CHARACTERIZATION OF THE FUNDAMENTAL PROPERTIES OF WIRELESS CSMA MULTI-HOP NETWORKS

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Abstract

A wireless multi-hop network consists of a group of decentralized and self-organized wireless devices that collaborate to complete their tasks in a distributed way. Data packets are forwarded collaboratively hop-by-hop from source nodes to their respective destination nodes with other nodes acting as intermediate relays. These networks can be used to collect and exchange data for a variety of applications in both civilian and military fields. Existing and future applications in wireless multi-hop networks will greatly benefit from better understanding of the fundamental properties of such networks. This thesis is concerned with wireless multi-hop networks operating with distributed Media Access Control (MAC) protocols - Carrier Sense Multiple Access (CSMA) protocols. In recent decades, CSMA protocols have become prevailing with widespread adoption.

In this thesis we explore two fundamental properties of wireless CSMA multi-hop networks, connectivity and capacity. We start off with investigating the connectivity of wireless CSMA multi-hop networks. A network is said to be connected if and only if there is at least one (multi-hop) path between any pair of nodes in the network. Despite that interference is a major performance-limiting factor, the research results reported in the literature is very limited on the connectivity properties of wireless multi-hop networks where the impact of interference is taken into account. Therefore in this thesis we investigate the critical transmission power for asymptotic connec-

tivity in large wireless CSMA multi-hop networks under the SINR model that can account for the mutual interference due to multiple concurrent transmissions. The critical transmission power is the minimum transmission power each node needs to transmit to guarantee that the resulting network is connected asymptotically almost surely. Both upper bound and lower bound of the critical transmission power are obtained analytically. The two bounds are tight and differ by a constant factor only. The comparison with previous work assuming no interference shows that the transmission power only needs to be increased by a constant factor to combat interference and maintain connectivity. This result is also in contrast to the previous results considering the connectivity of ALOHA networks under the SINR model.

Next we shift our focus to the capacity of wireless CSMA multi-hop networks. First, we develop a distributed routing algorithm where each node makes routing decisions based on local information only. This makes the routing algorithm compatible with the distributed nature of large wireless CSMA multi-hop networks. Second, we demonstrate that by carefully choosing controllable parameters of the CSMA protocols, together with our routing algorithm, a network running distributed CSMA protocols is able to achieve the order-optimal throughput scaling law of $\Theta\left(\frac{1}{\sqrt{n}}\right)$, which is the same as that of large networks employing centralized routing and scheduling algorithms. Note that scaling laws are only up to order and most network design choices have a significant effect on the constants preceding the order while not affecting the scaling law. Therefore we take a further step to analyze the pre-constant by giving an upper and a lower bound of throughput. The tightness of the bounds is validated using simulations.

To   Hui and Jiayi

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Statement of Originality

I hereby declare that this thesis, submitted in fulfillment of the requirements for the award of Doctor of Philosophy, in the School of Electrical and Information Engineering, the University of Sydney, is my own work unless otherwise referenced or acknowledged. The document has not been previously submitted for the award of any other qualification at any educational institution. Most of the results contained herein have been published, accepted for publication, or submitted for publication, in journals or conferences of international standing. My contribution in terms of published material is listed in "Related publications" chapters.

The original motivation to pursue research in this field was provided by thesis supervisor: Professor Guoqiang Mao from the University of Sydney.

Related Publications

[J1]   T. Yang, G. Mao and W. Zhang, "Connectivity of Large-Scale CSMA Networks," *IEEE Transactions on Wireless Communications*, vol. 11, pp. 2266-2275, 2012.

[J2]   T. Yang, G. Mao, W. Zhang and Xiaofeng Tao, "Transport Capacity of Distributed Wireless CSMA Networks," *IEEE Transactions on Wireless Communications*, Vol. 13, No. 10, pp. 5635-5647, 2014.

[C1]   T. Yang, G. Mao and W. Zhang, "Connectivity of Wireless CSMA Multi-Hop Networks," in *Proceedings of IEEE ICC*, 2011, pp. 1-5.

[C2]   T. Yang, G. Mao and W. Zhang, "Capacity of Interference-limited Three Dimensional CSMA Networks," in *Proceedings of IEEE ICC*, 2012.

[C3]   T. Yang, G. Mao and W. Zhang, "A necessary condition for connected wireless CSMA multi-hop networks," in *Proceedings of IEEE PIMRC*, 2012, pp. 1528-1533.

[C4]   T. Yang, G. Mao and W. Zhang, "An upper bound on transmission capacity of wireless CSMA networks," in *Proceedings of IEEE Globecom*, 2012, pp. 458-463.

[C5]   T. Yang, G. Mao and W. Zhang, "Connectivity of Wireless Information-Theoretic Secure Networks," accepted to appear in *Proceedings of IEEE Globecom*, 2014.

Contents

Chapter 1

Introduction

This chapter gives a brief review of two fundamental problems of the wireless multi-hop networks, followed by the research problems addressed in this thesis. The motivations behind each research problem and our main contributions for each problem are included. A concise outline for the remainder of the thesis is given in the last section of this chapter.

1.1   Fundamental Problems in Wireless Multi-hop Networks

In recent decades, technological advances have made it plausible to envisage the development of massively large communication systems composed of low-cost and ubiquitous wireless devices. These networks, often referred to as wireless multi-hop networks, can be used to collect and exchange data for a variety of applications in both civilian and military fields [1], such as human communication, environmental and habitat monitoring, security and surveillance. There are two defining features that characterize a wireless multi-hop network:

- Wireless devices (or, henceforth wireless nodes) are self-organized to form a network and collaborate to complete their tasks in a distributed way;

- Data packets are forwarded collaboratively hop-by-hop from source nodes to their respective destination nodes with other nodes acting as intermediate relays .

Various questions are of interest in this context of wireless multi-hop networks. The first and most fundamental one deals with connectivity, which expresses a global property of the network: can information be transferred through the network from sources to destinations? In other words, are any two nodes in the network connected by at least one (multi-hop) path of adjacent links? The second question naturally arises following the first one: what is the network capacity in terms of sustainable information flow under the given connectivity regime? In the remainder of this section, we first introduce these two fundamental problems which have attracted significant attention from researchers and developers in the field, then the research problems addressed in this thesis and the motivations behind.

1.1.1   Connectivity

Connectivity is considered as one of the most fundamental properties of wireless multi-hop networks as it is a prerequisite for providing many network functions, e.g. routing, localization, and topology control [2–5]. The research on connectivity problems in wireless multi-hop networks dates back to the work of Gilbert *et al.* [6] in 1961 who considered a random network formed by connecting pairs of nodes of a Poisson point process (p.p.) on an infinite plane if their Euclidean distance is smaller than or equal to a certain threshold, known as the *transmission range*. Gilbert's connection model is often referred to as the unit disk model (UDM). Using the above network

model, they established the existence of a critical transmission range, above which the network contains a connected component formed by an unbounded collection of nodes, i.e., the network *percolates* [7,8]; in contrast, below the critical transmission range, only components of finite size exist. Since their pioneering work [6], percolation models have been an important branch in the field of modeling connectivity of wireless multi-hop networks. Percolation models mostly deal with networks on infinite plane with nodes distributed following a Poisson p.p.. Studying random network models on an infinite plane allows the possibility of observing a phase transition in connectivity behavior: depending on some critical parameters such as node density or transmission range, the network percolates or only components of finite size are formed.

In this thesis, our focus is on studying asymptotic connectivity of finite networks that grow sufficiently large. A wireless multi-hop network is said to be *connected* if and only if (iff) there is at least one (multi-hop) path between any pair of nodes in the network. Significant results have been obtained in the study of connectivity problem using the random geometric graph and the UDM, which is usually obtained by randomly and uniformly distributing n nodes in a given area and connecting any two nodes iff their Euclidean distance is smaller than or equal to the transmission range $r(n)$. Particularly in the late 1990s and early 2000s, Penrose [9] and Gupta and Kumar [2] proved that in a disk of unit area, the above network with a transmission range of $r(n) = \sqrt{\frac{\log n + c(n)}{\pi n}}$ is asymptotically almost surely (a.a.s.) connected as $n \to \infty$ iff $c(n) \to \infty$. An event ξ_n depending on n is said to occur a.a.s. iff the probability that the event occurs approaches one as $n \to \infty$. Under the same model as above, Philips *et al.* [10] provided a necessary condition on the average node degree required for connectivity (the degree of a node is the total number of its neighbors); other work [11–13] provided upper and lower bounds on the node degree required

for guaranteeing an asymptotically connected network as $n \to \infty$.

Although the UDM has been widely used in many connectivity studies, it is far less than a realistic model. More realistic models have recently been considered for studying connectivity, including the log-normal shadowing connection model [4, 14–17] which takes a shadow fading effect into account, and the SINR (signal-to-interference-plus-noise ratio) model [18–20] that can account for the mutual interference due to multiple concurrent transmissions. In most recent work [21–23], the connectivity problem is investigated under a generic random connection model, where nodes directly connect to each other probabilistically depending on the Euclidean distances between them.

1.1.2 Capacity

Capacity of a communication system is the maximum data-rate in bits per second that can be reliably and sustainably transferred from transmitter to receiver. In wireless multi-hop networks, due to the multi-hop communication nature between nodes and that wireless channel is shared by multiple transmitter-receiver pairs, capacity of a network becomes much more complex to define and analyze and perhaps one of the most challenging problem in information theory [3].

The most general approach in the field, pioneered by Gupta and Kumar [24], studies the so-called *transport capacity*, which quantifies the *end-to-end throughput* that can be transported over a physical distance for randomly chosen source-destination pairs in the network. An alternative approach is to evaluate a metric termed *transmission capacity* (TC). The TC, first proposed by Weber *et al.* in [25], quantifies the maximum spatial density of single-hop concurrent transmissions, subject to a constraint on *outage probability* (OP) related to SINR threshold.

Transport capacity

Significant results have been obtained on characterizing the asymptotic *scaling law* of the transport capacity as the network becomes sufficiently large. Particularly, Gupta and Kumar [24] showed that in a network of n nodes uniformly and independently and identically distributed (i.i.d.) on an area of unit size and each node is capable of transmitting at W bits per second and employing a common transmission range, the achievable per-node throughput is $\Theta\left(\frac{W}{\sqrt{n}}\right)$[1]if nodes are optimally and deterministically placed to maximize capacity; the achievable per-node throughput is only $\Theta\left(\frac{W}{\sqrt{n \log n}}\right)$ when nodes are randomly located, by using a specific multi-hop communication strategy. Gupta and Kumar's work sparked an enormous research interest in the field. On one side, several alternative strategies have also been developed to achieve the same bound as $\Theta\left(\frac{W}{\sqrt{n \log n}}\right)$ [26, 27]. On the other side, research has focused on seeking bounds on the capacity scaling law that is independent of communication strategies. With assumptions made only on radio propagation process, it was established by many researchers [28–31] that $\Theta\left(\frac{1}{\sqrt{n}}\right)$ is an upper bound on the per-node throughput of wireless multi-hop networks. All of these results suggest that a $\sqrt{\log n}$ factor in denominator is the price to pay for the randomness due to the locations of the nodes. Franceschetti *et al* [32] considered the same network as that in [24] and showed that by using the so-called highway routing protocol and a centralized/deterministic Time Division Multiple Access (TDMA) protocol, the per-node throughput can reach $\Theta\left(\frac{1}{\sqrt{n}}\right)$ even when nodes are randomly located. Hence, the gap between capacity scaling law of randomly located and deterministically located nodes is closed. Since then, a number of solutions have been proposed to achieve the above upper bounds of scaling law under various network settings and using various routing and scheduling algorithms [24, 27, 32–41].

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[1]The notation Θ, Ω and O shall be formally defined in Chapter 3.

Moreover, some papers showed that a throughput higher than $\Theta\left(\frac{1}{\sqrt{n}}\right)$ can be achieved by changing the assumptions on the basic network settings. Notably, Grossglauser and Tse [37] considered a network with mobile nodes and showed that mobility of nodes can be exploited to considerably increase the throughput to $\Theta\left(1\right)$ at the expense of large delay. Their work [37] has sparked huge interest in studying the capacity-delay tradeoffs in mobile networks assuming various mobility models and the obtained results often vary greatly with the different mobility models being considered (see [26, 42–46] and references therein for examples). In [47], Chen *et al.* studied the capacity of wireless networks under a different traffic distribution. In particular, they considered a set of n randomly deployed nodes transmitting to a single sink or multiple sinks where the sinks can be either regularly deployed or randomly deployed. They showed that with a single sink, the transport capacity is given by $\Theta\left(W\right)$; with k sinks, the transport capacity increases to $\Theta\left(kW\right)$ when $k = \mathrm{O}(n\log n)$ or $\Theta\left(n\log nW\right)$ when $k = \Omega\left(n\log n\right)$. It was established in [35, 39] that a throughput of $\Theta\left(1\right)$ can also be achieved by deploying (randomly placed) infrastructure/base stations. Furthermore, there is also significant amount of work studying the impact of infrastructure nodes [48] and multiple-access protocols [49, 50] on the capacity and the multicast capacity [51]. We refer readers to [3] for a comprehensive review of related work.

Transmission capacity

The scaling laws provide an insight on how the performance of networks is determined by different network features, such as network size, mobility of nodes and infrastructure support. However, a finer view of capacity limits is however needed when the focus is on the impact of different techniques and channel states on the capacity of large networks, for example channel inversion and fading [52].

For a given constraint ϵ on the OP, i.e., the probability that SINR of a transmission falls below the target SINR, the TC is expressed as

$$\mathrm{TC}\left(\epsilon\right) \triangleq \left(1 - \epsilon\right) \sup \left\{\eta : \mathrm{OP} < \epsilon\right\} \tag{1.1.1}$$

where η is the spatial density of concurrent transmitters in the network. In words, the TC is the maximum number of possible successful concurrent transmissions per unit area, subject to a constraint on the OP.

Different technologies have been proposed to improve the network capacity using TC as the metric, including multiple antennas [53], guard zone around each receiver [54] and information cancellation [55]. Notably, Andrews *et al.* [56] made an extension of the original TC metric, termed random access transport capacity, which quantifies the *average* maximum rate of successful end-to-end transmissions, multiplied by the communication distance, and normalized by the network area. Ganti *et al.* [57] analyzed asymptotic OP and TC for generic isotropic node distributions and generic fading as the spatial density of concurrent transmitters goes to zero.

Combining with a homogeneous Poisson distribution for concurrent transmitters, the TC framework yields very good analytical tractability for detailed study of the network capacity in terms of system parameters and design choices, such as fading and interference cancellation techniques. This is generally very difficult to do by studying the transport capacity alone. In essence, the TC metric is more the description of a given technology through its achieved SINR than of technology-independent fundamental limits [3].

1.2 Research Problems Addressed in This Thesis

1.2.1 Connectivity under SINR model

In large wireless multi-hop networks which are distributed, some transmissions necessarily occur at the same time in the same frequency band. Interference is the main performance-limiting factor and the SINR is the relevant figure of merit. Due to the randomness nature of such networks, SINRs are not tightly controllable and subject to considerable uncertainty. If a node attempts to improve the SINR of its own transmission by raising its transmission power, it causes an increase on interference to all the other nodes. The interference can be mitigated quite efficiently with centralized control, for example coordinating the channelization or the transmission power of individual nodes [58]. However, centralized control is not compatible with the kind of networks investigated in this thesis, which requires more distributed operation protocols. In Chapter 4, we investigate the connectivity problem in wireless multi-hop networks where distributed MAC protocol - CSMA is assumed. We analytically derive a sufficient and a necessary condition on the critical transmission power for connectivity in the presence of interference.

1.2.2 Transport capacity of distributed networks

Despite the great achievements on characterizing the scaling law of per-node throughput of wireless multi-hop networks assuming centralized MAC protocol [24, 27, 33, 35, 36, 38–40], limited work exists on analyzing capacity of large networks operating in distributed/decentralized fashion. Chau *et al.* [34] took the lead in studying the capacity of networks employing the distributed CSMA protocols and showed that these networks can achieve the same order-optimal throughput of $\Theta\left(\frac{1}{\sqrt{n}}\right)$ as networks employing centralized TDMA schemes. While the use of CSMA for scheduling

in [34] constitutes a significant advance compared with the centralized protocols considered in previous work, the routing scheme in [34] still relies on the highway system proposed in [32] where the centralized coordination is needed to identify the highways. Moreover, the deployment of highway system in CSMA networks requires two different carrier-sensing ranges to be used, which exacerbates the hidden node (HN) problem in CSMA networks. In Chapter 5, we investigate the transport capacity of wireless CSMA networks. First, we develop a distributed routing scheme, together with carefully tuning controllable parameters in CSMA protocols, to achieve the order-optimal throughput scaling law of $\Theta\left(\frac{1}{\sqrt{n}}\right)$, which is the same as that of large networks employing centralized routing and scheduling algorithms. Note that scaling laws are only up to order and most network design choices have a significant effect on the constants preceding the order while not affecting the scaling law. Therefore we take a further step to analyze the pre-constant by giving an upper and a lower bound of throughput. The tightness of the bounds is validated using simulations.

1.3 Thesis Outline

The rest of the thesis is organized as follows. In Chapter 2, we discuss relevant work to the aforementioned research problems in the literature. In Chapter 3, we describe the basic elements commonly needed in the subsequent chapters to formulate each research problem. Chapter 4 and 5 comprise the major contributions of this thesis in which we investigate the connectivity and transport capacity problems, respectively. In Chapter 6, we conclude this thesis.

Chapter 2

Literature Review

The focus of this thesis is on studying asymptotic behavior of finite networks that grow sufficiently large. Be more specific, we investigate the connectivity and capacity of wireless multi-hop networks employing distributed CSMA protocol. In this chapter, we first discuss some closely related work in the field of connectivity and capacity of wireless multi-hop networks. We then identify the main challenge of charactering properties of CSMA networks, due to which different approaches have been used to rise the challenge. The existing approaches are then discussed.

2.1   Connectivity

Unit disk model

The literature is rich in studying connectivity using the UDM. It is usually considered that a network is formed by randomly and uniformly distributing n nodes in a given area and connecting any two nodes iff their Euclidean distance is smaller than or equal to a certain threshold $r(n)$. Significant outcomes have been achieved for both asymptotically infinite n [2,9,59] and for finite n [60,61]. Particularly, Penrose [9]

and Gupta and Kumar [2] proved that under the UDM and in a unit-area disc, the above network with a transmission range of $r(n) = \sqrt{\frac{\log n + c(n)}{\pi n}}$ is a.a.s. connected as $n \to \infty$ iff $c(n) \to \infty$. Philips *et al.* [10] proved that the average node degree must grow logarithmically with the area of the network to guarantee a connected network, where nodes are distributed on a square according to a Poisson distribution with a constant density. The result by Philips *et al.* in fact provides a necessary condition on the average node degree required for connectivity. The work [11–13] advanced the results in [10] by providing upper and lower bounds on the node degree required for guaranteeing an asymptotically connected network as $n \to \infty$. Most of the results for finite n are empirical results [60, 61].

Log-normal connection model

The work [4, 14–16] investigated the necessary condition for the same network as considered in [2, 9, 59] to be a.a.s. connected under the log-normal connection model, where two nodes are directly connected if the received power at one node from the other node, whose attenuation follows the log-normal model [62], is greater than a given threshold. These results however rely on the assumption that the node isolation events are independent, which is yet to be proved.

