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THE UNIVERSITY OF SYDNEY

DOCTORAL THESIS

Competitive Bidding in Supply Chains

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*A thesis submitted in partial fulfilment of the requirements
for the degree of Doctor of Philosophy*

in the

Discipline of Business Analytics
The University of Sydney Business School

March 2015

Declaration of Authorship

I, Lusheng SHAO, declare that this thesis titled, “Competitive Bidding in Supply Chains”, is my own. I confirm that:

- This work was done wholly while in the candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, it has been clearly stated.
- Where I have consulted the published work of others, it is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Date:

Abstract

Competitive Bidding in Supply Chains

by Lusheng SHAO

This thesis is primarily concerned with the competition between suppliers for a buyer's procurement business with consideration of subcontracting, commitment and capacity reservation.

Under the circumstance where suppliers face diseconomies of scale, it may be cost effective for a buyer to split an order across different suppliers. Even when the buyer chooses only one supplier, the winning supplier may subcontract part of the work to the others subsequently. Motivated by these observations, Chapter 2 studies a supplier bidding game where a buyer requests quotes from two competing suppliers. We consider two procurement scenarios: (1) *Order Splitting* where each supplier submits a function bid which specifies different payments for different quantities, and the buyer may split the order; (2) *Single-Sourcing Commitment* where the buyer commits to purchasing from only one supplier before suppliers submit their bids, and the winning supplier may subsequently subcontract with the losing one. The aim is to study the role of subcontracting and single-sourcing commitment in supplier competition.

The second and third papers investigate the competitive behaviour of suppliers with capacity reservation. Capacity reservation is vital when suppliers need to invest in capacity to meet a further order and the future demand is unpredictable. To hedge against financial risks, the suppliers often require a buyer to reserve capacity in advance by paying an upfront fee. In Chapter 3, we consider a discrete version of this problem in which competing suppliers each choose a reservation price and an execution price for blocks of capacity, and the buyer, facing a known distribution of demand, needs to decide which blocks to reserve. Chapter 4 studies a continuous version of the problem where we allow general cost functions for suppliers. The suppliers compete by offering the price functions (for both reservation and execution) and the buyer decides how much to reserve from each supplier.

This thesis sheds light on how suppliers compete with each other by considering a variety of factors. We believe an in-depth look at the competitive behaviour of suppliers will deepen our understanding of a buyer's procurement process, and hence has the potential to help a buyer make a better sourcing decision.

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To my family

Chapter 1

Introduction and Literature Review

1.1 Introduction

1.1.1 Motivations

The concept of “supply chain management” integrates supply and demand management both within and across firms. According to the Council of Supply Chain Management Professionals (CSCMP), supply chain management encompasses the planning and management of all activities including procurement, conversion and logistics management. It also includes coordination and collaboration with channel partners, which may be suppliers, intermediaries, third-party service providers, or customers (CSCMP, 2014). By coordinating the upstream and downstream entities, the objective of supply chain management is to deliver the right service or product to the right customers at the right time and in the right place.

An important area of study in supply chain management revolves around competition. Broadly speaking, competition in supply chains can be classified into two categories: *horizontal* competition between firms in the same echelon; and *vertical* competition between upstream and downstream partners. In the past few decades, research on vertical competition has received special attention from the operations management (OM) community, while this thesis focuses on horizontal competition.

In a procurement setting, a buyer can now easily have access to many different suppliers thanks to the advances in information & communication technologies (ICT). As a result, a growing number of buyers have switched from bilateral negotiation to (reverse) auctions. For example, Sun Microsystems awards almost 70% of its annual procurement contracts using online auctions, worth \$2.7 billion (Carbone, 2007). Procurement auctions are particularly relevant in a commodity market where quality is not a major concern since a buyer is roughly indifferent between purchasing from one supplier and

the other. Therefore, cost becomes a critical factor for the buyer's sourcing decision. This thesis is mainly concerned with such a horizontal competition setting where a buyer purchases a homogeneous service or product and requests quotes from multiple suppliers.

As an important business practice, subcontracting has become an edge for many firms to succeed in tough economic times. For example, in the contract cleaning industry, "...subcontracting is now the way many cleaning companies generate decent margins on their turnover ... " (Wynhausen, 2008). In South China, there are a number of component contracting manufacturers who share production capacity with each other on an *ad hoc* basis (Feng and Lu, 2012a). The big garment factories in Bangladesh usually take orders based on subcontracting capacity (Lahiri and Passariello, 2013). According to a quote from the managing director of Synergies Worldwide, "... [garment factories] may have a two-million capacity and they will take orders for four-million ... ".

To facilitate the cooperation and coordination among manufacturers and subcontractors in developing countries and countries with economies in transition, the United Nations Industrial Development Organization (UNIDO) has established Subcontracting and Partnership Exchange (SPX) Centers in more than 20 countries worldwide (UNIDO, 2013). The main objective is to help local enterprises successfully meet the challenges of globalization and take advantage of the emerging opportunities that evolve from industrial subcontracting, outsourcing and supply chain opportunities.

Subcontracting is not equivalent to outsourcing. In an outsourcing situation, outsourcing firms do not have production or service facilities, and hence rely completely on contractors. However, subcontracting is generally carried out when a prime contractor reaches a capacity limit in its production or service process. In order to meet the customer's order on time, the remaining work is subcontracted to an external specialized party for a temporary period of time. The purpose of subcontracting is to bring down the overall production cost since otherwise the prime contractor has to use more expensive resources. For example, it may run night shifts, hire more people, or use advanced facilities. The implication is that if the prime contractor does everything by itself, the production cost will be higher for the quantities beyond the existing standard capacity. Thus, contractors face some form of diseconomies of scale when bidding for a buyer's work, which is unique to a subcontracting setting. Even though the research on outsourcing is quite established, the topic of subcontracting has not been thoroughly studied. In Chapter 2, we are particularly interested in how contractors compete with each other for an outsourcing firm's business when they face diseconomies of scale. Specifically, we incorporate subcontracting and single-sourcing commitment into the supplier competition model, which allows us to evaluate their impact on supplier bidding behaviours and supply chain performance.

In industries, such as semi-conductors and electricity, where suppliers (manufacturers

or generators) face large investment costs and high demand uncertainty, risk management becomes critical for their operations. Semi-conductor manufacturers usually have limited opportunities to postpone production decisions until the time accurate demand forecasts are available. Generators normally make their sites available ahead of time in order to generate power later. Roughly speaking, there exist two types of risk involved in these industries: financial risk from capacity installation, and inventory risk from demand uncertainty. On one hand, suppliers are unwilling to take all the financial risks from capacity installation without knowing how much capacity is needed in the future. On the other hand, buyers remain hesitant to assume all the demand risks by making firm commitment on later purchases. Therefore, an appropriate mechanism would be beneficial in managing these risks.

One way of risk sharing between suppliers and buyers is capacity reservation. The buyer reserves capacity in advance by paying the supplier an upfront fee (i.e. a reservation price), then the supplier builds capacity accordingly. After discovering the actual demand, the buyer can purchase any amount up to the reserved level of capacity and pays only for the dispatched amount (i.e. an execution price). This partial commitment of capacity reservation helps reduce the risks for both parties: the supplier's financial risk is diminished by receiving a reservation payment for capacity installation; and the buyer's demand risk is mitigated by freely choosing how much capacity to use after it knows the actual demand.

A notable example of this sort is the UK's Short Term Operational Reserve (STOR) (National Grid, 2014). In order to balance the supply and demand of electricity on short time scales, the UK's National Grid has contracts in place with generators and large energy users to provide temporary extra power, or reduction in demand. STOR is procured via competitive tender with three tender rounds per year. There are two forms of payment in a STOR contract: (1) Availability Payments: service providers are paid to make their units/sites available for the STOR service within an Availability Window. (2) Utilisation Payments: service providers are paid for the energy delivered as instructed by National Grid. Chapter 3 and Chapter 4 study the strategic behaviour of suppliers in a situation where capacity reservation is required for the buyer as in the STOR market.

1.1.2 Models

This thesis studies a competitive situation in which multiple suppliers bid for a buyer's work and the buyer's demand is uncertain. The main contents of this thesis are organized in three chapters.

Chapter 2 studies the role of subcontracting and single-sourcing commitment in supplier bidding. Under the circumstance where suppliers face diseconomies of scale, it

may be cost effective for a buyer to split an order across different suppliers. Even when the buyer chooses only one supplier, the winning supplier may subcontract part of the work to the others subsequently. Inspired by these observations, this chapter studies two procurement scenarios. In the first scenario, the buyer does not commit to purchasing from only one supplier, implying that it may split the order. We find that the buyer's ordering decision in equilibrium is supply chain optimal, each supplier's profit equals its marginal contribution to the supply chain system, and the buyer takes the remaining profit. In the second scenario, the buyer makes a single-sourcing commitment before suppliers submit their bids, and the winning supplier may subsequently subcontract with the losing one. We demonstrate that the more bargaining power the winning supplier has in the subcontracting stage, the more (less) profit the buyer (each supplier) makes. Counter-intuitively, the buyer may be better off to allow subcontracting when the winning supplier's bargaining power exceeds a threshold. Finally, by comparing the equilibrium outcomes of these two scenarios, we show that the buyer prefers to commit to single-sourcing.

Chapter 3 examines the bidding behaviour of suppliers with capacity reservation. When a firm faces an uncertain demand, it is common to procure supply using some type of option or two-part contract. A typical setting of this problem involves capacity being purchased in advance, with a separate payment made that applies only to the part of capacity that is needed. We consider a discrete version of this problem in which competing suppliers each choose a reservation price and an execution price for blocks of capacity, and the buyer, facing a known distribution of demand, needs to decide which blocks to reserve. We first show how to solve the buyer's (combinatorial) problem efficiently and then establish that suppliers can do no better than offer blocks at execution prices that match their execution costs (making profits only from the reservation portion of their bids). Second, we find that, when the suppliers have equal-size blocks, in equilibrium the buyer selects the welfare maximizing set of blocks, each supplier makes a profit equal to its marginal contribution to the supply chain system, and the buyer takes the remaining profit. Finally, we provide a procedure to construct an equilibrium for suppliers in the case with unequal-size blocks.

Chapter 4 investigates a similar setting to Chapter 3 where a buyer first reserves capacity from suppliers before demand materializes and then decides how much capacity to use after observing the actual demand. The critical distinction of this chapter is that we allow general cost functions for suppliers, and the suppliers compete by offering the price functions (for both reservation and execution). Similar to Chapter 3, we find that it is optimal for suppliers to set execution prices to be execution costs, thus they make profits only from the buyer's reservation payments. We also show that, in a class of equilibria, the buyer's reservation choice is first best, each supplier's profit equals its marginal contribution to the supply chain system and the buyer takes the remaining

profit. Comparing with the existing literature, we highlight the significant impact of the suppliers' strategy space on supplier bidding behaviours and supply chain performance.

1.1.3 Contributions

Previous research on supplier competition has begun to consider the sophisticated choices that are often available: for example the use of the schedules of prices and quantities, or the use of reservation prices. This thesis aims to contribute to this literature by building models which better reflect the more complex issues in competition and supplier bidding mechanisms that are important in practice, e.g., subcontracting, commitment and capacity reservation.

We model an important setting where, from the supply chain point of view, it is optimal for a buyer to split its order across multiple suppliers. The splitting of orders is driven by the diseconomies of scale for suppliers (Chapter 2) or the trade-off between flexibility and cost efficiency for the buyer (Chapter 3 and Chapter 4). Surprisingly, the topic of supplier competition in this setting has not received as much attention as it deserves, so this thesis aims to fill this gap.

Many papers on competition do not consider the buyer's optimization problem explicitly. Instead, they directly assume that the demand functions satisfy certain conditions so that the equilibrium analysis is tractable. However, in this research we explicitly model the buyer's ordering behaviour, which turns out to be a non-trivial task. Particularly, in Chapter 3, we find that the buyer's problem is to maximize a non-monotonic submodular function. In general it is NP hard to maximize a submodular function, but we are able to develop a dynamic programming approach to solve the buyer's problem in polynomial time.

Chapter 2 and Chapter 4 involve the supply function competition in which suppliers each offer a function bid (i.e. a schedule of prices and quantities). As opposed to the competition models where the bidding strategy simply involves choosing a price, the analysis is much more challenging. The supply function equilibrium (SFE) literature deals with similar problems in that each bidder quotes a supply function which specifies the quantities it is willing to supply at different prices. In the SFE literature, the buyer's problem is relatively straightforward since the buyer chooses a clearing price to equate the demand with the aggregate supply. In our models, however, the buyer's optimization problem has to be addressed explicitly. As a result, the solution approach is different.

This thesis examines the buyer's procurement problem from the supplier competition's perspective. We believe a closer look at how suppliers behave when competing for a buyer's business will deepen our understanding of a buyer's procurement process. Ultimately, from a broader perspective, we hope this thesis will not only assist suppliers in making better bidding decisions but also help buyers make better sourcing decisions.

1.2 Literature Review

This section reviews the relevant literature. Since the detailed reviews will be provided in each chapter, we will focus on more general related papers at this stage. Our research is related to the literature on competition in supply chains and that on auctions.

1.2.1 Competition in supply chains

Many OM papers concern coordination and collaboration issues between upstream and downstream players in supply chains, e.g., information sharing, channel coordination, cost and/or lead time reduction, and collaborative R&D. For general reviews on these topics, we refer readers to Tsay et al. (1998), Lariviere (1999), and Cachon (2003). As we focus primarily on the horizontal competition between firms, we will restrict our attention to the literature on this topic. Specifically, we will review the literature on price and/or quantity competition, price and capacity competition, price and service competition, and price and inventory competition.

Price and/or quantity competition

There is a large literature on price and/or quantity competition in both economics and operations management, originating from the canonical Bertrand and Cournot competition models. The following quote from Kreps and Scheinkman (1983) clearly describes the distinction between Bertrand and Cournot models:

“...it is easier to explain what we mean by reviewing the stories associated with Cournot and Bertrand. The Cournot story concerns producers who simultaneously and independently make production quantity decisions, and who then bring what they have produced to the market, with the market price being the price that equates the total supply with demand. The Bertrand story, on the other hand, concerns producers who simultaneously and independently name prices. Demand is allocated to the low-price producers, who then produce up to the demand they encounter. Any unsatisfied demand goes to the second lowest price producers and so on ...”.

As an extension of Bertrand competition and Cournot competition, supply function competition has received considerable attention from researchers in economics and operations research, in which firms each offer a schedule of prices and quantities. In the following, we will divide the price and/or quantity competition models into four strands: competition in a B2B setting; competition in a B2C setting; chain-to-chain competition; and supply function equilibrium.

B2B: Supplier versus supplier competition is concerned with suppliers selling products to downstream distributors or retailers rather than end consumers (see Elmaghraby

(2000) for an excellent review). In this line of research, the buyer's optimization problem is usually modelled explicitly. Some papers study the competitive behaviour of suppliers with consideration of supply disruptions (Babich et al., 2007), commissions and sales targets (Gallego and Talebian, 2014a), or fixed capacity (Gallego and Talebian, 2014b). In addition, Jiang and Wang (2009) examine the competitive and coordinative behaviour of suppliers who provide complementary components to an assembly. Wu and Kleindorfer (2005) and Martínez-de Albéniz and Simchi-Levi (2009) examine supplier competition in an option market. Cachon and Kok (2010) study a setting where two manufacturers sell *differentiated* products to a common retailer and examine how different contract forms affect the competition between manufacturers. More recently, several papers study supplier competition under asymmetric information. Özer and Raz (2011) consider a supply chain with two heterogeneous suppliers and one buyer where the “big” supplier (with low variable cost) has incomplete information regarding the “small” supplier's cost. Zhao et al. (2014) study an outsourcing problem with a focus on information sharing where two service providers compete for a client's service contract. Lee and Yang (2013) extend Cachon and Kok (2010) by incorporating asymmetric demand information into the competition model. Different from Lee and Yang (2013), Li et al. (2014) consider a setting where competing suppliers supply a *homogeneous* product to a new vendor buyer, and investigate the trade-off between cost advantage and information rent.

B2C: The research on competition in a B2C setting studies how firms compete in a consumer market. In this literature, demand is usually taken as a primitive and exogenously given, hence the buyer's optimization problem is not relevant. An exception is by Caro and Martínez-de Albéniz (2012) who study price and product competition with consumer satiation, and explicitly model the customer behaviour based on utilization maximization. The demand models used in this literature include Multiplicative Competitive Interaction (MCI), Attraction, Multinomial Logit (MNL), Linear, Multiplicative and Exponential models (Cooper and Nakanishi, 2010). For comprehensive reviews on demand models, readers can refer to Anderson et al. (1992) and Huang et al. (2013). Several papers in this area study a one-shot game: Netessine and Shumsky (2005) analyze an inventory control problem in the airline industry under both horizontal competition and vertical competition. Federgruen and Yang (2009) examine the effect of the retailer's pricing power on supplier investment in supply reliability. Allon and Federgruen (2009) analyze the equilibrium behaviour of service providers in industries where they cater to multiple market segments. In addition to the above papers, there also exists a class of supermodular games in which the payoff functions of players are supermodular: see Topkis (1998) for an excellent treatment on supermodular games. Diverting from the above one-shot games, Adida and Perakis (2009) study dynamic competition in a make-to-order system where two firms compete in an uncertain market. Martínez-de

Albéniz and Talluri (2011) study dynamic pricing competition in an oligopoly with fixed capacity.

Chain-to-chain: In a chain-to-chain competition model, multiple supply chains compete by offering differentiated products to a common market. There is a rich literature on channel competition in economics and marketing (see e.g., McGuire and Richard, 1983; Moorthy, 1988; Choi, 1996), which explores the effect of channel coordination and product differentiation on channel performance. In the operations management field, Shou and Li (2009) study supply chain competition in the presence of supply disruptions. Anderson and Bao (2010) consider price competition within integrated and decentralized chains. Ha and Tong (2008) investigate the contracting and information sharing decisions in two competing supply chains. Ha et al. (2011) study the incentive of vertical information sharing in competing supply chains with the production technologies that exhibit diseconomies of scale.

Supply function equilibrium: Supply function equilibrium (SFE) models study the competitive behaviour of bidders who each submit a supply function, and have widespread application in electricity markets. Following the seminal paper by Klemperer and Meyer (1989), various extensions have been made by considering production capacity constraint (Holmberg, 2008), cost asymmetry (Anderson and Hu, 2008), multi-step cost functions (Holmberg et al., 2013), forward contracting (Anderson and Hu, 2012), or private information (Vives, 2011), etc.

To provide a basic understanding of the SFE literature, we now review the basic SFE model. Suppose the demand function is $D(p, \theta)$, where θ indicates a random shock. Each bidder submits a supply function which maps prices into outputs: $q : [0, +\infty) \rightarrow (-\infty, +\infty)$. After knowing the demand, the buyer determines the market clearing price that equates the demand with the aggregate supply. Assume there are two bidders i and j , then we have $D(p(\theta), \theta) = q_i(p(\theta)) + q_j(p(\theta))$. Given the bidder j 's strategy $q_j(p)$, the bidder i 's problem is to maximize its profit by choosing $q_i(p)$,

$$\max_{q_i(p)} \pi_i = pq_i - C(q_i) = p[D(p, \theta) - q_j(p)] - C[D(p, \theta) - q_j(p)].$$

Solving the first order condition with respect to p yields

$$\frac{dq_j}{dp} = \frac{q_i}{p - C'(q_i)} + \frac{\partial D}{\partial q}.$$

Then we can solve the above differential equations and characterize the equilibrium for bidders. In general, there exist multiple equilibria.

Price and capacity competition

Perhaps, a starting point of research on price and capacity competition is to investigate how capacity constraints affect the competitive equilibrium of firms by assuming *exogenous* capacity. A well-known result is given by Levitan and Shubik (1972) who show that there may not exist a *pure* strategy equilibrium, but there always exists an equilibrium (pure or mixed strategy). With an *endogenous* capacity decision, Kreps and Scheinkman (1983) study a two-stage game where in the first stage, firms build capacity, then they compete in a consumer market by choosing sale prices in the second stage. They show that there exists a unique (pure strategy) equilibrium which is equivalent to the Cournot outcome. Davidson and Deneckere (1986) explore the role of rationing rules in equilibrium outcomes. Acemoglu et al. (2009) examine a setting with a homogeneous product and demonstrate that there exist multiple pure strategy (subgame perfect) equilibria. Ángeles de Frutos and Fabra (2011) study the role of demand uncertainty in firm competition. Note that the competition models with capacity and price involve discontinuous payoff functions, which is a result of Bertrand-like effects where undercutting the price of other players is beneficial. See Dasgupta and Maskin (1986) for a general treatment on the equilibrium existence problems with discontinuous payoff functions.

Price and service competition

There are a number of papers that study price and service competition between firms. Service is a broad concept which can be measured based on the criteria of quality, fill rate, or delivery time. Tsay and Agrawal (2000) study a distribution system in which a manufacturer supplies a common product to two independent retailers, who then compete in a consumer market by setting service levels and retail prices. So (2000) analyzes the impact of time guarantees on price competition between firms. Cachon and Harker (2002) present both an EOQ model and a queueing model of competition between two firms that face scale economies. Ha et al. (2003) consider an EOQ model where two suppliers compete for supply to a customer on pricing and deliver-frequency decisions. Bernstein and Federgruen (2004) develop a stochastic inventory model where retailers compete by choosing retail prices, fill rates and inventory levels. More recently, Jin and Ryan (2012) study a setting where a buyer (OEM) outsources the manufacturing of a product to multiple make-to-stock suppliers who compete on price and service (i.e. fill rate).

Besides the above papers, there also exists a line of research studying the competition between service providers who are modelled as queues (Hassin and Haviv, 2003). Cachon and Zhang (2007) study a strategic queueing model in which two servers choose their processing rates and highlight the important role of the buyer's allocation policy.

Benjaafar et al. (2007) consider a buyer who outsources a fixed demand for a good or service to a set of potential suppliers, and compare two competition mechanisms of supplier allocation and supplier selection. Afanasyev and Mendelson (2009) model the competition between two service providers by considering delay-sensitive customers. Shang and Liu (2011) investigate the competitive behaviour of firms in industries where customers are sensitive to both promised delivery time and on-time delivery rate. Li et al. (2012) consider a two-stage game in which two service providers first select service rates and then set prices.

Price and inventory competition

Inventory competition has received tremendous attention over the past few decades. For a review, we refer readers to Chen and Simchi-Levi (2012). This literature, generally assuming exogenous prices, considers the stock-out based substitution where customers may turn to a different retailer when one retailer runs out of inventory. Some papers consider one-shot inventory competition (Netessine and Rudi, 2003), Stackelberg game (Serin, 2007), multi-period inventory competition (Nagarajan and Rajagopalan, 2009), assortment and inventory decision (Honhon et al., 2010), or asymmetric demand information (Jiang et al., 2011). A natural extension of inventory competition models is to consider the competition over both price and inventory. Zhao (2008) investigates a supply chain system with a common supplier selling to downstream retailers who are engaged in both price and inventory competition. Zhao and Atkins (2008) examine the simultaneous price and inventory competition between competing newsvendors. Kok and Xu (2011) study the strategic assortment planning and pricing decision for a product category with heterogeneous product types from two brands.

1.2.2 Auction and competitive bidding

Our model setup is similar to those in auctions where multiple bidders compete for an auctioneer's goods. There are a variety of criteria to distinguish different forms of auctions. According to auction formats, auctions can be conducted in the open outcry way or using sealed bids. Submitted bids might contain just a single attribute (price only) or multiple attributes (price and qualities). According to the number of units to sell, there are single-unit and multi-unit auctions. For a comprehensive literature review on single-unit auctions, readers can refer to Milgrom (2004) and Krishna (2009). In the following, we will focus our attention on multi-unit auctions.

If each bidder demands just a single unit the auction is called a *unit-demand* multi-unit auction, otherwise it is a *multi-demand* multi-unit auction. For the latter, the marginal evaluation of each unit is normally assumed to be decreasing. With an *exogenous* total amount, a number of auctions have been developed, for example, menu

auction (Bernheim and Whinston, 1986), share auction (Wilson, 1979), Vickery auction (Vickrey, 1961), uniform price auction (Bresky, 2013), discriminatory price auction (Menezes and Monteiro, 1995), and split award auction (Anton and Yao, 1989, 1992). This literature examines the efficiency and profit allocation of a given auction format. With an *endogenous* total amount, Chen (2007) studies a setting where suppliers compete for a newsvendor's business who each have private cost information, and proposes a supply contract auction. Dasgupta and Spulber (1989) examine a similar procurement problem and develop a quantity auction. More recently, Duenyas et al. (2013) develop a simple modified version of the standard open-descending auction, which is shown to be optimal for the buyer. These papers focus on designing an optimal auction mechanism for a buyer. The auction literature generally assumes that bidders have private information, while this thesis considers a full information setting.

Chapter 2

Supplier Competition with Subcontracting and Commitment

2.1 Introduction

Advances in information and communication technology have opened new channels for a firm's procurement function. A growing number of companies are re-thinking their sourcing strategies and switching from bilateral negotiation to auctions, because they can now easily access many different suppliers and maintain reasonable competition. Procurement auction helps buyers in efficient cost discovery and savings in negotiation and contracting costs (Tunca and Wu, 2009). It also leads to standardization of sourcing procedures, avoid bribery, and reduce order cycles. The electronic marketplaces, such as Alibaba and FreeMarkets (acquired by Ariba in 2004, who then was acquired by SAP in 2012), provide an excellent procurement platform for buyers to run auctions and for suppliers to bid for the buyers' business. Many large industrial players use reverse auctions to procure direct and indirect materials. For example, Sun Microsystems awarded almost 70% of its annual procurement contracts using online auctions, worth \$2.7 billion (Carbone, 2007). GM utilized internet tools to purchase most of its components, which enables decreased lead time and lower purchase cost (Burke et al., 2007).

With multiple units to buy, a firm's critical decision is whether to split an order among multiple suppliers (*multi-sourcing*) or to award the entire order to a single supplier (*single-sourcing*) (Elmaghraby, 2000). In practice, both multi-sourcing and single-sourcing are often used. There has been a debate on the ideal number of suppliers to fulfil product demand. Using a multi-sourcing strategy, firms may benefit from improving supply reliability (Federgruen and Yang, 2008), avoiding hold-up problems (Seshadri, 1995), and so forth. In contrast, a notable advantage of single-sourcing is that it helps a firm maintain a sustainable and long-term relationship with its supplier. The existing research on sourcing strategies has been focused on the *ex post* decision for a buyer

after it receives bids from suppliers. Thus, it is interesting to investigate how *ex ante* commitment on sourcing strategies affects competition between suppliers.

Subcontracting has recently become a prominent business practice in many industries¹. While the most common examples of subcontracting exist in building and civil engineering, the opportunities for subcontractors abound, ranging from manufacturing to cleaning industries. In this research, we are interested in subcontracting based on the short term need for additional production or service capacity. When the available capacity of a prime contractor is insufficient to execute an order and further installation of in-house capacity is neither feasible nor desirable, the contractor depends on a subcontractor to meet the balance of an order. For example, in South China, there are a number of contracting manufacturers who share production capacity by means of subcontracting (Feng and Lu, 2012a). The large garment factories in Bangladesh usually take orders based on subcontracting capacity. According to a quote from the managing director of the Bangladesh office of Synergies Worldwide, "... [garment factories] may have a two-million capacity and they will take orders for four-million ..." (Lahiri and Passariello, 2013).

The incentives of subcontracting include reducing costs, meeting deadlines to avoid punishment, and mitigating risks, etc. In this way, the buying firm receives the same product or service, yet the overall cost for the prime contractor is lower than that if it does everything by itself. Altogether, we claim that subcontracting is primarily driven by diseconomies of scale. This is particularly true in a short-term market. To meet a client's order on their own, contractors may have to use more expensive resources. For example, they may run night shifts by paying their employees at higher rates, or use more advanced facilities at higher costs. In the Bangladesh garment industry, if factories miss a shipping deadline, they may have to rush the shipment by airfreight, at their own expense, or give the buyer a 5% discount as a penalty (Lahiri and Passariello, 2013). The implication is that the contractors face higher marginal costs when the customer's order exceeds their existing capacity. Therefore, a model of competition in this situation should take into account the fact that contractors face diseconomies of scale when they bid for a firm's procurement business. Surprisingly, this topic has not been thoroughly studied, and this work aims to fill this important gap. Throughout this chapter, we shall use the term "supplier" to refer to a contractor and the term "buyer" to refer to a buying firm.

Under circumstances where suppliers face diseconomies of scale, it may be cost effective for a buyer to split an order across different suppliers. However, dealing with more

¹There is a slight difference between subcontracting and outsourcing. Subcontracting typically refers to the situation where a prime contractor procures an item or service that the contractor is normally capable of producing using its own facilities and that requires the contractor to make specifications available to a subcontractor. Outsourcing refers to the special case where the contractor has no in-house facilities and depends on the subcontractor for the entire product volume (Van Mieghem, 1999).

suppliers incurs a higher managerial cost, which may prevent the buyer from working with multiple suppliers. If the buyer awards its entire order to a single supplier, the winning supplier may subcontract part of the work to the others later. Motivated by these observations, we are interested in whether the buyer has incentive to make a single-sourcing commitment before running a bidding process. It is also intriguing to know how the availability of subcontracting affects supplier competition. Specifically, we ask the following research questions: (1) How do suppliers compete if the buyer does not commit to single-sourcing? (2) If the buyer commits to single-sourcing, how does each supplier's bidding strategy change if subcontracting between competing suppliers is considered? Is the buyer always worse off to allow subcontracting? (3) Should the buyer commit to single-sourcing when purchasing an item using an auction?

To answer these questions, we consider a stylized model where two suppliers compete for a single buyer's business. We find that, if the buyer does not make a single-sourcing commitment, the supply chain is coordinated in equilibrium, each supplier makes a profit equal to his marginal contribution, and the buyer takes the remaining profit. With single-sourcing commitment, we demonstrate that the bargaining power split at the subcontracting stage plays a vital role in determining the subgame perfect Nash equilibrium. Specifically, if the winning supplier has more bargaining power in subcontracting, the buyer (each supplier) will make more (less) profit. Our results also show that, counter-intuitively, subcontracting may benefit the buyer when the winning supplier's bargaining power is higher than a threshold. Finally, we show that the buyer is better off to make a single-sourcing commitment whether or not subcontracting is allowed.

Our model applies to both service and manufacturing industries where suppliers face diseconomies of scale. In general, subcontracting is undertaken when a prime contractor has arrived at a capacity limit in its production or service process. In order to meet the customer's demand, the remaining work is subcontracted to an external party for a temporary period of time. In the following, we discuss two industries where our model can be readily applied.

Contract manufacturing in the biopharmaceutical industry During the past 25 years, the biopharmaceutical industry has boomed to become a US\$167-billion market today (Martin, 2013). More than most high-technology industries, the biopharmaceutical sector faces a combination of high costs, competition, and uncertain demand, which has led to increasing use of contract manufacturers. The main challenge facing these contract manufacturers is that they lack the manufacturing plants, bioreactors, and other equipment needed to make sufficient amounts of biopharmaceuticals. Industrial data show that building a new manufacturing plant requires five years and hundreds of millions of dollars—typically from US\$200 to US\$400 million (Kamarck, 2006). Thus, these manufacturers instead begin to partner on an *ad hoc* basis to share capacity whenever

they run beyond capacity. This study is fascinated by the way the proactive strategy of capacity sharing affects the competition between contract manufacturers. Capacity sharing in this context is equivalent to subcontracting in the sense that contract manufacturers temporarily jointly produce for an order. Our model can be calibrated to address the following questions: Does a biopharmaceutical firm benefit from making a commitment to sourcing from only one contract manufacturer? If so, should the firm allow contract manufacturers to share capacity?

Contract cleaning industry Cleaning services are important in developed economies. For example, in Australia, the commercial contract cleaning market continued to grow at a rate of 2.2% annually from 2009 to 2014, and was worth AU\$8 billion in 2013 (IBIS, 2014). A critical resource in cleaning is labor work; thus, to fulfil a client's requirement, cleaning firms may need to hire additional people. However, future clients may not need as high a workload. As a result, many cleaning firms use subcontracting in order to generate decent margins on their turnover. Clearly, the main driver of subcontracting is to lower the overall cost for cleaning firms. However, it remains unclear whether this also benefits the buyer. In practice, some firms allow subcontracting, such as Westfield Group; while others do not, such as Mirvac Group (Wynhausen, 2008). This study questions whether a buyer has incentive to pre-commit to outsourcing its cleaning business to a single cleaning firm, and whether the buyer benefits from subcontracting between cleaning firms.

The remainder of this chapter is organized as follows: We review the relevant literature in Section 2.2 and present the model setup in Section 2.3. Two procurement scenarios are studied: under the first scenario of order splitting, the buyer does not make a single-sourcing commitment, thus it may split the order; under the second scenario of single-sourcing commitment, the buyer commits to purchasing from only one supplier but the winning supplier has the option to subcontract with the losing supplier later. We examine these two procurement scenarios in Section 2.4 and Section 2.5, respectively. Section 2.6 makes a complete comparison between them based on each player's profit. Finally, we conclude and discuss the managerial insights in Section 2.7.

2.2 Related Literature

This chapter primarily serves as a complement to the supply chain contract literature. For reviews on this literature, refer to Lariviere (1999) and Cachon (2003). Most of the work on contract design and supply chain coordination considers one-to-one or one-to-many supply chains with linear total production costs. Thus, research on many-to-one supply chains with nonlinear production costs would be beneficial (Cachon, 2003). Recently, several papers have made progress in this area. Cachon and Kok (2010) consider

a setting in which two manufacturers sell differentiated products to a retailer, and examine how different contract forms affect competition between manufacturers. Gallego and Talebian (2014b) study supply chain coordination under competition between capacitated suppliers. Li et al. (2014) examine how asymmetric demand information affects supplier competition, with a focus on the trade-off between cost advantage and information rent.

The current study contributes to this literature by examining a situation in which suppliers face diseconomies of scale, as in the earlier discussions. This topic has not been well explored. More importantly, this study incorporates *ex-post* subcontracting and *ex-ante* single-sourcing commitment into the supplier competition model, which allows evaluation of their effects on supplier bidding behaviors and supply chain performance.

Subcontracting has received some attention from the economics and operations management communities. Kamien et al. (1989) examine a setting where two symmetric firms supply a market with price-dependent demand, and can subcontract with each other. Van Mieghem (1999) studies a competitive stochastic investment game in which manufacturers first decide on their capacity investment levels, and then choose their production and sales with the option of subcontracting. Vairaktarakis and Aydinliyim (2007) analyze a capacity competition game in which a set of manufacturers outsource their workload to a subcontractor. In contrast to this literature, the current study is concerned with the implications of single-sourcing commitment and subsequent subcontracting on supplier bidding.

This study's comparison between the two procurement scenarios resembles that between delegation and control mechanisms in the principal-agent literature, in which a buyer purchases multiple products that are each produced by different privately informed suppliers. In the control case (similar to this study's order splitting scenario), the buyer purchases from each supplier directly; while, in the delegation case (similar to this study's single-sourcing commitment scenario), the buyer purchases only from a primary supplier who is delegated to purchase the other products. The main result is that the buyer prefers the control case if the contract offered can be arbitrarily complex, because there exists a cascading effect of information rent in the delegation case (Mookherjee, 2006). Enis et al. (2013) question the use of complex contracts and focus on simple contract forms. They identify conditions under which one mechanism dominates the other. Our model differs from the aforementioned studies in that it investigates the suppliers' competitive behavior, rather than designing an optimal mechanism for the buyer. In our commitment model, the competitors in the bidding game can be collaborators later in subcontracting. This feature does not exist in the mechanism design models. Moreover, our result is in sharp contrast with theirs—we find that single-sourcing commitment outperforms order splitting from the buyer's perspective.

There exists a large literature on procurement auctions. See Elmaghraby (2000) for a collection of papers on supplier competition and procurement auctions. In a setting where a buyer purchases multiple units of an item, various auction formats can be applied, such as split-award auction (Anton and Yao, 1989), multi-unit auction (Ausubel and Milgrom, 2006), and menu auction (Bernheim and Whinston, 1986). Assuming an exogenous purchase quantity, this literature focuses on price discovery and considers how to divide the order among bidders. With an endogenous purchase amount, Chen (2007) proposes a supply contract auction in which the buyer first designs a quantity-payment schedule, and then auctions off this contract to a cohort of suppliers. Thinking of the contract as an “object”, suppliers each bid a lump-sum fee. Dasgupta and Spulber (1989) study a similar problem and develop a quantity auction in which suppliers each bid a supply quantity instead of a lump-sum fee. However, our intention is not to design an optimal auction mechanism for the buyer. Instead, the buyer—as a Stackelberg follower in our model—decides how much to purchase from each supplier after receiving the suppliers’ bids.

2.3 Model Setup

The supply chain we investigate consists of two suppliers (“he”) and a single buyer (“she”), where the buyer purchases a homogeneous product or service from these two suppliers. Each player is risk neutral and maximizes their own profit. We consider two procurement scenarios: (1) *order splitting*, in which the buyer does not make any commitment regarding the number of winners in the bidding game. Each supplier submits a function bid that specifies different payments for different quantities². Given the available function bids, the buyer decides how much to purchase from each supplier; and (2) *single-sourcing commitment*, in which the buyer commits to purchasing from only one supplier before the suppliers submit their bids, and the winning supplier can subsequently opt to subcontract with the losing supplier.

The buyer has an expected revenue of $R(q)$ if she purchases q units of an item. We assume $R(q)$ is a finite, increasing and concave function of $q \geq 0$ and $R(0) = 0$. This revenue function covers several operations and economics models. The notable example in operations management is the newsvendor setting where a buyer places orders with suppliers prior to knowing the actual demand. After the demand is observed, the buyer sells the product at an exogenous price r . For simplicity, we assume zero penalty costs for unsatisfied demand and zero salvage costs for excessive orders. The demand D follows a distribution with the support $[d_1, d_2] \subset [0, \infty)$. The cdf of D , $F(d)$, is a continuous, strictly increasing and differentiable function over (d_1, d_2) . Define its pdf $f(d) := F'(d)$,

²In some contexts, a function bid is referred to as a nonlinear contract, e.g., quantity discount contract. Throughout we will use the term of “function bid”.

thus we have $f(d) > 0, \forall d \in (d_1, d_2)$. The revenue function in this setting can be written as

$$R(q) := r\mathbb{E}[\min(D, q)] = r \left(q - \int_0^q F(\tau) d\tau \right),$$

which can be readily checked to be increasing and concave with $R(0) = 0$. For convenience, we use the newsvendor model in the following analysis but the results apply more generally.

Denote by i, j the indices of suppliers, whose total production costs are $C_i(q)$ and $C_j(q)$, respectively, where $i = 1, 2$ and $j = 3 - i$. We make the following assumption on supplier costs.

Assumption 2.1. *For $i = 1, 2$, the cost function $C_i(q)$ is twice differentiable, increasing and strictly convex, i.e. $C_i'(q) > 0$ and $C_i''(q) > 0, \forall q > 0$. Also, $C_i(0) = 0$.*

First, if there is no production, no cost will be incurred. Second, we focus on the convex cost functions that have been used in the operations management literature (Ha et al., 2011). The diseconomies of scale are also supported by much empirical evidence for the industries in which firms use the cheapest resource first, then the second cheapest, and so forth (Baldick et al., 2004; Mollick, 2004).

In a situation with order splitting, the timeline is as follows: First, each supplier submits a function bid $T_i(q)$. Second, before knowing the actual demand, the buyer decides how much to purchase from each supplier. Third, the suppliers with positive orders begin production and deliver the product to the buyer. Finally, after discovering the actual demand, the buyer sells the product at the retail price, r . In a situation with single-sourcing commitment, the timeline differs in that the winning supplier is allowed to subcontract with the losing supplier after the buyer selects the winning supplier. See Figure 2.1 for the detailed sequence of events.

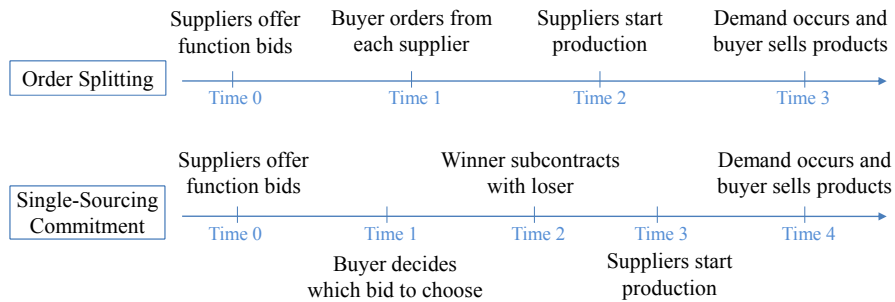


FIGURE 2.1: The timeline under each scenario

In our setting, the suppliers are Stackelberg leaders, and the buyer is a follower who has limited control over contract design. This setting is consistent with relatively mature markets. See Shi et al. (2013) and Chen et al. (2014) for discussions on power relationships in supply chains. Moreover, the production technologies and business environments in mature markets are relatively transparent; thus, this chapter considers a

complete information setting in which each supplier is fully aware of each other's cost and the product information (including demand distribution and retail price).

2.3.1 Supply chain optimal solutions

As a preliminary step, we examine the integrated supply chain problems which are equivalent to the buyer's problems if suppliers charge only their production costs. Define

$$\Pi(q_i, q_j) = r\mathbb{E}[\min(D, q_i + q_j)] - C_i(q_i) - C_j(q_j),$$

as the supply chain profit when the buyer purchases q_i, q_j from suppliers i, j , respectively. We choose the order quantity from each supplier to maximize the supply chain profit:

$$\max \{\Pi(q_i, q_j) : q_i, q_j \geq 0\}. \quad (2.1)$$

We can show that the objective function is jointly concave in q_i and q_j . We focus on the cases where no supplier dominates the other in terms of cost efficiency.

Assumption 2.2. *For supplier $i = 1, 2$, the cost functions $C_i(q)$ are chosen such that it is optimal for the buyer to split her order from the supply chain point of view.*

Assumption 2.2 implies that there exists an interior solution for the problem in (2.1), and the optimal solution (\bar{q}_i, \bar{q}_j) is characterized by the first order conditions:

$$r[1 - F(\bar{q}_i + \bar{q}_j)] = C'_i(\bar{q}_i) = C'_j(\bar{q}_j). \quad (2.2)$$

Let $\bar{Q} = \bar{q}_i + \bar{q}_j$ be the supply chain optimal total quantity, and $\Pi = \Pi(\bar{q}_i, \bar{q}_j)$ be the supply chain optimal profit. Since we will need to introduce a significant amount of notations it is convenient to collect all of them in Table 2.1.

