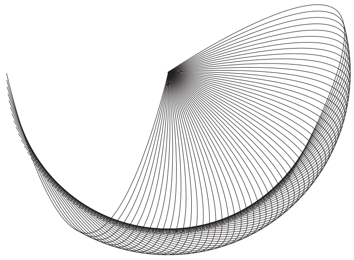
The Possibilities of the Parabola



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A thesis submitted in fulfillment of the requirements for the Degree of
Masters of Education
(Research)

Faculty of Education and Social Work

University of Sydney

2014

AUTHOR’S DECLARATION

This is to certify that:

I. this thesis comprises only my original work towards the Degree of Masters of Education (Research)
II. due acknowledgement has been made in the text to all other material used
III. the thesis does not exceed the word length for this degree
IV. no part of this work has been used for the award of another degree
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Name: ……………………………………………………………………………..

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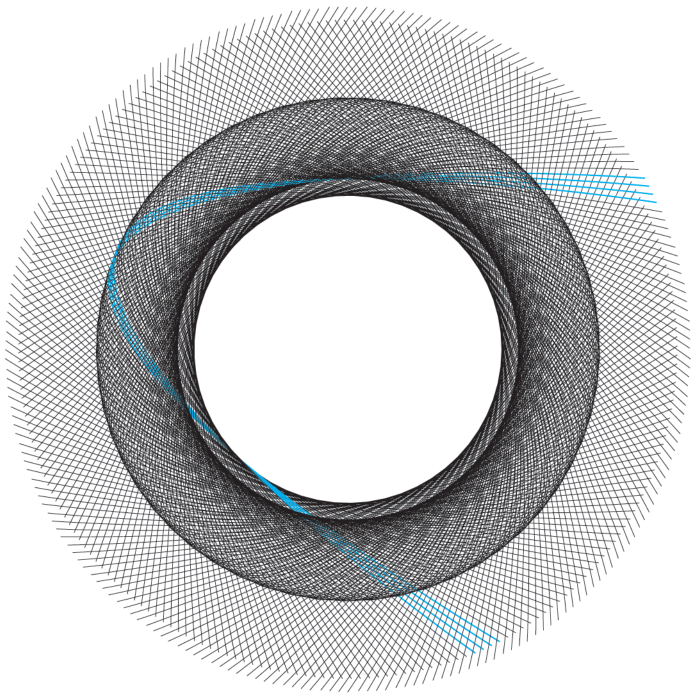
ACKNOWLEDGEMENTS

I would like to thank several people for their contribution to this project. Firstly, from the University of Sydney, I would like to thank Dr Marianne Hulsbosch for empowering me to undertake this study. Secondly and most importantly, I would like to thank Dr Robyn Gibson for providing clarity, ongoing support and encouragement. Her guidance was unfailing and I am truly grateful. I would like to thank the participating school for providing the opportunity to test my ideas, and I would state my gratitude towards my teaching colleagues for always believing I could make the ideas real. In particular, I would like to thank Jane Martin, whose input was invaluable as she provided all the theoretical mathematical guidance, Mike Hobson, whose technological prowess augmented the challenges I faced in pushing curriculum boundaries and Julian Kennard for his marvellous photography. I have the greatest respect for my colleagues. They have profoundly enriched my professional and personal life. I would like to thank all the students who participated in the POP journey and all the folk who gave us feedback. We had a lot of fun. With much appreciation, I would like to thank Alexia Lennon for providing a quiet space to write, good food and warm slippers. Last but not least, I would like to thank my husband and children. If they read anything I have written about this topic, it might begin to make more sense!



ABSTRACT

One curve, several students, a range of aesthetic variables…

The present study details an investigation into the provision of opportunities for secondary school students to develop an understanding of visual aesthetics by manipulating a single mathematical curve: the parabola. The study documents the establishment of a robust cross-curricular relationship between Mathematics and Visual Design by providing a pedagogical model for learning that relies on the conduit between the two subject areas being explicitly linked. Although the central concept is the marriage of disparate themes, it was operationalised by the development and delivery of activities sequenced to grow appreciation and understanding of the links between unrelated curricula. While this account aims to foster cross-curricular discourse and action, the product output simultaneously provided avenues for the presentation and exhibition of student work; a gallery of which is included in the study. Documenting the inter-disciplinary approach to learning and teaching has resulted in an exploration of the complexities we employ to discover meaning in a range of contexts not singularly reliant on art or language. A convergence is presented in that mathematical rules unite with the rules of art and design in the attempt to project new concepts into new situations where a space for originality exists. Here, the students have been encouraged to imagine new, effective ways of bringing ideas to form (Richmond, 2009). Naturally, developing explicit appreciation/action situations required critical and creative thinking to coincide with lateral *and* literal approaches to gaining knowledge and understanding of aesthetics. The study presents a reflexive account of the delivery of coursework entitled *The Possibilities of the Parabola*, from concept to completion.

The story begins…

> There is a curiously sharp sense of joy – or perhaps better expressed, a sense of mild ecstasy – that comes when you find the particular form for your creation... I have often wondered about this special sense of joy; it so often seems out of proportion to what actually has happened. I propose that it is the experience of this-is-the-way-things-are-meant-to-be. If only for that moment, we participate in the myth of creation. Order comes out of disorder, form out of chaos, as it did in the creation of the universe. The sense of joy comes from our participation, no matter how slight, in being as such. The paradox is that at that moment we also experience more vividly our own limitations. We discover the *amor fati* that Nietzsche writes about – the love of one's fate. No wonder it gives a sense of ecstasy! (May, 1975, p. 122)

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BEGINNING OF THE STORY

MY PART

My role as teacher/researcher is underpinned by a personal curiosity for idea-smashing. Blending dissimilar themes to create a unified whole has been a foundation for originality in all my professional pursuits both inherent and external to my teaching experience. I have always looked for opportunities to add *if* to *what*. I have also encouraged others to join me on these journeys because participation in the new is almost always accompanied by a sense of joy.

BACKGROUND

The initial idea for the *Possibilities of the Parabola* (POP) unit of work was borne from a component of a 100 hour NSW school developed Stage Five elective titled *Thinking Hyperbolically!* (TH!). Appropriately named because the aim of each individual task and the collective whole was to explore the linguistic definition of ‘hyperbole’ using collaborative approaches to learning and teaching. The intention of the exclamation mark included in the title signified the challenging and excitable nature of ideas that push curriculum boundaries. Developing an interdisciplinary approach to the delivery of innovative learning and teaching activities was my aim and I am grateful for the enthusiasm gifted by a range of colleagues collaborating in making this work. The course ran for several years and drew a solid cohort of very interested individuals. In general, *TH!* appealed to the more able and lateral thinking students of Mathematics as well as students interested in translating complex ideas into creative visual form. Its successful evolution was due to the ongoing enthusiastic collaboration between myself in art/design and my colleagues in the maths and ICT departments.

The *Possibilities of the Parabola* (POP), although a derivative project, has applied the rich learning approach explored in *TH!* and continues to cross boundaries in terms of curriculum content. POP also provides an avenue for creativity and innovation in learning and teaching, using mathematics as its point of departure, whilst collecting a variety of digital technological skills throughout the journey. When considering learning with a community technology focus, Jeff Utecht (2010a) proposes “autonomy is the key. Educators can initiate, curate and guide. But meaningful learning requires learner-driven activity” (p. 60). The design courses attracted students with varied levels of ability and technological skill. This is not new. The range is consistent with Utecht's (2010b) view that "learners need to experience confusion and chaos in the learning process. Clarifying this chaos is the heart of learning" (para. 2).

RATIONALE

Why Not? Intuition was my driver for this study in the pursuit of an aesthetic experience. Elegance was my destination. Intuition in this context can be defined as the non-inferential knowledge, a risky perception I had as a teacher, that students under my guidance might achieve an experience that was different to a traditional model of learning. Understanding the word elegance, as it is applied to both physical and theoretical contexts was an intentional outcome driven by my intuitive motivation. This is a story that needs to be told because it has provided fodder for much personal thought, professional development and student achievement. The story infers a level of innovation and creativity in learning and teaching that is meaningful in a cross-curricular context.

> Dewey tells us that while science states meaning, the Arts express meaning. Meaning is not limited to what is assertable. Dewey goes on to say that that the aesthetic cannot be separated from the intellectual, for the intellectual to be complete it must bear the stamp of the aesthetic. Having a nose for telling questions and a feel for incisive answers are not empty metaphors (Eisner, 2002, para 33).

One could justify that the type of basic mathematics learnt in secondary school prepares a person for successful interactions with numeric activities in later life. My question is how to marry this basic mathematics with creativity so that the experience is memorable and not empty.

Christopher Nokes, Head of Visual Arts at Vaughan Road Academy, Toronto, created a pedagogical model that allowed him to teach mathematics as a design process where collaborative maths tests were developed and structured as design problems. His pedagogical practice refers pointedly to ideas formulated by D. N. Perkins in *Knowledge as design* (1986) whose "view, aside from his insights through a designer's lens, is his holistic, integrative approach to education" (Nokes, 2005, p. 31). The ideas that Perkins raises are related to the transfer of knowledge from one context to another. Nokes (2005) incorporated this approach as he developed a program based on a "holistic integrated design education (or HIDE). Transforming students from the traditional, isolated receiver of information to the holistic child" (p. 33). It is fair to acknowledge the view that teachers have recognised that they need to make school an embracing and creative experience for kids and when the imagination of children is engaged, there is no end to what they might achieve because they see a purpose in doing it (Robinson, 2010). To support and acknowledge the notion of 'elegance' contributing to an aesthetic experience, curriculum must be dynamic. This study tracks the development of dynamic, inter-disciplinary curriculum via a journey into Cartesian graphing, digital design and computer-assisted manufacturing (CAM). I use the term 'elegance' as it often describes beautiful solutions articulated in art and design contexts as well as in mathematical proofs. Elegance may be applied as both description of internal experience as well as external output of the experience.

Within the facilitation and delivery of an inter-disciplinary course combining mathematics and visual design, it is possible to merge the descriptive aspect of elegance with the notion of aesthetic appreciation while actively immersed in a moment of mathematical, experiential circumstance. At secondary school level, subjects are often individualised because of the nature of planned curriculum and the traditional physical spaces in which learning and teaching take place. Australian Curriculum rationale states:

> Twenty-first century learning theories emphasise the importance of supporting authentic and ubiquitous (anywhere, anyhow) learning, and providing students with opportunities, resources and spaces to develop their creative and critical thinking skills (Newton & Fisher 2009; McGuinness 1999, 2010; ACARA, 2014a, p.253).

It can be argued that the role of teaching in this context is to enhance both the production of the aesthetic form, that is, something students can see, hold or be part of and also facilitate the aesthetic experience, that is, the journey from concept to completion, including activities related to producing unexpectedly beautiful and unique art and design (artefact). True and concerted attempts to cross reference the learning experience in order to align with an *aesthetic* experience for students, where all their senses are to be operating at their peak, is often difficult to facilitate in a planned curriculum.

LITERATURE REVIEW

One curve, several students, a range of aesthetic variables.

MODELS OF PEDAGOGY

The nature of cross-curricular models of pedagogy requires an acknowledgement of the theory of multiple intelligences as frameworks for understanding. Constructivist approaches require the promotion of learning contexts in which students play an active role. Vigotsky (1978) argues against an instructionist model where information is 'transmitted' via a traditional lecture or teacher text-book style environment. Constructivism advocates that a teacher should collaborate with his or her students in order to help facilitate the construction of meaning in students. Learning therefore becomes a reciprocal experience for both the students and teacher. Studies by Cossentino and Shaffer (1999) suggest that the re-conceptualisation of the classroom as a studio space where "cross-curricular explorations promote deep understanding through the confluence of both disciplines when subject content is still recognisable" (Shaffer, 1999, pp. 101-102) is underpinned by constructivist theory. Environments such as these may increase engagement and invoke the wonder of learning. The maintenance of the integral aspects of each discipline to produce a hybrid result affirms the teachers' responsibilities to provide student opportunities for meaningful learning. Beyond Cossentino and Shaffer (1999) and Muller and Ward (2006) who specifically investigated algebra via the sculptural weight distribution practiced by Alexander Calder, there is a gap in teacher biography that examines alternate pedagogies related specifically to Cartesian plotting and graphing.

Crossing curriculum boundaries assists in the provision of opportunities for students to express themselves according to their strengths. Diversified teacher instruction combined with passionate philosophies and beliefs about stretching learning topologically, provides an education environment in which quality learning experiences benefit student and teacher simultaneously. In contrast to arts integration, mathematic integration, apart from mandated consideration of *numeracy* in all curriculum design, can have the same effect in terms of recognising the diversity in students' strengths. Using Eisner's (2006) argument that if the notion of artistic intelligences is to be taken seriously, the concept of understanding mathematics might be considered in the same light, in that integration into the Arts "broadens the opportunities that youngsters have for understanding the subject matter" (p. 37). Making an evident connection between design and art by applying a Visual Arts approach to the study of Design and Technology warrants alignment with Eisner's education theories. Eisner was a champion of arts integrated learning, a seminal writer on embedding arts in the curriculum. His views on the development of critical thinking skills via arts integration justify his inclusion within a triangulated relationship between visual arts, design and mathematics. Eisner's view that small differences can have large effects is evident in the learning narrative I was participating in while undertaking this study.

So, let's imagine that students can create incredibly beautiful aesthetic forms by first engaging in some rigid and structured maths activities! In a visiting lecture on Visual and Environmental Studies at the Carpenter Centre, Harvard University in 1999, Zachary C. Sifuentes accords that aesthetic insight be fostered in the classroom. "I've learned the past few years that Harvard students don't really need to be taught. They need to be given the opportunity to learn something new, something complicated, something fascinating" (Topczewska, 2012, p. 1). Creativity can be aligned with Sifuentes' view in that three of its identifiable features of are acknowledged as: using imagination, pursuing purposes; and being original (Robinson, 1999). One pedagogical intention for pursuing the POP project and hence, this study, was to create an original experience for students (and teachers) in the attempt to push old notions of curriculum outside their comfortable borders. When used in the service of learning and teaching, creativity is the hackneyed word suggesting loose, abstract individualism, often applied to the output from students in a Visual Arts context and not to the designed curriculum input from teachers.

Politicians see creativity as the driver of innovation and entrepreneurship, yet within schooling the term is most often engaged in the delivery of the Arts and notionally applied in other subject areas. There is vagueness to the concept of creativity possibly because it is "large, unwieldy and hard to grasp" (Kaufman & Sternberg, 2006, p. 3). My attempt to address the nefarious topic of creativity and encouragement of students to engage in opportunities to create within an inter-disciplinary context, were aligned with Robinson's argument that creativity is "imaginative activity fashioned so as to produce outcomes that are both original and of value" (Robinson, 1999, p. 30). Creative thinking often challenges the status quo and yet creative and critical thinking skills are a mandated developmental requirement identified by ACARA as being important for the twenty-first century learner. (ACARA, 2014a). Contemporary Visual Arts students regularly, sometimes unconsciously, engage with cross-curricula ideas. This often takes the form of collaborative creative development and is not solely dependent on individualistic cognitive skill. As Fischer et al. (2005) argue: "Much human creativity arises from activities that take place in a social context in which interactions with other people and the artefacts that embody group knowledge are important contributors to the process. Creativity does not happen inside a person's head, but in the interaction between a person's thoughts and a socio-cultural context" (2005, p. 485). The role of the teacher in the development of a learning environment such as I developed for the delivery of POP is to manufacture a situation within which aesthetic understanding is collaboratively encouraged, controlled and prominent. If one is to teach creatively and collaboratively, students will acquire firm, not flimsy, notions of their own potential creativity. The research evidence suggests that creativity can be developed, taught and supported in schools (Kaufman & Sternberg, 2006).

THE PROBLEM

Aesthetics. May (1975) defines aesthetics as insights that "emerge not chiefly because they are 'rationally true' or even helpful, but because they have certain form, the form that is beautiful because it completes an incomplete Gestalt" (p. 62). In Visual Arts and Design Education, aesthetics may be defined subjectively as *what looks good*, that is, having visual appeal. But how does one know *what looks good*? Kant views this through an *a priori* lens: "A priori knowledge or justification is independent of experience" (Russell, 1961, p. 685). Hence, in terms of the acquisition of an aesthetic experience and it's placement in space and time, *a priori*, in the context of this study, relates to the mathematical rule underpinning the creative process in that "the objects of sense must obey geometry" (p. 686)

> Kant holds that Euclidean geometry is known *a priori*, although it is synthetic, ie. not deducible from logic alone. Geometrical proofs, he considers, depends on the figures; we can *see*, for instance, that, given two intersecting straight lines at right angles to each other, on *y*, one straight line at right angle to both can be drawn through their point of intersection. This knowledge is not derived from experience. But the only way in which my intuition can anticipate what will be found in the object is if it contains only the form of my sensibility, antedating in my subjectivity all the actual impressions. The objects of sense must obey geometry, because geometry is concerned with our ways of perceiving, and

therefore we cannot perceive otherwise. (Russell, 1961, p. 686).

The concept of the aesthetic in a mathematical sense may be considered as insightful and much more significantly aligned with a unified experience. In his discussion on gestalt psychology and its relationship to architecture and design, Victor G Popow (2000) quotes a theory by Nikos A. Salingaros, Mathematics Professor at the University of Texas, “Man's visual system is especially receptive to patterns” (Popow, 2000, p. 2). In Salingaros’ treatise, patterns are defined as “regularity in some dimension” and mathematics is a science of patterns. He adds “Thus I began to see how mathematics, patterns and gestalt psychology can be intertwined and applied to architecture” (p. 2). Popow’s paper referred to mankind’s need to “generate patterns out of some basic inner need” (p. 2) and by applying basic psychology we might ask, are human beings insecure in a perceived wildly chaotic universe? And if symmetry and patterns are preferred or considered ‘beautiful’ does this conversely imply that random design, empty walls, little colour, or no pattern be judged as non-preferable or even ‘ugly’? (Popow, 2000). The parabola is an ancient curve, becoming known as the graph of a quadratic function with the advent of coordinate geometry and its analysis, attributed to Descartes. The inherent rules guiding Cartesian plotting and graphing are constant. Their application is crucial to solutions to a myriad of theoretical and real world problems. The algebra on which we rely so heavily today did not prevent early mathematicians reasoning out solutions for equations. Considering geometry as a manipulative technique to teach mathematical concepts outside of the generally accepted mathematics curriculum, the validity of Kantian notions of perception related to *a priori* knowledge of geometry as it is applied to physics, can be utilised in validating the percepts augmenting the experience of working with the parabola as a foundation for creating visual arts and design projects. Kant’s notion of two views, subjective and objective, colliding in space to create experience, is necessarily reliant on both empirical and inferred knowledge. What one already knows and what one is caused to experience by engaging in that knowledge.

Certain stimulation is required to gain a full understanding of aesthetics in relation to form.

> The making of a form from the simplest sandcastle to the most advanced architectural achievement is a process in which aesthetic satisfactions are pervasive. Our need for variety and for stimulation is met, in part, through the aesthetics of human action” (Whitehead, 2006, p. 16).

An aesthetic experience in an education context must be motivated by stimulating action, exploration, awareness and surprise for it to be worthwhile and evocative (Eisner, 1985). Hutchings and Gale (2005), in a similar vein, cite Maxine Greene: "Sometimes I think that what we want to make possible is the living of lyrical moments, moments at which human beings (freed to feel, to know and to imagine) suddenly understand their own lives in relation to all that surrounds" (p. 1). In their conversation *Aesthetic literacy across the curriculum*, they also cite Elliot Eisner (2002):

> Do students recognise the aesthetic features of inquiry in science and in the social studies, or do they separate the aesthetic from what they study in general and assign it to the realm of the Arts alone? What would we need to teach in each of the fields students study to help them understand the role that the aesthetic plays in a particular field? (p. 1)

This statement is pertinent in that it relates to participation in a journey towards discovery of expression that perhaps is not self evident for students interested in maths and science as opposed to the Arts. May's (1975) definition, and his view that "scientists themselves, particularly the physicists, have told us that the creativity of science is bound up with the freedom of human beings to create in the free, pure sense" (p. 7) is explored within this study and serves as an anchor to examine the teaching of aesthetics in Visual Art and Design by using Mathematics as the first step in the creation of form. Therefore the problem explored in this study relates to how students might gain a better understanding of the concept of aesthetics, in both appreciation and experiential contexts, using mathematics as the foundation tool. Additionally, the problem attracts questions associated with vocalised general views held by many (non-mathematicians), that the study of maths is an unpleasant experience, not related to a positive aesthetic experience at all. Thus, the problem is how to make the study of mathematics aesthetically pleasing.

