(0) = y_0$, $\forall x_0 \in F^{-1}(y_0)$, and $\forall$ half open path $\gamma : [0, \delta)$ with $\delta \leqslant 1$, satisfying $F(\gamma(t)) = \sigma(t)$, $\forall t \in [0, \delta)$, the limit $\lim\limits_{t \to \delta - 0} \gamma(t)$ exists.

It is known ([D] or [Ch]) that if a local homeomorphism F is weakly proper, then it possesses the Uniqueness Path-Lifting Property. According to F. E. Browder [Br] and R. Plastock [Pl], a map between path connected, locally path-connected and locally simply connected topological spaces is a covering map if and only if it is a local homeomorphism and possesses the Uniqueness Path-Lifting Property.

The following deformation lemma is due to J. N. Corvellec [Co]: Let X be a metric space, $f : X \to R^1$ be continuous, and let $a < c$, and $\sigma > 0$ be constants. Assume that $f^{-1}[a, c]$ is complete and that $|df|(x) > \sigma$, $\forall x \in f^{-1}((a, c])$. Then there is a deformation $\eta : [0, 1] \times f_c \to f_c$, where $f_c = \{x \in X | f(x) \leqslant c\}$, such that

$$f(\eta(t, x)) \leqslant f(x),\ x \in f_a \Rightarrow \eta(t, x) = x, \text{ and } \eta(1, f_c) \subset f_a.$$

i.e., f_a is a strong deformation retract of f_c. More precisely, in fact, by the same proof, there is a continuous function t on f_c such that $0 < t(x) \leqslant \sigma^{-1}(f(x) - a)$, $\forall x \in f^{-1}(a, c]$, there exists a path $\gamma : [0, t(x)] \to f^{-1}[a, c]$ such that $\gamma(0) = x$, $\gamma(t(x)) \in f^{-1}(a)$, and $\varrho(\gamma(t), x) \leqslant t$, $\forall t \in [0, t(x)]$.

PROOF OF THEOREM 1. We are going to verify:

(1) $\forall y_0 \in Y$, there exists a neighborhood $V = V(y_0)$ such that V is path-connected and simply connected.

(2) the map F is weakly proper.

Step 1. (The choice of V). Since there are U, $g : U \to R^1$ and $\beta > 0$ satisfying (H_1), without loss of generality, we may assume that U is closed. $\forall c > 0$, define $V = \{y \in U | g(y) \leqslant c\}$. V is a closed neighborhood of y_0, so is complete. Now we apply the Deformation Lemma due to Corvellec, V

is contractible and path connected. Combining with the fact that F is a local homeomorphism, X is also locally path-connected and locally simply connected.

Step 2. (The weak properness of F) First we need

LEMMA 1. *Let V be defined above. Then under the assumptions of Theorem 1 there are a continuous $t: F^{-1}(V) \to R^1$, and a path $\alpha: [0, t(x)] \to F^{-1}(V)$ such that $\alpha(0) = x$, $F(\alpha(t(x))) = y_0$, $0 \leqslant t(x) \leqslant$ $\leqslant \frac{1}{k\beta} g(F(x))$, and $\varrho(\alpha(t), x) \leqslant t$, $\forall x \in F^{-1}(V)$.*

PROOF. Define $f = g \circ F : F^{-1}(V) \to R^1$. Again f is continuous, and $F^{-1}(V)$ is complete. Provided by a result due to Ioffe [Io], we have

$$|df|(x) \geqslant |dg|(y) \times sur(F, x) \geqslant \beta k := \alpha > 0,$$

$$\forall x \in F^{-1}(V \backslash \{y_0\}) = f^{-1}(0, c].$$

Now, we apply the Deformation Lemma due to Corvellec, there exist a continuous function $t: f_c \to R^1$ satisfying $0 < t(x) \leqslant \alpha^{-1} f(x)$, and a path $\sigma: [0, t(x)] \to f_c$ such that $\sigma(0) = x$, and $\sigma(t(x)) \in f^{-1}(0)$. Noticing that $f_c = F^{-1}(V)$, and $f_0 = F^{-1}(y_0)$, the lemma is proved.