Consider the case where supplier i is the sole supplier where $j = 1, 2$. Let

$$\Pi(q, 0) = r\mathbb{E}[\min(D, q)] - C_i(q)$$

be the supply chain profit when the buyer purchases q units from supplier i only. The supply chain problem in this case is

$$\max \{\Pi(q, 0) : q \geq 0\}.$$

It can be readily shown that $\Pi(q, 0)$ is concave in q , so the optimal solution \bar{Q}_i can be found from the first order condition:

$$r[1 - F(\bar{Q}_i)] = C'_i(\bar{Q}_i). \quad (2.3)$$

TABLE 2.1: The summary of notations

Notations	Interpretations
\bar{Q}	Supply chain optimal total quantity
\bar{q}_i	Supply chain optimal quantity for supplier i (see (2.2))
\bar{Q}_i	Single-sourcing supply chain optimal quantity for supplier i (see (2.3))
Π	Supply chain optimal profit
Π_i	Single-sourcing supply chain optimal profit for supplier i
q_i^D	Dual-sourcing buyer's optimal order from supplier i (see (2.6))
q_i^S	Single-sourcing buyer's optimal order from supplier i (see (2.7))
q_i^*	Buyer's global optimal order from supplier i
$\hat{q}_i^* := \hat{q}_i^*(T_j)$	Buyer's global optimal order from supplier i when i offers at cost
$\hat{q}_j^* := \hat{q}_j^*(T_j)$	Buyer's global optimal order from supplier j when i offers at cost
Γ_i	Optimal supply chain surplus contributed by i given $T_j(q)$ (see (2.12))
π_B^D	Dual-sourcing buyer's optimal profit
π_B^i	Single-sourcing buyer's optimal profit for supplier i
π_B^*	Buyer's global optimal profit
$T_i^*(q)$	Supplier i 's bid in equilibrium under order splitting
π_i^*	Supplier i 's profit in equilibrium under order splitting
Π^*	Supply chain profit in equilibrium under order splitting
T_i^n	Supplier i 's bidding payment in equilibrium under SC-N
Q_i^n	Supplier i 's bidding quantity in equilibrium under SC-N
π_i^n	Supplier i 's profit in equilibrium under SC-N
π_B^n	Buyer's profit in equilibrium under SC-N
Π^n	Supply chain profit in equilibrium under SC-N
$\hat{q}_j := \hat{q}_j(Q_i)$	Optimal subcontracted amount to j if buyer orders Q_i from i (see (2.30))
$\Delta_i(\hat{q}_j; Q_i)$	Optimal total gain from subcontracting if the buyer orders Q_i from i
$G(Q_i)$	Optimal supply chain profit if the buyer orders Q_i from i
T_i^s	Supplier i 's bidding payment in equilibrium under SC-S
Q_i^s	Supplier i 's bidding quantity in equilibrium under SC-S
π_i^s	Supplier i 's profit in equilibrium under SC-S
π_B^s	Buyer's profit in equilibrium under SC-S
Π^s	Supply chain profit in equilibrium under SC-S

Define Π_i as the optimal supply chain profit when supplier i is the sole supplier, so $\Pi_i = \Pi(\bar{Q}_i, 0)$. Note that we have $\Pi > \max(\Pi_1, \Pi_2)$, and $\bar{Q} > \bar{Q}_i > \bar{q}_i$ where $i = 1, 2$. We refer to $\Pi - \Pi_j$ as the supplier i 's *marginal contribution* to the supply chain system.

Throughout we use the following example to illustrate the main results derived in this chapter.

Example 2.1. *The buyer's demand follows a uniform distribution $D \sim U(0, 1)$, thus, $F(q) = q$ for $q \in [0, 1]$. The retail price is $r = 1$. Therefore, we have*

$$r\mathbb{E}[\min(D, q)] = r \left(q - \int_0^1 F(\tau) d\tau \right) = q - \frac{1}{2}q^2.$$

The production cost functions are quadratic with $C_1(q) = \frac{1}{2}q^2$ and $C_2(q) = q^2$. We solve the integrated supply chain problems as follows:

- If both suppliers are available, the supply chain problem is

$$\max \left\{ (q_1 + q_2) - \frac{1}{2}(q_1 + q_2)^2 - \frac{1}{2}q_1^2 - q_2^2 : q_1, q_2 \geq 0 \right\}.$$

The optimal solution is $(\bar{q}_1, \bar{q}_2) = (2/5, 1/5)$, the supply chain optimal total quantity is $\bar{Q} = 3/5$, and the chain optimal profit is $\Pi = 3/10$.

- If supplier 1 is the sole supplier, the supply chain problem is

$$\max \left\{ q - \frac{1}{2}q^2 - \frac{1}{2}q^2 : q \geq 0 \right\}.$$

The optimal quantity is $\bar{Q}_1 = 1/2$, and the single-sourcing supply chain optimal profit is $\Pi_1 = 1/4$.

- If supplier 2 is the sole supplier, the supply chain problem is

$$\max \left\{ q - \frac{1}{2}q^2 - q^2 : q \geq 0 \right\}.$$

The optimal quantity is $\bar{Q}_2 = 1/3$, and the single-sourcing supply chain optimal profit is $\Pi_2 = 1/6$.

2.4 Order Splitting

Under a situation with order splitting, the buyer does not commit to purchasing from only one supplier. We make the following assumption on each supplier's bidding strategy.

Assumption 2.3. The supplier i 's strategy is chosen from \mathbb{T}_i being the set of functions $T_i(q)$ satisfying: (a) $T_i'(q) > 0$ and $T_i''(q) > 0$ for $q > 0$; (b) $T_i(0) = 0$ and we allow for a discontinuity at 0 with an upward jump; (c) $T_i'(q) \geq C_i'(q)$ for $q > 0$.

First, we assume in (a) that the supplier bids are strictly increasing and convex. Also they are twice differentiable everywhere except at the origin. This condition will facilitate the equilibrium analysis, but the equilibrium outcomes hold in more general cases as we will show later. Second, the discontinuity at $q = 0$ is allowed in order to take account of the fact that if supplier i is not chosen, the buyer does not pay anything to him. Finally, we assume in (c) that the suppliers, as rational players, do not set the marginal price below the marginal cost for any quantity produced. From (b), (c) and Assumption 2.1, we can show that $T_i(q) \geq C_i(q)$ for $q \geq 0$.

Given the supplier bids, the buyer decides how much to purchase from each supplier. Then knowing the buyer's best response and given the competitor's bid, each supplier decides on the bidding strategy to maximize his profit. Following the backward induction approach, we start with the buyer's problem, and then characterize the equilibrium for suppliers.

2.4.1 Buyer's problem

Suppose the supplier bids are $\{T_i(q), T_j(q)\}$, then define the buyer's profit when ordering q_i from supplier i and q_j from supplier j as follows:

$$\pi_B(q_i, q_j) = r\mathbb{E}[\min(D, q_i + q_j)] - T_i(q_i) - T_j(q_j). \quad (2.4)$$

Given $\{T_i(q), T_j(q)\}$, the buyer's problem is to maximize her profit by choosing the purchase amount from each supplier:

$$\max \{ \pi_B(q_i, q_j) : q_i, q_j \geq 0 \}. \quad (2.5)$$

We can show that π_B is jointly concave in q_i and q_j provided they are strictly positive. Due to the possible discontinuity of $T_i(q)$ at $q = 0$ where $i = 1, 2$, we consider two cases regarding the buyer's ordering decision: (1) the *dual-sourcing* strategy where the buyer purchases from both suppliers; and (2) the *single-sourcing* strategy where the buyer purchases from only one supplier.

If the buyer adopts the dual-sourcing strategy, the optimal solution (q_i^D, q_j^D) must satisfy the first order conditions:

$$T'_i(q_i^D) = T'_j(q_j^D) = r[1 - F(q_i^D + q_j^D)]. \quad (2.6)$$

Let $\pi_B^D = \pi_B(q_i^D, q_j^D)$ be the buyer's dual-sourcing optimal profit.

If the buyer adopts the single-sourcing strategy for supplier i , her problem is

$$\max \{ \pi_B(q, 0) = r\mathbb{E}[\min(D, q)] - T_i(q) : q \geq 0 \}.$$

Again π_B is concave in q , and the optimal solution q_i^S can be found by solving the first order condition,

$$T'_i(q_i^S) = r[1 - F(q_i^S)]. \quad (2.7)$$

Let $\pi_B^i = \pi_B(q_i^S, 0)$ be the buyer's single-sourcing optimal profit when purchasing from supplier i . From (2.6) and (2.7) we can show $q_i^S \geq q_i^D$.

There are three local maxima for the buyer's choice, and the buyer will choose the sourcing strategy which gives her the largest profit, i.e. $\pi_B^* = \max(\pi_B^D, \pi_B^i, \pi_B^j)$. Note

that the buyer's optimal choice may not be unique. Following Cachon and Kok (2010), we assume the buyer breaks ties in favour of purchasing from more suppliers, that is, if $\pi_B^D = \max(\pi_B^i, \pi_B^j)$, the buyer will purchase from both suppliers. If $\pi_B^i = \pi_B^j > \pi_B^D$, we assume the buyer will randomly select a winning supplier.

2.4.2 Equilibrium Analysis

In this section, we study the equilibrium for suppliers. We begin by investigating each supplier's best response in choosing $T_i(q)$.

2.4.2.1 Suppliers' best responses

Given supplier j 's bid $T_j(q)$, suppose the buyer's profit of purchasing from supplier j only is π_B^j and the optimal order quantity is q_j^S . Note that both π_B^j and q_j^S are fixed for the supplier i 's best response problem.

Suppose the supplier i 's bid is $T_i(q)$, and given $T_i(q)$ (as well as $T_j(q)$), denote by (q_i^*, q_j^*) the buyer's optimal choice, that is,

$$(q_i^*, q_j^*) = \arg \max \{ \pi_B(q_i, q_j) : q_i, q_j \geq 0 \}. \quad (2.8)$$

Then the supplier i 's profit from offering $T_i(q)$ is given by,

$$\pi_i(T_i(q); q_i^*, q_j^*) = T_i(q_i^*) - C_i(q_i^*).$$

Anticipating the buyer's optimal ordering decision, supplier i aims to maximize his profit by choosing $T_i(q)$:

$$\max \{ \pi_i(T_i(q); q_i^*, q_j^*) : T_i(q) \in \mathbb{T}_i \}, \quad (2.9)$$

subject to (2.8). We need to consider the local maxima for the buyer's optimal choice as discussed in the previous section.

Our first observation is that supplier i will set the value of $T_i(q_i^*)$ as high as possible, provided that the buyer purchases q_i^* from supplier i . Specifically, given a certain slope $T_i'(q_i^*)$, since π_i increases in $T_i(q_i^*)$, supplier i will keep increasing the value of $T_i(q_i^*)$ (holding the slope constant) until the buyer turns to the single-sourcing strategy j . Therefore, for an optimal solution of $T_i(q)$, the buyer's optimal profit of choosing (q_i^*, q_j^*) must equal π_B^j , i.e. $\pi_B(q_i^*, q_j^*) = \pi_B^j$ (here undercutting behaviour occurs, but for convenience of exposition we shall use equality). Plugging this equation into (2.9), we rewrite the supplier i 's best response problem as follows:

$$\mathbf{P1:} \max \left\{ \pi_i(T_i(q); q_i^*, q_j^*) = r\mathbb{E}[\min(D, q_i^* + q_j^*)] - C_i(q_i^*) - T_j(q_j^*) - \pi_B^j : T_i(q) \in \mathbb{T}_i \right\}, \quad (2.10)$$

subject to (2.8), and $\pi_B(q_i^*, q_j^*) = \pi_B^j$.

Notice that $T_i(q)$ only appears in the constraints so supplier i can do no better than the solution to the following relaxed problem when the constraints are dropped:

$$\mathbf{P0}: \quad \max_{q_i, q_j \geq 0} r\mathbb{E}[\min(D, q_i + q_j)] - C_i(q_i) - T_j(q_j) - \pi_B^j. \quad (2.11)$$

The above problem can be thought of as solving the optimal marginal supply chain surplus contributed by supplier i when he charges only his cost. Denote by $(\hat{q}_i^*(T_j), \hat{q}_j^*(T_j))$ the optimal solution to **P0**. To shorten notations, we shall use \hat{q}_i^* for $\hat{q}_i^*(T_j)$ and \hat{q}_j^* for $\hat{q}_j^*(T_j)$ unless ambiguity arises. We write Γ_i for the optimal value of **P0**, so

$$\Gamma_i = r\mathbb{E}[\min(D, \hat{q}_i^* + \hat{q}_j^*)] - C_i(\hat{q}_i^*) - T_j(\hat{q}_j^*) - \pi_B^j. \quad (2.12)$$

Note that this gives the maximum possible profit supplier i can make.

Obviously if $\hat{q}_i^* = 0$, then $\Gamma_i = 0$, meaning that the marginal supply chain surplus contributed by supplier i is zero. In this trivial case, supplier i will make zero profits for sure, and any bid $T_i \in \mathbb{T}_i$ is optimal for supplier i . In the following, we will focus on the cases where $\hat{q}_i^* > 0$. We provide the necessary conditions for the supplier i 's best response in Lemma 2.1.

Lemma 2.1 (Necessary conditions for best responses). *Given the supplier j 's bid $T_j(q)$, suppose $T_i(q) \in \mathbb{T}_i$ is optimal for supplier i , then, assuming $\hat{q}_i^* > 0$,*

- (i) if $\hat{q}_j^* = 0$, then $T_i'(\bar{Q}_i) = C_i'(\bar{Q}_i)$ and $\pi_B(\bar{Q}_i, 0) = \pi_B^j$;
- (ii) if $\hat{q}_j^* > 0$, then $T_i'(\hat{q}_i^D) = C_i'(\hat{q}_i^D)$ and $\pi_B(\hat{q}_i^D, \hat{q}_j^D) = \pi_B^j$,

where \hat{q}_i^D and \hat{q}_j^D satisfy $r[1 - F(\hat{q}_i^D + \hat{q}_j^D)] = C_i'(\hat{q}_i^D) = T_j'(\hat{q}_j^D)$.

Given the supplier i 's optimal bid $T_i(q)$ (as well as $T_j(q)$), the buyer's optimal choice equals the solution to **P0**, i.e. $(q_i^*, q_j^*) = (\hat{q}_i^*, \hat{q}_j^*)$.

Proof of Lemma 2.1. First, consider the relaxed problem **P0** with the optimal solution $(\hat{q}_i^*, \hat{q}_j^*)$. In our discussion of the buyer problem in (2.5) we have observed that there are three local maxima. Using the same argument, we consider three cases for **P0**: If the buyer purchases from supplier j only, then her optimal order is q_j^S (by definition); If the buyer purchases from supplier i only, then her optimal order is \bar{Q}_i , which is obtained by solving the first order condition $r[1 - F(\bar{Q}_i)] = C_i'(\bar{Q}_i)$; If the buyer purchases from both suppliers, then the optimal solution $(\hat{q}_i^D, \hat{q}_j^D)$ must satisfy the first order conditions: $r[1 - F(\hat{q}_i^D + \hat{q}_j^D)] = C_i'(\hat{q}_i^D) = T_j'(\hat{q}_j^D)$. Comparing these three local maxima, we can obtain the global maximum $(\hat{q}_i^*, \hat{q}_j^*)$. As discussed earlier, the first case has $\hat{q}_i^* = 0$ and leads to the trivial condition that any bid $T_i(q)$ is optimal for supplier i with zero profits. Thus we focus on the cases where $(\hat{q}_i^*, \hat{q}_j^*) = (\bar{Q}_i, 0)$ or $(\hat{q}_i^*, \hat{q}_j^*) = (\hat{q}_i^D, \hat{q}_j^D)$.

Second, we show that if $T_i \in \mathbb{T}_i$ is optimal, then $(\hat{q}_i^*, \hat{q}_j^*)$ must be an optimal choice for the buyer, and supplier i achieves the maximum profit of Γ_i (hence establishing the last part of the lemma). Suppose otherwise and given the optimal bid $T_i(q)$ (as well as $T_j(q)$) the buyer's optimal choice is $(\check{q}_i, \check{q}_j)$ where $(\check{q}_i, \check{q}_j) \neq (\hat{q}_i^*, \hat{q}_j^*)$. We know from the constraints of **P1** that

$$\pi_B(\check{q}_i, \check{q}_j) = r\mathbb{E}[\min(D, \check{q}_i + \check{q}_j)] - T_i(\check{q}_i) - T_j(\check{q}_j) = \pi_B^j. \quad (2.13)$$

Moreover, from the optimality of $(\hat{q}_i^*, \hat{q}_j^*)$ for **P0**, we have

$$r\mathbb{E}[\min(D, \check{q}_i + \check{q}_j)] - C_i(\check{q}_i) - T_j(\check{q}_j) \leq r\mathbb{E}[\min(D, \hat{q}_i^* + \hat{q}_j^*)] - C_i(\hat{q}_i^*) - T_j(\hat{q}_j^*). \quad (2.14)$$

Using (2.13) and (2.14), we show the supplier i 's profit as follows:

$$\begin{aligned} T_i(\check{q}_i) - C_i(\check{q}_i) &= r\mathbb{E}[\min(D, \check{q}_i + \check{q}_j)] - C_i(\check{q}_i) - T_j(\check{q}_j) - \pi_B(\check{q}_i, \check{q}_j) \\ &\leq r\mathbb{E}[\min(D, \hat{q}_i^* + \hat{q}_j^*)] - C_i(\hat{q}_i^*) - T_j(\hat{q}_j^*) - \pi_B^j = \Gamma_i. \end{aligned}$$

However, if we were to choose a $T_i \in \mathbb{T}_i$ such that $(\hat{q}_i^*, \hat{q}_j^*)$ is an optimal choice for the buyer, for example, we could choose

$$T_i(q) = C_i(q) + \Gamma_i \text{ for } q > 0, \quad \text{and} \quad T_i(0) = 0,$$

then the supplier i 's profit would be Γ_i . So the choice of T_i leading to $(\check{q}_i, \check{q}_j)$ cannot be strictly better for supplier i than a choice leading to $(\hat{q}_i^*, \hat{q}_j^*)$. Therefore, it is optimal for supplier i to choose $T_i \in \mathbb{T}_i$ such that $(\hat{q}_i^*, \hat{q}_j^*)$ is an optimal choice for the buyer.

Third, if $(\hat{q}_i^*, \hat{q}_j^*)$ is optimal for the buyer, then it must satisfy the first order conditions. We consider two cases: (i) $(\hat{q}_i^*, \hat{q}_j^*) = (\bar{Q}_i, 0)$, i.e. $\hat{q}_j^* = 0$; and (ii) $(\hat{q}_i^*, \hat{q}_j^*) = (\hat{q}_i^D, \hat{q}_j^D)$, i.e. $\hat{q}_j^* > 0$. In case (i), from (2.7) we know that if the buyer's optimal order is \bar{Q}_i , then $r[1 - F(\bar{Q}_i)] = T_i'(\bar{Q}_i)$, which together with $r[1 - F(\bar{Q}_i)] = C_i'(\bar{Q}_i)$ yields $T_i'(\bar{Q}_i) = C_i'(\bar{Q}_i)$. Moreover, $\pi_B(\bar{Q}_i, 0) = \pi_B^j$ follows from the constraints of **P1**. Similarly, we can show the necessary conditions for case (ii) are $T_i'(\hat{q}_i^D) = C_i'(\hat{q}_i^D)$ and $\pi_B(\hat{q}_i^D, \hat{q}_j^D) = \pi_B^j$. This completes the proof. \square

Given $T_j(q)$, supplier i essentially competes on how to share the buyer's demand with supplier j . Supplier i can first decide his desired demand allocation, and then chooses $T_i(q)$ so that the buyer's optimal choice is indeed his desired order split, i.e. $(q_i^*, q_j^*) = (\hat{q}_i^*, \hat{q}_j^*)$. Because of the possible discontinuity of supplier bids, supplier i needs to consider both the case when the buyer chooses to purchase from him only and the case when the buyer splits her order. In the former case, from (i) of Lemma 2.1, we obtain $r[1 - F(\bar{Q}_i)] = T_i'(\bar{Q}_i)$, indicating that $(\bar{Q}_i, 0)$ is a boundary solution. Similarly,

in the latter case, we have $r[1 - F(\hat{q}_i^D + \hat{q}_j^D)] = T_i'(\hat{q}_i^D) = T_j'(\hat{q}_j^D)$ from (ii) of Lemma 2.1, implying that $(\hat{q}_i^D, \hat{q}_j^D)$ is an interior solution for the buyer's problem in (2.8).

To guarantee $(\hat{q}_i^*, \hat{q}_j^*)$ is globally optimal for the buyer, further conditions are needed. As an example, we provide a set of sufficient conditions in the following lemma for the supplier's best response.

Lemma 2.2 (Sufficient conditions for best responses). *Suppose that $T_i(q) \in \mathbb{T}_i$ satisfies the following conditions: (i) if $\hat{q}_j^* = 0$, then $T_i(\bar{Q}_i) - C_i(\bar{Q}_i) = \Gamma_i$ and $T_i'(\bar{Q}_i) = C_i'(\bar{Q}_i)$; or (ii) if $\hat{q}_j^* > 0$, then*

$$T_i(\hat{q}_i^D) - C_i(\hat{q}_i^D) = T_i(\bar{Q}_i) - C_i(\bar{Q}_i) = \Gamma_i \quad (2.15)$$

$$T_i'(\hat{q}_i^D) = C_i'(\hat{q}_i^D) \quad \text{and} \quad T_i'(\bar{Q}_i) = C_i'(\bar{Q}_i), \quad (2.16)$$

where $\hat{q}_i^D > 0$ and $\hat{q}_j^D > 0$ satisfy $r[1 - F(\hat{q}_i^D + \hat{q}_j^D)] = C_i'(\hat{q}_i^D) = T_j'(\hat{q}_j^D)$. Then $T_i(q)$ is optimal for supplier i .

Proof of Lemma 2.2. We begin with case (ii) where $\hat{q}_j^*, \hat{q}_j^* > 0$. First, we show that, given a bid $T_i(q) \in \mathbb{T}_i$ satisfying (2.15) and (2.16), the buyer's optimal choice is $(\hat{q}_i^D, \hat{q}_j^D)$. From $r[1 - F(\hat{q}_i^D + \hat{q}_j^D)] = C_i'(\hat{q}_i^D) = T_j'(\hat{q}_j^D)$ and (2.16), we obtain $r[1 - F(\hat{q}_i^D + \hat{q}_j^D)] = T_i'(\hat{q}_i^D) = T_j'(\hat{q}_j^D)$, which implies that $(\hat{q}_i^D, \hat{q}_j^D)$ is an interior solution for the buyer's problem from the concavity of her profit function. Using (2.4) and (2.15), we obtain the buyer's profit from choosing $(\hat{q}_i^D, \hat{q}_j^D)$ as follows:

$$\pi_B(\hat{q}_i^D, \hat{q}_j^D) = r\mathbb{E}[\min(D, \hat{q}_i^D + \hat{q}_j^D)] - T_i(\hat{q}_i^D) - T_j(\hat{q}_j^D) = \pi_B^j.$$

Similarly, from $T_i'(\bar{Q}_i) = C_i'(\bar{Q}_i)$ and (2.3) we obtain $r[1 - F(\bar{Q}_i)] = T_i'(\bar{Q}_i)$, which implies that the buyer's optimal order is \bar{Q}_i if supplier i is the sole supplier. Again using (2.4) and (2.15), we obtain the buyer's optimal profit is π_B^j when purchasing from only supplier i . Furthermore, the buyer's optimal profit when purchasing from supplier j only is also π_B^j (by definition). So all three local maxima give the buyer the same profit π_B^j , and thus the buyer's globally optimal choice is $(\hat{q}_i^D, \hat{q}_j^D)$ according to the tie-breaking rule.

Second, since the buyer orders \hat{q}_i^D from supplier i , the supplier i 's profit is $T_i(\hat{q}_i^D) - C_i(\hat{q}_i^D) = \Gamma_i$, which is the maximum profit supplier i can achieve. Therefore, such a $T_i(q)$ must be optimal for supplier i . This completes the proof for case (ii). The same reasoning applies to case (i) and we omit the details here to avoid repetition. \square

Lemma 2.2 shows that the conditions on the bid $T_i(\cdot)$ involve the slopes and the values at \hat{q}_i^D and \bar{Q}_i . On one hand, with the slope conditions in (2.16), the local optimal solutions for the buyer's choice will be $(\hat{q}_i^D, \hat{q}_j^D)$ and $(\bar{Q}_i, 0)$. On the other hand, the value conditions in (2.15) ensure that the buyer makes the same profit from these

two choices. According to the tie-breaking rule in relation to the buyer's optimal choice, the buyer will purchase \hat{q}_i^D from supplier i and \hat{q}_j^D from supplier j . This gives supplier i a profit of Γ_i . Since Γ_i is the maximum profit supplier i can make, such a $T_i(\cdot)$ must be optimal. Note that the choice of $T_i(q)$ is not unique, and there will be a continuum of equilibria as we show in the next section.

2.4.2.2 Equilibrium characterization

We now characterize the equilibrium for suppliers. A Nash equilibrium is a pair of bids $\{T_i^*(q), T_j^*(q)\}$, which are mutual best responses for each supplier. The equilibrium outcome is the buyer's optimal choice (q_i^*, q_j^*) . Let $\pi_B^*, \pi_i^*, \pi_j^*$ be the equilibrium profits of the buyer, supplier i , and supplier j , respectively. We begin by showing that no equilibrium exists where the buyer adopts the single-sourcing strategy.

Lemma 2.3. *There exists no equilibrium where the buyer purchases from only one supplier.*

Proof of Lemma 2.3. Suppose otherwise and there exists an equilibrium $\{T_i^*(q), T_j^*(q)\}$ where the buyer purchases from one supplier. Then from (i) in Lemma 2.1 we have $T_i^*(\bar{Q}_i) = C_i'(\bar{Q}_i)$. Suppose supplier i wins, and by definition we have the buyer's profit and the supplier i 's profit as follows:

$$\pi_i^* = T_i^*(\bar{Q}_i) - C_i(\bar{Q}_i) \quad \text{and} \quad \pi_B^i = r\mathbb{E}[\min(D, \bar{Q}_i)] - T_i^*(\bar{Q}_i).$$

Thus, we obtain $\pi_i^* = \Pi_i - \pi_B^i$. We also know $\pi_i^* \geq 0$, so $\pi_i^* = \max(0, \Pi_i - \pi_B^i)$. Again from (i) in Lemma 2.1, we have $\pi_B^i = \pi_B^j$ in equilibrium, from which we deduce $\pi_i^* = \max(\Pi_i - \Pi_j, 0)$ and $\pi_B^* = \min(\Pi_i, \Pi_j)$.

Using part (c) of Assumption 2.3, we obtain $T_i^*(\bar{q}_i) - C_i(\bar{q}_i) \leq T_i^*(\bar{Q}_i) - C_i(\bar{Q}_i)$ because $\bar{q}_i < \bar{Q}_i$. This together with $T_i^*(\bar{Q}_i) - C_i(\bar{Q}_i) = \max(\Pi_i - \Pi_j, 0)$ yields

$$T_i^*(\bar{q}_i) - C_i(\bar{q}_i) \leq \max(\Pi_i - \Pi_j, 0), \quad \text{for } i = 1, 2, j = 3 - i. \quad (2.17)$$

We now show that (\bar{q}_i, \bar{q}_j) gives the buyer a larger profit:

$$\begin{aligned} \pi_B(\bar{q}_i, \bar{q}_j) &= r\mathbb{E}[\min(D, \bar{q}_i + \bar{q}_j)] - T_i^*(\bar{q}_i) - T_j^*(\bar{q}_j) \\ &\geq r\mathbb{E}[\min(D, \bar{q}_i + \bar{q}_j)] - C_i(\bar{q}_i) - \max(\Pi_i - \Pi_j, 0) - C_j(\bar{q}_j) - \max(\Pi_j - \Pi_i, 0) \\ &= \Pi - \max(\Pi_i - \Pi_j, 0) - \max(\Pi_j - \Pi_i, 0) \\ &= \Pi - [\max(\Pi_i, \Pi_j) - \min(\Pi_i, \Pi_j)] \\ &> \min(\Pi_i, \Pi_j) = \pi_B^*, \end{aligned}$$

where the first inequality follows from (2.17). Thus, it is suboptimal for the buyer to choose only one supplier. A contradiction. \square

Note that from the supply chain point of view, it is optimal for the buyer to split her order. Lemma 2.3 shows that allowing suppliers to compete by offering function bids does not support the equilibrium where the buyer chooses single-sourcing. Later, we will establish a stronger result: the buyer's optimal choice in equilibrium is always first best.

Having eliminated the existence of equilibrium where the buyer chooses the single-sourcing strategy, we next focus on characterizing the equilibrium where the buyer chooses the dual-sourcing strategy. We need to consider the buyer's single-sourcing profit if she purchases from only one supplier, since this places a floor on the buyer's attainable profit. Lemma 2.4 provides a necessary condition for the equilibrium.

Lemma 2.4 (Equilibrium profit equivalence). *Suppose $\{T_i^*(q), T_j^*(q)\}$ is a Nash equilibrium, then the buyer's dual-sourcing profit is equal to the single-sourcing profit from each supplier. Formally,*

$$\pi_B^D = \pi_B^i = \pi_B^j. \quad (2.18)$$

Proof of Lemma 2.4. In the equilibrium where the buyer chooses dual-sourcing, we must have $\pi_B^D \geq \max(\pi_B^i, \pi_B^j)$ since otherwise the buyer will choose only one supplier. Suppose the result of the lemma does not hold and there exists a supplier i with $\pi_B^i < \pi_B^D$. Define $\delta = \pi_B^D - \pi_B^i > 0$. We show that supplier j has an incentive to deviate from $T_j^*(q)$. Consider a new bid $T_j(q)$ for supplier j where $T_j(q) = T_j^*(q) + \delta/2$ for $q > 0$ and $T_j(0) = 0$, which clearly satisfies Assumption 2.3. Given the new bid $T_j(q)$ (as well as $T_i^*(q)$), the buyer's dual-sourcing problem is,

$$\max \{ r\mathbb{E}[\min(D, q_i + q_j)] - T_i^*(q_i) - T_j^*(q_j) - \delta/2 : q_i, q_j > 0 \},$$

We can show that the optimal solution is (q_i^D, q_j^D) where

$$(q_i^D, q_j^D) = \arg \max \{ r\mathbb{E}[\min(D, q_i + q_j)] - T_i^*(q_i) - T_j^*(q_j) : q_i, q_j > 0 \},$$

and the buyer's optimal profit is $\pi_B^D - \delta/2$. If the buyer adopts the single-sourcing strategy for j , her optimal profit is $\pi_B^j - \delta/2$. Furthermore, by assumption the buyer makes a profit of $\pi_B^i = \pi_B^D - \delta$ if she uses the single-sourcing strategy i . Therefore, the buyer will choose (q_i^D, q_j^D) due to $\pi_B^D \geq \pi_B^j$. The supplier j then makes $\delta/2$ more profit, showing that supplier j has an incentive to deviate from the proposed bid. A contradiction. \square

This lemma indicates that in equilibrium the buyer is indifferent between purchasing from only one supplier and purchasing from both suppliers. The intuition is as follows: if

the buyer's single-sourcing profit is smaller, then the other supplier can slightly increase the bid price but makes sure that the buyer's choice remains unchanged. Hence, in equilibrium the buyer's profits of single-sourcing and dual-sourcing must be identical. We are now in a position to characterize the equilibria for suppliers.

Proposition 2.5 (Equilibrium characterization). *A pair of bids $\{T_1^*(q), T_2^*(q)\}$ satisfying Assumption 2.3, is a Nash equilibrium if and only if, for $i = 1, 2$ and $j = 3 - i$,*

$$(a) \quad T_i^{*'}(\bar{q}_i) = C_i'(\bar{q}_i) \text{ and } T_i^{*'}(\bar{Q}_i) = C_i'(\bar{Q}_i);$$

$$(b) \quad T_i^*(\bar{q}_i) = C_i(\bar{q}_i) + \Pi - \Pi_j \text{ and } T_i^*(\bar{Q}_i) = C_i(\bar{Q}_i) + \Pi - \Pi_j.$$

In such an equilibrium, the buyer's optimal choice is first best, i.e. $(q_i^, q_j^*) = (\bar{q}_i, \bar{q}_j)$, and the profit split amongst players is: $\pi_B^* = \Pi_i + \Pi_j - \Pi$ and $\pi_i^* = \Pi - \Pi_j$. The total supply chain profit is $\Pi^* = \Pi$.*

Proof of Proposition 2.5. Necessity: Suppose $\{T_1^*(q), T_2^*(q)\}$ is an equilibrium, then from the necessary conditions for best responses in (ii) of Lemma 2.1, we know $T_i^{*'}(\hat{q}_i^D) = C_i'(\hat{q}_i^D)$, where \hat{q}_i^D and \hat{q}_j^D satisfy the equations: $r[1 - F(\hat{q}_i^D + \hat{q}_j^D)] = T_i^{*'}(\hat{q}_i^D) = C_j'(\hat{q}_j^D)$, for $i = 1, 2, j = 3 - i$. To find an equilibrium, we jointly solve the above equations, and obtain that the buyer's optimal choice in equilibrium is (\bar{q}_i, \bar{q}_j) . This gives $T_i^{*'}(\bar{q}_i) = C_i'(\bar{q}_i)$ as we require for condition (a) of Proposition 2.5. In the following, we prove the other results in two steps.

First, we show that, in equilibrium, if the buyer purchases from supplier i only, the optimal order must be \bar{Q}_i , i.e. $T_i^{*'}(\bar{Q}_i) = C_i'(\bar{Q}_i)$. Suppose otherwise and the buyer purchases \hat{Q}_i when supplier i is the sole supplier where $\hat{Q}_i \neq \bar{Q}_i$, so we have $T_i^{*'}(\hat{Q}_i) = C_i'(\hat{Q}_i)$. By definition, the supplier i 's profit in equilibrium is $\pi_i^* = T_i^*(\bar{q}_i) - C_i(\bar{q}_i)$. As we show earlier from (2.6) and (2.7), we have $\hat{Q}_i \geq \bar{q}_i$, which together with (c) of Assumption 2.3 yields,

$$T_i^*(\hat{Q}_i) - C_i(\hat{Q}_i) \geq T_i^*(\bar{q}_i) - C_i(\bar{q}_i). \quad (2.19)$$

Moreover, we know from Lemma 2.4 that $\pi_B^i = \pi_B^D$. To write explicitly,

$$r\mathbb{E}[\min(D, \hat{Q}_i)] - T_i^*(\hat{Q}_i) = r\mathbb{E}[\min(D, \bar{q}_i + \bar{q}_j)] - T_i^*(\bar{q}_i) - T_j^*(\bar{q}_j). \quad (2.20)$$

From the optimality of \bar{Q}_i for the single-sourcing supply chain problem, we obtain $r\mathbb{E}[\min(D, \bar{Q}_i)] - C_i(\bar{Q}_i) > r\mathbb{E}[\min(D, \hat{Q}_i)] - C_i(\hat{Q}_i)$. Define the difference:

$$\delta = r\mathbb{E}[\min(D, \bar{Q}_i)] - C_i(\bar{Q}_i) - \left(r\mathbb{E}[\min(D, \hat{Q}_i)] - C_i(\hat{Q}_i) \right) > 0. \quad (2.21)$$

Now consider a new bid $T_i(q) \in \mathbb{T}_i$ for supplier i such that: $T_i'(\bar{Q}_i) = C_i'(\bar{Q}_i)$, $T_i'(\bar{q}_i) = C_i'(\bar{q}_i)$, and the values of T_i at these two points are given by the following two

equations:

$$\begin{aligned} r\mathbb{E}[\min(D, \bar{Q}_i)] - T_i(\bar{Q}_i) &= r\mathbb{E}[\min(D, \hat{Q}_i)] - T_i^*(\hat{Q}_i) + \delta/2, \\ r\mathbb{E}[\min(D, \bar{q}_i + \bar{q}_j)] - T_i(\bar{q}_i) - T_j^*(\bar{q}_j) &= r\mathbb{E}[\min(D, \bar{q}_i + \bar{q}_j)] - T_i^*(\bar{q}_i) - T_j^*(\bar{q}_j) + \delta/4. \end{aligned} \quad (2.22)$$

Given $T_i(q)$ (as well as $T_j^*(q)$), we can show that the buyer will purchase from supplier i only (by comparing the three local maxima as before and using (2.20)). Then, using (2.21) and (2.22), we show the supplier i 's profit when offering $T_i(q)$ is:

$$T_i(\bar{Q}_i) - C_i(\bar{Q}_i) = T_i^*(\hat{Q}_i) - C_i(\hat{Q}_i) + \delta/2 \geq T_i^*(\bar{q}_i) - C_i(\bar{q}_i) + \delta/2 = \pi_i^* + \delta/2,$$

where the inequality follows from (2.19). This implies that supplier i can improve his profit by offering $T_i(q)$. A contradiction. Therefore, in equilibrium we must have $T_i^*(\bar{Q}_i) = C_i'(\bar{Q}_i)$.

Second, we prove condition (b) which pertains to the profit split amongst players. Let $\pi_i(\bar{Q}_i) = T_i^*(\bar{Q}_i) - C_i(\bar{Q}_i)$, and by definition we have $\pi_i^* = T_i^*(\bar{q}_i) - C_i(\bar{q}_i)$. Lemma 2.4 shows that $\pi_B^D = \pi_B^i = \pi_B^j$ in equilibrium, which can be rewritten as follows:

$$\Pi - \pi_i^* - \pi_j^* = \Pi_i - \pi_i(\bar{Q}_i) = \Pi_j - \pi_j(\bar{Q}_j), \quad (2.23)$$

where we have used the earlier result that if the buyer purchases from supplier i only, the optimal order must be \bar{Q}_i .

From part (c) of Assumption 2.3, we know $\pi_i^* \leq \pi_i(\bar{Q}_i)$ and $\pi_j^* \leq \pi_j(\bar{Q}_j)$, which together with (2.23) yields $\pi_i^* \geq \Pi - \Pi_j$ and $\pi_j^* \geq \Pi - \Pi_i$. We now show that in equilibrium no supplier's profit can be greater than his marginal contribution. Suppose otherwise and $\pi_i^* = \Pi - \Pi_j + \delta_i$ and $\pi_j^* = \Pi - \Pi_i + \delta_j$, where $\delta_i + \delta_j > 0$. From (2.23) we obtain

$$\pi_i(\bar{Q}_i) = \Pi - \Pi_j + \delta_i + \delta_j \quad \text{and} \quad \pi_j(\bar{Q}_j) = \Pi - \Pi_i + \delta_i + \delta_j.$$

Without loss of generality, suppose $\delta_i > 0$. Then we show there exists a new bid $\tilde{T}_j(q)$ for supplier j such that the buyer chooses supplier j only, and both the buyer and supplier j are better off. For example, if a $\tilde{T}_j(q) \in \mathbb{T}_j$ satisfies the following conditions:

$$\tilde{T}_j(\bar{q}_j) = T_j^*(\bar{q}_j) - \frac{\delta_i}{4}; \quad \tilde{T}_j(\bar{Q}_j) = T_j^*(\bar{Q}_j) - \frac{\delta_i}{2}; \quad \tilde{T}_j'(\bar{q}_j) = C_j'(\bar{q}_j); \quad \tilde{T}_j'(\bar{Q}_j) = C_j'(\bar{Q}_j),$$

then the buyer will purchase from supplier j only. To see this, if the buyer chooses dual-sourcing, her optimal profit is $\pi_B^* + \delta_i/4$. If the buyer chooses to purchase from supplier j only, the buyer's optimal profit is $\pi_B^* + \delta_i/2$. If the buyer purchases from supplier i only, the buyer's optimal profit will be π_B^* (since the supplier i 's bid remains

unchanged). So the buyer is better off not to purchase from supplier i , leaving supplier i zero profits. Therefore, supplier i cannot make more profit than $\Pi - \Pi_j$, and hence in equilibrium we must have $\pi_i^* = \Pi - \Pi_j$. By definition we obtain $T_i^*(\bar{q}_i) = C_i(\bar{q}_i) + \Pi - \Pi_j$. Then from (2.23) we deduce $T_i^*(\bar{Q}_i) = C_i(\bar{Q}_i) + \Pi - \Pi_j$. This completes the proof for necessity.

Sufficiency: We first show the buyer's choice is (\bar{q}_i, \bar{q}_j) given the bids $\{T_i^*(q), T_j^*(q)\}$. Condition (a) ensures that the buyer's optimal dual-sourcing choice is (\bar{q}_i, \bar{q}_j) and the buyer's optimal single-sourcing quantity is \bar{Q}_i . Using condition (b), we calculate the buyer's profit when purchasing from both supplier as follows: $\pi_B^D = \Pi_i + \Pi_j - \Pi$. If the buyer purchases from only one supplier, then $\pi_B^i = \pi_B^j = \Pi_i + \Pi_j - \Pi$. Therefore, the buyer will choose (\bar{q}_i, \bar{q}_j) according to the tie-breaking rule. Then the supplier i 's profit is $\Pi - \Pi_j$.

