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# Capitulation and Transfer Kernels

par K. W. GRUENBERG et A. WEISS

RÉSUMÉ. On sait que pour une extension galoisienne finie $K/k$ d'un corps de nombres, le noyau du morphisme d'extension $\mathrm{Cl}_k \to \mathrm{Cl}_K$ s'identifie au noyau $X(H)$ du transfert $H/H' \to A$, où $H = \mathrm{Gal}(\tilde{K}/k)$, $A = \mathrm{Gal}(\tilde{K}/K)$ et $\tilde{K}$ est le corps de classes de Hilbert de $K$. Lorsque le groupe $G = \mathrm{Gal}(K/k)$ est abélien, H. Suzuki a montré que $|G|$ divise $|X(H)|$.

Nous appelons noyau de transfert pour $G$ tout groupe abélien fini $X$ qui s'écrit $X(H)$ pour un certain groupe $H$ tel que $A \hookrightarrow H \twoheadrightarrow G$. Après avoir caractérisé les noyaux de transfert en termes de représentations entières de $G$, nous montrons que $X$ est un noyau de transfert pour le groupe abélien $G$ si et seulement si on a $|G|X = 0$ et $|G|$ divise $|X|$, ce qui fournit une nouvelle démonstration du résultat de Suzuki.

ABSTRACT. If $K/k$ is a finite Galois extension of number fields with Galois group $G$, then the kernel of the capitulation map $\mathrm{Cl}_k \to \mathrm{Cl}_K$ of ideal class groups is isomorphic to the kernel $X(H)$ of the transfer map $H/H' \to A$, where $H = \mathrm{Gal}(\tilde{K}/k)$, $A = \mathrm{Gal}(\tilde{K}/K)$ and $\tilde{K}$ is the Hilbert class field of $K$. H. Suzuki proved that when $G$ is abelian, $|G|$ divides $|X(H)|$. We call a finite abelian group $X$ a transfer kernel for $G$ if $X \cong X(H)$ for some group extension $A \hookrightarrow H \twoheadrightarrow G$.

After characterizing transfer kernels in terms of integral representations of $G$, we show that $X$ is a transfer kernel for the abelian group $G$ if and only if $|G|X = 0$ and $|G|$ divides $|X|$. Our arguments give a new proof of Suzuki's result.

Let $K/k$ be a finite unramified Galois extension of number fields with Galois group $G$. The capitulation kernel for $K/k$ is the kernel of the natural homomorphism of ideal class groups $\mathrm{Cl}_k \to \mathrm{Cl}_K$. Suzuki [S] proved that when $G$ is abelian, its order $|G|$ divides the order of the capitulation kernel. This remarkable result encapsulates much of the information previously available about capitulation. We refer to the surveys [J] and [M]

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for relevant background. Our aim here is to explain a new approach to Suzuki's theorem.

The transition to group theory (reviewed in § 1) allows one to interpret capitulation kernels as transfer kernels, by which we mean the following: given a finite group $G$, then a finite abelian group $X$ is a *transfer kernel for* $G$ if there exists a group extension $A \hookrightarrow H \twoheadrightarrow G$ with $A$ finite abelian so that $X$ is isomorphic to the kernel of the transfer homomorphism $H/[H,H] \to A$. We shall prove the following result.

**Theorem 1.** *If $G$ is a finite abelian group, then the finite additive group $X$ is a transfer kernel for $G$ if, and only if, $|G|X = 0$ and $|G|$ divides $|X|$.*

We outline what follows. In §1 we translate the problem into an equivalent one on $G$-module extensions over $\Delta G$, the augmentation ideal of the integral group ring $\mathbb{Z}G$. Then §2, the core of the paper, is an analysis of the common structural properties of transfer kernels for $G$. This makes possible the proof of Theorem 1 in §3. In our final §4 we collect some comments and questions.

## 1. Translations

We begin with the classical result of E. Artin.