According to Csíkszentmihályí (1990), the good things in life do not come only through the senses. Some of the most exhilarating experiences we undergo are generated inside the mind, triggered by information that challenges our ability to think. He considers *wonder* – as noted by Sir Francis Bacon 400 years ago to be the seed of knowledge – as the reflection of the purest form of pleasure.

Recognising and acknowledging moments of "flow" as proposed by positive psychologist Mihály Csíkszentmihályí (1990), being "the way people describe their state of mind when consciousness is harmoniously ordered" (p. 6) is important to explore when observing students as "one can draw upon an almost infinite range of opportunities for enjoyment – for instance...mathematics" (Csiksentmihalyi, 1990, p. 6) and not rely solely on art or play. Compositional forces and the "whole qualities" (Arnheim, 1943, p. 71) of things, including experience in this instance, refer to the principles of Gestalt psychology. Philosophical romanticism was driven primarily by aesthetic motives without denying passion and individual experience. "Most romantics ardently favoured revolutionary principles" (Russell, 1961, p. 653). This study aimed to measure individual experience by participatory immersion and observation of the combination of revolutionary pedagogical variables consciously applied within a semi-rigid curriculum context.

Parabola. The mathematical function used in this project to demonstrate the relationship between visual and mathematical concepts derives from Cartesian graphing. This is the function that plots points on the x,y axes to produce a curve. The curve in question is commonly known as a *parabola*.

> It's easy to draw a curve. Artists do it all the time; architects lay out a sweep of new buildings in the curve of a crescent, or a modern close. A baseball pitcher throws a curve ball and when they shoot for goal, the ball follows a curve. But what if we were to ask 'What is a curve?' the answer is not so easy to frame. Mathematicians have studied curves for centuries and from many vantage points. It began with the Greeks and the curves they studied are now called the 'classical curves' (Crilly, 2007a, p. 88).

Terms and definitions were explored throughout the pedagogical delivery in order to increase the relevance of the relationship between the physical curve and its mathematical foundation. These derive from Alex Bogomolny (2004), John Conway (1995), Ichiro Kanaya, Yuya Nakano & Kosuke Sato (2007) in their design discourse: *Classification for aesthetic curves and surfaces for industrial designs* and Fred Attneave's (1954) ideas related to the gestalt principles of perceptual organisation. The curve is useful in the statement of principles. For instance, the purity achieved by the application of Gestalt theory can denote a junction of calm and action. A curve can provide a reinterpretation of a subject not foreseen in the basic concept of the original (Arnheim, 1954). Our perception of form, related to responding to design and visual arts, is dependent on the collision of intellectual and emotional stimulation. The constructive use of vision to ascertain a preference for one form over another is reliant, intrinsically, on the relationship of sight with all the senses, including those essentially considered ancillary to the process of seeing (de Sausmarez, 1964). "There must be a primary concern for visual response to what is taking place and decisions will be dependent on this, inevitably arising out of the differences in psychological make-up" (p. 10). Amédée Ozenfant, co-founder of the 20th century art movement known as Purism, refers to the dynamic experience embedded in the act of looking, as "the geometry of sensation"; in that the behavior of the forces or energies contained in colours, shapes and lines are of primary importance to coherent visual perception based on "human neural and psychophysiological reaction more than the process of intellection" (p. 16).

The role of teacher-researcher in the process of encouraging the perception of links between the pure mathematical concepts and the process of visual artmaking or designing was to facilitate the retention of the integrity and quality of the original curve in the students' product or visual output. Therefore, focus and intention was to facilitate the acknowledgement of this relationship and measure the level of student aesthetic experience as well as the aesthetic degree inherent in the students' artmaking or design output. Restriction to the use of parabolic curves in the POP design development was linked to the principle of simplicity "which favors the visual priority of circular shape" (Arnheim, 1954, p. 175). Curves inherently represent human motor skills by signifying simple fluid motion. "The perfection of circular shapes attracts attention" (p. 175). According to Arnheim, curves are seen to be humanised shapes as there is a "dominance of the motor factor over visual organisation" (p. 403). When drawing the human figure, children generally begin with the circle of the head, developing genetically from what Arnheim terms, *"the primordial circle"*, later citing Piaget, who attests "early shapes are topological rather than geometrical, i.e. they aim at such general, nonmetric properties as roundness… not at specific ideal embodiments" (p. 175)". Similarly, research experiments referred to later in this study demonstrate the preference people have for curves in the same way as "experiments by Charlotte Rice have shown that young children often pick the circles from a collection of different shapes even though they have been asked to look for diamonds" (Arnheim, 1954, p. 175).

Beauty, symmetry, iteration and elegance. These themes were explored in the context of both Mathematics and Visual Design. Stylianou & Grzegorczyk (2005) provide findings in their research on the links between art and mathematics that display:

> … familiarity with symmetry concepts [which] allow(s) students to use abstraction as a tool in their artistic creations… students reported improved attitudes towards mathematics and its relevance to their lives in general – a bonus finding for a liberal arts course (p.1).

Similarly, Kanaya, Nakano and Sato (2007) make reference to the definition of the word 'beautiful' as applied to specific mathematical theory. Iteration is repetition. Repetition creates certainty and provides information that is unambiguous, as Attneave (1954) attests in his ideas related to redundancy in visual perception. Bertrand Russell's (1919) pure thoughts on mathematics underpin the trajectory of this pedagogical exploration:

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty — a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest excellence, is to be found in mathematics as surely as poetry. (p. 49)

Where 'elegance' in a mathematics context is associated with the experience of searching for and possibly finding a beautiful and simply articulated solution to a problem, elegance in design is most often related to the visual appeal or aesthetics of the solution to the design problem. These two concepts suggest that elegance can be experienced. As Kerdeman (2009) argues "no matter the setting or the moment, aesthetic experience vitalises our lives with meaning and joy" (p. 88) in that this type of experience can only be "shaped or directed by the contribution and initiative of individuals" (p. 88). She goes on to say "personal growth, meaningful work, being moved by beauty – all testify to the transformations that accrue in aesthetic experience" (p. 90).

Evidential experience has shown me that students love to dabble in symmetry, especially when the activity relates to the discovery of ways to express both equal and unequal outcomes: Symmetry and Asymmetry. Many productive lessons can be and have been spent with students exploring symmetry from a maths or visual design perspective.

> Symmetry is an aspect of mathematics that is strongly linked to art and design… findings suggest that when working with repetitious geometric art, design students' approach to symmetry often goes beyond the one-to-one

correspondence between specific pieces of art and symmetry concepts. (Stylianou & Grzegorczyk, 2005, p. 1).

It is necessary to acknowledge the brevity of the literature review presented in this section. It is condensed due to the significant amount of the literature being revisited in the discussion of the findings. This was a conscious and deliberate decision in order to tighten the study content rather than increase repetition. I have selected key pieces of literature that are in my literature review and then I have revisited or introduced new literature in the discussion findings.

RESEARCH METHODOLOGY

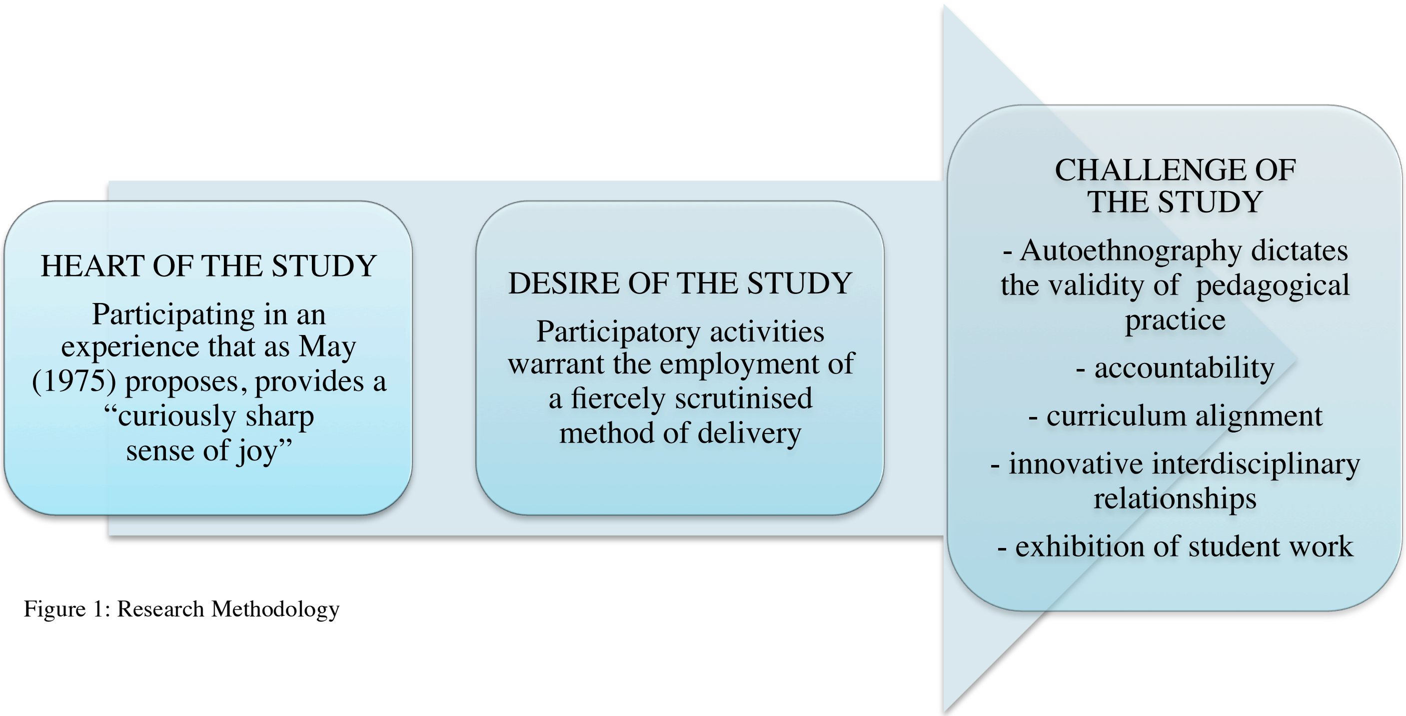


Figure 1: Research Methodology

PRIMARY RESEARCH QUESTION

The study was guided by the following research question:

Can Visual Design students in Stage 5 develop an understanding of aesthetics via the exploration of Cartesian plotting and graphing?

The study was undertaken at a secular, co-educational Independent school located in the inner city of Sydney, New South Wales. The participants in the study were students in Stage Five at the school, i.e. students of ages fourteen, fifteen and sixteen in Years nine and ten. The study was aimed at students selecting the Stage 5 electives in Visual Arts or Design and Technology, mandated for 100 hours of face-to-face teaching by the NSW Board of Studies. For logistical reasons, the students participating in the final study (not the pilot studies) were enrolled in a Design and Technology Stage 5 elective. These students came to the course with prior knowledge of Technology as a subject taught mandatorily in Stage 4, i.e. secondary school Years seven and eight.

My own explorations as teacher-researcher were born from a slightly different perspective in that I wanted to understand Mathematics more accurately in order to devise activities for students that were engaging while most importantly acknowledging the relationship between maths, art and design more explicitly. It was important to continue to develop these relationships from an equal perspective and not to lose the integrity of the Mathematics in order to simply produce wild or wicked aesthetics without any rigid foundation. The aesthetic experience and the journey towards gaining this experience were explored from the angle of objectivity *and* subjectivity. It was possible that the work produced by students may engender the experience itself. This experience could certainly be augmented by the response of an external audience viewing the work. And it was possible that the student as audience would provide a relational aspect to a full understanding of the aesthetic experience. Amartya Sen refers to Polyani's (1967) basic insight in the foreword of *The tacit dimension* (Polyani, 1967) "We know more than we can tell" (p. x) in that we can't always express this type of knowledge in exact words. Elliot Eisner (2002) qualifies this conundrum, because "not everything knowable can be articulated in propositional form" (para 32), Student experience may be communicated via body language, nuance, psycho/social expression and even silence. My teacher-researcher role necessitated constant and sensitive vigilance in terms of acknowledging and interpreting the relationship between the student(s) and the work they produced.

RESEARCH STRATEGY

Autoethnography.

> "A wonderful fact to reflect upon, that every human creature is constituted to be that profound secret and mystery to every other."
>
> Charles Dickens, *A Tale of Two Cities*

Autoethnography transgresses conventions. If ethnography is the study of culture then my choice of autoethnography as a research strategy is to place myself within a defined, localised culture – the classroom – in order to explore the key theme of the study. Immersion requires relationships to be established and nurtured. "Autoethnography is an autobiographical genre of writing and research that displays multiple layers of consciousness, connecting the personal to the cultural" (Ellis & Bochner, 2000, p. 737). This type of research method features concrete action and self-reflection appearing as "relational and institutional stories affected by history, social structure and culture, which themselves are dialectically revealed through action, feeling, thought, and language" (p. 737). This study is a metastory. My story was documented because it involved me immersing myself in a pedagogical culture that is steeped in stereotype. I wished to record how far the curriculum could be pushed to ascertain experience across traditionally separate areas of learning and teaching while still retaining links to summative and formative assessment within a required set syllabus.

The teacher/student relationship in the context of the delivery of an innovative unit of work necessitates particular reliance based on clarity, purpose and a collective intention to learn. By default, the researcher (myself) is heavily involved in the group. The authenticity of the narrative being told necessarily requires constant and vigilant referencing in order to retain its validity and relevance. This study does not lie in the lap of empiricism. Nor does the content of the study lie within a rigid curriculum. The autoethnographic approach aims to encourage a "form that will allow readers to feel the moral dilemmas, think with [my] story instead of about it, join actively in the decision points that define an autoethnographic project, and consider how their own lives can be made a story worth telling" (Ellis & Bochner, 2000, p. 735). Autoethnography requires a storytelling approach and the story in this situation is authenticated by constant referencing. This augments and validates the relevance of having documented the journey. In *Wording pictures - Discovering heartful autoethnography*, Ellis (2000) defines autoethnography as overlapping art and science, "part *auto* or self and part *ethno* or culture. Yet it is something different from both of them, or greater than its parts" (Knowles & Cole, 2008, p. 130). In *Connection – the foundation of learning*, Gilbert (2014) asks us to take a moment to remember a teacher who made an impact on you. She proposes that connecting with students and creating positive relationships has a lasting effect on the lives of young people. Christopher Knoell (2012) further attests:

> Teachers must never overlook the importance of cultivating student-teacher relationships in their classrooms. Student-teacher relationships are built through

> purposeful and continual effort, primarily on the part of the teacher. It is in the relationship between teacher and student where learning takes root and begins to grow; and the degree to which a teacher invests in those interactions not only affects learning outcomes and student behavior in the classroom, but also potentially impacts each student's future achievements and success. (p. 82)

Thus, immersion by way of participatory instruction and observation necessarily directed my focus for the delivery of the POP courses. Each cohort would bring different stories to the table. My role as teacher-researcher was to "locate myself along the continuum of art and science" in order to "capture experience" (Ellis & Bochner, 2000, p. 750). The inextricable connection between life and narrative was unable to be ignored in this study. "Life both anticipates telling and draws meaning from it. Narrative is both about living and part of it." (Ellis & Bochner, 2000, p. 746)

The central concept of the study was to immerse myself, observe and document the cultural shift that may occur by adopting a non-traditional praxis related to teaching mathematics and an inter-disciplinary post-modern approach to teaching art and design – within a middle school environment. "Ethnography puts together two different words: 'ethno' means folk, while 'graph' derives from 'writing'. Ethnography refers, then, to social scientific writing about particular folks" (Silverman, 2006, p. 68). In consideration of facts and values being truths of human activity, it was necessary to record from direct experience in order to ascertain how one affected the other. The autoethnographic methodology cannot underestimate or exclude the potential influence of the observer on the actions of the observed. In this study, the immersive circumstance and transformative potential could not be completely free of my own values and those of the participants.

Action perspective. Analysis of the collected data takes into account the intellectual capacity of student learning while simultaneously tracking the experience of the *curve to creation* journey. Aspects of action research methods were utilised for this particular study. They are a valid addition due to the nature of the inherent work. Schön's (2006) illustration depicting the "topology of professional landscapes" (Whitehead, 2006, p. 17) where the practitioners occupy the "swampy lowlands" of conceptual theory supports Whitehead (2006) in that it is necessary to regard practitioners "as competent professionals whose practical knowledge is key to developing human capabilities, their own and other people's" (p. 46). Thus, this study had a twofold aim: to develop my own and students' capabilities.

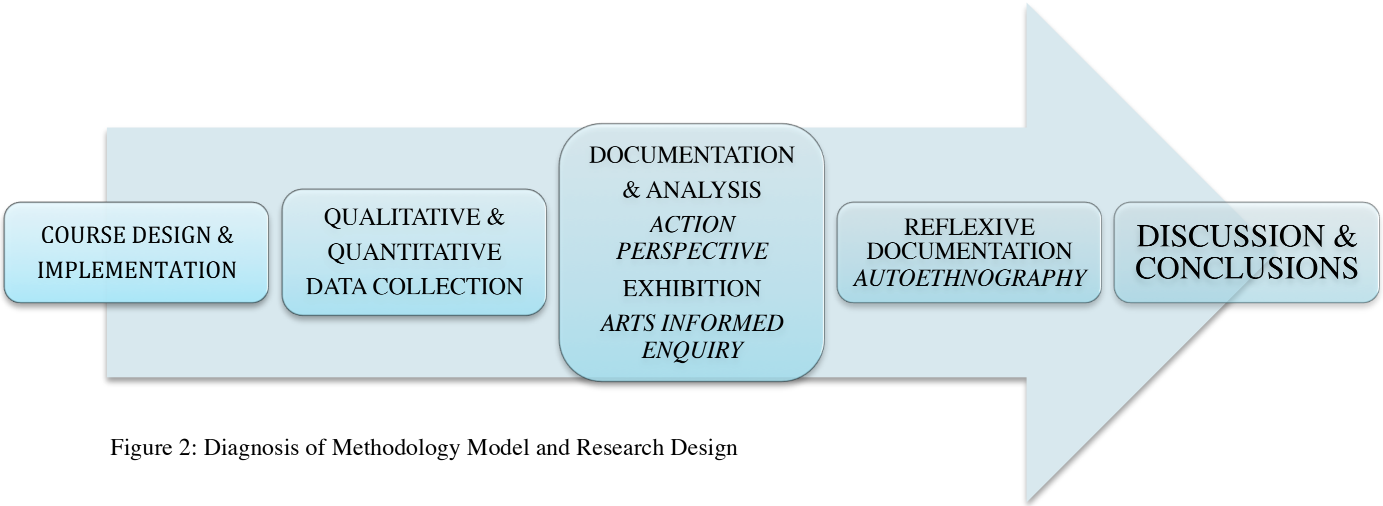


Figure 2: Diagnosis of Methodology Model and Research Design

Arts-Informed Inquiry is addressed in the methodology because artefacts were produced within the body of this study. Arts-Informed inquiry acknowledges findings being presented beyond the academy. The artefacts produced within the study were critically analysed and assessed in the program of learning. They were also subject to external scrutiny via physical and virtual exhibition. My experiments and student achievement were necessarily entwined in this study. This was unchartered territory. It has several narrative stands and a range of possible methods of data collection methods and interpretation. A marriage of methodologies is deemed valid due to the goal of connecting student work to an intended audience with the specific desire to evoke an aesthetic response. Arts informed enquiry maximises this communicative potential. However, I did not intend to lose my own purpose, as teacher, in the exploration. My presence as teacher/researcher must be transparent and apparent. It necessarily must address "the intersection of a researcher's life with that of those researched" (Knowles & Cole in Neilsen, Cole & Knowles, 2001, p.215).

Access. Although 'focussed ethnography' is the strategy typically used in a school context, my current role as teacher with access to Stage 5 students warranted active immersion rather than passive observation. Ethnographic style interviews were used to collect data informally as part of the immersion process. Similarly, through immersion, observational notes, recording a personal journal, and the documentation of student comments during course delivery provided rich fodder for the study to present a highly descriptive narrative, including personal and formal analytical approaches to the topic.