SINR model

Despite the significant impact of interference due to concurrent transmissions on connectivity, limited work exists on analyzing connectivity under the SINR model. In [20, 63], the authors studied connectivity from the perspective of channel assignment. Specifically, channel/time slots are assigned to each link for all active links to be simultaneously transmitting while satisfying the SINR requirement. The most relevant work is by Dousse *et al.* [18, 19], in which the impact of interference on the

connectivity was investigated from the *percolation* perspective. In their work, nodes were assumed to transmit independently, which corresponds to the ALOHA protocol. Unlike in ALOHA, where each node accesses the channel independently with a prescribed probability, nodes of CSMA networks suffer from a *spatial correlation problem*, which means that the activity of a node is dependent on the activities of other nodes due to the carrier-sensing operation. This correlation problem makes the analysis of interference and capacity of CSMA networks more challenging than that of ALOHA networks. Therefore, although both ALOHA and CSMA are distributed MAC protocols, the results obtained for ALOHA networks are not directly applicable to CSMA networks. In Chapter 4, we shall show results obtained in CSMA networks are actually in stark contrast to the results obtained in these two papers [18, 19].

Random connection model

The random connection model is a generalization of the UDM. Under this model, two nodes separated by a Euclidean distance x are directly connected with probability $g(x)$, where $g : [0, \infty) \to [0, 1]$ satisfies the properties of integral boundedness, rotational invariance and non-increasing monotonicity [7,8], independent of the event that another pair of nodes are directly connected. Mao *et al.* [21] investigated the connectivity problem under the random connection model and established the requirements for the same network as considered in [2,9,59] to be a.a.s. connected. Ng *et al.* [23] studied the connectivity problem under the random connection model as well but from the perspective of percolation, and they derived the analytical bounds of critical node density for percolation in 2-Dimension and 3-Dimension networks.

A critical assumption used in the analysis of connectivity under the UDM, the log-normal connection model and the random connection model is that connections are independent, i.e., the event that a pair of nodes are directly connected and the

event that another distinct pair of nodes are directly connected are independent. Under the SINR model that we consider in this thesis however, due to the presence of interference, connections are *correlated*. That is to say, the existence of a direct connection between a pair of nodes depends not only on the Euclidean distance between them but also on both the locations and the activities of all the other nodes in the network.

2.2   Transport capacity

In addition to the work mentioned in Section 1.1.2 on the study of network capacity, in this section we further review work closely related to the research and theoretical analysis in this thesis.

Limited work exists on analyzing capacity of large networks running distributed routing and scheduling algorithms, despite their extensive deployment in real networks. Byun *et al* [64] showed that networks with slotted ALOHA protocol can have order-optimal throughput. However, the ALOHA protocol has become obsolete [65]. The more advanced distributed MAC protocols, e.g. CSMA and CSMA/CA (Carrier Sense Multiple Access with Collision Avoidance) [62] have become prevailing with widespread adoption. Reference [34] discussed in Section 1.1.2 is among the first work studying the capacity of networks employing distributed and randomized CSMA protocols and showed that these networks can achieve the same order-optimal throughput of $\Theta\left(\frac{1}{\sqrt{n}}\right)$ as networks employing centralized TDMA schemes. In our previous work [41], we studied the achievable throughput of three dimensional CSMA networks and provided a lower bound on the scaling law of throughput. Ko *et al* [66] showed that in CSMA networks, by jointly optimizing the transmission range and packet generation rate, the end-to-end throughput and end-to-end delay can scale as

$\Theta\left(\frac{1}{\sqrt{n\log n}}\right)$ and $\Theta\left(\frac{n}{\sqrt{\log n}}\right)$, respectively.

Improving spatial frequency reuse of CSMA networks is an important problem that has also been extensively investigated, see [67–69] for the relevant work. However, high level of spatial frequency reuse does not directly lead to increased end-to-end throughput because the latter performance metric also critically relies on the communication strategies, e.g., routing algorithm and scheduling scheme, used in the network. In this thesis we focus on the study of achievable end-to-end throughput.

2.3   Other related work

Extensive research efforts have been devoted to modeling the spatial distribution of concurrent transmitters observing carrier-sensing constraints and the distribution of interference resulting from these transmitters. A major challenge in analyzing the performance of CSMA networks is that in CSMA networks, the locations of concurrent transmitters are correlated, i.e., a minimum separation distance is imposed among concurrent transmitters due to the carrier sensing mechanism. Therefore, even if all nodes are initially distributed following a Poisson p.p., the set of concurrent transmitters cannot be obtained by independent thinning of the Poisson p.p.. The set of concurrent transmitters no longer forms a Poisson p.p. but a more complicated p.p..

Busson *et al.* [70] proposed to use the Matérn hard-core p.p. to model the set of concurrent transmitters in CSMA networks. Haenggi [71] considered two types of hard-core p.p. and compared the *mean* interference generated by the two types of hard-core p.p. with the mean interference generated by a Poisson p.p. of the same node density. It was shown that the gap is negligible for one type of hard-core p.p., but increases exponentially with the minimum separation distance for the other

one. While the hard-core p.p. captures a key property of the concurrent transmitter set, i.e., two concurrent transmitters have to be separated by a minimum distance, such hard-core point processes and the associated interference are very challenging to characterize analytically. Therefore, *approximation* is often used in order to obtain closed-form analytical results [50,57,58,72,73]. In [58,72], homogeneous Poisson p.p. was used to approximate the spatial distribution of the set of concurrent transmitters in CSMA networks. Alfano *et al.* [50] considered an approach where the distribution of concurrent transmitters is approximated by an inhomogeneous Poisson p.p. whose local intensity depends on the distance from the desired transmitter. Ganti *et al.* [57] analyzed asymptotic OP and TC for generic isotropic node distributions and generic fading as the spatial density of concurrent transmitters goes to zero. To be specific, they showed the procedure to obtaining two constants γ and κ such that, for general node distribution and fading distribution, the success probability p_s, viz. the complement of the OP, can be approximated by $p_s \sim 1 - \gamma \eta^{\kappa}$ when $\eta \to 0$, where η is the spatial density of concurrent transmitters ($f(x) \sim g(x)$ means that $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$.). Nguyen and Baccelli in a more recent work [74] proposed to use the Random Sequential Absorption (RSA) p.p. as a more natural model for representing the spatial distribution of concurrent CSMA transmitters. They studied the RSA p.p. by characterizing its generating functional and derived upper and lower bounds for the generating functional. Furthermore, they derived the network performance metrics, viz., average medium access probability and average transmission success probability (two commonly used metrics in the study of transmission capacity), in terms of the generating functional. The work [74] and [50] studied the transmission capacity by investigating the transmission success probability and the medium access probability of a *typical* node, which quantifies the spatial *average performance* of the network. In comparison, the transport capacity often quantifies

the throughput that can be achieved by *every* source-destination pair (a.a.s.), which is often associated with the *worst case* performance.

In this thesis, we circumvent the above difficulty involving accurate modeling of the spatial distribution of concurrent transmitters in CSMA networks by pursuing bounds on performance metrics. Different from the above results [57, 72, 73], which have to resort to approximations of the spatial distribution of concurrent transmitters and empirical validation of the accuracy of such approximations, the results established in this thesis are analytically rigorous.

Chapter 3

Network Models

In this chapter, we describe the basic concepts and notations commonly required in the subsequent chapters to formulate each research problem.

This thesis is concerned with the asymptotic behavior of random networks that grow sufficiently large. In general, there are two network models that are considered in the study of asymptotic properties of networks of growing size: the *extended network model* where the network size scales with the network area while the node density is fixed; and the *dense network model* where the network size scales with the node density while the network area is fixed. By appropriate scaling of the distances, the results obtained under one model can often be extended to be applicable under the other one. Throughout this thesis, we consider the extended network model. Specifically, we consider a network with nodes deployed on a box $B_n \subset \mathbb{R}^2$ of size $\sqrt{n} \times \sqrt{n}$ following either a uniform distribution or a homogeneous Poisson distribution. These two random node location models have been widely used in the field of wireless multi-hop networks.

Uniform distribution

Under this model, a network comprises a set of n nodes which are independently, identically distributed (i.i.d.) in a given bounded region in $\mathbb{R}^2$ following a uniform distribution.

Homogeneous Poisson distribution

Under the homogeneous Poisson distribution with density λ, the random set of nodes satisfying the following three properties [75]:

- The number of points $\mathcal{N}(\mathbf{A})$ located in $\mathbf{A} \subset \mathbb{R}^2$ is a Poisson random variable with the expected value $\mathrm{E}[\mathcal{N}(\mathbf{A})] = \lambda |\mathbf{A}|$ where $|\mathbf{A}|$ is the Lebesgue measure of $\mathbf{A}$. That is,

$$\Pr\{\mathcal{N}(\mathbf{A}) = k\} = \frac{(\lambda |\mathbf{A}|)^k}{k!} e^{-\lambda |\mathbf{A}|} \tag{3.0.1}$$

  for integer $k \geq 0$;

- The number of nodes in any two non-overlapping regions are independent of each other;

- Conditioned on a given number of nodes in a region, these nodes are uniformly distributed in the region.

In the thesis, we consider that a total number of n nodes are uniformly i.i.d. on B_n in Chapter 4; and consider that nodes are distributed according to a homogeneous Poisson distribution with unit density on B_n in Chapter 5. We are mainly concerned with the events that occur inside B_n a.a.s. as $n \to \infty$. The following notations are used throughout the thesis concerning the asymptotic behavior of positive functions:

- $f(n) = \mathrm{O}(g(n))$ if that there exist a positive constant c and an integer n_0 such that $f(n) \leq cg(n)$ for any $n > n_0$;

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- $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$;

- $f(n) = \Theta(g(n))$ if that there exist two constants c_1, c_2 and an integer n_0 such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for any $n > n_0$;

- $f(n) = o(g(n))$ if $\lim\limits_{n \to \infty} \frac{f(n)}{g(n)} = 0$.

3.1 SINR model

Let $\boldsymbol{x}_k$, $k \in \Gamma$, be the location of node k, where Γ represents the set of indices of all nodes in the network. Throughout the thesis, we also refer to a node by its location. Let P_k be the transmission power used by node k, for $k \in \Gamma$. A node j can successfully receive the transmitted signal from a node i iff the SINR at $\boldsymbol{x}_j$, denoted by SINR $(\boldsymbol{x}_i \to \boldsymbol{x}_j)$, is above a prescribed threshold β, i.e.

$$\text{SINR}(\boldsymbol{x}_i \to \boldsymbol{x}_j) = \frac{P_i \ell(\boldsymbol{x}_i, \boldsymbol{x}_j)}{N_0 + \gamma \sum\limits_{k \in \mathcal{T}_i} P_k \ell(\boldsymbol{x}_k, \boldsymbol{x}_j)} \geq \beta, \tag{3.1.1}$$

where $\mathcal{T}_i \subseteq \Gamma$ denotes the subset of nodes transmitting at the same time as node i, i.e., interferers, and N_0 is the background noise power. The function $\ell(\boldsymbol{x}_i, \boldsymbol{x}_j)$ is the power attenuation from $\boldsymbol{x}_i$ to $\boldsymbol{x}_j$. We consider that the attenuation function ℓ depends on Euclidean distance only and is a power-law function [18, 19], i.e.,

$$\ell(\boldsymbol{x}_i, \boldsymbol{x}_j) = \|\boldsymbol{x}_i - \boldsymbol{x}_j\|^{-\alpha} \tag{3.1.2}$$

where α is the path-loss exponent which typically varies from 2 to 6 [62, p139]. In this thesis we assume $\alpha > 2$. Since in many practical situations the background noise is typically negligibly small compared to the interference due to multiple concurrent transmissions [3, 58], we ignore N_0. The coefficient $0 \leq \gamma \leq 1$ is the inverse of

the processing gain of the system and it weighs the impact of interference. In a broadband system using CDMA, γ depends on the orthogonality between codes used during concurrent transmissions and $\gamma < 1$; in a narrow-band system, $\gamma = 1$ [3, 18]. When $\gamma = 0$, the SINR model degrades to the UDM. In this thesis, we assume that all data transmissions are conducted over one common wireless channel, i.e., $\gamma = 1$, which corresponds to a narrow-band system.

3.2   CSMA protocol

The general idea of CSMA protocol is that nearby nodes will not be scheduled to transmit simultaneously. Each node has to sense the channel to guarantee that there is no other ongoing transmissions in its vicinity. This exclusion rule is realized as follows: a node j is said to be in the contention domain of node i if the received power by node i from node j is above a certain detection threshold [76], i.e.,

$$P_j \left\| \boldsymbol{x}_i - \boldsymbol{x}_j \right\|^{-\alpha} > \tau_i,$$

where τ_i is the detection threshold adopted by node i. The node i is allowed to transmit if there is no other transmitting node in its contention domain, or in other words, the node i senses the medium idle.

To prevent the situation where several nearby nodes start transmitting simultaneously when their common neighbor stops its transmission, hence causing a collision, a *backoff* mechanism is often employed such that a node sensing the channel idle will wait a random amount of time before starting its transmission. The following backoff mechanism is considered in this thesis. Each node senses the channel continuously and maintains a countdown timer, which is initialized to a non-negative random value. The timer of a node counts down when it senses the channel idle;

when the channel is sensed as busy, the node freezes its timer. A node initiates its transmission when its countdown timer reaches zero *and* the channel is sensed as idle. After finishing its transmission, the node resets its countdown timer to a new random value for the next transmission. The distribution of the random initial countdown timer will be specified in Chapter 5 when necessary.

Chapter 4

Connectivity

In this chapter, we investigate the critical transmission power for connectivity in wireless CSMA networks under the SINR model. The critical transmission power is the minimum transmission power each node needs to transmit to guarantee that the resulting network is connected asymptotically almost surely. Specifically, we consider a network with n nodes uniformly i.i.d. on the box $B_n \subset \mathbb{R}^2$ and each node is capable of performing carrier-sensing operation. A pair of nodes are directly connected iff the SINR requirements can be met at both ends of a link, i.e., both (4.2.1) and (4.2.2) are satisfied. We provide a sufficient condition and a necessary condition, i.e. an upper bound and a lower bound on the critical transmission power, required for having an a.a.s. connected CSMA network as $n \to \infty$. The two bounds differ by a constant factor only, as $n \to \infty$. Compared with that considering the UDM without interference, the transmission power only needs to be increased by a constant factor to combat interference and maintain connectivity. This result is also in stark contrast with previous results considering the connectivity of ALOHA networks under the SINR model. The results of this chapter appear in [J1, C1, C3].

4.1 Introduction

Due to the nature of wireless communications, signals transmitted at the same time will mutually interfere with each other. The SINR model described in Section 3.1 has been widely used to capture the impact of interference on network connectivity [3,18, 24]. Under the SINR model, the existence of a directional link between a pair of nodes is determined by the strength of the received signal from the desired transmitter, the interference caused by other concurrent transmissions and the background noise.

Dousse *et al.* [18] use the SINR model to analyze the impact of interference on connectivity from the percolation perspective. They consider a network where all nodes are distributed in $\mathbb{R}^2$ following a homogeneous Poisson p.p. with a constant intensity λ and an attenuation function ℓ with bounded support. Recall that the coefficient $0 \leq \gamma \leq 1$ appears in (3.1.1) and (4.2.2) weighs the impact of interference. By letting $\mathcal{T}_j = \Gamma / \{i, j\}$, i.e. all other nodes in the network transmit simultaneously with node i irrespective of their relative locations to $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$, it is shown that there exists a very small positive constant γ' such that if $\gamma > \gamma'$ there is no infinite connected component in the network, i.e., the network does not percolate. Further, when $\gamma < \gamma'$, there exists $0 < \lambda' < \infty$ such that percolation can occur when $\lambda > \lambda'$. An improved result by the same authors [19] shows that under the more general conditions that $\lambda > \lambda_c$ and the attenuation function has unbounded support, percolation occurs when $\gamma < \gamma'$. Here λ_c is the critical node density above which the network with $\gamma = 0$ (i.e. UDM with no interference) percolates [8, p48]. These results suggest that percolation under the SINR model can happen iff γ is sufficiently small. They assume that each node transmits randomly and independently, irrespective of any nearby transmitter. This corresponds to the ALOHA-type MAC protocol [3], which however has become obsolete [65].

The more advanced multiple access strategies, e.g. CSMA and CSMA/CA [62]

have become prevailing with widespread adoption. With CSMA protocol, nearby nodes will not be scheduled to transmit simultaneously, i.e., a minimum separation distance is imposed among concurrent transmitters. Therefore, it is natural to expect that CSMA could improve the performance of ALOHA by alleviating interference, particularly under heavy traffic. On the other hand, due to the difficulty in finding the accurate distribution of concurrent transmitters and the associated interference, as discussed in Section 2.3, in this chapter, we use an entirely different approach. Particularly, we investigate the bounds on interference, instead of an accurate characterization of interference distribution.

Our major contributions can be summarized as follows:

- We show that the interference experienced by any receiver in the network is upper bounded. Based on this result, we further show that for an arbitrarily chosen SINR threshold, there exists a transmission range R_0 such that a pair of nodes are directly connected if their Euclidean distance is smaller than or equal to R_0. On that basis, we derive a sufficient condition, i.e., an upper bound on the critical transmission power, for the CSMA network to be a.a.s. connected under the SINR model as $n \to \infty$.

- We provide a necessary condition, i.e., a lower bound on the critical transmission power, for the CSMA network to be a.a.s. connected. The two bounds are tight and differ from each other by a constant factor only.

- We show that the transmission power only needs to be increased by a constant factor to combat interference and maintain connectivity compared with that considering UDM without interference. This result is in stark contrast with previous results considering the connectivity of ALOHA networks [18,19] under the SINR model which shows that connectivity is much harder to achieve in

the presence of interference and is impossible in a narrow band system where $\gamma = 1$.

The remainder of this chapter is organized as follows. Section 4.2 defines network models and notations; In Section 4.3 we first derive an upper bound on the interference in CSMA networks, and a sufficient condition for connectivity is obtained based on the upper bound; Section 4.4 derives a necessary condition for connectivity; finally Section 4.5 summarizes the chapter.

4.2 Network Model and Notations

In this chapter, we consider a decentralized wireless multi-hop network with nodes uniformly i.i.d. on a $\sqrt{n} \times \sqrt{n}$ box $B_n \subset \mathbb{R}^2$ and each node is capable of performing carrier sense.

4.2.1 Connection model

For the connection model, we consider the SINR model which has been widely used to capture the impact of interference on network connectivity [3, 18, 24]. As commonly done in the connectivity analysis [2, 8, 9, 18, 19], the impact of small-scale fading is ignored and only bidirectional communication links are considered. In this chapter, we assume all nodes use the same transmit power P. A node j is directly connected to node i iff

$$\text{SINR}\,(\boldsymbol{x}_i \to \boldsymbol{x}_j) = \frac{P\,\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^{-\alpha}}{N_0 + \sum\limits_{k \in \mathcal{T}_i} P\,\|\boldsymbol{x}_k - \boldsymbol{x}_j\|^{-\alpha}} \geq \beta; \qquad (4.2.1)$$

similarly, node i is directly connected to node j iff

$$\text{SINR}\,(\boldsymbol{x}_j \to \boldsymbol{x}_i) = \frac{P\,\|\boldsymbol{x}_j - \boldsymbol{x}_i\|^{-\alpha}}{N_0 + \sum\limits_{k \in \mathcal{T}_j} P\,\|\boldsymbol{x}_k - \boldsymbol{x}_i\|^{-\alpha}} \geq \beta. \qquad (4.2.2)$$

Therefore node i and node j are directly connected, i.e. a bidirectional link exists between node i and node j, iff both (4.2.1) and (4.2.2) are satisfied. Since in many practical situations the background noise is typically negligibly small compared to the interference due to multiple concurrent transmissions [3, 58], we ignore N_0.