**Proposition 1.** *The capitulation kernel for $K/k$ is a transfer kernel for $G$.*

Here is a sketch of the proof. Let $\widetilde{K}$ be the Hilbert class field of $K$ and $A = \mathrm{Gal}(\widetilde{K}/K)$. If $H = \mathrm{Gal}(\widetilde{K}/k)$ then there is a commutative square

$$\begin{array}{ccc} \mathrm{Cl}_k & \xrightarrow{\simeq} & H/[H,H] \\ \text{capitulation}\downarrow & & \downarrow\text{transfer} \\ \mathrm{Cl}_K & \xrightarrow{\simeq} & A \end{array}$$

from which the proposition follows by taking kernels.

**Proposition 2.** *The finite additive group $X$ is a transfer kernel for $G$ if, and only if, there exists a $G$-module extension $A \rightarrowtail B \twoheadrightarrow \Delta G$ with $A$ finite and $X \simeq \mathrm{H}^{-1}(G,B)$.*

*Proof.* This result is clear from the functorial relationship between group extensions over $G$ and $G$-module extensions over $\Delta G$ (cf. [G] §10.5). As this is not the usual approach in the literature we sketch it here.

A group extension

$$(1) \qquad A \overset{i}{\rightarrowtail} H \overset{\pi}{\twoheadrightarrow} G$$

yields the $G$-module extension

$$A \overset{j}{\rightarrowtail} B \overset{\tau}{\twoheadrightarrow} \Delta G \tag{2}$$

where $B = \Delta H/\Delta A.\Delta H$, $\tau$ is induced from $\pi : \Delta H \to \Delta G$ and $j(a)$ is the appropriate coset of $i(a) - 1$.

Conversely, given (2), let

$$\widetilde{H} = \{b \in B \mid \tau(b) = g - 1, \text{ for some } g \in G\}.$$

Then $\widetilde{H}$ is a group with multiplication $x.y = \tau(x)y + x + y$, its identity element is 0 and the inverse of $x$ is $x^{-1} = -g^{-1}x$, where $\tau(x) = g - 1$. The module homomorphism $\tau$ gives the group homomorphism $\widetilde{H} \twoheadrightarrow G$ via $x \mapsto \tau(x) + 1$ with kernel $A$.

If $B$ arises from the group extension (1), then the *hidden group* $\widetilde{H}$ in $B$ gives an extension equivalent to $H$:

$$\begin{array}{ccccc} & & H & & \\ & \nearrow & \Big\downarrow \sigma & \searrow & \\ A & & & & G \\ & \searrow & & \nearrow & \\ & & \widetilde{H} & & \end{array} \tag{3}$$

where $\sigma(h) = (h - 1) + \Delta A.\Delta H$. The $G$-coinvariants on $B$, namely $B_G = B/(\Delta G)B$, are naturally isomorphic to $\Delta H/(\Delta H)^2$, whence to $H/[H,H]$ and so to $\widetilde{H}/[\widetilde{H},\widetilde{H}]$. Notice that $\widetilde{H}/[\widetilde{H},\widetilde{H}] \simeq B_G$ is just $x[\widetilde{H},\widetilde{H}] \mapsto x + (\Delta G)B$.

We claim there is a commutative square

$$\begin{array}{ccc} H/[H,H] & \xrightarrow{\simeq} & B_G \\ {\scriptstyle \text{transfer}}\Big\downarrow & & \Big\downarrow{\scriptstyle \widehat{G}} \\ A^G & \xrightarrow{\simeq} & B^G \end{array}$$

where $\widehat{G}$ is the norm endomorphism $\Sigma_{g\in G}\, g$ and the lower isomorphism is induced by $j$.

In view of (3) we may replace $H$ by $\widetilde{H}$ and view $j$ as inclusion. Take a transversal $t_g$, $g \in G$, for $A$ in $\widetilde{H}$. If $x \in \widetilde{H}$ with $\tau(x) = k - 1$, then the image of $x$ under transfer is $\Pi_g\, t_{kg}^{-1}.x.t_g$, which is the same as $\Sigma_g\, t_{kg}^{-1}.x.t_g$, because each factor is in $A$. Now

$$\begin{aligned} t_{kg}^{-1}.x.t_g &= (-(kg)^{-1}t_{kg}).(kt_g + x) \\ &= (kg)^{-1}(kt_g + x) - (kg)^{-1}t_{kg} \\ &= g^{-1}t_g + (kg)^{-1}x - (kg)^{-1}t_{kg} \end{aligned}$$

and so the transfer image is $\widehat{G}x$ as required.