Survey. The mode of the survey strategy was in the form of observation, interview and formal pre and post testing (online). According to Glasow (2005):

> In survey research, independent and dependent variables are used to define the scope of study, but cannot be explicitly controlled by the researcher. Before conducting the survey, the researcher must predicate a model that identifies the expected relationships among these variables. The survey is then constructed to test this model against observations of the phenomena. (p. 1)

The results of pre and post testing are included in the appendices. The key themes addressed through findings are interspersed throughout the project story and discussion. The conclusion section identifies and provides graphic analysis of the responses from the audience to the completed POP artefacts.

Observation & Participation. Silverman (2006) states that observation is almost self explanatory since "the observer looks, listens and records" (p. 67).

> Participant observation, ethnography and fieldwork are all used interchangeably... they can all mean spending long periods watching people, coupled with talking to them about what they are doing, thinking and saying, designed to see how they understand their world. (Delamont, 2007, p. 206)

According to May (1975), a sense of joy comes from participation. As a teacher, my practice was by default, participatory. I was curious to undertake a range of activities that may lead to an experience related to the sense of joy that May (1975) proposes is inherent in immersion.

Immersion was necessary for the collection of data for this study. My observation of student activity and achievement in the pilot and final courses provided much material for analysis. Informal interviews during the delivery of the course provided feedback on aspects of the evolutionary nature of such a content rich, interdisciplinary model of learning. By default, the immersive nature of this experience for me as teacher/researcher provided opportunity to record outcomes that had previously been untested. Those related to the acquisition of knowledge through experiential activities. Here I was looking for emerging themes. The level of student experience could be measured by their reaction and response to the final output of the artefacts. A level of quantitative data has been recorded (see Appendix 14), however the body of the data analysis is included in the way the story has been ethnographically told. Immersion in the study allowed for observational recording of the level of student achievement and detailed discussion of data analysis and student experience can be found in the Appendices 14, 17 and 18.

SUBSIDIARY RESEARCH QUESTION

Will the pedagogy and learning activities designed and implemented, lead to the deepening of students' aesthetic experience?

The ability to devise, amend, deliver and interpret strategies to use in the classroom to engender an aesthetic experience is dependent on a range of variables: self-knowledge, motivation, empathetic approaches, patience, willingness to consider alternatives (flexibility) and perception of when enough is enough.

> In a sense, all social research is a form of participant observation, because we cannot study the social world without being a part of it. From this point of view, participant observation is not a particular research technique but a mode of being-in-the-world characteristic of researchers. (Silverman, 2006, p. 69)

The flexibility of a semi-participatory observational approach to data collection is dangerous in that it may lead to "an open and unstructured research design which increases the possibility of coming across unexpected issues" (Silverman, 2006, p. 68). Therefore acknowledging the rich or poor representativeness of the sample was considered throughout the study. It was impossible to ignore the fact that there are always periods of disengagement in a classroom, at least for some students. Their responses are included in the analysis of the experience. These responses collided with evidence from my participatory observation throughout the pilot courses as well.

Participants and representativeness. The sample set (not including pilot courses) numbers 34 students. Inclusion of pilot course members incurs a set of approximately 100 students over three years.

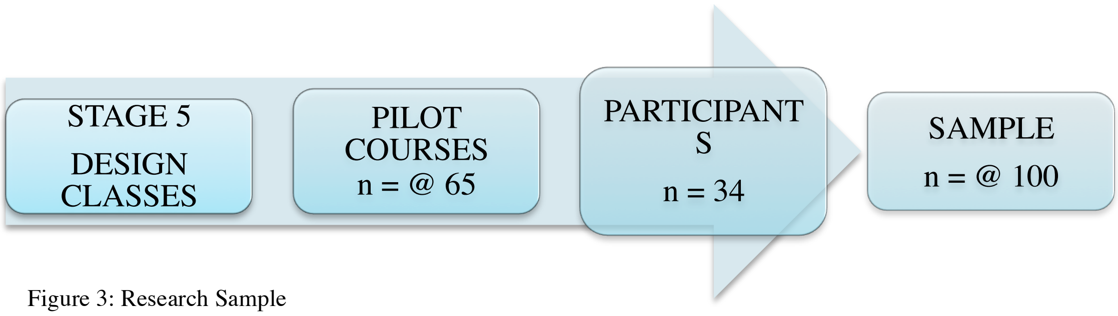


Figure 3: Research Sample

The strengths of a semi-participatory, observational approach have been validated in terms of a historical relationship between teacher and students, familiarity with the culture of the specific independent school, access to current technologies allowing for manual to digital application, experimentation and direct instruction (teacher to student cohort) and the dependability of teacher-researcher motivation in the development of pedagogical delivery. Weaknesses of the described approach emerged in dependence on the collective culture of Stage 5 cohorts from pilot to the most recent participants, adherence to required syllabus outcomes, student motivation and possible failure to engage. The latter criterion has been addressed in the discussion section of the study. The study sample is not beautifully representative of Stage 5 students in all schools, nationally or in New South Wales. The socio economic status of the sample and school itself are factors to be considered, therefore the data set dependencies are not indicative of a wider experience range. However, the autoethnographic approach to the study provides an opportunity for implementing similar course structure in wider contexts according to individual delivery variables. Secondary quantitative analysis inherent in the study discussion indicates levels of competency related to comparative data such as OECD learning and teaching statistics or at the very least, numeracy statistics from collective testing such as NAPLAN. However, analysis in this study shows the numeric value of these results have little impact on the acquisition of an aesthetic experience. They simply validate the relevance of inter-disciplinary opportunities to increase retention of mathematical content and its relationship with studies of art and design.

Data collection. A mixed-method approach included participatory activities in the form of semi-structured observation, interviews and student online questionnaires (pre and post testing). Some quantitative data was collected by way of closed answer questions embedded within the pre and post test to determine factors related to perception of mathematics ability and student understanding of the notion of the "aesthetic" and the "aesthetic experience". The responses to the online survey and

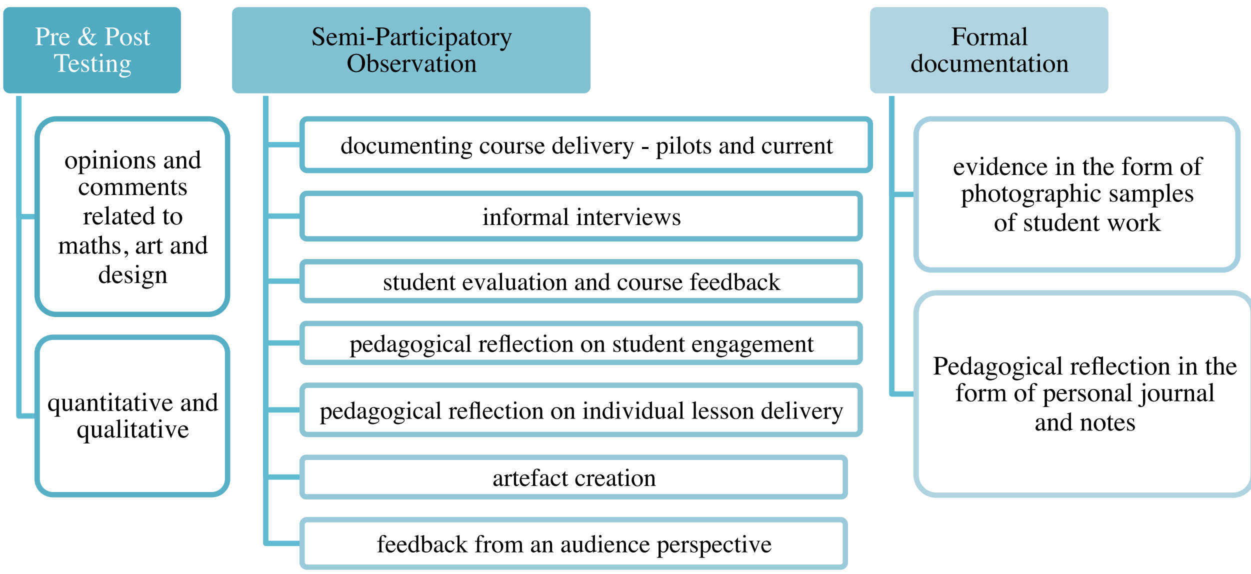


Figure 4: Data Collection & Management

questionnaire are analysed in the discussion section of the study. Additional graphic representations in the form of *word clouds* have been utilised in support of the data analysis.

Qualitative Analysis. The implication for gathering and interpreting hard data as opposed to soft data for this work was important to consider. Taking the richer, ethnographic approach resulted in a deeper understanding of the gestalt experience during the process of documenting student progress. Thus the approach was necessarily interpretivistic. "All research originates from some view of reality, which means that there are different ways of confirming our understanding (ie knowledge)" (Hart, 1998, p. 51). If epistemology is defined as "what can be accepted as real" and ontology describes "what reality is", then this was imperative in that I gathered alternative views and applied different strategies using qualitative methods in order for the data to be validated and true to the experience.

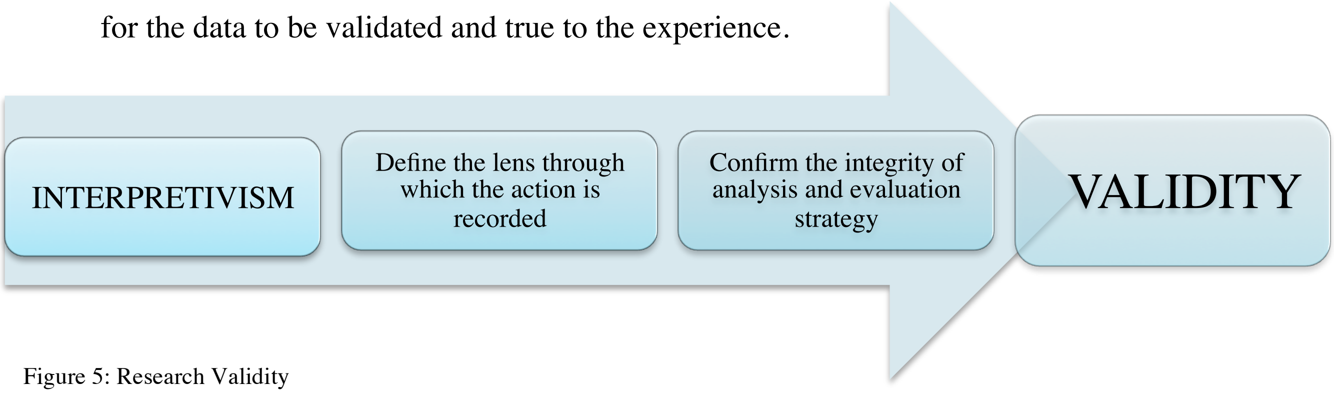


Figure 5: Research Validity

Quantitative Analysis. The notion of how to visualise data in this study was dependent on the provision of closed questions inherent in the pre and post test situation. These were developed as a Likert-type scale, requesting whether the students were proud of what they achieved and/or how the mathematics they applied was integral to the experience of achievement. This type of data is valuable to quickly ascertain the intensity of their experience over time while also accommodating neutral statements. Analysis of the non-respondents was also collated and has been included in the discussion section and appendices of the study.

Retrospective. The study was necessarily retrospective, in alignment with autoethographical models. Although the collective intention was prospective, in that the study looks for outcomes, the intepretivistic nature of documentation leans towards a more reflexive approach based on recording actual events. Since the study presents analyses of experience, or personal stories, it does not strictly test a preconceived theory in order to prove the theory "as is common in deductive research methods. Instead, the researcher begins with a general field of study and allows the theory to emerge from the data" (Pace, 2012, p. 7). Thus, the grounded approach relies on gathering data from the pilot POP experiences that evolved and supported the coursework for the project on which this study is based. The theory emerged from the data and "it refers to theory that explains how and why something happened – theory that yields conjectures and a potential basis for subsequent research" (Pace, 2012, p. 7).

Longitudinal. Similarly, a longitudinal study provided the opportunity for pilot programs to be developed and delivered in order to observe and record ethnographic elements such as cohort similarities and differences, amendment to pedagogical delivery and access to emerging technologies. The benefits of a longitudinal study allowed for the evolution of practice as compared to a cross sectional study that would have been dependent on factors that are unpredictable and uncertain.

Conclusions drawn from research relate to the overarching aim of the study: to document the journey I took as teacher-researcher to ascertain who benefitted from the *Possibilities of the Parabola* experience. Self reflection warranted questions connected to how the experience was manifested and how it contributed to student knowledge and understanding of 'aesthetics'. These results have validated the reasons behind my actions however it will always be the necessary scrutiny of others "who will endorse, or refute, my claim to knowledge" (Whitehead, 2006, p. 18). The next chapter outlines the story of the delivery of the *POP* project from its initial concept and pilot activities to completion of the most current iteration.

THE STORY

THE PROJECT

This is the story of a curve. It tracks the development of knowledge and understanding of the curve as it occurs within participatory, evolutionary learning contexts. The story describes the manner in which complexity encourages learning environments to be viewed as communities of practice where learning takes place collectively within a multifarious system of relational actions and responses. In the context of this study, *design* is incorporated in the complexities of arts education as well as being situated firmly in the area of applied technologies. The ideas underpinning the structure, scope and sequence of learning and teaching within this study are transferable between arts and design, and technology environments. This is not a study of arts and design based practices but rather a story of how a single mathematical curve can be introduced as the foundation for creating and making. Gaining a full understanding of how and why this curve exists in theory and practice has essentially provided mathematics integration by experience and not simply syllabus outcomes. For the purposes of this study, circumstance prescribes that the coursework lies within a Design and Technology context. However, the mathematics integration can be described as a synthesised cross-curricular approach that fosters and further develops knowledge of a non-arts based subject area through the understanding, appreciation, and production of artefact.

DEPARTURE POINT

The first parabolic 2D designs were created as part of an original *TH!* task related to Cartesian plotting and graphing. This was a brief introduction to coordinate plotting on a two dimensional plane – the *Cartesian Plane* – named for 17th century French mathematician René Descartes' significant contribution to geometry. "This was by no means his sole contribution to mathematics, but it was his most important" (Russell, 1961, p. 545). Students explored the mathematical explanation of the graphing of functions, concentrating on the intercept method. Intercepts are where a graphed line crosses the x and y axes of a 2D plane. (Graphing straight lines is an outcome inherent in the *Mathematics Stage 4 syllabus* therefore students had prior knowledge of a coordinate system.)

The class activities began with graphing mathematical functions by hand. (See Appendix 1). During this exposure to Cartesian plotting and graphing, the TH! students provided the basis for the ideas that would form the first POP pilot project. Using specialist maths software, TH! students continued to plot and graph curves in a digital environment. These explorations fell outside the initial signature curve of *parabola* and were related to the next creatively challenging graphing activity. That's another story. Nevertheless, it was the parabola that thrived in the development of an alternative Stage Five elective course currently aligned with the *NSW BOS Design and Technology (DT) syllabus*.

Using the 'hype' from hyperbolically, the DT course was appropriately named *Design Hype*. Ideas related to the development of the POP unit of work were aimed at the continued effort to drive innovative curriculum, while simultaneously abiding by requirements related to the achievement of set outcomes. (See Appendix 2). The ideas settled in the DT environment because my position as Head of Design and Technology (and one of the Stage Five teachers) allowed me some flexibility and freedom in the design and delivery of the pilot POP units of work and its current iteration on which this study is based. However, POP includes appropriate flexibility providing relevancy for application in a Visual Arts or Visual Design context. For the purposes of an auto-ethnographic approach to this study, the context of Design and Technology syllabus requirements must be used within which formative and summative assessment took place in a typical creative classroom environment. Conscious planning, development and description of this study required me (the teacher-researcher) to be "a complete member of the social world under study" whilst simultaneously engaging in "analytical reflexivity, demonstrating an awareness of the reciprocal influence between [myself, my] setting and [my] informants" (Pace, 2012, p. 5). My teaching responsibility required my constant vigilance related to student outcomes and achievement as well as observing and collecting data for this study. Hence, the stretching of curriculum boundaries without losing sight of specific aspects of the rationale accompanying the *Design and Technology syllabus* provided by the NSW Board of Studies.

Hybrid Thinking. More generally, when considering designing from a mathematical standpoint, it was necessary to explore the current Mathematics syllabus requirements for students in the same stage. The Australian Curriculum, interpreted through a NSW Board of Studies filter, provides a range of support for developing cross-curricular understanding between mathematics and design in particular. General capabilities related to Numeracy are mandated across all new and proposed Australian national curricula. Extracting detail to support a project such as POP was necessary to fulfill the intention of the pedagogical practice and as a consequence, student experience, acquisition of knowledge and understanding. The current Mathematics syllabus explicitly promotes General Capabilities in Numeracy: "In Measurement and Geometry, there is an opportunity to apply understanding to design" (ACARA, 2012a, p. 2). This capability can be openly linked to research activities inherent in the POP project addressing the Design and Technology syllabus aim "to engage students in technological innovation and the world of design" (p. 10) Similarly, the *Australian curriculum for the Arts* "aims to develop students' creativity, critical thinking, aesthetic knowledge… expressing and communicating ideas… use of innovative arts practices with available and emerging technologies" (ACARA, 2014b, p. 4). The use of CADCAM technologies in the hybrid Maths/Design context of POP, provided a natural progression from hand working when applying mathematical concepts in art and design. General Capabilities within all Australian Curriculum guidelines broadly mandate the application and use of Information and Communication Technology. The Mathematics curriculum specifies that "students develop ICT capability when they investigate, create and communicate mathematical ideas and concepts using fast, automated, interactive and multimodal technologies" (p. 2). The quick response to appreciating the elegance of Cartesian plotting and graphing when experimenting in a digital environment using dynamic geometry software, supported the opportunity for a personal aesthetic experience, as the output was immediate and visually appealing. General capabilities outlined in *The Arts curriculum* suggest that students "can enhance their ICT capability as they generate ideas and explore concepts and possibilities by exploiting available technologies" (p. 15). Although the impression of *exploiting* conjures a negative connotation, in current learning and teaching environments, the use of ICT technologies can and does enhance the speed at which many creative tasks can be actioned while also providing a sensational personal moment if the product of learning is visually aesthetic.

Inclusions in the rationale for *The Australian curriculum: Technologies* propose "this learning area encourages students to apply their knowledge and practical skills and processes when using technologies and other resources to create innovative solutions, independently and collaboratively, that meet current and future needs" (ACARA, 2013b, p. 4). As the POP pilot courses and current program have evolved, the move to online task delivery and instruction via the school's Moodle site was a necessary transition. Online activities and assessment have reinforced the current cross-curriculum priority addressing sustainability. Students have the opportunity to model best practice in terms of digital task submission. Avenues for collaboration have been expanded due to the community nature of utilising online forum models to share research at the investigatory stage of the project. Learning in a digital environment maintains the curriculum aim stating "All young Australians should develop capacity for action and a critical appreciation of the processes through which technologies are developed and how technologies can contribute to societies" (ACARA, 2013b, p. 4). Digital learning and teaching facilitated the delivery of the POP pre-test with data transferal rendered fast and accurate. Later in the study, I document the level of student appreciation for the use of new and emerging technologies in production.

The aim of the POP pre-test was to ascertain levels of engagement with mathematics, art and design before the unit of work proceeded. Similar questions were posed in the post-test after the POP project was completed (See Appendix 3).

Design and Technology projects developed for students in Stage Five in NSW are required to address a range of focus areas of design. The *Possibilities of the Parabola* design project was described as focusing on Conceptual Product Design in the program of learning register. The time allocated for POP was twelve weeks although some flexibility was necessary according to pressures of external commitments and school based activities preventing students from attending all scheduled classes. The description of the POP unit of work has not changed between pilot and the current course, however, individual tasks have evolved and improved over time in order to satisfy course requirements more adequately and complete all tasks within an appropriate time period. Ongoing changes were also necessary when considering the dynamics of the class cohort. Each year has provided a different experience for students with much diversity in terms of interest, engagement and skill.

WHY CURVE?