The remaining task is to show that no supplier has an incentive to deviate from the proposed bid. Suppose to the contrary and there exists a supplier i who deviates by choosing a different bid $\tilde{T}_i(q)$ and makes a higher profit $\tilde{\pi}_i > \Pi - \Pi_j$. Let $(\tilde{q}_i, \tilde{q}_j)$ be the buyer's optimal choice given the new bid $\tilde{T}_i(q)$ (as well as $T_j^*(q)$). Thus,

$$\tilde{\pi}_i = \tilde{T}_i(\tilde{q}_i) - C_i(\tilde{q}_i) > \Pi - \Pi_j. \quad (2.24)$$

First, the supplier i 's profit when the buyer purchases from him only is no greater than $\Pi_i - \pi_B^j = \Pi - \Pi_j$ since otherwise the buyer will choose supplier j only and make a profit of π_B^j . Thus, the only way of improving his profit is when the buyer purchases from both suppliers, i.e. when $\tilde{q}_i, \tilde{q}_j > 0$. However, we will show that the buyer is better off not to purchase from supplier i . The buyer's profit from choosing $(\tilde{q}_i, \tilde{q}_j)$ is:

$$\begin{aligned} \pi_B(\tilde{q}_i, \tilde{q}_j) &= rE_D[\min(D, \tilde{q}_i + \tilde{q}_j)] - \tilde{T}_i(\tilde{q}_i) - T_j^*(\tilde{q}_j) \\ &< rE_D[\min(D, \tilde{q}_i + \tilde{q}_j)] - C_i(\tilde{q}_i) - \Pi + \Pi_j - T_j^*(\tilde{q}_j), \end{aligned} \quad (2.25)$$

where the inequality follows from (2.24). Note that the right hand side of (2.25) is maximized when $\tilde{q}_i = \bar{q}_i$ and $\tilde{q}_j = \bar{q}_j$ due to $T_j^{*'}(\bar{q}_j) = C_j^{*'}(\bar{q}_j)$ and the concavity of the buyer's profit function. Furthermore, by definition we have $T_j^*(\bar{q}_j) = C_j(\bar{q}_j) + \Pi - \Pi_i$. Then, from (2.25) we deduce $\pi_B(\tilde{q}_i, \tilde{q}_j) < \Pi - \Pi + \Pi_j - \Pi + \Pi_i = \Pi_i + \Pi_j - \Pi$. On the other hand, if the buyer chooses supplier j only, her profit is $\pi_B^j = \Pi_i + \Pi_j - \Pi$. So the buyer will purchase from supplier j only, leaving supplier i zero profits. Therefore, no supplier has an incentive to deviate. This completes the proof for sufficiency. \square

Several points are worth mentioning. First, Proposition 2.5 reveals that the buyer's choice is first best in equilibrium, which is in contrast with Anton and Yao (1989) who study a similar setting but with a deterministic demand and show that there exist multiple demand allocations. If the buyer's demand is fixed, each supplier knows exactly

the order quantity from his competitor, so each supplier has the power to control the demand allocation by submitting a bid such that his desired demand allocation occurs. However, thanks to demand uncertainty, the total order quantity becomes endogenous, and neither supplier knows exactly the buyer's demand allocation without knowing the competitor's bid. As a result, the equilibria in which the buyer's choice is not first best are eliminated. This equilibrium refinement resonates with that which occurs in the supply function equilibrium (SFE) literature (Klemperer and Meyer, 1989).

Second, we show that the profit split in equilibrium is a Vickrey-Clarke-Groves (VCG) result: supplier i makes a profit which equals his marginal contribution to the supply chain system, $\Pi - \Pi_j$, and the buyer takes the remaining profit, $\Pi_i + \Pi_j - \Pi$. The literature on VCG mechanisms concerns designing efficient mechanisms to elicit agents to truthfully reveal their private information and the agents each are allocated a profit in proportion to their marginal contributions (Ausubel and Milgrom, 2006). The distinction of our study is that we are interested in how suppliers compete with each other rather than designing an mechanism for a buyer, and each supplier is paid according to their bids.

Third, Proposition 2.5 establishes the necessary and sufficient conditions for equilibrium bids. There are two critical points for the supplier i 's bid, $(\bar{q}_i, T_i^*(\bar{q}_i))$ and $(\bar{Q}_i, T_i^*(\bar{Q}_i))$. When choosing the bid, each supplier needs to coordinate the case when the buyer purchases from both suppliers with the case when the buyer purchases from him only. The condition (b) in Proposition 2.5 states that each supplier makes the same profit under these two cases. The equilibrium bidding strategy is not unique, despite the fact that both the profit split and the buyer's ordering decision are the same in all equilibria. As an example, Corollary 2.6 shows an equilibrium where each supplier's bid resembles a cost-plus-lump-sum contract.

Corollary 2.6 (Equilibrium with cost-plus-lump-sum bids). *It is an equilibrium for the suppliers to make offers $\{T_1^*(q), T_2^*(q)\}$ where, for $i = 1, 2$ and $j = 3 - i$, $T_i^*(0) = 0$ and $T_i^*(q) = C_i(q) + \Pi - \Pi_j$ for $q > 0$.*

Proof of Corollary 2.6. The result can be implied by Proposition 2.5. □

In this equilibrium, the bidding functions are constructed by shifting up the cost functions by their marginal contributions. These bids can be thought of as cost-plus-lump-sum contracts where each supplier adds a constant margin to his cost function everywhere except at the origin.

To fully describe the equilibria in Proposition 2.5, we now characterize two extreme equilibria where the bids are non-smooth. In doing so, we relax condition (a) in Assumption 2.3 and keep conditions (b) and (c) unchanged. Suppose $T_i(q)$ may be non-smooth but is continuous and convex for $q > 0$.

We first show an equilibrium of this form, which is the lower envelope of the equilibria characterized in Proposition 2.5. Define, for $i = 1, 2$,

$$q_i^0 = \frac{C'_i(\bar{Q}_i)\bar{Q}_i - C'_i(\bar{q}_i)\bar{q}_i - C_i(\bar{Q}_i) + C_i(\bar{q}_i)}{C'_i(\bar{Q}_i) - C'_i(\bar{q}_i)}, \quad (2.26)$$

which we will show is the threshold quantity for price changes. We have $\bar{q}_i < q_i^0 < \bar{Q}_i$. To see this,

$$\begin{aligned} q_i^0 - \bar{q}_i &= \frac{C'_i(\bar{Q}_i)(\bar{Q}_i - \bar{q}_i) - C_i(\bar{Q}_i) + C_i(\bar{q}_i)}{C'_i(\bar{Q}_i) - C'_i(\bar{q}_i)} > 0, \\ \bar{Q}_i - q_i^0 &= \frac{C_i(\bar{Q}_i) - C_i(\bar{q}_i) - C'_i(\bar{q}_i)(\bar{Q}_i - \bar{q}_i)}{C'_i(\bar{Q}_i) - C'_i(\bar{q}_i)} > 0, \end{aligned}$$

where the inequalities follow from the convexity of the cost function $C_i(q)$. We summarize in Lemma 2.7 the equilibrium for lower envelope piecewise linear bids.

Lemma 2.7 (Equilibrium for lower envelope piecewise linear bids). *It is an equilibrium for the suppliers to make offers $\{\underline{T}_1(q), \underline{T}_2(q)\}$ where, for $i = 1, 2$ and $j = 3-i$, $\underline{T}_i(0) = 0$ and*

$$\underline{T}_i(q) = \begin{cases} C_i(\bar{q}_i) + \Pi - \Pi_j + C'_i(\bar{q}_i)(q - \bar{q}_i), & 0 < q \leq q_i^0, \\ C_i(\bar{Q}_i) + \Pi - \Pi_j + C'_i(\bar{Q}_i)(q - \bar{Q}_i), & q > q_i^0, \end{cases}$$

where q_i^0 is given by (2.26). *The buyer's optimal choice and the profit split in this equilibrium are the same with those in Proposition 2.5.*

Proof of Lemma 2.7. The proof is similar to that of Proposition 2.5 (for sufficiency). The only difference is that the buyer's profit function is not differentiable at q_i^0 . However, it turns out to be insignificant. The detailed proof is omitted to avoid redundancy. \square

In this equilibrium, each supplier price-discriminates between different order sizes by setting different unit prices. Opposite to the quantity discount contract, the bid in our setting is a quantity premium contract, which is driven by the production diseconomies of scale. We now summarize in Lemma 2.8 the equilibrium for upper envelope piecewise linear bids.

Lemma 2.8 (Equilibrium for upper envelope piecewise linear bids). *It is an equilibrium for the suppliers to make offers $\{\bar{T}_1(q), \bar{T}_2(q)\}$ where, for $i = 1, 2$ and $j = 3-i$, $\bar{T}_i(0) = 0$ and*

$$\bar{T}_i(q) = \begin{cases} C_i(\bar{q}_i) + \Pi - \Pi_j, & 0 < q \leq \bar{q}_i, \\ C_i(\bar{q}_i) + \Pi - \Pi_j + \frac{C_i(\bar{Q}_i) - C_i(\bar{q}_i)}{\bar{Q}_i - \bar{q}_i}(q - \bar{q}_i), & \bar{q}_i < q \leq \bar{Q}_i, \\ \infty, & q > \bar{Q}_i. \end{cases}$$

The buyer's optimal choice and the profit split in this equilibrium are the same with those in Proposition 2.5.

Proof of Lemma 2.8. The proof is similar to that of Proposition 2.5 (for sufficiency). The distinction is that the buyer's profit function is not differentiable at both $q_i = \bar{q}_i$ and $q_i = \bar{Q}_i$. However, we can show that, at each point the left derivative of the buyer's corresponding profit function is positive, while the right derivative is negative. Therefore, the buyer's optimal solution when using the dual-sourcing strategy is (\bar{q}_i, \bar{q}_j) , and the buyer's optimal order when purchasing from only supplier i is \bar{Q}_i . We then can show that the buyer's globally optimal choice is (\bar{q}_i, \bar{q}_j) . The other proofs of Proposition 2.5 (for sufficiency) carry over to this lemma and are omitted here. \square

Lemma 2.8 shows that in the above equilibrium supplier i charges the same price for any quantity $q \in (0, \bar{q}_i]$, while the price is linearly increasing with the slope of $\frac{C_i(\bar{Q}_i) - C_i(\bar{q}_i)}{\bar{Q}_i - \bar{q}_i}$ for $q \in (\bar{q}_i, \bar{Q}_i]$. Any quantity beyond \bar{Q}_i has the marginal price of positive infinity.

The bids $\underline{T}_i(q)$ and $\bar{T}_i(q)$ provide the lower boundary and the upper boundary of the bids characterized in Proposition 2.5. Any bid that crosses the two points, $(\bar{q}_i, C_i(\bar{q}_i) + \Pi - \Pi_j)$ and $(\bar{Q}_i, C_i(\bar{Q}_i) + \Pi - \Pi_j)$, has the slopes of $C'_i(\bar{q}_i)$ and $C'_i(\bar{Q}_i)$ at these two points, and satisfies Assumption 2.3 (and hence is smooth), can be an equilibrium strategy for supplier i . See Figure 2.2 for the illustration of the equilibrium bids.

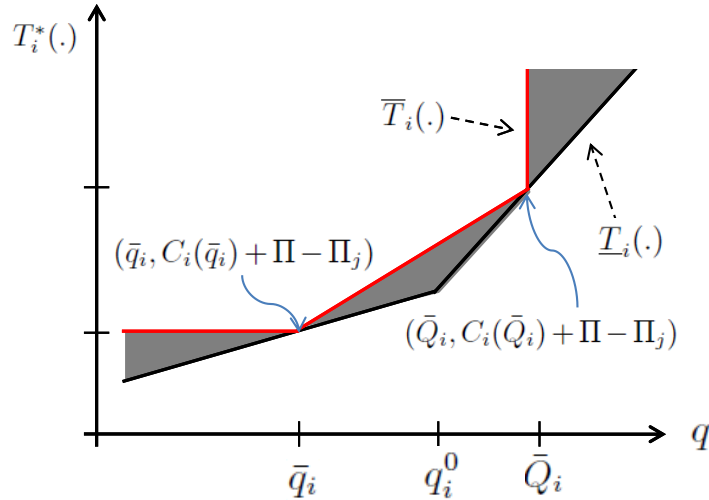


FIGURE 2.2: The illustration of the supplier bids in equilibrium

Example 2.2 (Continued). We return to the example to illustrate how to work out the equilibria. We know the supply chain optimal choice is $(\bar{q}_1, \bar{q}_2) = (2/5, 1/5)$, and the supply chain optimal profit is $\Pi = 3/10$. For the single-sourcing supply chain problems, we have $\Pi_1 = 1/4$; $\Pi_2 = 1/6$, and $\bar{Q}_1 = 1/2$; $\bar{Q}_2 = 1/3$. Based on Proposition 2.5, we know the buyer's choice in equilibrium is $(2/5, 1/5)$, and the profit split in equilibrium is: $\pi_1^* = \Pi - \Pi_2 = 2/15$, $\pi_2^* = \Pi - \Pi_1 = 1/20$, and $\pi_B^* = \Pi - \pi_1^* - \pi_2^* = 7/60$. Using the results in Corollary 2.6 and Lemma 2.7, we can work out the equilibrium with cost-plus-lump-sum bids and the equilibrium for lower envelope piecewise linear bids, respectively:

Equilibrium with cost-plus-lump-sum bids The suppliers' equilibrium bids are: $T_1^*(0) = T_2^*(0) = 0$, and

$$T_1^*(q) = \frac{1}{2}q^2 + \frac{2}{15} \quad \text{and} \quad T_2^*(q) = q^2 + \frac{1}{20}, \quad \text{for } q > 0.$$

Equilibrium with lower envelope piecewise linear bids The supplier 1's piecewise linear bid is: $\underline{T}_1^*(0) = 0$ and

$$\underline{T}_1(q) = \begin{cases} \frac{2}{25} + \frac{2}{15} + \frac{2}{5}(q - \frac{2}{5}), & 0 < q \leq \frac{9}{20}, \\ \frac{1}{8} + \frac{2}{15} + \frac{1}{2}(q - \frac{1}{2}), & q > \frac{9}{20}. \end{cases}$$

The supplier 2's piecewise linear bid is: $\underline{T}_2^*(0) = 0$ and

$$\underline{T}_2(q) = \begin{cases} \frac{1}{25} + \frac{1}{20} + \frac{2}{5}(q - \frac{1}{5}), & 0 < q \leq \frac{4}{15}, \\ \frac{1}{9} + \frac{1}{20} + \frac{2}{3}(q - \frac{1}{3}), & q > \frac{4}{15}. \end{cases}$$

The above equilibrium bids are depicted in Figure 2.3. We observe that both bids of supplier i pass through the two points, $(\bar{q}_i, C_i(\bar{q}_i) + \Pi - \Pi_j)$ and $(\bar{Q}_i, C_i(\bar{Q}_i) + \Pi - \Pi_j)$, and have the same slopes at these two points. In Figure 2.4 we plot the contours of the buyer's profits under these two equilibria. Note that the buyer's profit function is discontinuous at $q_1 = 0$ or $q_2 = 0$. We observe from Figure 2.4 that, at each equilibrium, there are three local maxima for the buyer's problem, i.e. an interior maximum $(2/5, 1/5)$ as well as two maxima at boundaries $(0, 1/2)$ and $(1/3, 0)$. This matches the result in Lemma 2.4, which states that the buyer's dual-sourcing profit and the single-sourcing profit from each supplier are the same in equilibrium.

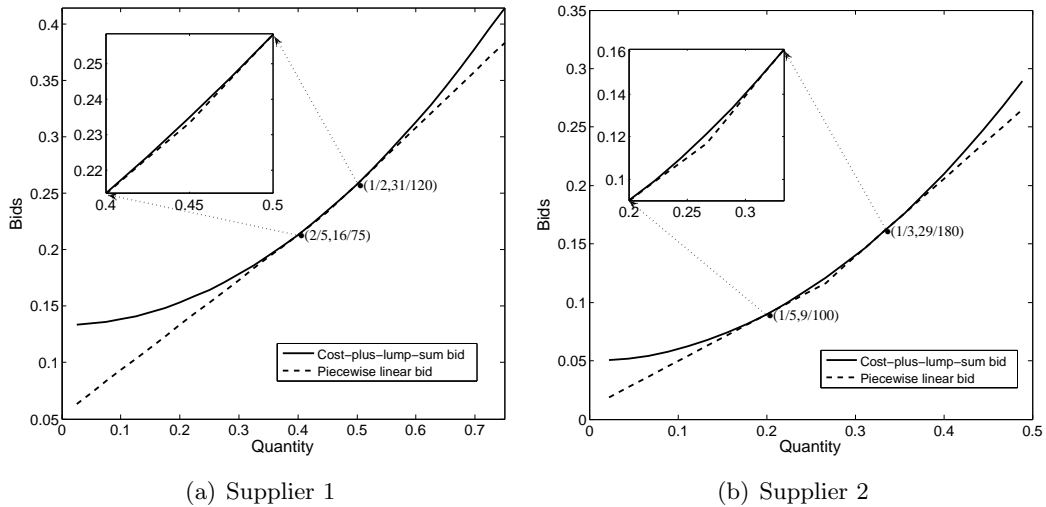


FIGURE 2.3: The supplier bids under the two equilibria

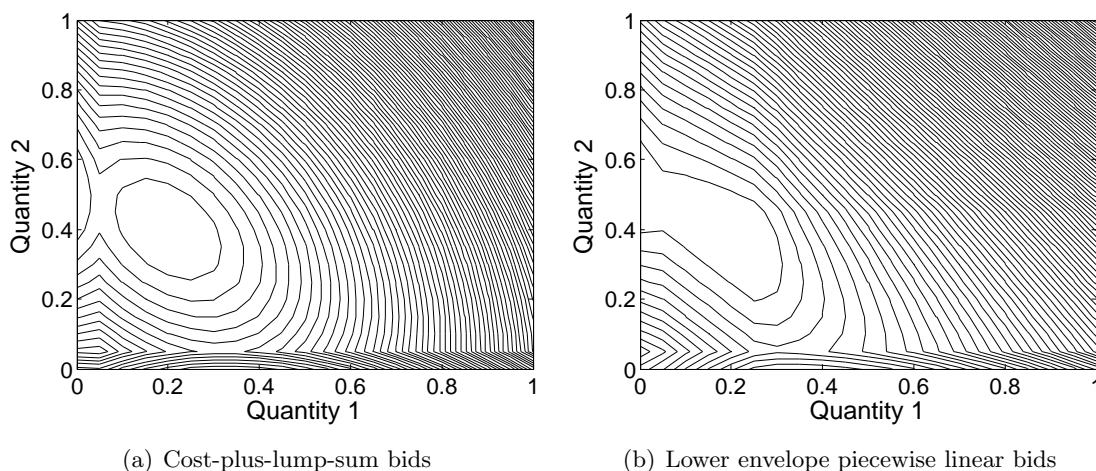


FIGURE 2.4: The contours of buyer profit under the two equilibria

2.5 Single-Sourcing Commitment

In a situation with single-sourcing commitment, the buyer commits to purchasing from only one supplier before suppliers submit their bids. To examine whether the buyer benefits from subsequent subcontracting, we first consider the case in which suppliers are disallowed to subcontract. This serves as a benchmark for the case in which suppliers are allowed to subcontract.

As with the scenario of order splitting, each supplier submits a function bid and the buyer decides the order quantity from the chosen supplier. Since in this scenario the buyer will choose only one supplier, suppliers each essentially optimize the slope and the value of only one point on their bidding functions. For easier exposition, we assume supplier i offers a payment-quantity bid (T_i, Q_i) . This simplification of bidding strategy does not change the equilibrium outcomes because supplier i can always choose a function bid such that the buyer purchases Q_i from him and pays T_i to him.

If the buyer selects the bid (T_i, Q_i) from supplier i where $i = 1, 2$, then her expected profit is

$$\pi_B(T_i, Q_i) = r\mathbb{E}[\min(D, Q_i)] - T_i. \quad (2.27)$$

The buyer chooses the bid which gives her a larger profit. In the event of a tie, she will randomly select a supplier as the winning supplier.

2.5.1 Competition without subcontracting (SC-N)

In this benchmark case, suppliers are not allowed to subcontract with each other. Given the supplier j 's bid (T_j, Q_j) , we write down the supplier i 's profit function with the offer

(T_i, Q_i) as follows,

$$\pi_i(T_i, Q_i) = \begin{cases} T_i - C_i(Q_i), & \pi_B(T_i, Q_i) > \pi_B(T_j, Q_j), \\ \frac{1}{2}[T_i - C_i(Q_i)], & \pi_B(T_i, Q_i) = \pi_B(T_j, Q_j), \\ 0, & \pi_B(T_i, Q_i) < \pi_B(T_j, Q_j). \end{cases}$$

Supplier i maximizes his own profit by choosing an optimal bid (T_i, Q_i) . We now characterize the equilibrium for suppliers in Proposition 2.9.

Proposition 2.9 (Equilibrium without subcontracting). *Suppose subcontracting is disallowed, then there exists a unique Nash equilibrium $\{(T_1^n, Q_1^n), (T_2^n, Q_2^n)\}$ where, for $i = 1, 2$ and $j = 3 - i$, $Q_i^n = \bar{Q}_i$ and $T_i^n = C_i(\bar{Q}_i) + \max(0, \Pi_i - \Pi_j)$. The supplier i 's profit is $\pi_i^n = \max(0, \Pi_i - \Pi_j)$ and the buyer's profit is $\pi_B^n = \min(\Pi_i, \Pi_j)$. The total supply chain profit is $\Pi^n = \max(\Pi_i, \Pi_j)$.*

Proof of Proposition 2.9. We first show that it is optimal for supplier i to set $Q_i = \bar{Q}_i$. For any Q_i , since $\pi_i(T_i, Q_i)$ is increasing in T_i , supplier i will set T_i as high as possible provided that the buyer still chooses his bid. That is, supplier i chooses T_i such that $\pi_B(T_i, Q_i) = \pi_B(T_j, Q_j) + \epsilon_i$ with $\epsilon_i > 0$ as small as possible. Thus, for any Q_i , we obtain $T_i = r\mathbb{E}[\min(D, Q_i)] - \pi_B(T_j, Q_j) - \epsilon_i$. Substituting it into π_i yields,

$$\pi_i(Q_i) = r\mathbb{E}[\min(D, Q_i)] - C_i(Q_i) - \pi_B(T_j, Q_j) - \epsilon_i,$$

which is maximized at $Q_i = \bar{Q}_i$. Therefore, it is optimal for supplier i to set $Q_i = \bar{Q}_i$.

Next we derive the equilibrium for suppliers. Knowing $Q_i = \bar{Q}_i$, we observe that if supplier i wins, the sum of the buyer's profit and the supplier i 's profit will be Π_i . So we can rewrite the buyer's profit when supplier i wins: $\pi_B(T_i, \bar{Q}_i) = \Pi_i - \pi_i(T_i, \bar{Q}_i)$. Noting that $\pi_i, \pi_j \geq 0$, we obtain the suppliers' best response functions as follows,

$$\pi_i = \max[0, \Pi_i - (\Pi_j - \pi_j) - \epsilon_i] \quad \text{and} \quad \pi_j = \max[0, \Pi_j - (\Pi_i - \pi_i) - \epsilon_j],$$

where ϵ_i and ϵ_j are infinitesimally close to zero. Jointly solving the above equations yields the supplier i 's profit in equilibrium: $\pi_i^n = \max(0, \Pi_i - \Pi_j)$. By definition we obtain $T_i^n = C_i(\bar{Q}_i) + \max(0, \Pi_i - \Pi_j)$. Then the buyer's profit is $\pi_B^n = \Pi_i - \max(0, \Pi_i - \Pi_j) = \min(\Pi_i, \Pi_j)$ and the supply chain profit is $\Pi^n = \max(\Pi_i, \Pi_j)$. If $\Pi_i > \Pi_j$, then supplier i wins out and gets a profit of $\Pi_i - \Pi_j$, and the supplier j 's profit is 0. If $\Pi_i = \Pi_j$, the buyer will randomly select a supplier, and each supplier gets zero profits. This completes the proof. \square

Proposition 2.9 shows that each supplier bids a quantity which maximizes the supply chain profit when he is the sole supplier. Thus, the bidding quantity is independent of the competitor's bid. As a result, the two-dimensional supplier competition model with both

quantity and price reduces to the one-dimensional price competition model. The above equilibrium is similar to the Bertrand competition equilibrium where the undercutting behaviour occurs.

2.5.2 Competition with subcontracting (SC-S)

In the case with subcontracting, the game involves two stages: in the first stage, suppliers compete by offering quantity-payment bids; in the second stage, subcontracting may occur between the winning supplier and the losing supplier. Following the backward induction approach, we first examine the subcontracting game. Without loss of generality, suppose supplier i wins the bidding game and his bid is (T_i, Q_i) . Thus, supplier i has to deliver an amount of Q_i to the buyer and gets a payment of T_i .

2.5.2.1 Subcontracting game

We model the subcontracting arrangement in a Nash bargaining framework (Mas-colell et al., 1995). Another way to represent the split of bargaining power is to consider changing the leader in a Stackelberg game where one supplier designs a take-it-or-leave-it subcontract that is offered to the other. Either the winning or losing supplier could be the leader, thus we could consider two cases. We will show these two extreme cases are covered by the Nash bargaining game.

Assume the winning supplier's bargaining power is $\alpha \in [0, 1]$ and the losing supplier's is $1 - \alpha$. The total gain from subcontracting is the difference in the production costs when subcontracting occurs. Given the winning bid (T_i, Q_i) , the total gain from subcontracting is

$$\Delta_i(q; Q_i) = C_i(Q_i) - C_i(Q_i - q) - C_j(q),$$

where q denotes the amount subcontracted to supplier j .

To formulate the bargaining problem, we need to specify both suppliers' disagreement points, i.e. their profits when the negotiation breaks down. Denote by d_i and d_j the disagreement points of supplier i and supplier j , respectively. By definition, $d_i = T_i - C_i(Q_i)$ is the supplier i 's profit when he does not subcontract, and $d_j = 0$ because supplier j , as the losing supplier in the bidding game, will make zero profits if subcontracting does not occur. In the bargaining problem, the two suppliers negotiate the profit split and choose the subcontracted quantity q . Let $\hat{\pi}_i$ and $\hat{\pi}_j$ be the profits of supplier i and supplier j , respectively. For any given q , the problem is to choose the profit split to maximize the Nash product $\Omega(\hat{\pi}_i, \hat{\pi}_j) = (\hat{\pi}_i - d_i)^\alpha (\hat{\pi}_j - d_j)^{1-\alpha}$:

$$\max \{ \Omega(\hat{\pi}_i, \hat{\pi}_j) : \hat{\pi}_i \geq d_i, \hat{\pi}_j \geq d_j, \hat{\pi}_i + \hat{\pi}_j \geq T_i - C_i(Q_i) + \Delta_i(q; Q_i) \}, \quad (2.28)$$

where the last constraint states that the sum of both suppliers' profits cannot exceed the total surplus $T_i - C_i(Q_i) + \Delta_i(q; Q_i)$.

From the KKT conditions, it is straightforward to show the bargaining leads to the profit split of $\hat{\pi}_i = T_i - C_i(Q_i) + \alpha\Delta_i(q; Q_i)$ and $\hat{\pi}_j = (1 - \alpha)\Delta_i(q; Q_i)$. We can see that the gain from subcontracting, $\Delta_i(q; Q_i)$, is allocated to the suppliers proportionally according to their bargaining power split. As α increases, the supplier i 's profit $\hat{\pi}_i$ increases and the supplier j 's profit $\hat{\pi}_j$ decreases. As a result, the bilateral coordination of both suppliers is achieved by choosing q to maximize the total gain from subcontracting:

$$\max\{\Delta_i(q; Q_i) : 0 \leq q \leq Q_i\}. \quad (2.29)$$

Since $\Delta_i(q; Q_i)$ is concave in q , the optimal solution \hat{q}_j can be obtained by solving the first order condition,

$$C'_i(Q_i - \hat{q}_j) = C'_j(\hat{q}_j). \quad (2.30)$$

If the buyer's order quantity from supplier i is Q_i , then it is optimal for supplier i to subcontract $\hat{q}_j(Q_i)$ to supplier j . When $Q_i = \bar{Q}$, we have $\hat{q}_j(\bar{Q}) = \bar{q}_j$. Note that \hat{q}_j is an increasing function of Q_i , and does not depend on the bargaining power split. For notational simplicity, we will suppress Q_i unless ambiguity arises. Thus we write $\hat{q}_j := \hat{q}_j(Q_i)$ and $\Delta_i(\hat{q}_j; Q_i) := \Delta_i(\hat{q}_j(Q_i); Q_i)$. We restrict our attention to the cases with $C'_i(\bar{Q}_i) > C'_j(0)$ so that, as we will show later, the equilibrium subcontracting quantity is strictly positive. This assumption highlights the contrast between the case with subcontracting and that without subcontracting. From the envelope theorem, we can show that $\Delta_i(\hat{q}_j; Q_i)$ is increasing in Q_i .

We write down the profits of the winning supplier i and the losing supplier j as follows,

$$\hat{\pi}_i = T_i - C_i(Q_i) + \alpha\Delta_i(\hat{q}_j; Q_i) \quad \text{and} \quad \hat{\pi}_j = (1 - \alpha)\Delta_i(\hat{q}_j; Q_i).$$

If there is a tie for the buyer's choice, the buyer will randomly select a winning supplier. Thus, the supplier i 's profit is

$$\hat{\pi}_t = \frac{1}{2} [T_i - C_i(Q_i) + \alpha\Delta_i(\hat{q}_j; Q_i) + (1 - \alpha)\Delta_j(\hat{q}_i; Q_j)],$$

where $(1 - \alpha)\Delta_j(\hat{q}_i; Q_j)$ is the supplier i 's reservation profit when he loses in the bidding game.

If we model subcontracting in a Stackelberg framework where either the winning supplier i or the losing supplier j offers a subcontract (t, q) to the other where t denotes the payment and q denotes the quantity, then we can show that the optimal subcontracted amount is \hat{q}_j as well. In the case where the winning supplier i takes the

lead by offering the subcontract, he will extract all the gain from subcontracting and leave zero profits to the losing supplier j . Therefore, the winning supplier i 's profit is $\hat{\pi}_i = T_i - C_i(Q_i) + \Delta_i(\hat{q}_j; Q_i)$, and the losing supplier j 's profit is $\hat{\pi}_j = 0$. This corresponds to the Nash bargaining game with $\alpha = 1$. Similarly, we can show that the Stackelberg game where the losing supplier j offers the subcontract is equivalent to the Nash bargaining game with $\alpha = 0$.

2.5.2.2 Bidding game

We now examine the competitive bidding problem in the first stage. Given the supplier j 's bid (T_j, Q_j) , supplier i may win, lose, or there is a tie for the buyer's supplier selection decision. We write down the supplier i 's profit when offering the bid (T_i, Q_i) as follows,

$$\pi_i(T_i, Q_i) = \begin{cases} T_i - C_i(Q_i) + \alpha \Delta_i(\hat{q}_j; Q_i), & \pi_B(T_i, Q_i) > \pi_B(T_j, Q_j), \\ \frac{1}{2} [T_i - C_i(Q_i) + \alpha \Delta_i(\hat{q}_j; Q_i) + (1 - \alpha) \Delta_j(\hat{q}_i; Q_j)], & \pi_B(T_i, Q_i) = \pi_B(T_j, Q_j), \\ (1 - \alpha) \Delta_j(\hat{q}_i; Q_j), & \pi_B(T_i, Q_i) < \pi_B(T_j, Q_j). \end{cases}$$

The supplier i 's objective is to maximize his profit by choosing (T_i, Q_i) , taking account of the subcontracting opportunity in the later stage as well as the buyer's supplier selection decision. As opposed to the case without subcontracting, the supplier i 's reservation profit (when he loses) is positive, i.e. $(1 - \alpha) \Delta_j(\hat{q}_i; Q_j) > 0$.

We next characterize the equilibrium for suppliers. First, define a useful function of Q_i : $G(Q_i) := \max_q \{r \mathbb{E}[\min(D, Q_i)] - C_j(q) - C_i(Q_i - q) : 0 \leq q \leq Q_i\}$. Then, it is straightforward to show that

$$G(Q_i) = r \mathbb{E}[\min(D, Q_i)] - C_j(\hat{q}_j) - C_i(Q_i - \hat{q}_j),$$

where \hat{q}_j is given in (2.30). Provided the total order quantity is Q_i , then $G(Q_i)$ gives the optimal supply chain profit when subcontracting is considered. We can show that $G(Q_i)$ is increasing in Q_i since the increase in Q_i enlarges the feasible region of the maximization problem and hence improves the optimal profit. Note that when $Q_i = \bar{Q}$, we have $G(\bar{Q}) = \Pi$. We now summarize the equilibrium for suppliers in Proposition 2.10.

Proposition 2.10 (Equilibrium with subcontracting). *Suppose subcontracting is allowed, then there exists a unique subgame perfect Nash equilibrium $\{(T_1^s, Q_1^s); (T_2^s, Q_2^s)\}$ where, for $i = 1, 2, j = 3 - i$, Q_i^s is obtained by solving the equation,*

$$r [1 - F(Q_i^s)] = (1 - \alpha) C_i'(Q_i^s) + \alpha C_j'(\hat{q}_j(Q_i^s)), \quad (2.31)$$

where $\hat{q}_j(Q_i^s)$ is given in (2.30), and $T_i^s = C_i(Q_i^s) + \max[0, G(Q_i^s) - G(Q_j^s)] + (1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$. The profit split is

$$\begin{aligned}\pi_i^s &= \max[0, G(Q_i^s) - G(Q_j^s)] + (1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s), \\ \pi_B^s &= \min[G(Q_i^s), G(Q_j^s)] - (1 - \alpha)\Delta_i(\hat{q}_j; Q_i^s) - (1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s),\end{aligned}$$

and the total supply chain profit is $\Pi^s = \max[G(Q_i^s), G(Q_j^s)]$.

Proof of Proposition 2.10. Given the supplier j 's bid (T_j, Q_j) , supplier i will set T_i as high as possible for any given Q_i provided that the buyer still chooses him. Formally, we have $\pi_B(T_i, Q_i) = \pi_B(T_j, Q_j) + \epsilon_i$, where $\epsilon_i > 0$ is infinitesimally close to zero. Plugging it into $\pi_i(T_i, Q_i)$, we cancel out T_i to obtain

$$\pi_i(Q_i) = r\mathbb{E}[\min(D, Q_i)] - C_i(Q_i) + \alpha[C_i(Q_i) - C_i(Q_i - \hat{q}_j) - C_j(\hat{q}_j)] - \pi_B(T_j, Q_j) - \epsilon_i,$$

which is a concave function of Q_i . Taking the first derivative w.r.t. Q_i yields,

$$\begin{aligned}\frac{\partial \pi_i(Q_i)}{\partial Q_i} &= r[1 - F(Q_i)] - C_i'(Q_i) + \alpha \left[C_i'(Q_i) - C_i'(Q_i - \hat{q}_j) \left(1 - \frac{d\hat{q}_j}{dQ_i} \right) - C_j'(\hat{q}_j) \frac{d\hat{q}_j}{dQ_i} \right] \\ &= r[1 - F(Q_i)] - (1 - \alpha)C_i'(Q_i) - \alpha C_i'(Q_i - \hat{q}_j),\end{aligned}$$

where the second equality follows from (2.30). Then we obtain the first order condition (2.31), which together with (2.30) gives Q_i^s .

In the following, for convenience, we work directly with the supplier profits instead of the payments. Using $Q_i = Q_i^s$ we rewrite the supplier i 's profit as follows:

$$\pi_i(Q_i^s) = r\mathbb{E}[\min(D, Q_i^s)] - C_i(Q_i^s) + \alpha\Delta_i(\hat{q}_j; Q_i^s) - \pi_B(T_j, Q_j^s) - \epsilon_i.$$

We know that $\pi_i(Q_i^s) \geq (1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$ since this is the profit he obtains when he loses. Note that if supplier j wins, the sum of the buyer's profit and the supplier j 's profit is $r\mathbb{E}[\min(D, Q_j^s)] - C_j(Q_j^s) + \alpha\Delta_j(\hat{q}_i; Q_j^s)$. Therefore, the buyer's profit when she selects supplier j can be written as $\pi_B(T_j, Q_j^s) = r\mathbb{E}[\min(D, Q_j^s)] - C_j(Q_j^s) + \alpha\Delta_j(\hat{q}_i; Q_j^s) - \pi_j(T_j, Q_j^s)$. Therefore, the best response functions for both suppliers are

$$\begin{aligned}\pi_i &= \max \left\{ (1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s), r\mathbb{E}[\min(D, Q_i^s)] - C_i(Q_i^s) + \alpha\Delta_i(\hat{q}_j; Q_i^s) \right. \\ &\quad \left. - (r\mathbb{E}[\min(D, Q_j^s)] - C_j(Q_j^s) + \alpha\Delta_j(\hat{q}_i; Q_j^s) - \pi_j) - \epsilon_i \right\}, \\ \pi_j &= \max \left\{ (1 - \alpha)\Delta_i(\hat{q}_j; Q_i^s), r\mathbb{E}[\min(D, Q_j^s)] - C_j(Q_j^s) + \alpha\Delta_j(\hat{q}_i; Q_j^s) \right. \\ &\quad \left. - (r\mathbb{E}[\min(D, Q_i^s)] - C_i(Q_i^s) + \alpha\Delta_i(\hat{q}_j; Q_i^s) - \pi_i) - \epsilon_j \right\},\end{aligned}$$

where $\epsilon_i > 0$ and $\epsilon_j > 0$ are infinitely close to 0. Jointly solving the above best responses yields $\pi_i^s = \max[0, G(Q_i^s) - G(Q_j^s)] + (1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$. By definition the bid price of

supplier i is given by $T_i^s = \pi_i^s + C_i(Q_i^s)$. The buyer's profit is $\pi_B^s = \min[G(Q_i^s), G(Q_j^s)] - (1 - \alpha)\Delta_i(\hat{q}_j; Q_i^s) - (1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$. The supply chain profit in equilibrium is $\Pi^s = \max[G(Q_i^s), G(Q_j^s)]$. In equilibrium, if $G(Q_i^s) > G(Q_j^s)$, the supplier i wins; if $G(Q_i^s) = G(Q_j^s)$, the buyer randomly selects a supplier. This completes the proof. \square

Proposition 2.10 reveals that the supplier i 's bidding quantity Q_i^s can be obtained by solving the simultaneous equations (2.31) and (2.30). Each supplier i 's profit consists of two components: the reservation profit $(1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$ when he loses, and the supply chain profit difference, $\max[0, G(Q_i^s) - G(Q_j^s)]$. If $G(Q_i^s) > G(Q_j^s)$, supplier i wins the bidding game, and supplier j makes a profit equal to his reservation profit, $(1 - \alpha)\Delta_i(\hat{q}_j; Q_i^s)$.

We now discuss the two extreme cases with $\alpha = 0$ and $\alpha = 1$. In the case with $\alpha = 0$ where the winning supplier gains nothing from subcontracting, we obtain from (2.31) that $Q_i^s = \bar{Q}_i$. If supplier i wins, he will subcontract $\hat{q}_j(\bar{Q}_i)$ to supplier j as we show in (2.30). Supplier i makes a profit of $\pi_i^s = \max[0, G(\bar{Q}_i) - G(\bar{Q}_j)] + \Delta_j(\hat{q}_i; \bar{Q}_j)$, and the buyer's profit is $\pi_B^s = \min[G(\bar{Q}_i), G(\bar{Q}_j)] - \Delta_i(\hat{q}_j; \bar{Q}_i) - \Delta_j(\hat{q}_i; \bar{Q}_j)$. As opposed to the case without subcontracting, suppliers compete less aggressively because of the secondary opportunity arising from subcontracting even when losing the bidding game.

In the other extreme case with $\alpha = 1$, we obtain from (2.31) that $Q_i^s = Q_j^s = \bar{Q}$, implying that both suppliers bid the supply chain optimal total quantity. If supplier i wins, then he will subcontract $\hat{q}_j(\bar{Q}) = \bar{q}_j$ to supplier j . Each supplier's actual production quantity equals the first best quantity. Supplier i makes a profit of $\pi_i^s = 0$ and the buyer's profit is $\pi_B^s = \Pi$. The supply chain is coordinated and the buyer takes all the supply chain profit. In this case, suppliers compete relentlessly, anticipating the big gain from subcontracting once winning the bidding game.

2.5.2.3 Sensitivity analysis for α

In this subsection, we evaluate how α affects the bidding quantities and the profit split in equilibrium. For clarity, we will explicitly write the bidding quantities and each player's profit as functions of α whenever necessary, e.g., $Q_i^s(\alpha)$. To facilitate the sensitivity analysis, we make the following assumption.

Assumption 2.4. *Both $(1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$ and $|Q_i^s - Q_j^s|$ decrease in α .*

Note that the supplier i 's reservation profit when he loses is $(1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$. When $\alpha = 0$ the reservation profit is $\Delta_j(\hat{q}_i; \bar{Q}_j)$ which is greater than 0; when $\alpha = 1$ the reservation profit becomes 0. We assume that $(1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$ decreases in α , since it is sensible that if the winning supplier has more bargaining power, the losing supplier will make a lower profit. Also, as shown earlier, $|Q_i^s - Q_j^s| = 0$ if $\alpha = 1$, and $|Q_i^s - Q_j^s| \geq 0$ if $\alpha = 0$, we assume that $|Q_i^s - Q_j^s|$ decreases in α . This eliminates the cases where the

sign of $Q_i^s - Q_j^s$ changes as α varies. We can show that these two assumptions are satisfied in the cases where suppliers have symmetric quadratic cost functions³. Making these two assumptions will facilitate the sensitivity analysis for α but they are not required for the qualitative results that we give in this chapter.

Corollary 2.11 (Sensitivity analysis for α). *For $i = 1, 2$, $Q_i^s(\alpha)$ increases in α . Under Assumption 2.4, as α increases, both $\pi_B^s(\alpha)$ and Π^s increase, while $\pi_i^s(\alpha)$ decreases.*

Proof of Corollary 2.11. We first show that the supplier i 's bidding quantity in equilibrium, Q_i^s , increases in α , where Q_i^s is obtained by solving the following simultaneous equations:

$$r[1 - F(Q_i^s)] = (1 - \alpha)C_i'(Q_i^s) + \alpha C_j'(\hat{q}_j) \quad (2.32)$$

$$C_i'(Q_i^s - \hat{q}_j) = C_j'(\hat{q}_j). \quad (2.33)$$

Taking the derivatives of both sides of (2.33) w.r.t. Q_i^s , we obtain

$$C_i''(Q_i^s - \hat{q}_j) \left(1 - \frac{d\hat{q}_j}{dQ_i^s}\right) = C_j''(\hat{q}_j) \frac{d\hat{q}_j}{dQ_i^s},$$

from which we obtain,

$$\frac{d\hat{q}_j}{dQ_i^s} = \frac{C_i''(Q_i^s - \hat{q}_j)}{C_i''(Q_i^s - \hat{q}_j) + C_j''(\hat{q}_j)}.$$

We now take the derivatives of both sides of (2.32) w.r.t. α , and obtain

$$\begin{aligned} -rf(Q_i^s) \frac{dQ_i^s}{d\alpha} &= -C_i'(Q_i^s) + (1 - \alpha)C_i''(Q_i^s) \frac{dQ_i^s}{d\alpha} + C_j'(\hat{q}_j) + \alpha C_j''(\hat{q}_j) \frac{d\hat{q}_j}{dQ_i^s} \frac{dQ_i^s}{d\alpha} \\ &= -C_i'(Q_i^s) + (1 - \alpha)C_i''(Q_i^s) \frac{dQ_i^s}{d\alpha} + C_j'(\hat{q}_j) + \frac{\alpha C_j''(\hat{q}_j) C_i''(Q_i^s - \hat{q}_j)}{C_i''(Q_i^s - \hat{q}_j) + C_j''(\hat{q}_j)} \frac{dQ_i^s}{d\alpha}. \end{aligned}$$

Thus we have

$$\frac{dQ_i^s}{d\alpha} = \frac{C_i'(Q_i^s) - C_j'(\hat{q}_j)}{rf(Q_i^s) + (1 - \alpha)C_i''(Q_i^s) + \frac{\alpha C_j''(\hat{q}_j) C_i''(Q_i^s - \hat{q}_j)}{C_i''(Q_i^s - \hat{q}_j) + C_j''(\hat{q}_j)}} > 0.$$

Therefore, we have established that Q_i^s is increasing in α .