Proposition 2 follows by taking kernels. □

## 2. Transfer kernels

Let $G$ be a finite group, not necessarily abelian. Put $\Lambda = \mathbb{Z}G/(\widehat{G})$ and identify $\Lambda_G$ with $\mathbb{Z}/|G|\mathbb{Z}$. If $M$ is a $\mathbb{Z}G$-module, then $d_G(M)$ denotes the minimum number of module generators of $M$.

**Theorem 2.** *The following are equivalent:*

**(a)** *$X$ is a transfer kernel for $G$;*

**(b)** *$X$ is isomorphic to the cokernel of a homomorphism $\varphi : U_G \to \Lambda_G^m$, where $m \geq d_G(\Delta G)$ and $U$ is a finitely generated $G$-submodule of $\mathbb{Q}\Lambda^{m-1}$;*

**(c)** *$X$ is isomorphic to $M_G$ for some finitely generated $G$-module $M$, where $\widehat{G}M = 0$ and $\mathbb{Q}M$ contains a $\mathbb{Q}G$-copy of $\mathbb{Q}\Lambda$;*

**(d)** *$|G|X = 0$ and there exists a surjective homomorphism $X \twoheadrightarrow M_G$ with $M$ as in* **(c)**.

*Proof.* **(a)** $\Rightarrow$ **(b)**. Using Proposition 2 we may, and shall, assume $X \simeq \mathrm{H}^{-1}(G,B)$, where $A \rightarrowtail B \twoheadrightarrow \Delta G$. Take a free resolution of $B$, so determining $m$ and $S$ in the following diagram:

$$\begin{array}{ccccc} S & \xrightarrow{=} & S & & \\ \downarrow & & \downarrow & & \\ R & \rightarrowtail & \mathbb{Z}G^m & \twoheadrightarrow & \Delta G \\ \downarrow & & \downarrow & & \downarrow \| \\ A & \rightarrowtail & B & \twoheadrightarrow & \Delta G \end{array} \tag{4}$$

Now $\mathrm{H}^{-1}(G,B) \simeq \mathrm{H}^0(G,S)$ (we use Tate cohomology throughout) and the exact sequence $S^G \hookrightarrow S \twoheadrightarrow U$ gives

$$\mathrm{H}^{-1}(G,U) \xrightarrow{\delta} \mathrm{H}^0(G,S^G) \longrightarrow \mathrm{H}^0(G,S),$$

where $\delta$ is the connecting homomorphism. Since $A$ is finite, $\mathbb{Q}S = \mathbb{Q}R \simeq \mathbb{Q}\oplus\mathbb{Q}G^{m-1}$, whence $S^G \simeq \mathbb{Z}^m$ and $\mathbb{Q}U \simeq \mathbb{Q}\Lambda^{m-1}$. Note that $U$ is $\mathbb{Z}$-torsion-free and so $U$ is contained in $\mathbb{Q}U$. Also $\widehat{G}U = 0$ gives $\mathrm{H}^{-1}(G,U) = U_G$ and $\mathrm{H}^0(G,U) = 0$. Thus $U_G \xrightarrow{\delta} \Lambda_G^m \twoheadrightarrow X$ is exact.

**(b)** $\Rightarrow$ **(c)**. The exact sequence $\Delta G \rightarrowtail \Lambda \twoheadrightarrow \Lambda_G$ of $G$-modules stays exact when we apply $\mathrm{Hom}(U,-)$ because $U$ is $\mathbb{Z}$-free. So we obtain the exact sequence

$$\mathrm{Hom}_G(U,\Lambda^m) \longrightarrow \mathrm{Hom}(U_G,\Lambda_G^m) \longrightarrow \mathrm{H}^1(G,\mathrm{Hom}(U,\Delta G^m)),$$

where the right hand term is 0 because it is isomorphic to $\mathrm{H}^0(G,\mathrm{Hom}(U,\mathbb{Z}^m))$, which is 0 because $\mathrm{Hom}_G(U,\mathbb{Z}) = 0$ as $U_G$ is finite.