Curves are, by their very nature, easy on the eye. According to Chris, a contributor to the science blog *Mixing Memory,* “people dig curves” (Chris, 2007). It seems that curves are the preferred choice over sharp angles. Chris refers to the findings of Moshe Bar and Maital Neta published in *Psychological Science*, (2006) in his argument supporting the curve as a preferred choice. Bar and Neta (2006) hypothesise that "sharp transitions in a contour might convey a sense of threat, on either a conscious or a non-conscious level, and thus trigger a negative bias” (p. 645) This preference, they argue, should show up very quickly (after viewing something for a few milliseconds) (para 2). Results from Bar and Neta’s research provide a basis for my pedagogical practice related to setting activities within the POP unit of work in which students gather data to justify the situation in which their design activities are positioned. POP students in the pilot and current courses were required to explore their own and other human responses to curves used in art and design before considering any mathematical or engineering theory.

Bar and Neta (2006) used images of over 180 objects made from “meaningless patterns and not associated with positive or negative emotions”(Chris, 2007). Each pair contained either curved or sharp contours. Participants were exposed to the images for less than a second and were required to state which image they preferred. “People liked the familiar objects more than the meaningless patterns, but for both item types, they liked the curved items more than the sharp-angled items. The items with both curved and sharp-angled contours fell in between the two” (Bar & Neta, 2006, para. 2). The implications for design and art can be evaluated with reference to human response and it was with this idea that the students undertaking the POP project began their journey of exploration into this type of contour. Chris (2007) surmises that these results have particular implications for art and design while also being of great importance to advertising and marketing. As the participants were exposed to the images for very short periods of time, they were not conscious or aware of the reasons for their personal choice. Perceptually, the formal structure of the simple shape of curves has often resulted in the perceiver’s response being similar to the creator’s intention.

> "Isomorphism," that is, the structural kinship between the stimulus pattern and the expression it conveys, can be shown most neatly in simple curves. If we compare a section of a circle with a section of a parabola, we find that the

> circular curve looks more rigid, the parabolic one more gentle. What is the cause of this difference? It derives from the geometric structure. The constant curvature of the circle obeys a single condition: it is the locus of all points equidistant from one center. A parabola satisfies two such conditions: It is the locus of all points that are equidistant from one point and one straight line. Because of this twofold dependence the curvature of the parabola varies; that of the circle is constant. The parabola may be called a compromise between two structural demands. Either condition yields to the other. In other words, the rigid hardness of the circular line and the gentle flexibility of the parabola can be derived from the inherent make-up of the two curves. (Arnheim, 1954, p. 450)

Alignment. All unit tasks and activities within the POP design project are aligned with *Stage 5 NSW Design and Technology syllabus* (2003) outcomes. (See Appendix 2). As a precursory activity when starting the POP design project, students discussed the nature and definitions of design, technology and appropriate terminology, opportunities for new and better solutions and requirements of end-users and stakeholders. They analysed designed products with respect to inherent form and function and discussed design in contemporary settings, identifying and summarising information from a range of sources. 'Form' and 'Function' activities were undertaken in order to review and familiarise students with specific Design and Technology terminology. This not only encouraged the appropriate use of electronic information and communication tools, it provided for many, a first glimpse into the global forum for creative practice on all levels. Students accessed a range of websites to assist in their research, some of which were supplied. They were searching for artefacts or designs that used curves. Students analysed the designs according to a scale of form and function. The notion of high and low aesthetic characteristics was introduced into the ensuing discussion.

> People who spend much time with art come to appreciate increasingly the affective, historical, and cultural aspects of the work they are viewing, occasionally more than they enjoy its purely visual aspects. As one professional involved in the Arts expressed it: "[Works of] art that I personally respond to…have behind them a lot of conceptual, political and intellectual activity…. The visual representations are really signposts to this beautiful machine that has been constructed, unique on the earth, and is not just a rehashing of visual elements, but is really a new thought machine that an artist, through visual means and combining his eyes with his perception, has created". (Csikszentmihalyi, 1990, pp. 117 – 118)

Students began to formulate their own perception of aesthetic understanding, which in some cases allowed them to enter the experience of the creator whose design was the object of analysis. The requirement to design and create, inherent within the POP project, was an attempt to encourage students themselves, to be part of this ‘beautiful machine’.

Ways of creating. Student research tasks in the area of Visual Arts and Design have the potential to enhance learning by suggesting ways of creating. “No matter the setting or the moment, aesthetic experience vitalises our lives with meaning and joy” (Kerdeman, 2009, p. 88). It is a difficult task in a classroom to shape an environment in which pure creativity is flowing. However, this study aimed to track the formation of such a situation. Unit Task One formed part of the summative assessment for the POP project. (See Appendix 4 and Figure 6). Collectively, the class deconstructed sample designs into simple terms; their understanding of form (aesthetics) – how it looks, and function – how it works. Working in pairs or small groups, students undertook an introductory research activity that required analysis of the use of the “curve” in a variety of found designs in more detail. This brief but intense research activity was recorded and began the journey of documentation for the POP project.

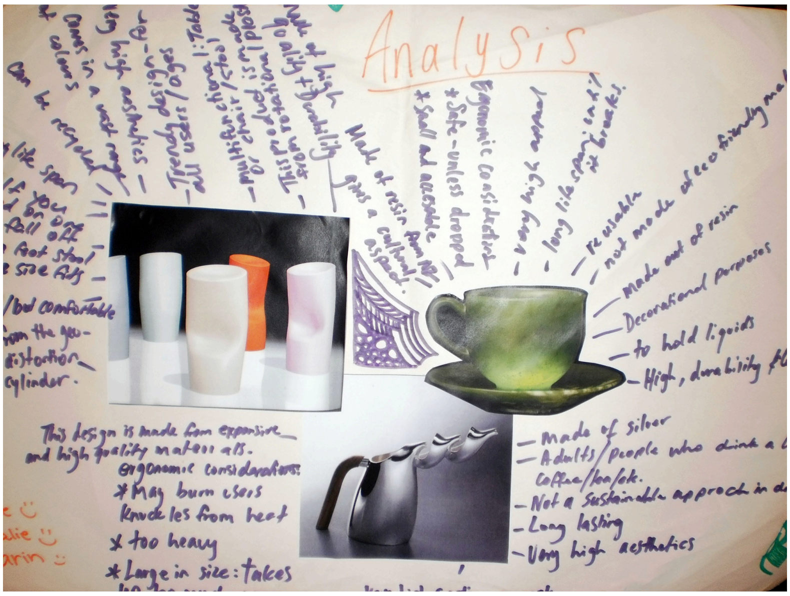


Figure 6: Form and Function Analysis

Task One (Part A) – Why Curve? supplied an effective conduit between looking at something and understanding what it does and how it was made. The results of the student peer presentations validated the idea that increased exposure to creative and innovative design (and art) is an expansive activity. “So even though a child might not develop an early interest in a domain in order to become creative in it later, it does help a great deal to become exposed early to the wealth and variety of life” (Csikszentmihalyi, 1996, p. 163). Although Csikszentmihalyi may be referring to the early years, curiosity in a classroom context at any age, particularly in the action of researching a topic, can lead to a small but not insignificant aesthetic experience. Certainly, some of the students in the POP course became excited about developing their own design ideas according to the high aesthetic principles being presented and discussed during the *Why Curve?* activity. The "scope to develop high order thinking" (DT Syllabus, 2003, p. 8) was evident. Constructivism advocates that a teacher should collaborate with his or her students in order to help facilitate the construction of meaning in students. Active interaction was the key to facilitating appropriate discussion related to the student presentations. This was not a context necessitating passive adjudication. There were many students across all cohorts participating in the POP projects, whose intention was to impress their peers and teacher with detailed analyses of curved designs. (See Appendix 5). Understandably, these students could be described as true investigators and already demonstrated skills in high order thinking. "Each child becomes interested in pursuing whatever activity gives him or her an edge in the competition for resources – the attention and admiration of significant adults being the most important resource involved" (Csikszentmihalyi, 1996, p. 158). The significant adult in this situation was me, the teacher, in the cultural context of a class of students at the beginning of a journey in shared knowledge. They were already on their way to attaining a range of aesthetic experiences.

It is important to acknowledge the difference in student's attitudes to learning at the beginning of a unit of work and as they progress through a variety of activities and assessments. My observation of their findings during the initial POP research task was that an early curiosity often drove creativity and allowed for a more confident approach to idea exploration in their own work. Csikszentmihalyi (1996) draws our attention to "a more than usually keen curiosity about one's surroundings" (Csikszentmihalyi, p. 156). being essential for later creativity. Strong engagement from the point of departure is necessary for a healthy approach to the imminent mathematics.

WHY PARABOLA?

According to Alex Bogomolny (2004), who writes on the website *cut-the-knot.org*:

> the term **parabola** comes [Schwartzman (1994), p. 158] from Greek *para* "alongside, nearby, right up to," and *-bola*, from the verb *ballein* 'to cast, to throw'. Understandably, *parallel* and many of its derivatives start with the same

root. The word *parabola* may thus mean 'thrown parallel' in accordance with the definition". (Bogomolny, 2004, para 4)

Common language has benefitted from the descriptive names of such mathematical theorem. The Greek *hyperbola* – thrown beyond, *ellipse* – falling short and *parabola* – thrown beside, are all related to mathematical notions of distance or connection.

> In rhetoric, 'hyperbolic' speech is the kind that goes beyond the facts, 'elliptic speech' falls short of them, while a 'parable' is a story that exactly fits the facts. I might add, that the word 'parabola' became 'parler' in French, meaning any kind of talk, and has given us the English words 'parlor' and 'parliament'; while in Portuguese it suffered a metathesis to 'palabre', which has given us the English 'palaver'. It is amusing [to me, at least] to contemplate the facts that 'parable', 'parlor', and 'palaver' are all etymologically the same word as 'parabola'. I could go on about the etymologies and history of many mathematical words for ages, but will stop here before I bore too many readers. (Conway, 1995, para 2)

It was important to dispel the idea of *boredom* and its relationship with mathematics being the front-runner description in the opinion of some students. In an attempt to expose the relationship between subject areas and teachers whose expertise relates specifically to those, a new collaboration was born. Maths teachers were invited into design classes to introduce the theory underpinning the creation of parabolic curves. In researching *The arts in schools*, Sir Ken Robinson (1982) provided analysis of the importance of the Arts being disciplined forms of enquiry and that "the uniqueness of human existence consists, above all, in our capacity to appraise and communicate with each other about our various experiences of the world" (Robinson, 1982, p. 18). The point in question is how to vary experiences. Inviting pure mathematics into an arts context was a challenging idea for the students. What resulted was a contagion produced by the excitement exuded by the mathematics teachers as they instructed. They were in their element, having their own aesthetic experience. Effectively, their language corresponded with the language of the Arts "in which our idea of beauty, grace, harmony, balance, harshness, stridency and ugliness are conceived, formulated and expressed. We call this our aesthetic awareness and mode of discourse" (Robinson, 1982, p. 18).

Connections. Each time this collaboration took place, I observed the approval of students as they acknowledged firstly, that a single teacher was not required to possess **all** the knowledge and secondly, that their teachers had relationships based on shared pedagogical intention. The human aspect of these initial activities added to the rich learning experience for the students. Their appraisal of the connections between seemingly disparate disciplines was formed more cohesively. Robinson (1982) considers Aristotle from *Nicomachean ethics book I*, 1094, b25:

> … who saw it as a mark of the educated person to be able to recognise the different ways in which our perceptions of the world are organised and communicated and to understand the various conventions and standards of judgement in each of them. (p. 19)

Subsequent peer-to-peer discussion revealed that our inter-disciplinary exposure inherent in the early POP activities symbiotically benefitted the mathematics teachers, particularly in the provision of methods of introducing the theory related to conic sections.

The parabola, one of the conic section family members of the classical curves, depicts primarily a relationship between variables x and y. The parabola, often called the U-shaped graph, apart from having useful properties such as symmetry and a vertex (being the minimum or maximum value) that represents the axis of symmetry, retains simplicity of form that easily supports the acquisition of understanding related to the notion of aesthetics. As stated earlier, people like curved shapes. The "beautifulness of the shape of curves and surfaces are well-studied topics, however, most conventional researchers seem to avoid a mathematical approach" (Kanaya et al., 2007, p. 7). The gestalt law of continuity maintains that straight or curving lines should follow the smoothest path possible. Points along the lines are seen as belonging together, being united. Other gestalt principles include symmetry. This principle connotes stability, consistency, structure and order. In 1958, Fred Attneave proposed many of the gestalt principles of perceptual organization pertain essentially to information distribution.

> The good gestalt is a figure with some high degree of internal redundancy. That the grouping laws of similarity, good continuation, and common fate all refer to conditions which reduce uncertainty is clear... (Attneave, 1954, p. 186)

Therefore it is a facile, natural task for the human brain to understand and comprehend qualities of a curve.

Non-linear. At this point in the timeline of POP course delivery, a non-linear approach to assessment tasks was necessary due to the availability of the maths teachers for instruction. In the first pilot POP course, students were required to research designs containing indiscriminate curves. Although this allowed for broad discussion on curved shapes in general, the subsequent iterations of POP contained a

Part B inclusion to this initial task in order to narrow the focus on curves to the single theory of *parabola*. Therefore Task One, having only undertaken Part A, remained incomplete. It was necessary to introduce the theory of Cartesian plotting and graphing before completing Part B in order for students to have some knowledge, both conceptually and visually, of the parabola. Crilly (2007b) attests “It’s easy to draw a curve… But what if we were to ask ‘What is a curve?’ the answer is not so easy to frame” (p. 88).

Enter the maths teacher. *Design Hype* students were introduced to the mathematical functions associated with graphing a parabola. Students were shown a brief presentation displaying a variety of curves used in building designs (See Appendix 6). The application of the parabola in scientific contexts was also a main point for discussion. Students of all mathematical ability gained an understanding of light and data transmission by way of diagrammatic explanation. Aspects of their newly acquired knowledge emerged in the depth of research they undertook as Part Two of Task One. The implication of constant cross-referencing of information as the POP project progressed supported a deeper level of relational experience as well as transferable knowledge across key learning areas (such as Science and Technology). This approach is grounded on Piaget’s theory of cognitive development where children are active agents and gain knowledge by “acting or otherwise operating on objects to discover their properties” (Australian Curriculum, 2012, p. 224). This constructivist approach allows students to make and coordinate meaning for themselves that is valid and relevant for their individual experience.

At this point in the POP lesson sequence, it was necessary to introduce the design brief for the project (Task Two) (See Appendix 7). This activity was inherent within *Design and Technology syllabus* outcomes. Writing a proposal outlined the requirements of the design project and formed the basis for much of the evaluative process that took place in consequent research, development and production tasks. However we would return to flesh out the specific criteria for the proposal after completing additional exploratory activities related to curves. My intention at this stage was simply to introduce the concept of *vessel*: a hollow or concave article for holding liquid or other contents.

Experiential. Set as Task Three in the POP lesson sequence, students learnt, under the guidance of a mathematics collaboration, to plot and graph a parabola by the *table* *of values* method. Using a resource entitled *Graphing workbook*, students calculated the solutions to the functions, then continued to plot the coordinate x, y values on the supplied graphs (See Appendix 1). Undertaking this activity by hand was an appropriate method of introduction as it built on the experience of former graphing activities inherent within the *Stage 4 Mathematics syllabus*. The directions on the worksheets were clear and unambiguous. Students of higher ability, who finished the task quickly, were encouraged to assist peers in their attempts. The communication occurring at this time is worth commenting upon. In an arts and design context, able maths students were presented with the opportunity to take pride in their ability outside of the typically scientific or academic setting. This student-centred approach supported an experiential context for learning and was helpful in situations where time was short. However, it reported a danger in that some students may have coerced their peers to complete the work for them. Thus, a gap in knowledge became apparent as the project progressed. A subject-centred approach brought our focus back to the parabola.

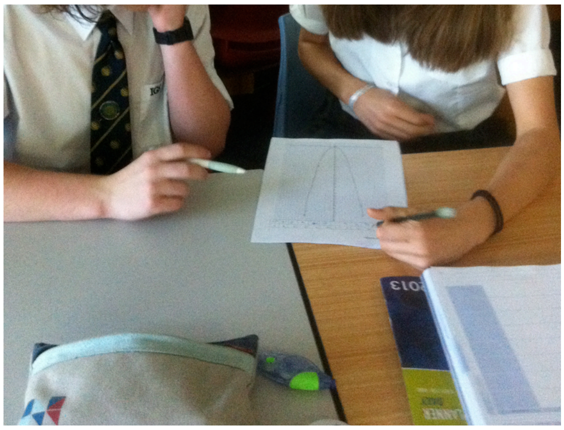


Figure 7: Peer to peer graphing assistance

The creation of inter-disciplinary experiential learning environments and opportunities may not be so aligned with current performance based movements in education such as high stakes testing, and pre-specified intended outcomes and standards. “To aspire for less is to court professional irresponsibility. We like our data hard and our methods stiff—we call it rigor” (Eisner, 2002, para. 10). Eisner further suggests that the current movement and the messages its policies send to students may undermine deeper educational values. “The values about which I speak include the promotion of self initiated learning, the pursuit of alternative possibilities, and the anticipation of intrinsic satisfactions secured through the use of the mind” (Eisner, 2002, para. 11). Chronologically, the hand graphing activity was extremely useful for placing the mathematics theory in the context of art and design as the integrity of hidden historical and conceptual connections was beginning to emerge. A general perception of value was likewise emerging. Creative projects derived from mathematics are usually quite different from mainstream art education but are not dissimilar to design. The aim of the POP project was to include a measure of visceral response while maintaining a highly intellectual approach to designing and making. At this early stage of the project, ideas were formulating but not yet actualised. Students began to ask questions about what they would be making and when would they start. Pacing the exploration of the parabola was necessary for students to retain a rigid foundation in the maths before plunging headlong into production. My design teaching experience has taught me that the latter often produces sub-standard work.

The very nature of constructivist learning is to enable students to "construct knowledge out of their exploratory actions on the environment" (Csíkszentmihályí, 1990, p. 149). The assumption that students learn best in conditions reliant on transmitted pedagogy when information is transmitted directly from teachers or books to students cannot be applied in a cross-curricular context. "Teacher-textbook transmitting styles are anachronistic to the field of classroom pedagogy and the way educators have since understood how students learn" (Csíkszentmihályí, 1990, p. 23). While it may be necessary for the majority of mathematics teachers to employ textbook teaching, a myriad of creative resources are available in support of textbook content. My collaborators introduced me to a range of mathematic software, they used to draw diagrams that not only had potential in the acquisition of theoretical knowledge; it also contained the power of the visual. Typically, students enjoyed using digital tools that provided instant response to their actions. It added to the intensity of their learning experience. This is what we explored next in the POP project.

No uncertainty. Returning to the curves, the common visual character of curves is that they are evenly distributed. Ichiroh Kanaya (2007) considers "If the changes of the curvature are constant (mathematically, if the second derivative of the curve is monotonic increasing/decreasing), the curve is *beautiful*" (p. 2). In mathematics, the term *monotonic* can be defined as a function that preserves a given order. The parabola employs monotonic schemes that are uniformly convergent. Visually, there is no uncertainty related to its form.

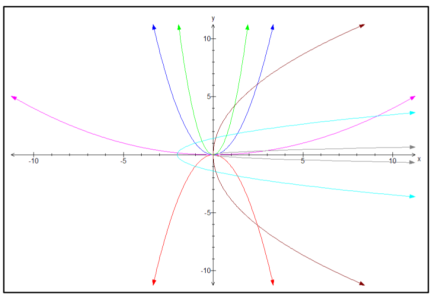


Figure 8: *FX Draw* sample

Students were now required to plot and graph parabolic curves using specialist software, in this case, *FX draw*. Some students were familiar with the interface due to their mathematics experience yet the functions were different to typical vector drawing software utilised in an art/design environment. They were supplied with an electronic worksheet displaying graphing and analysis tasks to complete (See Appendix 8). Direct instruction on the use of the software was used to collectively complete the first graph. All students across the chronology of the POP project delivery were able to complete the basic parabola set (See Appendix 9). Students continued to experiment with making new curves from invented equations, effectively making mini digital 2D artworks derived from numbers. (Figure 8).