4.2.2 Carrier-sensing range

In this chapter, we assume all nodes use the same detection threshold τ. From the power-law path loss, given by (3.1.2), a *minimum* Euclidean distance is imposed between any two concurrent transmitters, known as the *carrier-sensing range* and given by

$$R_c = (P/\tau)^{1/\alpha} \tag{4.2.3}$$

One may alternatively consider a scenario where a node transmit when the aggregated interference is below τ, which forms a trivial extension of the scenario considered in this chapter.

4.3 A Sufficient Condition on the Critical Transmission Power

A major technical challenge in connectivity analysis under the SINR model is due to the correlation problem. We shall resort to a technique, called *coupling*, to handle the connection correlations. The coupling technique amounts to building the connection between a more complicated model and a simpler model with established results such that if a property, e.g. connectivity, is true in the simpler model, it will also be true in the more complicated one. It then immediately follows that if the network is

connected under the simpler model, then it is also connected under the complicated counterpart.

We first establish an upper bound on the interference experienced by any receiver in the network. On that basis, we show that for an arbitrarily chosen SINR threshold β, there exists a transmission range R_0 such that a pair of nodes are directly connected if their Euclidean distance is smaller than or equal to R_0. Then we can use existing results on connectivity under the UDM to analyze connectivity under the SINR model.

4.3.1 An upper bound on interference and the associated transmission range

The following theorem provides an upper bound on the interference.

Theorem 4.1. *Consider a CSMA network with nodes distributed arbitrarily on a finite area in $\mathbb{R}^2$. Denote by r_0 the Euclidean distance between a receiver and its nearest transmitter in the network, which is also the intended transmitter for the receiver. When $r_0 < R_c$, the maximum interference experienced by the receiver is smaller than or equal to $N(r_0) = N_1(r_0) + N_2$, where*

$$N_1(r_0) = \frac{4P\left(\frac{5\sqrt{3}}{4}R_c - r_0\right)^{1-\alpha}\left(\frac{\sqrt{3}}{4}(3\alpha - 1)R_c - r_0\right)}{R_c^2(\alpha - 1)(\alpha - 2)}$$

$$+\frac{3P}{(R_c - r_0)^\alpha} + \frac{3P}{\left(\sqrt{3}R_c - r_0\right)^\alpha} + \frac{3P\left(\frac{3}{2}R_c - r_0\right)^{1-\alpha}}{(\alpha - 1)R_c} \qquad (4.3.1)$$

$$N_2 = \frac{3P}{R_c^\alpha} + \frac{3P(\frac{3}{2})^{1-\alpha}}{(\alpha - 1)R_c^\alpha} + \frac{3P}{\left(\sqrt{3}R_c\right)^\alpha} + \frac{3P\left(\frac{5}{4}\right)^{1-\alpha}(3\alpha - 1)}{(\alpha - 1)(\alpha - 2)\left(\sqrt{3}R_c\right)^\alpha} \qquad (4.3.2)$$

Proof. See Appendix A. □

Remark 4.1. The upper bound in Theorem 4.1 is valid for any node distribution. For a sparse network or a network where nodes are placed in a coordinated or planned manner, replacing R_c with the minimum distance among concurrent transmitters, Theorem 4.1 can be extended to be applicable.

Remark 4.2. The assumption that $r_0 < R_c$ is valid in most wireless systems which not only require the SINR to be above a threshold but also require the received signal to be of sufficiently good quality. However Theorem 4.1 does not critically depend on the assumption. For $r_0 \geq R_c$, so long as there exists a positive integer c such that $r_0 < cR_c$ the upper bound can be revised to accommodate the situation by changing the range of the summation in (A.0.2) (in Appendix A) from $[3, \infty]$ and $[2, \infty]$ to $[c + 2, \infty]$ and $[c + 1, \infty]$ respectively and revising the results accordingly.

The following result can be obtained as a ready consequence of Theorem 4.1.

Corollary 4.1. *Under the same settings as in Theorem 4.1, assume that the SINR threshold is β. There exists a transmission range $R_0 < R_c$ such that a pair of nodes are directly connected if their Euclidean distance is smaller than or equal to R_0, given implicitly by*

$$PR_0^{-\alpha}/N(R_0) = \beta. \qquad (4.3.3)$$

Proof. Theorem 4.1 established that the interference experienced by a receiver z at r_0 from its transmitter w, denoted by $I(r_0)$ is upper bounded by $N(r_0)$. Note that, for $r_0 < R_c$, $N(r_0)$ is increasing with r_0 and $Pr_0^{-\alpha}$ is decreasing with r_0. Therefore, using (4.3.3) the SINR of a receiver at $r_0 \leq R_0$ from its transmitter, denoted by SINR (r_0), satisfies SINR $(r_0) = \frac{Pr_0^{-\alpha}}{I(r_0)} \geq \frac{Pr_0^{-\alpha}}{N(r_0)} \geq \beta$.

By symmetry, when the transmission occurs in the opposite direction, i.e. from z to w, the interference generated by the set of nodes that are transmitting at the

same time as z is also upper bounded by $N(r_0)$. Therefore the SINR at w is also greater than or equal to β.

Finally the existence of a (unique) solution to (4.3.3) can be proved by noting that $\frac{Pr_0^{-\alpha}}{N(r_0)} \to \infty$ as $r_0 \to 0$, $\frac{Pr_0^{-\alpha}}{N(r_0)} \to 0$ as $r_0 \to R_c^-$ and that $\frac{Pr_0^{-\alpha}}{N(r_0)}$ is monotonically decreasing with r_0. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Corollary 4.1 relates R_0 to transmission power P and allows the computation of R_0 given P and the converse. A more convenient way to study the relation between P and R_0 is by noting that $P = \tau R_c^\alpha$ and considering R_0 as a function of R_c. Using (4.3.1), (4.3.2) and letting $\frac{R_c}{R_0} = x$, (4.3.3) can be rewritten as

$$
\begin{aligned}
\frac{1}{\beta} &= \frac{4\left(\frac{5\sqrt{3}}{4}x - 1\right)^{1-\alpha}\left(\frac{\sqrt{3}}{4}(3\alpha-1)x - 1\right)}{x^2(\alpha-1)(\alpha-2)} \frac{3}{(x-1)^\alpha} + \frac{3}{\left(\sqrt{3}x - 1\right)^\alpha} + \frac{3\left(\frac{3}{2}x - 1\right)^{1-\alpha}}{(\alpha-1)x} \\
&\quad + \frac{3}{x^\alpha} + \frac{3\left(\frac{3}{2}\right)^{1-\alpha}}{x^\alpha(\alpha-1)} + \frac{3}{\left(\sqrt{3}x\right)^\alpha} + \frac{3\left(\frac{5}{4}\right)^{1-\alpha}(3\alpha-1)}{(\alpha-1)(\alpha-2)\left(\sqrt{3}x\right)^\alpha}.
\end{aligned}
\tag{4.3.4}
$$

Figure 4.3.1 shows the ratio $\frac{R_c}{R_0}$ as a function of β. Different curves represent different choices of the path loss exponent α. For instance, when $\beta = 10$ and $\alpha = 4$, we have $\frac{R_c}{R_0} = 3.6$.

4.3.2 A sufficient condition on the critical transmission power

Based on the transmission range R_0 derived in Corollary 4.1, we obtain another main result:

Theorem 4.2. *Consider a CSMA network with a total of n nodes uniformly i.i.d. on $B_n \subset \mathbb{R}^2$. A pair of nodes are directly connected iff both (4.2.1) and (4.2.2) ($\gamma = 1$ and $N_0 = 0$ in (3.1.1)) are satisfied. As $n \to \infty$, the above network is a.a.s.*
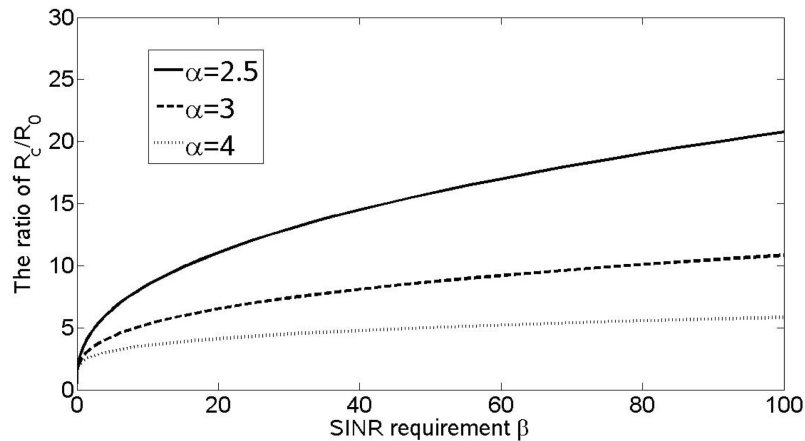
Figure 4.3.1: Variation of the ratio $\frac{R_c}{R_0}$ with the SINR requirement β when the path loss exponent α equals to 2.5, 3, 4, respectively.

connected if the transmission power

$$P = \tau b_1^\alpha \left(\log n + c\left(n \right) \right)^{\frac{\alpha}{2}}, \tag{4.3.5}$$

where $b_1 = b'/\sqrt{\pi}$, $c\left(n \right) = o\left(\log n \right)$ and $c\left(n \right) \to \infty$ as $n \to \infty$ and $\infty > b' > 1$ is the solution to (4.3.4).

Proof. By proper scaling of distances, the results in [2,9] show that, for a network with a total of n nodes uniformly i.i.d. on a $\sqrt{n} \times \sqrt{n}$ square and a pair of nodes are directly connected iff their Euclidean distance is smaller than or equal to a given threshold $r\left(n \right)$ (i.e., UDM), the network is a.a.s. connected as $n \to \infty$ iff $r\left(n \right) = \sqrt{\frac{\log n + c(n)}{\pi}}$ where $c\left(n \right) \to \infty$ as $n \to \infty$. Using this result, (4.3.4) (letting $b' = \frac{R_c}{R_0}$), Corollary 4.1 and Theorem 4.1, the result in the theorem follows. □

The implication of Theorem 4.2 is that in CSMA networks, since the interference is bounded above by a constant almost surely as shown in Theorem 4.1, to meet an arbitrarily high β (albeit constant with the increase in n), the power needs to be increased only by a constant factor compared with that under the UDM to maintain

the same set of connections. This result is in contrast to the ALOHA networks considered in [18, 19] in which percolation occurs only for a sufficiently small γ.

4.4 A Necessary Condition on the Critical Transmission Power

Section 4.3 derives a sufficient condition for a connected CSMA network as $n \to \infty$ in the presence of interference. A logical question arises: what is the necessary condition for the same CSMA network to be connected as $n \to \infty$.

In a CSMA network, any set of nodes can transmit simultaneously as long as the carrier-sensing constraints are satisfied. Further, in a large-scale network, scheduling is often performed in a distributed manner. In the absence of accurate global knowledge of which particular set of nodes are simultaneously transmitting at a particular time instant, it is natural that a node sets its transmission power to be above *the minimum transmission power* required for a network to be connected under *any* scheduling algorithm (It is trivial to show that, see also the proof of Lemma 4.1, when the transmission power increases, connectivity will also improve). Denote that minimum power by P'_Ω where Ω represents the set of all scheduling algorithms satisfying the carrier-sensing constraints. In this section, we investigate P'_Ω, i.e., a necessary condition required for connectivity as $n \to \infty$. This is done by analyzing the transmission power required for the above network to have no isolated node which is a necessary condition for having a connected network. The following lemma is required for the analysis of P'_Ω:

Lemma 4.1. *Denote by P_Ω (respectively, P_ω) the minimum transmission power required for the network to have no isolated node under any scheduling (respectively,*

under a particular scheduling ω). We have

$$P'_\Omega \geq P_\Omega = \max_{\omega \in \Omega} P_\omega.$$

Proof. We prove the lemma by showing that the minimum transmission power required for the network to have no isolated node under *any* scheduling has to be greater than or equal to the minimum transmission power required for the same network to have no isolated node under a particular scheduling.

Define a set of nodes that can simultaneously transmit while satisfying the carrier-sensing constraints as an *independent set*. Obviously the independent set depends on the transmission power of nodes. As the transmission power decreases, other things being equal, R_c will decrease and the number of nodes that can simultaneously transmit will increase or remain the same.

Denote by ϕ' a set of nodes that are scheduled to transmit simultaneously in the CSMA network. It follows that ϕ' must be an independent set. *Given ϕ'*, a node $v \in \phi'$ is *isolated* if there is no node in the network that can successfully receive from it when the nodes in ϕ' are simultaneously transmitting. Further, as explained in the last paragraph, the independent set depends on the transmission power. When the transmission power is decreased from P_1 to P_2, where $P_2 \leq P_1$, if ϕ' is an independent set at power level P_1, it will also be an independent set at power level P_2. Based on the above observation and using (4.2.1) and (4.2.2), a decrease in the transmission power will cause a decrease in the SINR, it readily follows that if a node $v \in \phi'$ is isolated at power level P_1 when the set of active transmitters is ϕ', it will also be isolated at power level P_2 when the set of active transmitters is ϕ'. For any transmission power less than $P_\Omega = \max_{\omega \in \Omega} P_\omega$, there exists a scheduling that will result the network to have an isolated node at that power level. Therefore, P_Ω has

to satisfy $P_\Omega = \max_{\omega \in \Omega} P_\omega$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Remark 4.3. As an easy consequence of Lemma 4.1, the probability that a CSMA network has no isolated node is a non-increasing function of the transmission power.

Now the task becomes constructing a particular scheduling which gives as large P_ω as possible, i.e. a tight lower bound on P'_Ω. Next we construct such a scheduling ω heuristically.

4.4.1 Construction of scheduling ω

Obviously, ω needs to satisfy the constraint on the minimum separation distance between concurrent transmitters imposed by the carrier-sensing requirement. Meanwhile, ω needs to schedule as many concurrent transmissions as possible to maximize interference, hence P_ω.

We start with a lemma that is required for the construction of ω. We place B_n on the Cartesian coordinate system on the plane in a way that B_n coincides with the square $\left[-\frac{\sqrt{n}}{2}, \frac{\sqrt{n}}{2}\right]^2$.

Lemma 4.2. *Partition B_n into non-overlapping hexagons of equal side length s_n such that the origin o coincides with the centre of a hexagon and two diagonal vertices of this hexagon, whose Euclidean distance is $2s_n$, are located on y axis, as shown in Figure 4.4.1. We call a hexagon an* interior hexagon *if it is entirely contained in B_n. When $s_n = \sqrt{(2\log n)/5}$, a.a.s. each interior hexagon is occupied by at least one node as $n \to \infty$.*

Proof. Because nodes are uniformly i.i.d., the probability that an arbitrary interior hexagon is empty is $\left(1 - \frac{3\sqrt{3}s_n^2}{2n}\right)^n$. Let ξ_i be the event that an interior hexagon i is empty, where $i \in \Xi$ and Ξ denotes the set of indices of all interior hexagons. There are at most $\frac{2n}{3\sqrt{3}s_n^2}$ interior hexagons.
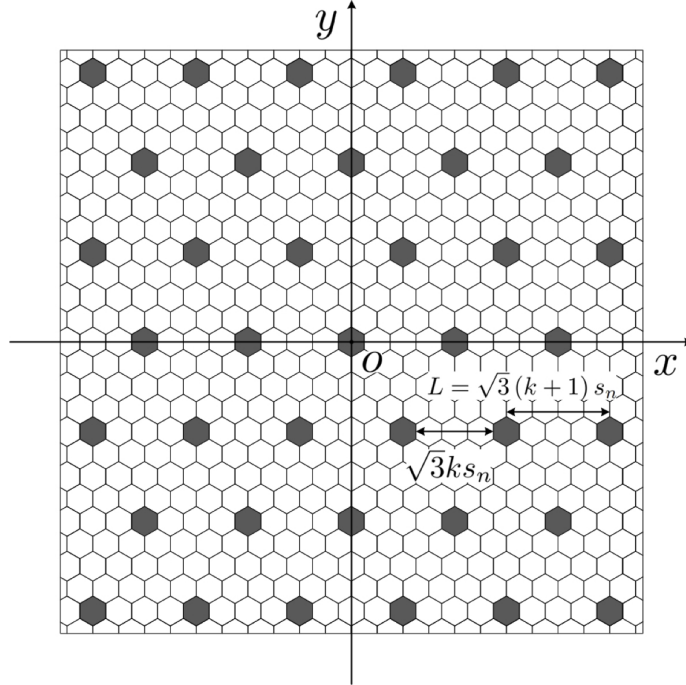
Figure 4.4.1: An illustration of the hexagonal partition of the network area. The shaded hexagons represent simultaneously active hexagons, where $k = 3$.

Denote by A_n the event that there is at least one empty interior hexagon in B_n. It follows that $\Pr\{A_n\} = \Pr\{\cup_{i\in\Xi}\xi_i\}$. Using the union bound, we have $\Pr\{\cup_{i\in\Xi}\xi_i\} \leq \sum_{i\in\Xi}\Pr\{\xi_i\} \leq \frac{2n\left(1-\frac{3\sqrt{3}s_n^2}{2n}\right)^n}{3\sqrt{3}s_n^2}$. Using the fact that $1-x \leq \exp(-x)$ and $s_n = \sqrt{\frac{2\log n}{5}}$, we have $\lim_{n\to\infty}\Pr\{A_n\} \leq \lim_{n\to\infty}\frac{2ne^{-\frac{3\sqrt{3}s_n^2}{2}}}{3\sqrt{3}s_n^2} = \lim_{n\to\infty}\frac{5n}{3\sqrt{3}n^{\frac{3\sqrt{3}}{5}}\log n} = 0$ which completes the proof. $\square$

Hereinafter, we declare a hexagon to be *active* if there is a node transmitting in it. We consider a scheduling ω that uses the hexagons as the basic unit for scheduling. Due to the minimum separation distance, any two simultaneously active hexagons should be separated by a minimum Euclidean distance (depending on the carrier-sensing range given in (4.2.3)). Let k be an integer and represent the minimum number of inactive hexagons between two closest simultaneously active hexagons (see Figure 4.4.1). Any two nodes inside the two active hexagons are separated

by a Euclidean distance of at least $\sqrt{3}ks_n$. With a bit twist of terminology, we further define a *maximal independent set* for scheduling to be the set of hexagons that a) includes as many hexagons as possible; and b) closest hexagons in the set are separated by exactly k adjacent hexagons. Figure 4.4.1 illustrates such a maximal independent set with $k = 3$.

We define ω such that only hexagons belonging to the same maximal independent set can be active at the same time. No nodes in the same hexagon can be scheduled to transmit simultaneously. (Note that if a hexagon intersecting the border of B_n has node(s) in it, it is also included into the maximal independent set and its node(s) are treated in the same way as other nodes in interior hexagons.) As a consequence of the CSMA constraint and the definition of k, we have

$$\sqrt{3}ks_n \geq R_c \geq \sqrt{3}\left(k-1\right)s_n. \tag{4.4.1}$$

4.4.2 Probability of having no isolated node

In this subsection, we derive a lower bound on P_ω for ω defined in the previous subsection. This is done by analyzing the event that the network has no isolated node under ω. The following theorem summarizes another major outcome of this chapter:

Theorem 4.3. *Under the same setting in Theorem 4.2 and the scheduling algorithm ω, a necessary condition on P_ω for the CSMA network to have no isolated node a.a.s. as $n \to \infty$ is*

$$P_\omega \geq \tau b_2^\alpha \left(\log n\right)^{\frac{\alpha}{2}} \tag{4.4.2}$$

where $b_2 = \sqrt{6/5}\left(b-1\right)$ and b is the smallest integer satisfying the inequality:
$\frac{2\left(\sqrt{3}(b+1)+1\right)^{1-\alpha}\left(\sqrt{3}(\alpha-1)(b+1)+1\right)}{(b+1)^2(\alpha-1)(\alpha-2)} \leq \frac{1}{\beta}\left(\frac{2\pi}{5}\right)^{\frac{\alpha}{2}}$.