Second, we show that each supplier's profit is decreasing in α . Without loss of generality, assume $Q_i^s(\alpha) \geq Q_j^s(\alpha)$ and supplier i wins. Then the supplier j 's profit is $\pi_j^s = (1 - \alpha)\Delta_i(\hat{q}_j; Q_i^s)$, which is decreasing in α according to Assumption 2.4. We now examine the supplier i 's profit: $\pi_i^s = G(Q_i^s(\alpha)) - G(Q_j^s(\alpha)) + (1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$. Note first that $(1 - \alpha)\Delta_j(\hat{q}_i; Q_j^s)$ is decreasing in α . We next show that $G(Q_i^s(\alpha)) - G(Q_j^s(\alpha))$

³See Appendix 2.8 for the proof.

is decreasing in α as well. We have

$$\begin{aligned} \frac{\partial [G(Q_i^s(\alpha)) - G(Q_j^s(\alpha))]}{\partial \alpha} &= G'(Q_i^s(\alpha)) \frac{dQ_i^s(\alpha)}{d\alpha} - G'(Q_j^s(\alpha)) \frac{dQ_j^s(\alpha)}{d\alpha} \\ &\leq [G'(Q_i^s(\alpha)) - G'(Q_j^s(\alpha))] \frac{dQ_j^s(\alpha)}{d\alpha} \leq 0, \end{aligned}$$

where the first inequality follows that $Q_i^s(\alpha) - Q_j^s(\alpha)$ is decreasing in α and the second inequality follows that $G(z)$ is concave in z . Therefore, the supplier i 's profit π_i^s is decreasing in α . The buyer's profit is $\pi_B^s = G(Q_j^s) - (1-\alpha)\Delta_i(\hat{q}_j; Q_i^s) - (1-\alpha)\Delta_j(\hat{q}_i; Q_j^s)$, which is increasing because $G(Q_j^s)$ is increasing and both $(1-\alpha)\Delta_i(\hat{q}_j; Q_i^s)$ and $(1-\alpha)\Delta_j(\hat{q}_i; Q_j^s)$ are decreasing. The supply chain profit in equilibrium is $\Pi^s = \max[G(Q_i^s), G(Q_j^s)]$, which is increasing in α . This completes the proof. \square

Corollary 2.11 states that each supplier's bidding quantity increases in α . Both the buyer's profit and the supply chain profit increase in α , while each supplier's profit decreases in α . The intuition of the above result is that, if the winning supplier has more bargaining power, the losing supplier's secondary opportunity arising from subcontracting becomes smaller, while the winning supplier's profit becomes larger. As a result, suppliers bid more aggressively, anticipating that they can make more profit by subcontracting once they win. Therefore, each supplier becomes worse off while the buyer is better off in equilibrium.

Example 2.3 (Continued). *Back to the earlier example, we can calculate the bidding quantities of both suppliers:*

$$Q_1^s = \frac{3}{6-\alpha} \quad \text{and} \quad Q_2^s = \frac{3}{9-4\alpha}.$$

The profit split in equilibrium is given as follows:

$$\pi_1^s = \frac{3(7-2\alpha)}{2(6-\alpha)^2} - \frac{3}{2(9-4\alpha)}, \quad \pi_2^s = \frac{3(1-\alpha)}{2(6-\alpha)^2}, \quad \pi_B^s = \frac{3}{2(9-4\alpha)} - \frac{3(1-\alpha)}{2(6-\alpha)^2}, \quad \Pi^s = \frac{3(7-2\alpha)}{2(6-\alpha)^2}.$$

We plot the bidding quantities and each player's profit in Figure 2.5.

We can see that as α increases, both Q_1^s and Q_2^s increase. Both the buyer's profit π_B^s and the total supply chain profit Π^s increase in α as well, while each supplier's profit (π_1^s and π_2^s) decreases in α . When $\alpha = 0$, $Q_1^s(0) = 1/3$ and $Q_2^s(0) = 1/2$; when $\alpha = 1$, $Q_1^s(1) = Q_2^s(1) = 3/5$. Only when $\alpha = 1$ is the supply chain coordinated. In equilibrium, supplier 1 wins in the bidding game, and subcontracts $\hat{q}_2 = (1/3)Q_1^s$ to supplier 2 subsequently. As a result, supplier 1 produces an amount of $Q_1^s - \hat{q}_2 = 2/(6-\alpha)$, and supplier 2 produces an amount of $\hat{q}_2 = 1/(6-\alpha)$.

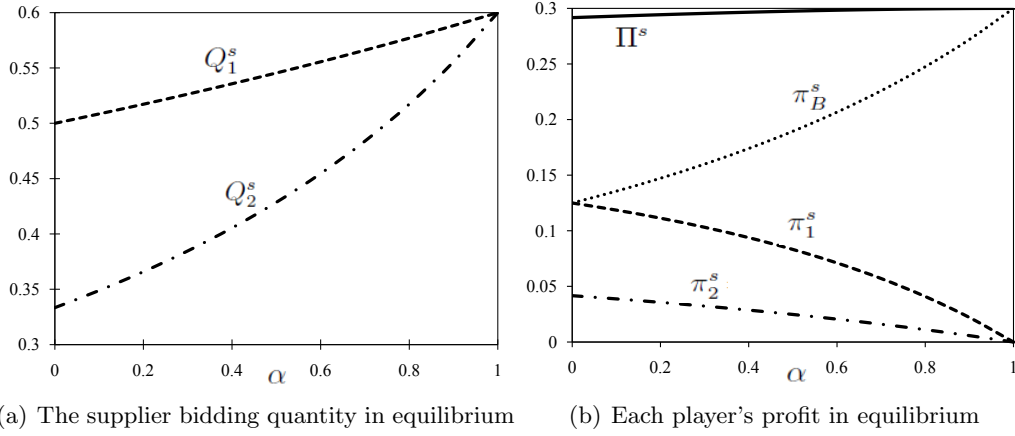


FIGURE 2.5: The bidding quantities and each player's profit as functions of α under single-sourcing commitment with subcontracting

2.6 Comparisons

In this section, we answer the questions regarding whether the buyer benefits from subsequent subcontracting and single-sourcing commitment.

2.6.1 Non-subcontracting vs. subcontracting

We now make the first comparison between the single-sourcing commitment without subcontracting and that with subcontracting, according to the buyer's equilibrium profit.

Theorem 2.12. *Under Assumption 2.4, there exists a threshold $\alpha^* \in (0, 1)$ such that if $\alpha > \alpha^*$ the buyer making single-sourcing commitment is better off to allow subcontracting; otherwise the buyer is better off to disallow subcontracting.*

Proof of Theorem 2.12. We have shown in Corollary 2.11 that the buyer's profit $\pi_B^s(\alpha)$ is increasing in α under the commitment scenario with subcontracting. Note also that the buyer's profit is continuous in α , so we only need to show when $\alpha = 0$, $\pi_B^n > \pi_B^s(0)$; and when $\alpha = 1$, $\pi_B^n < \pi_B^s(1)$. Without subcontracting, the buyer's profit is $\pi_B^n = \min(\Pi_i, \Pi_j)$. With subcontracting, if $\alpha = 0$, the buyer's profit is $\pi_B^s(0) = \min[\Pi_j - \Delta_i(\hat{q}_j; \bar{Q}_i), \Pi_i - \Delta_j(\hat{q}_i; \bar{Q}_j)] < \pi_B^n$, where the inequality follows from $\Delta_i(\hat{q}_j; \bar{Q}_i) > 0$ and $\Delta_j(\hat{q}_i; \bar{Q}_j) > 0$. In this case, the buyer is better off to disallow subcontracting. If $\alpha = 1$, the buyer's profit is $\pi_B^s(1) = \Pi > \pi_B^n$, which implies that the buyer is better off to allow subcontracting. Therefore, there must exist a threshold α^* such that $\pi_B^s(\alpha^*) = \pi_B^n$. If $\alpha > \alpha^*$ the buyer is better off to allow subcontracting. This completes the proof. \square

This result is somewhat counter-intuitive since conventional wisdom may suggest that the availability of subsequent subcontracting dampens the competition between suppliers in the first stage's bidding game. However, we show that, if the winning

supplier's bargaining power exceeds a threshold, the buyer prefers to allow suppliers to subcontract with each other. We also observe that $Q_i^s(\alpha) \geq Q_i^n$ where the equality holds only when $\alpha = 0$. This implies that the supply chain performance is higher with subcontracting (due to $G(Q_i^s) \geq \Pi^n$). However, whether the supply chain performance improvement benefits the buyer or not depends on its magnitude. With a smaller α , the efficiency improvement is smaller but each supplier i 's reservation profit $(1-\alpha)\Delta_j(\hat{q}_i; Q_j^s)$ when he loses is larger. As a result, the efficiency improvement cannot counteract the increase of the suppliers' reservation profits. For example, in the extreme case with $\alpha = 0$, we have

$$\begin{aligned}\pi_B^s(0) &= \min [G(\bar{Q}_i), G(\bar{Q}_j)] - \Delta_i(\hat{q}_j; \bar{Q}_i) - \Delta_j(\hat{q}_i; \bar{Q}_j) \\ &= \min [\Pi_i - \Delta_j(\hat{q}_i; \bar{Q}_j), \Pi_j - \Delta_i(\hat{q}_j; \bar{Q}_i)],\end{aligned}$$

which is less than $\pi_B^n = \min(\Pi_i, \Pi_j)$. Thus, the buyer is better off to disallow subcontracting. In the other extreme case with $\alpha = 1$, we have $\pi_B^s(1) = \Pi > \pi_B^n$, implying that the buyer benefits from subcontracting. Therefore, there exists a threshold $\alpha^* \in (0, 1)$, above which the buyer prefers to allow subcontracting.

One may wonder why suppliers still subcontract with each other if both are worse off than they are with non-subcontracting. This results from the phenomenon of the prisoner's dilemma. In our model, non-subcontracting is strictly dominated by subcontracting in the second stage's game, which implies that non-subcontracting is off the equilibrium path. However, both suppliers are worse off in the overall game. The prisoner's dilemma in a dynamic game has been observed in other contexts. A notable example is in a Cournot competition setting with forward markets (Allaz and Vila, 1993). Allaz and Vila (1993) show that, in equilibrium, each firm sells forward, which makes them worse off than if the forward market does not exist.

2.6.2 Order splitting vs. single-sourcing commitment

We now make the second comparison between order splitting and single-sourcing commitment according to each player's equilibrium profit. We show that the buyer (each supplier) is always better off (worse off) with single-sourcing commitment.

Theorem 2.13. *From the buyer's perspective, single-sourcing commitment outperforms order splitting, while the reverse is true from the suppliers' perspective, whether or not subcontracting is allowed.*

Proof of Theorem 2.13. We need to compare the order splitting scenario with the two cases of single-sourcing commitment based on both the buyer's profit and each supplier's

profit. First, we compare order splitting with single-sourcing commitment *without* subcontracting. Comparing the buyer's profits under these two scenarios, we have

$$\pi_B^* - \pi_B^n = \Pi_i + \Pi_j - \Pi - \min(\Pi_i, \Pi_j) = \max(\Pi_i, \Pi_j) - \Pi < 0,$$

indicating that the buyer's profit is lower under order splitting. In terms of each supplier's profit, we have

$$\pi_i^* - \pi_i^n = \Pi - \Pi_j - \max(0, \Pi_i - \Pi_j) = \Pi - \max(\Pi_i, \Pi_j) > 0,$$

showing that each supplier's profit is higher under order splitting.

Second, we compare order splitting with single-sourcing commitment with subcontracting. Since under the latter scenario, the buyer's profit is smallest and each supplier's profit is largest when $\alpha = 0$, if we can show the result holds when $\alpha = 0$, then the result will hold for any value of α . We begin by comparing the buyer's profits under these two scenarios. We get the difference:

$$\begin{aligned} \pi_B^s(0) - \pi_B^* &= \min [\Pi_i - \Delta_j(\hat{q}_i; \bar{Q}_j), \Pi_j - \Delta_i(\hat{q}_j; \bar{Q}_i)] - (\Pi_i + \Pi_j - \Pi) \\ &= \Pi - \max [\Pi_i + \Delta_i(\hat{q}_j; \bar{Q}_i), \Pi_j + \Delta_j(\hat{q}_i; \bar{Q}_j)] \\ &= G(\bar{Q}) - \max [G(\bar{Q}_i), G(\bar{Q}_j)] > 0, \end{aligned}$$

where the inequality follows from $\bar{Q} > \max(\bar{Q}_i, \bar{Q}_j)$. Thus, the buyer makes a higher profit under single-sourcing commitment. Now we make a comparison based on the supplier i 's profit. We get the difference:

$$\begin{aligned} \pi_i^s(0) - \pi_i^* &= \max [\Pi_i + \Delta_i(\hat{q}_j; \bar{Q}_i), \Pi_j + \Delta_j(\hat{q}_i; \bar{Q}_j)] - \Pi \\ &= \max [G(Q_i), G(Q_j)] - G(\bar{Q}) < 0. \end{aligned}$$

So each supplier is worse off under the scenario of single-sourcing commitment. Therefore, we have established that the buyer benefits from single-sourcing commitment while the reverse is true for each supplier. This completes the proof. \square

Theorem 2.13 shows that the buyer prefers to make a single-sourcing commitment, regardless of subcontracting. In particular, when $\alpha = 0$, we have

$$\begin{aligned} \pi_B^s(0) - \pi_B^* &= \min [\Pi_i - \Delta_j(\hat{q}_i; \bar{Q}_j), \Pi_j - \Delta_i(\hat{q}_j; \bar{Q}_i)] - (\Pi_i + \Pi_j - \Pi) \\ &= \Pi - \max [\Pi_i + \Delta_i(\hat{q}_j; \bar{Q}_i), \Pi_j + \Delta_j(\hat{q}_i; \bar{Q}_j)] \\ &= G(\bar{Q}) - \max [G(\bar{Q}_i), G(\bar{Q}_j)] > 0. \end{aligned}$$

Therefore, the buyer is better off making a single-sourcing commitment even when the winning supplier has no bargaining power in subcontracting.

The intuitive reason that the buyer is better off under single-sourcing commitment *without* subcontracting, as opposed to order splitting, is relatively straightforward. If the buyer does not commit to single-sourcing, the competition between suppliers will be dampened because each supplier feels “assured” that they will win at least part of the buyer’s business. However, when comparing order splitting with single-sourcing commitment *with* subcontracting, the intuition is less clear. Under both scenarios, both suppliers produce for the buyer. Specifically, under order splitting, the buyer directly purchases from both suppliers; while, under the latter scenario, the losing supplier signs the subcontract with the winning supplier later. Our results suggest that suppliers compete less aggressively under order splitting. Therefore, the buyer is better off making a commitment, while each supplier becomes worse off with commitment.

Example 2.4 (continued). *For the example we introduce earlier, we plot in Figure 2.6 each player’s profit and the supply chain profit against α under each scenario. Since the profit splits under order splitting and single-sourcing commitment without subcontracting have nothing to do with α , each player’s profit and the supply chain profit are constants under these two scenarios.*

From Sub-figure 2.6(a) we observe that there exists an intersection between π_B^n and π_B^s , and the threshold value is $\alpha^ = 0.35$. When $\alpha > 0.35$, the buyer is better off to allow subcontracting. We also observe the buyer’s profit under the scenario of order splitting is always lower than that under the scenario of single-sourcing commitment whether or not subcontracting is allowed. As Sub-figures 2.6(c) and 2.6(d) show, however, both suppliers’ profits are lower with single-sourcing commitment. Sub-figure 2.6(b) indicates that the supply chain is coordinated under order splitting, which obviously outperforms the other two scenarios from the supply chain’s perspective.*

2.7 Conclusions

Many operations management models assume that firms face economies of scale or have linear total costs, whereas this study focuses on the case in which firms face diseconomies of scale, as in the examples discussed earlier. Although research on supply chain contracts is a well-established field, the strategic interaction among suppliers facing production diseconomies has not been well explored. This chapter attempts to fill this gap by incorporating *ex-post* subcontracting and *ex-ante* commitment into the supplier competition model. In doing so, it is possible to study how these factors affect the competition dynamics for suppliers, while also providing valuable guidance on procurement practices.

The results in this research are rich and insightful. The first model considered is the order splitting scenario, in which the buyer does not make a single-sourcing commitment.

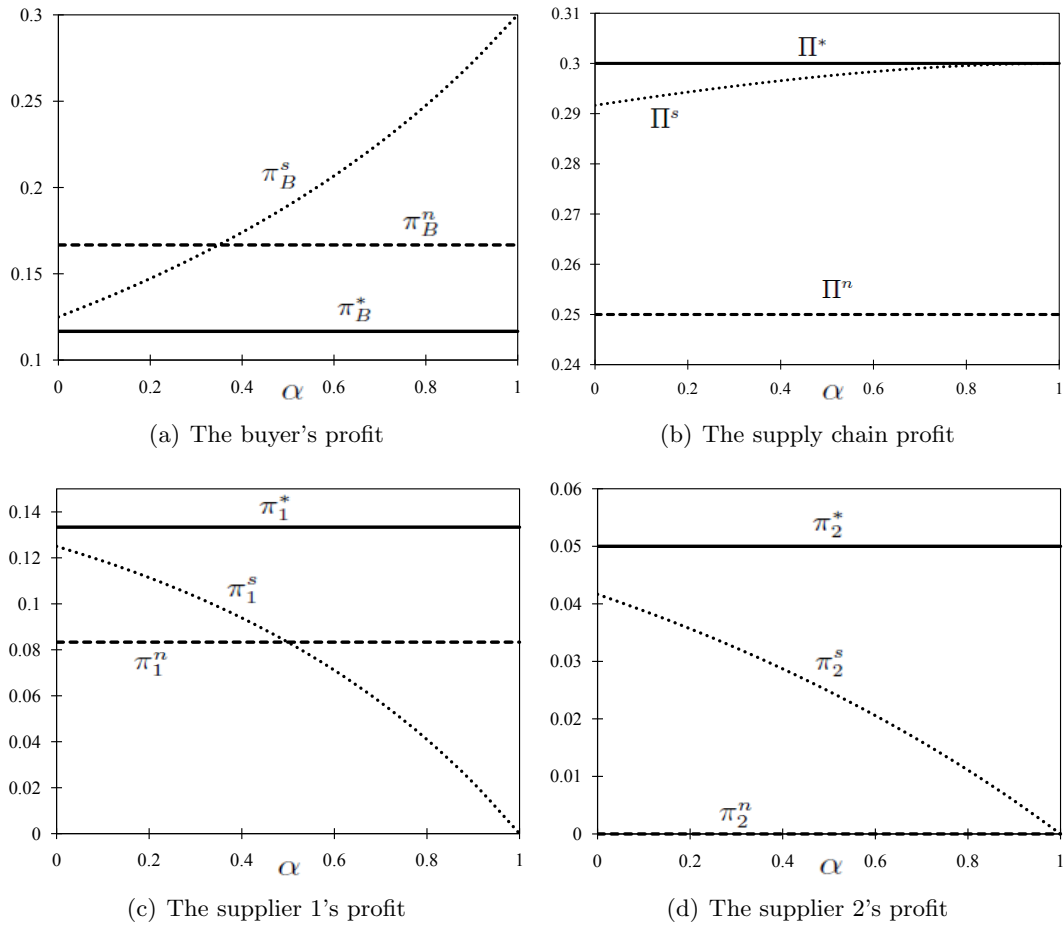


FIGURE 2.6: Each player's profit and the supply chain profit as functions of α under different scenarios

We find that, in equilibrium, the buyer's optimal choice is always first best. Each supplier's profit equals its marginal contribution to the supply chain system, and the buyer takes the remaining profit. The second model is single-sourcing commitment in which the buyer commits to purchasing from only one supplier. In this case, we study whether the buyer has incentive to allow subsequent subcontracting between suppliers. The analysis shows that, the buyer benefits from subcontracting when the winning supplier's bargaining power in subcontracting exceeds a threshold. It also shows that the buyer prefers to commit to single-sourcing, whether or not subcontracting is considered.

Based on the analytical results, this study's recommendation for procurement practitioners is to commit to single-sourcing prior to running a bidding process. For example, they could announce to suppliers that they will not split orders. This result is robust to the case with a positive managerial cost of dealing with a supplier. If the administrative cost is taken into account, the buyer will have an even greater preference for single-sourcing commitment. In addition, the buyer should allow subcontracting when the winning supplier's bargaining power in subcontracting is high. In practice, it may

be difficult for the buyer to identify the extent of the bargaining power enjoyed by the winning supplier. A rule of thumb is to consider the competition intensity in the subcontracting market. If there are many subcontractors in the market, the winning supplier may have high bargaining power. In this case, we recommend the buyer allow subcontracting.

Regarding subcontracting, one may think that subcontractors may not participate in the bidding game. Specifically, suppliers have an outside option of subcontracting, and the subcontractors' strategic behaviour is not considered. In this situation, subcontracting only changes each supplier's cost structure for the bidding game, and the game theoretical models of outsourcing may provide some explanations (Feng and Lu, 2012b; Wu and Zhang, 2014).

2.8 Appendix

We will show that Assumption 2.4 is satisfied with symmetric suppliers having quadratic costs. Suppose suppliers have the same cost, i.e. $C_i(q) = C_j(q) = C(q)$. Then each supplier's equilibrium bidding quantity will be the same and we write $Q_i^s = Q_j^s = Q^s$. From Proposition 2.10 we know there will exist a unique symmetric equilibrium. Let $\hat{q} := \hat{q}(Q^s)$ be the optimal subcontracted amount given the total amount of Q^s . From $C'(Q^s - \hat{q}) = C'(\hat{q})$, we can easily obtain $\hat{q} = \frac{1}{2}Q^s$, and so $\frac{d\hat{q}}{dQ^s} = \frac{1}{2}$. Then

$$\frac{dQ^s}{d\alpha} = \frac{C'(Q^s) - C'(\frac{1}{2}Q^s)}{rf(Q^s) + (1 - \alpha)C''(Q^s) + \frac{\alpha}{2}C''(\frac{1}{2}Q^s)}.$$

Each supplier makes a profit of $(1 - \alpha)\Delta(\hat{q}; Q^s) = (1 - \alpha) [C(Q^s) - 2C(\frac{1}{2}Q^s)]$. We take the derivative of the supplier profit w.r.t. α , and obtain

$$\begin{aligned} \frac{\partial[(1 - \alpha)\Delta(\hat{q}; Q^s)]}{\partial\alpha} &= - \left[C(Q^s) - 2C(\frac{1}{2}Q^s) \right] + (1 - \alpha) \left[C'(Q^s) - C'(\frac{1}{2}Q^s) \right] \frac{dQ^s}{d\alpha} \\ &= - \left[C(Q^s) - 2C(\frac{1}{2}Q^s) \right] + \frac{(1 - \alpha) [C'(Q^s) - C'(\frac{1}{2}Q^s)]^2}{rf(Q^s) + (1 - \alpha)C''(Q^s) + \frac{\alpha}{2}C''(\frac{1}{2}Q^s)} \\ &< - \left[C(Q^s) - 2C(\frac{1}{2}Q^s) \right] + \frac{[C'(Q^s) - C'(\frac{1}{2}Q^s)]^2}{C''(Q^s)} \end{aligned}$$

Therefore, the sufficient condition on $C(q)$ for $\frac{\partial(1-\alpha)\Delta(\hat{q};Q^s)}{\partial\alpha} < 0$ is

$$\left[C'(q) - C'(\frac{1}{2}q) \right]^2 - C''(q) \left[C(q) - 2C(\frac{1}{2}q) \right] \leq 0. \quad (2.34)$$

We now show that the quadratic cost function $C(q) = aq^2 + bq$ where $a, b > 0$ can satisfy the condition (2.34):

$$\begin{aligned} & \left[C'(q) - C'\left(\frac{1}{2}q\right) \right]^2 - C''(q) \left[C(q) - 2C\left(\frac{1}{2}q\right) \right] \\ &= (2aq + b - aq - b)^2 - 2a \left(aq^2 + bq - \frac{1}{2}aq^2 - bq \right) = 0. \end{aligned}$$

Hence, we have $(1 - \alpha)\Delta(\hat{q}; Q^s) = (1 - \alpha) [C(Q^s) - 2C(\frac{1}{2}Q^s)]$ is decreasing in α , which can be verified as follows:

$$\begin{aligned} \frac{\partial(1 - \alpha)\Delta(\hat{q}; Q^s)}{\partial\alpha} &= -\frac{1}{2}a(Q^s)^2 + \frac{(1 - \alpha)(aQ^s)^2}{rf(Q^s) + (2 - \alpha)a} \\ &< -\frac{1}{2}a(Q^s)^2 + \frac{(1 - \alpha)(aQ^s)^2}{(2 - 2\alpha)a} = 0. \end{aligned}$$

Chapter 3

Supplier Competition with Reservation Bidding

3.1 Introduction

In recent years, increasingly high demand uncertainty and long lead times have challenged the operations of supply chains. One of the challenges revolves around how to share various risks amongst supply chain members. Risk sharing is particularly important when suppliers need to invest heavily in capacity installation and the future demand is highly unpredictable. To hedge against financial risks, suppliers often require a buyer to reserve capacity in advance by paying an upfront fee (i.e. a reservation price) and build capacity accordingly. After knowing the actual demand, the buyer decides how much capacity to use and pays only for the used capacity (i.e. an execution price). This contract arrangement is referred to as “supply option” in the operations management literature (Martínez-de Albéniz and Simchi-Levi, 2009).

In an option contract, the buyer’s demand risk is mitigated by freely choosing how much capacity to use after knowing the actual demand, while the supplier’s financial risk is diminished by receiving a reservation payment in advance. The underlying assumption is that the supplier only builds the capacity that is reserved, so this becomes the limit for later production. This same basic model can represent a situation in which an early decision is required in order to obtain components with a long lead time: once the components are available actual production can take place quickly, but the components ordered imply a limit on this production.

In a competitive situation with capacity reservation, each supplier quotes a reservation price and an execution price. The fundamental trade-off for a buyer is to balance flexibility and cost efficiency. A flexible bid has a lower reservation price but a higher

⁰This chapter is based on a joint work with Professor Edward Anderson and Professor Bo Chen.

execution price, while an inflexible bid has a higher reservation price but a lower execution price. If there is no demand uncertainty then the buyer will accept the bids with the lowest total prices. But if there is a small chance of demand occurring, the buyer will purchase the bids with very low reservation prices but high execution prices. Therefore, with demand following a certain distribution, it is natural for the buyer to purchase a portfolio of supply options. This research is concerned with how suppliers compete with each other in such a supply option market. We believe that a closer look at how suppliers compete will deepen our understanding of a firm's procurement strategy, and hence has the potential to help a buyer make a better sourcing decision.

This underlying model has numerous applications. First, in general industrial procurement context, it is common to use these types of contracts. For example, Hewlett-Packard adopts a portfolio of procurement contract for electronic and memory components, amongst which 30% is in the form of option contracts (Martínez-de Albéniz and Simchi-Levi, 2005). Within an electricity market context we can see the selection of a mix of generation capacity as falling into this framework. Instead of a single uncertain demand with a known distribution we consider a range of electricity demand occurring over time, but the mathematical model is entirely equivalent. The base-load generation has the lowest overall cost, while peaking generation has lower costs for a fixed amount of installed capacity, but higher costs for generation. This makes it appropriate as part of the portfolio mix for use when demand is high.

In this chapter, we consider a discrete problem which involves a choice between "blocks" of capacity offered, without the possibility of the buyer choosing to reserve only part of a block. Thus the problem for the buyer is to select the right set of suppliers. An example of this sort occurs within the UK's system for purchasing Short Term Operating Reserve (STOR) for electricity supply. This is a scheme under which the UK's National Grid maintains a reserve generation ability in case of sudden demand variations or plant failures. Part of the Operating Reserve is made up by contracts that are bid for within the STOR. Tenders are assessed on the basis of reservation prices (called availability prices) and execution prices (called utilization prices) together with a consideration of response times and geographical location.

Figure 3.1 shows the STOR bidding data from Round 18 (2012: Season 6.5) with accepted bids shown as circles (in green). This illustrates the portfolio selection and one can see that there is a curved boundary for the bids accepted.

The problems of procurement and contract design for supply options have received a considerable amount of attention. With exogenous option contracts, several papers study the buyer's procurement strategy (See e.g., Barnes-Schuster et al., 2002; Burnetas and Ritchken, 2005; Martínez-de Albéniz and Simchi-Levi, 2005). More recently, Fu et al. (2010) examine the value of portfolio procurement when a buyer can purchase from suppliers using fixed price contracts, option contracts or spot purchases. Lee

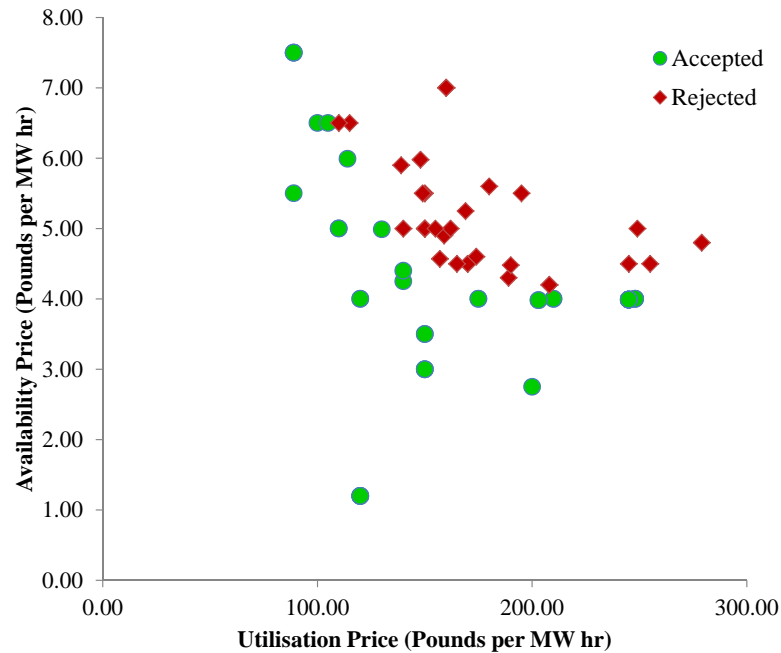


FIGURE 3.1: The STOR bids submitted in Round 18, Season 6.5, 2012.

et al. (2013) examine the buyer's optimal procurement decision with capacity limits for suppliers. More relevant to our work are the models of contract design for options. With just a single supplier, several papers model a Stackelberg game between the supplier (the leader) and a single buyer (the follower). For example, Wu et al. (2002) consider a single supplier operating along side a spot market, and show that it is best for the supplier to offer execution prices equal to its costs. Pei et al. (2011) analyze the structure and pricing of option contracts in the presence of spot trading and asymmetric information of the buyer's valuation. With multiple suppliers competing with each other in an option market, Wu and Kleindorfer (2005) extend the result of Wu et al. (2002) (again in the case where a spot market provides an alternative source of supply/demand for the buyer/supplier).

The paper that is closest to our model is Martínez-de Albéniz and Simchi-Levi (2009), but with one very significant difference: they assume that each supplier has (infinitely) scalable capacity so that execution prices and reservation prices are bid and the buyer can then decide how much capacity to reserve from each supplier. In our model (as in the STOR example) where capacity comes as a block, the buyer has to reserve it all or none. As a result, we have a combinatorial style auction with blocks of capacity reserved from selected suppliers. It is interesting that the result of this assumption is to improve the overall performance of the supply chain (in a sense that we will make clear later).

The general arrangement, in which an auction takes place in a multi-dimensional setting and players have private information about their own costs, is often treated as a type of mechanism design problem. Chao and Wilson (2002) discuss some fundamental

questions of an auction for reserve in which the requirement is to specify both a scoring rule (determining which capacity is to be used) and a payment rule for the winning suppliers. Schummer and Vohra (2003) propose an Expected Vickrey-Clarke-Groves (EVCG) mechanism which arranges the payments to give each supplier their contribution to expected overall costs. There are a continuum of EVCG mechanisms with different amounts paid to the suppliers “up front” after demand is realised, but they all have the characteristic of inducing truthful revelation of the actual costs (both for reservation and execution). Our approach is different from this literature in several ways: we are concerned with optimizing the revenue for the buyer rather than overall welfare; suppliers are paid exactly as they bid; and we consider a complete information setting.

In this chapter we consider the optimal behaviour for suppliers who know their costs (both for reservation and execution) and want to determine their prices in a competitive market. In Section 3.2 we first set up the model and describe the sequence of decisions more precisely. Then we discuss the buyer’s problem of choosing an optimal set of suppliers, and show that it is straightforward to find an optimal solution in the case where all the suppliers offer blocks of the same size. In Section 3.3 we turn to the problem faced by a supplier knowing the bids of the other players. We show that suppliers offer at their execution costs, making profits only on the reservation component in their bids. This result fails when suppliers own more than one block, or the buyer can reserve just part of a block. In Section 3.4 we characterize the equilibrium for the suppliers and show that, provided all the blocks have the same size, at a Nash equilibrium the buyer selects exactly those suppliers necessary to give an efficient outcome for the supply chain as a whole. For this equilibrium result we require the buyer’s optimal profit to be submodular as a function of the set of supplier bids available. This submodularity result requires a complex proof and this is relegated to the Appendix¹. Moreover, we provide a procedure to construct an equilibrium for the case with unequal size blocks. Finally we conclude in Section 3.5.

3.2 Model Setup and Buyer’s Problem

3.2.1 Model Setup

We suppose that demand is a non-negative random variable D with cdf F , so $F(t) = \Pr[D \leq t]$. Before demand occurs, the buyer can reserve capacity that is offered by a number of different suppliers in blocks. Later we will restrict our attention to the case where all blocks have the same size, but initially we allow any size blocks. After demand occurs the buyer will meet the demand (up to the total amount of capacity reserved)

¹This proof is the work of Professor Bo Chen and is included here for convenience.

and at this point pays an additional (execution) price for the capacity that is needed. For the total demand that can be met, the buyer will be paid a retail price ρ .

Suppose that there are n suppliers, and supplier $i \in N := \{1, 2, \dots, n\}$ sets a reservation price r_i and an execution price p_i . The suppliers each try to maximize their expected profits given their reservation costs e_i , execution costs c_i , and block sizes $K_i > 0$. We write $\mathcal{C} = \{(c_i, e_i, K_i) : i \in N\}$. Given the set of supplier bids $\mathcal{B} = \{(p_i, r_i, K_i) : i \in N\}$, the buyer decides which blocks to select. For convenience of presentation we assume without loss of generality that all execution prices are distinct. We label the bids so that $p_1 < \dots < p_n$. Note that both p_i and r_i are prices per unit so the buyer needs to pay an amount $r_i K_i$ to reserve block i . If not all of the block is required when demand occurs, say the amount required is x_i where $x_i < K_i$, then the buyer pays an execution cost of $p_i x_i < p_i K_i$.

Following the backward induction approach, we first consider an optimal policy for the buyer and then analyze the supplier's best response as well as the equilibria for the suppliers.

3.2.2 Buyer's combinatorial problem

After receiving the supplier bids in \mathcal{B} , the buyer makes a two-stage decision: reservation and execution. Provided that the buyer has reserved a set of bids and made the reservation payment, when demand occurs the buyer decides which capacity to execute. Our first observation is that the blocks that are used will be those that have the cheapest execution prices. In other words, the blocks will be used in the increasing order of execution price. Since the sale price is ρ , any bid with an execution price greater than ρ will not be used for sure. If the observed demand is higher than the total reserved amount, there will be lost sales. On the other hand, if the actual demand is lower, the buyer will not execute the excessive capacity.

Knowing the execution decision, the buyer needs to choose a set of bids that will maximize her expected profit given the supplier bids in \mathcal{B} . Suppose the buyer's reservation set is $S = \{j_1, \dots, j_v\}$ where $v \geq 1$ and $j_1 < \dots < j_v$. Hence from our observation above, when demand is known the buyer will first use block j_1 , then j_2 etc., till all the demand is met, or all the blocks have been used.

It is convenient to write Y_i for the total capacity of the first i blocks in S , so

$$Y_i = \sum_{m=1}^i K_{j_m}, \quad \text{for } i = 1, \dots, v, \quad (3.1)$$

and $K_{j_i} = Y_i - Y_{i-1}$ with $Y_0 = 0$. Then Y_v is the total size of the blocks in S and we will write this as $Y(S)$.

Denote by $x_i(t)$ the capacity dispatched from supplier j_i if the actual demand is t . It is easy to see that

$$x_i(t) = \min((t - Y_{i-1})^+, K_{j_i}),$$

where $(z)^+ = \max(z, 0)$. Thus the buyer's expected profit from reserving S is given by

$$\Pi(S) = \sum_{i=1}^v ((\rho - p_{j_i})\mathbb{E}_D[x_i(D)] - r_{j_i}K_{j_i}),$$

where $\mathbb{E}_D[x_i(D)]$ is the expected amount that the buyer uses from supplier j_i . Note that

$$\begin{aligned} \mathbb{E}_D[x_i(D)] &= \mathbb{E}_D[\min((D - Y_{i-1})^+, K_{j_i})] \\ &= [1 - F(Y_i)]K_{j_i} + \int_{Y_{i-1}}^{Y_i} (t - Y_i)f(t)dt \\ &= K_{j_i} - \int_{Y_{i-1}}^{Y_i} F(t)dt. \end{aligned}$$

We rewrite the buyer's expected profit as follows:

$$\Pi(S) = \sum_{i=1}^v \left((\rho - p_{j_i}) \left[K_{j_i} - \int_{Y_{i-1}}^{Y_i} F(t)dt \right] - r_{j_i}K_{j_i} \right).$$

We now define a price function $\tilde{p}_S(t)$ associated with the set of bids S so that $\tilde{p}_S(t)$ is the execution price that applies at quantity t . Hence

$$\tilde{p}_S(t) = p_{j_i}, \quad \text{for } Y_{i-1} < t \leq Y_i$$

with $\tilde{p}_S(t) = 0$ if $t \leq 0$. With this definition we can rewrite the buyer's profit as

$$\Pi(S) = \sum_{i=1}^v (\rho - p_{j_i} - r_{j_i})K_{j_i} - \int_0^{Y(S)} (\rho - \tilde{p}_S(t))F(t)dt.$$

It will be helpful to break down $\Pi(S)$ into the contribution to the buyer's profit made by different subsets. For any set $S \subseteq N$ with $|S| = v$ and $u \geq 0$, we define $\Pi(u, S)$ to be the contribution to the buyer's profit from S if demand up to an amount u is met from some other set of blocks. Then

$$\begin{aligned} \Pi(u, S) &= \sum_{i=1}^v (\rho - p_{j_i} - r_{j_i})K_{j_i} - \int_u^{u+Y(S)} (\rho - \tilde{p}_S(t-u))F(t)dt \\ &= \sum_{i=1}^v (\rho - p_{j_i} - r_{j_i})K_{j_i} - \int_0^{Y(S)} (\rho - \tilde{p}_S(\tau))F(\tau+u)d\tau. \end{aligned}$$

So $\Pi(u, S)$ is non-increasing in u as F is non-decreasing. Note that we have $\Pi(0, S) = \Pi(S)$.

For two subsets $R, S \subseteq N$, we write $R \preceq S$ if R is larger than S , and the blocks in S have higher prices than the corresponding blocks in R . Specifically, we have $R \preceq S$ if $Y(R) \geq Y(S)$ and $\tilde{p}_R(t) \leq \tilde{p}_S(t)$ for $t \leq Y(S)$. We now show a preliminary result that deals with differences in $\Pi(u, S)$ as u varies.

Lemma 3.1. *For any subsets $R, S \subseteq N$, with $R \preceq S$ and $a > b$, we have*

$$\Pi(b, S) - \Pi(a, S) \leq \Pi(b, R) - \Pi(a, R).$$

Proof of Lemma 3.1. We observe that

$$\begin{aligned} \Pi(b, S) - \Pi(a, S) &= \int_0^{Y(S)} (\rho - \tilde{p}_S(\tau))(F(\tau + a) - F(\tau + b))d\tau \\ &\leq \int_0^{Y(S)} (\rho - \tilde{p}_R(\tau))(F(\tau + a) - F(\tau + b))d\tau, \end{aligned}$$

since $\tilde{p}_R(\tau) \leq \tilde{p}_S(\tau)$ and $F(\tau + a) \geq F(\tau + b)$. Hence, provided that ρ is greater than any of the prices (so the integrand is positive) we can use $Y(R) \geq Y(S)$ to obtain

$$\Pi(b, S) - \Pi(a, S) \leq \int_0^{Y(R)} (\rho - \tilde{p}_R(\tau))(F(\tau + a) - F(\tau + b))d\tau = \Pi(b, R) - \Pi(a, R).$$

This completes the proof. \square

Lemma 3.1 shows that the profit gain by shifting the blocks in a set forward from a to b is higher when the execution prices of bids in the set are lower.

Next we will show the submodularity of the set function Π . Submodularity is an important concept in combinatorial optimization. We begin with a definition of it.

Definition 3.2. A set function $\Phi : 2^V \rightarrow R$ is submodular if and only if, for all subsets $A, B \subseteq V$, we have: $\Phi(A) + \Phi(B) \geq \Phi(A \cup B) + \Phi(A \cap B)$.

Note that if a function is submodular and $\Phi(\emptyset) = 0$ (which we always assume), then for any two disjoint sets $A, B \subseteq V$, $\Phi(A \cup B) \leq \Phi(A) + \Phi(B)$. That is, submodularity implies subadditivity. However, the reverse does not hold in general. For a supermodular function, we simply replace \geq with \leq .

There are some variations of the above definition. For example, the set function Φ is submodular if and only if for all $A, B \subseteq V$ and $j, k \in V \setminus A$, we have

$$\Phi(A \cup \{k\}) - \Phi(A) \geq \Phi(A \cup \{j, k\}) - \Phi(A \cup \{j\}),$$

from which we observe that submodular functions present the property of “diminishing return” and may be seen as a discrete analogue to *concave* functions.

Let us introduce the following useful notations: for any $S \subseteq N$ and $1 \leq y \leq x \leq v$,

$$S_x = \{i \in S : i \leq x\}; \quad S^y = \{i \in S : i \geq y\}; \quad S_x^y = S_x \cap S^y. \quad (3.2)$$

We are now in a position to show that the buyer's profit function $\Pi(S)$ is submodular.

