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Subjective Wellbeing, Mean Reversion and Risk

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Statement of Originality

This is to certify that to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or other purposes. I certify that the intellectual content of this thesis is the product of my own work and that all the assistance received in preparing this thesis and sources have been acknowledged.

Paul Barnsley

Abstract

This thesis examines the possibility of directly measuring utility via reported subjective wellbeing and considers the structure of utility functions such measurements imply. Chapter one surveys the practical and theoretical arguments in favour of such an approach, and outlines the key technical difficulties associated with obtaining usable utility data from self-reported subjective wellbeing. Chapter two links subjective wellbeing to the model presented by Robson (2001) whereby restrictions on utility are predicted by evolutionary models. A specific model of mean reversion is then suggested, demonstrating the relationship between mean reversion and time preference, and data from the British Household Panel Survey (BHPS) is used to estimate the speed at which habituation occurs. Links between evolutionary models and the problem of mapping from reported subjective wellbeing to underlying utility are then used to generate a method for recovering cardinal utility data from ordinal subjective wellbeing using observed response frequency. This approach is then demonstrated using the BHPS data. Chapter three considers the aggregation of subjective wellbeing through time and introduces the concept of ‘peak aversion’, a preference for smoothness in utility across states and through time, and relates it to traditional risk aversion measures and general state-dependent utility. The application of peak aversion to subjective wellbeing, Quality Adjusted Life Years (‘QALYs’), tortious compensation and social choice theory are then considered. An empirical estimate of the strength of the preference is obtained using data on differences in Standard Gamble and Time Tradeoff QALYs and is used to calculate the curvature of the inequality averse social welfare function and appropriate QALY weightings based on severity of illness.

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Chapter 1

Background: Subjective Wellbeing and Utility

1.1 Introduction

Directly reported subjective wellbeing ('SWB') has been the subject of increasing academic and popular interest in recent years. This interest reflects a belief that surveys can measure a variable which is, at least, a proxy for 'happiness', and that the measure of happiness so derived is relevant to social policy.

While this thesis is a mostly theoretical, rather than empirical, look at the emerging field of subjective wellbeing, we will briefly attempt to address the reader's potential skepticism as to the practicality and relevance of obtaining estimates of subjective wellbeing. Once we establish a *prima facie* case that sensible SWB estimates are both obtainable and informative, we will go on to discuss some of the theoretical problems associated with utilising the data so gathered.

This introductory chapter is laid out as follows: Section 1.2 presents arguments as to the usefulness of subjective wellbeing data as an adjunct to existing revealed preference, methods of analysing individual preferences. Section 1.3 considers, in turn, the theoretical and practical arguments against the possibility of gathering usable subjective wellbeing data. Section 1.5 provides a more detailed analysis of the distinctions between economists' 'utility' and subjective wellbeing researchers' 'happiness'. Finally, section 1.6, which makes up the bulk of this chapter, provides a discussion of a series of technical difficulties associated with the gathering and application of subjective wellbeing data in practice. Of these difficulties, the issues of boundedness of scale, granularity of scale, mapping problems and data scale

confusion provide a gateway into the more formal model of SWB formation presented in chapter two. Section 1.7 provides a brief summary of the portions of the empirical SWB literature relevant to the remainder of the thesis, while section 1.8 provides a brief summary of the conclusions at which we arrive.

1.2 Why Study Subjective Wellbeing?

As Kahneman and Krueger (2006) point out, economists have, since the early twentieth century, preferred to determine people's preferences by looking at their actions. This revealed preference approach remains an effective tool for predicting consumers' actions and measuring their welfare, particularly in a market context.

Direct measurement of SWB, acting as a proxy for underlying utility, has the potential to augment traditional revealed preference analysis in four main ways.

First, SWB can be used to disentangle 'decision utility', the preferences that determine the choices made by an individual, from 'experienced utility', the level of utility actually experienced as the result of a choice. Obviously, in the case of a rational, forward-looking individual, these two measures will generally rank experiences identically. However, given the experimentally demonstrated violations of rational preferences raised by behavioral economics, and the preference reversals displayed by impatient individuals¹ through time, there are a number of situations in which it is useful to distinguish between what people like and what they choose. A direct measure of SWB allows us to talk about preferences and choices as something more than a tautology, and gives us a benchmark against which to test theories of rationality. Such a distinction also allows us to identify areas where market failure may generate a case for intervention which is *ex post*, if not *ex ante*, Pareto-improving.

Second, SWB has the potential to provide a direct cardinal measure of preference strength, as distinct from the purely ordinal data provided by most observed decisions. Given the paradoxes and debates that surround choice under uncertainty and time-preference, a measure with the ability to go beyond a binary ranking of states offers significant potential to enhance our understanding of choice. In addition, SWB measurement improves our ability to evaluate welfare – by enabling us to adopt rankings other than the Pareto criterion, SWB can be used to consider potentially welfare-enhancing changes which involve the redistribution of utility.

¹That is, individuals who display certain forms of time preference.

Third, SWB sidesteps the significant difficulties associated with valuing non-market goods and nonpecuniary events. The Existing methods of valuing such events which rely on revealed preference,² such as measuring the impact of public goods and negative externalities on nearby house-prices, are useful but often fail to provide a complete picture. This is true particularly where markets are illiquid or heterogenous. SWB analysis provides a valuable additional perspective as to the causes of changes in aggregate utility, giving us a more sophisticated (or at least differently-flawed) proxy for national welfare than changes in per-capita GDP.

Fourth, SWB provides a time-separable representation of utility flows. This enables us to determine the structure of payoffs through time, and the consequences of optimising behaviour as a function of individuals' life-cycles.

What these four classes of benefit have in common is that they render traditional, rational-actor models testable and falsifiable. They enable us to directly determine when our hypotheses hold in individual instances, rather than in aggregate, and they open up a conversation about the normative consequences of both occasional and systematic deviations from individual rationality.

1.3 Can We Study Subjective Wellbeing?

Having opened with some grand claims for the potential effectiveness of SWB data, it now falls to us to demonstrate that we can, in practice, hope to obtain usable data. Prior to presenting the case for the defence, though, it is important to consider the various senses in which we could understand the link between reported subjective wellbeing and utility.³

1. The strongest relationship we could claim between reported subjective wellbeing and utility would be that the reported value of subjective wellbeing was equal to, or equivalently, a linear transformation of, the welfare-relevant value of utility. In this case SWB and utility would rank states equally and changes in SWB would be cardinally comparable to changes in utility.
2. A weaker version of claim 1 would be that reported SWB is a monotonic, but nonlinear transformation of utility. In this case SWB and utility would still rank all states equivalently, but cardinal differences in SWB would not be comparable to those in utility.

²See the discussion in Frijters, Johnston, and Shields (2008)

³This discussion is picked up in our analysis of differing normative utility measures in 1.5, below.

3. The weakest version of the utility/SWB relationship would be the claim that SWB is a monotonic transformation of utility further modified by random errors. This is equivalent to recognising a positive correlation between utility and SWB. According to this claim, SWB would rank states according to utility in expectation, though any two randomly selected states could be incorrectly ranked. Cardinal differences in utility would, obviously, not be captured in SWB data.

Even the weakest of these potential relationships allows us, in practice, to draw substantial links between the factors which alter subjective wellbeing and those which alter utility. One way of characterising the weak relationship between utility and SWB is that, if a specific individual reports a change in their SWB we conclude it is, *ceteris paribus*, more likely than not that utility has moved in the same direction. In chapter two, below, we formally consider the problem faced by an individual trying to signal utility using subjective wellbeing.

1.3.1 A Theoretical Defence

To begin with we should consider the theoretical relationship between the concept that SWB research attempts to measure and ‘utility’ as it is traditionally conceived of by economists.

To a great extent, economists’ ‘utility’ is defined by the uses to which it is put, rather than by any underlying appeal to philosophical principles. One of these approaches is to conclude that utility is simply the value which a utilitarian social planner would seek to collectively maximise. Another is to suggest that utility is no more than a means of operationalising rational choice theory; that the actions people take maximise utility and they take those actions because they lead to utility being maximised. The difficulty with this, somewhat tautological, approach to preferences and outcomes is that it defines away, *ab initio*, the possibility of *ex ante* regrettable choices. This is the distinction between decision utility and experienced utility alluded to above; the fact that individuals *do* routinely experience regret in situations other than negative realisations of *ex ante* risk indicates that a distinction between predicted and resulting utility is worth drawing.

We argue, below in 1.5, that the conception of ‘happiness’ embodied in subjective wellbeing data cannot be systematically distinguished from the notion of ‘the good’ which traditional utilitarians seek to maximise. We further note that, to the extent that utility and happiness are purely subjective phenomena, they must, by definition, exist within the mind of the individual. We might, as in 1.6 and chapter two, below, produce technical

arguments about the relationship between experienced happiness or utility and the values reported in response to a survey, but it makes no sense to speak, in general terms, of utility as something which is unknown to the individual allegedly guided by its dictates.

Assuming, for the sake of argument, that subjective wellbeing *is* conceptually identical to utility, and that our surveying techniques are capable of eliciting levels of wellbeing accurately, then we would have something approaching Edgeworth's Hedonometer. The development of such an instrument ought to be of great interest to economists, as it was to Edgeworth, and the readings it produced of fundamental value to the pursuit of our profession. So objections to the relevance of SWB data would seem, of necessity, to turn on one of the two problems assumed-away above: either that subjective wellbeing is systematically and normatively different from utility, or that surveys cannot correctly elicit its true level. We deal with the first, philosophical, question in 1.5, below, while the second, more practical, complaint is addressed in 1.3.2 and 1.6, and in chapter two, below.