Proof. The main strategy used is to couple the network under the SINR model with the associated network under UDM. Then, an upper bound on the probability of having no isolated node in the network under the SINR model is obtained by using existing results for UDM.

Denote the Euclidean distance between the centers of two closest hexagons in a maximal independent set by $L = \sqrt{3}\,(k+1)\,s_n$. See Figure 4.4.1 for an illustration. Divide the hexagons belonging to the same maximal independent set as a hexagon h_i into tiers of increasing Euclidean distance from the centre of h_i using a similar strategy as that in the proof of Theorem 4.1. The mth tier of h_i has at most $6m$ hexagons. Further, we declare that the mth tier of h_i is *complete in a given area* if all the $6m$ hexagons are entirely enclosed in this given area. Recall that B_n coincides with the square $\left[-\frac{\sqrt{n}}{2}, \frac{\sqrt{n}}{2}\right]^2$. Denote by $C_A \subset B_n$ a square $\left[-\frac{\sqrt{cn}}{2}, \frac{\sqrt{cn}}{2}\right]^2$ $(0 < c < 1$ and the exact value of c will be decided later in this paragraph). The hexagon containing the origin o has a number of $t = \left\lfloor \frac{\frac{c\sqrt{n}}{2} - \frac{\sqrt{3}s_n}{2}}{L} \right\rfloor$ complete tiers in C_A. As c increases, t increases as well. For the hexagons located in C_A but near the border of C_A, the number of complete tiers in B_n decreases with an increase in c. We choose the value of c such that each hexagon inside C_A has at least t complete tiers in B_n, and the value of t is maximized. Let C_A' be the union of hexagons entirely contained in C_A. With a little bit abuse of terminology, we use C_A (C_A') to denote both the area itself and the size of the area. We can obtain $\lim_{n \to \infty} \frac{C_A'}{C_A} = 1$.

Consider an arbitrarily node i transmitting inside a hexagon h_i in C_A'. If there is no node that can receive from it, then node i is isolated. Let I_{min} be the *minimum* interference that could possibly be experienced by a potential receiver of node i under ω. Note that the Euclidean distance between the transmitter inside a hexagon in the mth tier of h_i and the centre of hexagon h_i is less than $mL + s_n$ (see Figure 4.4.1).

4.4. A Necessary Condition on the Critical Transmission Power

Using Lemma B.1 provided in Appendix B gives

$$
\begin{aligned}
I_{min} &\geq \sum_{m=1}^{t} 6m \left(mL + s_n\right)^{-\alpha} P \\
&= 6Ps_n^{-\alpha} \sum_{m=1}^{t} m \left(\sqrt{3}m\left(k+1\right)+1\right)^{-\alpha} \\
&= 6Ps_n^{-\alpha} \int_1^t \lfloor x \rfloor \left(\sqrt{3}\lfloor x \rfloor\left(k+1\right)+1\right)^{-\alpha} \mathrm{d}x \qquad (4.4.3) \\
&\geq 6Ps_n^{-\alpha} \int_1^t x \left(\sqrt{3}x\left(k+1\right)+1\right)^{-\alpha} \mathrm{d}x \qquad (4.4.4)
\end{aligned}
$$

where $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x. (4.4.4) is obtained due to the fact that $x \left(\sqrt{3}x\left(k+1\right)+1\right)^{-\alpha}$ is a decreasing function when $x > \frac{1}{\sqrt{3}(k+1)(\alpha-1)}$ and $\sqrt{3}\left(k+1\right)\left(\alpha-1\right) > 1$ for $\alpha > 2$ and $k \geq 1$. Therefore $x \left(\sqrt{3}x\left(k+1\right)+1\right)^{-\alpha}$ is a decreasing function when $x > 1$. Further, noting that $\lim\limits_{n\to\infty} t = \lim\limits_{n\to\infty} \left\lfloor \frac{\frac{\sqrt{cn}}{2} - \frac{\sqrt{3}s_n}{2}}{L} \right\rfloor = \infty$, it follows that

$$
\begin{aligned}
&\lim_{n\to\infty} 6 \int_1^t x \left(\sqrt{3}x\left(k+1\right)+1\right)^{-\alpha} \mathrm{d}x \\
&= \frac{2\left(\sqrt{3}\left(k+1\right)+1\right)^{1-\alpha} \left(\sqrt{3}\left(\alpha-1\right)\left(k+1\right)+1\right)}{\left(k+1\right)^2 \left(\alpha-1\right)\left(\alpha-2\right)} \triangleq f\left(k\right).
\end{aligned}
$$

The above equation implies that for an arbitrarily small positive constant ε, there exists a positive integer n_ε such that when $n \geq n_\varepsilon$,

$$
\text{RHS of (4.4.4)} \geq Ps_n^{-\alpha}\left(f\left(k\right)-\varepsilon\right) \triangleq J_n. \qquad (4.4.5)
$$

Let d be the Euclidean distance between node i and its receiver. By (3.1.1), (4.2.2), it follows that *only* when $\frac{Pd^{-\alpha}}{J_n} \geq \beta$, the transmission from node i to its receiver could *possibly* be successful. In other words, if there is no node within a Euclidean distance of $R = \left(\beta J_n/P\right)^{-\frac{1}{\alpha}}$ to node i, then it is isolated.

4.4. A Necessary Condition on the Critical Transmission Power

Denote by M and M^{SINR} the (random) number of isolated nodes in the CSMA network on B_n and in $C'_A \subset B_n$ respectively. Denote by M^{UDM} the (random) number of isolated nodes in the area $C'_A \subset B_n$ in a network with a total of n nodes uniformly i.i.d. on the square B_n under UDM with the transmission range R. Based on the discussion in the last paragraph and using the coupling technique, it can be shown that $\Pr\{M \geq 1\} \geq \Pr\{M^{\text{SINR}} \geq 1\} \geq \Pr\{M^{\text{UDM}} \geq 1\}$. Consequently,

$$\Pr\{M = 0\} \leq \Pr\{M^{\text{UDM}} = 0\}. \tag{4.4.6}$$

It remains to find the value of $\Pr\{M^{\text{UDM}} = 0\}$. We first consider a network with a total of n nodes distributed on B_n under UDM with a transmission range $r(n)$. It is well-known that when the average node degree in the above network equals to $\log n + \zeta(n)$ and $\lim_{n \to \infty} \zeta(n) = \zeta$ where ζ is a constant ($\zeta = \infty$ is allowed), the probability that there is no isolated node in the above network asymptotically converges to $e^{-e^{-\zeta}}$ as $n \to \infty$ [8,77]. Further, it was shown in [22,77] that boundary effect has an asymptotically vanishing impact on the number of isolated nodes. Let Z be a random integer representing the number of nodes located inside $C_A \subset B_n$. It follows from the distribution of nodes that $\text{E}[Z] = cn$ and $\text{Var}[Z] = cn(1-c)$. Let $M^{r(n)}$ be the number of isolated nodes within C_A in the above network with a transmission range $r(n)$. Based on the above results, conditioned on that $Z = cn$ we have (here we have omitted some trivial discussions involving the situation that cn is not an integer)

$$\lim_{n \to \infty} \Pr\left\{M^{r(n)} = 0 \,\middle|\, Z = cn\right\} = e^{-ce^{-\zeta}} \tag{4.4.7}$$

Using Chebyshev's inequality, for $0 < \delta < \frac{1}{2}$, we obtain that

$$\lim_{n \to \infty} \Pr\left\{ |Z - cn| \geq (cn)^{\frac{1}{2}+\delta} \right\} \leq \lim_{n \to \infty} \frac{\text{Var}\,[Z]}{\left((cn)^{\frac{1}{2}+\delta}\right)^2} = 0 \qquad (4.4.8)$$

Let $g(n) = (cn)^{\frac{1}{2}+\delta}$. Using the following two equations: $\log(n + g(n)) + \zeta(n) = \log n + \log\left(1 + \frac{g(n)}{n}\right) + \zeta(n)$ and $\lim_{n\to\infty} \log\left(1 + \frac{g(n)}{n}\right) + \zeta(n) = \lim_{n\to\infty} \zeta(n) = \zeta$ and (4.4.7), it can be shown that $\lim_{n\to\infty} \Pr\left\{ M^{r(n)} = 0 \middle| Z = cn + g(n) \right\} = e^{-ce^{-\zeta}}$. Hence, for any integer y satisfying $-g(n) \leq y \leq g(n)$, it can be shown that $\lim_{n\to\infty} \Pr\left\{ M^{r(n)} = 0 \middle| Z = cn + y \right\} = e^{-ce^{-\zeta}}$. This equation, together with (4.4.8), allows us to conclude that when $r(n) = \sqrt{\frac{\log n + \zeta(n)}{\pi}}$,

$$\lim_{n \to \infty} \Pr\left\{ M^{r(n)} = 0 \right\} = e^{-ce^{-\zeta}}. \qquad (4.4.9)$$

As a result of (4.4.6), a necessary condition for $\lim_{n\to\infty} \Pr\{M = 0\} = 1$ is that $\lim_{n\to\infty} \Pr\left\{ M^{\text{UDM}} = 0 \right\} = 1$. Using the fact that $\lim_{n\to\infty} \frac{C'_A}{C_A} = 1$ and (4.4.9), it follows that a necessary condition for the network under the SINR model to a.a.s. have no isolated node is that $R \geq \sqrt{\frac{\log n + \zeta(n)}{\pi}}$ and $\zeta(n) \to \infty$ as $n \to \infty$. As denoted $R = (\beta J_n / P)^{-\frac{1}{\alpha}}$, together with the value of J_n given by (4.4.5) and the value of s_n given by Lemma 4.2, we obtain that $f(k) \leq \frac{1}{\beta}\left(\frac{2\pi}{5} \frac{\log n}{\log n + \zeta(n)} \right)^{\frac{\alpha}{2}} + \varepsilon$. Letting $n \to \infty$ and then $\varepsilon \to 0$ in the above inequality yields $f(k) \leq \frac{1}{\beta}\left(\frac{2\pi}{5} \right)^{\frac{\alpha}{2}}$. Based on the above equation, together with (4.2.3) and (4.4.1), Theorem 4.3 results. $\qquad\square$

The following corollary is obtained as a ready consequence of Theorem 4.3 and Lemma (4.1).

Corollary 4.2. *A necessary condition required for the CSMA networks to be a.a.s. connected as $n \to \infty$ under any scheduling algorithm, i.e., a lower bound on P'_Ω, is given by*
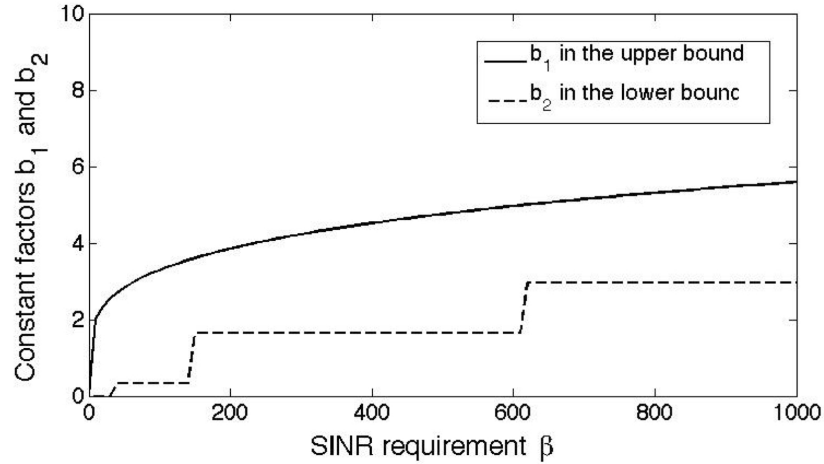
Figure 4.4.2: A plot of the two constant factors b_1 and b_2 in the upper bound (4.3.5) and in the lower bound (4.4.10) when $\alpha = 4$.

$$P'_{\Omega} \geq \tau b_2^{\alpha} \left(\log n\right)^{\frac{\alpha}{2}}. \tag{4.4.10}$$

Comparing the lower bound on P'_{Ω} in (4.4.10) with the upper bound in (4.3.5) and noting that $c(n) = o(\log n)$, it can be shown that, given an arbitrary β, the two bounds differ by a constant factor only as $n \to \infty$. Figure 4.4.2 shows a plot of the two constant factors, viz. b_1 and b_2, in (4.3.5) and in (4.4.10) respectively as a function of β when $\alpha = 4$. The curve representing b_2 is a step function due to the granularity caused by the integer k in the scheduling algorithm ω.

4.5   Summary

In this chapter, we studied the connectivity of wireless CSMA networks considering the impact of interference. We showed that, different from ALOHA networks, the aggregated interference experienced by any receiver in CSMA networks is upper bounded even when the coefficient γ in (3.1.1) equals to 1.

An upper bound and a lower bound were obtained on the critical transmission

power required for having an a.a.s. connected CSMA network. The two bounds are tight and differ by a constant factor only. The results suggested that any pair of nodes can be connected for an arbitrarily high SINR requirement so long as the carrier-sensing capability is available. Compared with that considering UDM without interference, the transmission power only needs to be increased by a constant factor to combat interference and maintain connectivity. This is an optimistic result compared with previous results on the connectivity of ALOHA networks under the SINR model.

The gap between the two bounds can be further narrowed by considering more finer geometric shapes than hexagons. However such improvement is possibly of minor importance. The implication of the results in this chapter is that there exists a spatial and temporal scheduling algorithm in a large-scale CSMA network that allows as many as possible concurrent transmissions, and meanwhile, allows any pair of nodes in the network to be connected under an arbitrarily high SINR requirement. We also introduce a hexagon-based scheduling algorithm that allows the CSMA network to be connected. However, it remains a major challenge to find the optimum scheduling algorithm that gives the minimum delay and the maximum capacity under a specific traffic distribution.

Chapter 5

Transport Capacity

In the preceding chapter, we have investigated the connectivity of wireless CSMA multi-hop networks under the SINR model. In this chapter, we take a further step by studying the transport capacity of large wireless CSMA multi-hop networks. Different from previous studies which rely on the use of centralized scheduling and/or centralized routing algorithm to achieve the optimal capacity scaling law, we show that the optimal capacity scaling law can be achieved using distributed routing and scheduling algorithms. Specifically, we consider a network with nodes Poissonly distributed with unit intensity on a $\sqrt{n} \times \sqrt{n}$ square $B_n \subset \mathbb{R}^2$. Furthermore, each node chooses its destination randomly and independently and transmits following the CSMA protocol. By resorting to the percolation theory and by carefully tuning the three controllable parameters in CSMA protocols, i.e. transmission power, carrier-sensing threshold and count-down timer, we show that a throughput of $\Theta\left(\frac{1}{\sqrt{n}}\right)$ is achievable in distributed CSMA networks. Furthermore, we derive the pre-constant preceding the order of the transport capacity by giving an upper and a lower bound of the transport capacity. The tightness of the bounds is validated using simulations. The results of this chapter appear in [J2].

5.1 Introduction

In a wireless multi-hop network, nodes communicate with each other via wireless multi-hop paths, and packets are forwarded collaboratively hop-by-hop by intermediate relay nodes from sources to their respective destinations. Studying the capacity of these networks is an important problem. Capacity of large wireless networks has been extensively investigated with a particular focus on the throughput scaling laws when the network becomes sufficiently large [24, 27, 28, 32, 34–39, 78].

In the ground-breaking work [24] by Gupta and Kumar, it was shown that in a static network of n nodes uniformly and i.i.d. on an area of unit size and each node is capable of transmitting at W bits/second and using a fixed and identical transmission range, the achievable per-node throughput is $\Theta\left(\frac{W}{\sqrt{n \log n}}\right)$ when each node chooses its destination randomly and independently. If nodes are optimally and deterministically placed to maximize capacity, the achievable per-node throughput becomes $\Theta\left(\frac{W}{\sqrt{n}}\right)$. As discussed in Chapter 1, with assumptions made only on radio propagation process, it was established by many researchers [28–31] that $\Theta\left(\frac{1}{\sqrt{n}}\right)$ is an upper bound on the per-node throughput of wireless multi-hop networks, regardless of the scheduling and routing algorithm being employed. A network is said to achieve the optimal capacity scaling law if it achieves a per-node throughput of $\Theta\left(\frac{1}{\sqrt{n}}\right)$. A number of solutions have been proposed to achieve the optimal capacity scaling law under various network settings and using various routing and scheduling algorithms [24, 27, 32–34, 36, 38–41]. In [32], Franceschetti *et al.* considered the same network as that in [24] except that nodes are allowed to use two different transmission ranges. They showed that by using a routing scheme based on the so-called "highway system" and a centralized/deterministic TDMA protocol, the per-node throughput can reach $\Theta\left(\frac{1}{\sqrt{n}}\right)$ even when nodes are randomly located. Specifically, the highway system is formed by nodes using the smaller transmission range, whereas the larger

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transmission range is used for the last mile, i.e., between the source (or destination) and its nearest highway node. The existence of highway system was established using the percolation theory. The above work of Franceschetti *et al.* [32] and Gupta and Kumar [24], and most other work in the field [27, 33, 35, 36, 38–40], established the capacity of wireless multi-hop networks using centralized scheduling and routing schemes, which may not be appropriate for large-scale networks being investigated in [24, 28, 32].

Chau *et al.* [34] took the lead in studying the throughput of CSMA networks. They showed that CSMA networks can achieve the optimal capacity scaling law $\Theta\left(\frac{1}{\sqrt{n}}\right)$, the same order as networks using centralized TDMA, if multiple backoff countdown rates are used in the distributed CSMA protocol and packets are routed using the highway system proposed in [32]. While the use of distributed CSMA for scheduling in [34] constitutes a significant advance compared with the centralized TDMA considered in previous work, the routing scheme in [34] still relies on the highway system, which needs centralized coordination to identify the highway nodes and to establish the highway. The centralized routing scheme used in [34] is not compatible with the distributed CSMA scheduling scheme. In this sense, the routing and scheduling scheme in [34] is not entirely distributed and may not be suitable for large-scale networks. Furthermore, the deployment of the highway system in CSMA networks in [34] requires two different carrier-sensing ranges to be used: a smaller carrier-sensing range used by the highway nodes and a larger carrier-sensing range used by the remaining nodes to access the highway. The use of two different carrier-sensing ranges may exacerbate the hidden node (HN) problem, which shall be formally defined in Section 5.4. To conquer the potential HN problem brought by the use of two different carrier-sensing ranges, the entire frequency bandwidth is divided into two sub-bands for use by the two types of nodes employing different

carrier-sensing ranges respectively. This imposes additional hardware requirements on the nodes and also causes spectrum waste.

Based on the above observations, we are motivated to develop a distributed scheduling and routing algorithm to achieve the order-optimal throughput in CSMA networks in this chapter. Specifically, by resorting to the percolation theory and by carefully tuning the three controllable parameters in CSMA protocols, i.e., transmission power, carrier-sensing threshold and count-down timer, we show that a throughput of $\Theta\left(\frac{1}{\sqrt{n}}\right)$ is achievable in distributed CSMA networks operating with one frequency band. More important, we analyze the pre-constant preceding the order of the transport capacity by giving an upper and a lower bound of the transport capacity. The tightness of the bounds is established using simulations.