Lemma 3.3. *The set function $\Pi(X)$ with $X \subseteq N$ is submodular.*

Proof of Lemma 3.3. To prove submodularity of $\Pi(X)$, it is sufficient to establish the following result: for any $X \subseteq N$ and $\alpha, \beta \in N \setminus X$

$$\Pi(X \cup \{\alpha, \beta\}) + \Pi(X) \leq \Pi(X \cup \{\alpha\}) + \Pi(X \cup \{\beta\}).$$

Without loss of generality let $\alpha < \beta$. Using the notations in (3.2), we can partition X into three subsets X_{j_α} , $X_{j_\beta}^{j_\alpha+1}$ and $X^{j_\beta+1}$ where $j_t = \max\{i \in X : i < t\}$ for $t = \alpha, \beta$. Then, cancelling the common term $\Pi(X_{j_\alpha})$ we have

$$\begin{aligned} \Pi(X \cup \{\alpha, \beta\}) - \Pi(X \cup \{\beta\}) &= \Pi(Y(X_{j_\alpha}), \{\alpha\}) + \Pi(Y(X_{j_\alpha}) + K_\alpha, X_{j_\beta}^{j_\alpha+1} \cup \{\beta\} \cup X^{j_\beta+1}) \\ &\quad - \Pi(Y(X_{j_\alpha}), X_{j_\beta}^{j_\alpha+1} \cup \{\beta\} \cup X^{j_\beta+1}), \\ \Pi(X \cup \{\alpha\}) - \Pi(X) &= \Pi(Y(X_{j_\alpha}), \{\alpha\}) + \Pi(Y(X_{j_\alpha}) + K_\alpha, X_{j_\beta}^{j_\alpha+1} \cup X^{j_\beta+1}) \\ &\quad - \Pi(Y(X_{j_\alpha}), X_{j_\beta}^{j_\alpha+1} \cup X^{j_\beta+1}). \end{aligned}$$

Now observing that $X_{j_\beta}^{j_\alpha+1} \cup \{\beta\} \cup X^{j_\beta+1} \preceq X_{j_\beta}^{j_\alpha+1} \cup X^{j_\beta+1}$ and from Lemma 3.1, we deduce

$$\begin{aligned} &\Pi(Y(X_{j_\alpha}) + K_\alpha, X_{j_\beta}^{j_\alpha+1} \cup X^{j_\beta+1}) - \Pi(Y(X_{j_\alpha}), X_{j_\beta}^{j_\alpha+1} \cup X^{j_\beta+1}) \\ &\leq \Pi(Y(X_{j_\alpha}) + K_\alpha, X_{j_\beta}^{j_\alpha+1} \cup \{\beta\} \cup X^{j_\beta+1}) - \Pi(Y(X_{j_\alpha}), X_{j_\beta}^{j_\alpha+1} \cup \{\beta\} \cup X^{j_\beta+1}). \end{aligned}$$

Thus, we have

$$\Pi(X \cup \{\alpha\}) - \Pi(X) \leq \Pi(X \cup \{\alpha, \beta\}) - \Pi(X \cup \{\beta\}),$$

and the result is established. \square

Lemma 3.3 demonstrates the economic phenomenon of diminishing returns of the buyer's profit as a set function. It implies that the marginal contribution of a supplier to the buyer profit decreases as the buyer's earlier reservation set becomes larger.

Having characterized the property of the buyer's objective function Π , we now maximize Π by choosing a set of blocks in N . This is a combinatorial optimization problem and is formulated as follows:

$$\Pi^*(N) = \max_{S \subseteq N} \Pi(S). \quad (3.3)$$

In general, to maximize a monotonic submodular function is challenging and NP-hard, and it is even harder for a non-monotonic submodular function. However, under the special case with equal-size blocks ($K_i = K$), we are able to propose an efficient algorithm to solve the buyer's problem (3.3) in polynomial time.

3.2.3 The case with equal-size blocks

In the case where all the suppliers have equal-size blocks, we can establish stronger results. Without loss of generality we take $K_i = 1$ so that we can rewrite

$$\Pi(S) = \sum_{i=1}^v ((\rho - p_{j_i})\bar{F}(i) - r_{j_i}),$$

where $\bar{F}(i) = 1 - \int_{i-1}^i F(t)dt$. Since the integral here is the average of F in the range $(i-1, i)$ the \bar{F} function can be seen as a form of average probability that demand exceeds a point in the range $(i-1, i)$. Similarly,

$$\begin{aligned} \Pi(u, S) &= \sum_{i=1}^v (\rho - p_{j_i} - r_{j_i}) + \int_0^{Y(S)} (\rho - \tilde{p}_S(\tau))F(\tau + u)d\tau \\ &= \sum_{i=1}^v ((\rho - p_{j_i})\bar{F}(i + u) - r_{j_i}). \end{aligned}$$

We also write

$$\Delta(u, S) = \Pi(u, S) - \Pi(u + 1, S) = \sum_{i=1}^v (\rho - p_{j_i}) [F(i + u + 1) - F(i + u)],$$

which is the loss from shifting each block in S back by one place. Note from Lemma 3.1 that for any subsets $R, S \subseteq N$, we have $R \preceq S$ if $\Delta(u, S) \geq \Delta(u, R)$. Using the notations in (3.2), we obtain

$$\Pi(X) - \Pi(X \cup \{i\}) = \Delta(|X_i|, X^i) - \Pi(|X_i|, \{i\}).$$

We now consider a version of the buyer's problem where the buyer is restricted to choose k blocks where $k = 1, \dots, n$. Denote by $N(k)$ the buyer's optimal choice set when restricted to choose k blocks. That is,

$$N(k) = \arg \max_{S \subseteq N, |S|=k} \Pi(S), \quad (3.4)$$

so the buyer's optimal profit in this case is given by $\Pi(N(k))$. We first show a property of $N(k)$.

Lemma 3.4. For $k = 1, \dots, n - 1$, the optimal buyer's choice when restricted to k blocks can be chosen as a subset of the optimal buyer's choice when restricted to $k + 1$ blocks. That is, $N(k) \subset N(k + 1)$.

Proof of Lemma 3.4. Let $U = N(k) = \{j_1, \dots, j_k\}$ where $j_1 < \dots < j_k$, and let $V = N(k + 1) = \{h_1, \dots, h_{k+1}\}$ where $h_1 < \dots < h_{k+1}$. We write $U_+(m) = \{j_{m+1}, \dots, j_k\}$, $U_-(m) = \{j_1, \dots, j_m\}$, $V_+(m) = \{h_{m+1}, \dots, h_{k+1}\}$, and $V_-(m) = \{h_1, \dots, h_m\}$. Define q as the minimum value such that $V_+(q) \preceq U_+(q - 1)$, thus we must have $j_{q-1} < h_q$. In the case that $j_k < h_{k+1}$ we will take $q = k + 1$. We will show that $U \cup \{h_q\}$ is just as good a choice of blocks for the buyer as V .

We first show that $\tilde{V} = V_-(q) \cup U_+(q - 1)$ is as good a choice as V . Since U is the best choice with k elements we cannot improve by using the set $U_-(q - 1) \cup V_+(q)$ which also has k elements. Hence

$$\Pi(q - 1, V_+(q)) \leq \Pi(q - 1, U_+(q - 1)).$$

But, as $V_+(q) \preceq U_+(q - 1)$ we know that $\Delta(q - 1, V_+(q)) \geq \Delta(q - 1, U_+(q - 1))$ and so

$$\begin{aligned} \Pi(q, U_+(q - 1)) &= \Pi(q - 1, U_+(q - 1)) - \Delta(q - 1, U_+(q - 1)) \\ &\geq \Pi(q - 1, V_+(q)) - \Delta(q - 1, V_+(q)) = \Pi(q, V_+(q)). \end{aligned}$$

This is precisely the inequality we need to show that $\Pi(\tilde{V}) \geq \Pi(V)$.

Second, we show that $\Pi(U \cup \{h_q\}) \geq \Pi(\tilde{V})$. To do this we need to replace the first $q - 1$ elements $V_-(q - 1)$ of \tilde{V} with $U_-(q - 1)$. Since U is the best choice with k elements we cannot improve by using the set $V_-(q - 1) \cup U_+(q - 1)$ which also has k elements. Comparing the Π values for these two sets we see they differ only on the first $q - 1$ elements. Thus $\Pi(U_-(q - 1)) \geq \Pi(V_-(q - 1))$ and so

$$\Pi(U_-(q - 1) \cup \{h_q\} \cup U_+(q - 1)) \geq \Pi(V_-(q - 1) \cup \{h_q\} \cup U_+(q - 1)) = \Pi(\tilde{V}).$$

Combining the above two steps, we establish the result required. \square

We next show a property of $\Pi(N(k))$ which is a discrete analogue of concavity.

Lemma 3.5. For $k = 1, \dots, n - 1$, we have

$$\Pi(N(k)) - \Pi(N(k - 1)) \geq \Pi(N(k + 1)) - \Pi(N(k)).$$

Proof of Lemma 3.5. Denote by α and β the k th and $(k + 1)$ th reserved blocks, respectively. From Lemma 3.4, we know $N(k) = N(k - 1) \cup \{\alpha\}$ and $N(k + 1) =$

$N(k-1) \cup \{\alpha, \beta\}$. Then we obtain

$$\begin{aligned}
& \Pi(N(k)) - \Pi(N(k-1)) - \Pi(N(k+1)) + \Pi(N(k)) \\
= & \Pi(N(k-1) \cup \{\alpha\}) - \Pi(N(k-1)) - \Pi(N(k-1) \cup \{\alpha, \beta\}) + \Pi(N(k-1) \cup \{\alpha\}) \\
\geq & \Pi(N(k-1) \cup \{\beta\}) - \Pi(N(k-1)) - \Pi(N(k-1) \cup \{\alpha, \beta\}) + \Pi(N(k-1) \cup \{\alpha\}) \\
\geq & 0,
\end{aligned}$$

where the first inequality follows from $\Pi(N(k-1) \cup \{\alpha\}) \geq \Pi(N(k-1) \cup \{\beta\})$ because α is the best choice for the k th block, and the second inequality follows from the submodularity of $\Pi(S)$ in Lemma 3.3. This completes the proof. \square

The results in Lemma 3.4 and Lemma 3.5 allow us to solve the buyer's problem using a dynamic programming approach. We solve the problem (3.4) recursively, starting with $k = 1$ and then increasing k one at a time. At each stage there are less than n options to consider as we add each of the possible blocks into the reservation set one at a time. Once we stop making an improvement by adding another block then we have found an optimal solution. The question of finding an efficient computational approach for a similar problem is also considered by Schummer and Vohra (2003) who show that it can be solved using a linear program (either a transportation problem or a shortest path problem). In terms of time complexity, our dynamic programming approach is more efficient.

We now give an example to illustrate how the optimal buyer choice set evolves as k increases.

Example 3.1. *The buyer's demand follows a discrete uniform distribution with $\Pr(D = i) = 1/10, i = 1, \dots, 10$. The retail price is $\rho = 10$. The bids offered by the suppliers are given in Table 3.1.*

TABLE 3.1: The bids submitted by suppliers

	Bid 1	Bid 2	Bid 3	Bid 4	Bid 5	Bid 6	Bid 7	Bid 8	Bid 9	Bid 10
p_i	3	3.2	3.3	3.7	3.8	4	4.2	4.5	4.7	5
r_i	4	3.8	3.7	3.7	3.5	3.4	3.2	2.5	2	1

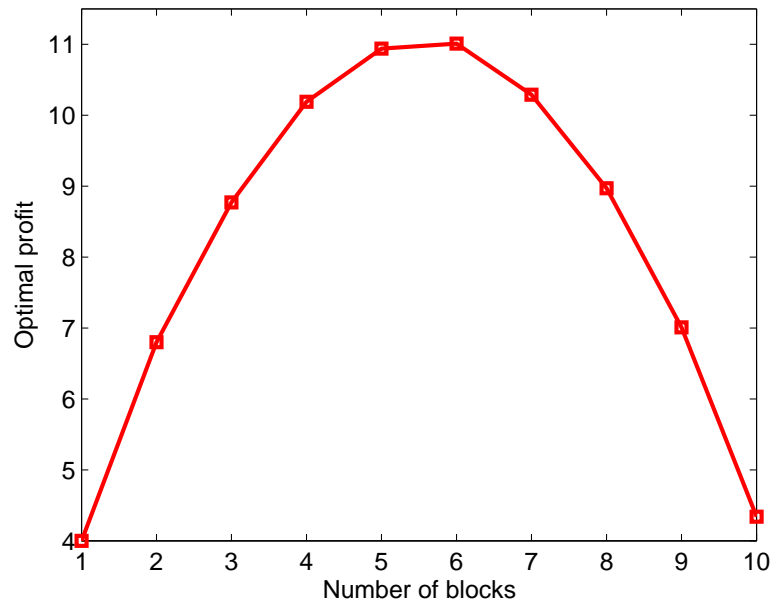
Table 3.2 lists the buyer's reservation sets as k increases from 1 to 10. The number in each cell indicates whether the bid is chosen by the buyer (1 if the bid is chosen; 0 otherwise). As we can see, a block which is chosen when the buyer is restricted to choose k blocks will still be chosen when restricted to choose $k + 1$ blocks.

We also plot in Figure 3.2 the optimal buyer profit $\Pi(N(k))$ against k . We observe that $\Pi(N(k))$ is a discrete analogue of some concave function. The buyer profit $\Pi(N(k))$ increases as k increases until when $k = 6$. Therefore, the optimal number of reserved

TABLE 3.2: The optimal buyer choice sets

	Bid 1	Bid 2	Bid 3	Bid 4	Bid 5	Bid 6	Bid 7	Bid 8	Bid 9	Bid 10
1	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	1	1	1
4	0	0	1	0	0	0	0	1	1	1
5	0	1	1	0	0	0	0	1	1	1
6	1	1	1	0	0	0	0	1	1	1
7	1	1	1	0	0	0	1	1	1	1
8	1	1	1	0	1	0	1	1	1	1
9	1	1	1	0	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1

blocks is 6, and the optimal reservation set includes the first three and the last three blocks as shown in Table 3.2.

FIGURE 3.2: The optimal buyer profit as a function of k

Having established that the buyer's profit function $\Pi(X)$ is submodular, we next show that, with equal-size blocks, this property is inherited by the set function $\Pi^*(X)$ in (3.3) which takes the best buyer profit given a set of available blocks X .

Lemma 3.6. *When blocks are of equal-size, then $\Pi^*(X)$ is submodular for $X \subseteq N$.*

Proof of Lemma 3.6. See the Appendix. □

We now demonstrate with an example that this submodularity property does not hold when supplier blocks have different sizes.

Example 3.2. Suppose that demand takes only the single value 10 and the selling price ρ is 50. We have 5 blocks available $\{a, b, c, g, h\}$ with (p_i, r_i, K_i) triples as follows: $a = b = c = (1, 10, 4)$ and $g = h = (1, 7, 5)$. Then $\Pi(\{a, b\}) = 49 \times 8 - (10 \times 8) = 312$, $\Pi(\{a, g\}) = 49 \times 9 - (10 \times 4) - (7 \times 5) = 366$, $\Pi(\{a, b, c\}) = 49 \times 10 - (10 \times 12) = 370$, $\Pi(\{a, b, g\}) = 49 \times 10 - (10 \times 8) - (7 \times 5) = 375$, $\Pi(\{g, h\}) = 49 \times 10 - (10 \times 7) = 420$. Hence we see that

$$\begin{aligned}\Pi^*(\{a, b, c\}) &= \Pi(\{a, b, c\}) = 370, \\ \Pi^*(\{a, b, c, g\}) &= \Pi^*(\{a, b, c, h\}) = \Pi(\{a, b, g\}) = 375, \\ \Pi^*(\{a, b, c, g, h\}) &= \Pi(\{g, h\}) = 420.\end{aligned}$$

Thus $\Pi^*(\{a, b, c\}) + \Pi^*(\{a, b, c, g, h\}) > \Pi^*(\{a, b, c, g\}) + \Pi^*(\{a, b, c, h\})$ contradicting submodularity.

Based on the submodularity in Lemma 3.6, we now provide a further property of the set function $\Pi^*(X)$. This result will be used for the equilibrium analysis in Section 3.4.

Corollary 3.7. For any set $A \subseteq S$, we have

$$\sum_{i \in A} (\Pi^*(S) - \Pi^*(S \setminus \{i\})) \leq \Pi^*(S) - \Pi^*(S \setminus A). \quad (3.5)$$

Proof of Corollary 3.7. We prove it by induction. Note that it is trivial when $|A| \leq 1$. If $|A| = 2$, let $A = \{j, k\}$. From Lemma 3.3, we obtain

$$\Pi^*(S \setminus \{j\}) + \Pi^*(S \setminus \{k\}) \geq \Pi^*(S) + \Pi^*(S \setminus \{j, k\}),$$

which can be rearranged to show the result required.

Suppose (3.5) holds for a given subset $B \subseteq S$ with $|B| > 2$, that is,

$$\sum_{i \in B} (\Pi^*(S) - \Pi^*(S \setminus \{i\})) \leq \Pi^*(S) - \Pi^*(S \setminus B). \quad (3.6)$$

With any block $l \in S \setminus B$, we have

$$\begin{aligned}\sum_{i \in B \cup \{l\}} (\Pi^*(S) - \Pi^*(S \setminus \{i\})) &= \sum_{i \in B} (\Pi^*(S) - \Pi^*(S \setminus \{i\})) + \Pi^*(S) - \Pi^*(S \setminus \{l\}) \\ &\leq \Pi^*(S) - \Pi^*(S \setminus B) + \Pi^*(S) - \Pi^*(S \setminus \{l\}),\end{aligned}$$

where the inequality follows from (3.6). Moreover, from Lemma 3.3 we obtain,

$$\Pi^*(S \setminus B) + \Pi^*(S \setminus \{l\}) \geq \Pi^*(S) + \Pi^*(S \setminus (B \cup \{l\})).$$

Combining the above two inequalities yields,

$$\sum_{i \in B \cup \{l\}} (\Pi^*(S) - \Pi^*(S \setminus \{i\})) \leq \Pi^*(S) - \Pi^*(S \setminus (B \cup \{l\})).$$

Hence, by induction, we complete the proof. \square

Corollary 3.7 shows that the overall contribution of a set is larger than the sum of each bid's marginal contribution to the optimal buyer's profit.

3.3 Suppliers' Best Responses

In this section, we examine each supplier's best response problem. Denote by $\mathcal{B}_{-i} = \mathcal{B} \setminus \{(p_i, r_i, K_i)\}$ the set of bids submitted by the suppliers except i . Let N_{-i}^* be the optimal buyer selection when block i is not available, so

$$N_{-i}^* = \arg \max_{S \subseteq N \setminus \{i\}} \Pi(S).$$

We look at how supplier i responds to \mathcal{B}_{-i} by choosing a reservation price r_i and an execution price p_i . Note that only when block i is selected by the buyer, can supplier i make profit. Suppose that the supplier i 's offer is (p_i, r_i) , and given this offer (as well as \mathcal{B}_{-i}), denote by $X \cup \{i\}$ the buyer's optimal set. Let u be the ranking of block i in $X \cup \{i\}$. Similar to our previous notations, we denote by X_u the subset of bids in X with execution prices lower than p_i and by X^u the subset of bids in X with execution prices higher than p_i . So the profit for the buyer when bid i is included is,

$$\begin{aligned} \Pi(X \cup \{i\}) &= \Pi(X_u) + \Pi(Y(X_u) + K_i, X^u) + \Pi(Y(X_u), \{i\}) \\ &= \Pi(X_u) + \Pi(Y(X_u) + K_i, X^u) + (\rho - p_i) \left[K_i - \int_0^{K_i} F(\tau + Y(X_u)) d\tau \right] - r_i K_i. \end{aligned}$$

The supplier i 's profit from offering (p_i, r_i) is

$$\pi_i(p_i, r_i) = (p_i - c_i) \left[K_i - \int_0^{K_i} F(\tau + Y(X_u)) d\tau \right] + (r_i - e_i). \quad (3.7)$$

Given \mathcal{B}_{-i} , the supplier i 's best response problem is to maximize his profit by choosing (p_i, r_i) ,

$$\max_{(p_i, r_i)} \pi_i(p_i, r_i), \quad (3.8)$$

subject to

$$X \cup \{i\} = \arg \max_{S \subseteq N} \Pi(S), \quad (3.9)$$

where the constraint (3.9) is to ensure that the buyer selects block i . We now show an optimal strategy for supplier i .

Theorem 3.8. *Given \mathcal{B}_{-i} , the supremum of the expected profit for supplier i is achieved when $p_i = c_i$ and is constant for p_i in an interval containing c_i .*

Proof of Theorem 3.8. Notice that for any given value of p_i , it is optimal for supplier i to set r_i as high as possible, subject to the proviso that bid i is still chosen by the buyer. Alternatively, for any p_i there will be a maximum possible value of r_i that depends on p_i . This maximum value for r_i gives the maximum expected profit for supplier i , and is such that the buyer is indifferent between including block i and excluding it (here undercutting behaviour occurs, but for convenience of exposition we shall use equality). So we have $\Pi(X \cup \{i\}) = \Pi(N_{-i}^*)$, from which we deduce,

$$r_i K_i = \Pi(X_u) + \Pi(Y(X_u) + K_i, X^u) + (\rho - p_i) \left[K_i - \int_0^{K_i} F(\tau + Y(X_u)) d\tau \right] - \Pi(N_{-i}^*).$$

Plugging the above equation into π_i we cancel out r_i to obtain

$$\pi_i = \Pi(X_u) + \Pi(Y(X_u) + K_i, X^u) + (\rho - c_i) \left[K_i - \int_0^{K_i} F(\tau + Y(X_u)) d\tau \right] - \Pi(N_{-i}^*) - e_i K_i.$$

Since this depends on the choice of p_i only through the set X which can take only finitely many values, π_i will be piecewise constant as a function of p_i . The next step is to show that c_i is in an interval that achieves the maximum supplier profit.

We write $\bar{\mathcal{B}} = \mathcal{B}_{-i} \cup \{(c_i, d_i, K_i)\}$ and let $\Pi_{\bar{\mathcal{B}}}(S)$ be the buyer's profit when choosing the bids in S under $\bar{\mathcal{B}}$. Let L^* be the optimal buyer selection under $\bar{\mathcal{B}}$, thus,

$$L^* = \arg \max_{S \subseteq N} \Pi_{\bar{\mathcal{B}}}(S). \quad (3.10)$$

We will show that supplier i can do no better than set $p_i^* = c_i$ and $r_i^* = e_i + (\Pi_{\bar{\mathcal{B}}}(L^*) - \Pi(N_{-i}^*)) / K_i$. With this offer supplier i makes profit only from the reservation component of the offer, thus

$$\pi_i(p_i^*, r_i^*) = K_i(r_i^* - e_i) = \Pi_{\bar{\mathcal{B}}}(L^*) - \Pi(N_{-i}^*).$$

Suppose a different offer $(p'_i, r'_i) \neq (p_i^*, r_i^*)$ gives a higher profit to supplier i . We write $\mathcal{B}' = \mathcal{B}_{-i} \cup \{(p'_i, r'_i, K_i)\}$. With this offer, denote the buyer's optimal choice set by L' . Thus, we have

$$\pi_i(p'_i, r'_i) > \Pi_{\bar{\mathcal{B}}}(L^*) - \Pi(N_{-i}^*),$$

and the buyer's optimal profit from selecting L' is no less than that from selecting N_{-i}^* ,

$$\Pi_{\mathcal{B}'}(L') \geq \Pi(N_{-i}^*).$$

Combining the above two inequalities yields

$$\pi_i(p'_i, r'_i) + \Pi_{\mathcal{B}'}(L') > \Pi_{\bar{\mathcal{B}}}(L^*).$$

However, the optimality of L^* implies that $\Pi_{\bar{\mathcal{B}}}(L^*) \geq \pi_i(p'_i, r'_i) + \Pi_{\mathcal{B}'}(L')$. Therefore, we arrive at a contradiction, implying the result required. \square

Theorem 3.8 shows that it is optimal for suppliers to set execution prices to be execution costs. This result mirrors what have been found in other circumstances (Wu et al., 2002; Wu and Kleindorfer, 2005), and significantly facilitates the equilibrium analysis as we will see in Section 3.4.

3.3.1 Two extensions

In this subsection, we discuss two extensions of our basic model.

- (1) **Partial reservation:** Each supplier owns a single block and the blocks can be of different sizes. Every supplier chooses an execution price and a reservation price for his capacity block and the buyer is allowed to reserve any portion of a block. Compared with the baseline model, the difference in this extension is that the buyer is not restricted to reserve a block all or none.
- (2) **Multiple blocks with a common owner:** Each supplier owns multiple unit-blocks and can choose different prices for (possibly) different unit-blocks. The buyer can freely choose the offered blocks.

One of the key results obtained for the baseline model is that a best response for supplier i is to set $p_i = c_i$ and the suppliers make profits only through reservation payments. In the above two extensions, however, this result does not hold in general, as we will demonstrate with the following example.

Example 3.3. *Suppose the other bids are $\mathcal{B}_{-i} = \{(1, 3, 1); (4, 1, 1)\}$ where the first number in each triple is the execution price, the second is the reservation price, and the last is the block size. The demand distribution is given by $\Pr(D = k) = 1/5$ for $k = 1, \dots, 5$. The retail price is $\rho = 15$. Supplier i has a block of size 3 and $c_i = e_i = 2$, so we can write $i = (2, 2, 3)$ for block i . We also write $\bar{\mathcal{B}} = \mathcal{B}_{-i} \cup \{(2, 2, 3)\}$. Under $\bar{\mathcal{B}}$, we can show that the buyer will choose all the three blocks which include five units, and the buyer's optimal profit is*

$$\Pi_{\bar{\mathcal{B}}}^* = \left[\frac{5}{5}(15 - 1) - 3 \right] + \sum_{k=2}^4 \left[\frac{k}{5}(15 - 2) - 2 \right] + \left[\frac{1}{5}(15 - 4) - 1 \right] = 29.6.$$

If supplier i is unavailable, the buyer will choose both blocks in \mathcal{B}_{-i} and make a profit of

$$\Pi_{\mathcal{B}_{-i}}^* = \frac{5}{5}(15 - 1) - 3 + \frac{4}{5}(15 - 4) - 1 = 18.8.$$

So we could expect that the maximum profit supplier i can achieve is $29.6 - 18.8 = 10.8$. In fact, in the baseline model where the buyer is restricted to reserve the whole block, the supplier i 's optimal profit is indeed 10.8. However, the optimal profit of supplier i is less than 10.8 in the two extended models.

First, we consider the extension of partial reservation. We show that it is suboptimal for supplier i to choose $p_i^* = 2$. Since the buyer can decide how much to reserve from supplier i , there are three cases regarding the buyer's choice.

- If the buyer's optimal selection includes only one unit from supplier i , then an optimal strategy for supplier i is $p_i^* = 2$ and $r_i^* = 8.2$, which indeed leads to the buyer to reserve only one unit from supplier i (plus the bids in \mathcal{B}_{-i}) and make a profit of 18.8. The supplier i 's profit is

$$\pi_i = \frac{4}{5}(p_i^* - c_i) + r_i^* - e_i = 6.2.$$

- If the buyer chooses two units from supplier i , the optimal solution is $p_i^* = 15$ and $r_i^* = 0$. In this case, the supplier i 's profit is

$$\pi_i = \left(\frac{4}{5} + \frac{3}{5}\right)(p_i^* - c_i) + 2(r_i^* - e_i) = 9.$$

- If the buyer chooses three units from supplier i , the optimal solution is again $p_i^* = 15$ and $r_i^* = 0$. The supplier i 's profit is

$$\pi_i = \left(\frac{4}{5} + \frac{3}{5} + \frac{2}{5}\right)(p_i^* - c_i) + 3(r_i^* - e_i) = 9.6.$$

Comparing the above three cases, we find the optimal strategy is to set $p_i^* = 15$ and $r_i^* = 0$. Note that the optimal execution price is not within the interval of $[1, 4]$. In this case, the buyer reserves three units from supplier i and makes a profit of 18.8, and supplier i makes a profit of 9.6.

If p_i is forced to take the value of 2, then it is optimal for supplier i to set $r_i^* = 5.6$. In this case the buyer will reserve just two units and the supplier i 's expected profit is

$$\pi_i = \frac{4}{5}(p_i^* - c_i) + r_i^* - e_i + \frac{3}{5}(p_i^* - c_i) + r_i^* - e_i = 7.2.$$

If we impose the condition of $1 \leq p_i \leq 4$, then it is optimal for supplier i to set $p_i^* = 4$ and $r_i^* = 4.4$. In this case the buyer will reserve two units and the supplier i 's profit is

$$\pi_i = \frac{4}{5}(p_i^* - c_i) + r_i^* - e_i + \frac{3}{5}(p_i^* - c_i) + r_i^* - e_i = 7.6.$$

Therefore, we have established that it is not optimal for supplier i to set $p_i^* = 2$, or any value in the range $[1, 4]$.

Second, we consider the extension where a single supplier owns multiple unit-blocks that can be offered at different prices. We can think of supplier i owning three unit-blocks with identical costs. In contrast with the partial reservation extension, supplier i can choose different prices for different blocks. To differentiate these three unit-blocks, we denote the supplier i 's bids by $\{(p_{i1}, r_{i1}), (p_{i2}, r_{i2}), (p_{i3}, r_{i3})\}$.

We find the optimal solution is: $p_{i1}^* \in [1, 4]$, $r_{i1}^* = 9.8 - 0.8p_{i1}^*$, and $p_{i2}^* = p_{i3}^* = 15$, $r_{i2}^* = r_{i3}^* = 0$. Given these bids, the buyer will reserve three unit-blocks from supplier i (plus the bids in \mathcal{B}_{-i}). The supplier i 's profit is

$$\begin{aligned} \pi_i &= \frac{4}{5}(p_{i1}^* - c_i) + r_{i1}^* - e_i + \frac{2}{5}(p_{i2}^* - c_i) + r_{i2}^* - e_i + \frac{1}{5}(p_{i3}^* - c_i) + r_{i3}^* - e_i \\ &= \frac{4}{5}(p_{i1}^* - 2) + 9.8 - 0.8p_{i1}^* - 2 + \frac{2}{5}(15 - 2) + 0 - 2 + \frac{1}{5}(15 - 2) + 0 - 2 \\ &= 10. \end{aligned}$$

This shows that it is optimal for supplier i to set different prices for different blocks, and hence the result of $p = c$ does not hold in this setting.

If we impose the constraint of $p_{i1} = p_{i2} = p_{i3} = 2$, the optimal solution is $r_{i1}^* = r_{i2}^* = r_{i3}^* = 5.6$. The buyer will reserve two unit-blocks and the supplier i 's profit is 7.2 as we show in Extension (1). If we impose the constraint of $1 \leq p_{ij} \leq 4$ where $j = 1, 2, 3$, then the optimal solution for supplier i is: $p_{i1}^* = p_{i2}^* = p_{i3}^* = 4$ and $r_{i1}^* = r_{i2}^* = r_{i3}^* = 4.4$. In this case, the buyer will reserve two unit-blocks and the supplier i 's profit is 7.6 as shown in Extension (1).

3.4 Equilibrium Analysis

Having established the best response for each supplier we are now in a position to characterize the Nash equilibrium for the suppliers. For the case where suppliers have equal-size blocks, we are able to fully characterize the equilibria for suppliers. When the supplier blocks are of different sizes, we propose an algorithm to find an equilibrium.

Throughout the following analysis, we will slightly violate the earlier notations. We use the notation $\Pi_{\mathcal{B}}(S)$ for the buyer's profit when supplier offers are given by \mathcal{B} and the buyer selects the set $S \subseteq N$. Similarly, $\Pi_{\mathcal{B}}^*(S)$ is the maximum profit achieved from a subset of S (i.e. $\Pi_{\mathcal{B}}^*(S) = \max_{X \subseteq S} \Pi_{\mathcal{B}}(X)$). We will also write $S(\mathcal{B})^*$ and $S(\mathcal{B})_{-i}^*$

for the optimal buyer choices from amongst the set S and $S \setminus \{i\}$, respectively, assuming that supplier offers are given by \mathcal{B} .

As we show for each supplier's best response, the choice of bid at optimal solutions is made in such a way that there is no unique optimum for the buyer. Moreover, because supplier profits will depend on buyer choices, it is important to have a definite algorithm to break ties.

Assumption 3.1. *We assume that the costs in \mathcal{C} are chosen such that $N(\mathcal{C})^*$ is unique under \mathcal{C} , where $N(\mathcal{C})^* = \arg \max_{X \subseteq N} \Pi_{\mathcal{C}}(X)$. There are some given positive weights, $\zeta_1, \zeta_2, \dots, \zeta_n$, that are only used to break ties, with $\zeta_i \neq \zeta_j$, for $i \neq j$, and with $\zeta_i > n + \zeta_j$ for $i \in N(\mathcal{C})^*$ and $j \notin N(\mathcal{C})^*$.*

Assumption 3.1 states that in the event of a tie the buyer chooses the set of suppliers X to maximize the sum of the weights of selected suppliers, $\sum_{i \in X} \zeta_i$. Thus, when it makes no difference to the buyer profit, the buyer will always prefer to choose an additional block, and to choose a set of blocks with more elements from $N(\mathcal{C})^*$.

We first prove a key characteristic of an equilibrium set of bids: at equilibrium the optimal buyer profit remains the same when any individual block is removed.

Lemma 3.9. *Suppose $\mathcal{B} = \{(p_j^*, r_j^*, K_i) : j \in N\}$ is a Nash equilibrium, then for $i = 1, 2, \dots, n$, we have*

$$\Pi_{\mathcal{B}}^*(N) = \Pi_{\mathcal{B}}^*(N \setminus \{i\}). \quad (3.11)$$

Proof of Lemma 3.9. From optimality we must have $\Pi_{\mathcal{B}}^*(N) \geq \Pi_{\mathcal{B}}^*(N \setminus \{i\})$, so we suppose (3.11) does not hold and there exists some i with $\delta := \Pi_{\mathcal{B}}^*(N) - \Pi_{\mathcal{B}}^*(N \setminus \{i\}) > 0$. Note that this implies that block i is in the buyer selection under \mathcal{B} . We now consider a new bid for supplier i , with a higher reservation price: $(p_i^*, r_i^* + \delta/(2K_i))$. The buyer now makes a choice from the set $\mathcal{B}' = \mathcal{B} \setminus \{(p_i^*, r_i^*, K_i)\} \cup \{(p_i^*, r_i^* + \delta/(2K_i))\}$.

If the buyer chooses $N(\mathcal{B})^*$ then the profit to the buyer is reduced by $\delta/2$ from $\Pi_{\mathcal{B}}^*(N)$, and if the buyer chooses a set excluding i then its profit is the same as before. Hence

$$\Pi_{\mathcal{B}'}(N(\mathcal{B})^*) = \Pi_{\mathcal{B}}^*(N) - \delta/2 > \Pi_{\mathcal{B}}^*(N \setminus \{i\}) = \Pi_{\mathcal{B}'}(N \setminus \{i\})$$

We also know $\Pi_{\mathcal{B}'}^*(N) \geq \Pi_{\mathcal{B}'}(N(\mathcal{B})^*)$. Thus, we have $\Pi_{\mathcal{B}'}^*(N) > \Pi_{\mathcal{B}}^*(N \setminus \{i\})$, which implies that after the change the optimal buyer selection still includes i . Hence the change in offer will increase the profit for supplier i , which contradicts the fact that \mathcal{B} is an equilibrium. This establishes the result we require. \square

Lemma 3.9 shows that the buyer is indifferent between choosing a block and not choosing it. According to our tie-breaking rule, the buyer will reserve N^* instead of N_{-i}^* in equilibrium. The intuition of this property is the following: if the buyer's profit from reserving N^* is greater than that from reserving N_{-i}^* , supplier i can always increase the

reservation price a little but makes sure that the buyer still chooses his bid. Therefore, in equilibrium the marginal profit of each bid to the buyer must be zero.

3.4.1 The case with equal-size blocks

In this section, we consider the problem of finding a Nash equilibrium for the problem where n suppliers each offer an equal-size block. Without loss of generality we take $K_i = 1$, for $i = 1, \dots, n$. For this case, we are able to fully characterize the equilibrium for the suppliers, thanks to the submodularity property of $\Pi^*(X)$ as shown in Lemma 3.6. From Theorem 3.8 we can assume that each supplier chooses an execution price $p_i = c_i$.

We begin by showing that the property of an equilibrium is inherited by the solution in which unselected suppliers reduce reservation prices to reservations costs. Also this does not change the set selected by the buyer.

Lemma 3.10. *Suppose $\mathcal{B} = \{(c_i, r_i^*) : i = 1, \dots, n\}$ is an equilibrium, and for some set S that are not selected by the buyer, we let*

$$\hat{\mathcal{B}} = \mathcal{B} \setminus \{(c_i, r_i^*) : i \in S\} \cup \{(c_i, e_i) : i \in S\},$$

then $\hat{\mathcal{B}}$ is also an equilibrium and $\Pi_{\hat{\mathcal{B}}}^(N) = \Pi_{\mathcal{B}}^*(N)$.*

Proof of Lemma 3.10. We establish this by induction. As usual, denote by $N(\mathcal{B})^*$ the buyer's optimal choice given the set of bids \mathcal{B} . Consider any $j \in N \setminus N(\mathcal{B})^*$, and let $\mathcal{B}_j = \mathcal{B} \setminus \{(c_j, r_j^*)\} \cup \{(c_j, e_j)\}$. We first show \mathcal{B}_j is an equilibrium and then establish our result inductively.

It is trivially true if $r_j^* = e_j$. When $r_j^* > e_j$ we have $\Pi_{\mathcal{B}_j}^*(N) \geq \Pi_{\mathcal{B}_j}^*(N \setminus \{j\})$ and $\Pi_{\mathcal{B}_j}^*(N \setminus \{j\}) = \Pi_{\mathcal{B}}^*(N \setminus \{j\}) = \Pi_{\mathcal{B}}^*(N)$, and so $\Pi_{\mathcal{B}_j}^*(N) \geq \Pi_{\mathcal{B}}^*(N)$. On the other hand if $\Pi_{\mathcal{B}_j}^*(N) > \Pi_{\mathcal{B}}^*(N)$ then supplier j must be selected by the buyer under \mathcal{B}_j , from which we deduce that supplier j can make non-zero profit by decreasing r_j^* by ϵ , where $0 < \epsilon < r_j^* - e_j$. This contradicts that \mathcal{B} is an equilibrium, and hence we have established that $\Pi_{\mathcal{B}_j}^*(N) = \Pi_{\mathcal{B}}^*(N)$. Next we show no supplier has an incentive to change his bid given in \mathcal{B}_j . We consider three cases:

- (a) First, supplier j has no incentive to change his bid. Under \mathcal{B}_j supplier j makes no money, but it will not be selected if its price increases.
- (b) Next, consider some $k \neq j$ with $k \in N \setminus N(\mathcal{B})^*$. Since both j and k are not in $N(\mathcal{B})^*$ we have $\Pi_{\mathcal{B}}^*(N \setminus \{j, k\}) = \Pi_{\mathcal{B}}^*(N)$, and so $\Pi_{\mathcal{B}_j}^*(N \setminus \{j, k\}) = \Pi_{\mathcal{B}_j}^*(N)$. Now submodularity implies that

$$\Pi_{\mathcal{B}_j}^*(N \setminus \{j\}) + \Pi_{\mathcal{B}_j}^*(N \setminus \{k\}) \geq \Pi_{\mathcal{B}_j}^*(N \setminus \{j, k\}) + \Pi_{\mathcal{B}_j}^*(N),$$

which together with $\Pi_{\mathcal{B}_j}^*(N \setminus \{j\}) = \Pi_{\mathcal{B}}^*(N \setminus \{j\}) = \Pi_{\mathcal{B}}^*(N) = \Pi_{\mathcal{B}_j}^*(N)$ yields $\Pi_{\mathcal{B}_j}^*(N \setminus \{k\}) \geq \Pi_{\mathcal{B}_j}^*(N)$. We also know that $\Pi_{\mathcal{B}_j}^*(N \setminus \{k\}) \leq \Pi_{\mathcal{B}_j}^*(N)$. Hence, we have $\Pi_{\mathcal{B}_j}^*(N \setminus \{k\}) = \Pi_{\mathcal{B}_j}^*(N)$. Now we can use this to establish $\Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{j\}) = \Pi_{\mathcal{B}_k}^*(N \setminus \{j\}) = \Pi_{\mathcal{B}}^*(N) = \Pi_{\mathcal{B}}^*(N)$, where $\mathcal{B}_{j,k} = \mathcal{B}_j \setminus \{(c_k, r_k^*) \cup \{(c_k, e_k)\}$. And similarly, $\Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{k\}) = \Pi_{\mathcal{B}_j}^*(N) = \Pi_{\mathcal{B}}^*(N)$ and clearly $\Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{j, k\}) = \Pi_{\mathcal{B}}^*(N)$. Now submodularity implies

$$\Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{j\}) + \Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{k\}) \geq \Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{j, k\}) + \Pi_{\mathcal{B}_{j,k}}^*(N),$$

from which we deduce $\Pi_{\mathcal{B}_{j,k}}^*(N) \leq \Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{j, k\})$. However, we know $\Pi_{\mathcal{B}_{j,k}}^*(N) \geq \Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{j, k\})$. Thus, they must be equal. If supplier k wishes to make a positive profit under \mathcal{B}_j it must ensure it is selected. But at any price higher than e_k the buyer profit is, for any subset $S \subseteq N$,

$$\Pi_{\mathcal{B}_j}(\{k\} \cup S) < \Pi_{\mathcal{B}_{j,k}}(\{k\} \cup S) \leq \Pi_{\mathcal{B}_{j,k}}^*(N) = \Pi_{\mathcal{B}_{j,k}}^*(N \setminus \{j, k\}) = \Pi_{\mathcal{B}_j}^*(N \setminus \{j, k\})$$

and so k will not be selected.

- (c) Finally, we show no supplier $m \in N(\mathcal{B})^*$ has an incentive to change his bid in \mathcal{B}_j . Now

$$\Pi_{\mathcal{B}_j}^*(N) = \Pi_{\mathcal{B}}^*(N) = \Pi_{\mathcal{B}}^*(N \setminus \{m\}) \leq \Pi_{\mathcal{B}_j}^*(N \setminus \{m\}),$$

where the last inequality follows from the fact that for any particular selection by the buyer, the buyer profit can only improve in moving from \mathcal{B} to \mathcal{B}_j . Hence no supplier $m \in N(\mathcal{B})^*$ is able to improve his profit by increasing r_m^* since this reduces the buyer's profit if m is selected and the buyer will find it preferable to choose the optimal solution excluding m .