1.3.2 A Practical Defence

How do we know if subjective wellbeing is an accurate proxy for utility? Given that experienced utility is inherently unobservable, we will never have a concrete answer to that question. Adherents to a strong form of individual rationality might conclude that any evidence that decision utility leads to suboptimal experienced utility is itself evidence that experienced utility is being incorrectly measured. As a defence of individual rationality this is unfalsifiable and hence somewhat inconclusive.

However, once we conclude that utility can neither be directly observed nor consistently inferred from revealed preference, how do we confirm that SWB is capable of serving as its proxy? We will treat the question in two parts, first establishing that SWB is a meaningful measure of *something*, and then providing evidence that that 'something' correlates with underlying utility as most people would understand it.

As to the first point: people, almost universally, appear to understand what is being asked when questioned about their "satisfaction", "contentment" or "happiness". As Kahneman and Krueger (2006) note, over 99% of respondents to the 1998 General Social Survey were willing and able to provide an answer (other than "don't know") to the question "Taken all together, how would you say things are these days? Would you say that you are very happy, pretty happy, or not too happy?" By way of comparison, 83% of respondents provided answers to questions about their income.

Answers about subjective wellbeing also display a relatively high test-retest correlation for individuals,⁴ an indication that the answers provided are something other than random guesses.

So, if answers to questions about subjective wellbeing are both coherent and consistent, are they also significant, from a utilitarian point of view? There is a variety of evidence suggesting that reported SWB correlates closely with other variables which we might think of as signifying high levels of utility. It is difficult to improve on the survey provided by Kahneman and Krueger (2006), which cites studies of individuals with high subjective wellbeing showing that these persons tend to:

- recover faster from wounds and the common cold;
- show greater neurological activity in the portion of the brain associated with pleasure;
- smile more often;
- be judged as happier by those around them; and
- more frequently express positive emotions.

Further, statistical analysis as to the determinants of SWB returns coefficients broadly consistent with what we would predict. Income increases wellbeing, as does good health, the deaths of friends and family reduce it.⁵ Education, freedom and a high relative position in society all correlate positively with one's sense of wellbeing.

Obviously, there is some risk of circularity here – we cannot both look to subjective wellbeing research to tell us about the structure of our utility functions while simultaneously using its consistency with our *ex ante* predictions as to their structure to test its validity. We would, however, be skeptical of a measure which told us that health and wealth tended to reduce our utility; we have, as a starting point, a model which puts the same signs on the key arguments of a utility function as does traditional microeconomics.

So, in summary, economics seeks to maximise the collective good; when asked about their individual 'good', under the labels of 'happiness', 'satisfaction' or the local translations thereof, people understand the question, base their answers at least partially on the kinds of things we think ought to

⁴Kahneman and Krueger (2006), at page 7, cite a correlation of 0.77 for two surveys taken four weeks apart.

⁵One of the key claims of SWB research, however, is that these changes tend to be transitory in nature. This process is formally modeled in chapter two, below.

make them happy, and appear, to neuroscientists and to their friends, to be telling the truth. While we might raise all manner of technical objections as to the precise nature of the data gathered, and the uses to which it is put, it does seem that, *ceteris paribus*, economics should be willing to consider policies which tend to lead to more people reporting more ‘good’.

1.4 Which Notion of ‘Subjective Wellbeing’?

A variety of different survey questions attempt to operationalise the concept of subjective wellbeing. Respondents can be asked about their “happiness”, “satisfaction”, whether with life overall or with a specified domain, about “joy” or whether life is overall “worthwhile”, among many others. We consider the extent to which word choice, particularly when applied across cultures, might bias the outcomes of SWB research in section 1.6.8, below. But bias outcomes relative to what standard? Thompson and Marks (2008) divide SWB measures into three broad categories: hedonic (or affective), dealing with day to day feelings and moods, evaluative, a cognitive judgement weighing up different aspects of ones life, and eudaimonic, capturing higher level notions of the inherent *value* of the SWB experienced.

The use to which we seek to put SWB is instrumental: we argue below that SWB can serve as a useful proxy for utility, and, more generally, that public policy can proceed on a virtuous circle of shared understanding that the correct version of SWB is the one which enables SWB to serve as a tool for public policy. This purpose suggests that we are more interested in evaluative questions (satisfaction rather than joy or worth, more or less) but that any question has the potential to convey the proper instrumental information to respondents if they understand the use to which their answers will be put. There is, however, a significant and important literature on optimal survey design which is beyond the scope of this thesis.

1.5 Which Notion of ‘The Good’?

We argue above that changes in reported subjective wellbeing do tend, on average, to correlate with changes in ‘utility’. This claim, however, elides the question as to which measure of utility we consider to be socially relevant. Even if we accept that policies which maximise reported happiness also tend to maximise some conceptions of utility, is this sufficient to persuade us to advocate those policies?

We might begin to answer that question by returning to the distinction

between experienced utility and decision utility, drawn in 1.2, above. If individuals habitually make decisions which fail to optimise the hedonic pleasure they experience, should a benevolent social planner privilege the level of pleasure experienced over the amount anticipated?

Given that decision utility is primarily a predictive tool, designed to describe an individual's actions, it is difficult to assign much moral significance, *from a utilitarian perspective*, to the choices it dictates, in circumstances where those choices deviate from those that would be adopted by the individual *ex post*.⁶ Where an individual makes choices which they later regret, that regret is a more concrete object for utilitarianism and utilitarian policy to consider than the inaccurate predictions which brought about the regrettable choice. As such, when experienced utility and decision utility suggest different state rankings or different policy responses, utilitarians should be comfortable with seeking to maximise experienced utility.

We now turn to the argument that experienced utility is an inherently impoverished measure of the utilitarian good – that a focus on transitory pleasure ignores deeper aspects of happiness, aspects which may cause temporary pain or which yield lesser immediate pleasures, but which should nonetheless be treated as being more significant and more valuable. A life of religious asceticism, for instance, might produce less pleasure than a life of cheerful hedonism, but more 'good', according to the relevant standard. Or a week spent in the library, reading James Joyce, might be both more demanding, in the sense of moment-to-moment sensations, but also more rewarding, when viewed appropriately, than a week spent reading celebrity gossip.

We can distinguish two arguments making up this strand of reasoning. The first suggests that certain classes of experience deserve a higher weighting in the utilitarian calculus *even if the individual experiencing them never experiences any compensatory pleasure*. This view holds that listening to classical music is *inherently* better than listening to rap, or perhaps visa versa, and views a calculus that treats equal pleasures derived from each activity equally as willfully blind to the true nature of 'the good'. This view, whatever its other recommendations, is a significant deviation from utilitarianism as it is usually conceived. If valuing an activity relies on an appeal to some objective view as to that activity's quality, independent of the actor's preferences, then it ceases, to that extent, to be utilitarian, and must be justified as an alternative normative theory from the ground up. While the value of such an approach may be worth debating, it is not something we will further pursue here.

⁶Leaving aside situations where the *ex post* and *ex ante* optimal choices differ because of the resolution of uncertainty; decision utility is not to be criticised for urging people to take a fair, but ultimately unfortunate, gamble.

The second form of the “happiness isn’t everything” argument is more traditionally utilitarian in nature. It holds that the individual, themselves, would in some sense recognise the superiority of Joyce-reading to gossip-reading, even if they gain more transitory pleasure from pursuing ‘lowbrow’ entertainment. Advocates for the normative significance of subjective well-being can incorporate this possibility in their normative framework in three ways:

1. By arguing that the Joyce-advocates are failing to distinguish between decision utility and experienced utility. If people are choosing to read gossip while ultimately deciding that they would have preferred to read Joyce, we can conclude that they are simply failing to maximise the experienced utility measured by SWB data.
2. Alternatively, the Joyce problem can be viewed as an aspect of aggregating experience utility through time. Much as investment can be rational even when it results in reduced utility today, Joyce can be seen as both unpleasant and worthwhile, even within a pure SWB framework. Provided the increased future satisfaction generated by Joyce outweighs the immediate sacrifice of gossip-reading, maximising happiness can be consistent with choosing Joyce. We should be no more surprised when an individual makes decisions leading to a temporary reduction in pleasure than when a consumer decides to defer consumption. On this view, the debate is not one about whether mere ‘happiness’ is enough, but one about impatience and time preference, an issue dealt with in detail in chapter three, below.
3. Finally, a defender of the SWB approach could concede the semantic point that neither ‘happiness’ nor ‘contentment’, as they are typically meant, capture the full range of notions of ‘the good’ an individual might seek to maximise, but point out that, provided the policy role SWB is intended to play is clear to the people surveyed, it doesn’t much matter what we label the variable we are measuring. That is: any concept of ‘the good’ under utilitarianism is subjective in nature. If an individual is never aware of it, it therefore does not exist. By definition, then, individuals are fully aware of how much ‘good’ they have access to, *regardless of which concept of ‘the good’ we wish to apply*. Closing the gap between full and narrow conceptions of happiness, then, is simply a matter of correctly communicating what the measure is for. If the Joyce-reader reports that they are unhappy, because, while they are glad they are reading Joyce, they are find it hard-going, then they have answered the wrong question. Similarly, it seems odd to imagine that a question of the form “All things considered, how satisfied are you with your life as a whole these days?”

could elicit an answer like “very satisfied (but I am wasting my life with these gossip magazines)”; if reliance on subjective wellbeing data becomes widespread, that sort of misinterpretation is likely to become as rare as misreporting one’s preferred political party or favourite brand of margarine because one assumed the question related only to the candidate’s physical attractiveness or the colour of packaging it was contained in.

Once we begin to use ‘happiness’ as an instrumental concept, to determine individuals’ satisfaction with the current state of the world, the correct response to being asked to measure it is defined by that instrumental role – much as the electoral mechanism clarifies precisely what it means to give a party one’s ‘vote’. Subjective wellbeing then becomes, in a sense, a Borda-count vote as to the current state of the world. Ultimately, the question of whether ‘happiness’ is the thing we should seek to maximise becomes irrelevant once we have clearly and publicly labeled the thing we should seek to maximise as ‘happiness’.