The following is a detailed summary of our contributions:

- We develop a distributed routing and scheduling algorithm that is able to achieve the order-optimal throughput in CSMA networks. More specifically, the routing decision relies on the use of local neighborhood knowledge only and each node competes for channel access in a distributed and randomized manner using CSMA protocols.

- We demonstrate that by jointly tuning the carrier-sensing threshold and the transmission power, the HN problem can be eliminated even for nodes using different carrier-sensing thresholds, different transmission powers and one common frequency band. This is different from the techniques used in the previous work [34] where nodes using different carrier-sensing ranges have to use different frequency band for transmission. The technique developed provides guidance on setting the carrier-sensing threshold and the transmission power to avoid the HN problem in CSMA networks in a more general setting.

- As pointed out in [3], the pre-constant is important to fully understand the impact of various parameters on network capacity. We analyze the pre-constant preceding the order of the transport capacity by giving an upper and a lower bound of the transport capacity.

- Extensive simulations are carried out which validate the tightness of our analytical results.

The rest of this chapter is organized as follows. Section 5.2 defines notations and concepts used in the later analysis; Section 5.3 describes the routing algorithm and analyzes the traffic load of each node; Section 5.4 presents the solution for obtaining a hidden node free CSMA network; Section 5.5 optimizes the medium access probability for each node by tuning the backoff timer and analyzes the per-node throughput under our proposed communication strategy; Finally, Section 5.6 summarizes this chapter.

5.2 Definitions and Notations

5.2.1 Data rate

Despite the common knowledge that a higher SINR can lead to an increased link capacity, in reality transmission from a transmitter to a receiver can only occur at one of a set of preset data rates after the SINR threshold is met [68,69]. Therefore for a transmitter-receiver pair, when its associated SINR is above β, it is considered that the transmitter can transmit to the receiver at a fixed rate of

$$W = \log_2\left(1 + \beta\right) \text{ bits per second.} \tag{5.2.1}$$

5.2.2 Definition of throughput

Each node sends packets to an independently and randomly chosen destination node via multiple hops. A node can be a source node, a destination node for another source node, a relay node or a mixture.

The per-node throughput or equivalently the transport capacity of the network, denoted by $\lambda(n)$, is defined as the *maximum* rate that could be achieved a.a.s. by *all* source-destination pairs simultaneously. Similar as that in [24], we say that a per-node throughput of $\lambda(n)$ is *feasible* if there is a temporal and spatial routing and scheduling scheme such that every node can send $\lambda(n)$ bits per second *on time average* to its destination a.a.s., i.e., there exists a sufficiently large positive number μ such that in every finite time interval $[(j-1)\mu, \, j\mu]$ every node can send $\mu\lambda(n)$ bits to its destination a.a.s..

5.3 Routing Algorithm and Traffic Load

In this section we describe the routing algorithm to be used and analyze the traffic load for each node under the algorithm. The routing algorithm chooses the sequence of nodes to deliver a packet from its source to its destination without considering physical layer implementation details.

To begin the construction of our routing algorithm, we partition B_n of size $\sqrt{n} \times \sqrt{n}$ into *squares* of side length $c_1 \log n$ where c_1 is a positive constant. Each of these squares is then further subdivided into smaller *cells* of constant side length c. The values of c_1 and c will be specified later. See Fig. 5.3.2 for an illustration. Following common terminology used in the percolation theory, we also refer to these cells as *sites* and use the two terms cells and sites exchangeably. We call a site *open* if it contains at least one node, and *closed* otherwise. Due to the Poisson distribution of

nodes with unit intensity, a site is open with probability $p \triangleq 1 - e^{-c^2}$. Furthermore, the event that a site is open or closed is independent of the event that another distinct site is open or closed. The total number of sites in a square is $\left(\frac{c_1}{c} \log n\right)^2$, the total number of sites in B_n is $\left(\frac{\sqrt{n}}{c}\right)^2$ and the total number of squares in B_n is $\left(\frac{\sqrt{n}}{c_1 \log n}\right)^2$. The techniques to handle the situation that $\frac{c_1}{c} \log n$, $\frac{\sqrt{n}}{c}$ and $\frac{\sqrt{n}}{c_1 \log n}$ are not integers are well-known [32]. Therefore in this paper we ignore some trivial discussions involving the situations that $\frac{c_1}{c} \log n$, $\frac{\sqrt{n}}{c}$ and $\frac{\sqrt{n}}{c_1 \log n}$ are not integers and consider them to be integers.

Before we can further explain our routing algorithm, we need to first establish some preliminary results. The network area B_n can be sliced into horizontal rectangles of size $c_1 \log n \times \sqrt{n}$, where each horizontal rectangle consists of $\frac{\sqrt{n}}{c_1 \log n}$ squares. Denote by H_i the ith horizontal rectangle where $1 \leq i \leq \frac{\sqrt{n}}{c_1 \log n}$. We call two sites *adjacent* if they share a common edge. We define *a left to right open path* in H_i as a sequence of distinct and adjacent open sites that starts from an open site on the left border of H_i and ends at an open site on the right border of H_i. The following theorem, due to [8, Theorem 4.3.9], gives a lower bound on the number of open paths contained in H_i.

Theorem 5.1. *[8, Theorem 4.3.9]Consider site percolation with parameter $p = 1 - e^{-c^2}$. For c sufficiently large, there exist constants c_1 and ω_1 independent of n, satisfying*

$$\frac{5}{6} < p < 1, \tag{5.3.1}$$

$$2 + c_1 \log\left(6\left(1 - p\right)\right) < 0 \tag{5.3.2}$$

and

$$\omega_1 \log \frac{p}{1 - p} + c_1 \log\left(6\left(1 - p\right)\right) + 2 < 0, \tag{5.3.3}$$
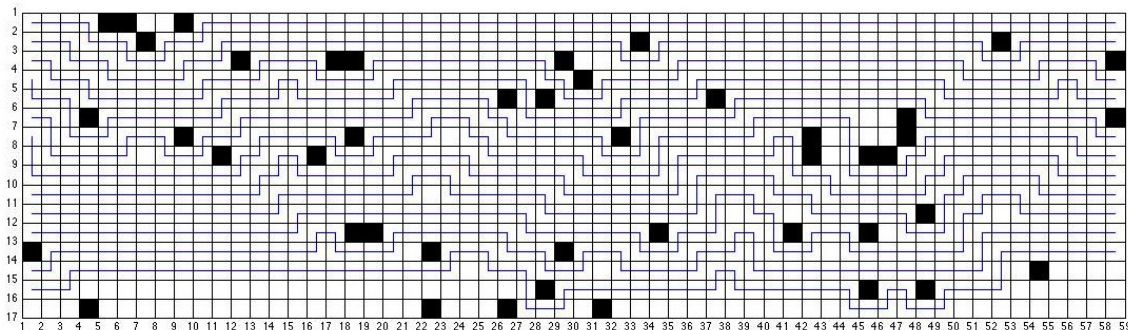
48

Figure 5.3.1: An illustration of left-to-right open paths in a rectangle obtained by computer simulations. Black cells represent closed sites while white cells represent open sites.

such that a.a.s. there exist at least $\omega_1 \log n$ left to right disjoint open paths in every horizontal rectangle.

Fig. 5.3.1, drawn from a simulation, further gives an intuitive illustration of the open paths in a horizontal rectangle.

By symmetry, if we partition B_n into $\frac{\sqrt{n}}{c_1 \log n}$ vertical rectangles. Each one is of size $c_1 \log n \times \sqrt{n}$ and consists of $\frac{\sqrt{n}}{c_1 \log n}$ squares. Denote by V_j the jth, $1 \leq j \leq \frac{\sqrt{n}}{c_1 \log n}$ vertical rectangle. It can also be established that a.a.s. there are at least $\omega_1 \log n$ top to bottom disjoint open paths in every V_j, $1 \leq j \leq \frac{\sqrt{n}}{c_1 \log n}$. The following result can be readily established:

Corollary 5.1. *There are a.a.s at least $\omega_1 \log n$ left-to-right open paths and $\omega_1 \log n$ top-to-bottom open paths in every square.*

We are now ready to describe our routing algorithm. Denote by SD_i the line segment connecting node i to its destination. The packets generated by source node i are routed along the squares intersecting SD_i. A square will only serve the traffic of a source-destination pair if the associated SD line intersects the square. Note that it is trivial to establish that a.a.s. every square has at least one node. The routing can be divided into three stages:

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In the first stage, a source node S, if it is not a node located in an open site that forms one of the open paths, will transmit its packet to a node in a *randomly chosen* open site that forms an open path. If there are multiple nodes in an open site, a node will be designated randomly to relay all traffic passing through the site. If the source node is already in a site that forms an open path, this stage of routing can be omitted and the routing proceeds directly to the next stage. The maximum distance between the source node and its next-hop node in this stage is bounded by $\sqrt{2}c_1 \log n$ because the distance between any two nodes located in a square is at most $\sqrt{2}c_1 \log n$.

In the second stage, the packet will be routed to the adjacent square intersecting the SD line along one of these left-to-right open path or top-to-bottom open paths until the packet reaches a node in the next square. Depending on the location of the open path containing the relay node and the location of the adjacent square, the packet may be routed along a left-to-right open path (when the adjacent square is on the left or on the right of the current square) or along a top-to-bottom open path (when the adjacent square is on the top or on the bottom of the current square). If the packet needs to be switched from a left-to-right open path to a top-to-bottom open path (e.g., when the previous square is on the left of the current square but the next square is on the bottom of the current square), a top-to-bottom open path is chosen randomly from the at least $\omega_1 \log n$ open path available. The above process continues until the packet reaches the square that contains the destination node. In this stage, the maximum distance between a node and its next-hop node is bounded by $\sqrt{5}c$ because the distance between any two nodes located in two adjacent cells is at most $\sqrt{5}c$.

In the third stage, after reaching the square containing the destination node, if the destination node is located on one of the open paths, the packet will be routed
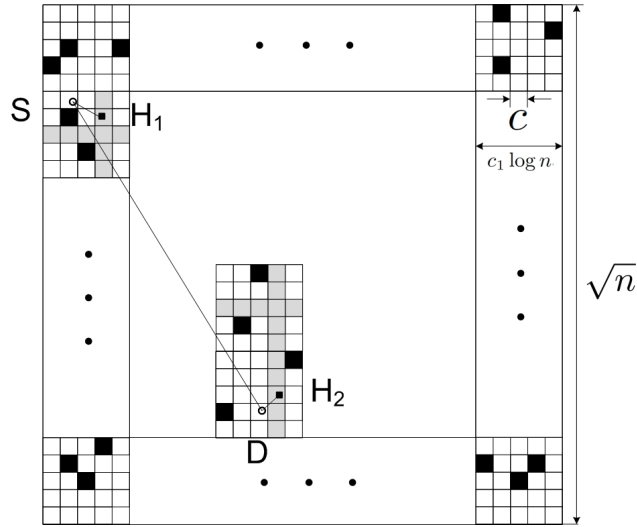
Figure 5.3.2: An illustration of partition of B_n and the routing algorithm. Black square represents a closed site and white square represents an open site. Grey square represents an open site that forms an open path. S and D, indicated by two small hollow circles, are a pair of source and destination nodes. H_1 and H_2, indicated by two small black squares, are two nodes located in open sites that form open paths. First S transmits its packets to H_1 using a transmission range of up to $\sqrt{2}c_1 \log n$. Then the packets will be routed along the open paths to H_2, using a transmission range of up to $\sqrt{5}c$. Finally, H_2 transmits the packets to the destination D. If H_1 itself is a source node, then it transmits its packet directly to the next-hop node along the open path, using a transmission range of up to $\sqrt{5}c$.

along a multi-hop path to the destination via open paths; if the destination is not located on one of the open paths, the packet will be transmitted to the destination directly and the maximum transmission distance is bounded by $\sqrt{2}c_1 \log n$.

The same route is used for all packets belonging to the same source-destination pair.

The feasibility of the above routing algorithm is guaranteed by Corollary 5.1. A node only needs neighborhood information of nodes no more than $\sqrt{5}c_1 \log n$ away to make a routing decision. The required information for making a proper routing decision is vanishingly small compared with that in the highway algorithm. Furthermore, compared with the network size, the required information is also vanishingly
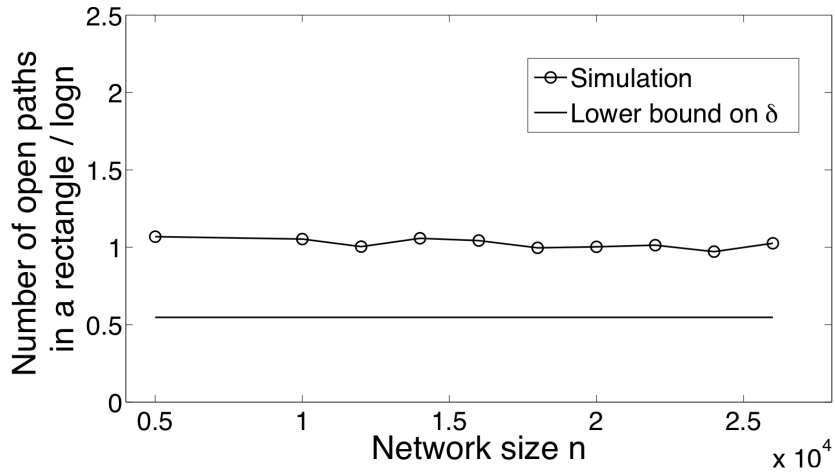
Figure 5.3.3: An illustration of the number of left-to-right open paths in a horizontal rectangle as the network size varies. Vertical axis shows the ratio of the number of open paths to $\log n$.

small as $n \to \infty$. Therefore the routing algorithm can be executed in a distributed manner.

Corollary 5.2 is a ready consequence of Theorem 5.1:

Corollary 5.2. *Let $c = 1.7308$ and $c_1 = 3$, a.a.s. there are at least $0.5474 \log n$ left-to-right open paths in every horizontal rectangle.*

In the rest of this paper, we carry out analysis assuming that c and c_1 take values specified in Corollary 5.2 and $\omega_1 = 0.5474$. Fig. 5.3.3 shows simulation results of the number of open paths in a horizontal rectangle as the network size n varies. Each random simulation is repeated a large number of times and the average result is shown. The confidence interval is very small and negligible, and thus not plotted in the figure. The lower bound on the number of open paths suggested in Corollary 5.2 is also plotted for comparison. As shown in Fig. 5.3.3, the lower bound is reasonably tight.

After establishing the routing algorithm, next we analyze the traffic load for each node under the algorithm, which forms a key step in analyzing the network capacity.

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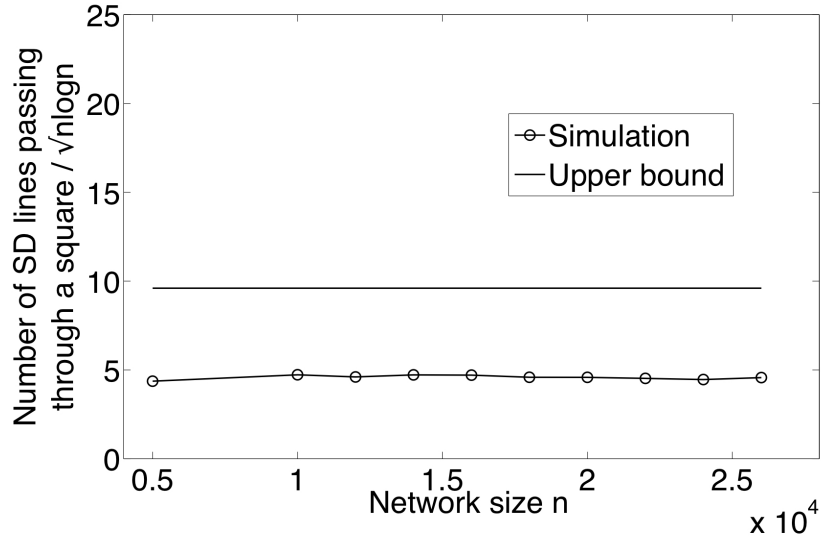
Figure 5.3.4: The number of SD lines passing through a square versus the upper bound in Lemma 5.1. Vertical axis shows the ratio of the number of SD lines passing through a square to $\sqrt{n}\log n$.

Lemma 5.1 shows that the random number of SD lines passing through an arbitrarily chosen square, including the SD lines originating from and ending at the square, is upper bounded.

Lemma 5.1. *For an arbitrary square in B_n, the random number of SD lines passing through it, denoted by Y, satisfies that*

$$\lim_{n\to\infty} \Pr\left\{Y \le \omega_2\sqrt{n}\log n\right\} = 1 \tag{5.3.4}$$

where $\omega_2 = 3.2\left(1 + \epsilon\right)\left(1 + \delta_1\right)c_1$, ϵ *and* δ_1 *are arbitrarily small positive constants.*

Proof. See Appendix C. □

As a way of establishing the tightness of the bound in Lemma 5.1, Fig. 5.3.4 shows simulation results of the number of SD lines passing a square in comparison with the upper bound in Lemma 5.1.

Using Corollary 5.1 and Lemma 5.1, the following result can be readily established:

Lemma 5.2. *Each relay node needs to carry the traffic of at most $\frac{\omega_2 \sqrt{n}}{0.5474}$ source-destination pairs a.a.s.*

Note that a node not on an open path does not need to carry the traffic of other source-destination pairs.

5.4 A Solution to HN Problem

Our routing algorithm described in the last section needs to use two different transmission ranges of lengths $\Theta(1)$ and $\Theta(\log n)$ respectively. The use of two different transmission ranges in CSMA networks will exacerbate the HN problem. See Fig. 5.4.1 for an illustration. Assume that the same carrier-sensing threshold is used by node A and B. The transmission of A using a lager transmission power (node B using a smaller transmission power, respectively) can be detected by nodes located within a distance R_A (R_B, respectively), and $R_A > R_B$. Consequently B can detect A's transmission but node A cannot detect node B's. Therefore even when node B is transmitting, node A still can start its own transmission, thereby resulting in a collision and causing the HN problem. In [34], the problem was addressed by letting nodes operate on two frequency bands, namely, short-range transmissions operate on one frequency band while long-range transmissions operate on the other. Their solution may result in lower spectrum usage because long-range transmission is used less frequently and also pose additional hardware requirements on nodes. Therefore, we present a solution by jointly tuning the transmission power and the carrier-sensing threshold.
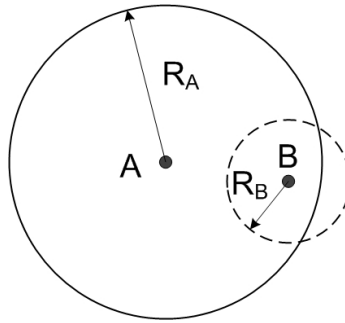
Figure 5.4.1: An illustration of the HN problem when nodes use different transmission power.

5.4.1 A formal definition of the HN problem

Under the SINR model, a set of concurrent transmissions (or links) are said to form an *independent set* if the SINRs are *all* above the SINR threshold β. Let $\mathcal{F}$ be the set of all independent sets. Because of the random and distributed nature of the carrier-sensing operations by individual nodes, the set of simultaneous transmissions observing the carrier-sensing constraint, denoted by $\mathcal{S}^{CS}$, may or may not belong to $\mathcal{F}$, i.e., some transmissions observing the carrier-sensing constraints may still cause the SINRs at some receivers to be above β. Let $\mathcal{F}^{CS}$ be the set of all $\mathcal{S}^{CS}$s. Let Ψ be the set of concurrent transmissions in a CSMA network. More formally, a HN problem is said to occur if $\Psi \in \mathcal{F}^{CS}$ but $\Psi \notin \mathcal{F}$. A CSMA network is said to be *hidden node free* if its carrier-sensing operations and transmission powers are carefully designed such that all $\Psi \in \mathcal{F}^{CS}$ also meets the condition that $\Psi \in \mathcal{F}$.