Thus we have shown that \mathcal{B}_j forms an equilibrium, and hence by induction $\hat{\mathcal{B}}$ is also an equilibrium. We have also established inductively that $\Pi_{\hat{\mathcal{B}}}^*(N) = \Pi_{\mathcal{B}}^*(N)$. \square

We now show there exists an equilibrium where each supplier adds a margin to the reservation cost which equals its marginal contribution, i.e. $\Pi_{\mathcal{C}}^*(N) - \Pi_{\mathcal{C}}^*(N \setminus \{i\})$. In equilibrium, the buyer selects the set of bids matching the supply chain optimal choice.

Theorem 3.11. *Suppose suppliers have equal-size blocks, then it is a Nash equilibrium for the suppliers when they offer $p_i^* = c_i$ and $r_i^* = e_i + \Pi_{\mathcal{C}}^*(N) - \Pi_{\mathcal{C}}^*(N \setminus \{i\})$, where $i = 1, \dots, n$. At this equilibrium the buyer's reservation set is $N(\mathcal{C})^*$.*

Proof of Theorem 3.11. We write $\mathcal{B}^* = \{(p_i^*, r_i^*), i \in N\}$. Let M^* be the optimal buyer selection under \mathcal{B}^* , i.e. $M^* = N(\mathcal{B}^*)^*$. We begin by showing that $M^* = N(\mathcal{C})^*$. Suppose otherwise and $M^* \neq N(\mathcal{C})^*$, thus (from our tie breaking rule) we have $\Pi_{\mathcal{B}^*}^*(N) >$

$\Pi_{\mathcal{B}^*}(N(\mathcal{C})^*)$. To write explicitly,

$$\Pi_{\mathcal{C}}(M^*) - \sum_{i \in M^*} (r_i^* - e_i) > \Pi_{\mathcal{C}}(N(\mathcal{C})^*) - \sum_{i \in N(\mathcal{C})^*} (r_i^* - e_i).$$

From the definition of r_i^* we have $r_i^* \geq e_i$, and $r_i^* = e_i$ if $i \notin N(\mathcal{C})^*$. Hence the above inequality is equivalent to

$$\Pi_{\mathcal{C}}(M^*) > \Pi_{\mathcal{C}}^*(N) - \sum_{i \in N \setminus M^*} (r_i^* - e_i) = \Pi_{\mathcal{C}}^*(N) - \sum_{i \in N \setminus M^*} (\Pi_{\mathcal{C}}^*(N) - \Pi_{\mathcal{C}}^*(N \setminus \{i\})),$$

which directly contradicts Corollary 3.7. Therefore, the buyer's optimal choice must match the supply chain optimal set given the bids in \mathcal{B}^* .

The next step is to show that \mathcal{B}^* is a Nash equilibrium for the suppliers by demonstrating that no supplier i can increase his profit by offering a different bid (p'_i, r'_i) . Without loss of generality, due to Theorem 3.8, we can assume that $p'_i = c_i$; so to show that the choice of r_i^* is the best response we will show that a unilateral increase of the reservation price from r_i^* cannot improve the profit of supplier i since block i will no longer be chosen by the buyer.

Recall that $N(\mathcal{C})_{-i}^*$ is the optimal buyer choice under \mathcal{C} from the set $N \setminus \{i\}$. In fact, as we have noticed $r_j^* = e_j$ for $j \notin N(\mathcal{C})^*$,

$$\Pi_{\mathcal{B}^*}(N(\mathcal{C})_{-i}^*) = \Pi_{\mathcal{C}}(N(\mathcal{C})_{-i}^*) - \sum_{j \in N(\mathcal{C})_{-i}^*} (r_j^* - e_j) = \Pi_{\mathcal{C}}^*(N \setminus \{i\}) - \sum_{j \in N(\mathcal{C})^* \setminus \{i\}} (r_j^* - e_j),$$

where the second equality follows that $N(\mathcal{C})^* \setminus \{i\} \subseteq N(\mathcal{C})_{-i}^*$ as shown in Lemma 3.15 (see the Appendix). Now it follows from the definition of r_i^* that

$$\Pi_{\mathcal{C}}^*(N \setminus \{i\}) - \sum_{j \in N(\mathcal{C})^* \setminus \{i\}} (r_j^* - e_j) = \Pi_{\mathcal{C}}^*(N) - \sum_{j \in N(\mathcal{C})^*} (r_j^* - e_j) = \Pi_{\mathcal{B}^*}(N(\mathcal{C})^*) = \Pi_{\mathcal{B}^*}^*(N).$$

where the last equality follows from $M^* = N(\mathcal{C})^*$. Thus $\Pi_{\mathcal{B}^*}(N(\mathcal{C})_{-i}^*) = \Pi_{\mathcal{B}^*}^*(N)$. We also know $\Pi_{\mathcal{B}^*}^*(N \setminus \{i\}) \geq \Pi_{\mathcal{B}^*}(N(\mathcal{C})_{-i}^*)$. Then we have established that $\Pi_{\mathcal{B}^*}^*(N \setminus \{i\}) \geq \Pi_{\mathcal{B}^*}^*(N)$. This implies (from the optimality of $\Pi_{\mathcal{B}^*}^*(N)$) that

$$\Pi_{\mathcal{B}^*}^*(N \setminus \{i\}) = \Pi_{\mathcal{B}^*}^*(N). \quad (3.12)$$

Finally we show that block i will not be selected by the buyer if r_i^* is increased by any amount $\delta > 0$. It is clear that this will decrease the buyer profit from any set including block i , but the maximum buyer's profit if i is not selected, given by $\Pi_{\mathcal{B}^*}^*(N \setminus \{i\})$, is unaltered. Using (3.12) we see that the buyer's profit is $\Pi_{\mathcal{B}^*}^*(N \setminus \{i\})$, which is larger than any profit available when i is selected, establishing the result we require and completing the proof that \mathcal{B}^* is an equilibrium. \square

The equilibrium in Theorem 3.11 may not hold in the case where supplier blocks are of unequal sizes, which we will demonstrate using Example 3.4.

Example 3.4 (Example 3.2 (continued)). *Suppose that the 5 blocks are offered by 5 different suppliers. If the result in Theorem 3.11 holds, then we would expect blocks a , b and c to be offered at costs (so $p_i^* = 1$ and $r_i^* = 10$ where $i = a, b, c$), and blocks g and h to raise their reservation price by their contribution to supply chain profit which is equal to $420 - 375 = 45$. Thus for these two blocks we have $p_i^* = 1$ and $r_i^* = 16$ where $i = g, h$. But now $\Pi_{\mathcal{B}^*}(\{g, h\}) = 330$ which is less than $\Pi_{\mathcal{B}^*}(\{a, b, c\}) = 370$. Hence the blocks g and h are not selected, and their profits can be improved by a reduction of reservation price to a point where they enter the selection set for the buyer. Thus this cannot be an equilibrium. In fact there is a whole set of equilibria where $r_g^* = 12 + \delta$ and $r_h^* = 12 - \delta$ where $\delta \in [0, 5]$.*

We now study the profit split in equilibrium. From Theorem 3.11, we know each supplier i 's profit is given by,

$$\pi_i^* = \Pi_{\mathcal{C}}^*(N) - \Pi_{\mathcal{C}}^*(N \setminus \{i\}), \quad \text{for } i = 1, \dots, n.$$

The buyer's choice set is supply chain optimal. Then, the buyer's profit is equal to the supply chain optimal profit less the sum of the supplier profits,

$$\Pi_B^* = \Pi_{\mathcal{C}}^*(N) - \sum_{i=1}^n \pi_i^* = \Pi_{\mathcal{C}}^*(N) - \sum_{i=1}^n (\Pi_{\mathcal{C}}^*(N) - \Pi_{\mathcal{C}}^*(N \setminus \{i\})).$$

Next we will show a stronger result: essentially, at any equilibrium the buyer's reservation choice is supply chain optimal and the profit split amongst players is the same.

Theorem 3.12. *In the case of equal-size blocks, at any equilibrium with $p_i = c_i$, the buyer chooses the supply chain optimal set $N(\mathcal{C})^*$, and supplier i makes a profit $\pi_i^* = \Pi_{\mathcal{C}}^*(N) - \Pi_{\mathcal{C}}^*(N \setminus \{i\})$.*

Proof of Theorem 3.12. Suppose $\mathcal{B} = \{(c_i, \tilde{r}_i^*) : i = 1, \dots, n\}$ is an equilibrium. We first show at equilibrium $N(\mathcal{C})^*$ is an optimal buyer choice, i.e. $\Pi_{\mathcal{B}}^*(N) = \Pi_{\mathcal{B}}(N(\mathcal{C})^*)$. Suppose otherwise, $N(\mathcal{B})^* \neq N(\mathcal{C})^*$ and $\Pi_{\mathcal{B}}^*(N) > \Pi_{\mathcal{B}}(N(\mathcal{C})^*)$. Define

$$\hat{\mathcal{B}} = \mathcal{B} \setminus \{(c_i, \tilde{r}_i^*) : i \in N(\mathcal{C})^* \setminus N(\mathcal{B})^*\} \cup \{(c_i, e_i) : i \in N(\mathcal{C})^* \setminus N(\mathcal{B})^*\},$$

From the definition, we obtain,

$$\Pi_{\hat{\mathcal{B}}}(N(\mathcal{B})^*) = \Pi_{\mathcal{C}}(N(\mathcal{B})^*) - \sum_{j \in N(\mathcal{B})^*} (\tilde{r}_j^* - e_j); \quad \Pi_{\hat{\mathcal{B}}}(N(\mathcal{C})^*) = \Pi_{\mathcal{C}}(N(\mathcal{C})^*) - \sum_{j \in N(\mathcal{C})^* \cap N(\mathcal{B})^*} (\tilde{r}_j^* - e_j).$$

Thus the difference in buyer profit under $\hat{\mathcal{B}}$ between $N(\mathcal{B})^*$ and $N(\mathcal{C})^*$ is given by

$$\Pi_{\hat{\mathcal{B}}}(N(\mathcal{B})^*) - \Pi_{\hat{\mathcal{B}}}(N(\mathcal{C})^*) = \Pi_{\mathcal{C}}(N(\mathcal{B})^*) - \Pi_{\mathcal{C}}(N(\mathcal{C})^*) - \sum_{j \in N(\mathcal{B})^* \setminus N(\mathcal{C})^*} (\tilde{r}_j^* - e_j). \quad (3.13)$$

We know $\Pi_{\hat{\mathcal{B}}}(N(\mathcal{B})^*) = \Pi_{\mathcal{B}}(N(\mathcal{B})^*) = \Pi_{\mathcal{B}}^*(N) = \Pi_{\hat{\mathcal{B}}}^*(N)$ from Lemma 3.10, and thus we have $\Pi_{\hat{\mathcal{B}}}(N(\mathcal{B})^*) \geq \Pi_{\hat{\mathcal{B}}}(N(\mathcal{C})^*)$, which together with (3.13) yields $\Pi_{\mathcal{C}}(N(\mathcal{C})^*) \leq \Pi_{\mathcal{C}}(N(\mathcal{B})^*)$. This contradicts the assumption that under \mathcal{C} there is a unique maximum $N(\mathcal{C})^*$.

We next show that at any equilibrium the profit split is the same. From the above result and the tie breaking rule, we know the buyer's reservation set is $N(\mathcal{C})^*$, i.e. $N(\mathcal{B})^* = N(\mathcal{C})^*$. Thus,

$$\Pi_{\mathcal{B}}^*(N) = \Pi_{\mathcal{B}}(N(\mathcal{C})^*) = \Pi_{\mathcal{C}}(N(\mathcal{C})^*) - \sum_{j \in N(\mathcal{C})^*} (\tilde{r}_j^* - e_j).$$

We then show supplier i makes a profit of $\Pi_{\mathcal{C}}(N(\mathcal{C})^*) - \Pi_{\mathcal{C}}(N(\mathcal{C})_{-i}^*)$. Clearly, if $i \notin N(\mathcal{C})^*$, his profit in equilibrium will be 0 because he will not be chosen by the buyer, implying the result required. We now consider $i \in N(\mathcal{C})^*$.

First, for any $j \in N(\mathcal{B})_{-i}^* \setminus N(\mathcal{B})^*$, we must have $\tilde{r}_j^* = e_j$. Suppose otherwise and $\tilde{r}_j^* > e_j$, then supplier j can decrease \tilde{r}_j^* by ϵ where $0 < \epsilon < \tilde{r}_j^* - e_j$. With the new bid of supplier j , the buyer will choose supplier j , which gives supplier j a positive profit. Therefore, in equilibrium $\tilde{r}_j^* = e_j$ must hold.

Now define

$$\hat{\mathcal{B}} = \mathcal{B} \setminus \{(c_i, \tilde{r}_i^*) : i \in N \setminus N(\mathcal{B})^*\} \cup \{(c_i, e_i) : i \in N \setminus N(\mathcal{B})^*\}.$$

From Lemma 3.10 we know $\hat{\mathcal{B}}$ is also an equilibrium and $\Pi_{\hat{\mathcal{B}}}^*(N) = \Pi_{\mathcal{B}}^*(N)$. Moreover, from Lemma 3.9, we have $\Pi_{\hat{\mathcal{B}}}^*(N \setminus \{i\}) = \Pi_{\hat{\mathcal{B}}}^*(N)$ and $\Pi_{\mathcal{B}}^*(N \setminus \{i\}) = \Pi_{\mathcal{B}}^*(N)$. So

$$\Pi_{\mathcal{B}}^*(N \setminus \{i\}) = \Pi_{\hat{\mathcal{B}}}^*(N \setminus \{i\}). \quad (3.14)$$

We have

$$\begin{aligned} \Pi_{\hat{\mathcal{B}}}(N(\mathcal{C})_{-i}^*) &= \Pi_{\mathcal{C}}(N(\mathcal{C})_{-i}^*) - \sum_{j \in N(\mathcal{C})_{-i}^* \cap N(\mathcal{B})^*} (\tilde{r}_j^* - e_j) \\ &\geq \Pi_{\mathcal{C}}(N(\mathcal{B})_{-i}^*) - \sum_{j \in N(\mathcal{B})^* \setminus \{i\}} (\tilde{r}_j^* - e_j) \end{aligned} \quad (3.15)$$

where the inequality follows from $\Pi_{\mathcal{C}}^*(N \setminus \{i\}) \geq \Pi_{\mathcal{C}}(N(\mathcal{B})_{-i}^*)$. From Lemma 3.15 (in the Appendix), we know the absence of i does not cause the buyer to leave out any bid j

where $j \in N(\mathcal{B})^*$ and $j \neq i$. Thus we obtain $N(\mathcal{B})^* \setminus \{i\} \subseteq N(\mathcal{B})_{-i}^*$ and hence

$$\Pi_{\mathcal{C}}(N(\mathcal{B})_{-i}^*) - \sum_{j \in N(\mathcal{B})^* \setminus \{i\}} (\tilde{r}_j^* - e_j) \geq \Pi_{\mathcal{B}}(N(\mathcal{B})_{-i}^*) = \Pi_{\mathcal{B}}^*(N \setminus \{i\}) = \Pi_{\hat{\mathcal{B}}}^*(N \setminus \{i\}). \quad (3.16)$$

Putting together (3.15) and (3.16) and since, by definition, $\Pi_{\hat{\mathcal{B}}}^*(N \setminus \{i\}) \geq \Pi_{\hat{\mathcal{B}}}^*(N(\mathcal{C})_{-i}^*)$ we have shown $\Pi_{\hat{\mathcal{B}}}^*(N \setminus \{i\}) = \Pi_{\hat{\mathcal{B}}}^*(N(\mathcal{C})_{-i}^*)$. Equality here also establishes that (3.15) is an equality and hence that

$$\Pi_{\hat{\mathcal{B}}}^*(N(\mathcal{C})_{-i}^*) = \Pi_{\mathcal{C}}(N(\mathcal{C})_{-i}^*) - \sum_{j \in N(\mathcal{C})^* \setminus \{i\}} (\tilde{r}_j^* - e_j). \quad (3.17)$$

Using (3.14) and (3.17) we obtain

$$\begin{aligned} \Pi_{\hat{\mathcal{B}}}^*(N \setminus \{i\}) &= \Pi_{\hat{\mathcal{B}}}^*(N(\mathcal{C})_{-i}^*) \\ &= \Pi_{\mathcal{C}}(N(\mathcal{C})_{-i}^*) - \sum_{j \in N(\mathcal{C})^*} (\tilde{r}_j^* - e_j) + \tilde{r}_i^* - e_i \\ &= \Pi_{\mathcal{C}}(N(\mathcal{C})_{-i}^*) - \Pi_{\mathcal{C}}(N(\mathcal{C})^*) + \Pi_{\mathcal{B}}(N(\mathcal{B})^*) + \tilde{r}_i^* - e_i, \end{aligned}$$

where we have used our earlier result that $N(\mathcal{B})^* = N(\mathcal{C})^*$. Knowing that $\Pi_{\hat{\mathcal{B}}}^*(N \setminus \{i\}) = \Pi_{\hat{\mathcal{B}}}^*(N)$ in equilibrium, we obtain $\tilde{r}_i^* - e_i = \Pi_{\mathcal{C}}(N(\mathcal{C})^*) - \Pi_{\mathcal{C}}(N(\mathcal{C})_{-i}^*)$, which gives the profit for supplier i as required. This completes the proof. \square

The result in Theorem 3.12 is appealing and unexpected. It shows that essentially the supply chain is coordinated in the sense that the buyer's ordering decision in equilibrium maximizes the total supply chain profit.

3.4.2 The case with unequal-size blocks

In this subsection, we construct an equilibrium for the suppliers who have unequal-size blocks. Let

$$\mathcal{B}^{(0)} = \mathcal{C} = \{(c_1, e_1, K_1), \dots, (c_n, e_n, K_n)\}, \quad (3.18)$$

and denote by $N(\mathcal{C})^* = \{j_1, \dots, j_m\}$ the optimal buyer choice when each supplier offers at costs.

Define $\mathcal{B}^{(k)}$ recursively by

$$\mathcal{B}^{(k)} = \mathcal{B}^{(k-1)} \setminus \{(c_{j_k}, e_{j_k}, K_{j_k})\} \cup \{(c_{j_k}, r_{j_k}, K_{j_k})\} \quad (3.19)$$

where

$$r_{j_k} = \Pi_{\mathcal{B}^{(k-1)}}^*(N) - \Pi_{\mathcal{B}^{(k-1)}}^*(N \setminus \{j_k\}) + e_{j_k}, \quad \text{for } k = 1, \dots, n. \quad (3.20)$$

So, at each stage k we increase r_{j_k} by the maximum amount that we can without k dropping out of the buyer's choice. In this way, we are able to construct an equilibrium for suppliers.

Theorem 3.13. *Suppose the sets $\mathcal{B}^{(k)}$, for $k = 1, \dots, m$ are defined from (3.18), (3.19) and (3.20), then $\mathcal{B}^{(m)}$ is an equilibrium.*

Proof of Theorem 3.13. For $i \notin N(\mathcal{C})^*$ the offers are at $r_i = e_i$, and there is no possibility of one of these suppliers making more money by reducing its offer in order to be accepted. We show that for each $i \in N(\mathcal{C})^*$ this block is chosen by the buyer and will not be chosen if r_i is increased. This is enough to show that none of these suppliers can make more money.

First we show that $N(\mathcal{B}^{(k)})^* = N(\mathcal{B}^{(k-1)})^*$. Since the difference in these two sets of offers is an increase in r_{j_k} , the only possible change in buyer selection will involve block j_k not being selected. But

$$\Pi_{\mathcal{B}^{(k)}}^*(N \setminus \{j_k\}) = \Pi_{\mathcal{B}^{(k-1)}}^*(N \setminus \{j_k\}) = \Pi_{\mathcal{B}^{(k-1)}}^*(N) - (r_{j_k} - e_{j_k}) = \Pi_{\mathcal{B}^{(k)}}(N(\mathcal{B}^{(k-1)})^*).$$

Thus the best possible result when the buyer leaves out block j_k and makes other choices arbitrarily, matches what the buyer achieves by continuing to choose $N(\mathcal{B}^{(k-1)})^*$, the selection from the previous stage. Given our tie breaking rule, this is enough to establish inductively that $N(\mathcal{B}^{(m)})^* = N(\mathcal{B}^{(0)})^*$.

Now we show that increasing r_{j_k} will mean that the buyer no longer selects this block. This is a consequence of the fact that with offers given by $\mathcal{B}^{(m)}$ there is an equally good buyer selection that leaves out block j_k . Specifically we have

$$\begin{aligned} \Pi_{\mathcal{B}^{(m)}}(N(\mathcal{B}^{(k-1)})_{-j_k}^*) &\geq \Pi_{\mathcal{B}^{(k-1)}}(N(\mathcal{B}^{(k-1)})_{-j_k}^*) - \sum_{i>k} (r_{j_i} - e_{j_i}) \\ &= \Pi_{\mathcal{B}^{(k-1)}}^*(N \setminus \{j_k\}) - \sum_{i>k} (r_{j_i} - e_{j_i}) \\ &= \Pi_{\mathcal{B}^{(k-1)}}^*(N) - \sum_{i \geq k} (r_{j_i} - e_{j_i}) \\ &= \Pi_{\mathcal{B}^{(k-1)}}(N(\mathcal{B}^{(k-1)})^*) - \sum_{i \geq k} (r_{j_i} - e_{j_i}) \\ &= \Pi_{\mathcal{B}^{(m)}}(N(\mathcal{B}^{(m)})^*) = \Pi_{\mathcal{B}^{(m)}}^*(N). \end{aligned}$$

Thus we have $\Pi_{\mathcal{B}^{(m)}}(N(\mathcal{B}^{(k-1)})_{-j_k}^*) = \Pi_{\mathcal{B}^{(m)}}^*(N)$, which implies that any increase in r_{j_k} will cause the buyer not to choose block j_k . Therefore no supplier unilaterally deviates and this completes the proof. \square

The equilibrium constructed in this procedure ensures that the buyer's optimal choice matches the supply chain optimal set. Note that in the algorithm we have assumed

the order of blocks $\{j_1, \dots, j_m\}$ is fixed. As the example below demonstrates, different orders may give different equilibria. Therefore, we could expect there may be multiple equilibria with different profit splits despite the fact that in this class of equilibria the buyer's optimal choice is the same.

Example 3.5. *Demand is fixed at 10 and the retail price is 10. There are four blocks: $i = (0, 3, 3)$, $j = (0, 1.5, 7)$, $k = (0, 3, 2)$, and $l = (0, 3, 8)$. The first number denotes the execution cost, the second number is the reservation cost, and the last is the block size. The supply chain optimal solution is $\{i, j\}$, which gives $\Pi_C(\{i, j\}) = 80.5$. We have*

$$\begin{aligned}\Pi_C^*(\{j, k, l\}) &= \Pi_C(\{j, k\}) = 73.5, \\ \Pi_C^*(\{i, k, l\}) &= \Pi_C(\{k, l\}) = 70, \\ \Pi_C^*(\{i, j, k, l\}) &= \Pi_C(\{i, j\}) = 80.5, \\ \Pi_C^*(\{i, j, l\}) &= \Pi_C^*(\{i, j, k\}) = \Pi_C(\{i, j\}) = 80.5.\end{aligned}$$

We can see the marginal contribution of each of blocks l and k is 0. The marginal contribution of block i is $80.5 - 73.5 = 7$, while the supplier j 's marginal contribution is $80.5 - 70 = 10.5$. Note that when block i is absent, block j is still chosen; while when block j is absent, block i is not chosen. In fact, the marginal contribution 10.5 (of block j) is actually the joint contribution of both block i and block j .

- If we start with block i , then the following bids form an equilibrium: $i = (0, 3 + 7/3, 3)$, $j = (0, 1.5 + 3.5/7, 7)$ and $k = (0, 3, 2)$, $l = (0, 3, 8)$. In this equilibrium, the buyer will choose blocks $\{i, j\}$. The supplier i 's profit is 7, which equals his marginal contribution. The supplier j 's profit is 3.5, which equals the joint contribution of both i and j less the supplier j 's contribution (i.e. $3.5 = 10.5 - 7$).
- If we start with block j , then the following bids form an equilibrium: $i = (0, 3, 3)$, $j = (0, 1.5 + 10.5/7, 7)$ and $k = (0, 3, 2)$, $l = (0, 3, 8)$. In this equilibrium, the buyer will choose blocks $\{i, j\}$. Supplier j makes a profit of 10.5, while the supplier i 's profit is 0, which is less than his marginal contribution.

3.5 Conclusions

This chapter has analyzed a model of competition between suppliers who offer two-part contracts to a buyer under demand uncertainty. In this model each supplier offers a block of capacity and quotes two prices: a reservation price and an execution price. The buyer needs to decide which blocks of capacity to reserve in advance of knowing the customer demand. Thus the buyer's optimization problem becomes combinatorial: choosing the right subset of suppliers.

We first show that, when supplier blocks have the same size, the optimal buyer profit function is submodular. Based on this result, we propose a dynamic programming approach to solve the buyer's optimization problem in polynomial time. We then demonstrate that the submodularity property does not carry over to the case with unequal-size blocks. If competing suppliers know the other bids that they face, it is optimal for each supplier to bid their execution costs and make money only from the reservation margin. This result mirrors what has been found in other circumstances (Wu and Kleindorfer, 2005). However, this result does not hold when the buyer is not restricted to reserve a block all or none, or when each supplier owns multiple blocks and can choose different prices for different blocks.

By using a submodularity result on the buyer's optimal profit as a function of the available set of supplier bids, we are able to analyze the equilibrium behaviour for the suppliers. When the blocks are of equal size, the equilibrium is essentially unique: the buyer's choice at equilibrium matches that which achieves the maximum overall profit in the supply chain and each supplier makes a profit equal to his marginal contribution to the supply chain system (i.e., the difference between the overall supply chain profit when that supplier is present and when it is absent). The fact that this model achieves a supply chain optimal outcome at equilibrium is unexpected. The equivalent non-combinatorial problem analyzed by Martínez-de Albéniz and Simchi-Levi (2009) has a total loss up to 25% in comparison with the supply chain optimal outcome (and even more than this when the demand distribution is not log-concave).

We have shown that our equilibrium result is sensitive to the setting that each supplier has the same capacity. In the case that suppliers have blocks of different sizes, the equilibrium is no longer unique and we characterize a set of equilibria that will imply different profit values for the suppliers depending on which equilibrium occurs.

3.6 Appendix

In this appendix we prove the submodularity result that we need in order to show the equilibrium behaviour of suppliers. This result was established by Professor Bo Chen and is included here for reference.

For convenience of exposition, in our notation system (3.2) we will also use a *sequence* to represent the *set* of elements in the sequence. Let us introduce a very useful technical lemma. For any mutually disjoint sets $X, X_{\text{in}}, X_{\text{out}} \subseteq N$, denote by

$$\Theta[X; X_{\text{in}} \rightarrow X_{\text{out}}] \equiv \Pi(X \cup X_{\text{in}}) - \Pi(X \cup X_{\text{out}})$$

the buyer's profit change when blocks of X_{out} are replaced by blocks of X_{in} while blocks of X are kept. The following lemma characterizes how the buyer's profit change depends on the presence of a special block $m \in N$ in the set of blocks that are kept.

Lemma 3.14. *Let $X, X_{\text{in}}, X_{\text{out}}, \{m\} \subseteq N$ be mutually disjoint and $d_m = \Theta[X \cup \{m\}; X_{\text{in}} \rightarrow X_{\text{out}}] - \Theta[X; X_{\text{in}} \rightarrow X_{\text{out}}]$. Then*

$$d_m \geq 0, \quad \text{if } m > \max\{i : i \in X_{\text{out}}\} \text{ and } |X_{\text{out}}| \geq |X_{\text{in}}|; \quad (3.21)$$

$$d_m \leq 0, \quad \text{if } m > \max\{i : i \in X_{\text{in}}\} \text{ and } |X_{\text{in}}| \geq |X_{\text{out}}|. \quad (3.22)$$

Proof. Let $\sigma_1 = X_{\text{out}}[m-]$ and $\sigma_2 = X_{\text{out}}[m+]$; $\sigma'_1 = X_{\text{in}}[m-]$ and $\sigma'_2 = X_{\text{in}}[m+]$; $X_1 = X[m-]$ and $X_2 = X[m+]$. Since $\Pi(X_1 \cup \sigma_1)$ and $\Pi(X_1 \cup \sigma'_1)$ are canceled out in their respective two Π -terms of d_m , we have

$$\begin{aligned} d_m &= \Pi[|X_1| + |\sigma'_1|, m] - \Delta[|X_1| + |\sigma'_1|, X_2 \cup \sigma'_2] \\ &\quad - \Pi[|X_1| + |\sigma_1|, m] + \Delta[|X_1| + |\sigma_1|, X_2 \cup \sigma_2]. \end{aligned} \quad (3.23)$$

If $m > \max\{i : i \in X_{\text{out}}\}$ and $|X_{\text{out}}| \geq |X_{\text{in}}|$, then $\sigma_2 = \emptyset$ and $|\sigma'_1| \leq |\sigma_1| = |\sigma|$. According to (3.23), we get

$$d_m = \Delta[|X_1| + |\sigma'_1|, \underbrace{m \cdots m}_s X_2] - \Delta[|X_1| + |\sigma'_1|, X_2 \cup \sigma'_2],$$

where $s = |\sigma_1| - |\sigma'_1| = |\sigma'_2| + |X_{\text{out}}| - |X_{\text{in}}| \geq |\sigma'_2|$, which together with Lemma 3.1 implies $d_m \geq 0$.

Symmetrically, if $m > \max\{i : i \in X_{\text{in}}\}$ and $|X_{\text{in}}| \geq |X_{\text{out}}|$, then $\sigma'_2 = \emptyset$ and $|\sigma_1| \leq |\sigma'_1| = |\sigma'|$. According to (3.23), we get

$$d_m = \Delta[|X_1| + |\sigma_1|, X_2 \cup \sigma_2] - \Delta[|X_1| + |\sigma_1|, \underbrace{m \cdots m}_s X_2],$$

where $t = |\sigma'_1| - |\sigma_1| = |\sigma_2| + |X_{\text{in}}| - |X_{\text{out}}| \geq |\sigma_2|$, which together with Lemma 3.1 implies $d_m \leq 0$. \square

Let $N_0 \subseteq N$ be the optimal buyer choice when offer $\ell \in N$ is unavailable. Suppose availability and then acceptance of offer ℓ has resulted in a new optimal buyer choice N_1 with $\ell \in N_1$. The following lemma demonstrates a limited impact created by the additional block ℓ .

Lemma 3.15. *Inclusion of $\ell \notin N_0$ into N_1 will result in neither inclusion of another $k \notin N_0$ into N_1 nor exclusion of more than one element of N_0 from N_1 . More formally, we have*

$$N_1 \setminus \{\ell\} \subseteq N_0 \quad \text{and} \quad |N_1 \setminus \{\ell\}| \geq |N_0| - 1. \quad (3.24)$$

Proof. Let $I, J, K \subseteq N \setminus \{\ell\}$ be mutually disjoint such that $N_0 = I \cup J$ and $N_1 = I \cup K \cup \{\ell\}$. We prove the lemma by showing that $K = \emptyset$ and $|J| \leq 1$. To this end, assume $J \cup K \neq \emptyset$. Then respective optimality of N_0 and N_1 implies that, for any partition $\{J_0, J_1\}$ of J and $\{K_0, K_1\}$ of K , we have

$$\Pi(N_0) - \Pi(I \cup J_0 \cup K_0) \geq 0 \quad \text{and} \quad \Pi(N_1) - \Pi(I \cup J_1 \cup K_1 \cup \{\ell\}) \geq 0,$$

and, provided $J_1 \cup K_0 \neq \emptyset$, at least one of the two inequalities is strict according to the Tie-Breaking Assumption, since it is impossible that the total weight of elements in $J_0 \cup K_0$ plus the total weight of elements in $J_1 \cup K_1$ is smaller than the total weight of elements in $J \cup K$. Therefore, if $J_1 \cup K_0 \neq \emptyset$, then summation of the two inequalities is strictly positive, namely:

$$\Theta[I \cup J_0; J_1 \rightarrow K_0] - \Theta[I \cup K_1 \cup \{\ell\}; J_1 \rightarrow K_0] > 0. \quad (3.25)$$

or equivalently (with different combinations of terms):

$$\Theta[I \cup K_0; K_1 \cup \{\ell\} \rightarrow J_0] - \Theta[I \cup J_1; K_1 \cup \{\ell\} \rightarrow J_0] > 0, \quad (3.26)$$

Denote $j = \max\{i : i \in J\}$ and $k = \max\{i : i \in K\}$.

If $|J| \geq |K| + 2$, then $J \neq \emptyset$ and hence $j \in N$. In (3.26) we let $J_1 = \{j\}$, $K_0 = \emptyset$ and hence $J_0 = J \setminus \{j\}$, $K_1 = K$, yielding

$$\Theta[I; K \cup \{\ell\} \rightarrow J_0] - \Theta[I \cup \{j\}; K \cup \{\ell\} \rightarrow J_0] > 0,$$

which contradicts (3.21).

We remain to consider $|J| \leq |K| + 1$ with $K \neq \emptyset$, which implies $k \in N$. Suppose first $|J| \leq |K|$. If $k > \ell$, then in (3.26) we let $J_1 = \emptyset$, $K_0 = \{k\}$ and hence $J_0 = J$, $K_1 = K \setminus \{k\}$, yielding

$$\Theta[I \cup \{k\}; K_1 \cup \{\ell\} \rightarrow J] - \Theta[I; K_1 \cup \{\ell\} \rightarrow J] > 0,$$

which contradicts (3.22). On the other hand, if $\ell > k$, then then in (3.25) we let $J_1 = J$, $K_0 = K$ and hence $J_0 = \emptyset = K_1$, yielding

$$\Theta[I; J \rightarrow K] - \Theta[I \cup \{\ell\}; J \rightarrow K] > 0,$$

which contradicts (3.21).

We are left with the final case where $|J| = |K| + 1$ and $j, k \in N$. If $k > \ell$, then in (3.26) we let $J_1 = \{j\}$, $K_0 = \{k\}$ and hence $J_0 = J \setminus \{j\}$, $K_1 = K \setminus \{k\}$, yielding

$$\Theta[I \cup \{k\}; K_1 \cup \{\ell\} \rightarrow J_0] > \Theta[I \cup \{j\}; K_1 \cup \{\ell\} \rightarrow J_0].$$

However, the left-hand side above is at most $\Theta[I; K_1 \cup \{\ell\} \rightarrow J_0]$ according to (3.22), while the right-hand side is at least this same amount according to (3.21), demonstrating a contradiction. On the other hand, if $\ell > k$, then in (3.25) we let $J_1 = J \setminus \{j\}$, $K_0 = K$ and hence $J_0 = \{j\}$, $K_1 = \emptyset$, yielding

$$\Theta[I \cup \{j\}; J_1 \rightarrow K] > \Theta[I \cup \{\ell\}; J_1 \rightarrow K].$$

However, the left-hand side above is at most $\Theta[I; J_1 \rightarrow K]$ according to (3.22), while the right-hand side is at least this same amount according to (3.21), demonstrating another contradiction. \square

Now we are ready to prove Lemma 3.6: set function $h(X) = \max_{S \subseteq X} \Pi(S)$ defined on the subsets of N is submodular. We show that, for every $X \subseteq N$ and $\ell, k \in N \setminus X$ and $\ell < k$,

$$h(X \cup \{\ell, k\}) + h(X) \leq h(X \cup \{\ell\}) + h(X \cup \{k\}). \quad (3.27)$$

Suppose $\Pi(A) = h(X)$, $\Pi(\bar{A}) = h(X \cup \{\ell\})$, $\Pi(B) = h(X \cup \{k\})$ and $\Pi(\bar{B}) = h(X \cup \{\ell, k\})$. Then inequality (3.27) is equivalent to

$$\Delta_B \equiv \Pi(\bar{B}) - \Pi(B) \leq \Delta_A \equiv \Pi(\bar{A}) - \Pi(A). \quad (3.28)$$

Without loss of generality, we assume

$$\{\ell, k\} \subseteq \bar{B} \quad (3.29)$$

since $\ell \notin \bar{B}$ implies $\Pi(\bar{B}) = \Pi(B)$ and $k \notin \bar{B}$ implies $\Pi(\bar{B}) = \Pi(\bar{A})$. In either case, inequality (3.28) becomes trivial. Consequently, we have

$$\ell \in \bar{A} \text{ and } k \in B, \quad (3.30)$$

since otherwise, we would have $\bar{A} \subseteq A \subseteq X$ or $B \subseteq A \subseteq X$, contradicting (3.29) due to Lemma 3.15, which states that whenever a new block becomes available and accepted, no more other new blocks will be accepted and at most one block originally accepted will be replaced.

Let $\bar{B} = X_0 \cup \{k, \ell\}$ for some $X_0 = \{j_1, \dots, j_m\} \subseteq X$ with $m \geq 0$. (NB: $X_0 = \emptyset$ if $m = 0$.) Then according to Lemma 3.15 and (3.30), we have (a1) $\bar{A} = X_0 \cup \{\ell\}$ or (a2) $\bar{A} = X_0 \cup \{\ell, i\}$ for some $i \in X \setminus \bar{B}$; and (b1) $B = X_0 \cup \{k\}$ or (b2) $B = X_0 \cup \{k, j\}$ for

some $j \in X \setminus \bar{B}$. Note that we may have $i = j$. Consequently, as follows we only have four possible combinations, in which combination of (a2) and (b2) can be easily further specified into the following three sub-cases: where $i \neq j$ and $i, j \in X \setminus \bar{B}$:

$$\begin{aligned} \bar{A} &= X_0 \cup \{\ell, j\}, B = X_0 \cup \{k, j\} \quad \text{and} \quad A = X_0 \cup \{j\}; \\ \bar{A} &= X_0 \cup \{\ell, j\}, B = X_0 \cup \{k, j\} \quad \text{and} \quad A = X_0 \cup \{i, j\}; \\ \bar{A} &= X_0 \cup \{\ell, i\}, B = X_0 \cup \{k, j\} \quad \text{and} \quad A = X_0 \cup \{i, j\}. \end{aligned}$$

1. $\bar{A} = X_0 \cup \{\ell\}$ and $B = X_0 \cup \{k\}$

Since $\bar{A} = X_0 \cup \{\ell\}$, we have $A = X_0$ or $A = X_0 \cup \{u\}$ for some $u \in X \setminus \bar{B}$ according to Lemma 3.15. If $A = X_0$, then (3.28) is directly implied by Lemma 3.3. If $A = X_0 \cup \{u\}$, then

$$\Delta_B \leq \Pi(\bar{B}) - \Pi(B \cup \{u\}) \leq \Pi(\bar{B} \setminus \{k\}) - \Pi(B \cup \{u\} \setminus \{k\}) = \Delta_A,$$

where the first inequality is due to the optimality of B , while the second one follows from $\ell < k$ and (3.22).

2. $\bar{A} = X_0 \cup \{\ell\}$ and $B = X_0 \cup \{k, j\}$

As in the previous subsection, either $A = X_0$ or $A = X_0 \cup \{u\}$ for some $u \in X \setminus \bar{B}$ according to Lemma 3.15. Now since $B = X_0 \cup \{k, j\}$, there cannot be either $A = X_0$ or $u \neq j$ according to Lemma 3.15. Hence we must have $A = X_0 \cup \{j\}$. Therefore, inequality (3.28) is directly implied by (3.22).

3. $\bar{A} = X_0 \cup \{\ell, i\}$ and $B = X_0 \cup \{k\}$

Let $\bar{A}' = \bar{A} \setminus \{i\}$ and $B' = B \cup \{i\}$. Respective optimality of \bar{A} and of B imply

$$\Pi(\bar{A}) - \Pi(\bar{A}') \geq 0 \quad \text{and} \quad \Pi(B) - \Pi(B') \geq 0,$$

and at least one of the two inequalities is strict according to the Tie-Breaking Assumption, since otherwise the total weight of elements in $\{\ell, i, k\}$ would be larger than itself. Therefore, summation of the two inequalities with recombination of the four terms yield

$$\Theta[X_0 \cup \{i\}; \ell \rightarrow k] - \Theta[X_0; \ell \rightarrow k] > 0,$$

which together with Lemma 3.14 implies that $i < \ell$. Consequently, we have $k > \ell, i$ and hence

$$\Delta_B \leq \Theta[X_0 \cup \{k\}; \ell \rightarrow i] = \Pi(X_0 \cup \{\ell\}) - \Pi(X_0 \cup \{i\}) \leq \Delta_A,$$

where the first and last inequality are respectively due to the optimality of B and \bar{A} (with $A = X_0 \cup \{i\}$ due to Lemma 3.15 and the forms of \bar{A} and B), and the equality is implied by (3.21) and (3.22).

4. $\bar{A} = X_0 \cup \{\ell, j\}$, $B = X_0 \cup \{k, j\}$ **and** $A = X_0 \cup \{j\}$

According to (3.22) we have

$$\Delta_B \leq \Pi(\bar{B} \setminus \{k\}) - \Pi(A) \leq \Delta_A,$$

where the last inequality is due to the optimality of \bar{A} .