1.6 Difficulties in Applying Subjective Wellbeing Data

If we accept the above arguments that SWB data is obtainable, meaningfully related to utility and practically useful, we must then consider the theoretical difficulties which arise in applying the data to practical questions of welfare evaluation.

1.6.1 Interpersonal Comparisons

The initial question is whether interpersonal comparisons of SWB are possible, and, relatedly, whether underlying utility itself is interpersonally comparable. We might divide this objection into two related questions. First, can we conclude that an individual who reports a wellbeing of ‘5’ on a 0–10 scale is at the same point on their internal utility scale as any other individual reporting a ‘5’. This amounts to a question about the consistency of mapping schema between individuals, and is discussed in more detail in chapter two, below. The second, more philosophical, objection questions whether any meaningful interpersonal comparison of a purely subjective sensation such as utility is possible. One could concede the existence of a constant, identical mapping from equal levels of utility to equal levels of reported SWB between individuals, but reject the underlying claim that utility levels can be meaningfully equal.

To some extent, this second objection is an unanswerable assertion. It is entirely possible, notwithstanding humans' near-identical neurological makeup, that any one person's experience of the colour green and utility ten percent above an hedonically neutral state is as different from any other's as their experiences of the same films and foodstuffs. While there is no practical way to reject this possibility, we can make a series of observations as to its usefulness as a restriction on welfare analysis.

The first observation is that economists habitually assume interpersonal equality of decision-making apparatus. To the extent that individuals satisfice, adopt rational ignorance strategies or otherwise operate according to bounded rationality, one would expect the degree of deviation from rationality arising from limitations in mental capacity to be at least as personal a characteristic as the quantity of pleasure experienced as a result of one's decisions. If *this* assumption of cognitive uniformity is either accurate or a harmless 'white-lie' when conducting welfare analysis, it seems *prima facie* reasonable to carry it across to the measurement of utility.

Secondly, economists normally assume that individuals have extremely detailed knowledge of their own utility maps, and typically that this knowledge is at a cardinal scale. This, *ex hypothesi*, distinguishes utility from virtually every other subjective sensation; we have no expectation that an individual is capable of unerringly comparing differences in 'greenness' as between multiple shades, and still less that they could consistently make statements of the form "the difference in quality between movie x and movie y is three times the difference in quality between movie y and movie z". We treat utility, intrapersonally, as much more ordered and well-behaved than competing subjective measurement scales. While it does not necessarily follow that it be well-behaved in the same way *between* individuals, those proposing strong incomparability are suggesting key differences in a fundamental and complicated piece of mental architecture.

The third observation is that, while interpersonal comparability is an unprovable hypothesis, it remains a useful starting point for measuring welfare. Even if we cannot be sure that two individuals who report a subjective wellbeing of '4' are experiencing identical subjective hedonic sensations, it seems reasonable to proceed as if this is *on average* likely to be true. The first individual may be a *bon vivant*, capable of appreciating life's benefits and lamenting its woes more deeply than the second individual. Or the reverse may be the case. But the burden on the individual opposing a policy that can be shown to increase measured welfare is surely not that they simply raise the *possibility* of error, but rather that they demonstrate its *systematic* likelihood.

Alternatively, a social planner can simply conclude that, while a *bon vi-*

vant may feel their losses in some sense more deeply than the average person, this is not a sense in which the social planner ought to be interested. On this analysis, interpersonal comparability becomes a kind of ‘legal fiction’, whereby treating individuals *as if* they possessed equal ranges and distributions of feeling becomes an aspect of the equal treatment axiom that lies at the heart of utilitarianism.

1.6.2 Boundedness of Scale

When individuals are asked to estimate their subjective wellbeing, they are asked to do so according to a scale which is bounded at both ends. We typically assume, when modeling utility, that individuals display nonsatiation at least in relation to money, implying that utility is unbounded above. More generally, if we assume that, for any possible allocation of the goods⁷ which make up the arguments of the individual’s utility function, utility is increasing in at least one of those goods then we will have utility which is unbounded above.⁸ The converse argument can be applied to the existence of a lower bound for utility, though in this case we can more naturally imagine some sort of ‘worst-case-scenario’ which generates utility equal to some lower bound.

If we assume, for the moment, that underlying utility itself is unbounded in at least one direction, this raises an obvious set of problems. How do individuals map from an unbounded scale to a bounded one? How will they report the realisation of utility values above or below the level they had previously assigned to the top or bottom of the reported scale?

There is a risk that individuals will find themselves ‘bumping-against’ the top or, less cheerfully, the bottom of the provided scale, particularly as they are resurveyed through time, or that they will, anticipating this problem, incorporate unarticulated nonlinearity into the relationship between utility and reported SWB.

1.6.3 Granularity of Scale

The issue of the granularity of the reporting scales provided is closely related to the issue of boundedness, above. If we consider underlying utility to be continuous as well as unbounded, dividing it into a fixed number of ‘buckets’ for purposes of reporting will lead to, at a minimum, errors in rounding. The nature of this mapping, from utility to reported SWB,

⁷Where ‘goods’ are broadly defined.

⁸Cowen and High (1988) provide a discussion of satiation in money and upper-bounded utility in the context of the St Petersburg Paradox.

will determine the scale and tractability of the resulting data. This issue, together with the problem of boundedness discussed above, is considered as an aspect of the individual’s mapping problem, in 2.5, below.

Even where the mapping from utility is perfectly linear – in the sense that the SWB scale is cardinally related to underlying utility and experienced utility values are assigned to the closest available number on the SWB scale – granularity will still collapse unlike situations together along the SWB scale, leading to unbiased measurement error.

1.6.4 The Mapping Problem

In chapter two, below, we formally consider the problem faced by an individual trying to accurately signal the level of their underlying utility function by means of a bounded, granular scale. We demonstrate that boundedness in combination with granularity can be expected to introduce systematic errors (or non-linearities) into SWB estimates. On this basis, we are able, in section 2.5.4 to suggest error-correction procedures that should be applied to ‘raw’ SWB data prior to treating it cardinally across the reported scale.

1.6.5 Data Scale Confusion

Until recently, the most common practice in handling reported SWB data was to treat responses across a numerical, or even qualitative scale⁹ as producing cardinal data, capable of generating arithmetic means, and even ratio-scale based percentage comparisons. Such usage rests on the assumption that underlying utility is being mapped in a linear fashion onto the proffered subjective wellbeing scale, so that the portions of the underlying scale covered by each reported SWB value are equal in size.

Where there is nonlinear compression of the cardinal differences between utility values at one or both ends of the reported scale, measures of central tendency which assume linearity will understate the significance for underlying utility of SWB values at the extremes of the scale. This understatement will, in turn, lead to an unarticulated concave transformation of socially-significant utility.

As the field of subjective wellbeing analysis has matured, the implicit assumption that SWB data is cardinal and defined up to a ratio scale has become less common, but to the extent that the underlying mapping remains unobservable, assumptions about its structure need to be made for most practical forms of analysis. As such, while we are now less likely to be

⁹Such as “Not happy, Happy, Very Happy”.

told, as we were by *The Economist* (Anon (2002)), that “Danes are five times happier than Italians”, difficulties with the precise nature of the data produced by SWB surveys remain common.

Questions as to data scale are really questions about the structure of underlying utility and the mapping from utility to reported SWB. We deal with each of these issues in more detail in chapter two, below.

1.6.6 Random Error

As noted in 1.3.2, above, test-retest correlations for the SWB scores of single individuals have been shown to be significant, but not high enough for any variation to be explicable solely by genuine changes in the individuals’ situations. Examples of this kind of variability are provided Schwarz (1987), who shows that the weather on the day the survey was taken has a significant impact on reported levels of SWB. Similarly, Schwarz (1987), also cites an experiment where individuals were asked to perform a photocopying task prior to answering questions about their wellbeing. Half the group was randomly selected to discover a dime in the photocopier while performing the task, and this group displayed significantly increased levels of SWB.

We can conclude from these examples either that subjective wellbeing fails to conform to underlying utility, or that underlying utility is, itself, highly variable through time in response to apparently minor factors. For our purposes, not much turns on this distinction, except perhaps to note the potential for predicted falls in utility to be wiped-out by a carelessly placed penny or unseasonably warm weather; whether the variability arises in the transition from utility to SWB or at its source, large random errors will be introduced when trying to measure the effect of significant policy variables.

Ultimately, though, given large enough sample sizes – and many of the data sets used for SWB research, such as the World Values Survey, are very large indeed – random error of this sort will tend to wash-out across the entire population surveyed. It is no objection to theories of demand based on prices and income that purchasing decisions can be swayed by prominent placement and pleasing packaging, and no objection to the basic notion of subjective wellbeing that it can be skewed by warm days and found money.

1.6.7 Strategic Misreporting

Another potential barrier to the adoption of SWB data as a tool for determining policy is that, as such adoption becomes widespread, individuals may face an incentive to strategically misstate their level of subjective

wellbeing in the hopes of influencing policy in their favour.

Similar misstatement, whether due to strategic or expressive motivations, appears in contingent valuation surveys in relation to environmental goods.¹⁰ Such surveys sometimes yield stated willingness-to-pay values that vastly exceed an individual's own level of wealth. This effect arises notwithstanding the small marginal effect any individual valuation can expect to have on final policy.

Two points can be made here in defence of SWB analysis. First, the relationship between stated SWB and actual policy decisions is much more attenuated than in the case of contingent valuation surveys, which typically solicit a statement of preference-strength in relation to a specific, identified policy goal. The policy role of overall statements about subjective wellbeing, on the other hand, is limited to a statistical search for its determinants based on the characteristics of an individual who reports a particular value.