For a CSMA network in which uniform transmission power is in use, by setting the carrier-sensing range to be a constant multiple of the transmission range, the hidden node problem can be effectively eliminated [34,79]. For our routing algorithm using two transmission ranges of lengths $\Theta(1)$ and $\Theta(\log n)$, if the carrier-sensing range is set to be $\Theta(\log n)$, although the hidden node problem can be eliminated, the number of concurrent transmissions (hence the spatial frequency reuse) will be

reduced compared with a carrier-sensing range of $\Theta(1)$, which in turn causes a reduced capacity. Therefore we manage to have transmissions with different lengths to coexist concurrently instead. In this way, the capacity will be maximized while eliminating the HN problem.

More specifically, let P_i^k be the transmission power used for the kth transmission by node i where the same transmitter may use different power when transmitting to different receiver. The transmitter also uses different carrier-sensing threshold when different transmission power is used. Denote by τ_i^k the carrier-sensing threshold used for P_i^k. Furthermore, let the transmission power of a transmitter be such that the power received at its intended receiver is at least $\bar{P}$ ($\bar{P}$ is a constant not depending on n and the value of $\bar{P}$ will be specified shortly later in this section). In the following analysis, for the simplicity of notation, we drop off the superscript of P_i^k. The following lemma specifies the relation between P_i and τ_i required for two transmitters to be able to sense each other's transmission.

Lemma 5.3. *Let the values of P_i and τ_i be chosen such that the following condition is met*

$$P_i = \bar{P}/\tau_i. \tag{5.4.1}$$

For two arbitrary transmitters located at $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ respectively, they can sense each other's transmission iff

$$\|\boldsymbol{x}_i - \boldsymbol{x}_j\| < \left(\frac{\bar{P}}{\tau_i \tau_j}\right)^{\frac{1}{\alpha}} = \left(\frac{P_i P_j}{\bar{P}}\right)^{\frac{1}{\alpha}}. \tag{5.4.2}$$

Proof. When node i located at $\boldsymbol{x}_i$ transmits using power P_i, the power received at node j at location $\boldsymbol{x}_j$ is given by $P_i \|\boldsymbol{x}_i - \boldsymbol{x}_j\|^{-\alpha}$. Let τ^j be the carrier-sensing threshold of node j. The transmission of node i can be detected iff $P_i \|\boldsymbol{x}_i - \boldsymbol{x}_j\|^{-\alpha} > \tau^j$. Using (5.4.1), node j can detect node i's transmission iff $\frac{\bar{P}}{\tau_i} \|\boldsymbol{x}_i - \boldsymbol{x}_j\|^{-\alpha} > \tau_j$ or

equivalently $\|\boldsymbol{x}_i - \boldsymbol{x}_j\| < \left(\frac{\bar{P}}{\tau_i \tau_j}\right)^{\frac{1}{\alpha}} = \left(\frac{P_i P_j}{\bar{P}}\right)^{\frac{1}{\alpha}}$. Using a similar argument, node i can detect node j's transmission iff (5.4.2) is met. $\qquad\qquad\square$

Lemma 5.3 shows that by carefully choosing the carrier-sensing threshold according to the transmission power for each transmitter, a major cause of the hidden node problem: a node A senses another node B's transmission but node B cannot sense node A's transmission can be eliminated. In the next several paragraphs, we shall demonstrate how to choose $\bar{P}$, which determines the minimum power received at a receiver, such that the SINR requirement can also be met.

In the first and third stages of our routing algorithm, the maximum distance between a transmitter and a receiver is $\sqrt{2}c_1 \log n$ while the the maximum distance between a transmitter and a receiver in the second stage is $\sqrt{5}c$. Accordingly, for the first and third stages, we let the transmission power be

$$P^h = \bar{P}\left(\sqrt{2}c_1 \log n\right)^{\alpha}, \tag{5.4.3}$$

while for the second stage, the transmission power is set to be at

$$P^l = \bar{P}\left(\sqrt{5}c\right)^{\alpha}. \tag{5.4.4}$$

The received signal power of all transmissions is at least $\bar{P}$. Furthermore, Theorem (4.1) established in Section (4.3) helps to obtain an upper bound on the interference experienced by any receiver in the network. Consider a CSMA network with nodes distributed arbitrarily on a finite area in $\mathbb{R}^2$ where all nodes transmit at the same power P and use the same carrier-sensing threshold τ. Let r_0 be the distance between a receiver and its transmitter. The maximum interference experienced by the receiver

is smaller than or equal to $N_1 \left(d, r_0 \right) + N_2 \left(d \right)$ where

$$N_1 \left(d, r_0 \right) = \frac{4 \left(\frac{5\sqrt{3}}{4} d - r_0 \right)^{1-\alpha} \left(\frac{\sqrt{3}}{4} \left(3\alpha - 1 \right) d - r_0 \right)}{d^2 \left(\alpha - 1 \right) \left(\alpha - 2 \right)}$$
$$+ \frac{3}{\left(d - r_0 \right)^{\alpha}} + \frac{3}{\left(\sqrt{3}d - r_0 \right)^{\alpha}} + \frac{3 \left(\frac{3}{2} d - r_0 \right)^{1-\alpha}}{\left(\alpha - 1 \right) d} \qquad (5.4.5)$$

and

$$N_2 \left(d \right) = \frac{3}{d^{\alpha}} + \frac{3 \left(\frac{3}{2} \right)^{1-\alpha}}{\left(\alpha - 1 \right) d^{\alpha}} + \frac{3}{\left(\sqrt{3}d \right)^{\alpha}}$$
$$+ \frac{3 \left(\frac{5}{4} \right)^{1-\alpha} \left(3\alpha - 1 \right)}{\left(\alpha - 1 \right) \left(\alpha - 2 \right) \left(\sqrt{3}d \right)^{\alpha}} \qquad (5.4.6)$$

and $d = \left(\frac{P}{\tau} \right)^{\frac{1}{\alpha}}$.

Noting that $N_1 \left(d, r_0 \right)$ is a monotonically increasing function of r_0, it can be readily established that in the CSMA network analyzed in this chapter in which two sets of transmission power, carrier-sensing threshold and the maximum transmission range are employed, the maximum interference (for any value of n) is bounded by

$$N_1 \left(\left(\frac{P^h}{\tau^h} \right)^{\frac{1}{\alpha}}, \sqrt{2}c_1 \log n \right) + N_2 \left(\left(\frac{P^h}{\tau^h} \right)^{\frac{1}{\alpha}} \right) + N_1 \left(\left(\frac{P^l}{\tau^l} \right)^{\frac{1}{\alpha}}, \sqrt{5}c \right) + N_2 \left(\left(\frac{P^l}{\tau^l} \right)^{\frac{1}{\alpha}} \right)$$
$$(5.4.7)$$

where τ^l and τ^h are the carrier-sensing threshold chosen for P^l and P^h respectively according to (5.4.1).

Remark 5.1. At the expense of more analytical efforts, a tighter bound on interference can be established that the maximum interference in the CSMA network considered in this chapter is bounded by $N_1 \left(\left(\frac{P^l}{\tau^l} \right)^{\frac{1}{\alpha}}, \sqrt{5}c \right) + N_2 \left(\left(\frac{P^l}{\tau^l} \right)^{\frac{1}{\alpha}} \right)$ for *any value* of n. Because for a sufficiently large network, which is the focus of this chapter,

the difference between this bound and the upper bound in (5.4.7) is negligibly small. It is easy to conclude that using (5.4.5) and (5.4.6) when $\alpha > 2$, the contribution of the first two terms $N_1\left(\left(\frac{P^h}{\tau^h}\right)^{\frac{1}{\alpha}}, \sqrt{2}c_1 \log n\right) + N_2\left(\left(\frac{P^h}{\tau^h}\right)^{\frac{1}{\alpha}}\right)$, attributable to transmissions using a larger transmission power, become vanishingly small compared with the last two terms as $n \to \infty$. The following theorem provides guidance on how to choose $\bar{P}$ to meet the SINR requirements for all concurrent transmissions in a large CSMA network.

Theorem 5.2. *For an arbitrarily high SINR requirement β, there exists a value of $\bar{P}$ for sufficiently large n such that the SINR of all transmissions in a CSMA network, in which each transmitter sets its transmission power and carrier sensing threshold according to the relationship in Lemma 5.3, is greater than or equal to β. Furthermore, the value of $\bar{P}$ is given implicitly by the following equation*

$$\frac{\bar{P}}{N_1\left(\left(\frac{P^l}{\tau^l}\right)^{\frac{1}{\alpha}}, \sqrt{5}c\right) + N_2\left(\left(\frac{P^l}{\tau^l}\right)^{\frac{1}{\alpha}}\right)} = \beta. \tag{5.4.8}$$

Proof. Noting that the minimum received power is $\bar{P}$, the theorem becomes an easy consequence of the interference upper bound established earlier in the section. $\square$

As a brief summary of the results of this section, Theorem 5.2 gives guidance on how to choose $\bar{P}$ to meet the SINR requirement. When the value of $\bar{P}$ is fixed, the transmission powers are then determined using (5.4.3) and (5.4.4) respectively. Finally, the carrier sensing threshold associated with each transmission power is determined using Lemma 5.3, which ensures that nodes can sense each other's transmission. It can be readily established that the CSMA network whose transmission power and carrier sensing threshold are chosen following the above steps are immune from the HN problem.

5.5 backoff Timer Setting and Capacity Analysis

In the last section, we demonstrated how to choose the transmission power and the carrier sensing threshold to solve the HN problem. In the CSMA network in which nodes may use two different transmission powers, a potential problem that may arise is that nodes using the larger transmission power may potentially contend with more nodes for transmission opportunities. Therefore nodes using the larger transmission power may not get a fair transmission opportunity compared with nodes using the smaller transmission power. This may potentially causes nodes using the larger transmission power to become a bottleneck in throughput which reduces the overall network capacity. In this section, we demonstrate how to choose another controllable parameter in CSMA protocols, i.e., backoff timer, to conquer the difficulty.

Same as that in references [34] and [80], we consider a CSMA protocol in which the initial backoff timer is a random variable following an exponential distribution. Nodes using different transmission power may however choose different mean value to use in the exponential distribution governing their respective random initial backoff timer. The following theorem provides the basis for choosing these mean values.

Theorem 5.3. *Let δ_2 and δ_3 be two small positive constants. If transmissions using a low transmit power P^l set their initial backoff time to be exponentially distributed with mean $\lambda_l = 1$ and transmissions using a high transmission power P^h set their initial backoff time to be exponentially distributed with mean $\lambda_h = \frac{1}{\log^2 n}$, then*

(i) a.a.s. each low power transmission can be active with a constant probability greater than or equal to

$$\omega_3 = \frac{1}{\pi \left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2 + (1+\delta_2)\,10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}} + 1} \tag{5.5.1}$$
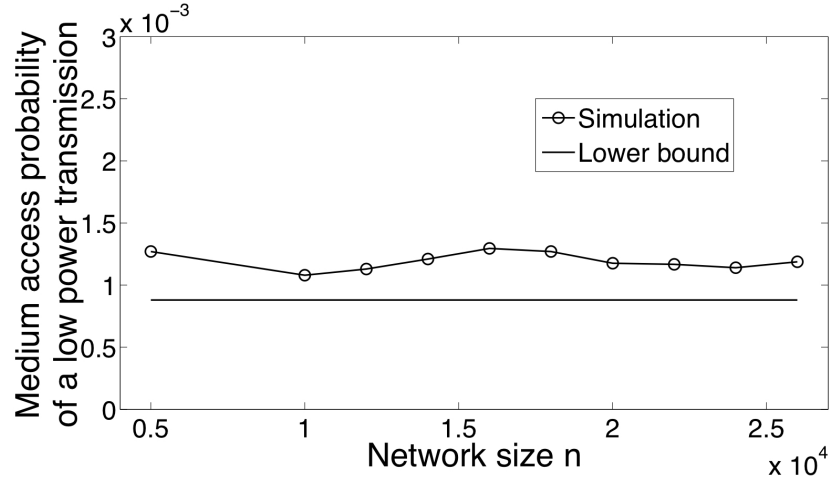
Figure 5.5.1: A comparison between the simulation result on the medium access probability of a node using the low power transmission with the lower bound in Theorem 5.3 when $\beta = 10$ and $\alpha = 4$.

(ii) a.a.s. each high power transmission can be active with a probability greater than or equal to

$$\omega_4 = \frac{1}{\pi \left(\sqrt{10}c_1\bar{P}^{\frac{1}{\alpha}}\right)^2 \log^4 n + (1 + \delta_3)\, 4\pi c_1^4 \bar{P}^{\frac{2}{\alpha}} \log^4 n + 1}$$

Proof. See Appendix D. □

Fig. 5.5.1 shows the transmission opportunity (or the medium access probability) of a node using P^l versus the lower bound in Theorem 5.3 for different values of n.

On the basis of the results established in this section and in the earlier sections, we present the following theorem which forms the major result of this chapter.

Theorem 5.4. *The achievable per-node throughput in the CSMA network is greater than or equal to*

$$\frac{0.5474\omega_3}{\omega_2\sqrt{n}}W; \tag{5.5.2}$$

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and is smaller than or equal to

$$\frac{1}{0.52c \left(5\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}} + 1\right) \sqrt{n}} \times W$$

a.a.s. as $n \to \infty$, where ω_2 is given in Lemma 5.1 and ω_3 is given by (5.5.1).

Proof. We first show that the achievable per-node throughput is lower bounded by $\frac{0.5474\omega_3}{\omega_2\sqrt{n}}W$. Let $\lambda_1(n)$ ($\lambda_2(n)$, respectively) be the per-node throughput that can be achieved in the first and the third (the second, respectively) stages of our routing algorithm. Obviously the final per-node throughput $\lambda(n)$ satisfies $\lambda(n) = \min\{\lambda_1(n), \lambda_2(n)\}$. In the following, we analyze $\lambda_1(n)$ and $\lambda_2(n)$ separately.

As an easy consequence of Lemma 5.2, a.a.s. each relay node carries the traffic of at most $\frac{\omega_2\sqrt{n}}{0.5474}$ source-destination pairs. According to the first statement of Theorem 5.3, a.a.s. each relay node on an open path can access the channel with a probability of at least ω_3, which is a constant independent of n. The conclusion then readily follows that $\lim_{n\to\infty}\Pr\left\{\lambda_1(n) \geq \frac{0.5474\omega_3}{\omega_2\sqrt{n}}W\right\} = 1$.

For the second stage of the routing, note that a source or a destination node *not* on an open path does not need to carry traffic for other source-destination pairs. Using the second statement of Theorem 5.3, conclusion follows that $\lambda_2(n) = \Omega\left(\frac{1}{\log^4 n}\right)$.

Combining the above two results on $\lambda_1(n)$ and $\lambda_2(n)$ and noting that the capacity bottleneck lies in the first and the third stages, the first statement in this theorem is proved.

We now further show that the achievable per-node throughput is upper bounded by $\frac{W}{0.52c\left(5\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}}+1\right)\sqrt{n}}$. The upper bound is to be established using a result proved in [?, Corollary 6], which shows that the per-node throughput is equal to the product of the average number of simultaneous transmissions and the link capacity divided by the product of the average number of transmissions required to deliver a packet

to its destination and the number of source-destination pairs. We first analyze the average number of transmissions required for a packet to reach its destination. The average distance between a randomly chosen source-destination pair is $0.52\sqrt{n}$ [81]. A packet moves by one cell in each hop on an open path where the contribution of the last mile transmission between a source (a destination) and an open-path node is vanishingly small compared with $0.52\sqrt{n}$. Thus a.a.s. the average number of hops traversed by a packet is at least $\frac{0.52\sqrt{n}}{c}$. Next we analyze the average number of simultaneous transmissions. Since there is at most one node in a cell acting as an open path node, there are at most $\frac{n}{c^2}$ open path nodes in the network. Let η_i^l be the event that a transmission of node i using the low transmit power is active. Following the same procedure in obtaining (D.0.1), (D.0.2) and (D.0.3), we have that $\Pr\left\{\eta_i^l\right\} \leq \frac{1}{5\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}}+1}$. Therefore, the average number of simultaneous transmissions is at most $\frac{n}{c^2} \times \frac{1}{5\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}}+1}$. Note that when a non-open-path node transmits with P^h, the number of simultaneous transmissions will only reduce. As a ready consequence of the above analysis and [?, Corollary 6], an upper bound on the per-node throughput results. $\qquad\square$

The lower bound on the per-node throughput provided in Theorem 5.4 is order optimal in the sense that the throughput is of the same order as the known result on the optimum per-node throughput [32] of networks under the same settings. Furthermore, Theorem 5.4 gives the pre-constant preceding the order of the per-node throughput: $\frac{0.5474\omega_3}{\omega_2}$. A detail examination of the pre-constant reveals that the pre-constant can be separated into the product of two terms: $\frac{0.5474}{\omega_2}$ and ω_3. The first term $\frac{0.5474}{\omega_2}$ is entirely determined by the routing algorithm, more specifically determined by how the routing algorithm distribute traffic load among relay nodes and among source-destination pairs. The second term ω_3 is entirely determined by the scheduling algorithm and some physical layer details, i.e., the SINR requirement,

Figure 5.5.2: A simulation of per-node throughput with $\alpha = 4$ and $\beta = 10$. For comparison, the upper and the lower bound obtained is also shown.

interference and propagation model. The above observation appears to suggest that impact of the routing algorithm and the scheduling algorithm can be decoupled and studied separately, and the two algorithms that determine the overall network capacity can be optimized separately.

Fig. 5.5.2 shows a comparison of the per-node throughput obtained from simulations, the upper and lower bounds obtained in Theorem 5.4 for different values of n. To facilitate comparison, Fig. 5.5.3 further shows the ratio of the per-node throughput obtained from simulations to the throughput lower bound and the ratio of the throughput upper bound to the throughput lower bound. As shown in the figures, the lower bound is fairly tight and the upper bound is also within a factor of 10 of the simulation result. The simulation results demonstrate that the pre-constant obtained in our study provides a pretty accurate characterization of the per-node throughput.
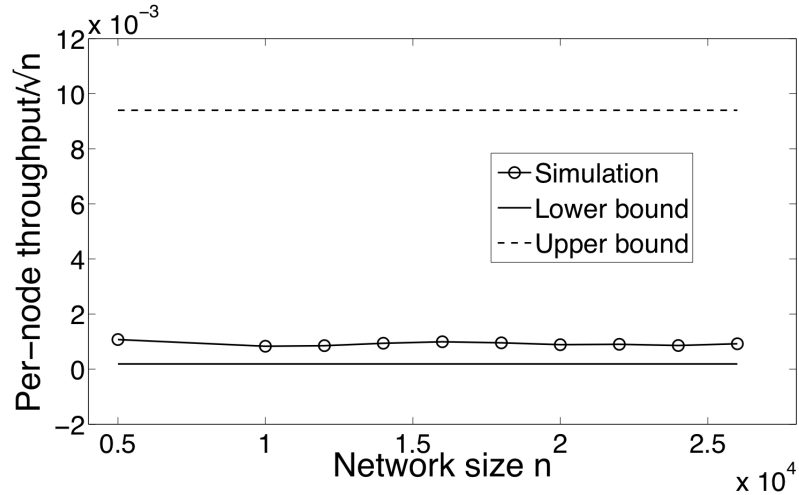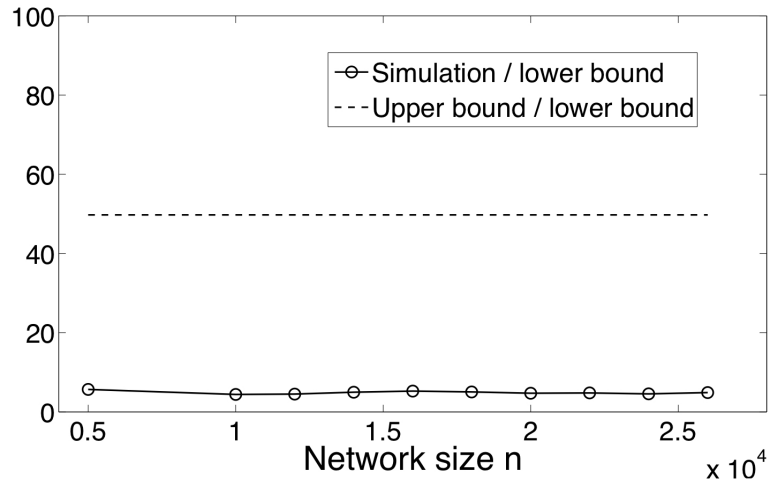
Figure 5.5.3: A simulation of per-node throughput with $\alpha = 4$ and $\beta = 10$. For comparison, the upper and the lower bound obtained are also shown. To facilitate comparison, both the per-node throughput obtained from simulations and the per-node throughput upper bound are normalized by the per-node throughput lower bound.