5. $\bar{A} = X_0 \cup \{\ell, j\}$, $B = X_0 \cup \{k, j\}$ **and** $A = X_0 \cup \{i, j\}$

If $j > i$, then with optimality of B ,

$$\begin{aligned} \Delta_A &\stackrel{(3.21)}{\geq} \Theta[X_0; \ell \rightarrow i] \stackrel{(3.22)}{\geq} \Theta[X_0 \cup \{k\}; \ell \rightarrow i] \\ &\geq \Theta[X_0 \cup \{k\}; \ell \rightarrow j] = \Delta_B. \end{aligned}$$

If $i > j$, then with optimality of \bar{A} ,

$$\begin{aligned} \Delta_B &\stackrel{(3.22)}{\leq} \Theta[X_0; \ell \rightarrow j] \stackrel{(3.21)}{\leq} \Theta[X_0 \cup \{i\}; \ell \rightarrow j] \\ &\leq \Pi(X_0 \cup \{\ell, j\}) - \Pi(X_0 \cup \{i, j\}) = \Delta_A. \end{aligned}$$

6. $\bar{A} = X_0 \cup \{\ell, i\}$, $B = X_0 \cup \{k, j\}$ **and** $A = X_0 \cup \{i, j\}$

We show that this case does not actually exist. If $i > j$, then immediately we have

$$\Delta_A \stackrel{(3.21)}{\geq} \Theta[X_0; \ell \rightarrow j] \stackrel{(3.22)}{\geq} \Theta[X_0 \cup \{k\}; \ell \rightarrow j] = \Delta_B.$$

Therefore we assume without loss of generality that $i < j$. Let $\bar{A}' = \bar{A} \cup \{j\} \setminus \{i\}$ and $B' = B \cup \{i\} \setminus \{j\}$. As we have argued before with the Tie-Breaking Assumption, respective optimality of \bar{A} and B imply that

$$\Pi(\bar{A}) - \Pi(\bar{A}') + \Pi(B) - \Pi(B') > 0, \quad (3.31)$$

or equivalently

$$\Theta[X_0 \cup \{\ell\}; i \rightarrow j] > \Theta[X_0 \cup \{k\}; i \rightarrow j], \text{ and } \Theta[X_0 \cup \{j\}; k \rightarrow \ell] > \Theta[X_0 \cup \{i\}; k \rightarrow \ell].$$

Therefore, with (3.21) and (3.22) we have the following respective implications:

$$\ell > i \Rightarrow k < j, \text{ and } i > \ell \Rightarrow k > j,$$

which together with the fact that $\ell < k$ and $i < j$ imply one of the following two situations:

$$i < \ell < k < j, \tag{3.32}$$

$$\ell < i < j < k. \tag{3.33}$$

Elements $\{i, j, k, \ell\}$ partition the sequence of elements in X_0 into five parts $(\sigma_1, \dots, \sigma_5)$. With canceling out, it is straightforward to derive the following:

$$\begin{aligned} & \Pi(\bar{A}) - \Pi(\bar{A}') + \Pi(B) - \Pi(B') \\ &= \begin{cases} \Delta[|\sigma_1| + |\sigma_2|; \sigma_3 k] - \Delta[|\sigma_1| + |\sigma_2|; \ell \sigma_3], & \text{if (3.32) holds;} \\ \Delta[|\sigma_1| + |\sigma_2|; \sigma_3 j] - \Delta[|\sigma_1| + |\sigma_2|; i \sigma_3], & \text{if (3.33) holds.} \end{cases} \end{aligned}$$

In either case, inequality (3.31) is contradicted by Lemma 3.1.

Chapter 4

Supply Option Competition with General Costs

4.1 Introduction

High demand uncertainty brings many operational challenges to supply chains, of which an important one revolves around how to share various risks amongst supply chain members. Risk sharing is particularly important in capital-intensive industries, such as petrochemical, electronics and semiconductors, in which manufacturers need to invest heavily in building capacity and the lead times are very long. Moreover, the costs of capacity investment in early stages are much higher relative to actual production costs (Kleindorfer and Wu, 2003).

Traditional approaches in capacity investment assume that all the financial risks from investment are imposed on the firms who build capacity (Wu et al., 2005a). However, capital-intensive firms are reluctant to do so without knowing in advance how much capacity will be required in the future. As a consequence, they tend to adopt a conservative capacity management policy, which in turn leads to the low availability of products and hence affects the buyer's capability to fulfil the customer's demand. On the other hand, the buyer may hesitate to take all the demand risks by making firm commitment for later purchases. Specifically, if it orders too little, there may be lost sales, while there will be surpluses if it orders too much. Therefore, these two types of risk provide a reason for buyers to reserve capacity from suppliers in advance.

We model capacity reservation in a supply option framework. In the first stage, before knowing the actual demand, a buyer reserves a certain amount of capacity by paying a *reservation price*. In the second stage, when observing the actual demand, the buyer executes the capacity up to the lesser of the reserved amount and the observed demand. At this stage, the buyer pays an *execution price* only for the amount dispatched.

The underlying assumption of this model is that suppliers have to install capacity before demand materializes, which can be justified when delayed capacity building is impossible.

Most literature on supply chain contracts assumes a constant marginal cost, while a more general setting would specify arbitrary cost functions (Cachon, 2003). Our model considers a setting where suppliers each have a two-dimensional cost structure: a reservation cost and an execution cost. We allow both costs to be general functions (including constant marginal costs as a special case), so our model is flexible enough to capture many practical settings in terms of cost modelling. With constant marginal costs, it is plausible to focus on simple contract forms as assumed in most of the literature. An exception is Hochbaum and Wagner (2014) who investigate the impact of general production costs on supply chain performance by considering price-only contracts within a one-to-one supply chain. With generic cost functions, however, we expect that some sophisticated bidding formats will be worthwhile, such as function bids which specify different prices for different quantities.

This chapter is concerned with how suppliers compete to supply a homogeneous item to a buyer in a supply option market. The buyer first reserves capacity before knowing the actual demand, and then decides how much capacity to use after observing the demand. The suppliers compete by offering the price functions (for both reservation and execution).

Chapter 3 studies a similar problem but with fundamental differences: each supplier owns a block of capacity and the buyer has to reserve a block all or none. Suppliers each choose a reservation price and an execution price for their blocks. Therefore, it is a combinatorial style auction. However, in this chapter we study the competitive bidding problem for the suppliers whose costs are characterized by general functions. Each supplier offers a function bid consisting of a reservation price function and an execution price function, and the buyer, after receiving function bids, decides how much to purchase from each supplier. In summary, Chapter 4 complements Chapter 3 by studying a situation where the buyer is able to reserve any amount from each supplier and suppliers can charge different prices for different quantities.

We find some similar results: for example, it is optimal for suppliers to set execution prices to be execution costs, thus they make profits only from the buyer's reservation payment. We also show that, in a class of equilibria, the buyer's reservation choice is supply chain optimal, each supplier's profit equals its marginal contribution to the supply chain system, and the buyer takes the remaining profit.

Supply options have been widely studied in the operations management literature (see e.g., Barnes-Schuster et al., 2002; Burnetas and Ritchken, 2005; Chen et al., 2005; Wu et al., 2005b; Fu et al., 2010; Secomandi and Wang, 2012). Perhaps, the initial step of study is to investigate a buyer's optimal purchasing decision (see e.g., Martínez-de Albéniz and Simchi-Levi, 2005). As an extension of this, several papers examine

option contract design problems in a Stackelberg game between a buyer and a supplier with a focus on the interaction between option markets and spot markets (see e.g., Wu et al., 2002; Pei et al., 2011). In a competitive setting, Wu and Kleindorfer (2005) incorporate capacity decisions in a supplier competition model and derive a Bertrand-Nash equilibrium.

The closest paper to ours is Martínez-de Albéniz and Simchi-Levi (2009) (thereafter, “MS”) who study a setting similar to ours except that marginal costs are constants and each supplier chooses a reservation price and an execution price for their *limitless* capacity. They show that, in the worst equilibrium, the efficiency loss is no greater than 25%. This chapter generalizes MS by considering general cost functions and allowing suppliers to submit function bids. By enlarging the strategy space of suppliers, we find a relatively clean and intuitive result as we discuss earlier. Comparing with MS, we also find that allowing suppliers to offer a (sophisticated) function bid makes the buyer worse off. Our findings highlight the significant impact of the suppliers’ strategy space on equilibrium outcomes.

This chapter studies a situation where suppliers each submit a function bid, which resembles that in the supply function equilibrium (SFE) literature (Klemperer and Meyer, 1989). The distinction is that we examine the buyer’s optimization problem explicitly; while in the SFE literature, the buyer’s problem is to choose a clearing price to equate the total supply with the demand, and each supplier’s best response is characterized by a differential equation.

This work is also loosely related to the auction literature. At the heart of auctions is the problem of decentralized resource allocation (Klagnanam and Parkes, 2003). A subset of this literature examines the efficiency and profit allocation of a given auction format, for example, menu auction (Bernheim and Whinston, 1986), share auction (Wilson, 1979), uniform price auction (Bresky, 2013), discriminatory price auction (Menezes and Monteiro, 1995), and split award auction (Anton and Yao, 1989, 1992). The above papers generally assume that the total purchase amount of a buyer is deterministic. With an endogenous purchase amount, Chen (2007) studies a procurement mechanism design problem for a newsvendor buyer and proposes a supply contract auction. Dasgupta and Spulber (1989) examine a similar procurement problem and develop a quantity auction. More recently, Duenyas et al. (2013) propose a simple modified version of the standard open-descending auction, which is shown to be optimal for the buyer. These papers focus on designing an optimal auction mechanism. Our research differs from the auction literature in that we consider a full information setting and look at how suppliers compete in a supply option market.

The rest of this chapter is organized as follows: Section 4.2 presents the model setup and examines the buyer’s problem. In the baseline model, we consider the case with two

suppliers, and study the equilibrium for suppliers in Section 4.3. Section 4.4 discusses the extension with more than two suppliers, and we conclude in Section 4.5.

4.2 Model Setup and Buyer's Problem

4.2.1 Model Setup

Consider a supply chain with a single buyer (“she”) and multiple suppliers (“he”) indexed by $i = 1, \dots, n$, where the buyer purchases a homogeneous item from the n suppliers. The buyer’s stochastic demand D has a cdf $F(d)$ and a pdf $f(d)$ over $[\underline{d}, \bar{d}]$ with $0 \leq \underline{d} < \bar{d} < \infty$. Let $\bar{F}(d) = 1 - F(d)$. The retail price ρ of the product is exogenously given.

Each supplier i ’s cost consists of two components: the marginal reservation cost $e_i(t)$ and the marginal execution cost $c_i(t)$. Note that $c_i(t)$ and $e_i(t)$ might be constants as assumed in the previous literature. Suppliers each maximize their own expected profits by choosing a reservation price function $r_i(t)$ and an execution price function $p_i(t)$ for $t \in [0, \bar{t}]$. Without loss of generality, we assume $\bar{t} = \bar{d}$ since the buyer will not reserve more than \bar{d} units in any case. On the other hand, if a supplier does not want to offer that much, he may simply set a very high price for the quantities beyond the desired amount so that the buyer will not reserve them for sure.

We model this game in a Stackelberg framework where the suppliers are leaders and the buyer is a follower. Each supplier has complete information about the buyer’s demand distribution and the supplier costs. The sequence of events is depicted in Figure 4.1.

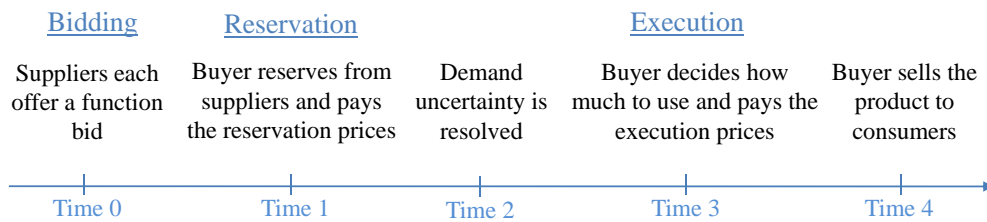


FIGURE 4.1: The timeline

First, suppliers each offer a function bid (consisting of a reservation price function and an execution price function) to the buyer. Second, prior to knowing the actual demand, the buyer decides the reserved amount from each supplier and pays the reservation price. After demand is observed, the buyer chooses what capacity to use and pays only for the amount used. If the demand exceeds the total amount of reserved capacity, there will be lost sales. Finally, the buyer sells the product to the consumer market at the retail price ρ .

We consider a stylized supply chain with two suppliers labelled by $i = 1, 2$. We use j to indicate the supplier other than i where $i, j = 1, 2, i \neq j$. In Section 4.4, we will

extend it to the case with more than two suppliers. Following the backward induction approach, we start by analyzing the buyer's optimal reservation behaviour.

4.2.2 Buyer's problem

Suppose the bids offered by suppliers are $\{(p_i(t), r_i(t)) : i = 1, 2\}$ for $t \in [0, \bar{d}]$. Note that both $p_i(t)$ and $r_i(t)$ can be constants. The buyer makes a reservation decision in the first stage and an execution decision in the second stage. We first show the buyer's execution policy in Remark 4.1.

Remark 4.1. After making a selection of capacity, when demand materializes, the buyer will use the capacity with the lowest execution price first.

Notice that the reservation payment becomes sunk when the buyer makes the execution decision. With any actual demand, the buyer will first use the capacity with the lowest execution price up until the lesser of the actual demand and the reserved amount. Based on this observation, we make the following assumption regarding the supplier bidding strategy.

Assumption 4.1. For $i = 1, 2$, both $p_i(t)$ and $r_i(t)$ are continuous, and $p_i(t)$ is non-decreasing.

Assumption 4.1 allows us to easily formulate the cumulative amount of capacity with execution prices below a given price.

Given the supplier bids $\{(p_i(t), r_i(t)) : i = 1, 2\}$, suppose the buyer's reservation choice is (t_1, t_2) . That is, the buyer reserves t_1 from supplier 1 and t_2 from supplier 2. For $i, j = 1, 2, i \neq j$, let

$$\gamma_i(x, t_j) = \sup\{0 \leq y \leq t_j : p_j(y) \leq p_i(x)\}, \quad \text{for } x \in [0, t_i] \quad (4.1)$$

be the dispatched amount from supplier j with execution prices below $p_i(x)$. In particular, if $p_j(t)$ is strictly increasing and continuous, then $\gamma_i(x, t_j) = \min\{p_j^{-1}(p_i(x)), t_j\}$. We now write the cumulative amount of capacity with execution price less than $p_i(x)$ as follows,

$$h_i(x, t_j) = x + \gamma_i(x, t_j), \quad \text{for } x \in [0, t_i], \quad (4.2)$$

where the term x indicates the dispatched amount from supplier i and the second term represents the dispatched amount from supplier j . In the special case with $p_1(t) = p_1$ and $p_2(t) = p_2$ where p_1, p_2 are constants, we obtain that: if $p_i < p_j$, then $h_i(x, t_j) = x$ and $h_j(x, t_i) = x + t_i$; if $p_1 = p_2$, we can apply some tie-breaking rule for the buyer's execution policy.

With the reservation choice (t_1, t_2) , we can write down the buyer's expected profit as follows,

$$\pi_B(t_1, t_2) = \sum_{i,j=1,2,j \neq i} \int_0^{t_i} \{[\rho - p_i(x)]\bar{F}(h_i(x, t_j)) - r_i(x)\} dx, \quad (4.3)$$

where $h_i(\cdot, \cdot)$ is given in (4.2) and $\bar{F}(z) = \Pr[D \geq z]$.

Given the supplier bids $\{(p_i(t), r_i(t)) : i = 1, 2\}$, the buyer aims to maximize her expected profit by choosing the reserved amount from each supplier:

$$\max \{ \pi_B(t_1, t_2) : t_1, t_2 \in [0, \bar{d}] \}. \quad (4.4)$$

We use the rules in Assumption 4.2 to break ties for the buyer's reservation choice.

Assumption 4.2. *In the event of a tie for the buyer's reservation choice, the buyer prefers to choose from more suppliers, and will randomly choose one if the candidate solutions imply the same number of suppliers.*

Solving the buyer's problem in (4.4), we characterize the buyer's optimal reservation choice in Proposition 4.2.

Proposition 4.2. *The buyer's optimal reservation choice (T_1, T_2) satisfies the following simultaneous equations: for $i, j = 1, 2, i \neq j$,*

$$\underbrace{(\rho - p_i(T_i))\bar{F}(h_i(T_i, T_j)) - r_i(T_i)}_{\text{left}} - \underbrace{\int_{\gamma_i(T_i, T_j)}^{T_j} [\rho - p_j(x)]f(h_j(x, T_i))dx}_{\text{right}} = 0. \quad (4.5)$$

Proof of Proposition 4.2. First, we observe that π_B is continuous as a function of (t_1, t_2) . From the extreme value theorem, there must exist an optimal solution in the compact and bounded set $[0, \bar{d}] \times [0, \bar{d}]$. Second, we show π_B is differentiable and smooth. The first partial derivative of π_B with respect to t_i is

$$\frac{\partial \pi_B}{\partial t_i} = (\rho - p_i(t_i))\bar{F}(h_i(t_i, t_j)) - r_i(t_i) - \int_0^{t_j} [\rho - p_j(x)]f(h_j(x, t_i)) \frac{\partial h_j(x, t_i)}{\partial t_i} dx,$$

which is continuous because both $p_i(t)$ and $r_i(t)$ are continuous. Furthermore, we have, for any $x \in [0, \bar{d}]$,

$$\left. \frac{\partial \pi_B}{\partial t_i} \right|_{t_i=0, t_j=x} = (\rho - p_i(0))\bar{F}(0) - r_i(0) > 0 \quad \text{and} \quad \left. \frac{\partial \pi_B}{\partial t_i} \right|_{t_i=\bar{d}, t_j=x} = -r_i(\bar{d}) < 0.$$

Thus, the optimal solution (T_1, T_2) must satisfy the first order conditions: for $i, j = 1, 2, i \neq j$,

$$(\rho - p_i(T_i))\bar{F}(h_i(T_i, T_j)) - r_i(T_i) - \int_0^{T_j} [\rho - p_j(x)]f(h_j(x, T_i)) \left(\frac{\partial h_j(x, t_i)}{\partial t_i} \Big|_{t_i=T_i} \right) dx = 0, \quad (4.6)$$

We now simplify (4.6) by considering the following two cases.

(1) If $p_i(T_i) < p_j(T_j)$, then from (4.1) we obtain $\gamma_i(T_i, T_j) < T_j$. Also,

$$\gamma_j(x, T_i) = \begin{cases} \sup\{y \geq 0 : p_i(y) \leq p_j(x)\}, & x \in [0, \gamma_i(T_i, T_j)), \\ T_i, & x \in [\gamma_i(T_i, T_j), T_j]. \end{cases}$$

Then we have,

$$\frac{\partial h_j(x, t_i)}{\partial t_i} \Big|_{t_i=T_i} = \frac{\partial(x + \gamma_j(x, t_i))}{\partial t_i} \Big|_{t_i=T_i} = \begin{cases} 0, & x \in [0, \gamma_i(T_i, T_j)), \\ 1, & x \in [\gamma_i(T_i, T_j), T_j]. \end{cases}$$

Therefore, the second term in (4.6) can be rewritten as,

$$\int_0^{T_j} [\rho - p_j(x)]f(h_j(x, T_i)) \left(\frac{\partial h_j(x, t_i)}{\partial t_i} \Big|_{t_i=T_i} \right) dx = \int_{\gamma_i(T_i, T_j)}^{T_j} [\rho - p_j(x)]f(h_j(x, T_i)) dx.$$

(2) If $p_i(T_i) \geq p_j(T_j)$, we have $\gamma_i(T_i, T_j) = T_j$ and $\gamma_j(x, T_i) < T_i$ for any $x \in [0, T_j]$.

Thus,

$$\frac{\partial h_j(x, t_i)}{\partial t_i} \Big|_{t_i=T_i} = \frac{\partial(x + \gamma_j(x, t_i))}{\partial t_i} \Big|_{t_i=T_i} = 0.$$

Combining the above two cases establishes the result required. \square

We can think of the first term of (4.5) as the unadjusted marginal profit from the last unit of reserved capacity, while the integral term represents a non-negative adjusting component, which measures the effect of T_j on T_i . It will be helpful to break down (4.5) into two cases:

- (1) $T_j = 0$ and (4.5) reduces to $(\rho - p_i(T_i))\bar{F}(T_i) - r_i(T_i) = 0$. In this case, the buyer reserves from supplier i only.
- (2) $T_1 > 0, T_2 > 0$. In this case, the buyer reserves from both suppliers. In particular, if $p_1(T_1) = p_2(T_2)$, then $h_1(T_1, T_2) = h_2(T_2, T_1) = T_1 + T_2$, and $(\rho - p_i(T_i))\bar{F}(T_1 + T_2) - r_i(T_i) = 0$ for $i = 1, 2$, from which we obtain $r_1(T_1) = r_2(T_2)$.

To find an optimal reservation choice, we also need to check the second order conditions. The second derivative of π_B with respect to t_i is given by,

$$\begin{aligned} \frac{\partial^2 \pi_B}{\partial t_i^2} &= -p'_i(t_i)\bar{F}(h_i(t_i, t_j)) - (\rho - p_i(t_i))f(h_i(t_i, t_j))\frac{\partial h_i(t_i, t_j)}{\partial t_i} - r'_i(t_i) \\ &\quad - \int_0^{t_j} [\rho - p_j(x)] \left[f'(h_j(x, t_i)) \left(\frac{\partial h_j(x, t_i)}{\partial t_i} \right)^2 + f(h_j(x, t_i)) \frac{\partial^2 h_j(x, t_i)}{\partial t_i^2} \right] dx \\ &= -p'_i(t_i)\bar{F}(h_i(t_i, t_j)) - (\rho - p_i(t_i))f(h_i(t_i, t_j))\frac{\partial h_i(t_i, t_j)}{\partial t_i} - r'_i(t_i) \\ &\quad - \int_0^{t_j} [\rho - p_j(x)]f'(h_j(x, t_i)) \left(\frac{\partial h_j(x, t_i)}{\partial t_i} \right)^2 dx, \end{aligned}$$

where the second equality follows from $\partial^2 h_j(x, t_i)/\partial t_i^2 = 0$. The cross partial derivative is given by,

$$\frac{\partial^2 \pi_B}{\partial t_i \partial t_j} = -(\rho - p_i(t_i))f(h_i(t_i, t_j))\frac{\partial h_i(t_i, t_j)}{\partial t_j} - (\rho - p_j(t_j))f(h_j(t_j, t_i))\frac{\partial h_j(t_j, t_i)}{\partial t_i}.$$

If $f' \geq 0$ and $r'_i(t) \geq 0$, then we can show the Hessian matrix is negative definite, implying that π_B is jointly concave in (t_i, t_j) . Thus, the local maximum characterized by (4.5) is also the global maximum. Note that there may exist multiple solutions for the buyer's problem. In this case, we need to compare all the solutions in order to obtain the global maximum.

We now give an example to illustrate how to calculate the buyer's optimal reservation choice.

Example 4.1. Suppose the supplier bids are $p_1(t) = t$, $r_1(t) = 1/4$, $p_2(t) = (1/2)t$, and $r_2(t) = 1/2$ for $t \in [0, 1]$. The demand D follows a uniform distribution over $[0, 1]$. Thus, $F(t) = t$ for $t \in [0, 1]$. The retail price is $\rho = 2$. We have the following necessary conditions for the buyer's optimal choice (T_1, T_2) :

$$\begin{aligned} (2 - T_1)(1 - T_1 - \min\{2T_1, T_2\}) - \frac{1}{4} - \int_{2T_1}^{T_2} (2 - (1/2)x)dx &= 0 \\ (2 - (1/2)T_2)(1 - T_2 - \min\{(1/2)T_2, T_1\}) - \frac{1}{2} - \int_{(1/2)T_2}^{T_1} (2 - x)dx &= 0 \end{aligned}$$

Solving the above equations yields the optimal solution $(T_1, T_2) = (0.6369, 0.1797)$.

4.3 Equilibrium Analysis

In this section, we study the equilibrium for suppliers. We begin with each supplier's best response.

4.3.1 Suppliers' best responses

Given the supplier j 's bid $(p_j(t), r_j(t))$, we now solve the supplier i 's best response problem. First, consider the buyer's optimal choice when there is only supplier j available. Denote by T_j^s the buyer's optimal reserved amount if supplier j is the sole supplier, that is,

$$T_j^s = \arg \max \left\{ \int_0^t \{[\rho - p_j(x)]\bar{F}(x) - r_j(x)\} dx : t \in [0, \bar{d}] \right\}.$$

Further denote by π_B^j the optimal buyer profit if supplier j is the sole supplier.

As a Stackelberg leader, each supplier is able to anticipate the buyer's reservation choice provided that the competitor's bid is observed. Notice that the buyer's optimization problem is embedded in the supplier i 's best response problem.

To distinguish from $\pi_B(t_1, t_2)$ in (4.3), we write $\pi_B(t_1, t_2; p_i(t), r_i(t))$ for the buyer's profit from choosing (t_1, t_2) provided the supplier i 's offer is $(p_i(t), r_i(t))$. We have

$$\pi_B(t_1, t_2; p_i(t), r_i(t)) = \sum_{i,j=1,2,j \neq i} \int_0^{t_i} \{[\rho - p_i(x)]\bar{F}(h_i(x, t_j)) - r_i(x)\} dx. \quad (4.7)$$

Suppose the optimal buyer choice is (T_1, T_2) given $(p_i(t), r_i(t))$ (as well as $(p_j(t), r_j(t))$), so

$$(T_1, T_2) = \arg \max \{ \pi_B(t_1, t_2; p_i(t), r_i(t)) : t_1, t_2 \in [0, \bar{d}] \}. \quad (4.8)$$

Then the supplier i 's expected profit by offering $(p_i(t), r_i(t))$ is

$$\pi_i(p_i(t), r_i(t); T_1, T_2) = \int_0^{T_i} \{ [p_i(x) - c_i(x)]\bar{F}(h_i(x, T_j)) + r_i(x) - e_i(x) \} dx. \quad (4.9)$$

Knowing the buyer's reservation behaviour, supplier i aims to maximize his expected profit by choosing $(p_i(t), r_i(t))$:

$$\max_{p_i(t), r_i(t)} \pi_i(p_i(t), r_i(t); T_1, T_2), \quad (4.10)$$

subject to (4.8) which pertains to the buyer's problem given the supplier i 's bid $(p_i(t), r_i(t))$ (as well as $(p_j(t), r_j(t))$).

Fix $p_i(t)$ and suppose the buyer's reservation choice is (T_1, T_2) , then supplier i will set $r_i(t)$ as high as possible subject to the proviso that the buyer still reserves T_i from supplier i . That is, for the supplier i 's best response, we must have $\pi_B(T_1, T_2; p_i(t), r_i(t)) = \pi_B^j$, from which we deduce,

$$\int_0^{T_i} r_i(x) dx = \int_0^{T_i} [\rho - p_i(x)]\bar{F}(h_i(x, T_j)) dx + \int_0^{T_j} \{ [\rho - p_j(x)]\bar{F}(h_j(x, T_i)) - r_j(x) \} dx - \pi_B^j.$$

Plugging the above equation into the supplier i 's profit function $\pi_i(p_i(t), r_i(t); T_1, T_2)$, we cancel out r_i and obtain

$$\begin{aligned} \pi_i(p_i(t); T_1, T_2) &= \int_0^{T_i} \{[\rho - c_i(x)]\bar{F}(h_i(x, T_j)) - e_i(x)\} dx \\ &\quad + \int_0^{T_j} \{[\rho - p_j(x)]\bar{F}(h_j(x, T_i)) - r_j(x)\} dx - \pi_B^j, \end{aligned}$$

where $h_i(x, T_j)$ and $h_j(x, T_i)$ are given in (4.2). Then we reformulate the supplier i 's problem as follows:

$$\mathbf{P1} : \quad \max_{p_i(t), r_i(t)} \pi_i(p_i(t); T_1, T_2), \quad (4.11)$$

subject to (4.8) and $\pi_B(T_1, T_2; p_i(t), r_i(t)) = \pi_B^j$.

Solving the problem **P1**, we characterize the supplier i 's best response in Lemma 4.3.

Lemma 4.3. *Given $(p_j(t), r_j(t))$, it is optimal for supplier i to set $p_i(t) = c_i(t)$ and choose $r_i(t)$ such that $\pi_B(\hat{T}_1, \hat{T}_2; c_i(t), r_i(t)) = \pi_B^j$ where (\hat{T}_1, \hat{T}_2) maximize the buyer profit and are determined by the simultaneous equations:*

$$(\rho - c_i(\hat{T}_i))\bar{F}(\bar{h}_i(\hat{T}_i, \hat{T}_j)) - e_i(\hat{T}_i) - \int_{\gamma_i(\hat{T}_i, \hat{T}_j)}^{\hat{T}_j} [\rho - p_j(x)]f(h_j(x, \hat{T}_i))dx = 0 \quad (4.12)$$

$$(\rho - p_j(\hat{T}_j))\bar{F}(\bar{h}_j(\hat{T}_j, \hat{T}_i)) - r_j(\hat{T}_j) - \int_{\gamma_j(\hat{T}_j, \hat{T}_i)}^{\hat{T}_i} [\rho - c_i(x)]f(h_i(x, \hat{T}_j))dx = 0, \quad (4.13)$$

where $\bar{h}_i(x, \hat{T}_j) = x + \sup\{0 \leq y \leq \hat{T}_j : p_j(y) \leq c_i(x)\}$ and $\bar{h}_j(x, \hat{T}_i) = x + \sup\{0 \leq y \leq \hat{T}_i : c_i(y) \leq p_j(x)\}$.

Proof of Lemma 4.3. First consider the buyer's problem when supplier i charges only his costs, i.e. $p_i(t) = c_i(t)$ and $r_i(t) = e_i(t)$. The buyer's profit with the reservation choice (t_1, t_2) is

$$\begin{aligned} \pi_B(t_1, t_2; c_i(t), e_i(t)) &= \int_0^{t_i} \{[\rho - c_i(x)]\bar{F}(\bar{h}_i(x, t_j)) - e_i(x)\} dx \\ &\quad + \int_0^{t_j} \{[\rho - p_j(x)]\bar{F}(\bar{h}_j(x, t_i)) - r_j(x)\} dx, \end{aligned}$$

where $\bar{h}_i(x, t_j) = x + \sup\{0 \leq y \leq t_j : p_j(y) \leq c_i(x)\}$ and $\bar{h}_j(x, t_i) = x + \sup\{0 \leq y \leq t_i : c_i(y) \leq p_j(x)\}$. Suppose (\hat{T}_1, \hat{T}_2) is the optimal buyer choice. Similar to the problem in (4.4), we can obtain the necessary conditions for (\hat{T}_1, \hat{T}_2) as shown in (4.12) and (4.13).

Second, observe that the objective function $\pi_i(p_i(t); T_1, T_2)$ does not contain $r_i(t)$. We now consider a relaxed problem by ignoring the constraints:

$$\mathbf{P0} : \quad \max_{p_i(t), T_1, T_2} \pi_i(p_i(t); T_1, T_2). \quad (4.14)$$

An optimal solution to the problem **PO** is $p_i(t) = c_i(t)$ and $(T_1, T_2) = (\hat{T}_1, \hat{T}_2)$.

The last step is to show that it is achievable to find an appropriate $r_i(t)$ such that the constraints are met. To guarantee (\hat{T}_1, \hat{T}_2) is the buyer's optimal choice, we need to examine the necessary conditions in (4.5). Putting (4.12), (4.13) and (4.5) together, we obtain that a necessary condition is $r_i(\hat{T}_i) = e_i(\hat{T}_i)$. Furthermore, we add a margin of $\pi_B(\hat{T}_1, \hat{T}_2; c_i(t), e_i(t)) - \pi_B^j$ to $e_i(t)$ in a segment before \hat{T}_i . Since we allow an arbitrary function for $r_i(t)$, in fact there exist multiple solutions for $r_i(t)$ such that the buyer's optimal choice is indeed (\hat{T}_1, \hat{T}_2) . \square

Lemma 4.3 gives an optimal solution with $p_i(t) = c_i(t)$ for the supplier i 's best response problem. There may be other solutions where $p_i(t) \neq c_i(t)$. This happens since supplier i can adjust $r_i(t)$ so that the supplier i 's profit remains unchanged. For the supplier i 's best response, we show that the optimal buyer choice is (\hat{T}_1, \hat{T}_2) , which maximizes the buyer's profit when supplier i charges only his costs. Similar to the buyer's problem in (4.4) we consider three cases regarding the buyer's optimal reservation choice characterized in (4.12) and (4.13). If $\hat{T}_i = 0$, then supplier j wins all the buyer's business even when supplier i sets prices to be costs. If $\hat{T}_j = 0$, then supplier j will win no business from the buyer, implying that it is optimal for supplier i to beat supplier j out of the market. If $\hat{T}_i, \hat{T}_j > 0$, the buyer reserves from both suppliers.

4.3.2 Equilibrium Characterization

Having established the best response for each supplier, we are now in a position to study the equilibrium for suppliers. An equilibrium is a pair of bids $\{(p_i^*(t), r_i^*(t)) : i = 1, 2\}$, which are mutual best responses for each supplier. The equilibrium outcome is the buyer's reservation choice (T_1^*, T_2^*) . Let $\pi_B^*, \pi_1^*, \pi_2^*$ be the equilibrium profits of the buyer, supplier 1, and supplier 2, respectively. If the buyer is constrained to reserve from supplier i only, we denote by π_B^{i*} , π_i^{s*} , and T_i^{s*} , the buyer's optimal profit, the supplier i 's optimal profit, and the reserved amount, respectively.

We first show that in equilibrium the buyer's profit is the same when either supplier is unavailable.

Lemma 4.4. *Suppose $\{(p_i^*(t), r_i^*(t)) : i = 1, 2\}$ is a Nash equilibrium, then*

$$\pi_B^* = \pi_B^{1*} = \pi_B^{2*}. \quad (4.15)$$

Proof of Lemma 4.4. From optimality we must have $\pi_B^* \geq \pi_B^{i*}$ where $i = 1, 2$. Suppose otherwise and (4.15) does not hold. Then there exists a supplier i and $\delta := \pi_B^* - \pi_B^{i*} > 0$. We will show that the other supplier j has an incentive to deviate from the proposed bid $(p_j^*(t), r_j^*(t))$. Consider a new bid $(p_j^*(t), \hat{r}_j^*(t))$ by lifting $\hat{r}_j^*(t)$ above $r_j^*(t)$ in a segment

before $\min(T_j^*, T_j^{s*})$ so that:

$$\int_0^{T_j^*} \hat{r}_j(x) dx = \int_0^{T_j^*} r_j^*(x) dx + \delta/2 \quad \text{and} \quad \int_0^{T_j^{s*}} \hat{r}_j(x) dx = \int_0^{T_j^{s*}} r_j^*(x) dx + \delta/2.$$

With this new bid (and the supplier i 's bid $(p_i^*(t), r_i^*(t))$), we can show that the buyer will still reserve T_j^* units from supplier j . Then supplier j makes $\delta/2$ more profit, implying that supplier j has an incentive to deviate. A contradiction. \square

Lemma 4.4 shows that, in equilibrium the buyer's optimal profit when reserving from two suppliers is equal to that when reserving from only one. The intuition is that if the buyer's profit of reserving from either supplier is lower, the other supplier can always increase his bid prices a little but makes sure that the buyer's reservation choice remains the same.

Now let us consider the supply chain optimal problems when both suppliers offer bids at costs. Let $\mathbf{c} = (c_i(t), c_j(t))$ and $\mathbf{e} = (e_i(t), e_j(t))$. With the reservation choice (t_1, t_2) for the buyer, we write the supply chain profit as follows,

$$\pi_B(t_1, t_2; \mathbf{c}, \mathbf{e}) = \sum_{i,j=1,2,j \neq i} \int_0^{t_i} \{[\rho - c_i(x)]\bar{F}(h_i(x, t_j)) - e_i(x)\} dx,$$

where

$$h_i(x, t_j) = x + \sup\{0 \leq y \leq t_j : c_j(y) \leq c_i(x)\}.$$

For tractability, we assume the supply chain profit function is jointly strictly concave in q_1 and q_2 . Suppose (\bar{T}_1, \bar{T}_2) is the supply chain optimal solution, which implies that

$$(\bar{T}_1, \bar{T}_2) = \arg \max \{ \pi_B(t_1, t_2; \mathbf{c}, \mathbf{e}) : t_1, t_2 \in [0, \bar{d}] \}. \quad (4.16)$$

Denote by Π the supply chain optimal profit, i.e. $\Pi = \pi_B(\bar{T}_1, \bar{T}_2; \mathbf{c}, \mathbf{e})$.

If supplier i is the sole supplier, the supply chain profit when the buyer orders t_i from supplier i is given by

$$\pi_B^s(t_i; c_i(t), e_i(t)) = \int_0^{t_i} \{[\rho - c_i(x)]\bar{F}(x) - e_i(x)\} dx.$$

Similarly, we assume $\pi_B^s(t_i; c_i(t), e_i(t))$ is strictly concave in t_i . Denote by \bar{T}_i^s the buyer's optimal choice when she purchases from supplier i only. Therefore, we have

$$\bar{T}_i^s = \arg \max \{ \pi_B^s(t_i; c_i(t), e_i(t)) : t_i \in [0, \bar{d}] \}.$$

Let Π_i^s be the supply chain optimal profit when supplier i is the sole supplier, i.e. $\Pi_i^s = \pi_B^s(\bar{T}_i^s; c_i(t), e_i(t))$. Note that we have $\Pi > \max(\Pi_i^s, \Pi_j^s)$ because there exists an

interior solution for the supply chain optimal problem in (4.16). We further assume $\bar{T}_i^s \geq \bar{T}_i$ where $i = 1, 2$. Though we do not give details here, it can be shown that this assumption is satisfied when $r_i(t)$ is non-decreasing in t .

We now define the following power function, which will be used for constructing an equilibrium: for $i, j = 1, 2, i \neq j$,

$$\Delta_i(t) = \frac{(\beta_i + 1)(\Pi - \Pi_j^s)}{\bar{T}_i} \left(\frac{\bar{T}_i - t}{\bar{T}_i} \right)^{\beta_i}, \quad \text{for } t \in [0, \bar{T}_i], \quad (4.17)$$

where $\beta_i > 0$ is a constant. We call $\Delta_i(t)$ the *price margin* imposed by supplier i . The definite integral of $\Delta_i(t)$ from 0 to \bar{T}_i is given by,

$$\int_0^{\bar{T}_i} \Delta_i(t) dt = \int_0^{\bar{T}_i} \frac{(\beta_i + 1)(\Pi - \Pi_j^s)}{\bar{T}_i} \left(\frac{\bar{T}_i - t}{\bar{T}_i} \right)^{\beta_i} dt = \Pi - \Pi_j^s. \quad (4.18)$$

The first derivative of $\Delta_i(t)$ is given by,

$$\frac{d\Delta_i(t)}{dt} = -\frac{\beta_i(\beta_i + 1)(\Pi - \Pi_j^s)}{\bar{T}_i^2} \left(\frac{\bar{T}_i - t}{\bar{T}_i} \right)^{\beta_i - 1} < 0, \quad \text{for } t \in [0, \bar{T}_i].$$

Thus, $\Delta_i(t)$ is decreasing in t for $t \in [0, \bar{T}_i]$. Moreover, with a larger β_i , $\Delta_i(t)$ decreases more steeply at the beginning and becomes flatter at the end.

We now construct a continuum of equilibria for suppliers using the supply chain optimal solutions and the price margin functions in (4.17).

Theorem 4.5 (Equilibrium with power functions). *There exists a pair of $\beta_1^* > 1$ and $\beta_2^* > 1$ such that for any $(\beta_1, \beta_2) \in [\beta_1^*, \infty) \times [\beta_2^*, \infty)$, the following strategy forms a Nash equilibrium: for $i, j = 1, 2, i \neq j$,*

$$p_i^*(t) = c_i(t) \quad \text{and} \quad r_i^*(t) = \begin{cases} e_i(t) + \Delta_i(t), & 0 \leq t \leq \bar{T}_i, \\ e_i(t), & \text{otherwise,} \end{cases}$$

where Δ_i is given in (4.17). At these equilibria, the buyer's reservation choice is (\bar{T}_1, \bar{T}_2) . The profit split amongst players is

$$\pi_B^* = \Pi_1^s + \Pi_2^s - \Pi \quad \text{and} \quad \pi_i^* = \Pi - \Pi_j^s. \quad (4.19)$$

Proof of Theorem 4.5. Part (a): We first show that given the bids $\{(p_i^*, r_i^*) : i = 1, 2\}$, the buyer's optimal choice is (\bar{T}_1, \bar{T}_2) . Let $\mathbf{p}^* = (p_i^*(t), p_j^*(t))$ and $\mathbf{r}^* = (r_i^*(t), r_j^*(t))$. We consider two cases: (1) $t_1, t_2 > 0$; and (2) $t_i > 0, t_j = 0$.

(1) If $t_1, t_2 > 0$, observe that

$$\begin{aligned}
& \pi_B(t_1, t_2; \mathbf{p}^*, \mathbf{r}^*) \\
&= \sum_{i,j=1,2,j \neq i} \int_0^{t_i} \{[\rho - p_i^*(x)]\bar{F}(h_i(x, t_j)) - r_i^*(x)\} dx \\
&= \sum_{i,j=1,2,j \neq i} \int_0^{t_i} \{[\rho - c_i(x)]\bar{F}(h_i(x, t_j)) - e_i(x)\} dx - \sum_{i,j=1,2,j \neq i} \int_0^{\min(t_i, \bar{T}_i)} \Delta_i(x) dx \\
&= \pi_B(t_1, t_2; \mathbf{c}, \mathbf{e}) - \sum_{i,j=1,2,j \neq i} \int_0^{\min(t_i, \bar{T}_i)} \Delta_i(x) dx. \tag{4.20}
\end{aligned}$$

Then the first partial derivative of π_B with respect to t_i is

$$\frac{\partial \pi_B(t_1, t_2; \mathbf{p}^*, \mathbf{r}^*)}{\partial t_i} = \frac{\partial \pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})}{\partial t_i} - \Delta_i(\min(t_i, \bar{T}_i)).$$

We obtain the derivative at \bar{T}_i as follows,

$$\left. \frac{\partial \pi_B(t_1, t_2; \mathbf{p}^*, \mathbf{r}^*)}{\partial t_i} \right|_{t_i=\bar{T}_i} = \left. \frac{\partial \pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})}{\partial t_i} \right|_{t_i=\bar{T}_i} - \Delta_i(\bar{T}_i) = 0, \tag{4.21}$$

where the second equality follows from the concavity of $\pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})$ and $\Delta_i(\bar{T}_i) = 0$. Therefore, \bar{T}_i satisfies the first order condition.