It follows that the given homomorphism $\varphi : U_G \to \Lambda_G^m$ lifts to a $G$-homomorphism $\beta : U \to \Lambda^m$ with $\beta_G = \varphi$ giving the commutative diagram

$$\begin{array}{ccccc} U & \xrightarrow{\beta} & \Lambda^m & \twoheadrightarrow & M \\ \downarrow & & \downarrow & & \\ U_G & \xrightarrow{\varphi} & \Lambda_G^m & \twoheadrightarrow & X \end{array}$$

with $M = \mathrm{Coker}\beta$. This induces an epimorphism $M \twoheadrightarrow X$ and hence, by taking $G$-coinvariants, an isomorphism $M_G \simeq X$. Finally, $\mathbb{Q}M \oplus \mathbb{Q}U$ contains (a $G$-copy of) $\mathbb{Q}\Lambda^m$, whence $\mathbb{Q}M$ contains $\mathbb{Q}\Lambda$.

**(c)** $\Rightarrow$ **(d)** is clear since $|G|$ annihilates $M_G = \mathrm{H}^{-1}(G, M)$.

**(d)** $\Rightarrow$ **(b)**. Choose $m \geq \max\{d_G(M),\ d_G(\Delta G),\ d(X)\}$ and take a $G$-free presentation $L \hookrightarrow \mathbb{Z}G^m \twoheadrightarrow M$ of $M$. Since $\widehat{G}M = 0$, $L^G = (\mathbb{Z}G^m)^G$, thereby giving the exact sequence $U \overset{i}{\hookrightarrow} \Lambda^m \twoheadrightarrow M$, where $U = L/L^G$, and hence the exact sequence $U_G \xrightarrow{i_G} \Lambda_G^m \twoheadrightarrow M_G$. Choose $\alpha : \Lambda_G^m \twoheadrightarrow X$ and let $\beta : X \twoheadrightarrow M_G$ be the given homomorphism. Thus $\mathrm{Coker}\, i_G \simeq M_G \simeq \mathrm{Im}\beta\alpha$, which implies, by the Lemma below, that $\mathrm{Im}\, i_G \simeq \mathrm{Ker}\beta\alpha$. Consequently $\mathrm{Ker}\alpha$ is isomorphic to a subgroup $D$ of $\mathrm{Im}\, i_G$. There exists a map of $U_G$ onto $D$ (remember we are dealing with finite abelian groups), giving the composite $\varphi : U_G \twoheadrightarrow D \hookrightarrow \Lambda_G^m$, with $\mathrm{Im}\varphi \simeq \mathrm{Ker}\alpha$. Again using the Lemma below, $\mathrm{Coker}\varphi \simeq \mathrm{Im}\alpha \simeq X$ and so $U_G \xrightarrow{\varphi} \Lambda_G^m \twoheadrightarrow X$ is exact. Finally, $\mathbb{Q}U \oplus \mathbb{Q}M \simeq \mathbb{Q}\Lambda^m$ shows that $\mathbb{Q}M \supseteq \mathbb{Q}\Lambda$ implies $\mathbb{Q}U \subseteq \mathbb{Q}\Lambda^{m-1}$.

**Lemma.** (i) *Given epimorphisms* $f_1, f_2 : \Lambda_G^m \twoheadrightarrow X$, *then* $\mathrm{Ker} f_1 \simeq \mathrm{Ker} f_2$.
(ii) *Given epimorphisms* $g_i : \Lambda_G^m \twoheadrightarrow X_i$, $i = 1, 2$, *with* $\mathrm{Ker} g_1 \simeq \mathrm{Ker} g_2$, *then* $X_1 \simeq X_2$.

*Proof.* In (i) the homomorphisms $f_1, f_2$ are free presentations of $X$ as $\mathbb{Z}/|G|\mathbb{Z}$-module. So Schanuel's Lemma and the Krull-Schmidt property give the result. For (ii), dualise with respect to $\mathbb{Q}/\mathbb{Z}$ and obtain $X_1^* \simeq X_2^*$ by (i).

**(b)** $\Rightarrow$ **(a)**. Our aim is to prove that the $X$ of (b) is a transfer kernel in the module-theoretic sense of Proposition 2.