Second, to the extent there is a 'squeaky wheel gets the grease' motivation to understate one's level of contentment when surveyed, the ability of such strategic misstatements to influence policy is heavily circumscribed by the existence of upper and lower bounds on the available reporting scale. In this instance, placing an arbitrary bound on an unbounded underlying scale has useful secondary consequences, and this result can be thought of as an aspect of the 'legal fiction' justification for interpersonal comparability discussed above in 1.6.1.

Overall, there appears to be no serious cause for concern about strategic misreporting, even should links between SWB and policy choices become widespread and widely-known.

1.6.8 Sensitivity to Word and Language Choice

The precise wording of the questions used to elicit information about subjective wellbeing may also tend to bias the answers received. In English, the concept of 'satisfaction', while closely related to 'happiness', has noticeably different connotations. A survey of "satisfaction" might tend to elicit information about a sort of 'negative unhappiness' – a freedom *from* things that make one unhappy, while enquiring about "happiness" directly might focus on a more positive sense of hedonic experience. The few surveys which add "joy" to the emotional mix presumably tend to exclude pleasures people associate with comfort but which they would be embarrassed to admit brought them actual joy. Obviously, these measures will be correlated, but they may not be identical, and may deviate systematically in measuring

¹⁰See, for example, the discussion in Sunstein (2000).

certain aspects of hedonic experience.

This problem is significantly worsened when international surveys require the notion of happiness to be translated into other languages and transposed to fit different sets of cultural assumptions. The linguistic and cultural nuances of asking about ‘happiness’ in different cultures is not a topic on which we are qualified to hold forth in any detail, but one striking example of these difficulties arises in Wolfers and Leigh (2006). They observe that, when asked about their level of “happiness”, Nigerians rank as the happiest people in the world, according to the World Values Survey, with Tanzanians ranking second. When asked about “life satisfaction”, though, Nigeria and Tanzania rank 37th and last respectively. Clearly the distinction between these concepts is much sharper in the language or culture of these two nations than it is in English and anglo-saxon cultures.

To some extent, as with the discussion of which notion of utility we should seek to maximise (in 1.5, above), we can hope that this problem will be self-correcting in the face of widespread familiarity with SWB surveys. Much as the construction “thinking about x” has become a topic marker unmoored from its original meaning in the international language of opinion-polling, individuals may come to understand the uses to which subjective wellbeing surveys are put, and to interpret the questions and use the proffered scale instrumentally, regardless of the precise language in which the question is phrased.¹¹

1.7 What Does the Existing Research Show?

Having suggested, with a number of caveats, that changes in subjective wellbeing can provide useful insight into changes in utility, we will now briefly outline some key issues raised by the existing literature, issues with which we will deal in more depth in chapters two and three, below.

As this thesis is primarily concerned with the theoretical issues arising from the translation of subjective wellbeing into utility, we will not here attempt anything approaching a full survey of the literature, which runs the gamut from happiness-inducing gardening to happiness-reducing citizen initiated referenda. Rather than representing a response to any particular study, the material presented in chapters two and three suggests changes to the existing means of manipulating and aggregating SWB data which will, we hope, provide a basis for assisting future research into these and related questions.

¹¹See our earlier discussion of this point in 1.5, above.

As noted in section 1.3.2, above, Kahneman and Krueger (2006) provide a summary of existing research showing that the key determinants of utility – income, health, health of friends and family – display the expected, positive, coefficients. Of particular relevance to the material presented in chapter two, however, is that the increases in subjective wellbeing generated by changes in these factors appears to be transitory, or at least to decrease through time.

This phenomenon, which we will refer to as ‘mean reversion’, is consistent with psychologists’ models of human behaviour, but is not typically incorporated into economists’ intertemporal utility functions. Easterlin (2005) considers the evidence for mean reversion in relation to earnings, including the oft-cited result that increasing in per-capita GDP in middle and high income countries do not appear to have lead to increases in average subjective wellbeing.¹² The author concludes that the SWB evidence supports a finding of significant, but not complete, mean reversion in response to permanent changes in income.

A more recent panel-data survey, Tella, New, and MacCulloch (2010), estimates that 65% of the current-year effect of a change in income dissipates over a four year period. In section 2.4, below, we introduce a formal model of this form of mean reversion. Conversely, Deaton (2008), argues, on the basis of gallup-poll data, that national SWB averages are increasing linearly in the log of income.

Evidence of some degree of mean reversion arises not only in relation to income – Oswald and Powdthavee (2008), consider adaptation to disability. They find such adaptation to be significant but incomplete, such that the decrease in subjective wellbeing associated with disability¹³ declines by roughly 16% each year for at least the first several years following the disabling incident.

Without taking a firm position on the question of whether the reversion suggested by SWB data is complete, incomplete or nonexistent, we argue that there is sufficient evidence to warrant the development of utility models which explain the existence of mean reversion, predict the behaviour of individuals in response to its effects and which consider optimal social welfare in its presence. In chapter two, below, we introduce models to each of these three ends, first by considering evolutionary models of utility which predict the existence of mean reversion, second by introducing a formal model of mean reversion and considering the behaviour it generates and its interac-

¹²Note that the arithmetic means for SWB data underlying these conclusions assume away some of the technical issues raised above and dealt with in more detail below.

¹³According to a particular definition of ‘disability’ and excluding separately defined ‘serious disabilities’.

tion with a pure rate of time preference, and thirdly with a (less-formal) discussion of the normative consequences of mean reversion for social planners.

1.8 Conclusions

In this chapter we have sought to address the desirability, viability and technical difficulty of obtaining and applying direct measures of subjective wellbeing.

In relation to desirability, we have suggested that measurements of subjective wellbeing provide a useful complement to more traditional revealed preference measures of individuals' wants and choices. Subjective wellbeing analysis enables economists to check whether individuals are acting consistently with the dictates of rational actor theory, and offers a useful measure of preference strength, which is absent from purely ordinal measures of demand or disapproval.

In relation to viability, we first addressed the threshold question of whether reported subjective wellbeing is, in any sense, a meaningful variable. Arguing on the basis of correlation within individual scores, high response rates and correlation with objective indicators of mental state, we concluded that subjective wellbeing measures some relatively stable variable and that this unobserved variable bears at least some resemblance to the 'utility' which economists implicitly seek to maximise.

The second question as to SWB's viability dealt with was whether notions such as 'wellbeing', 'contentment' or 'happiness' adequately capture the normative notion of 'utility'. We argued that, in the first place, such definitional distinctions were likely to be overstated in practice, and, secondly, that a clearly-communicated instrumental role for SWB would likely act to ensure the reporting of a sufficiently holistic, and normatively-accurate, conception of 'happiness'. To the extent that utilitarianism, ultimately, relies on subjective sensations, eliciting 'true' happiness is simply a matter of correctly defining the sensations to be reported, or of correctly communicating the consequences of a given response.

Moving beyond the conceptual relationship between utility and wellbeing, the third issue in relation to viability is the *statistical* relationship between the two concepts. While this issue will be considered in more detail in chapter two, below, this chapter has presented a range of possible statistical relationships between utility and wellbeing and argued that even the weakest of these, a purely ordinal correlation, preserves a significant role for SWB in public-policy, given the existing reliance on other measures

which are, on an individual level, only correlated with the preferences we are seeking to measure.

The fourth and final viability question is the extent to which the data generated by SWB surveys is interpersonally comparable. While conceding that this question is to some extent unresolvable, we argued firstly that assumptions resembling interpersonal comparability, and with similar normative consequences, are already made with respect to other neural processes, and secondly that an assumption of comparability, even if factually questionable, is a justifiable ‘legal-fiction’ on which to base policy choices.

Finally, we considered the technical difficulties associated with gathering and applying subjective wellbeing data. The problems of boundedness, granularity and mapping from underlying to reported SWB were all treated as aspects of a single reporting scale problem, and are dealt with in more detail in chapter two, below.

Similarly, although the questions of what scale data SWB research generates, and the appropriate measures of central tendency to be applied, affect researchers rather than subjects, they are also linked to the issue of mapping from underlying to reported scales, and are dealt with in detail in chapter two.

Sensitivity to changes in language and wording of the survey is dealt with primarily as a question of clarifying the instrumental role of SWB data. While, in certain cases, even instrumental concepts may translate poorly for cultural or linguistic reasons, in general we have argued that a transparent policy role for SWB should render linguistic disputes as irrelevant as the definitional issues discussed earlier.

The final technical difficulties addressed are random measurement error, which, while potentially large, is likely to be unbiased and partially negated by sufficiently large sample sizes, and strategic misreporting of one’s level of SWB, which we argue is likely to remain trivially low for the foreseeable future.

We have argued, then, that despite significant practical difficulties, subjective wellbeing data has normative significance at a theoretical and at a policy level, and that, properly handled, it could provide useful insights into individual behaviour and optimal policy.

1.9 Questions and Next steps

Chapter one has presented the case for SWB as a partial basis for non-Paretian utilitarian decision making and considered a number of practical

and technical difficulties associated with effectively using SWB data to assess the utility associated with different states of the world.

In chapters two and three, we broaden our enquiry somewhat in order to develop a methodological toolkit which enables us to return to a subset of those technical difficulties in a more rigorous fashion. We also consider in more detail the policy implications of the stylised features of utility we identify as part of this process, and apply the general principles identified to a range of specific situations in which policy makers attempt to measure and aggregate proxies for individual utility.

1.9.1 Recovering interval scale data from SWB estimates

The key problem we address in chapter two is the transformation of SWB data so as to give it interval scale properties. As we have discussed above, there is no reason to assume that the underlying utilities associated with different SWB responses will be linearly distributed along an arbitrary survey scale. This implies that measuring means and summing cardinal differences in SWB will be an inaccurate measurement of the underlying policy-relevant variable, since they will implicitly assume equality of the differences between different points on the scale.