LEMMA 2. *Under the assumptions of Theorem 1, if $0 < \delta \leqslant 1$ and $\gamma: [0, \delta) \to X$ is a path such that $\lim_{t \to \delta - 0} F(\gamma(t)) = y_0 \in Y$, then the limit $\lim_{t \to \delta - 0} \gamma(t)$ exists in X.*

PROOF. Suppose that $\gamma(t)$ has no limit as $t \to \delta - 0$. Then there exist $\varepsilon > 0$ and $t_n < t_n' < \delta$ with $t_n \to \delta - 0$, such that $\varrho(\gamma(t_n), \gamma(t_n')) \geqslant \varepsilon$, and $F(\gamma(t_n))$, $F(\gamma(t_n')) \to y_0$. Therefore we may assume $x_n = \gamma(t_n)$, and $x_n' =$ $= \gamma(t_n') \in F^{-1}(V)$, where $V = V(y_0)$ is defined in step 1. Let g and $f = g \circ F$ be defined as above w.r.t y_0, one has $f(x_n) \to 0$. For large n, we may assume $f(x_n) < \frac{1}{3} \alpha\varepsilon$. According to Lemma 1, there is a path $\sigma: [0, t(x_n)] \to F^{-1}(V)$, with $\sigma(0) = x_n$, $F(\sigma(t(x_n))) = y_0$ and $\varrho(\sigma(t), x_n) \leqslant t$, where $0 < t(x_n) \leqslant \alpha^{-1} f(x_n) < \frac{1}{3} \varepsilon$.

Denoting $z = \sigma(t(x_n))$, we have $\varrho(z, x_n) \leqslant \frac{1}{3} \varepsilon$.

Similarly, we have a path $\sigma': [0, t(x_n')] \to F^{-1}(V)$, satisfying

$$\sigma'(0) = x_n', \ F(\sigma'(t(x_n'))) = y_0$$

and $\varrho(z', x_n') \leqslant \frac{1}{3} \varepsilon$, where $z' = \sigma'(t(x_n'))$.

Thus $z \neq z'$, both $z, z' \in F^{-1}(y_0)$.

Now, we construct a path $\eta = \sigma' \circ \gamma \circ \sigma^*$, where $\sigma^*(t) = \sigma(1-t)$; i.e., η is a path starting from z, goes along σ^* to x_n, then along γ to x_n', and then along σ' to z'.

We have $\eta : [0, 1] \to F^{-1}(V)$ with $\eta(0) = z$ and $\eta(1) = z'$. Again, we write $f = g \circ F$ on $F^{-1}(V)$.

Applying the Deformation Lemma due to Corvellec, we obtain a path $\xi(t) = \zeta(1, \eta(t)) \subset f^{-1}(0) = F^{-1}(y_0)$, $\forall t \in [0, 1]$, where $\zeta : [0, 1] \times f_c \to f_c$ is the deformation satisfying $\zeta(1, f_c) \subset f_0$.

This contradicts with the assumption that F is a local homeomorphism. The lemma is proved.

As to the last assertion, it is a well known fact in elementary topology, which follows directly from the Uniqueness Path Lifting property and the Homotopy Path Lifting Theorem.

THEOREM 2. *Suppose that X and Y are complete, path-connected metric spaces, and that $F : X \to Y$ is a local homeomorphism. Assume* ($H1$) *and* ($H2$). *If X is path-connected after the removal of any discrete subset, and if there exists a continuous function $h : Y \to R^1$, satisfying the* (PS) *condition, and possessing a unique minimizer and a discrete set of maximizers as the only critical points, Then F is a global homeomorphism.*

PROOF OF THEOREM 2. Let Y_0 be the set of maximizers of h, and $Y_1 = Y \backslash Y_0$.