In each iteration of the POP project, the *FX draw* activity was the first time students experienced this type of data representation. The collective energy produced by almost total student engagement was delightful to observe. The concept of *elegance* in mathematics was thus emerging. As Attneave (1954) attests, the curve is attributed with a high degree of *redundancy* when perceived visually. Therefore the parabola and its relationship with an awareness of aesthetics is working with an archaic aspect of visual perception and is derived from the simplicity and elegance of mathematical proof.

Questions of order and relation. As the POP unit of work evolved, additional resources were accessed and implemented to augment learning activities. Online resources such as the *Khan Academy* were explored for mathematical explanations and a range of online design magazines were accessed to support student understanding of the elements and principles of design. Practical application of mathematical theory was introduced to the current POP course delivery where students formed cones from non-toxic modelling clay, which led them to the discovery of conic sections. Modelling in 3D enabled students to gain a coherent understanding of where in space the parabola actually occurs. To date, they had only experienced a 2D level of understanding (Figure 9).

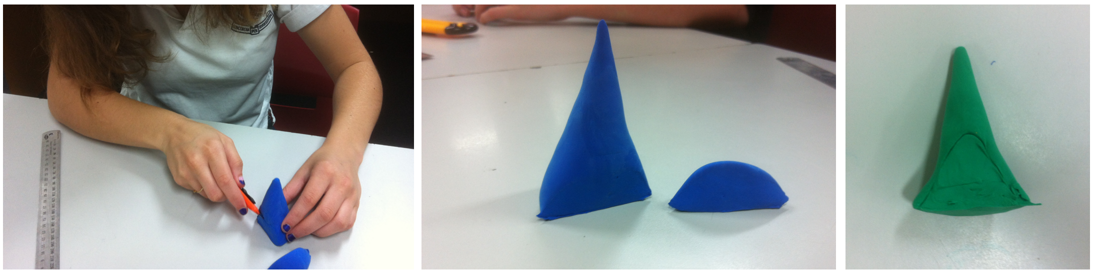


Figure 9: Making conic sections

Ideas related to the magnitude of creating art or design based on mathematical methods underpins Christopher Alexander's (1964) views on the *Synthesis of form*. Alexander attests that:

> Modern mathematics deals at least as much with questions of order and relation as with questions of magnitude. And though even this kind of mathematics may be a poor tool if used to prescribe the physical nature of forms, it can become a very powerful tool indeed if it is used to explore the conceptual order and pattern which a problem presents to its designer. (Alexander, 1964, p. 6)

One of the limitations with this explanation is that Alexander's work is based on pre-computational design. Contemporary studies in art and design make use of much mathematical theorem to inform praxis, frequently utilising current and emerging technologies. At a basic level, students may be introduced to connections between art, design and mathematics by the exploration of concept and theory such as tessellation (M. C. Escher) (Figure 10), iteration (fractal geometry), the golden ratio and such topics. More recently, digital approaches to art and design have included practices such as the interpretation and representation of data integrating GIS (Geographic Information System) mapping and algorithmic data visualisation. Art and design of this nature engages audiences both virtual (the online audience) and physical (within gallery spaces) in the contemporary arts environment. At a basic level, without coding, students were enacting their own thought collaboration to support these shifts in art and design syntheses.

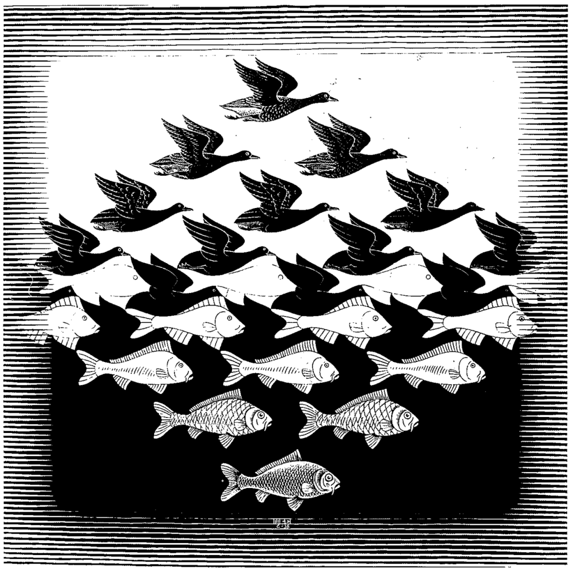


Figure 10: M.C. *Escher Sky and Water I* (Seckel, 2004)

Collaborations between mathematicians, physicists and artists have emerged in environments not typically reserved for the intellectually elite or culturally enlightened. Some of the ideas explored within these collaborations echo the journey I have taken with students to expand their understanding of the possibilities of association between maths and a range of non-maths contexts. Words and phrases appear to reiterate in the dialogues taking place internationally. This "collective psyche" phenomenon supports the idea that interdisciplinary rigour can be achieved at all levels of education. *A beautiful elsewhere* exhibition held in Paris in 2011 brought together prizewinning mathematicians and artists who asked the question "What does mathematics look like?" (Austen, 2011, para.1). Although ambitious, the collaborators included mathematicians and physicists with impressive credentials including three Fields medal winners (the international medal for outstanding discoveries in mathematics). The curator, Thomas Delamarre, hoped it would do nothing less than provide an "answer to the abstraction of mathematics" (Austen, 2011, para.2). The artists checked back with the maths experts throughout the process "to ensure that the underlying figures had not been distorted" (Austen, 2011, para.3). The activities undertaken by the POP students although on a much smaller scale, contained the potential to create forms of the same aesthetic significance as those exhibited in the Paris exhibition.

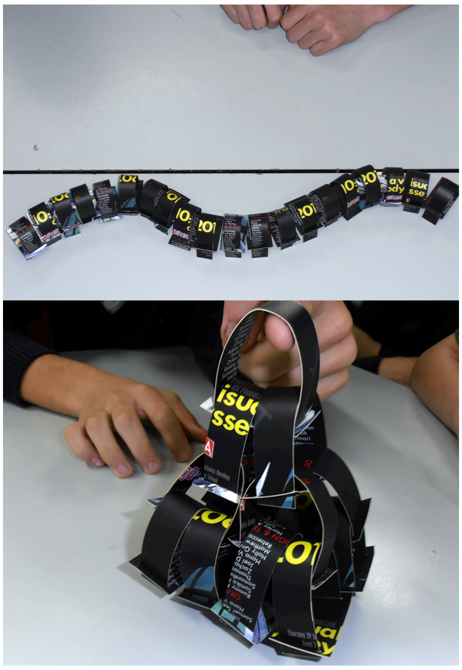


Figure 11: Parabola iterations group activity

Similar to the shaping and sectioning of cones, activities in POP pilot courses provided students with the opportunity to manipulate the parabola in order to ascertain a sense of repetition or iteration. This was a collaborative activity modelled on a game or competition where teams were required to design a form from a given set of small cardboard parabolas. As a group, the class critiqued the designs in terms of conceptual value and aesthetic appeal. The possibilities of how to work with this curve emerged collectively. The result of this activity became apparent in the individual design task later in the project as the group work supplied design ideas for less able or (less motivated) students (See Figure 11).

Concurrency. At this point in the POP project timeline, it was necessary to run concurrent assessment tasks to fulfil outcomes identified in the program. Essential research had to be completed without losing precious time dedicated to practical activities. Therefore, Task One - Part B was introduced, clarified and completed as homework. The pilot courses undertook this task as a straightforward assessment with a range of possible methods of presentation:

- interpreted information formatted as a word processed document

- in the form of an exported PDF document that will be shared with the other members of the class on Moodle
- poster presentation - hard copy (minimum of A3 size)
- digtial presentation - Prezi, Powerpoint or other.

The researched information was shared class-wide and provided essential content for examination revision purposes (See Appendix 10). This activity was necessary as the POP unit of work was formulated as an assessable NSW ROSA (Record of School Achievement) component for the 100 hour Stage 5 DT course. However the current POP class used a different model of research for this task. This different approach was much more transparent and richer in collaborative content. Individuals were required to contribute to an online forum on the school's Moodle site where all students in the POP course could read and respond to each other's entries. This model of submission complied with the collective agreement to consider sustainability at the local classroom level. Students could access the forum for exam revision in the same way as the previous years. Responsibility for this task was based on autonomy and personal motivation.

Research questions were scaffolded to address students of all learning ability. This guide was clear and unambiguous: How is the parabola used in the design context? What materials is the parabola made from in this context? Why is the parabola important in the design context? Consider:

its structural importance	its functional importance	its aesthetic importance

Most students completed the task without difficulty and found a variety of samples of parabolas used in a wide range of contexts, undeniably influencing the acquisition of knowledge related to this particular curve (See Appendix 11). Robinson (2010) proposes that curriculum must be made dynamic. Student use of the online forum was particularly pleasing for me to observe because some chose to clarify topic areas that we had already covered in class. Students engaging in POP research online were in charge of their own learning. They were effectively teaching each other and alerting me to the fact that I may have not delivered aspects of the POP content in a way that was accessible to all members of the class. "Critically reflective teachers recognise the error of assuming that good teaching is always signaled by the receipt of uniformly good student evaluations" (Whitehead, 2006, p. 17). Enhancing already learnt content allowed the entire class to create knowledge, collaborate, communicate, challenge each other and challenge me.

Criteria for Success. Aspects of the *NSW Visual Design syllabus* related to making describe visual design "as a system of symbolic communication through which particular forms of aesthetic, social, kinetic, mechanical and ergonomic information are transmitted" (Board of Studies NSW, 2004, p. 35). Student expectation is to employ strategies of planning and making visual design artworks "to communicate meaning" (p. 35). Similarly, Design and Technology students must realise the importance of establishing criteria for success for the development and completion of a design project including the impact of design restrictions and limitations. Understanding the parameters of the project is essential for creating appropriate criteria under which making and producing within the Arts/design context can be successful.

> The use of logical structures to represent design problems has an important consequence. A logical picture is easier to criticise than a vague picture since the assumptions it is based on are brought out into the open. Its increased precision gives us the chance to sharpen our conception of what the design process involves (Alexander, 1964, p. 8).

Students returned to Task Two of the POP design project. This task involved the establishment of appropriate criteria for developing and producing a vessel with reference to specific parameters and design requirements (See Appendix 12). Factors to be considered when establishing design criteria relate to explicit outcomes inherent within the DT syllabus; factors affecting a holistic approach to design purpose and production such as form, function, aesthetics, end-user aspirations and context, time factors: historical, contemporary and future considerations, quality and trends (*DT Syllabus*, 2003, p.12). The design brief was brief: To design, create and evaluate a vessel based on the study of the *parabola*. The target market was broad. The greatest challenge in setting a project combining mathematics and design was to not lose sight of the integrity of the maths. "The ultimate object of design is form" (Alexander, 1964, p. 15). At this stage, the students didn't know what they would be creating. They knew the context in which designing and making would occur but up until this moment, they had no understanding of the restrictions I placed on the project in terms of its design and construction. This was intentional. I did this in order to promote the scope of creativity via removal of certain construction techniques. The specifications included in the project required that the design:

- must be based on one or more parabolic curves;
- must be constructed without visible adhesives;
- maximum cubic size = 200mm (or by negotiation, determined by the materials used in construction).

Curiosity was aroused by the challenge although some students could not conceive the idea of making a structure without using glue. In his work on enhancing personal creativity, Csiksentmihalyi (1996) purports:

> the first step towards a more creative life is the cultivation of curiosity and interest, that is, the allocation of attention to things for their own sake. On this score, children tend to have the advantage over adults; their curiosity is like a constant beam that highlights and invests with interest anything within range. (p. 346)

The general buzz amongst the DH classes was associated with 'when can we start?' Thus the challenge for me was to manage the structured activities to enable the collective understanding to be firmly grounded in the mathematical theory underpinning the brief before rushing headlong into production.

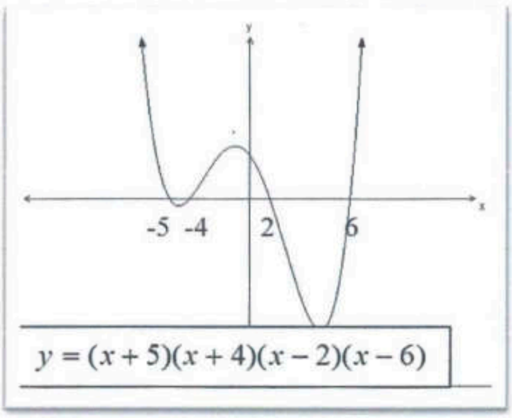
A gallery of curves. Using dynamic geometry software, students used newly acquired knowledge of the parabola to create a gallery of curves (minimum of 5) (See Figure 12). The creation of the curves was documented to demonstrate the progression of the design project. Using PMI analysis (Plus Minus Interesting), students were required to comment on the curves individually and outline the manner each might be able to be translated into a vessel design. These explorations were to form the basis of design ideas later in the unit of work. Observation of this activity led me to the conclusion that most students were excited about the translation of a range of mathematical functions (devised by them) into visual form. A sense of wonder was beginning to emerge as students took more risks in their exploration of the mathematics. Csiksentmihalyi (1996) proposes:

> without awe life becomes routine. Creative individuals are childlike in that their curiosity remains as fresh even at ninety years of age; they delight in the strange and the unknown. And because there is no end to the unknown, their delight also is endless. (p. 346)

Choose the five parabolic curves that appeal to you and paste them into the area provided. You must include the equation that produced the curve in the graphing exercise

Use the elements and principles of design to write a PMI analysis next to the curve outlining the potential for the curve to be used in a vessel design. PMI = Plus Minus Interesting

Example



Plus:
Sections of this curve could be used as a vessel. If the whole curve was to be used then it would have to be placed on an angle to be able to stand up.

Minus:
The dips at the base of the curve are quite narrow and may difficult to retrieve something out of it if it were a vessel. Using the whole curve would be unstable.

Interesting:
I like this one because of its combination of 3 parabolas although it looks as if there are only 2.

Figure 12: A gallery of curves

The creation of a gallery of curves brought Task Three to a close. A selection of students were very detailed in analysing the positive, negative and interesting components of their digital curve experiments while others were too busy playing. It was necessary for me to urge students to consider the value in what they were creating in reference to the project brief. In that way, the trajectory of the planned tasks was refocused on required assessment. Our next task was to increase the level of technology to lead the students out of a pure mathematical environment and into a 2D vector graphic world.

Original versions of vector experiment tasks were trialled in a Visual Arts context where students produced relief and intaglio prints. (See Appendix 13). The output for this activity melded into aspects of the DH POP course structure through its development. What we had to work with at this point was the knowledge of what and where a parabola is situated in the world both theoretically and in space. It is interesting to note that the forthcoming digital designs in the POP course were supported in strength by hand drawn experiments. These involved template parabola shapes (prepared by me) with a range of points of rotation along the curved edge. The potential for *spirographic* type images to be produced by hand, on paper, was a playful precursor to the more technical discipline involving drawing with vector software. This activity provided essential catch-up time for students still creating and analysing their gallery of curves. It did not reside so much in the realm of differentiation as in the area of possibility: what can we visually invent with the parabola? From both mathematical and visual reference to "invention", it can be argued that the experience of doing maths and creating art is purely subjective. "…mathematicians often feel they have created something of their own" (Zagier, 2011, p. 92) Students using the acrylic parabola templates to draw on paper with coloured pencils were engaged in their own journey of discovery, ignorant of the fact that their creations were not new. The point is:

> for most mathematicians… at each moment and for each problem, there is a huge number of possible deductions from the axioms and from what is already known. All of these deductions are in a sense, 'already there', but one has to constantly decide which direction to take, and it is precisely these different choices that reveal the abilities, tastes and personality of the individual. (Zagier, 2011, p. 92)

Therefore in the context of play and engagement, student collaboration, critiquing and complimenting their own and each other's work produced a delightful environment from which to launch into digital experimentation where we basically were doing the same. (See Figure 13, 14 & 15). Zagier (2011) validates his idea of the individual

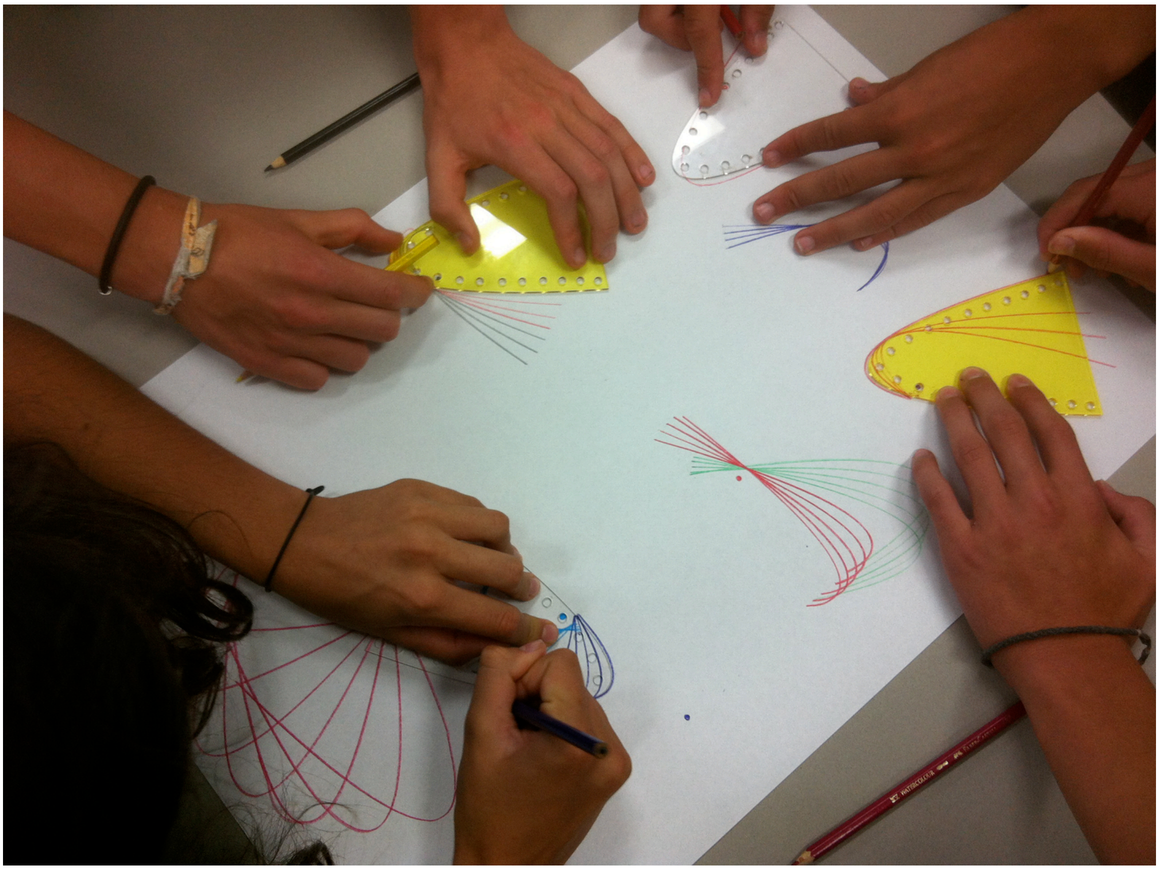
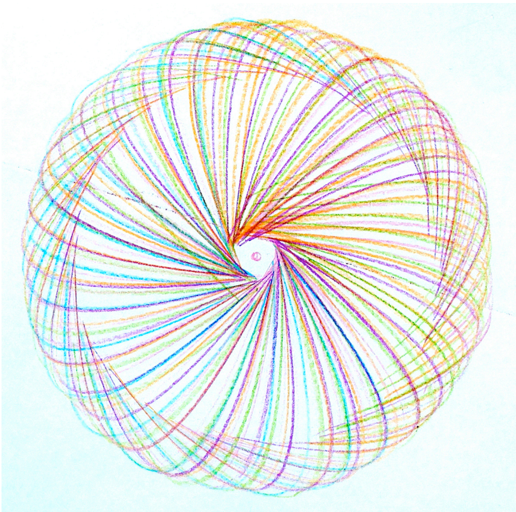


Figure 13: Hand drawing parabolas in groups

experience with the beautifully formulated saying by the French mathematician, Gustave Choquet: "the theorem one is seeking has existed from time immemorial, but that in order to *discover* it, one must *invent* a path" (p. 92). The DH class now faced working in a digital 2D environment in which to manipulate their chosen parabola. *Task Five* was rich in expectation.