Next we show there always exists a positive β_i^A such that: when $\beta_i \geq \beta_i^A$, \bar{T}_i is the unique solution to the first order condition. First, for $t_i > \bar{T}_i$, we have

$$\frac{\partial \pi_B(t_1, t_2; \mathbf{p}^*, \mathbf{r}^*)}{\partial t_i} = \frac{\partial \pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})}{\partial t_i} < 0, \tag{4.22}$$

where the inequality follows from the concavity of $\pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})$. Second, again from the concavity of $\pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})$, suppose $\left. \frac{\partial^2 \pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})}{\partial t_i^2} \right|_{t_i=\bar{T}_i} = K < 0$ and we have, for t_i close enough to \bar{T}_i where $t_i < \bar{T}_i$,

$$\frac{\partial \pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})}{\partial t_i} = K(t_i - \bar{T}_i) + O(t_i - \bar{T}_i)^2 > (-K/2)(\bar{T}_i - t_i).$$

Thus

$$\begin{aligned}
\frac{\partial \pi_B(t_1, t_2; \mathbf{p}^*, \mathbf{r}^*)}{\partial t_i} &= \frac{\partial \pi_B(t_1, t_2; \mathbf{c}, \mathbf{e})}{\partial t_i} - \Delta_i(t_i) \\
&> (-K/2)(\bar{T}_i - t_i) - \Delta_i(t_i) \\
&= (\bar{T}_i - t_i) \left(-\frac{K}{2} - \frac{(\beta_i + 1)(\Pi - \Pi_j^s)}{\bar{T}_i^2} \left(\frac{\bar{T}_i - t_i}{\bar{T}_i} \right)^{\beta_i - 1} \right).
\end{aligned}$$

Assume that t_i is close enough to \bar{T}_i so that $\frac{\bar{T}_i - t_i}{\bar{T}_i} < 1/2$, then we can choose a $\beta_i^A > 1$ such that

$$-\frac{K}{2} - \frac{(\beta_i^A + 1)(\Pi - \Pi_j^s)}{\bar{T}_i^2} \left(\frac{1}{2}\right)^{\beta_i^A - 1} > 0.$$

Then, for $t_i < \bar{T}_i$ and t_i close enough to \bar{T}_i , when $\beta_i \geq \beta_i^A$ we have

$$\frac{\partial \pi_B(t_1, t_2; \mathbf{p}^*, \mathbf{r}^*)}{\partial t_i} > 0. \quad (4.23)$$

Combining (4.21), (4.22) and (4.23), we show that when $\beta_i \geq \beta_i^A$, \bar{T}_i is the unique solution to the first order condition, and hence (\bar{T}_1, \bar{T}_2) is the unique maximum for the buyer's choice. Then the buyer's profit from choosing (\bar{T}_1, \bar{T}_2) is

$$\begin{aligned} \pi_B(\bar{T}_1, \bar{T}_2; \mathbf{p}^*, \mathbf{r}^*) &= \pi_B(\bar{T}_1, \bar{T}_2; \mathbf{c}, \mathbf{e}) - \sum_{i,j=1,2,j \neq i} \int_0^{\bar{T}_i} \Delta_i(x) dx \\ &= \Pi - (\Pi - \Pi_i^s) - (\Pi - \Pi_j^s) = \Pi_i^s + \Pi_j^s - \Pi. \end{aligned}$$

(2) If $t_j = 0$, then the buyer purchases from supplier i only. Similar to case (1), we can show there always exists a positive $\beta_i^B > 1$ such that when $\beta_i > \beta_i^B$, \bar{T}_i^s is the unique solution to the first order condition. Then the buyer's profit from choosing \bar{T}_i^s is

$$\begin{aligned} \pi_B(\bar{T}_i^s; p_i^*(t), r_i^*(t)) &= \pi_B(\bar{T}_i^s; c_i(t), e_i(t)) - \int_0^{\min(\bar{T}_i^s, \bar{T}_i)} \Delta_i(x) dx \\ &= \Pi_i^s - (\Pi - \Pi_j^s) = \Pi_i^s + \Pi_j^s - \Pi. \end{aligned}$$

Let $\beta_i^* = \max(\beta_i^A, \beta_i^B)$. For a $\beta_i \geq \beta_i^*$, combining case (1) and case (2), we obtain $\pi_B(\bar{T}_1, \bar{T}_2; \mathbf{p}^*, \mathbf{e}^*) = \pi_B(\bar{T}_i; p_i^*(t), r_i^*(t))$. According to Assumption 4.2, the buyer will choose (\bar{T}_1, \bar{T}_2) . Then we can calculate each supplier's profit $\pi_i^* = \Pi - \Pi_j^s$ for $i, j = 1, 2, j \neq i$.

Part (b): We next show that no supplier unilaterally deviates from the proposed offer. Suppose otherwise and supplier i improves his profit by making a different offer $(\hat{p}_i(t), \hat{r}_i(t))$, i.e. $\hat{\pi}_i > \Pi - \Pi_j^s$. Note that the buyer's profit cannot be less than $\pi_B^{j*} = \Pi_i^s + \Pi_j^s - \Pi$ since otherwise the buyer will reserve from supplier j only, so $\hat{\pi}_B \geq \Pi_i^s + \Pi_j^s - \Pi$. Combining the above two inequalities yields

$$\hat{\pi}_B + \hat{\pi}_i > \Pi - \Pi_j^s + \Pi_i^s + \Pi_j^s - \Pi = \Pi_i^s. \quad (4.24)$$

From Lemma 4.3, we assume $\hat{p}_i(t) = c_i(t)$ without loss of generality. Given the new offer $(\hat{p}_i(t), \hat{r}_i(t))$ (and $(p_j^*(t), r_j^*(t))$), suppose the buyer's reservation choice is (\hat{T}_1, \hat{T}_2) .

Using (4.7) and (4.9) we obtain

$$\begin{aligned}
& \hat{\pi}_B + \hat{\pi}_i \\
&= \int_0^{\hat{T}_i} \left\{ [\rho - \hat{p}_i(x)] \bar{F}(h_i(x, \hat{T}_j)) - \hat{r}_i(x) \right\} dx + \int_0^{\hat{T}_j} \left\{ [\rho - p_j^*(x)] \bar{F}(h_j(x, \hat{T}_i)) - r_j^*(x) \right\} dx \\
&\quad + \int_0^{\hat{T}_i} \left\{ [\hat{p}_i(x) - c_i(x)] \bar{F}(h_i(x, \hat{T}_j)) + \hat{r}_i(x) - e_i(x) \right\} dx \\
&= \int_0^{\hat{T}_i} \left\{ [\rho - c_i(x)] \bar{F}(h_i(x, \hat{T}_j)) - e_i(x) \right\} dx + \int_0^{\hat{T}_j} \left\{ [\rho - c_j(x)] \bar{F}(h_j(x, \hat{T}_i)) - e_j(x) \right\} dx \\
&\quad - \int_0^{\min(\hat{T}_j, \bar{T}_j)} \Delta_j(x) dx \tag{4.25}
\end{aligned}$$

Similar to Part (a) for the buyer's problem, we can show that the right-hand side of (4.25) is maximized at $\hat{T}_i = \bar{T}_i$ and $\hat{T}_j = \bar{T}_j$ (by checking the first order conditions as before). Thus, we have,

$$\begin{aligned}
\hat{\pi}_B + \hat{\pi}_i &\leq \int_0^{\bar{T}_i} \left\{ [\rho - c_i(x)] \bar{F}(h_i(x, \bar{T}_j)) - e_i(x) \right\} dx \\
&\quad + \int_0^{\bar{T}_j} \left\{ [\rho - c_j(x)] \bar{F}(h_j(x, \bar{T}_i)) - e_j(x) \right\} dx - \int_0^{\bar{T}_j} \Delta_j(x) dx \\
&= \Pi - (\Pi - \Pi_i^s) = \Pi_i^s,
\end{aligned}$$

which contradicts (4.24), showing that no supplier has an incentive to deviate unilaterally. This completes the proof. \square

In this class of equilibria with power functions, each supplier sets the execution price to be the execution cost and adds a margin to the reservation cost. The profit split amongst players is the same: each supplier makes a profit equal to his marginal contribution to the supply chain, and the buyer takes the remaining profit. The choice of β_i^* depends on the system parameters, e.g., demand distribution and cost functions. The intention of choosing a large enough β_i is to guarantee that the buyer will select the supply chain optimal solution (\bar{T}_1, \bar{T}_2) . Later in Subsection 4.3.3.2 we will demonstrate this point using an example.

Next we construct an equilibrium where the reservation payment has two components: a lump-sum payment and a marginal reservation price. This is equivalent to an equilibrium with power functions when β_i are infinitely large.

Corollary 4.6 (Equilibrium with lump-sum payments). *There exists an equilibrium where suppliers each set prices to be costs, i.e. $p_i^*(t) = c_i(t)$, $r_i^*(t) = e_i(t)$, and charge a lump-sum reservation payment $K_i = \Pi - \Pi_j^s$, for $i, j = 1, 2, i \neq j$. In this equilibrium, the buyer's reservation choice is first best. The profit split amongst players is given in (4.19).*

Proof of Corollary 4.6. The proof is similar to that of Theorem 4.5. We first show that, given the proposed bids, the buyer's optimal reservation choice is (\bar{T}_1, \bar{T}_2) . Second we demonstrate that no supplier makes more profit by unilaterally deviating from his bid. In fact, the proof is simpler because we do not need to check the second order optimality conditions for the buyer's problem. To avoid repetition, we omit the details here. \square

This result resonates with a result in Cachon and Kok (2010) (see Theorem 4), which shows that in the two-part tariff equilibrium, each manufacturer charges only their costs and extracts his full incremental profit via its fixed fee.

4.3.3 Special case with constant marginal costs

In this section, we consider a special case where suppliers have constant marginal costs. MS study a similar problem where each supplier chooses a pair of reservation price and execution price for their limitless capacity. We use an example provided in MS to demonstrate the difference in equilibrium between our model and theirs.

Example 4.2 (Example 1 in MS). *The buyer's demand is uniformly distributed over $[0, 1]$, so $F(t) = t$ for $t \in [0, 1]$. There are two suppliers and their costs are $(c_1, e_1) = (0, 60)$ and $(c_2, e_2) = (75, 5)$. The retail price is $\rho = 100$. We now examine the supply chain optimal problems.*

- *If the buyer reserves from two suppliers, the supply chain problem is:*

$$\max \left\{ \int_0^{t_1} [(\rho - c_1)\bar{F}(x) - e_1] dx + \int_0^{t_2} [(\rho - c_2)\bar{F}(x + t_1) - e_2] dx : t_1, t_2 \in [0, 1] \right\}.$$

The optimal solution is $(\bar{T}_1, \bar{T}_2) = (4/15, 8/15)$. The supply chain optimal profit is $\Pi = 32/3$.

- *If the buyer chooses supplier 1 only, the supply chain problem is:*

$$\max \left\{ \int_0^{t_1} [(\rho - c_1)\bar{F}(x) - e_1] dx : t_1 \in [0, 1] \right\}.$$

The optimal solution is $\bar{T}_1^s = 2/5$ and the supply chain optimal profit is $\Pi_1^s = 8$.

- *If the buyer chooses supplier 2 only, the supply chain problem is:*

$$\max \left\{ \int_0^{t_2} [(\rho - c_2)\bar{F}(x) - e_2] dx : t_2 \in [0, 1] \right\}.$$

The optimal solution is $\bar{T}_2^s = 4/5$ and the supply chain optimal profit is $\Pi_2^s = 8$.

4.3.3.1 Equilibrium with scalar prices

If suppliers each offer a pair of reservation price and execution price, in equilibrium these two suppliers bid infinitesimally close to each other. MS show that the following is a continuum of ϵ -equilibria parameterized with $p \in [50, 75]$:

$$(p_1^*, r_1^*) = (p_2^*, r_2^*) = \left(p, 60 - \frac{55}{75}p \right).$$

In equilibria, the buyer's reservation choice is

$$T_1^* = \frac{4}{15} \quad \text{and} \quad T_2^* = \frac{40}{3(100-p)}.$$

The profit split amongst players is summarized as follows:

$$\pi_B^* = \frac{8(150-p)^2}{225(100-p)}, \quad \pi_1^* = \frac{8p}{225}, \quad \pi_2^* = \frac{800(75-p)}{9(100-p)^2}, \quad \Pi^* = \frac{32(225-2p)}{9(100-p)} + \frac{800(75-p)}{9(100-p)^2}.$$

Note that all the above equilibria are inefficient (i.e. not supply chain optimal) except the one with $p = 75$. At this efficient equilibrium, each supplier offers a bid $(75, 5)$ which is identical to the supplier 2's cost. The supplier 2's profit is 0, the supplier 1's profit is $8/3$, and the buyer's profit equals 8.

We now demonstrate that the above strategies do not form an equilibrium if we allow suppliers to offer function bids. Suppose supplier 1 chooses the proposed bid $(p_1^*, r_1^*) = (p, 60 - (55/75)p)$, and we now examine the supplier 2's best response in choosing a function bid.

First, if supplier 1 is the sole supplier, the buyer's reserved amount will be the sum of T_1^* and T_2^* , and the buyer's profit is equal to π_B^* . Therefore, we have

$$T_1^{s*} = \frac{600-4p}{15(100-p)} \quad \text{and} \quad \pi_B^{1*} = \frac{8(150-p)^2}{225(100-p)}.$$

Second, we show that the following strategy for supplier 2 improves his profit: setting prices to be costs and charging a lump-sum payment,

$$(\tilde{p}_2, \tilde{r}_2, \tilde{K}_2) = \left(75, 5, \frac{32(75-p)}{9(100-p)} \right).$$

Given this offer (and the supplier 1's offer (p_1^*, r_1^*)), we can show that the interior solution for the buyer's problem is

$$(\tilde{T}_1, \tilde{T}_2) = \left(\frac{4}{15}, \frac{8}{15} \right),$$

and the buyer's profit from choosing $(\tilde{T}_1, \tilde{T}_2)$ is equal to π_B^{1*} . Also if the buyer purchases from only supplier 2, the buyer makes a profit of π_B^{1*} as well. According to the tie-breaking rule in Assumption 4.2, the buyer will select $(\tilde{T}_1, \tilde{T}_2)$. Then the supplier 2's

profit becomes

$$\tilde{\pi}_2 = \tilde{K}_2 = \frac{32(75-p)}{9(100-p)},$$

which is greater than π_2^* because,

$$\tilde{\pi}_2 - \pi_2^* = \frac{32(75-p)}{9(100-p)} - \frac{800(75-p)}{9(100-p)^2} = \frac{32(75-p)^2}{9(100-p)^2} \geq 0,$$

where the equality holds only when $p = 75$. Therefore, we have demonstrated that the equilibria in MS do not hold if we allow suppliers to offer a function bid.

4.3.3.2 Equilibrium with function bids

In our setting where suppliers each offer a function bid, we construct the following two equilibria: an equilibrium with power functions as shown in Theorem 4.5; and the equilibrium with lump-sum payments as shown in Corollary 4.6.

Equilibrium with power functions The following bids form a Nash equilibrium:

$$p_1^*(t) = 0 \quad \text{and} \quad r_1^*(t) = \begin{cases} 60 + \left(\frac{15}{2}\right)^3 \left(t - \frac{4}{15}\right)^2, & 0 \leq t \leq \frac{4}{15}, \\ 60, & \frac{4}{15} < t \leq 1. \end{cases}$$

$$p_2^*(t) = 75 \quad \text{and} \quad r_2^*(t) = \begin{cases} 5 + \left(\frac{15}{4}\right)^3 \left(t - \frac{8}{15}\right)^2, & 0 \leq t \leq \frac{8}{15}, \\ 5, & \frac{8}{15} < t \leq 1. \end{cases}$$

Equilibrium with lump-sum payments The following bids form a Nash equilibrium:

$$(p_1^*, r_1^*, K_1) = \left(0, 60, \frac{8}{3}\right) \quad \text{and} \quad (p_2^*, r_2^*, K_2) = \left(75, 5, \frac{8}{3}\right).$$

At both equilibria, the buyer's optimal reservation choice is $(T_1^*, T_2^*) = (4/15, 8/15)$. The profit split amongst players is given as follows:

$$\pi_B^* = \frac{16}{3} \quad \text{and} \quad \pi_1^* = \pi_2^* = \frac{8}{3}.$$

In these equilibria, each supplier's profit equals his contribution to the supply chain system and the buyer takes the remaining profit.

We now use this example to demonstrate that an inappropriate β_i may lead to the buyer not choosing the supply chain optimal choice. For example, if we set $\beta_i = 1$ for (4.17), then the bids (with linear marginal prices) are given as follows:

$$p_1^*(t) = 0 \quad \text{and} \quad r_1^*(t) = \begin{cases} 60 + 20 \left(1 - \frac{15}{4}t\right), & 0 \leq t \leq \frac{4}{15}, \\ 60, & \text{otherwise.} \end{cases}$$

$$p_2^*(t) = 75 \quad \text{and} \quad r_2^*(t) = \begin{cases} 5 + 10 \left(1 - \frac{15}{8}t\right), & 0 \leq t \leq \frac{8}{15}, \\ 5, & \text{otherwise.} \end{cases}$$

We will show the buyer may not choose the first best solution $(4/15, 8/15)$ given the linear bids. To illustrate, we plot in Figure 4.2 the contours for the buyer's profits under the above three strategies. For the strategy with linear marginal price (Subfigure 4.2(a)), there are a number of other interior solutions except (\bar{T}_1, \bar{T}_2) . Thus, we cannot guarantee that the buyer will choose the first best solution. For the strategies with power functions and lump-sum payments (Subfigures 4.2(b) and 4.2(c)), there are three maxima for the buyer choice, i.e. an interior maximum $(4/15, 8/15)$ and two maxima at boundaries $(0, 4/5)$ and $(2/5, 0)$. Then the buyer will choose $(4/15, 8/15)$ according to Assumption 4.2.

4.3.3.3 Comparisons

One of the key results in this chapter is that an optimal strategy for each supplier is to set execution prices to be execution costs. This result does not hold in general for the problem considered by MS as we show earlier.

Comparing with MS, we also find that the buyer makes a higher profit if each supplier submits a pair of reservation price and an execution price rather than a function bid. However, the suppliers are better off if they submit function bids. To see this, note that for this example the buyer profit is

$$\frac{8(150 - p)^2}{225(100 - p)} \geq \frac{64}{9} > \frac{16}{3},$$

and the supplier profits are

$$\begin{aligned} \frac{8p}{225} &\leq \frac{8}{3}, \\ \frac{800(75 - p)}{9(100 - p)^2} &\leq \frac{8}{9} < \frac{8}{3}. \end{aligned}$$

The result that the sophisticated contract makes the buyer worse off resonates with a result by Cachon and Kok (2010), who consider a setting where two manufacturers sell differentiated products to a retailer and study the preferences of different players over different contract types. One of their results is that complex contracts do not benefit the buyer when the manufacturers' products are not close substitutes.

In addition, we show that in our equilibria the buyer's reservation choice is first best, while in MS only the equilibrium with $p = 75$ is efficient because

$$\frac{32(225 - 2p)}{9(100 - p)} + \frac{800(75 - p)}{9(100 - p)^2} \leq \frac{32}{3}.$$

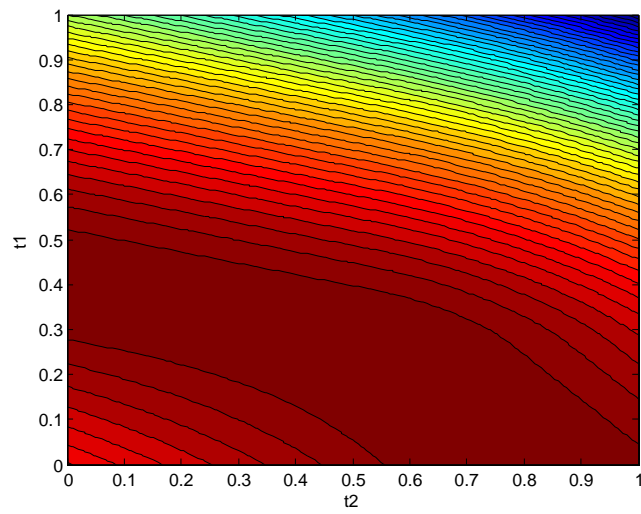
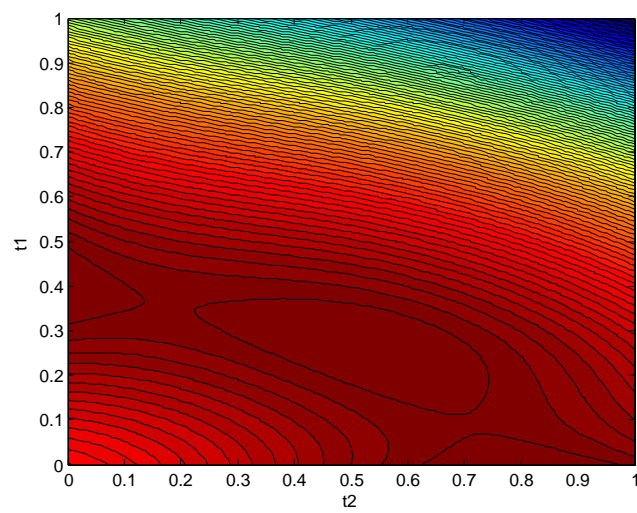
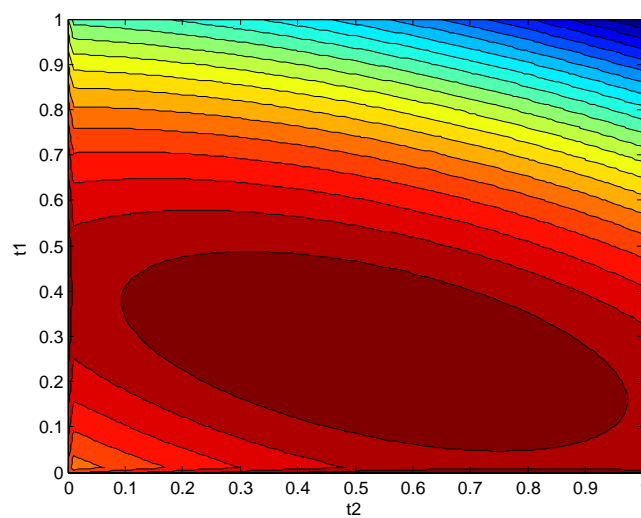
(a) Linear function ($\beta_i = 1$)(b) Power function ($\beta_i = 2$)(c) Lump-sum payment ($\beta_i = \infty$)

FIGURE 4.2: The contours of buyer profit for the three bidding strategies

In the equilibrium with $p = 75$, supplier 2 makes zero profits, while the buyer makes a higher profit than that when function bids are allowed. In fact, the supplier 2's profit predicted by our model goes to the buyer in the equilibrium given by MS (i.e. $8/3 + 16/3 = 8$).

4.4 The Case with Multiple Suppliers

In this section, we consider an extension in which there are n competing suppliers where $n > 2$. We write $N = \{1, \dots, n\}$ as the set of supplier indices. The buyer's reservation choice is denoted by a vector $\mathbf{x} = (x_1, \dots, x_n)$ where x_i is the reserved amount from supplier i . Let $\mathbf{x}_{-i} = (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)$.

Suppose the supplier bids are $\{(p_j(t), r_j(t)) : j = 1, \dots, n\}$. If the buyer's reservation choice is \mathbf{t} , then the cumulative amount of capacity with execution prices less than $p_i(x)$ is,

$$h_i(x, \mathbf{t}_{-i}) = x + \sum_{j \neq i} \gamma_j(x, t_j), \quad \text{for } x \in [0, t_i], \quad (4.26)$$

where $\gamma_i(x, t_j)$ is defined in (4.1). The buyer's profit from choosing \mathbf{t} is given by,

$$\pi_B(\mathbf{t}) = \sum_{i=1}^n \int_0^{t_i} \{[\rho - p_i(x)] \bar{F}(h_i(x, \mathbf{t}_{-i})) - r_i(x)\} dx, \quad (4.27)$$

where $h_i(\cdot, \cdot)$ is given in (4.26). Then the buyer's problem is to maximize her expected profit by choosing an optimal reservation choice \mathbf{t} :

$$\max \{ \pi_B(\mathbf{t}) : \mathbf{t} \in [0, \bar{d}]^n \}.$$

For $i = 1, \dots, n$, the first partial derivative with respect to t_i is given as follows:

$$\frac{\partial \pi_B}{\partial t_i} = (\rho - p_i(t_i)) \bar{F}(h_i(t_i, \mathbf{t}_{-i})) - r_i(t_i) - \sum_{j \neq i} \int_0^{t_j} [\rho - p_j(x)] f(h_j(x, \mathbf{t}_{-j})) \left(\frac{\partial h_j(x, \mathbf{t}_{-j})}{\partial t_i} \right) dx.$$

Denote by $\mathbf{T} = (T_1, \dots, T_n)$ the buyer's optimal reservation choice. Further let $\mathbf{T}_{-i} = (T_1, \dots, T_{i-1}, T_{i+1}, \dots, T_n)$. Similar to (4.5) the first order conditions are: for $i = 1, \dots, n$,

$$\underbrace{(\rho - p_i(T_i)) \bar{F}(h_i(T_i, \mathbf{T}_{-i})) - r_i(T_i)}_{\underbrace{\sum_{j \neq i} \int_{\gamma_i(T_i, T_j)}^{T_j} [\rho - p_j(x)] f(h_j(x, \mathbf{T}_{-j})) dx}_{=0}} = 0. \quad (4.28)$$

We now look at each supplier's best response problem. Given the other supplier bids $\{(p_j(t), r_j(t))\}_{j \neq i}$, we examine how supplier i responds by choosing $(p_i(t), r_i(t))$. To distinguish from $\pi_B(\mathbf{t})$ in (4.27), we write $\pi_B(\mathbf{t}; p_i(t), r_i(t))$ for the buyer's profit of

choosing \mathbf{t} provided that supplier i offers the bid $(p_i(t), r_i(t))$. We have

$$\pi_B(\mathbf{t}; p_i(t), r_i(t)) = \sum_{i=1}^n \int_0^{t_i} \{[\rho - p_i(x)]\bar{F}(h_i(x, \mathbf{t}_{-i})) - r_i(x)\} dx.$$

Suppose the optimal buyer reservation choice is \mathbf{T} given the supplier i 's bid $(p_i(t), r_i(t))$ (and the other supplier bids), that is,

$$\mathbf{T} = \arg \max \{ \pi_B(\mathbf{t}; p_i(t), r_i(t)) : \mathbf{t} \in [0, \bar{d}]^n \}. \quad (4.29)$$

Then we can write down the supplier i 's profit as follows,

$$\pi_i(p_i(t), r_i(t); \mathbf{T}) = \int_0^{T_i} \{ [p_i(x) - c_i(x)]\bar{F}(h_i(x, \mathbf{T}_{-i})) + r_i(x) - e_i(x) \} dx.$$

The supplier i 's best response problem is to maximize his profit by choosing $(p_i(t), r_i(t))$:

$$\max_{p_i(t), r_i(t)} \pi_i(p_i(t), r_i(t); \mathbf{T}), \quad (4.30)$$

subject to (4.29), which gives the buyer's optimal reservation choice given the supplier i 's offer $(p_i(t), r_i(t))$ (as well as $\{(p_j(t), r_j(t))\}_{j \neq i}$).

Before solving the supplier i 's best response problem, we consider the buyer's reservation problem when supplier i charges only his costs, i.e. when $(p_i(t), r_i(t)) = (c_i(t), e_i(t))$. Suppose the optimal buyer choice is $\hat{\mathbf{T}}$ in this case and the optimal buyer profit is denoted by $\pi_B(\hat{\mathbf{T}}; c_i(t), e_i(t))$. Further suppose the optimal buyer profit is π_B^{-i} if supplier i is unavailable. We now show an optimal response for supplier i in Lemma 4.7.

Lemma 4.7. *Given $\{(p_j(t), r_j(t))\}_{j \neq i}$, it is optimal for supplier i to set $p_i(t) = c_i(t)$ and $r_i(t) = e_i(t)$ as well as charge a fixed amount $K_i = \pi_B(\hat{\mathbf{T}}; c_i(t), e_i(t)) - \pi_B^{-i}$.*

Proof of Lemma 4.7. The proof is similar to that of Lemma 4.3 and is omitted. \square

Next we will find an equilibrium for suppliers. An equilibrium can be constructed using the results for the supply chain optimal solutions. We begin by examining the supply chain optimal problems where each supplier i sets prices to be costs, i.e. $(p_i(t), r_i(t)) = (c_i(t), e_i(t))$. Let $\mathbf{c} = (c_1(t), \dots, c_n(t))$ and $\mathbf{e} = (e_1(t), \dots, e_n(t))$. With the reservation choice \mathbf{t} we have the supply chain profit as follows:

$$\pi_B(\mathbf{t}; \mathbf{c}, \mathbf{e}) = \sum_{i=1}^n \int_0^{t_i} \{ [\rho - c_i(x)]\bar{F}(h_i(x, \mathbf{t}_{-i})) - e_i(x) \} dx.$$

Let $\bar{\mathbf{T}} = (\bar{T}_1, \dots, \bar{T}_n)$ be the optimal reservation choice, that is,

$$\bar{\mathbf{T}} = \arg \max \{ \pi_B(\mathbf{t}; \mathbf{c}, \mathbf{e}) : \mathbf{t} \in [0, \bar{d}]^n \}$$

We also denote by Π the supply chain optimal profit, i.e. $\Pi = \pi_B(\bar{\mathbf{T}}; \mathbf{c}, \mathbf{e})$.

For any subset $S \subset N$, denote by $\bar{\mathbf{T}}^{-S} = (\bar{T}_1^{-S}, \dots, \bar{T}_n^{-S})$ the supply chain optimal solution when the buyer purchases from only the suppliers in $N \setminus S$. Formally,

$$\bar{\mathbf{T}}^{-S} = \arg \max \left\{ \pi_B(\mathbf{t}^{-S}; \mathbf{c}, \mathbf{e}) : \mathbf{t} \in [0, \bar{d}]^n, \text{ and } t_i^{-S} = 0 \text{ for } i \in S \right\},$$

where

$$\pi_B(\mathbf{t}^{-S}; \mathbf{c}, \mathbf{e}) = \sum_{i=1}^n \int_0^{t_i^{-S}} \left\{ [\rho - c_i(x)] \bar{F}(h_i(x, \mathbf{t}_{-i}^{-S})) - e_i(x) \right\} dx.$$

Note we have $\bar{T}_i^{-S} = 0$ for $i \in S$. The optimal supply chain profit in this case is denoted by Π^{-S} where $\Pi^{-S} = \pi_B(\bar{\mathbf{T}}^{-S}; \mathbf{c}, \mathbf{e})$. Clearly, we have $\Pi \geq \Pi^{-S}$, for $S \subset N$.

To characterize an equilibrium for suppliers, we make the following assumption regarding the supply chain optimal profit.

Assumption 4.3. *The supply chain optimal profit is submodular, i.e. for $S \subset N$ and $j, k \notin S$, we have,*

$$\Pi^{-S} + \Pi^{-(S \cup \{j, k\})} \leq \Pi^{-(S \cup \{j\})} + \Pi^{-(S \cup \{k\})}. \quad (4.31)$$

We conjecture that this assumption holds when suppliers have non-decreasing marginal costs for both execution and reservation.

Conjecture 4.1. *If both the marginal reservation cost and the marginal execution cost are non-decreasing for each supplier, then the supply chain optimal profit is submodular.*

In support of the above conjecture we explore an example where suppliers have constant marginal costs.

Example 4.3. *Suppose the buyer demand D follows a uniform distribution over $[0, 1]$. There are three suppliers whose costs are: $c_1 = 1, e_1 = 3$; $c_2 = 2.5, e_2 = 2$; and $c_3 = 5, e_3 = 1$. The retail price is $\rho = 10$. We solve the supply chain optimal problems and summarize the optimal reservation choices and profits in Table 4.1. To save space, we skip the detailed calculations here (see Example 4.2 for similar calculations).*

Using the results in the table, we can show that:

$$\begin{aligned} \Pi + \Pi^{-\{2,3\}} &< \Pi^{-\{2\}} + \Pi^{-\{3\}} \\ \Pi + \Pi^{-\{1,3\}} &= \Pi^{-\{1\}} + \Pi^{-\{3\}} \\ \Pi + \Pi^{-\{1,2\}} &< \Pi^{-\{1\}} + \Pi^{-\{2\}} \\ \Pi &< \Pi^{-\{1\}} + \Pi^{-\{2,3\}} \\ \Pi &< \Pi^{-\{2\}} + \Pi^{-\{1,3\}} \\ \Pi &< \Pi^{-\{3\}} + \Pi^{-\{1,2\}}. \end{aligned}$$

TABLE 4.1: The supply chain optimal reservation choices and profits

Available Suppliers	S	\bar{T}_1^{-S}	\bar{T}_2^{-S}	\bar{T}_3^{-S}	Π^{-S}
{1, 2, 3}	\emptyset	1/3	4/15	1/5	2.1333
{1, 2}	{3}	1/3	2/5	0	2.1
{1, 3}	{2}	1/2	0	3/10	2.1
{2, 3}	{1}	0	3/5	1/5	2.05
{1}	{2, 3}	2/3	0	0	2
{2}	{1, 3}	0	11/15	0	2.0167
{3}	{1, 2}	0	0	4/5	1.6

Therefore, the submodularity property in Assumption 4.3 is satisfied.

The submodularity property in Assumption 4.3 may not hold when suppliers have decreasing marginal costs as we demonstrate with the following example.

Example 4.4. Suppose the demand is fixed with $D = 10$ and the retail price is $\rho = 20$. There are three suppliers. Supplier i and supplier j have the same costs with $c_i(t) = c_j(t) = e_i(t) = e_j(t) = 0$, for $t \in [0, 5]$. The supplier k 's costs are $c_k(t) = 0$ and $e_k(t) = 10 - t$ for $t \in [0, 10]$. So both supplier i and supplier j have the capacity of 5 and the supplier k 's capacity is 10.

We now look at the supply chain optimal problems. If all the three suppliers are available, the buyer will choose 5 units both from supplier i and supplier j . The supply chain optimal profit is $\Pi = 200$. If only suppliers k and i (or j) are available, the buyer will choose 5 units both from k and i (or j). The supply chain optimal profit is $\Pi^{-\{i\}} = \Pi^{-\{j\}} = 162.5$. If supplier k is the sole supplier, the buyer will choose 10 units from k and the supply chain optimal profit is $\Pi^{-\{i,j\}} = 150$. Therefore, we have $\Pi + \Pi^{-\{i,j\}} = 350 > \Pi^{-\{i\}} + \Pi^{-\{j\}} = 325$, which contradicts the submodularity property.

As a preliminary step, we show a result based on Assumption 4.3, which is similar to Corollary 3.7 in Chapter 3.

Corollary 4.8. Under Assumption 4.3, for any $S \subseteq N$, we have,

$$\Pi - \Pi^{-S} \geq \sum_{i \in S} (\Pi - \Pi^{-\{i\}}). \quad (4.32)$$

Proof of Corollary 4.8. The proof is the same with that of Corollary 3.7 in Chapter 3, and is omitted here. \square

We are now in a position to show an equilibrium. Following the similar idea as in the two-supplier case, we show the following strategies form an equilibrium, which is an extension of Corollary 4.6 to the case with more than two suppliers.

Theorem 4.9. *Under Assumption 4.3, there exists an equilibrium $\{(p_i^*(t), r_i^*(t)) : i \in N\}$ where, for $i = 1, \dots, n$, supplier i sets prices to be costs, i.e. $p_i^*(t) = c_i(t)$, $r_i^*(t) = e_i(t)$, and charges a lump-sum reservation payment $K_i = \Pi - \Pi^{-\{i\}}$. In this equilibrium, the buyer's reservation choice is $\bar{\mathbf{T}}$. The profit split is*

$$\pi_B^* = \Pi - \sum_{i=1}^n (\Pi - \Pi^{-\{i\}}) \quad \text{and} \quad \pi_i^* = \Pi - \Pi^{-\{i\}}. \quad (4.33)$$

Proof of Theorem 4.9. We first show that, given the supplier bids $\{(p_i^*(t), r_i^*(t), K_i) : i \in N\}$, the buyer will choose the first best solution $\bar{\mathbf{T}}$. Note that once the buyer decides to reserve from the suppliers in $N \setminus S$, the buyer's optimal choice must be $\bar{\mathbf{T}}^{-S}$ (due to $p_i^*(t) = c_i(t)$ and $r_i^*(t) = e_i(t)$). Therefore, the buyer's reservation choice problem boils down to the supplier selection problem, and is a combinatorial optimization program. Provided that the buyer chooses the suppliers in $N \setminus S$, then her profit is given as follows:

$$\begin{aligned} \pi_B^{-S} &= \Pi^{-S} - \sum_{i \in N \setminus S} (\Pi - \Pi^{-i}) \\ &= \Pi^{-S} + \sum_{i \in S} (\Pi - \Pi^{-i}) - \sum_{i \in N} (\Pi - \Pi^{-i}) \\ &\leq \Pi^{-S} + \Pi - \Pi^{-S} - \sum_{i \in N} (\Pi - \Pi^{-i}) \\ &= \Pi - \sum_{i \in N} (\Pi - \Pi^{-i}), \end{aligned}$$

where the inequality follows from Corollary 4.8. Note that the buyer's profit from choosing $\bar{\mathbf{T}}$ is $\pi_B^* = \Pi - \sum_{i \in N} (\Pi - \Pi^{-i})$. Therefore, the buyer's optimal choice must be $\bar{\mathbf{T}}$ according to Assumption 4.2. Moreover, we find that the buyer's profit when choosing $\bar{\mathbf{T}}^{-i}$ is,

$$\pi_B^{-i} = \Pi^{-i} - \sum_{j \neq i} (\Pi - \Pi^{-j}) = \Pi - \sum_{i=1}^n (\Pi - \Pi^{-i}) = \pi_B^*. \quad (4.34)$$

Next we show that no supplier has an incentive to deviate from the proposed strategy. Suppose otherwise and supplier i makes a higher profit by offering a different bid $(\tilde{p}_i(t), \tilde{r}_i(t), \tilde{K}_i)$. From Lemma 4.7, we assume $(\tilde{p}_i(t), \tilde{r}_i(t)) = (p_i^*(t), r_i^*(t))$ and $\tilde{K}_i > K_i$. We will show that the buyer is better off not to choose supplier i . First, from (4.34) the buyer's optimal profit when not choosing supplier i is π_B^* . Second, any set of bids that includes i will mean that the buyer makes a smaller profit than π_B^* . Therefore, the buyer will not choose supplier i , implying that no supplier has an incentive to unilaterally change his bid. This completes the proof. \square

4.5 Conclusions

In this chapter, we have considered a supplier competition game in a supply option market where suppliers have general cost functions. Suppliers each submit a function bid consisting of a reservation price function and an execution price function. Then the buyer decides how much capacity to reserve from each supplier before knowing the actual demand.

When observing the competitors' bids, we show an optimal strategy for a supplier is to set the execution price to be the execution cost and add a margin on the reservation cost. This implies that suppliers make profits only from the buyer's reservation payments. Regarding the strategic interaction amongst suppliers, we characterize a class of equilibria where the buyer's reservation choice is first best, each supplier's profit equals his marginal contribution to the supply chain and the buyer takes the remaining profit. The implication is that by allowing suppliers to compete using function bids, the supply chain is coordinated. This chapter complements Chapter 3 by allowing general cost functions for suppliers. Comparing with Martínez-de Albéniz and Simchi-Levi (2009) it also highlights the important impact of the strategy space on supplier competition.

Chapter 5

Concluding Remarks

5.1 Main Results

This thesis studies how suppliers compete with each other for a buyer's business by considering subcontracting, commitment, and capacity reservation.

In a procurement setting with multiple units to buy, one of the fundamental decisions for a buyer is whether to split an order across multiple suppliers (multi-sourcing) or to award its entire order to a single supplier (single-sourcing). Chapter 2 studies the role of subcontracting and single-sourcing commitment in supplier bidding. We show that the buyer prefers to commit to single-sourcing whether or not subcontracting is considered. The intuition is that the competition between suppliers is dampened if the buyer does not commit to single-sourcing. Regarding whether the buyer should allow subcontracting, we find the bargaining power of the winning (or losing) supplier at the subcontracting stage plays a vital role in determining the subgame perfect Nash equilibrium. Our analysis shows that, counter-intuitively, subcontracting does not necessarily hurt the buyer. The buyer is better off to allow subcontracting when the winning supplier's bargaining power exceeds a threshold.

Capacity reservation is important when suppliers need to invest in capacity to meet a further order and the future demand is uncertain. To hedge against financial risks, suppliers often require a buyer to reserve capacity in advance. In Chapter 3, we consider the situation where each supplier owns a "block" of capacity characterized by a reservation cost and an execution cost as well as a block size. Suppliers each offer a reservation price and an execution price. Then the buyer decides which blocks to reserve. We first show that the optimal buyer profit function is submodular when suppliers have equal-size blocks. This submodularity property fails when supplier blocks are of different sizes. Then we find that it is optimal for suppliers to set execution prices to be execution costs, thus they make profits only from the buyer's reservation payments. In the case with equal-size blocks, we show in equilibrium the buyer's choice matches

the supply chain optimal choice, each supplier's profit equals his marginal contribution and the buyer takes the remaining profit. We also provide a procedure to construct an equilibrium for the case with unequal-size blocks.

Chapter 4 complements Chapter 3 by studying a continuous version of the problem where suppliers face general cost functions. The suppliers each quote a function bid consisting of a reservation price function and an execution price function. Then the buyer, after receiving a set of function bids, decides how much to reserve from each supplier. We show that some results in Chapter 3 still hold in this setting. To be specific, it is optimal for suppliers to set execution prices to be execution costs. Under mild conditions, we are able to show that, in a class of equilibria, each supplier's profit equals his marginal contribution to the supply chain system and the buyer takes the remaining profit.

5.2 Future Research

This research opens up a number of follow-up questions. In this subsection, we discuss several extensions.

A natural extension for Chapter 2 is to consider the case with more than two suppliers. This requires dealing with each procurement scenario separately because they present different types of challenges. Under the order splitting scenario, this study shows that the equilibrium result will still hold if the supply chain optimal profit is sub-modular in the sense that each supplier makes a decreasing marginal contribution to the supply chain optimal profit as the number of available suppliers increases. Whether this condition is satisfied depends largely on supplier costs. It is challenging to characterize the cost functions for which the sub-modularity property holds. However, the main results carry over to the case with more than two *symmetric* suppliers. For the single-sourcing commitment scenario, the question of how suppliers compete in bidding relates directly to the modelling of subcontracting negotiation between the winning supplier and losing suppliers. The subcontracting negotiation may be addressed by drawing on the multilateral bargaining theory.

Another extension for Chapter 2 is the incomplete information setting where suppliers each have private information regarding their own costs. For example, supplier i may have the cost function parameterized by β_i which follows a known distribution, and the aim is to characterize the Bayesian Nash equilibrium for suppliers.

In Chapter 3, we formulate a one-shot competition game with reservation bidding in which suppliers each submit a reservation price and an execution price simultaneously, while the buyer makes a two-stage decision (of reservation and execution). It remains unclear how the competitive dynamics change if we separate the supplier bidding decision into two stages as well. In particular, we can study a dynamic game where suppliers

compete in the first stage by offering a reservation price, and then compete by choosing an execution price in the second stage. Moreover, it is intriguing to compare the dynamic game with the one-shot game. Doing so may have the potential to help a buyer design a better sourcing strategy.

Chapter 3 builds a two-dimensional price competition model where capacity is exogenous. It would be interesting to endogenize capacity decisions in addition to pricing decisions. Specifically, we can consider a one-shot game where suppliers each offer a three-dimensional bid which consists of an execution price, a reservation price and an offered quantity (which is no greater than its block size). A similar problem has been examined by Wu and Kleindorfer (2005) who focus on the interaction between option markets and spot markets, and consider only execution costs for suppliers. We leave these extensions for future research.

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