5.6   Summary

In this chapter, we studied the transport capacity of large wireless multi-hop CSMA networks. We showed that by carefully choosing the controllable parameters in the CSMA protocol and designing the routing algorithm, a network running distributed CSMA scheduling algorithm and each node making routing decisions based on local information only can also achieve an order-optimal throughput of $\Theta\left(\frac{1}{\sqrt{n}}\right)$, which is the same as that of large networks employing centralized routing and scheduling algorithms. Furthermore, we not only gave the order of the throughput but also derived the pre-constant preceding the order by giving an upper and a lower bound of the transport capacity. The tightness of the bounds was validated using simulations. Theoretical analysis was presented on tuning the carrier-sensing threshold and the transmission power to avoid HN problem and on tuning the backoff timer distribution to ensure each node gain a fair access to the channel in CSMA networks using non-

uniform transmission powers. The principle developed through the analysis was expected to be also helpful to set the corresponding parameters of CSMA networks in a more realistic setting.

Chapter 6

Conclusions

In this thesis, we considered wireless multi-hop networks operating with distributed MAC protocols: the CSMA protocols. The two fundamental properties, connectivity and capacity of wireless CSMA multi-hop networks were investigated. We conclude the thesis by summarizing our contributions in this chapter.

6.1   Critical Transmission Power for Connectivity

In chapter 4 we studied the connectivity of wireless CSMA networks under the SINR model. That is, we investigated the connectivity with the consideration of interference due to concurrent transmissions.

Firstly, we established an upper bound on the aggregated interference experienced by any receiver in CSMA networks even when the coefficient γ in (3.1.1) equals to 1. The obtained upper bound (Theorem 4.1) is valid for any node distribution in $\mathbb{R}^2$.

Secondly, we showed that for an arbitrarily chosen SINR threshold, there exists a transmission range such that a pair of nodes are directly connected if their Euclidean distance is smaller than or equal to this transmission range. On that basis, we derived an upper bound on the critical transmission power required for the CSMA network

to be a.a.s. connected under the SINR model as $n \to \infty$.

Thirdly, we derived a lower bound on the critical transmission power by heuristically constructing a scheduling algorithm who observes the carrier-sensing constraints. The two bounds are tight and differ by a constant factor only. The results suggested that the network can be connected for an arbitrarily high SINR requirement so long as the carrier-sensing capability is available. Compared with that considering UDM without interference, the transmission power only needs to be increased by a constant factor to combat interference and maintain connectivity. This result is optimistic compared with previous results on the connectivity of ALOHA networks under the SINR model [18, 19].

In summary, we studied the connectivity of wireless CSMA networks by pursuing bounds of the critical transmission power. Different from other work [57,72,73], which resorted to approximations of the spatial distribution of concurrent transmitters in CSMA networks and empirical validation of the accuracy of such approximations, the results generated by our method are analytically rigorous.

6.2 Transport Capacity

In Chapter 5 we investigated the transport capacity of large wireless CSMA multi-hop networks.

Firstly, we developed a distributed routing and scheduling algorithm that is compatible with large wireless multi-hop networks. The routing decision relies on the use of local neighborhood knowledge only and each node competes for channel access in a distributed and randomized manner using CSMA protocols.

Secondly, we demonstrated that by jointly tuning the carrier-sensing threshold and the transmission power, the HN problem can be eliminated even for nodes us-

ing different carrier-sensing thresholds, different transmission powers and a common frequency band. This is different from the techniques used in the previous work [34] where nodes using different carrier-sensing ranges have to transmit in different frequency band. The developed technique provided guidance on setting the carrier-sensing threshold and the transmission power to avoid the HN problem in CSMA networks in a general setting.

The pre-constant preceding the scaling law is important to fully understand the impact of various parameters on network capacity. Therefore, we not only showed that wireless CSMA networks can achieve the optimal capacity scaling law, but also analyzed the pre-constant preceding the order of the transport capacity by giving an upper and a lower bound of the transport capacity. The principle developed through the analysis is expected to be also helpful to set the corresponding parameters of CSMA networks in a more realistic setting.

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Appendix A

Proof of Theorem 4.1

A network on a finite area, denoted by $\mathbf{A} \subset \mathbb{R}^2$, can always be obtained from a network on an infinite area $\mathbb{R}^2$ with the same node density and distribution by removing those nodes outside $\mathbf{A}$. Such removal process will also remove all transmitters outside $\mathbf{A}$. Therefore the interference at a receiver on $\mathbf{A}$ is less than or equal to the interference experienced by its counterpart in a network on $\mathbb{R}^2$. It then suffices to show that the interference in a network on $\mathbb{R}^2$ is bounded.

Consider that an arbitrary receiver z is located at a Euclidean distance r_0 from its closest transmitter w, which is also the intended transmitter for z. We construct a coordinate system such that the origin of the coordinate system is at w and z is on the $+y$ axis, as shown in Fig. A.0.1.

The distance between any two concurrent transmitters is at least R_c, given by (4.2.3). Draw a circle of radius $R_c/2$ centered at each transmitter. Then the two circles centered at two closest transmitters cannot overlap except at a single point. Therefore the problem of determining the maximum interference can be transformed into one that determining the maximum number of equal-radius non-overlapping circles that can be packed into $\mathbb{R}^2$. The densest circle packing, i.e., fitting the
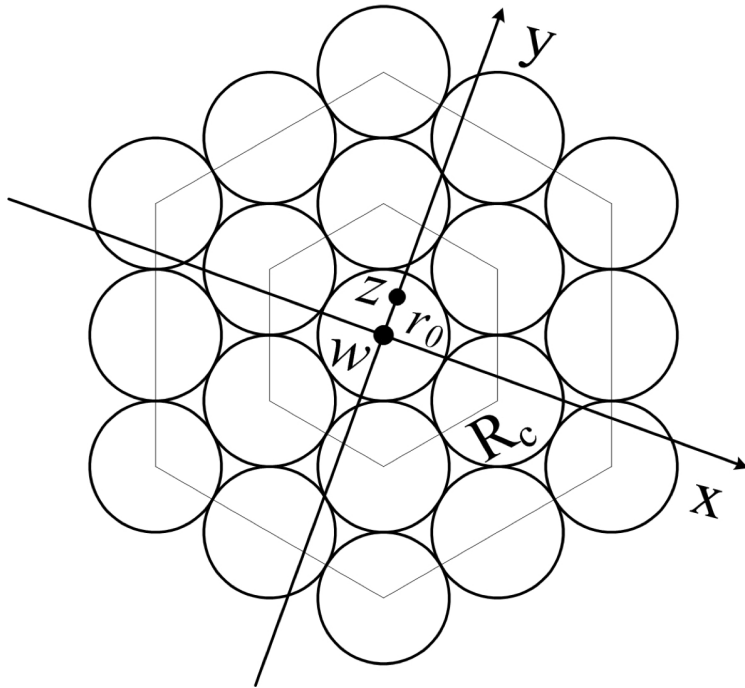
Figure A.0.1: An illustration of the densest equal-circle packing.

maximum number of non-overlapping circles into $\mathbb{R}^2$, is obtained by placing the circle centers at the vertices of a hexagonal lattice [82, p. 8], as shown in Fig. A.0.1.

Group the vertices of the hexagonal lattice into tiers of increasing distances from the origin. The six vertices of the first tier are within a Euclidean distance R_c to the origin. The $6m$ vertices in the mth tier are located at distances within $((m-1) R_c, m R_c]$ from the origin.

Let I_1 be the interference caused by transmitters, hereinafter referred to as interferers in this proof, above the x-axis at node z. Using the triangle inequalities gives $\|\boldsymbol{x}_i - \boldsymbol{z}\| \geq \|\boldsymbol{x}_i\| - r_0$ where $\boldsymbol{x}_i$ is the location of an interferer above the x-axis. Among the $6m$ interferers in the mth group, half of them are located above the x-axis. Among these interferers in the mth group above the x-axis, three of them are at a Euclidean distance of exactly $m R_c$ from the origin and the rest $3(m-1)$ interferers

Appendix A. Proof of Theorem 4.1

are at Euclidean distances within $[\frac{\sqrt{3}}{2}mR_c, \ mR_c]$. Hence, we have

$$I_1 \leq \sum_{m=1}^{\infty} \left(\frac{3\,(m-1)\,P}{\left(\frac{\sqrt{3}}{2}mR_c - r_0\right)^{\alpha}} + \frac{3P}{(mR_c - r_0)^{\alpha}} \right). \tag{A.0.1}$$

Look at the first summation in (A.0.1). Let U_m, $m = 3, \ldots, \infty$, be random variables uniformly and i.i.d. in $[m - 1/2, \ m + 1/2]$. It follows from the convexity of $\frac{3(m-1)P}{\left(\frac{\sqrt{3}}{2}mR_c - r_0\right)^{\alpha}}$ and Jensen's inequality (used in the second step) that

$$
\begin{aligned}
&\sum_{m=3}^{\infty} \frac{3\,(m-1)\,P}{\left(\frac{\sqrt{3}}{2}mR_c - r_0\right)^{\alpha}} \\
&= \sum_{m=3}^{\infty} \frac{3\,(\mathrm{E}\,[U_m] - 1)\,P}{\left(\frac{\sqrt{3}}{2}\mathrm{E}\,[U_m]\,R_c - r_0\right)^{\alpha}} \qquad\qquad\qquad (\text{A.0.2}) \\
&\leq \sum_{m=3}^{\infty} \mathrm{E}\left[\frac{3\,(U_m - 1)\,P}{\left(\frac{\sqrt{3}}{2}U_m R_c - r_0\right)^{\alpha}} \right] \\
&= \sum_{m=3}^{\infty} \int_{m-1/2}^{m+1/2} \frac{3\,(x-1)\,P}{\left(\frac{\sqrt{3}}{2}xR_c - r_0\right)^{\alpha}}\mathrm{d}x \\
&= 3P \int_{5/2}^{\infty} (x-1) \left(\frac{\sqrt{3}}{2}xR_c - r_0\right)^{-\alpha}\mathrm{d}x \\
&= \frac{4P\left(\frac{5\sqrt{3}}{4}R_c - r_0\right)^{1-\alpha}\left(\frac{\sqrt{3}}{4}(3\alpha - 1)R_c - r_0\right)}{R_c^2\,(\alpha - 1)\,(\alpha - 2)} \qquad (\text{A.0.3})
\end{aligned}
$$

Likewise, we also have $\sum_{m=2}^{\infty} \frac{3P}{(mR_c - r_0)^{\alpha}} \leq \frac{3P\left(\frac{3}{2}R_c - r_0\right)^{1-\alpha}}{(\alpha - 1)R_c}$. As a result of the last equation and (A.0.1), (A.0.3), (4.3.1), it follows that $I_1 \leq N_1\,(r_0)$.

Now we consider the total interference caused by interferers below the x-axis at node z, denoted by I_2. Let $\boldsymbol{x}_i$ be the location of an interferer below the x-axis, it follows from the triangle inequality that $\|\boldsymbol{x}_i - \boldsymbol{z}\| \geq \|\boldsymbol{x}_i\|$. Therefore

$$I_2 \ \leq \ \sum_{m=1}^{\infty} \left(\frac{3P}{(mR_c)^{\alpha}} + \frac{3\,(m-1)\,P}{\left(\frac{\sqrt{3}}{2}mR_c\right)^{\alpha}} \right)$$

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$$\leq \ \frac{3P}{R_c^\alpha} + \frac{3P(\frac{3}{2})^{1-\alpha}}{(\alpha-1)\,R_c^\alpha} + \frac{3P}{\left(\sqrt{3}R_c\right)^\alpha} + \frac{3P\left(\frac{5}{4}\right)^{1-\alpha}(3\alpha-1)}{(\alpha-1)\,(\alpha-2)\left(\sqrt{3}R_c\right)^\alpha} \quad \text{(A.0.4)}$$

Combining $I_1 \leq N_1\left(r_0\right)$ and (A.0.4), Theorem 4.1 is proved.

Appendix B

Lemma B.1

Lemma B.1 is needed in the proof of Theorem 4.3. Theorem B.1 is used to prove Lemma B.1.

Theorem B.1. *(Theorem 1 in [83]) Let $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_j$ be j arbitrary points in $\mathbb{R}^2$. Let $w_1, w_2, \ldots w_j$ be j positive numbers regarded as weights attached to these points, and define a position vector $\boldsymbol{c}$ by $\sum_{i=1}^{j} w_i \boldsymbol{v}_i = W \boldsymbol{c}$ where $W = \sum_{i=1}^{j} w_i$. Then for an arbitrary point $\boldsymbol{z}$, the following holds: $\sum_{i=1}^{j} w_i \|\boldsymbol{v}_i - \boldsymbol{z}\|^2 = \sum_{i=1}^{j} w_i \|\boldsymbol{v}_i - \boldsymbol{c}\|^2 + W \|\boldsymbol{z} - \boldsymbol{c}\|^2$*

Lemma B.1. *Consider a triangular lattice with unit side length and having a vertex located at the origin o. Define the 1st tier of points to be the six points placed at the vertices of the triangular lattice at a distance of 1 to the origin o. Let the mth tier of points be the $6m$ points placed at the vertices of the triangular lattice located at distances within $(m-1, m]$ from the origin o, as shown in Figure B.0.1. The total number of points from the 1st tier to the mth tier then equals to $j = 3m(1 + m)$. Let $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots \boldsymbol{v}_j$ be the location vectors of these j points and the points are ordered according to their distances to the origin o in a non-decreasing order. For an arbitrary point $\boldsymbol{z}$ located inside the hexagon formed by the 1st tier of six points, the following*
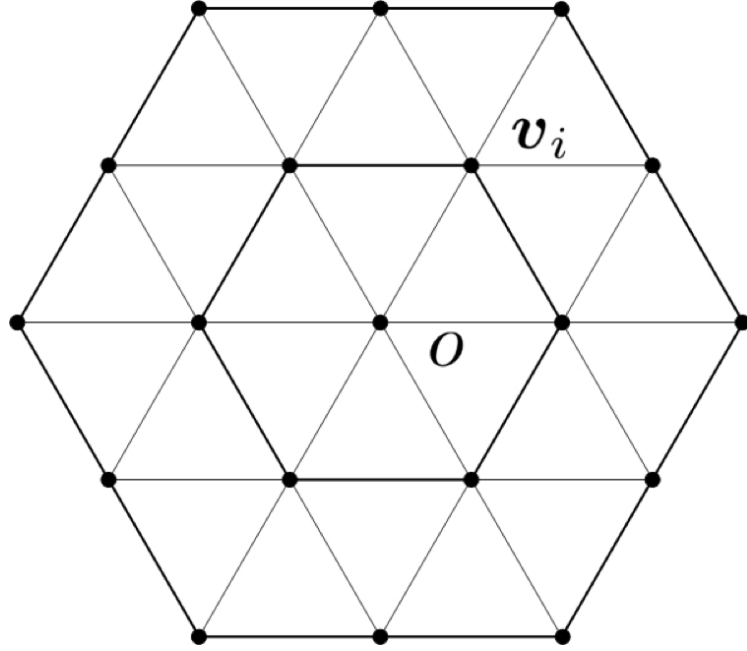
Figure B.0.1: An illustration of a triangular lattice

holds: $\sum_{i=1}^{j} \|\boldsymbol{v}_i - \boldsymbol{z}\|^{-\alpha}$ *is minimized when* $\boldsymbol{z}$ *is located at the origin o.*

Proof. Now we use Theorem B.1 to prove Lemma B.1. Letting all attached weights w_i equal to 1 and using Theorem B.1, for an arbitrary point $\boldsymbol{z}$ located inside the hexagon formed by the 1$^{\text{st}}$ tier of six points, we have

$$\sum_{i=1}^{6} \|\boldsymbol{v}_i - \boldsymbol{z}\|^2 = \sum_{i=1}^{6} \|\boldsymbol{v}_i - \boldsymbol{c}\|^2 + 6 \|\boldsymbol{z} - \boldsymbol{c}\|^2 \qquad (\text{B.0.1})$$

where $\boldsymbol{c}$ is given by $\sum_{i=1}^{6} \boldsymbol{v}_i = 6\boldsymbol{c}$. It is clear that $\boldsymbol{c}$ is the centroid of the six points. Since the hexagon has a unit side length, $\|\boldsymbol{v}_i - \boldsymbol{c}\|$ equals to 1. Let $x_i = \|\boldsymbol{v}_i - \boldsymbol{z}\|$ and $y = \|\boldsymbol{z} - \boldsymbol{c}\|$. The problem in Lemma B.1 can then be converted to the following constrained minimization problem:

$$\begin{aligned} \text{minimize} \quad & f(x_1, \ldots, x_6) = \sum_{i=1}^{6} x_i^{-\alpha} \\ \text{subject to} \quad & h(x_1, \ldots, x_6) = \sum_{i=1}^{6} x_i^2 - 6 - 6y^2 = 0 \end{aligned}$$

where the constraint is due to (B.0.1). Using the method of Lagrange multipliers, we first construct the Lagrangian in the following: $F(x_1, \ldots x_6, \Lambda) = f(x_1, \ldots, x_6) + \Lambda h(x_1, \ldots, x_6)$ where the parameter Λ is known as the Lagrange multiplier. Then find the gradient and set it to zero: $\nabla F(x_1, \ldots x_6, \Lambda) = \begin{pmatrix} -\alpha x_1^{-\alpha-1} + 2\Lambda x_1 \\ \vdots \\ -\alpha x_6^{-\alpha-1} + 2\Lambda x_6 \\ h(x_1, x_2, \ldots, x_6) \end{pmatrix}^T =$

0. Solving the above equation, it is obtained that $\Lambda = \frac{\alpha}{2}(1+y^2)^{-\frac{\alpha+2}{2}}$ and $x_1 = x_2 \ldots = x_6 = \left(\frac{2\Lambda}{\alpha}\right)^{\frac{-1}{\alpha+2}} = (1+y^2)^{\frac{1}{2}}$. Since $x_i = \|v_i - z\|$ denotes the Euclidean distance from v_i to z, only when $z = c$, we can have $x_1 = x_2 = \ldots = x_6 = 1$. It follows that the minimum of $f(x_1, x_2, \ldots, x_6)$ is obtained only when z is located at the origin o. Further, for the $6m$ points of the mth tier, using the same method, it can be shown that $\sum_{i=1}^{6m} \|v_i - z\|^{-\alpha}$ is minimized only when z is located at the origin o. The result follows. $\qquad\square$

Appendix C

Proof of Lemma 5.1

In the proof of Lemma 5.1, we will make use of a result established in the stochastic ordering theory [84]. For two real valued random variables X_1 and X_2, we say $X_1 \leq_{st} X_2$ iff for all $x \in (-\infty, \infty)$, $\Pr\{X_1 > x\} \leq \Pr\{X_2 > x\}$.