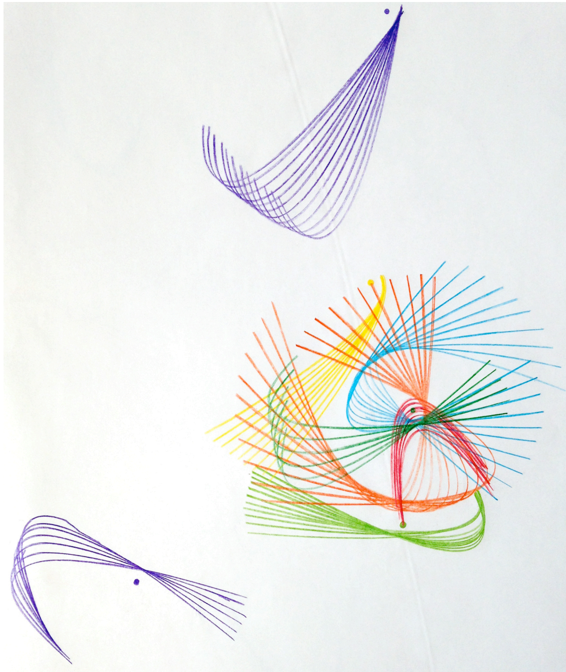


Figure 14 & 15: Hand drawn parabolas

Mathematical parent. Using the gallery created in Task 3, students chose a minimum of three individual parabolas to transcribe into vector graphics using vector drawing software. The individual functions of the curves were also to be recorded in order to identify the relationship between the completed design solution and its mathematical parent. I demonstrated the basic digital drawing techniques to refresh student knowledge of the software. Students were required to take notes, screen captures and recordings of their learning to provide themselves with customised *help*, if needed. This type of software has enormous potential, however, our POP project required a measure of control in this dimension, to achieve the set brief to create form.

> The form is a part of the world over which we have control, and which we decide to shape while leaving the rest of the world as it is. The context is that part of the world that puts demands on this form; anything on the world that makes demands on the form is context...in a problem of design, we want to satisfy the mutual demands the two make on one another. We want to put the context and the form into effortless contact or frictionless coexistence. (Alexander, 1964, p. 18)

Creating 2D vector curves became effortless for most students beyond creating the initial single parabola. We reviewed the elements and principles of design pertinent to the activity at hand: line, shape, scale, rotation, repetition, and as a class activity in the context of what we were working on, we defined the term *iteration*. Students simply defined the word as *repetition*; doing or making something again and again.

Mathematics as art. What happened next was complete engagement. Our simple curve was manipulated to produce what seemed like an infinite range of possible designs. The parabola was very quickly translated into art. Zagier (2011) suggests "when we speak of the 'art' aspect in mathematics, we are thinking less about the relationships between mathematics and the other arts…but rather about the fact that mathematics *itself* is an art" (p. 93). Of course, the context in which the DH students were making art was in the application of mathematics or more simply that the mathematics assumed a role of visual derivative. Therefore it is important to note that while students became consumed with iteration, pattern and a general sense of what looks amazing, their absorption in the parabola manipulation activity was purely aesthetic. In his chapter on *Habits of strength*, Csiksentmihalyi (1990) considers the protection of creative energy after it is awakened.

> We must erect barriers against distraction, dig channels so that energy can flow more freely, find ways to escape outside temptations and interruptions. If we do not, entropy is sure to break down the concentration that the pursuit of an interest requires. Then thought returns to its baseline state – the vague, unfocused, constantly distracted condition of the normal mind. (p. 351)

Without writing a sequence of code to produce the same visual effect, students explored the ways in which to control the applied digital technology to produce 2D imagery that surprisingly resulted in a three dimensional effect. This was an illusion incorporating aspects of the moiré pattern, also new to many students. Alexander's (1964) *Notes on the synthesis of form* supports this discovery, where designers organise their concept of form "under the driving force of some comparatively simple concept". (p. 29) The student voice during this time identified a range of levels of student engagement. The resounding comment from the students when describing their 2D explorations with the parabola was that "it looked so complex to do but was actually really easy" (See Appendix 14 for more student comments and Appendix 17 and 18 for detailed data analysis and discussion).

Figure 16 Class set of 2D parabola manipulations

Please note: A1 foldout is included as a separate file: ***Silk_MC_Thesis3***

MY STORY

DISCUSSION

The aesthetic experience. There is a breakthrough that occurs when the relationship between unconscious experience and consciousness is realised. Developers of the General Capabilities within the Australian Curriculum support the 2001 change by Anderson and Krathwohl (2001) to "Bloom's cognitive process of 'synthesis' to 'creativity'" (ACARA, 2013a, p. 253) and made it the highest level of intellectual functioning. They believed the ability to create required the production of an original idea or a product from a unique synthesis of discrete elements (Australian Curriculum, 2013, p. 58). The insight that occurs in the instance when 'creativity' is realised can be recognised as a breakthrough. "The unconscious seems to take delight (if I may so express it) in breaking through – and breaking up – exactly what we cling to most rigidly in our conscious thinking" (May, 1975, p. 59). The insight that May considers is "struggling to be born" is a dynamic struggle, observable learning and not a mere "expansion of awareness". There is also a sense of pride that a student may feel in addition to the "joy and gratification that is inseparable from the actualising of a new idea or vision" (p. 59). May returns us to Picasso's idea that "Every act of creation is first of all an act of destruction" (p. 60). Students experiencing a breakthrough by combining logic with experiment allow themselves to move from a state of not knowing to knowing and in the process, realising that to get to the 'knowing' may not be as difficult as it may once have seemed. In the case of manipulating a parabola to create an aesthetic form, students experienced a vividness and clarity that may be expressed as synthesis. By controlling their actions according to the elements and principles of design, students gained a more aesthetic experience – where all senses were operating at their peak – rather than simply entering a set of variables into a java script application and letting the computer do it for them. This is aligned with what May (1975) describes as "ecstacy – the uniting of unconscious experience with consciousness, a union that is not *in abstracto*, but a dynamic, immediate fusion" (p. 61).

Dynamic, immediate fusion. Back to the parabola and how to assess these experimental activities. Students created a layered digital file containing a range of iterative designs, a selection of which was to be analysed using the PMI model. (See

Appendix 15). One of the analysed designs would become the foundation for the 3D vessel and would be translated into CAD formats for final vessel construction. Students were required to document all design development and experiments by recording a series of annotated screen captures of their progress. They were also asked to record the equation and function used to create the original curves in their documentation annotation. This activity log was included in the student design folio and was ultimately a significant portion of the summative assessment for the project. With play, comes formality. My observations of the activities undertaken in Task 5 was that assessment requirements were almost in total contrast to actions described in the task itself, "I see around me a professional disease of taking everything too seriously" (Castiglioni, 2010).

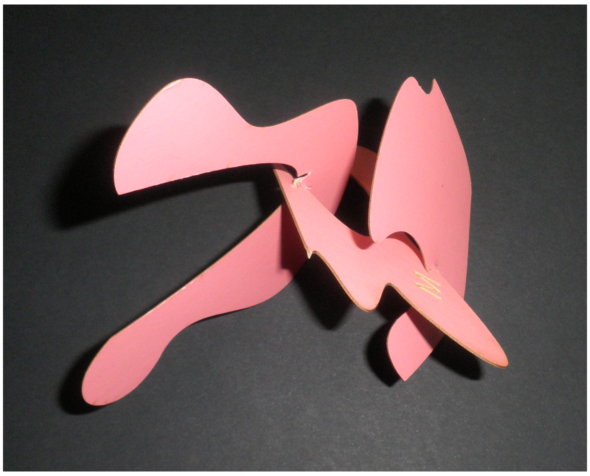


Figure 17: Closed path experiments

My attempt at maintaining a high level of whimsy for the remainder of the project was important if the acquisition of an aesthetic experience was to occur. However it was apparent that several students achieved this experience by striving to achieve in formal, theoretical tasks as well as the more experimental practical task of designing and making. In these experiments, students learned to manipulate vector paths by creating a curved shape to represent a letter of the alphabet. These were then laser cut and distributed back to the students to inform a sculptural construction based on individual names. (See Figure 17 and Appendix 16). Although there was value in exploring slotting and threading constructions techniques, there would be more effective methods to explore in future iterations of the course. Becoming a critically reflective teacher requires the elimination of the "Perfect Ten" Syndrome (Brookfield, 1995) where many teachers assume the worst when student evaluation of their teaching is "less than perfect" (p. 17). My observations of student confusion or lack of interest resulted in a certain streamlining of POP activities in order to attain knowledge, understanding and a sense of achievement. It was important for me to remember that during the years of teaching the POP project, each year group was new to the concept and my own ennui had to be curtailed in order to incite interest in others.

Most beautiful.

> Almost all mathematicians use words like "beauty" and "elegance", and they in fact use them more frequently than more scientific-sounding words like "convincing" or "correct". And what is even more interesting, is that this feeling of mathematical beauty often seems to be the best guide to follow when trying to figure out which direction to take through the labyrinth of mathematics: a sort of Ariadne's thread. An artist can make his or her choices (what should I write, what should I paint? what should I compose?) based on aesthetic criteria. A scientist hardly ever has that kind of luxury since nature cannot always be expected to make choices that will be pleasing to humans, and scientists have to stick to reality. Mathematics is somewhere in between: it is not absolutely necessary to adhere to aesthetic criteria when doing mathematics, and the right solution to a problem may not always be the most beautiful, but in the vast majority of cases, the right mathematical path always turns out to be the best one from an aesthetic point of view. There is no better general strategy, when you want to do good mathematics, than to look for the *most beautiful* solution. (Zagier, 2011, p. 93)

Conceptual principles inherent within the *Design and Technology syllabus* require students to consider parameters. The explicit parameters facing the student in their design development for the POP vessel was that the designed object was to be constructed without glue. The noun, a staple material and verb, an obvious technique, has been in use since the students learnt to 'make' as children. Restricted access to adhesive requires greater creativity in designing. In the DH class, discussion related to the 'triple S' approach to engineering design came into play: Strength, Stability and Serviceability. The transferal from 2D shape to 3D form was a challenging process for most students as it required a shift in perception as well as further developing skills in digital drawing techniques. In her Harvard Crimson article *Confined arts – Students negotiate the limits of their own potential*, Topczewska (2012) proceeds to quote Harvard professor Krzysztof Gajos. He "defines creativity in the same formulaic fashion he might create an algorithm. 'Creativity is novelty plus quality plus surprise,' he says" (p. 3). One could say that the pursuit of the aesthetic experience cannot be open to substitutability among the elements of an applied algorithm such as that suggested by Gajos. "The absence of substitutability promotes attention to the particular. Developing an awareness of the particular is especially important for those of us who teach since the distinctive character of how we teach is a pervasive aspect of what we teach" (Eisner, 2002, para.3). If we consider 'quality' as a variable in an arts or design education context, it is surely an imperative for teaching with parameters. To reduce student expectations is to invite mediocrity. In the POP course, we collectively set out to achieve quality under the guidance and assurance of the distinctive and pervasive intention of how the course was taught. The influence of computers on the Arts, design and mathematics is irrefutable. The digital contribution to the effectiveness of POP design and production activities resulted in fast, quality experimentation, allowing students to develop solutions to problems quickly and accurately.

Novelty + quality + surprise. All senses need to be operating at their peak within an aesthetic experience. This sensation is not simply a single aesthetic insight but a physical and cerebral sensation of achievement (quality), satisfaction (surprise), a feeling of having done something for the first time (novelty), revelation (it wasn't that difficult), and gratitude (taking part in an alternative way of learning). ACARA's (2013) view that learning in the Arts is sequential and cumulative can be aligned with Gajos' algorithm: creativity = novelty + quality + surprise. The pursuit of alternative possibilities as Eisner (2002) suggests allows students to experience a sense of novelty. Self initiated learning leading to the development of deep knowledge, an intention reiterated in the *Australian Curriculum* occurs only when innovative approaches to teaching have provided the opportunity for deep learning to occur. To know deeply is the culmination. The sequential elements may well be formed by an innovative interdisciplinary approach. A good place to start was graphing a curve because curves are inherently beautiful and the numeric rules that allow curves to be created have strong structural integrity that is also inherently elegant in both the mathematical sense and the visual.

In *Medical News Today*, Catherine Paddock (2014) reports findings from a new study suggesting that beauty may have a neurological basis.

> Using brain scans, researchers in the UK found appreciation of abstract beauty – such as in finding aspects of mathematics beautiful – excites the same emotion centres in the brain as the appreciation of beauty that comes from a more sensory experience – like listening to music or looking at great art". (Paddock, 2014, para.1)

Consolidation of such ideas can be supported by the experience of solving a difficult mathematical problem, which Zagier (2011) reveals is "a lot of fun". Further to this, there is "the joy inspired by the elegance and beauty of the results and arguments that one reads in the works of others or discovers for oneself". (p. 95) Returning to the DH workshop, observation of students revealed that although some had difficulty developing the design ideas for their vessels, the ease with which all students had embraced the 2D experiments was giving way to actual problem solving using two and three-dimensional perception. Flat was to become form.

KISS. Visualising this transformation was not possible without the construction of draft pieces or models. Aligning with the sustainable focus taken by the class, we recycled packaging cardboard to start modelling parabolic vessel design ideas. Economy of materials informed the layout of student designs using CAD software to communicate technologically. First iterations of student designs in both the pilot courses and the current POP project were produced using laser-cutting technology. The most recent class set of vessels incorporate laser engraving as well as computational cutting for manufacturing as the highly aesthetic results of student 2D experiments were considered valuable as an addition to the final product. Reductive designing techniques were introduced in order to rearrange the working properties and characteristics of rigid materials. In simple terms, we reduced the strength of bamboo plywood so that it could bend into the shape of a parabola. (Figure 18).

Figure 18: curved ply

The range of variables related to the width and depth of the curve was explored individually, as was the structural design surrounding the curves. The variety of student styles and approaches to tackling the design development gave me the opportunity to measure the breadth of motivation apparent in the most recent cohort. Similarly Zagier (2011) describes "the reasons why mathematics gives certain people such a deep feeling of joy" being "indeed true only for certain people: mathematics is not for everyone" (p. 96). Not all students fully engaged with the task of creating a form from a parabola because it seemed intellectually challenging and physically difficult. Some students set unachievable goals in terms of the concept. Some were afraid of the mandated use of CAD technology, others didn't really see the point. It was necessary for me (the teacher) to guide, encourage and model a KISS method of applying the mathematics to the design project: Keep It Simple, Stupid. Previous experience had taught me that they (students) would all enjoy the outcome and the accolades offered by an audience when viewing their work. Returning to Kerdeman (2009) "Personal growth, meaningful work, being moved by beauty – all testify to the transformations that accrue in aesthetic experience" (p. 90). Interacting with students as they work towards solving any type of problem contributes to their experience of delight in developing knowledge. The engagement shared by teacher and student when working towards finding a solution by ideation, calculation, experimentation and manipulation of materials leads to an integrative experience, a characteristic of a "threshold concept" an idea introduced by UK researchers, Jan Meyer and Ray Land in 2003. Their ideas on mastery and its relationship to the acquisition of knowledge are based on "[a] new way of understanding, interpreting, or viewing something […] a transformed internal view of subject matter, subject landscape, or even a world view" (Meyer & Land, 2005, p. 373). Learning and teaching within POP was animated by pedagogic strategies that addressed the quest for creative alternative practices to enable new understanding, for all students.

The liminal. These are the spaces between states, between knowing and not knowing, spaces where ideology and frameworks intersect. Meyer and Land (2005) suggest that states of liminality "tend to be transformative in function, and usually involve an individual or group being altered from one state into another" (p. 376). At times, throughout the POP course delivery, mastery, or the recognition of a threshold concept was explicitly observed. The KISS approach allowed all participants in the course to work towards "['conceptual gateways'] or 'portals' that lead to previously inaccessible, and initially perhaps, 'troublesome', ways of thinking about something" (Meyer & Land, 2005, p. 373). The action of *thinking* in order to *create* within the parameters of the POP project proved troublesome for some students. The liminal state in which they remained was not yet transformative, irreversible and integrative. However the troublesome aspect of their learning indicated these students' arrival at a "threshold" and this was the "pedagogically fertile" (p. 374) area in which to evolve the course. In her *Introduction to threshold concepts,* Glynis Cousin (2006) summarises Meyer and Land's idea of threshold concepts and its relevance in curriculum design. A threshold concept is transformative – marking "a turn in understanding a subject" and often irreversible in that "once understood, the learner is unlikely to forget it" (p. 4). Meyer and Land (2005) suggest that a concept can be unlearned "only through considerable effort" (p. 373). A reflective aspect to note is that teachers may have difficulty "retracing the journey back to their own days of 'innocence' when understanding of threshold concepts escaped them in their early stages of their own learning" (p. 373). Certainly this can be ascribed to the aforementioned possibility of ennui in my own approach to the POP course delivery and required constant attention to the evolution of content in order to satisfy the original pedagogical intention. A threshold concept is likewise integrative "in that it exposes the interrelatedness of phenomenon. Mastery of a threshold concept often allows the learner to make connections that were hitherto hidden from view" (Cousin, 2006, p. 4). The entire trajectory of POP coursework was to make those connections. However, there was space for the concept to be questioned, thus involving "forms of 'troublesome' knowledge; David Perkins (2006) defines this as 'that which appears counter-intuitive, alien (emanating from another culture or discourse), or seemingly incoherent'" (p. 4). In her own work, Cousin (2006) explored the troublesome nature of learning as it relates to the grasping of threshold concepts in that the difficulty of mastery is also prey to external forces, those other than the concept itself. "The idea of liminal states provides a useful metaphor to aid our understanding of the conceptual transformations students undergo, and the difficulties or anxieties that attend these transformations" (p. 4).

Flow. Considering intelligence as thinking new thoughts, the intellectual risk-taking requested by Australian syllabuses must be applied by pushing curriculum boundaries. Robinson (2010) suggests if all that we had in the way of intelligence could be accounted for with IQ tests, most of human culture would never have happened.

> Intelligence is self evidently diverse and multi-faceted. Human intelligence is wonderfully organic and what happens when you attempt to classify it is that it gets reduced to particular features. Each of us has our own unique intelligence. The human mind is tremendously sensitive and varied and rather than having a single scale where people say "how intelligent are you?", a better question is "how are you intelligent?" (Robinson, 2001, p. 66)

Observation of student activities as we increased the production process for final designs provided the opportunity to record levels of aesthetic experience. There was prolific student output, a plethora of material use and a healthy measure of positive peer critique at each step of production. Comments were very much in the vein of "wow, that looks great", "yours is really beautiful", "that's awesome" and so on. Occasionally, there were disappointments resulting in a student returning to the vector drawing stage to produce a better result. Iteration and symmetry were key aesthetic design principles considered in the designs. Those that adhered closely to simple mathematical foundations were clearly the most appealing artefacts. In reference to student output, there was no evident correlation between the work of students of higher intellectual capacity and those of lesser ability in that realm. Surprisingly, the latter students most often produced pieces considered highly aesthetic. There was no obvious differentiation of experiences, simply a range of irrefutably joyous moments, frequently unexpected because students found themselves in *flow*. A further point here is related to Csiksentmihalyi's (1996) views on the organisation of knowledge as it is passed down a generation.

> To be creative, a person must first understand the domain. If the knowledge in the domain is nearly incomprehensible, few young people will bother learning it, and thus the chances of creative innovations will be less." (p. 340)

Here Csiksentmihalyi argues about rigidity (memorising rules and focusing on a single way to solve a problem) verses the emphasis of mathematical thinking and understanding which more traditional parents and teachers may view as *dumbing down* "math and further eroding our children's comparative standing in this important domain" (p. 340). He goes on to say that the answer can be found in the middle ground. "To cope well with numbers it is essential to automate as many mental operations as possible – and this requires some memorising and practicing" (pp. 340-341). However Csiksentmihalyi does not discount the importance of intuition in the effective use of numbers and that there is no single right way of teaching as the transmission of knowledge must be appropriate to the skills of the learner. Similarly, the method should match the context. Within the POP course, the original mathematics was firmly grounded in an esoteric design experience devoid of rigidity but rich in beauty. The wonderful feeling described by Zagier (2011) in *A Passion for mathematics* where mathematics "which comes from the inside while at the same time describing something on the outside, is the only science in which one is able to find the truth… by looking inside oneself" (p. 96).