We use the isomorphism

$$\mathrm{H}^1(G, \mathrm{Hom}(U, \mathbb{Z}^m)) \simeq \mathrm{Hom}(\mathrm{H}^{-1}(G, U),\ \mathrm{H}^0(G, \mathbb{Z}^m))$$

given by integral duality: $\xi \mapsto (x \mapsto \xi.x)$. Hence $\varphi$ corresponds to a uniquely determined extension $\mathbb{Z}^m \rightarrowtail S \twoheadrightarrow U$ whose associated connecting homomorphism $\mathrm{H}^{-1}(G, U) \to \mathrm{H}^0(G, \mathbb{Z}^m)$ is $\varphi$ (e.g. 11.1 in [GW]). Thus $U_G \xrightarrow{\varphi} \Lambda_G^m \twoheadrightarrow \mathrm{H}^0(G, S)$ is exact and so $X \simeq \mathrm{H}^0(G, S)$.

Take a free presentation $R \hookrightarrow \mathbb{Z}G^m \twoheadrightarrow \Delta G$ of $\Delta G$ and embed $S$ in $R$ with cokernel $A$. This can be done because

$$\mathbb{Q}S \simeq \mathbb{Q}^m \oplus \mathbb{Q}U \subseteq \mathbb{Q}^m \oplus \mathbb{Q}\Lambda^{m-1} \simeq \mathbb{Q} \oplus \mathbb{Q}G^{m-1} \simeq \mathbb{Q}R.$$

Taking the pushout along $R \twoheadrightarrow A$ gives a diagram exactly like (4) except that $A$ might not be finite. In any case $\mathrm{H}^0(G,S) \simeq \mathrm{H}^{-1}(G,B)$.

It remains to find a submodule $L$ of $A$ so that $A/L$ is finite and $\mathrm{H}^{-1}(G,B) \simeq H^{-1}(G,B/L)$. First note that $\mathbb{Q}B^G = 0$ by the middle column of (4) and $\mathbb{Q}S^G \simeq \mathbb{Q}^m$. Hence $B_G$ is finite and so $A/A \cap \Delta G.B$ is finite. Pick a torsion-free $G$-submodule $L$ of finite index in $A \cap \Delta G.B$. Then $A/L$ is finite; also $\widehat{G}L = 0$, whence $L^G = 0$. The exact sequence $L \hookrightarrow B \twoheadrightarrow B/L$ then gives the exact sequence

$$\mathrm{H}^{-1}(G,L) \xrightarrow{0} \mathrm{H}^{-1}(G,B) \longrightarrow \mathrm{H}^{-1}(G,B/L) \longrightarrow \mathrm{H}^0(G,L) = 0,$$

which finishes the proof. □

## 3. Proof of Theorem 1

To prove Theorem 1 it suffices, in view of Proposition 2 and Theorem 2, to establish the equivalence of

(i) *$X$ is a finite additive group such that $|G|X = 0$ and $|G|$ divides $|X|$;*

with

(ii) *$X$ is isomorphic to $M_G$ for some finitely generated $G$-module $M$, where $\widehat{G}M = 0$ and $\mathbb{Q}M$ contains a $\mathbb{Q}G$-copy of $\mathbb{Q}\Lambda$.*

**(i) ⇒ (ii).** By (d) of Theorem 2 it suffices to prove $X$ has a transfer kernel for $G$ as a homomorphic image. We shall show that any image of $X$ of order $|G|$ is a transfer kernel. Change notation and call this image $X$. So we have $|X| = |G|$ and shall use induction on $|X|$: when $X = 0$, then $G = 1$ and so we can take $M = 0$.