An alternative approach, which recognises but does not entirely solve this problem is to treat SWB responses ordinally, so that no assumption is made about the distribution along the SWB scale except that it is strictly increasing in reported SWB. Treating SWB data this way avoids making inaccurate assumptions, but, if respondents *do* intend to report their SWB cardinally then ordinal approaches lead researchers to discard valuable cardinal information.

In chapter two, then, we explore a number of approaches which might allow us to recover the relationship between observed SWB and unobserved utility. This begins with a lengthy detour into the literature on evolution and restricted utility functions, which both provides a mathematical solution to a closely related problem and, coincidentally, points us towards a method of modelling an entirely different phenomenon recognised by the SWB literature: mean reversion.

Mean reversion, as we note above, is the observed tendency for reported SWB (and, we argue, therefore underlying utility) to return to, or towards, some “baseline” level following a positive or negative shock. Our analysis of evolutionarily restricted utility functions enables us to derive a range of predictive models for why mean reversion might arise and how it might function in practice. We show that there is a relationship between evolution-

ary incentives in the presence of mean reversion and positive pure rates of time preference, and discuss qualitatively how a SWB-informed utilitarian policy maker ought to optimise social welfare for a citizenry which rapidly adapts to changes in utility. We also consider empirical evidence as to the existence, and, importantly for our theoretical models, rate and structure of mean reversion.

This empirical exercise provides our motivation to return to the cardinalisation problem – armed with the related proofs we have adapted from the evolutionary utility literature. Mean reversion is fiendishly hard to measure if all we have are ordinal time series utility measures – so we consider the conclusions we might be able to draw by modelling the interaction between surveyor and respondent as a cooperative signalling game where the respondent attempts to signal the value of a continuous, unbounded variable using a bounded granular scale.

At the conclusion of chapter two, then, we hope to have achieved the following goals: provided a theoretical foundation for the phenomenon of mean reversion, derived a series of models for how mean reversion might operate in practice, discussed the policy implications of an “iron law of happiness”, added to the empirical evidence of mean reversion, particularly the rate at which it occurs, considered and modelled the individual’s problem in mapping from utility to SWB and derived and implemented an error-minimising solution to this problem as part of our measurement of mean reversion and for use in interpersonal aggregation of SWB.

1.9.2 Wellbeing, risk and aggregation through time

In chapter three we analyse a range of questions arising from quite a different aspect of SWB research: how should we aggregate utility and its proxies through time and across states of the world?

In much the same way as chapter two considers the validity of treating SWB as linear when aggregating it between persons, we ask whether individuals’ preferences permit linear aggregation through time. As with chapter two, this question leads to a lengthy detour, this time into individual attitudes to risk; we suggest that intertemporal and cross state aggregation are related questions and that the observed failings of traditional, expected utility maximising risk models suggest a unified, tractable and intuitively appealing approach to aggregation of wellbeing, however it is measured, through time and across states. We label this preference structure “peak aversion”.

Peak aversion gives us a model for how we might aggregate SWB data

across states and through time. We go on to suggest that it is possible to measure the strength of observed peak aversion, and that the existence of peak aversion also has direct implications, via Rawls' veil of ignorance argument, for how a social planner ought to weight individual proxies for utility in comparing distributions and states of the world.

From hypothetical social planner to real ones, we turn our attention to how the theoretical results suggested in chapter three apply to the assessment of health technologies using Quality Adjusted Life Years and to judges setting compensation for tortious injuries. We show that existing approaches are flawed and that proxies for utility, such as QALYs and compensation, should take into account the issues of risk preference and aggregation previously recognised in the theoretical literature, as well as those raised in this chapter.

At the end of chapter three, we hope to have suggested a formal model for aggregating SWB through time and states, as well as suggesting modifications to the implicit aggregation strategies employed in law and health care. More broadly we question the explicit state-based preference model in general use by economists, and show that individuals' attitudes to risk are more complex, and more relevant to policy setting than is generally accepted.

Chapter 2

Mapping: From Nature to Utility to Reported SWB

2.1 Introduction

Having made the argument that subjective wellbeing is a valid area of study, we now take something of a detour, turning our attention instead to the structure of the utility functions which underly and, hopefully, generate, responses to subjective wellbeing surveys. As will become clear, the structure of the special class of utility functions introduced below has implications for the individual's mapping problem, as discussed in section 1.6.4, above. By applying the analogous results from evolutionary utility theory, we resolve some of the measurement issues discussed in the second half of chapter one, above, demonstrating how ordinal SWB data can be cardinalised.

In addition to the technical links between these 'restricted' utility functions and the problem of drawing inferences about utility from reported SWB, the utility functions modeled in this chapter display 'mean reversion' – a tendency, previously introduced in section 1.7, for experienced utility to return to its previous level following permanent change in circumstances – a phenomenon which appears to arise from existing SWB data and which goes to the heart of the policy recommendations offered by many SWB practitioners. We use this mean-reverting behaviour, predicted by evolutionary utility models and observed in empirical data, as the basis for a formal model of the mean reversion. We show that anticipated mean reversion impacts on individuals' motivation to maximise utility and show that a positive rate of time preference can represent a solution to this problem. On this basis we are able to suggest the appropriate treatment of mean reversion and time preference from the point of view of a benevolent social planner.

At the conclusion of this chapter we will have, then, a clearer model of when and how analysts can draw conclusions about utility from reported SWB, and a tractable model of how mean reversion operates in terms of traditional utility functions and some understanding of its normative consequences. In arriving at these conclusions, the chapter is structured as follows:

Section 2.2 provides an introduction to existing restricted utility models. These models posit the existence of a number of utility states distinguishable by human cognition which is small relative to the number of relevantly distinct states of the world. They therefore call for utility functions which are, as a consequence of these neurological limitations, neither unbounded nor continuous, as they attempt to map multiple states of the world to each possible value of the utility function. These models treat the mapping from states of the world to restricted utility as a mechanism by which evolution seeks to optimise an individual's behaviour – effectively a form of signalling problem whereby evolution attempts to alert individuals to the optimal course of action by means of a restricted set of signals. This evolutionary signalling problem is ultimately useful as a lens through which to examine the mapping problem faced by individuals reporting their subjective wellbeing.

Having considered the existing models of restricted utility, section 2.3 presents a new model, based on more restrictive assumptions about the information available to evolution in designing a utility function. The alternative model derived in this section is not strictly necessary to the material which follows – the conclusions in relation to SWB hold in general terms even under the preexisting models – but this section makes a small contribution to the evolutionary utility literature by placing those models on what we hope is a more intuitively appealing footing.

The results suggested by the new model, and the preexisting models, are then discussed and, in 2.3.4, the cardinality and measurement scale of utility values produced by restricted utility functions are analysed, tying back to our discussion of data scale and SWB in section 1.6.5, above.

In section 2.4 we move on to derive models of the kind of mean reversion predicted by the restricted utility models, and observed in SWB data. The initial model presented, in section 2.4.1, draws directly on the structure of the restricted utility functions modelled in the earlier sections, while section 2.4.3 considers a more general mean reversion structure. Empirical evidence drawn from the British Household Panel Survey is then used to estimate the parameter values of this model in 2.4.4. We then discuss the relationship between mean reversion and time preference in the context of this model, and discuss the normative consequences of mean reversion and time preference

in the context of social choice theory. Arising from this discussion, we weigh the policy implications which would arise should SWB data demonstrate strong mean reversion of the sort modeled here.

Subsequently, in section 2.5, we return to a direct consideration of SWB data; we model the signalling problem faced by individuals declaring their subjective wellbeing on a bounded, discrete scale. We consider the relationship between the individual's SWB mapping problem and the role of evolution in restricted utility models, showing that there is an optimal mapping from utility to SWB based on a cooperative signalling model of the survey process. We derive this optimal mapping for the British Household Panel Survey and use it to cardinalise the data and to update the results obtained from ordinal SWB data in 2.4.4. We then qualitatively analyse the optimal mapping from utility to SWB in the absence of a known utility distribution and the possibility and consequences of an individual optimally changing their mapping in response to updated utility observations.

Finally, section 2.6 discusses the suggested adjustments to raw subjective wellbeing data as a consequence of the results derived in this chapter.

Section 2.7 briefly summarises our conclusions.

2.2 Restricted Underlying Utility

We typically conceive of utility as an unbounded, continuous function, capable of distinguishing between any two arbitrarily similar states of the world. In most approaches to choice under uncertainty, we further assume that utility is cardinal – capable of not only ranking any two states, but of revealing ratio-scaled differences between them.

The claims we make for unobservable utility, though, are not particularly consistent with the behaviour of our other, more easily observable, measurement apparatus. Humans are not capable of drawing infinitely fine distinctions between volumes of sound, or brightness of light, or saltiness, softness or lemon-scentedness. Still less are humans capable of internally reproducing cardinal scales like lumens, scoville-units and decibels.

The role for utility in our models of individual behaviour requires that it collapse a series of arguments, such as food, shelter and family, onto a single scale, and that each of these variables presents the would-be measurer with at least the same challenges as light and sound.¹ It seems unlikely, therefore, that nature has in fact equipped us with an hedonic apparatus

¹While raw volumes of food may be relatively easy to internally measure, its evolutionarily relevant components – calories, salts, and so forth, are not.

capable of measuring and distinguishing every possible situation and converting them to some unobservable cardinal scale, if only because such an approach would be, in evolutionary terms, enormously expensive in neurons and cranial capacity.

But if utility is not the continuous, cardinal concept usually used in our models, how *do* individuals go about ranking states of the world to see which one they prefer? In answering this question, we will begin by considering the underlying, evolutionary role of utility functions as a means of ranking states of the world in order of their desirability.