According to the critical point theory for continuous functionals, Y_1 is contractible, because the only critical point of $h_1 := h|_{Y_1}$ is the unique minimizer. Let $X_0 = F^{-1}(Y_0)$ and $X_1 = X \backslash X_0$, then X_0 is a discrete subset of X, and then X_1 is again path-connected. Provided by Theorem 1, F is a covering map, so is $F_1 := F|_{X_1}$. However, $F_1(X_1) \subset Y_1$, the later is contractible, F_1 is a global homeomorphism, according to Theorem 1. This implies that $\forall z \in Y_1$, $\#F^{-1}(z) = 1$. Therefore, $\#F^{-1}(y) = 1$ $\forall y \in Y$. This proves that F is a global homeomorphism.

REMARK 1. Comparing Theorem 2 with Theorem 6.1 in [Ka], the only difference is that in the definition of «nice» space, all functions g_{y_0} defined in ($H1$) are assumed to have the same property as h in Theorem 2, $\forall y_0 \in Y$; while in our case only one such h is needed.

We present here few examples:

EXAMPLE 1 (Hilbert Riemannian manifold). Let (M, g) be a Hilbert Riemannian manifold, modelled on a Hilbert space E with Riemannian metric g. It is known that the topology derived by the distance, which is defined by the length of the geodesics between two points, coincides with the given topology on M. With this distance d, M is a metric space (M, d).

Claim: (M, d) satisfies (H_1).

Indeed, $\forall p \in M$, let Ω be the domain of normal coordinates around p, and let exp be the exponential map. $B_r(\theta_p)$, $B_r(p)$ are the r-balls with centers at $\theta_p \in T_p(M)$, the tangent space at p, and at $p \in M$ respectively. We assume that $B_r(p) \subset \Omega$. Now, $\forall y_0 \in B_{r/2}(p) \setminus \{p\}$, we set $\delta = \frac{1}{2} \min\{d(y_0, p), r\}$ and $U = B_\delta(y_0)$. Thus $d(y, p) \leqslant r$, $\forall y \in U$, and then $\xi = \exp_p^{-1}(y)$ is well defined, and $d(y, p) = \|\xi\|$, where $\|\circ\| = \|\circ\|_p$ denotes the norm induced by g on $T_p(M)$. Let ξ_0 denotes the unit vector of ξ.

Define $H(y, t) = \exp_p(\xi - t\xi_0)$ and $h(y) = \|\exp_p^{-1}(y)\|$ $\forall (y, t) \in U \times [0, \delta)$. Since,

$$t < \delta \leqslant d(y_0, p) - d(y, y_0) \leqslant d(y, p) = \|\xi\|,$$

$H(y, t) \in B_r(p)$ is well defined. It is easy to verify: $d(H(y, t), y) = t$, and $h(H(y, t)) = \|\xi\| - t = h(y) - t$. Therefore, (H_1) is verified.

COROLLARY 1. *Suppose that (M, g) and (N, h) are Hilbert-Riemannian manifolds, and that $F: M \to N$ is a local homeomorphism satisfying the following condition: there exists $k > 0$, $\forall p \in M$, there exists $r = r(p) > 0$, such that*

$$k\varrho(q, p) \leqslant d(F(q), F(p)) \quad \forall q \in B_r(p),$$

In particular, if F is a C^1 mapping, $\|dF(p)^{-1}\| \geqslant k$ $\forall p \in M$. Then F is a covering map. Furthermore if $\pi_1(N) \subset F^*\pi_1(M)$, then F is a global homeomorphism.

PROOF OF COROLLARY 1. We only need to prove the C^1 case. Let ϱ and d be the induced distance on M and N resp. Then $(M, \varrho), (N, d)$ are complete, path connected, locally path-connected, locally simply connected metric spaces, satisfying (H_1). According to the Implicit Function Theorem, F is a local homeomorphism. It remains to verify:

$$sur(F, p) \geqslant k \quad \forall p \in M$$

Indeed, provided by the assumption on the boundedness of $\|dF^{-1}\|$, there are a neighborhood U of p and a constant $m \geqslant k$, such that $B_{mr}(\theta_{F(q)}) \subset dF(q)(B_r(\theta_q))$, $\forall q \in U$.