> Nowadays this point of view is often called Platonism from the famous passage in Plato's Meno in which Socrates uses a series of clever questions to lead an uneducated slave boy to understand and prove the theorem that a square drawn on the diagonal of a given square will have twice the area of the original square – though Plato drew from this the, [to us,] rather odd conclusion that the boy had an immortal soul and was simply remembering the proof from a previous life! (Zagier, 2011, pp. 96 -97)

The ‘wonderful feeling’ is akin to the joyful experience many POP students encountered on successful completion of their parabolic vessel designs (See pages 59 – 64).

The purest form of pleasure. The intuitive aspect of students’ application of the mathematical functions was to be explored in a more heightened sense in the next conceptual phase of the POP project. Observation of students in varying liminal states was a powerful reminder of the necessity for course content to be engaging and differentiated. One problem with vigorous conceptual projects lies in the possibility of some students remaining in a sate of pre-liminality “in which understandings are at best vague” (Cousin, 2006, p. 4). The necessarily affective and cognitive learning that took place in the earlier activities presumed a level of understanding as we moved into 3D modelling of the parabola. Students in the ‘troublesome’ state remained on an “unsafe journey” where the practice of mimicry constructs their own conditions of safety. This is lamentable for the teacher because it represents a default position, presenting a situation for the student where no actual mastery is at play. Cousin (2006) proposes a holding environment for the toleration of confusion; “teachers must demonstrate that they can tolerate learner confusion and can ‘hold’ their students through liminal states” (p. 5). Constant roving and observation is necessary in any practical classroom context. The provision of detailed and accurate instruction is also a deterrent to confusion. In the context of the POP project, the use of industry level software and adequate access to help was enhanced by strong peer support at the ground level (See Figure 19). Taking the parabola into the next phase was a journey of discovery for me also. This was the first time I developed the 3D modelling aspect of the course as the hardware was a new acquisition and not available during the pilot delivery; 3D printing technology.

Exhilarating experiences. The idea of playing around in a three dimensional environment was extremely appealing to me as a design teacher. Devising a context within which *play* was operative meant discarding certain aspects of required outcomes prescribed in syllabus documents. By pushing the curriculum boundaries in the context of the POP course, students were provided an opportunity to explore new and emerging technologies whilst simultaneously applying their mathematical and graphical knowledge. The point is that “playing with ideas is extremely exhilarating. Not only philosophy but the emergence of new scientific ideas is fuelled by the enjoyment one obtains from creating a new way to describe reality" (Csiksentmihalyi, 1990, p. 127). My intention at this stage was to engender the idea of the existence of many possibilities for the application of the parabola. In so doing, the curve may or may not be recognisable in the output, but we would know it was there. My aim was to have students explore, mostly in the not-knowledge of what they were doing. This collective liminal state resulted in the gaining of much knowledge related to 3D space and the elements and principles of design.

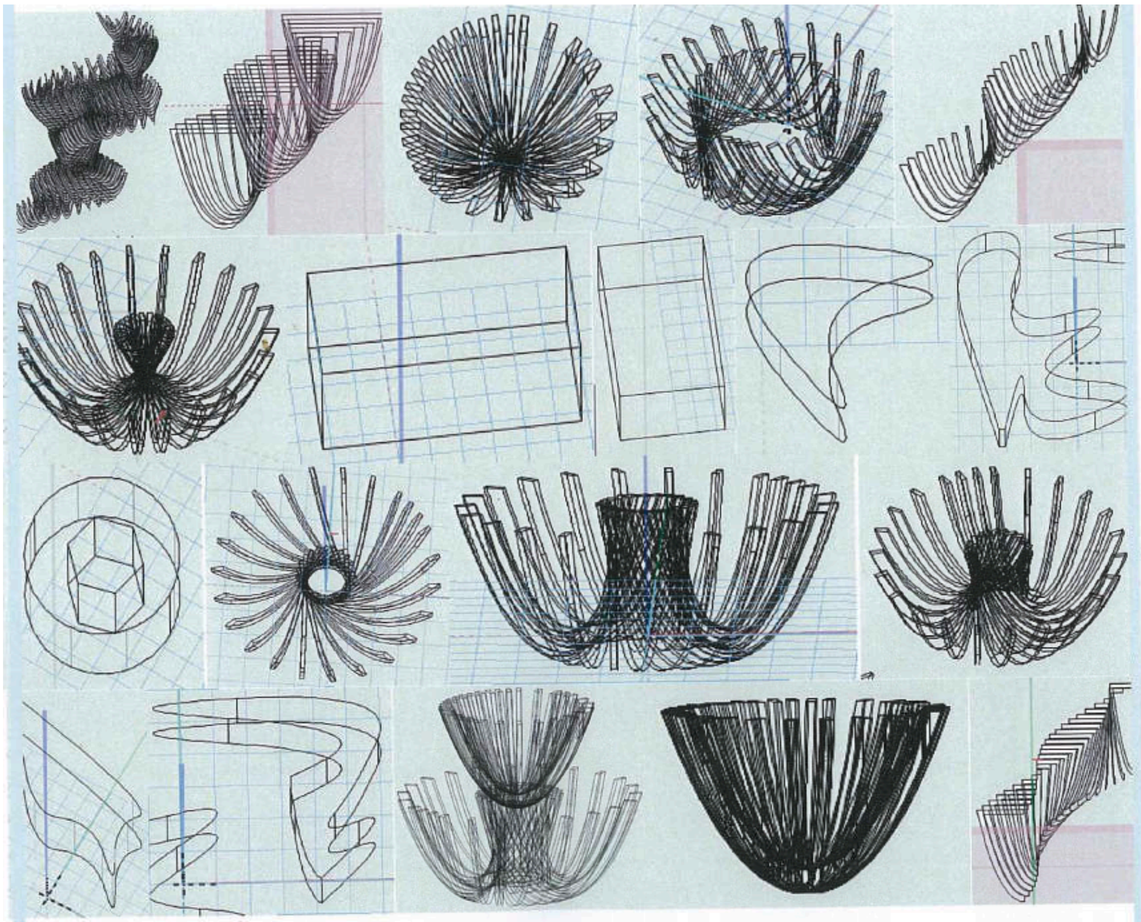


Figure 19: 3D modelling of parabola iterations

The application of mathematical concepts to a visual context varies according to the myriad of possibilities that can be explored. The exploration of new technologies has increased the capacity to work with tools and equipment that can directly use mathematical thinking as a point of departure. Reinforcing the alignment with current practice, more and more designers are choosing to work with mathematicians and vice versa. Designers are collaborating with mathematicians and physicists (and software developers) in order to use algorithms and algorithmic functions in design development.

The Australian Curriculum: Technologies requires students to apply a range of thinking skills in the development of design projects. In relation to thinking skills, the curriculum promotes:

> A core and fundamental dimension to both Design and technologies and Digital technologies is the way students learn to use higher order thinking skills to reflect, evaluate and validate their technological knowledge. Reflecting on learning in Technologies builds their technologies knowledge and deepens their understanding. (ACARA, 2012b, p. 9)

Both strands acknowledge the importance of students developing "a sense of pride, satisfaction and enjoyment in producing quality solutions that may be both functionally appropriate and aesthetically pleasing" (p. 9). Current curriculum documents make explicit connections between technologies and the Arts. Besides the obvious application of the elements and principles of design:

> aspects of aesthetics are incorporated into the design processes in Technologies learning activities. This occurs when students design products and environments including those with a focus on graphics technologies. Knowledge of materials, tools and equipment and the ways they can be used to create designed solutions provides links between Technologies and two and three-dimensional design in Visual Arts. (ACARA, 2013b, p. 30)

Aspects of aesthetics noted in the *Australian Curriculum* certainly could be supported by the inclusion of the idea of aesthetic experience as an enabler for learning. The use of new and emerging technology in the practice of producing aesthetic form heightened the experience for participants in the POP course. The work they produced was surprisingly metaphorical; conjuring an unexpected connection to biomimicry in design as well as powerful socio-environmental statements. The parabolic designs produced by a 3D printing process in white ABS plastic were reminiscent of bleached coral and other sea life-forms. Together, they made a definite statement. Singularly, they presented as fragile *and* strong, intelligent, and above all, elegant.

> The good things in life do not come only through the senses. Some of the most exhilarating experiences we undergo are generated inside the mind, triggered by information that challenges our ability to think, rather than from the use of sensory skills. As Sir Francis Bacon noted almost four hundred years ago, wonder – which is the seed of knowledge – is the reflection of the purest form of pleasure. Just as there are flow activities corresponding to every physical potential of the body, every mental operation is able to provide its own particular form of enjoyment. (Csiksentmihalyi, 1990, p. 117)

Wonderment and awe. Although we were running out of time and the possibilities of the parabola was seemingly becoming exhausted, the final product of our labour was breathtakingly beautiful. There were many variables in the success of the journey towards producing that beauty. Some students struggled with the recursive nature of the task, inherent in the challenging interface belonging to any 3D modelling applications. There were a number of returns to the instructions and the practice of creating in three-dimensional space, looping the knowledge around the intention (Figure 20). This is where the "threshold concept perspective refreshes the critique of a simplistic, linear, learning outcomes approach" (Cousin, 2006, p. 5).

> for the notion of learning as excursive, as a journey or excursion which will have intended direction and outcome but will also acknowledge (and indeed desire) that there will be deviation and unexpected outcomes within the excursion; there will be digression and revisiting (recursion) and possible further points of departure and revised direction. The eventual destination may be reached, or it may be revised. It may be a surprise. It will certainly be the point of embarkation for further excursion. (Cousin, 2006, p. 5)

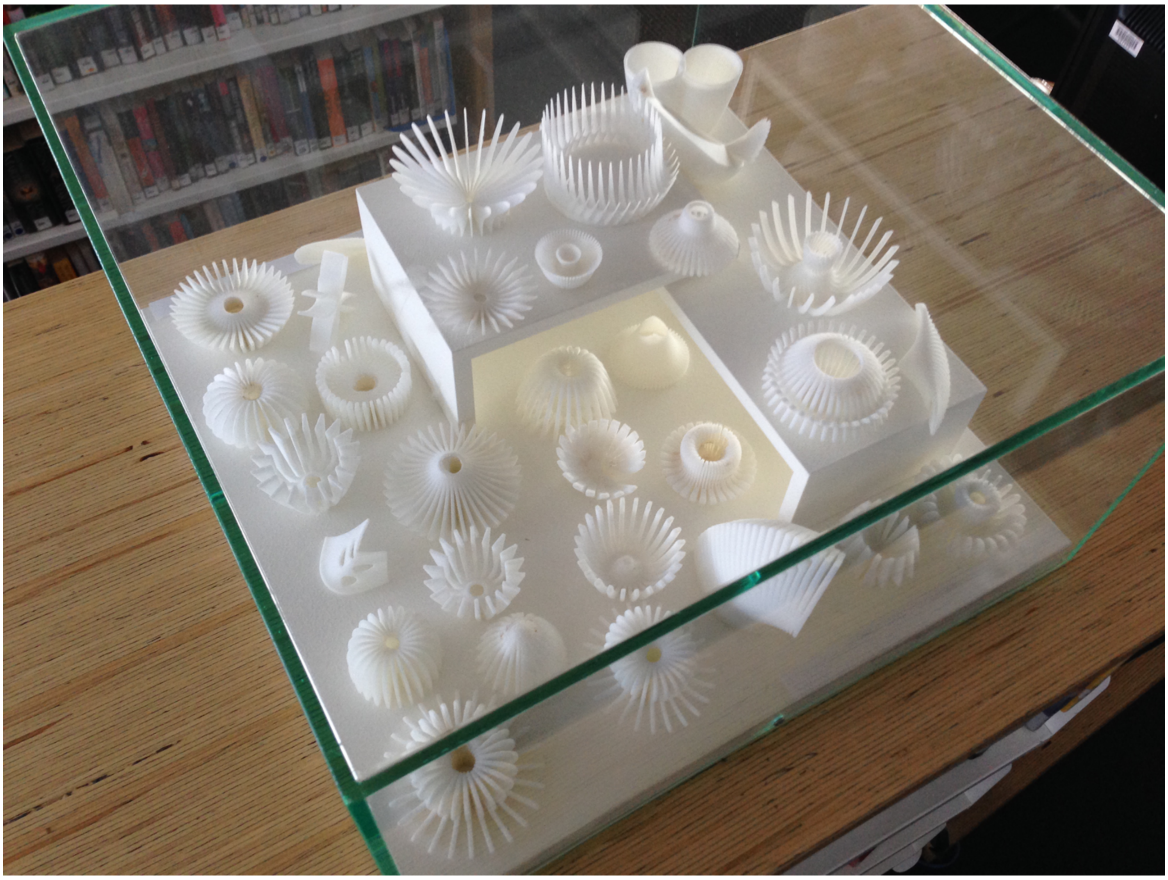


Figure 20: Exhibition 3D prints

Some students were hesitant to explore due to the complexity of the 3D modelling interface. Others took hold of the idea and ran with it, headlong, into an exploratory imaginative space where there were no boundaries whatsoever. These students exhibited moments of flow and from an observer's perspective they achieved a definite aesthetic experience. Teaching in a constructivist environment enhanced my practice as I continued to learn alongside the students. Eisner (1985) proposes that "[a]ll scientific inquiry culminates in the creation of form: taxonomies. Theories, frameworks, conceptual systems" (p. 26). Our collective experience would be rendered redundant without the fruit of our learning having been exposed. "The scientist, like the artist, must transform the content of his or her imagination in to some public, stable form, something that can be shared with others" (Eisner, 1985, p. 26). The next level of understanding of input required to measure the depth of the aesthetic experience lay in the exhibition of student work and the reaction of an audience. The following pages contain a gallery of work produced by the students during the *Possibilities of the Parabola* course. The work is produced by CAD CAM technologies such as laser cutting and 3D printing production techniques.

"Seeing all of the designs by every person to me was an aesthetic experience" Participating student (2013). The evidence supporting the range of student aesthetic experience is presented in detail in Appendix 17. The collected data analysed in Appendix 17 was specific to the study. It has been broken into individual questions, responses and detailed findings.

THE GALLERY

Figure 21 Student POP construction in the studio

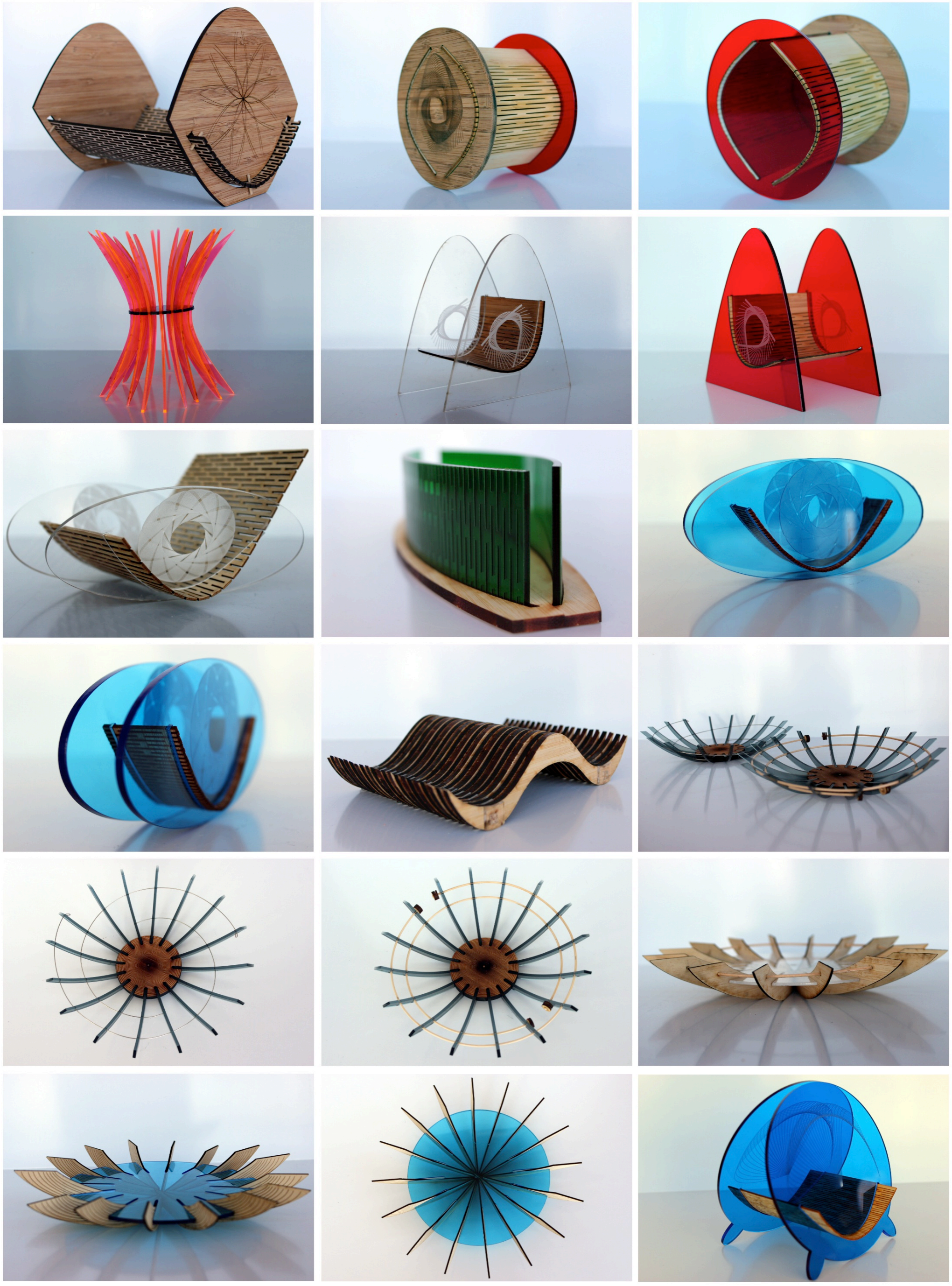


Figure 22: vessels

Figure 23: vessels

Figure 24: vessels

Figure 25: vessels

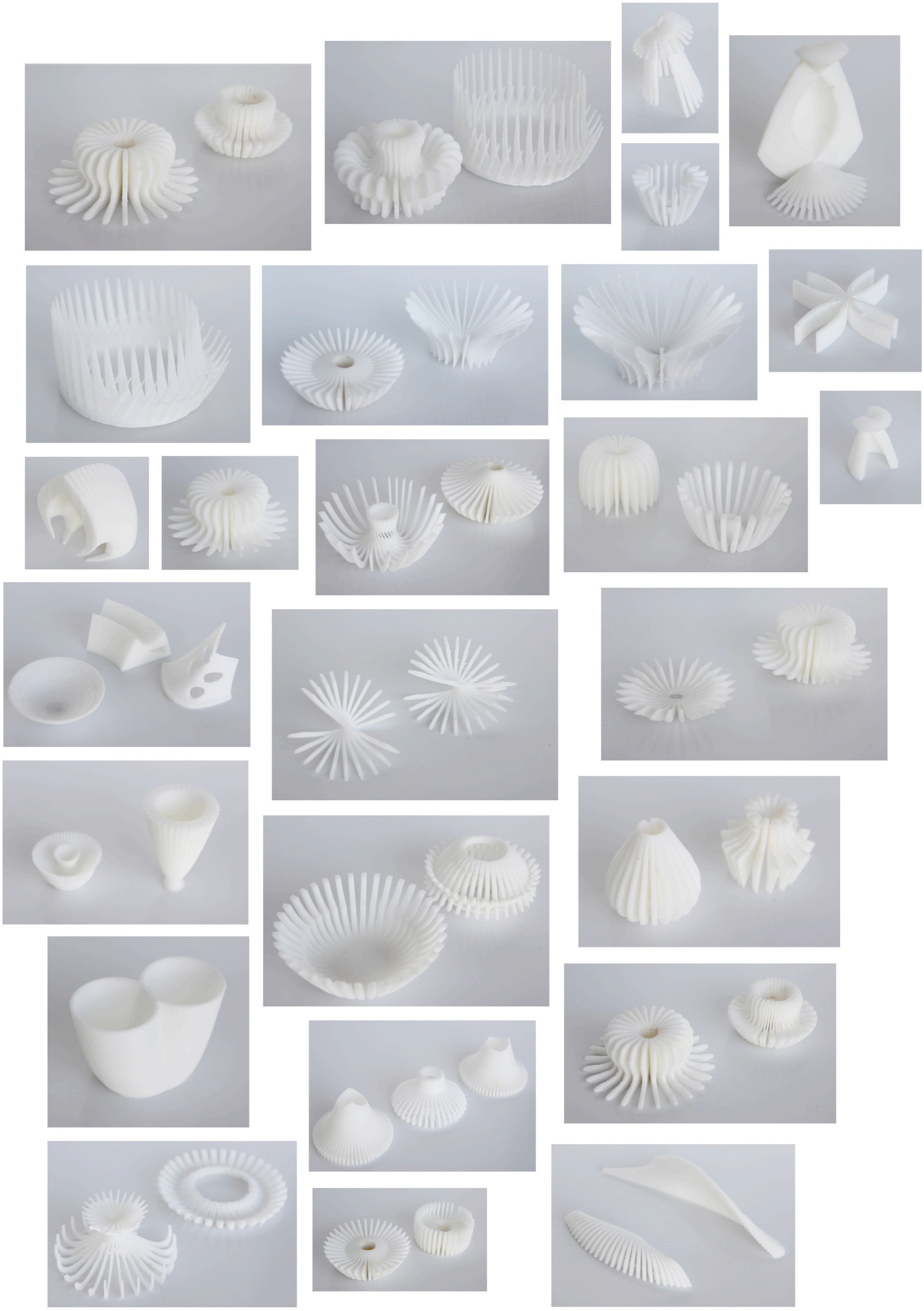


Figure 26: 3D prints

NOT THE END OF THE STORY

CONCLUSION

Gaining insights. The personal narrative can be included in the broad definition of autoethnography. Considering the term in its basic form as "cultural-level studies by anthropologists of their 'own people'" (Ellis & Bochner, 2000, p. 739) my position within the culture of the *Design Hype* class warrants inclusion in terms of familiarity and full membership of the group. My observation of the group supports an understanding of the range of variation within learning. Gaining insights into why and how students learn resulted in my own path towards understanding concepts of threshold. The "transformational or creative experience in what [is] termed the liminal space of learning" (Meyer & Land, 2005, p. 380) has been applied to my own praxis within a "subject-centred" (Palmer, 1998, p. 116) environment.