2.2.1 Utility and Evolutionary Fitness

At this stage we will adopt the conceit, first suggested by Robson (2001), of evolution as the conscious designer of human neural utility infrastructure. Evolution, for the purposes of this metaphor, seeks to provide humans with a utility function which will correctly signal to them the most beneficial course of action in response to any given set of choices. We assume, reasonably, that this evolutionary approach is preferable (from the point of view of genetic survivability) to simply ‘hard-wiring’ humans with specific responses to every imaginable situation; using a properly-calibrated utility function as a day-to-day proxy for evolution allows people to adapt to circumstances which would have been impossible to predict at the time of evolution’s ‘programming’.

That is: we can imagine situations in which the most plentiful species of food change rapidly relative to the speed at which dietary preferences can evolve. Hard-wiring humans to hunt a specific species, rather than providing them with an hedonic feedback mechanism based on the fat, sugar and protein content of whichever animal they decide is the best overall target, leaves them hamstrung, able to adapt to a changing environment only over evolutionary timescales.

Rather, we suggest, evolution would better encourage genetic fitness by structuring hedonic payoffs in order to motivate humans to seek the intermediate determinants of reproduction (calories, mates, survival) while leaving the individual to choose the precise means by which those goals are to be accomplished.

Note that throughout this chapter, when we consider ‘evolution’s’ actions and motivations, we are speaking consistently with the above metaphor, and evolution’s optimal ‘choices’ are, in fact, the evolutionary fitness-maximising characteristics which will be selected for in subsequent generations. The key point is that we would expect to observe such optimal adaptations in modern

humans, rather than competing suboptimal traits.

So: we posit that humans' utility functions have been 'built' by evolution so as to maximise their genetic fitness. Rather than preprogramming responses to every conceivable situation, evolution endows humans with an incentive function and hard-wires them to behave so as to maximise the value of that function. If this were the case, we would have something resembling a traditional principal-agent problem as between evolution and the individual humans whose behaviour evolution seeks to influence. Thought about this way, evolution's problem in constructing a survival-maximising utility function is analogous to a principal setting an optimal compensation structure so as to incentivise their agent to act in the principal's best interests. Evolution can take for granted that humans will, in expectation at least, adopt the course of action that maximises their utility, in much the same way as an agent will maximise compensation net of effort. Evolution must, therefore, ensure that the utility function returns the highest value in response to the evolutionarily optimal strategy.

If there are no further constraints on the structure of the utility function, this 'evolution-as-principal' model would simply collapse back into a traditional model of humans as rational-genetic-survival-maximisers; evolution will simply map utility to some linear transformation of the genetic benefits associated with a choice, and the humans will choose evolution's preferred option in every case.

However, we have argued in 2.2, above, that it is not reasonable to expect any human evaluation apparatus to cardinally distinguish an infinite number of states. We might, on the contrary, expect that evolution would be constrained, in designing its optimal utility function, by a human neurological inability to generate a genuinely continuous and unbounded range of utility values.

We can imagine two broad classes of constraint which evolution-as-principal might have to satisfy when constructing its utility-as-incentive function. First, potential utility functions might display an inability to distinguish between states of nature which are sufficiently close together. Or, alternatively, the utility function might be limited to taking some fixed, finite number of values, such that it is bounded and discrete rather than unbounded and continuous.

The first approach is analogous to the notion propounded by Edgeworth (1881) of 'just-noticeable-differences' in utility. For certain, sufficiently-small deviations from any utility level,² the difference would simply not be noticeable given an individual's cognitive limitations. This approach is applied in

²Or, more accurately, any state of nature corresponding to a given utility level.

Rayo and Becker (2007); the authors describe their approach as follows:

Consider a representative agent (i.e., a hunter-gatherer) who faces an abstract one shot project. To fix ideas, suppose this project amounts to an opportunity to collect fruit. The agent first observes the current state of nature s , which describes the physical configuration of the world, such as the presence of fruit and dangers in specific locations. Next, he selects a course of action $x \in X$, which represents the strategy adopted, such as traveling in a certain direction or climbing a particular tree. The combination of x and s randomly determines a level of output $y \in \mathbb{R}$, namely, the amount of fruit collected.

Denote the conditional probability distribution of output by $f(y|x; s)$, a function known by the agent. Once y is realized, the agent experiences a one-dimensional level of happiness given by $V(y)$.

Where $V(y)$ is the valuation function, which maps from level of evolutionary success (amount of fruit collected, in this example) to the level of utility experienced by the individual. Rayo and Becker then adopt a restricted utility approach whereby the individual is unable to distinguish between sufficiently similar outcomes, y , meaning that the valuation function, $V(y)$, returns identical values for y 's which fall within a certain distance of each other.

The authors then consider the optimal ($E[y]$ -maximising) structure of $V(y)$ where the agent acts so as to maximise $E[V(y)]$ and show that such an optimal structure implies that payoffs will experience reversion to the mean.

The key difficulty with general application of Rayo and Becker's just-noticeable-difference approach is that it leads to violations of transitivity in the preference structures it generates. If an individual is incapable of distinguishing payoffs which are less than ϵ apart, then for three payoff values, x , y and z , chosen such that:

$$\begin{aligned} |x - y| &< \epsilon \\ |y - z| &< \epsilon \\ |x - z| &\geq \epsilon \end{aligned}$$

then x will be indifferent to y , y will be indifferent to z but x will be preferred to z , violating transitivity. A related consequence of this observation is that it is impossible to construct an indifference map for individuals for

whom $\epsilon > 0$, as any point on the map will be judged indifferent to all points within a circle, radius ϵ , of itself.

This brings us to the second class of restricted utility function. This approach, suggested by Robson (2001), restricts the utility function to a finite number of values. As the underlying fitness payoffs provided by nature are assumed to be unbounded and continuous, this generates a step function relating the payoffs to utility, such that a range of fitness outcomes are assigned to each individual value of the utility function.³

Under the Robson model, evolution is faced with the problem of mapping a known distribution, but unknown realisations, of available outcomes in nature to a fixed number of utility values so as to maximise evolutionary fitness. That is, evolution must construct a mapping from states of nature, which the individual is unable to observe, to the utility function which motivates them, so as to cause them to make choices which are optimal from evolution's point of view. An optimal utility function will therefore correctly rank prospective states of the world according to their evolutionary benefit to the individual.

The above maximisation problem is equivalent to minimising the error that arises when the individual incorrectly identifies more than one possible strategy as providing the highest available payoff as a result of two or more payoffs being mapped to the same value of the utility function. That is, evolution selects for a utility function which minimises the errors arising in circumstances when, as a result of their limited ability to distinguish unlike states, an individual incorrectly ranks two unequal evolutionary payoffs as providing equal utility payoffs.

In the simplest case, where the utility function takes only two values, and only two payoffs are available to choose between each period, evolution attempts to minimise error by choosing the optimal value of the cutoff point between the low and high utility values, c . If both payoffs, x_1 and x_2 , fall on the same side of the cutoff, c , they will generate equal expected utility, and the individual will be unable to distinguish between them – leading the individual to choose randomly between the two options.

Robson (2001) initially examines such a two-value utility function. Formally, Robson models the behaviour of an organism required to choose between two lotteries, each being an independent draw from a known continu-

³If we combine the just-noticeable-difference approach considered above with upper and lower bounds on the values the utility function can take, we have something resembling this step-function model. The key distinction is that the just-noticeable difference method assumes an inability to distinguish between any two sufficiently close states of the world, while the step function model suggests such a difficulty will arise only where the two states are assigned to the same utility category.

ous cumulative distribution function, F . After the draws are made for each lottery, the organism must choose between them but, owing to its restricted underlying utility function, it can only determine whether each of the draws is above or below the cutoff in its two-value utility function, which Robson labels c . Where the utility functions fails to distinguish between the two lotteries the organism will, as we argue above, simply randomise between the two options.

Robson shows that the probability of error as a function of the cutoff between high and low utility, c , is given by:

$$\frac{1}{2}\Pr\{x_1, x_2 < c\} + \frac{1}{2}\Pr\{x_1, x_2 > c\}$$

which is equal to

$$\frac{1}{2}(F(c))^2 + \frac{1}{2}(1 - F(c))^2$$

where x_1 and x_2 are the outcomes of the two competing lotteries. Minimising this expression gives the result that the error-minimising value of the cutoff, c is achieved where $F(c) = \frac{1}{2}$, such that c is optimally located at the median of the payoff distribution.

This initial approach has the disadvantage of treating all errors arising from an incorrect choice of payoff equally. A subsequent version of the model takes into account the size of the errors generated by an incorrect choice. Under this assumption, c is optimally located at the mean, rather than the median of the distribution of x . Netzer (2009) provides a general solution to the model in the case of N utility thresholds.

2.3 Extending the Robson/Becker Model

We can begin by pointing out, in general terms, that both the Robson and Rayo/Becker models condition the optimal mapping from payoffs to utility on the distribution of payoffs in the environment. As such, where there is a shift in the distribution of the determinants of evolutionary success, the optimal mapping from payoffs to utility will, itself, be altered. Provided that the neural structures responsible for generating this mapping initially permit remapping in response to such changes, we can expect the stimulus necessary to generate a given hedonic state to change with, and in the same direction as, the level of environmental abundance.

That is, even without replicating the Rayo/Becker/Robson models' predictions in detail, we can predict that those models, which condition the relationship between utility and evolutionary payoffs on the prevailing distribution of those payoffs, will predict a changing relationship between outcomes and the utility they provide. We return to this result in our discussion of observed mean reversion in section 2.4, below.

We can, however, demonstrate this result more explicitly, by considering a variation on the existing models.

2.3.1 Deriving a Mapping from Experienced Utility

Robson's two-value utility function model conditions the optimal choice of c , the cutoff point beyond which the utility function returns a high value, on the distribution of the payoffs to different strategies, x , provided by nature.⁴ This same general relationship, between the optimal mapping from payoff to utility and the distribution of the payoff function, holds in all the models outlined above.