There exists $\delta > 0$, such that if $t < \delta$, then $B_{tk}(F(p)) \subset F(U) \cap \Omega$, where Ω is the domain of normal coordinates around $F(p)$ in N.

Therefore, $\forall y \in B_{tk}(F(p))$, $\eta = \exp_{F(p)}^{-1}(y) \in T_{F(p)}(N)$ is well defined.

Setting $\sigma(s) = \exp_{F(p)}(s\eta)$, $\lambda(s) = F^{-1}(\sigma(s))$, $\forall s \in [0, 1]$, and $x = F^{-1}(y)$, then $\lambda(0) = p$, $\lambda(1) = x$, and $\lambda'(0) = (dF(p))^{-1}\eta \in T_p(M)$.

We have

$$\begin{aligned}\varrho(x, p) &\leqslant \int_0^1 \|\lambda'(s)\|_{\lambda(s)}\, ds \\ &\leqslant \int_0^1 \|(dF)^{-1}(\lambda(s))\, \sigma'(s)\|_{\lambda(s)}\, ds \\ &\leqslant \frac{1}{k} d(y, F(p)) \\ &\leqslant t\,.\end{aligned}$$

This proves the Corollary.

REMARK 2. The conclusion in this Corollary could be extended to the case of Finsler-Banach manifold, provided we are able to define the exponential map on such a manifold.

LEMMA 3. *Suppose that* (Y, d) *is a metric space. If* $\forall y_0 \in Y$ *there exist a neighborhood* $U = U(y_0)$, *a norm space* $(E, \|.\|)$, *and a local homeomorphism* $\Phi : U \to E$ *such that*

(1) $d(u, v) \leqslant \|\Phi(u) - \Phi(v)\|$ $\forall u, v \in U$.

(2) $\Phi(y_0) = \theta$ *and* $\Phi(U)$ *is a star-shaped with respect to* θ.

Then T *satisfies* $(H1)$.

PROOF OF LEMMA 3. Now, we define $g(y) = \|\Phi(y)\|$ and choose $\delta \in (0, \|\Phi(u)\|)$, with $B_\delta(y_0) \subset U$, and then defined on $B_\delta(y_0) \times (0, \delta)$ the mapping:

$$H(u, t) = \Phi^{-1}\left(1 - \frac{t}{\|\Phi(u)\|}\right) \Phi(u)$$

if $u \in U \setminus \{y_0\}$, and $H(y_0, t) = y_0$.

The verification is trivial.

EXAMPLE 2. Let X and Y be subsets in Banach spaces E and F, resp. in which each point has a star-shaped neighborhood, and let ϱ and d be the induced metrics respectively.

COROLLARY 2. *Suppose that* $F:(X, \varrho) \to (Y, d)$ *is a local homeomorphism that* $d(F(x), F(y)) \geqslant k\varrho(x, y)$, *where* k *is a positive constant. Then* F *is a covering map.*

PROOF OF COROLLARY 2. It follows directly from Theorem 1 and Lemma 3.

One may consider other geometric objects.

EXAMPLE 3 (Infinite graph). Let E be a Banach space with norm $\|.\|$. Given a set of isolated points $A = \{a_1, a_2, \ldots\}$, and a set of closed segments connecting some pairs of these points $L = \{l_1, l_2, \ldots\}$.

If the intersection of any two segments is either empty or a point in A, then we denote them by a triple $\{A, L, E\}$. The triple determines a metric space (Y, d), in which

$$Y = \bigcup_{i=1}^{\infty} l_i$$

endowed with the reduced distance from E. It is called an infinite graph.