Theorem C.1. *[84, Theorem 1(a)]Suppose X_i follows a Binomial distribution with parameters $n_i \in \mathbb{N}$ and $p_i \in (0, 1)$, denote the distribution of X_i by $B(n_i, p_i)$, $i = 1, 2$, i.e., $X_i \sim B(n_i, p_i)$. We have $X_1 \leq_{st} X_2$ iff $(1 - p_1)^{n_1} \geq (1 - p_2)^{n_2}$ and $n_1 \leq n_2$.*

As an easy consequence of the above theorem, for three independent Binomial random variables $X_1 \sim B(n_1, p_1)$, $X_2 \sim B(n_1, p_2)$ and $X_3 \sim B(n_2, p_2)$ with $n_1 \leq n_2$ and $p_1 \leq p_2$, it can be concluded that $X_1 \leq_{st} X_2 \leq_{st} X_3$.

Now we are ready to prove Lemma 5.1. Let Y_i^j be the indicator random variable for the event that the SD_i passes through the jth square:

$$Y_i^j = \begin{cases} 1 & \text{if } SD_i \text{passes through the } j^{\text{th}}\text{square} \\ 0 & \text{otherwise.} \end{cases}$$

We shall derive an upper bound on $\Pr\{Y_i^j = 1\}$ for any $j \in \left[1, \frac{n}{\log^2 n}\right]$. Circumscribe
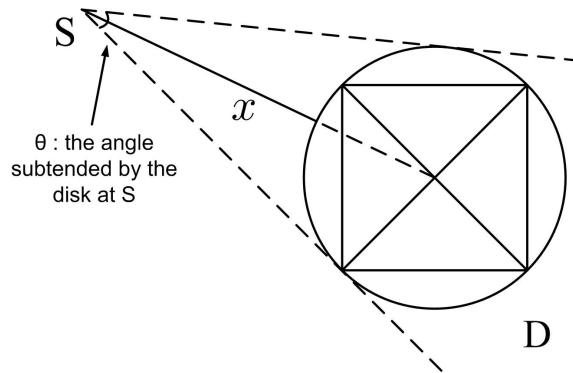
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Figure C.0.1: An illustration of a SD line intersecting the circumscribed circle

the jth square with a small circle of radius $\frac{\sqrt{2}}{2}c_1 \log n$, as shown in Fig. C.0.1. For a source S located outside the square and at a distance x from the center of the square, the angle $\theta(x)$ subtended by the circle at S is $\theta(x) = 2 \arcsin \frac{\frac{\sqrt{2}}{2}c_1 \log n}{x}$. Using the fact that $\arcsin x \leq 1.6x$ when $0 \leq x \leq 1$, we have

$$\theta(x) = 1.6 \arcsin \frac{\frac{\sqrt{2}}{2}c_1 \log n}{x} \leq 3.2 \frac{\frac{\sqrt{2}}{2}c_1 \log n}{x} \tag{C.0.1}$$

Noting that B_n is of size $\sqrt{n} \times \sqrt{n}$, the area of the sector formed by the two dashed tangents Fig. C.0.1 and the boarder of B_n is at most $\frac{\theta(x)}{2\pi}n$. If the destination of S, denoted by D, does not lie in this sector, then the associated SD line does not pass through the circle. Therefore, the probability that the SD line intersecting the circle is at most $\frac{\theta(x)}{2\pi}$. Considering that the circle is located in a $\sqrt{n} \times \sqrt{n}$ box B_n, the probability density that S is at a distance x from the circle can be shown to be upper bounded by $\frac{2\pi x}{n}$. It follows from the above analysis and (C.0.1) that

$$\Pr\{Y_i^j = 1\} \leq \int_0^{\sqrt{2n}} \frac{3.2 \times \frac{\sqrt{2}}{2}c_1 \log n}{2\pi x} \times \frac{2\pi x}{n} \mathrm{d}x = \frac{3.2 c_1 \log n}{\sqrt{n}} \tag{C.0.2}$$

Recall that Γ represents the set of indices of all nodes in the network. For a fixed square j, the total number of SD lines passing through it is given by $Y^j = \sum_{i=1}^{|\Gamma|} Y_i^j$,

which is the sum of i.i.d. Bernoulli random variables since the locations of nodes are independent and Y_i^j depends only on the locations of source and destination nodes of the ith source-destination pair. Therefore Y^j follows the Binomial distribution, i.e., $Y^j \sim B\left(|\Gamma|, \Pr\left\{Y_i^j = 1\right\}\right)$. As an easy consequence of the Poisson distribution of nodes, a.a.s. the total number of nodes $|\Gamma| \leq (1+\epsilon)n$, where ϵ is an arbitrarily small positive constant. Define another Binomial random variable $\tilde{Y}^j \sim B\left((1+\epsilon)n, \frac{4c_1 \log n}{\sqrt{n}}\right)$. It follows from Theorem C.1 that

$$Y^j \leq_{\text{st}} \tilde{Y}^j$$

It can be further shown that for any $0 < \delta_1 < 1$,

$$
\begin{aligned}
&\Pr\left\{Y^j > (1+\delta_1)(1+\epsilon)n\frac{3.2c_1 \log n}{\sqrt{n}}\right\} \\
\leq\ &\Pr\left\{\tilde{Y}^j > (1+\delta_1)(1+\epsilon)n\frac{3.2c_1 \log n}{\sqrt{n}}\right\} \\
=\ &\Pr\left\{\tilde{Y}^j > (1+\delta_1)\,\mathrm{E}\left[\tilde{Y}^j\right]\right\} \\
\leq\ &\exp\left(-\frac{\delta_1^2}{3}\mathrm{E}\left[\tilde{Y}^j\right]\right) &\text{(C.0.3)} \\
=\ &\exp\left(-\frac{3.2(1+\epsilon)\delta_1^2 c_1 \sqrt{n}\log n}{3}\right) &\text{(C.0.4)}
\end{aligned}
$$

where (C.0.3) results from the Chernoff bound. Using the union bound and the above result, we have

$$
\begin{aligned}
&\Pr\left\{\bigcup_{j=1}^{\frac{n}{c_1^2 \log^2 n}} Y^j > 3.2(1+\epsilon)(1+\delta_1)c_1\sqrt{n}\log n\right\} \\
\leq\ &\frac{n}{c_1^2 \log^2 n}\exp\left(-\frac{3.2(1+\epsilon)\delta_1^2 c_1 \sqrt{n}\log n}{3}\right) &\text{(C.0.5)}
\end{aligned}
$$

Noting that $\frac{n}{c_1^2 \log^2 n}\exp\left(-\frac{3.2(1+\epsilon)\delta_1^2 c_1 \sqrt{n}\log n}{3}\right) \to 0$ as $n \to \infty$, therefore a.a.s. $Y^j \leq$

$3.2 \left(1 + \epsilon\right) \left(1 + \delta_1\right) c_1 \sqrt{n} \log n$ for any $j \in \left[1, \frac{n}{\log^2 n}\right]$ which completes the proof of Lemma 5.1.

Appendix D

Proof of Theorem 5.3

Consider a node i on an open path located at $\boldsymbol{x}_i$ transmitting with power $P^l = \bar{P}\left(\sqrt{5}c\right)^\alpha$. Since the highest transmission power used in the network is $P^h = \bar{P}\left(\sqrt{2}c_1 \log n\right)^\alpha$, by (5.4.2), the furtherest transmitter that node i can sense is within a distance of $\sqrt{10}cc_1\bar{P}^{\frac{1}{\alpha}} \log n$. Denote by $\mathcal{D}\left(\boldsymbol{x}, r\right)$ a disk centered at $\boldsymbol{x}$ and with a radius of r. All nodes that are possibly competing with node i for transmission opportunities are located within $\mathcal{D}\left(\boldsymbol{x}_i, \sqrt{10}cc_1\bar{P}^{\frac{1}{\alpha}} \log n\right)$. Denote by $\mathcal{A}\left(\boldsymbol{x}, r_1, r_2\right)$ an annulus area centered at $\boldsymbol{x}$ with an inner radius r_1 and an outer radius r_2. A little reflection shows that all nodes using the low transmission power P^l *and* competing with node i must be located in $\mathcal{D}\left(\boldsymbol{x}_i, 5\bar{P}^{\frac{1}{\alpha}}c^2\right)$, and the nodes in $\mathcal{A}\left(\boldsymbol{x}_i, 5\bar{P}^{\frac{1}{\alpha}}c^2, \sqrt{10}cc_1\bar{P}^{\frac{1}{\alpha}} \log n\right)$ that compete with node $\boldsymbol{x}_i$ must use the high transmit power P^h. Note that in each open site that forms the open path, only one node serves as the relay node. Hence, there are at most $\frac{\pi\left(5\bar{P}^{\frac{1}{\alpha}}c^2\right)^2}{c^2} = \pi\left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2$ open path nodes in $\mathcal{D}\left(\boldsymbol{x}_i, 5\bar{P}^{\frac{1}{\alpha}}c^2\right)$ that use P^l. Let $\mathcal{N}\left(\boldsymbol{x}, r\right)$ be the random number of nodes located in $\mathcal{D}\left(\boldsymbol{x}, r\right)$. Next we provide an asymptotic upper bound on the number of nodes in $\mathcal{D}\left(\boldsymbol{x}_i, \sqrt{10}cc_1\bar{P}^{\frac{1}{\alpha}} \log n\right)$ for any node i on an open path. Denoting by $\mathcal{H}$ the set of indices of nodes on open paths, clearly $|\mathcal{H}| < \frac{n}{c^2}$. By Chernoff bound and the union bound, we have for an

Appendix D. Proof of Theorem 5.3

arbitrarily small positive constant δ_2,

$$
\begin{aligned}
&\Pr\left\{ \bigcup_{i \in \mathcal{H}} \mathcal{N}\left(\boldsymbol{x}_i, \sqrt{10}cc_1 \bar{P}^{\frac{1}{\alpha}} \log n\right) \geq (1 + \delta_2)\, 10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}} \log^2 n \right\} \\
= \ &\Pr\left\{ \bigcup_{i \in \mathcal{H}} \mathcal{N}\left(\boldsymbol{x}_i, \sqrt{10}cc_1 \bar{P}^{\frac{1}{\alpha}} \log n\right) \geq (1 + \delta_2)\, \mathrm{E}\left[\mathcal{N}\left(\boldsymbol{x}_i, \sqrt{10}cc_1 \bar{P}^{\frac{1}{\alpha}} \log n\right)\right] \right\} \\
\leq \ &\frac{n}{c^2} e^{-\frac{\delta_2^2}{3}\mathrm{E}\left[\mathcal{N}\left(\boldsymbol{x}_i, \sqrt{10}cc_1 \bar{P}^{\frac{1}{\alpha}} \log n\right)\right]}
\end{aligned}
\tag{D.0.1}
$$

where E denotes the expectation operator.

It can be readily shown that $\frac{n}{c^2} \exp\left\{ -\frac{\delta_2^2}{3}\mathrm{E}\left[\mathcal{N}\left(\boldsymbol{x}_i, \sqrt{10}cc_1 \bar{P}^{\frac{1}{\alpha}} \log n\right)\right] \right\}$ approaches 0 as $n \to \infty$. Therefore a.a.s. the number of nodes within a distance $\sqrt{10}cc_1 \bar{P}^{\frac{1}{\alpha}} \log n$ of an open path node is bounded above by $(1 + \delta_2)\, 10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}} \log^2 n$.

Next we analyze the transmission opportunity of an open path node. Denote by t_i the back off timer of node i at a particular time instant when the channel is idle. Denote by $\mathcal{C}_i$ the set of indices of nodes that compete with node i for transmission. Following the CSMA protocol, node i can become an active transmitter in the competition if

$$
t_i < \min_{j \in \mathcal{C}_i \setminus \{i\}} t_j.
$$

Let η_i^l be the event that a transmission of node i using the low transmit power is active. Using the "memoryless" property of an exponential distribution that for a timer following an exponential distribution, the amount of lapsed time does not alter the distribution of the remaining value of the timer, it can be shown that for any $i \in \mathcal{H}$,

$$
\begin{aligned}
&\Pr\left\{ \eta_i^l \right\} \\
= \ &\Pr\left\{ t_i < \min_{j \in \mathcal{C}_i \setminus \{i\}} t_j \right\}
\end{aligned}
$$

$$\geq \int_0^\infty \left(e^{-\lambda_l t}\right)^{\pi\left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2} \left(e^{-\lambda_h t}\right)^{(1+\delta_2)10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}}\log^2 n}$$
$$\times \lambda_l e^{-\lambda_l t}\mathrm{d}t \tag{D.0.2}$$

where in the above equation the term $\left(e^{-\lambda_l t}\right)^{\pi\left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2}$ represents the probability that at a randomly chosen time instant when the channel is idle, all $\pi\left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2$ open path nodes in $\mathcal{D}\left(\boldsymbol{x}_i, 5\bar{P}^{\frac{1}{\alpha}}c^2\right)$, which are competing for transmission opportunities with node i, have their respective back off timer larger than a particular value t; the term $\left(e^{-\lambda_h t}\right)^{(1+\delta_2)10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}}\log^2 n}$ represents the probability that all nodes using P^h in $\mathcal{D}\left(\boldsymbol{x}_i, \sqrt{10}cc_1\bar{P}^{\frac{1}{\alpha}}\log n\right)$, which are competing for transmission opportunities with node i, have their respective back off timer larger than t; the term $\lambda_l e^{-\lambda_l t}$ is the pdf of the back off timer of node i. It can be further shown from (D.0.2) that for any $i \in \mathcal{H}$,

$$\begin{aligned}
&\Pr\left\{\eta_i^l\right\} \\
\geq\ & \lambda_l \int_0^\infty e^{-\left(\pi\left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2\lambda_l + \lambda_h(1+\delta_2)10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}}\log^2 n + \lambda_l\right)t}\mathrm{d}t \\
=\ & \frac{\lambda_l}{\pi\left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2\lambda_l + \lambda_h(1+\delta_2)10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}}\log^2 n + \lambda_l} \\
=\ & \frac{1}{\pi\left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2 + \frac{\lambda_h}{\lambda_l}(1+\delta_2)10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}}\log^2 n + 1} \\
=\ & \frac{1}{\pi\left(5\bar{P}^{\frac{1}{\alpha}}c\right)^2 + (1+\delta_2)10\pi c^2 c_1^2 \bar{P}^{\frac{2}{\alpha}} + 1}. \tag{D.0.3}
\end{aligned}$$

Now we continue to prove the second part of Theorem 5.3. Consider that a node j transmits using the high power $P^h = \bar{P}\left(\sqrt{2}c_1\log n\right)^\alpha$. By (5.4.2), all nodes that are possibly competing with node j are located within $\mathcal{D}\left(\boldsymbol{x}_j, 2c_1^2\bar{P}^{\frac{1}{\alpha}}\log^2 n\right)$. Furthermore, among the nodes competing with node j, those open path nodes using the lower transmission power P^l must be located in $\mathcal{D}\left(\boldsymbol{x}_j, \sqrt{10}cc_1\bar{P}^{\frac{1}{\alpha}}\log n\right)$, and the

number of these open path nodes is *at most* $\frac{\pi\left(\sqrt{10}cc_1\bar{P}^{\frac{1}{\alpha}}\log n\right)^2}{c^2} = \pi\left(\sqrt{10}c_1\bar{P}^{\frac{1}{\alpha}}\log n\right)^2$.

Next we derive an upper bound on the number of nodes in $\mathcal{D}\left(\boldsymbol{x}_j, 2c_1^2\bar{P}^{\frac{1}{\alpha}}\log^2 n\right)$ competing with node j for any $j \in \mathcal{O}$ where $\mathcal{O}$ is the set of indices of nodes using the high power. It can be easily shown that $\lim_{n\to\infty}\Pr\left(|\mathcal{O}| < 2n\right) = 1$. Using the union bound and the Chernoff bound, we have for any small positive constant δ_3,

$$\Pr\left\{\bigcup_{j\in\mathcal{O}}\mathcal{N}\left(\boldsymbol{x}_j, 2c_1^2\bar{P}^{\frac{1}{\alpha}}\log^2 n\right) \geq (1+\delta_3)\,4\pi c_1^4\bar{P}^{\frac{2}{\alpha}}\log^4 n\right\}$$

$$= \Pr\left\{\bigcup_{j\in\mathcal{O}}\mathcal{N}\left(\boldsymbol{x}_j, 2c_1^2\bar{P}^{\frac{1}{\alpha}}\log^2 n\right) \geq (1+\delta_3)\,\mathrm{E}\left[\mathcal{N}\left(\boldsymbol{x}_j, 2c_1^2\bar{P}^{\frac{1}{\alpha}}\log^2 n\right)\right]\right\}$$

$$\leq 2ne^{-\frac{\delta_3^2}{3}\mathrm{E}\left[\mathcal{N}\left(\boldsymbol{x}_j, 2c_1^2\bar{P}^{\frac{1}{\alpha}}\log^2 n\right)\right]}$$

Obviously $2n\exp\left\{-\frac{\delta_3^2}{3}\mathrm{E}\left[\mathcal{N}\left(\boldsymbol{x}_j, 2c_1^2\bar{P}^{\frac{1}{\alpha}}\log^2 n\right)\right]\right\}$ approaches 0 as $n \to \infty$. Therefore a.a.s. the number of nodes competing with node j where $j \in \mathcal{O}$ is smaller than or equal to $(1+\delta_3)\,4\pi c_1^4\bar{P}^{\frac{2}{\alpha}}\log^4 n$. Let η_j^h be the event that node j, $j \in \mathcal{O}$, is active. It can be shown that for any $j \in \mathcal{O}$,

$$\Pr\left\{\eta_j^h\right\}$$

$$\geq \int_0^\infty \left(e^{-\lambda_l t}\right)^{\pi\left(\sqrt{10}c_1\bar{P}^{\frac{1}{\alpha}}\log n\right)^2}\left(e^{-\lambda_h t}\right)^{(1+\delta_3)4\pi c_1^4\bar{P}^{\frac{2}{\alpha}}\log^4 n}$$

$$\times \lambda_h e^{-\lambda_h t}\mathrm{d}t$$

$$= \frac{1}{\pi\left(\sqrt{10}c_1\bar{P}^{\frac{1}{\alpha}}\log n\right)^2\frac{\lambda_l}{\lambda_h} + 1\,(1+\delta_3)\,4\pi c_1^4\bar{P}^{\frac{2}{\alpha}}\log^4 n + 1}$$

$$= \frac{1}{\pi\left(\sqrt{10}c_1\bar{P}^{\frac{1}{\alpha}}\right)^2\log^4 n + (1+\delta_3)\,4\pi c_1^4\bar{P}^{\frac{2}{\alpha}}\log^4 n + 1} \tag{D.0.4}$$