Now let $X = X_1 \oplus \mathbb{Z}/p^s\mathbb{Z}$. Since $|G| = |X|$, so $G$ has an image $\overline{G} = G/G_1$ of order $p^s$ and then $|G_1| = |X_1|$. By induction, $X_1 \simeq (M_1)_{G_1}$ for an appropriate $M_1$. Define $M = \mathrm{Ind}_{G_1}^G(M_1) \oplus \bar{\Lambda}$, where $\bar{\Lambda} = \mathbb{Z}\overline{G}/(\widehat{\overline{G}})$ is a $G$-module by inflation. Then $\widehat{G}M = 0$ since $\widehat{G}_1 M_1 = 0$, whence

$$\begin{aligned} M_G = \mathrm{H}^{-1}(G,M) &= \mathrm{H}^{-1}(G_1,M_1) \oplus \mathrm{H}^{-1}(G,\bar{\Lambda}) \\ &= (M_1)_{G_1} \oplus \bar{\Lambda}_G = X_1 \oplus \bar{\Lambda}_{\bar{G}} = X. \end{aligned}$$

Also, since $\mathbb{Q}M_1 \supseteq \mathbb{Q}\Lambda_1$ so $\mathbb{Q}M \supseteq \mathbb{Q}(\mathrm{Ind}_{G_1}^G \Lambda_1) \oplus \mathbb{Q}\bar{\Lambda} \simeq \mathbb{Q}\Lambda$, as required.

**(ii) ⇒ (i).** We repeat the classical argument. Take a free $\mathbb{Z}G$-presentation $F \twoheadrightarrow M$ of $M$, with $F = \mathbb{Z}G^m$. Since $M_G$ is finite, the kernel of $F_G \twoheadrightarrow M_G$ is isomorphic to $F_G$ and so $M_G$ is the cokernel of an endomorphism $f$ of $F_G$. It follows that $\det f = \pm|M_G|$.

Since $F \to F_G$ maps $\mathrm{Ker}(F \to M)$ onto the image of $f$, there is a $\mathbb{Z}G$-endomorphism $\tilde{f}$ of $F$ such that $\tilde{f}_G = f$ and $\mathrm{Coker}\tilde{f}$ maps onto $M$. Now $\det \tilde{f}$ annihilates $\mathrm{Coker}\tilde{f}$ (recall that $\mathbb{Z}G$ is a commutative ring). So

$(\det \tilde{f})M = 0$ implies $\det \tilde{f} = n\widehat{G}$ for a suitable integer $n$ (since $\mathbb{Q}\Lambda \subseteq \mathbb{Q}M$).

Finally, with $\varepsilon$ denoting the augmentation on $\mathbb{Z}G$, $\varepsilon \det \tilde{f} = \det f$ and thus $n|G| = \varepsilon(n\widehat{G}) = \pm|M_G|$.

## 4. Remarks

**(1)** Is the converse of Proposition 1 true? This is a fundamental problem. An even stronger form of this is the following: given a group extension $A \to H \to G$ with $A$ abelian, does there exist an unramified Galois extension $L$ with Galois group $H$ so that $L$ is the Hilbert class field $\widetilde{K}$ of the fixed field $K$ of $A$?

It should be noticed that any group $H$ can be realised as the Galois group of an unramified extension $L/k$ ([L], p. 121). Then $L \subseteq \widetilde{K}$ and the difficulty lies in ensuring that $L = \widetilde{K}$.

**(2)** Suppose $X$ is a finite additive group such that $|G|X = 0$. Then

(a) if $X$ is a transfer kernel for $G$, $|G/[G,G]|$ divides $|X|$;
(b) if $|G|$ divides $|X|$, then $X$ is a transfer kernel for $G$.

Both these facts are variations of §3; for (b) one must first show that if, for each prime $p$, the $p$-primary part of $X$ is a transfer kernel for a Sylow $p$-subgroup of $G$, then $X$ is one for $G$.

However, neither (a) nor (b) has a converse if $G$ is a non-abelian $p$-group. This is obvious for (b) (take $A = 1$). For (a), if $X$ is $\mathbb{Z}$-cyclic and of order $|G/[G,G]|$, then $X$ cannot be a transfer kernel for $G$: for if $X \simeq M_G$ with $M$ as in (c) of Theorem 2, then lifting a generator of $M_G$ to $M$ gives a $G$-homomorphism $\Lambda \to M$ which becomes an isomorphism on coinvariants (by Nakayama's Lemma and $\mathbb{Q}M \supseteq \mathbb{Q}\Lambda$); then $|X| = |G|$ forcing $G$ to be abelian.

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