COROLLARY 3. *Suppose that* (X, ϱ) *and* (Y, d) *are two infinite graphs determined by the triples* $\{A, L, E\}$ *and* $\{B, M, F\}$ *respectively. Assume that* $F: X \to Y$ *is a mapping such that* F *maps* (A, L) *to* (B, M), *which is a local bijection, and is linear on each* $l \in L$. *If there are positive constants* m *and* $k(a)$ *such that*

$$m \leqslant \frac{|F(l)|}{|l|} \leqslant k(a),$$

if $a \in l$, $\forall l \in L$, $\forall a \in A$, *where* $|\circ|$ *is the length of a segment. Then* F *is a covering map. Furthermore if every loop in* Y *is the image of a loop in* X, *then* F *is a global homeomorphism. In particular, if* Y *is a tree, then* F *is a global homeomorphism.*

PROOF OF COROLLARY 3. The verification of (H_1) is divided into two cases: (1) $y_0 \in int(l)$ for some $l \in L$, (2) $y_0 \in A$. In the former, it is a special case of Corollary 2. In the later, there is $\delta > 0$ such that $A \cap B_\delta(y_0) = y_0$. Define $g_{y_0}(y) = \|y - y_0\|$, and $H(y, t) = y - \frac{t}{\|y - y_0\|}(y - y_0)$ $\forall y \in V = Y \cap \cap B_\delta(y_0)$.

EXAMPLE 4 (Abstract Polyhedra). Let E be a Banach space with norm $\|.\|$. Given a set of isolated points $A \subset E$. For any index set $I = = \{i_0, i_1, \ldots\}$ finite or infinite, let

$$\Delta_I = Cl\{conv(a_{i_0}, a_{i_1}, \ldots)\}$$

be a geometric simplex in E. A geometric complex Γ in E is defined to be a set of index sets satisfying:

(1) If $I' \subset I$, and $I \in \Gamma$; then $I' \in \Gamma$.

(2)

$$\Delta_{I \cap J} = \Delta_I \cap \Delta_J \qquad \forall I, J \in \Gamma$$

The subset

$$K = \bigcup_{I \in \Gamma} \Delta_I$$

endowed with the distance induced from E is called an infinite polyhedron.

Let (Z, d) be a metric space, and let $\tau : K \to Z$ be a surjective satisfying

$$C_1 \|\xi - \eta\| \leqslant d(\tau(\xi), \tau(\eta)) \leqslant C_2 \|\xi - \eta\|, \qquad \forall \xi, \eta \in K.$$

for some positive constants C_1, C_2. Let $Y = \tau(K)$ endowed with the induced metric d, then (Y, d) is a metric space. We call it an abstract polyhedron.

Let $(X, \varrho), (Y, d)$ be two abstract polyhedra determined by $\{A, \Gamma, E, \tau\}$, and $\{B, \Lambda, F, \eta\}$, respectively. $F : X \to Y$ is called bi-Lipschitzian, if

$F : \tau(\Delta_I) \to \eta(\Delta_J)$, is a surjective, $\forall I \in \Gamma$, and for some $J = J(I) \in \Lambda$, and is bi-Lipschitzian in each $\tau(\Delta_I)$, $I \in \Gamma$. i.e.,

$$C_1(I)\, \varrho(x, x') \leqslant d(F(x), F(x')) \leqslant C_2(I)\, \varrho(x, x'), \quad \forall x, x' \in \tau(\Delta_I),$$

where $C_1(I)$, $C_2(I)$ are positive constants depending on I.

COROLLARY 4. *Suppose that* (X, ϱ) *and* (Y, d) *are two complete path connected polyhedra. If* $F : X \to Y$ *is a bi-Lipschitzian map, and also a local surjection, and if there is a positive constant k such that*

$$C_1(I) \geqslant k \qquad \forall I \in \Gamma$$

Then F is a covering map. Moreover, let L and M be the graphs (1-*skeletons*) *determined by* Γ *and* Λ *respectively. If any loop in M is the image of a loop in L; then F is a global homeomorphism.*

The proof is similar to that of Corollary 3 and that of Lemma 3.

REMARK 3. An index set $I \in \Gamma$ is called maximal, if $\forall J \in \Gamma$, $J \cap I \neq \emptyset$, implies $J \subset I$. If we assume that the X is such an abstract polyhedron:

$\forall x \in X$, $\tau^{-1}(x)$ is in at most finite many Δ_I, where I is maximal in Γ.

Then the assumption on the local homeomorphism of F in Corollary 4 can be replaced by the assumption that the map: $I \to J = J(I)$, is one to one.

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