This is, in some ways, an unusual choice of structure, given that the actual payoffs provided by nature are, *ex hypothesi*, unobservable by the individual. That is, the individual is endowed by evolution with a discrete, bounded utility function which is intended to act as a proxy for the underlying, continuous, possibly unbounded payoff function; but the mapping between payoff and utility is allowed to change, dynamically, in response to the exact, unobservable distribution of the payoff function.

This approach would be reasonable if evolution, itself, had detailed knowledge of the distribution of payoffs available to the individual. In this case, the mapping function could be set 'at birth' based on these known probabilities. However, this would require that the mapping be locked-in over an evolutionary timescale, and any change to the distribution of payoffs – a natural disaster, say, or technological progress – would potentially render the prespecified mapping obsolete.

By way of analogy, Robson compares the evolutionary problem of generating a mapping to discrete (or otherwise restricted) utility to the taking of a reading from an old-fashioned voltmeter, noting that "to get an accurate reading from a voltmeter, one must first estimate the range into which the unknown voltage falls." But Robson's approach presupposes that we are *not* equipped only with a voltmeter, but that we start with some knowledge of

⁴Note that for the purposes of this discussion we ignore Robson's y -values and treat the x -values made available by nature as describing both the available strategies and their payoffs, as the one leads directly to the other in this model.

what sort of voltages one ought to expect.

On this basis, we propose a model whereby an individual's mapping is updated dynamically, *based only on the values of utility observable by the individual*. For ease of exposition, we will consider the two-value utility function case with two available strategies in each period, analogous to the Robson model outlined above. Each period, nature makes available a choice between two lotteries across evolutionary payoffs,⁵ x^1 and x^2 (where we assign the lotteries such that $E[x^2] \geq E[x^1]$) which the individual is able to evaluate only on the basis of the utility value returned by each of them. The utility function takes only two values, which we will normalise, without loss of generality, to 0 and 1. Evolution's aim is to choose c , the value of x above which the utility function returns 1 rather than 0, so as to minimise the error generated by incorrectly choosing the lottery which generates the lower expected payoff. If exactly one of the lotteries generates $U(x, c) = 1$ then the individual will (correctly) choose that lottery. In the event that each of the lotteries generates the same utility value, the individual will randomise between them.

The values of x^1 and x^2 will be drawn by nature from a distribution which is unknown to the individual (who lacks the perceptual tools to directly observe the x -values) and to evolution, which chooses to allow the individual to adapt to changing circumstances. We will also assume, for the purposes of this analysis, that the distribution from which the x 's are drawn is unbounded above but bounded below, and that this lower bound, perhaps the death of the individual and their genetic relations, is known to evolution. We will normalise this lower bound to $x = 0$.

In the initial iteration of this game, evolution's choice of c (the cutoff value for the utility function) is completely arbitrary. Without any knowledge of the x -values or the distribution from which they are to be drawn, evolution can only pick a random value for c that is greater than 0.

Following the observation of the initial set of x -values at time $t = 0$, evolution is able to determine,⁶ based on the U -values returned by the 'good' option, x^2 and the 'bad' option, x^1 , whether zero, one or two of the x -values lies below the initial, arbitrary value of c . Based on this observation, the evolution will then update the value of c for the next period, $t = 1$, based on the following difference equations:

⁵These lotteries can be thought of as degenerate lotteries delivering a single, certain payoff without altering our analysis.

⁶Or, less anthropomorphically, "the preprogrammed fitness-maximising mapping from payoff distribution to utility will alter to take into account".

$$\begin{aligned}
&\text{If } x_t^1 > c_t \text{ then } c_{t+1} = f_t(c_t) \\
&\text{If } x_t^2 < c_t \text{ then } c_{t+1} = g_t(c_t) \\
&\text{If } x_t^1 < c_t \text{ and } x_t^2 > c \text{ then } c_{t+1} = c_t
\end{aligned}$$

where $t = 0$ and $f_t'(c) > 0$, $g_t'(c) > 0$, $f_t(c) > c > g_t(c)$, $f_t(c) - c \geq c - g_t(c) \forall c \in \mathbf{R}_+$. In words: after the first set of signals have been observed, evolution updates the cut-point between utility 0 and utility 1 based only on the ordinal distribution of the observed strategy set around the initial (randomly selected) value of the cut-point as follows:

1. if all available strategies return utility of 1 (that is, the highest possible utility), evolution will increase its requirements for awarding high utility in subsequent periods by some increasing transformation, $f_t(c)$, of the initial cut-point value.⁷
2. similarly, if both available strategies yield utility of 0, evolution will respond by reducing the level of the cut-point by some decreasing transformation, $g_t(c)$; and
3. finally, if the initial cut-point between high and low utility lies between the two initial payoff values, then evolution leaves the cut-point unchanged.

As we have assumed that the distribution of payoffs is bounded below, but not above, and that this lower bound is known to evolution, evolution's changes to c will be asymmetric, with increases being larger than decreases towards the lower bound, explaining the condition that the difference between $f(c)$ and c be greater than that between c and $g(c)$ for every value of c .

After the first round of observations has been completed, and the cut-point, c , updated according to the above difference equations, a new set of payoffs, x_1^1 and x_1^2 will be produced. In this second iteration, though, evolution has information based not only on how the x -values relate to the new cut-point, c_1 , but also the information from period $t = 0$. This requires an additional set of difference equations to describe the evolution of f_t and g_t . Initially, for period $t = 1$ we will have $f_0(c) = f_1(c)$ and $g_0(c) = g_1(c) \forall c \in \mathbf{R}_+$, but after the x -values for period $t = 1$ have been observed, evolution can update these functions based on the success of its

⁷As the initial value of c is selected randomly, $f_0(c)$ is, itself, simply a random increasing transformation; its relationship to c is chosen only for ease of notation.

initial changes to c . The following equations describe the behaviour of f_t and g_t :

$$\begin{aligned}
&\text{If } x_t^1 > c_t \text{ and } x_{t-1}^1 > c_{t-1} \text{ then } f_{t+1}(c) = f_t(f_t(c)) \\
&\text{If } x_t^2 < c_t \text{ and } x_{t-1}^2 < c_{t-1} \text{ then } g_{t+1}(c) = f_t(g_t(c)) \\
&\text{If } x_t^1 > c_t \text{ and } x_{t-1}^2 < c_{t-1} \text{ then } g_{t+1}(c) = g_t(g_t(c)) \\
&\text{If } x_t^2 < c_t \text{ and } x_{t-1}^1 > c_{t-1} \text{ then } f_{t+1}(c) = g_t(f_t(c)) \\
&\text{Otherwise, if } x_t^1 < c_t \text{ and } x_t^2 > c_t \\
&\text{Or } x_{t-1}^1 < c_{t-1} \text{ and } x_{t-1}^2 > c_{t-1} \\
&\text{Then } f_{t+1}(c) = f_t(c) \text{ and } g_{t+1}(c) = g_t(c)
\end{aligned}$$

where $t = 1$. In words: evolution is increasing the rate at which it adjusts c between periods depending on the number of consecutive periods for which it is aware that c has been biased in the same direction. Conversely, where the previous period's adjustment process has caused the new c -value to overshoot both x -values, evolution reduces the subsequent rate of adjustment in the direction of overshoot. This process is designed to increase the speed with which c converges on the median of the unknown distribution of x -values.⁸ In situations where the value of c has successfully distinguished x_2 from x_1 , either at $t = 1$ or at $t = 2$, evolution leaves the rate of adjustment unchanged, as there is no evidence on which to base a change.

Note that the consecutive-period rates of adjustment, $f(f(c))$ and $g(g(c))$ have been arbitrarily fixed as a function of the initial rate of adjustment, $f(c)$ and $g(c)$. Since this choice of function, like the original choices of c , $f(c)$ and $g(c)$ is arbitrary, this choice has been made to simplify notation and hopefully to reflect the parsimony of a model of utility-function formation predicated on limited cognitive resources. In any case, none of the results drawn from this model turn on the precise specification of the adjustment rate.

In the spirit of parsimony, then, we will further restrict the existing model before setting out the behaviour of the utility function for the general time t case. For purposes of clarity, we will assume that both $f_0(c)$ and $g_0(c)$ are simply linear functions of c , such that $f_0(c) = \alpha_0 c$ and $g_0(c) = \beta_0 c$, $0 < \beta_0 < 1 < \alpha_0$. In this specification α_0 and β_0 are constants reflecting the

⁸While, as noted above, Netzer (2009) and Robson (2001) demonstrate that the mean of the x -distribution is the preferred error-minimising value for c , evolution is unable to observe any information related to the mean in skewed distributions where it differs from the median. In this model, evolution can only hope to target the median of the distribution.

recursive rates of adjustment, and we have $f(f(c)) = \alpha^2 c$ and $g(g(c)) = \beta^2 c$. We can rewrite the difference equations for $t = 0$ and $t = 1$ as follows:

$$\begin{aligned} \text{If } x_t^1 > c_t \text{ then } c_{t+1} &= \alpha_t c_t \\ \text{If } x_t^2 < c_t \text{ then } c_{t+1} &= \beta_t c_t \\ \text{If } x_t^1 < c_t \text{ and } x_t^2 > c_t \text{ then } c_{t+1} &= c_t \end{aligned}$$

For $t = 0$ and:

$$\begin{aligned} \text{If } x_t^1 > c_t \text{ and } x_{t-1}^1 > c_{t-1} \text{ then } \alpha_{t+1} &= \alpha_t^2 \\ \text{If } x_t^2 < c_t \text{ and } x_{t-1}^2 < c_{t-1} \text{ then } \beta_{t+1} &= \alpha_t \beta_t \\ \text{If } x_t^1 > c_t \text{ and } x_{t-1}^2 < c_{t-1} \text{ then } \beta_{t+1} &= \beta_t^2 \\ \text{If } x_t^2 < c_t \text{ and } x_{t-1}^1 > c_{t-1} \text{ then } \alpha_{t+1} &= \alpha_t \beta_t \\ \text{Otherwise, if } x_t^1 < c_t \text{ and } x_t^2 > c_t \\ \text{Or } x_{t-1}^1 < c_{t-1} \text{ and } x_{t-1}^2 > c_{t-1} \\ \text{Then } \alpha_{t+1} = \alpha_t \text{ and } \beta_{t+1} &= \beta_t \end{aligned}$$

For $t = 1$.