I have found the concept of freedom to be contradictory when related to ideas contiguous with rationality and order. In the context of counter-intuition, Michel Cassé (2011), astrophysicist, quotes Georg Cantor, German mathematician, (inventor of set theory) in his contribution to *Mathematics, a beautiful elsewhere*:

> the essence of mathematics lies in its freedom. However it is a freedom that is under surveillance, reined in by reason, and this is not without its contradictions because mathematicians are willing to submit to inexorably rigid rules in the free exercise of their activity. (Cassé, 2011, p. 31)

He goes on to suggest that rationality requires complete mental freedom, a context within which any established truth is questioned. However the rigidity imposed on conceptual freedom actually led to discovery, or truth, "comparable to the experimental method in vogue since the Renaissance" (p. 31). Alternatively, May (1975), in his chapter on *Creativity and the unconscious* proposes "scientists themselves, particularly the physicists, have told us that the creativity of science is bound up with the freedom of human beings to create in the free, pure sense" (May, 1975, p. 71). This idea of discovering something by engaging in imagination and indulging simply in the joy of discovery "runs the risk of radically upsetting our nicely worked-out theories, as it did when Einstein introduced his theory of relativity" (May, 1975, p. 71). Both understandings weave their way towards the idea that creativity of the spirit cannot be truly hindered by structure and presupposition and that there is obvious "interrelation between unconscious, irrational illumination and scientific discovery" (p. 72). The parameters for the brief supplied to students at the beginning of the *Possibilities of the Parabola* project specified that the construction of their parabolic vessels was to omit the use of adhesives. At a basic level, this was a very challenging construction technique yet a method that has been applied in many applied design disciplines for thousands of years. There were students who immediately embraced the challenge as they could see the structural potential in the early curves created in two dimensions. Intuition came into play for these particular young 'makers'; my part in their journey was to support their technological learning so that their ideas became manifest. "The risk of potential damage caused by intuition is limited by the proof or by experimentation; it is nevertheless intuition that kicks off the process of discovery" (Cassé, 2011, p. 71).

My own intuition led me to develop the *Possibilities of the Parabola* as a unit of work for secondary school students. The tasks and activities I developed for inclusion in the POP project were experimental in their application to a school based art/design context. Algorithmic studies in programming, even at secondary school level might easily have produced a similar outcome, however my intention was driven by personal interest in controlling the inputs to effect innumerable aesthetic experiences for both student and teacher. Documenting the account of the innovative development and delivery of tasks associated with POP provided the opportunity for personal and professional reflection while simultaneously enriching my entire teaching practice. This is an ongoing process. I began this inquiry in order to improve aspects of my own knowledge and experience and through the initial investigation, felt that it was valid to share this knowledge by encouraging others to partake in a similar journey. A guiding principle in my pedagogical practice has been curiosity. What if?

> This idea of developing human capability is core to action research. Sen (1999) describes capabilities as people's ability to think for themselves and make their own decisions about how they wish to live their lives. He also makes the point that realising these capabilities requires people to be free, and, in turn, to exercise their freedom to ensure the continued development of their own capabilities and the capabilities of others. (Whitehead, 2006, p. 46)

Discussion of the results of the POP Pre-test and Post-test can be found in the Appendix 17 section of this study. They are included to support the context of developing capabilities of others, that is, students. The tests included a range of questions related to student awareness and knowledge of maths, art, design and aesthetics, particularly the introduction of the idea of an ‘aesthetic experience’. It also included information about age and intention.

Artefact and audience. There does remain, however, a final input to the overall aesthetic nature of the POP project: the reaction from an external audience. To clarify, the vessel designs can be viewed in the context of artefact, designed object or artwork. In support of the aesthetic value of artworks:

> …the less mysterious explanation is threefold: first and foremost, that the aesthetic value of artworks lies only in the experience of them; second, that the context in which any property of an artwork is experienced can radically alter the experience of it; and third, that different observers, even fully qualified observers, can experience the same objective properties of artworks differently. If the value of artworks lies in the experience of them, and if that experience varies with context and observer, then one must experience artworks in order to appreciate their value. (Goldman, 2006, p. 333)

To conclude this study we must return to the student gallery, viewable online here: http://moodle.igssyd.nsw.edu.au/mod/lightboxgallery/view.php?id=4530

Mathematical beauty. To what extent is mathematical beauty in the eye of the beholder? asks Ken Clements (2006) when investigating the concept of elegance as it applies to mathematical problem-solving. He cites Gina Del Campo in support of this exploration.

> An elegant strategy has the property that it is recognised as a “very good” method by other problem solvers once they become aware of it. Elegance as we have described it implies not only a deeper than usual awareness of the structure of the problem, but also a creative ability to apply a procedure not suggested by the structure. While the construction of an “elegant” solution is a personal achievement, it is something which is readily recognised as “worthy” by others in a position to appreciate it. (Clements et al., 2006, p. 61)

Both Del Campo and Clements argue that although the sincere efforts of any problem solver should be valued, especially in an education context, “some solution strategies are more elegant (or, if you like, better) than others” (2006, p. 1).

In the context of *Possibilities of the Parabola*, the creation of aesthetic form via a theoretical mathematical conduit required flexibility, collaboration and a shared intention to succeed. The idea of freedom was a recurring theme in learning and teaching through POP. A significant contributing factor to the development of this project was the freedom to push curriculum boundaries, seek collaborations, experiment with new technologies and take risks in terms of student expectations. When I began, there was no indication of the project’s potential success. Intuition was my driver. Elegance was my destination.

This intended outcome seemingly mandated a healthy environment for the cross-fertilisation of ideas where the inter-disciplinary potential between the study of mathematics and visual art or design could be explicitly explored and applied. Curricula suggested that an aesthetic dimension is a value crucial to exploring new ways of thinking. The manufacture of a learning environment in which an aesthetic experience could occur, required the channelling of knowledge and skills gained by the promotion of new ways of thinking. This was a valid contribution to a continued development and understanding of the scope of critical and creative thinking; combined capabilities proposed by the *Australian Curriculum*. "Though the two are not interchangeable, they are strongly linked, bringing complementary dimensions to thinking and learning" (ACARA, 2014a, p. 250). Existing variables supplied the range of ideas that might lead students to an aesthetic experience through a multi-dimensional path of learning, meaning that there was more than one way to develop an understanding of aesthetics throughout the POP project. The fusion of thoughts, senses and emotions with diversity of personal, social and imagined experience was expected to result in the creation of exquisite form. Simple and complex forms were the product of sequential and cumulative learning experiences. These I consider are relevant responses to:

> challenges of the twenty-first century – with its complex environmental, social and economic pressures – [requiring] young people to be creative, innovative, enterprising and adaptable, with the motivation, confidence and skills to use critical and creative thinking purposefully. (ACARA, 2014a, p. 250)

Value gained by experiencing aesthetically while also developing an aesthetic voice is aligned to the development of new ways of thinking. If aesthetics is fusion and fusion requires new ways of thinking, sensing and experiencing, the role of the educator is to promote the circumstance whereby a student can engage with their aesthetic voice via a range of channels.

Subject-centred. Together we put a third 'thing' at the centre of our pedagogical circle: the parabola. This curve was the substance in a subject-centred model of learning and teaching. I did not want to be "whiplashed, with no way to hold the tension" (Palmer, 1998, p. 116) between student and teacher in fear of failing "to find the synthesis that might embrace the best of both" (p. 116). Palmer elaborates on the idea of subject-centred education, considering this environment to be modeled on the "community of truth, this is a classroom in which the best features of teacher – and student- centred education are merged and transcended by putting not teacher, not student, but subject at the centre of our attention" (p. 116). Our obsession with the parabola became the true driving force for the collective aesthetic experience brought on by an appreciative and exuberant audience.

The *Possibilities of the Parabola* exhibition took place at the end of the project. The work was displayed both physically and virtually in an online gallery space. Students, family members, the school community and professional persons were invited to comment on the POP student work via an online survey. Their feedback was anonymous. It was also very positive. The POP pilot courses had already gained gratifying exposure in that they produced highly creative output yet remained within the school community context. Feedback from the online gallery created specifically for the most recent POP project, the basis for this study, validated the intention to promote the potential of cross-curricular models of learning and teaching.

Validation also proved to be a contributing factor to the student aesthetic experience. Viewing their own and others' work, critically critiquing and analysing the work, in conjunction with inviting the external world to view their work without caution, opened the channel to objectivity in defence of their creative output. This took form out of context and demonstrated to students the idea of the object being "a product both of your subjectivity *and* external reality" (May, 1975, p. 118). Appreciation of the vessel artefacts was a worthy experience in itself. If "a particular work stops us for a moment as we become immersed appreciatively in its world and manner of being" (Richmond, 2009, p. 100), it provides an encounter for the viewer to embrace "the synthesis of feeling, form, and concept into a complete and satisfying whole [giving] a feeling of pleasure and accomplishment in a work that is replete and finely wrought" (p. 100).

Using this equation below, the parabola itself was manipulated to visually represent data gleaned from the views of the audience visiting the POP vessels site, and in this action, perhaps engender an aesthetic experience for the reader of this story (See Appendix 18).

$$\frac{1}{n} = \frac{x}{360} \longrightarrow x = \frac{360}{n} \longrightarrow x \times i = \% \text{ response}$$

n = numeric value of total feedback and i = numeric value of individual feedback

Figure 27: formula for graphic representation

The ethnographer's experience. This study has shown that my participation exists beyond the role of classroom teacher. Radical empiricism attributed to Jackson (1989) "refers to a process that includes the ethnographer's experiences and interaction with other participants as vital parts of what is being studied" (Ellis & Bochner, 2000, p. 741). I drew personal satisfaction and meaning from the exploration of the subject and was hopeful that the hybrid nature of the coursework would also be meaningful for students. Their creative output in the form of 'vessels' made a certain and resounding impact on the audience. The same impact validated my intention to promote the potential of specific inter-disciplinary approaches to learning and teaching. Writing directly from my experience permitted the freedom to rationalise my pedagogical objective while simultaneously discovering the evocative nature of transformative participation. In preparation for teaching this course, I anticipated a full and rich relationship to emerge between subject areas and student/teacher engagement. I was not prepared for the innumerable possibilities related to the creation of visual form from this single curve. These possibilities continue to excite and arouse.

> Mathematics is a specially sharpened tool used for cutting into the fat of the world, for slicing it up, or for carving into abstract concepts and removing the fillet. And there is no limit to its power. The constant evolution of mathematics and its continual reference to the eternal truth make it, paradoxically, a model of openness as well as closure. Mathematics is both inside of time – it experiences the human condition – and outside of time in that nothing exists except laws and formulae, there is no reality, no good or bad or time or yesterday or tomorrow…nothing but an eternal mathematical present… (Cassé, 2011, p. 31)

There is no limit also, to the power of engaging in critical inquiry through the context of the Arts. The "sense of vitality and the surge of emotion we feel when touched by one of the Arts can be secured in the ideas we explore with students" (Eisner, 2002, para. 38). It seems that the adjectival use of the word elegant, when related to a mathematical concept or theory holds a special meaning for those who consider themselves well-versed in algorithmic environments. There is then the added context of the use of the adjective, *elegant*, when describing a thing that is gracefully refined. However, introducing the possible production of an artefact that might be described in this way by utilising the similarly proven elegance of the mathematical curve might spark a marriage of disparate themes and therefore engender a deeper appreciation of the word itself while also contributing to an elegant emotional response. The noun, *elegance*, however, encompasses the entirety of the POP journey. A pleasingly superior process involving intelligence, sophistication, beauty, wonder, learning, appreciation and achievement.

In conclusion, the experience I gained from researching, collaborating, developing and delivering all tasks and activities directed towards the *Possibilities of the Parabola* has prescribed a variety of options in the way education can be less fragmented. The findings of the research are not typically recognisable as solid recommendations or statements of fact. Since the study was an immersive, observational exploration into the possibilities of idea smashing in a math/maker context, I have attempted to record and analyse as much collected data as possible to support the idea of student acquisition of an aesthetic experience. If there is a lack of research rigour evident in the way the research is theorised, the evidence supporting rigour is found in the samples of work. The images of student work represent the culmination of much collaborative interdisciplinary thought and action. Participants in the collaboration contributed knowledge and expertise from stereotypically disparate subject areas. I believe that the rigour was evidenced in the provision of work samples that clearly identify the inherent mathematical theory underpinning the entire study. The student responses through their learning journey clearly identify their growing awareness of the integration of ideas and the effect of such integration on the ways they learn, know and collect experiences of the world around them. Curriculum activities should not be isolated, they should be unified. I would like to think that some DH students will want to voluntarily pursue the ideas they have been taught "after the artificial incentives so ubiquitous in our schools are long forgotten" (Eisner, 2002, para. 38). In short, it can be stated that this curve will not be ignored.

The parabola remains a constant. It will contribute to the next narrative and the one after that. I have found that collaborating with mathematics teachers in order to establish an alternative approach to an arts/design project has made a resounding effect on the depth of learning for the majority of participating students. This learning includes aspects of both the Arts and Science. The constancy of the rigid mathematical rules applied to Cartesian plotting and graphing was playfully manipulated in an irregular circumstance. The Visual Art or Design studio environment is not generally perceived as rigid. Such environments are home to fluid and evolving idea generation leading to breakthrough moments of creativity and innovation. Although conceptually and theoretically the parabola cannot be changed without input data, there are many and varied possibilities of what one might do with the curve in the Arts context. Interdisciplinary learning is not restricted to the rigid curriculum model requiring delivery without change. That is why the constancy of the curve can contribute to a myriad of narratives in several dimensions. I believe it is the responsibility of teachers to search for innovative ways to blend ideas and encourage the retention of knowledge by way of creating and making. This is a contemporary STEAM (Science, Technology, Engineering, Maths and the Arts) approach. Possibilities of the Parabola is an example of STEAM curriculum in action. More of this type of course will contribute to a culture of integrated learning that satisfies a range of experiential knowledge acquisition models. One such context being the acquisition of an aesthetic experience.

Figure 28: Word cloud mapping audience feedback

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ETHICS APPROVAL

THE UNIVERSITY OF SYDNEY

Research Integrity
Human Research Ethics Committee

Tuesday, 8 April 2014

Dr Robyn Gibson
Education and Social Work - Research; Faculty of Education & Social Work
Email: robyn.gibson@sydney.edu.au

Dear Robyn

I am pleased to inform you that the University of Sydney Human Research Ethics Committee (HREC) has approved your project entitled "**The Possibilities of the Parabola**".

Details of the approval are as follows:

Project No.: **2014/157**

Approval Date: **8 April 2014**

First Annual Report Due: **8 April 2015**

Authorised Personnel: **Gibson Robyn; Silk Melissa;**

Documents Approved:

Date Uploaded	Type	Document Name
27/03/2014	Advertisements/Flyer	Consent form for Parents
27/03/2014	Advertisements/Flyer	Consent form for Participants
27/03/2014	Advertisements/Flyer	Information Statement for Parents
27/03/2014	Advertisements/Flyer	Information Statement for Under 18s
23/02/2014	Questionnaires/Surveys	Sample questions

HREC approval is valid for four (4) years from the approval date stated in this letter and is granted pending the following conditions being met:

Condition/s of Approval

- Continuing compliance with the National Statement on Ethical Conduct in Research Involving Humans.
- Provision of an annual report on this research to the Human Research Ethics Committee from the approval date and at the completion of the study. Failure to submit reports will result in withdrawal of ethics approval for the project.
- All serious and unexpected adverse events should be reported to the HREC within 72 hours.
- All unforeseen events that might affect continued ethical acceptability of the project should be reported to the HREC as soon as possible.
- Any changes to the project including changes to research personnel must be approved by the HREC before the research project can proceed.

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- Note that for student research projects, a copy of this letter must be included in the candidate's thesis.

Chief Investigator / Supervisor's responsibilities:

1. You must retain copies of all signed Consent Forms (if applicable) and provide these to the HREC on request.
2. It is your responsibility to provide a copy of this letter to any internal/external granting agencies if requested.

Please do not hesitate to contact Research Integrity (Human Ethics) should you require further information or clarification.

Yours sincerely

S. J. Assinder

Dr Stephen Assinder
Chair
Human Research Ethics Committee

This HREC is constituted and operates in accordance with the National Health and Medical Research Council's (NHMRC) National Statement on Ethical Conduct in Human Research (2007), NHMRC and Universities Australia Australian Code for the Responsible Conduct of Research (2007) and the CPMP/ICH Note for Guidance on Good Clinical Practice.

International Grammar School

February 2014

Possibilities of the Parabola

PERMISSION STATEMENT FROM MICHAEL MANISKA
PRINCIPAL, INTERNATIONAL GRAMMAR SCHOOL

I Michael Maniska give permission and consent to Melissa Silk to collect and interpret data from International Grammar School students who have been members of the Stage Five elective subject, *Design Hype*. This data will only be collected by those students who have volunteered to participate and whose parents have signed permission forms allowing their participation. I understand that the voluntary participant identity will remain anonymous within the completed research.

I give permission for this information to be recorded, collated and analysed in support a study being conducted by Melissa Silk, Head of Design and Technology at International Grammar School and will form the basis for the degree of Masters of Education (Research) at The University of Sydney under the supervision of Dr Robyn Gibson, Senior Lecturer, Visual & Creative Arts Education.

I give permission and consent to Melissa to approach and seek consent from selected students and parents. These students are selected possible voluntary participants in this study because they have selected the *Design Hype* course as part of their Stage 5 elective subjects. I understand that the students and parents will be provided with participation information containing details of the research project and all the methods of data collection involved. Parents and caregivers will also be provided with the opportunity to contact Melissa or her USYD supervisor for further clarification.

I understand that the intention of the *Possibilities of the Parabola* Unit of work is to establish a firm relationship between Mathematics and Visual Design and Technology curricula. The unit of work provides a pedagogical model for learning that relies on the conduit between the two subject areas being explicitly linked. Coursework involves student exploration of the inherent mathematical concepts related to plotting and graphing curves in order to transfer their understanding to a design context. The intended result of their work is to design, create and evaluate aesthetic forms based on manipulations of a single mathematical curve: the parabola.

Yours sincerely,

Michael Maniska
Principal

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