Now, for the purposes of deriving the general rule for updating c , let ζ_t represent a shorthand showing the state outcome at time $t - 1$. We will assign $\zeta = -1$ in the case that both x -values fell below the cut-point at time t , ie, where $x_{t-1}^2 < c_{t-1}$. Similarly, $\zeta = 1$ where both x -values lay above the cut-point, $x_{t-1}^1 > c_{t-1}$, and, in the final case where the cut-point successfully distinguished the values, $x_{t-1}^1 < c_t < x_{t-1}^2$ then we have $\zeta_t = 0$.

We can now present the difference equations for c , α and $\beta \forall t \in \mathbf{R}_+$:

$$\begin{aligned} \text{If } x_t^1 > c_t \text{ then } c_{t+1} &= \alpha_t c_t \text{ and } \zeta_{t+1} = -1 \\ \text{If } x_t^2 < c_t \text{ then } c_{t+1} &= \beta_t c_t \text{ and } \zeta_{t+1} = 1 \\ \text{Otherwise } c_{t+1} &= c_t \text{ and } \zeta_{t+1} = 0 \end{aligned}$$

for c and

If $x_t^1 > c_t$ and $\zeta_t = -1$ then $\alpha_{t+1} = \alpha_t^2$
 If $x_t^2 < c_t$ and $\zeta_t = 1$ then $\beta_{t+1} = \alpha_t \beta_t$
 If $x_t^1 > c_t$ and $\zeta_t = 1$ then $\beta_{t+1} = \beta_t^2$
 If $x_t^2 < c_t$ and $\zeta_t = -1$ then $\alpha_{t+1} = \alpha_t \beta_t$
 Otherwise $\alpha_{t+1} = \alpha_t$ and $\beta_{t+1} = \beta_t$

for α and β .

2.3.2 Analysis of Restricted Utility Model

We can get a sense of the behaviour of the model described above by considering its behaviour in the presence of constant x -values. This removes the additional complication of the x -values being drawn from an unknown distribution, and lets us focus on the key characteristics of c 's movement through time. If c is initially (randomly) set below both x -values, it will increase at an exponentially increasing rate, until it either ends up correctly positioned between the x -values, or it overshoots, ending up above both x -values. In this case, c will adjust downwards, and its future rate of upwards correction will be reduced. Thereafter, c will perform a series of increasingly dampened bounces, resembling the motion of a spring following Hooke's law about its equilibrium position, until it 'comes to rest' between the x -values.

Obviously this picture is greatly complicated in the case of variable x -values drawn from an unknown distribution, but the basic structure of c 's movement in relation to the expected value of the x -values remains unchanged.

The advantage of the above model is its ability to respond endogenously to changes in the distribution of the x values through time, while conditioning the behaviour of utility only on the variables directly observable by the human agent. If a period of unusual fecundity, or technological improvements, increase the median payoff, the above model will, through time, automatically adjust to target the new median value of x , even if the new distribution is not directly observable.

The chief cost imposed by the model's inability to condition the structure of the utility function on the actual x -values observed is that it is unable to target the size of the errors induced by incorrect ranking of the x -values, meaning that it will not converge, even with a constant distribution, on the optimal, mean, value of x .

We could, at the cost of significantly more computation, generalise this

model for the more realistic case of an n -valued rather than two-valued utility function, and for m , rather than two, potential strategies from nature in any period. The basic structure – that of attempting to optimise the cutoffs between the different discrete utility values based on the observed distribution of the x -values relative to the existing cutoffs – would remain unchanged.

What, then, can we draw from the proposed model, and how does it relate to the Robson (2001) and Netzer (2009) models?

All three models agree that, if utility is constrained to taking some finite number of values, it will minimise error by adopting a mapping based on the distribution of the payoffs to strategies made available by nature. In the error-size minimising model, evolution is able to build a system of payoffs which incorporates the additional future information gained by directly observing the range of available payoffs to minimise, not merely the possibility of choosing a sub-optimal strategy, but the expected error associated with doing so. In the model outlined above, evolution must construct a neural system to dictate the structure of utility through time without that system ever being able to observe the actual payoffs generated, other than through the blurry lens of the restricted utility function.

We could imagine a hybrid model, where evolution can condition the mapping from payoffs to utility either on the stream of payoffs observed up to the previous period.⁹ This model, provided the distribution remained constant, would quickly converge to the error-size minimising model as the utility function is able to infer the distribution of x by observing a stream of random x -values.

A further possible modification would be a model whereby evolution observes, and hence can condition the mapping to utility on, only the value of the payoff it actually receives. That is, the cutoffs for different levels of utility are determined, as in the model presented here, by the past values of x relative only to the past values of the cutoffs *and* by the value of whichever x is selected in any given period. In this case, evolution must try to infer the expected value of the highest value of x , given only the previous highest values.

We will not formally derive these hybrid models at this time.

⁹In the language of the model above, c_t is a function of $x_s^1, x_s^2, s=(t-1, t-2, \dots 0)$.

2.3.3 Practical Applications of the Restricted Utility Models

The Robson (2001) and Netzer (2009) models share with the model presented above the characteristic that the optimal mapping from outcomes to utility they generate is not fixed for changes to the distribution of payoffs in nature. Provided we accept the assumptions underlying these models (chiefly that cognitive limitations prevent the generation of a utility function capable of taking an infinite number of values, while payoffs in nature are not so restricted) we would expect to observe such dynamic mappings between utility and states of the world in practice and, in particular, we would expect shifts in the distribution of payoffs to be negatively correlated with shifts in the utility level assigned to a particular state.

The idea that continually higher payoffs will be required to maintain a given level of utility or, equivalently, that a given state of the world initially yielding above-average utility will tend to revert to the mean, is a widespread one. We deal, in 2.4.3, below, with a more explicit model of this kind of mean reversion. At this point we will simply note that, for any of the theories of restricted underlying utility examined here, we should not necessarily expect a given state of the world to generate in an individual a specific level of utility at different points in time. Where the mapping between utility and outcomes is variable, as the above models suggest that it will be over at least some time scale, we will be unable to confidently assign normative significance to reported changes in utility, or even to revealed preferences.

2.3.4 What Measurement Scale is Restricted Utility Data?

Before moving on to a detailed discussion of mean reversion and its normative consequences, we will consider the nature of the utility values generated by restricted underlying utility models like those considered above.

We assume, for these purposes, that the payoffs produced by nature are, in addition to being continuous and unbounded,¹⁰ cardinal in nature and defined up to a ratio scale. That is, we assume that the opportunities provided by nature, as perceived by evolution, can be represented as a single value, that the differences between these values adhere to a consistent scale and that a zero on this scale actually represents zero units of some meaningful concept. These are strong assumptions, but they are useful for purposes of illustration.

Having assumed that payoffs, as represented by the x -values, are continuous, unbounded and defined up to a ratio scale, we consider the nature

¹⁰We assume in the model laid out above that the payoff function is bounded below, but not above. This alternative assumption is also consistent with the results derived here.

of the utility values generated from the x -values through the operation of the utility function. The utility function, we have assumed, takes a finite number of values and as a consequence it adopts a mapping from x which collapses continuous ranges of payoffs into single utility values.

Now, even in the case of a purely linear mapping from outcomes to utility, this process will remove the cardinal significance of gaps between payoffs. Since we can not assume that equal utility values represent equal outcomes, we must also be aware that differences between payoffs assigned to neighboring utility values may vary and may, indeed, be smaller than differences between payoffs *within* a single utility value.¹¹

Provided that the mapping from payoffs to utility divides the payoff scale into equally-sized increments, though, equal differences in utility will be cardinally equal at least in expectation. However, as we are assuming that the payoff scale is unbounded in at least one direction, it is not possible for there to be a linear mapping from an infinite range to a finite number of categories.

We might alternatively assume that, while the true universe of possible payoffs is unbounded (a reasonable assumption given the possibility for technical advancement), the distribution of payoffs is bounded at any given time. In this case a linear mapping from payoffs to utility is possible and would generate utility values which, while not individually cardinally accurate, would at least be cardinal in expectation. These values would display the general characteristic of utility values produced according to a restricted utility functions of not being comparable, cardinally or ordinally, to utility values produced by a subsequent, different payoff distribution.

If, however, the payoff values are distributed non-uniformly, then an optimal mapping (from the point of view of minimising error from choices in a single period) will not be linear, and we would not expect the utility scale so generated to be cardinal even in expectation.

What are the consequences of ordinal, rather than cardinal utility? One key result is that normal, expected utility methods of evaluating state-based gambles or flows of utility through time become impossible. Ordinal, frequency-based approaches, like those proposed by Manski (1988), provide an alternative, but will generate errors relative to expected-value calculations based on underlying, cardinal payoff values. As such, a purely ordinal utility function will sacrifice accuracy relative to cardinal measures of evolutionary success.

¹¹A c -value of 50 in the above, two-value utility model, would suggest that a payoff of 80 was equal to a payoff of 51 and that both were equally higher than a payoff of 49, for example.

There are two possible means by which evolution could, effectively, ‘re-cardinalise’ the ordinal utility data resulting from a non-linear mapping. First, provided the number of values taken by the utility function is large relative to the range of the current distribution of payoff values, evolution can simply treat the number of utility values between any two states as a measure of cardinal difference. That is, evolution treats the ordinal utility values *as if* they were cardinal, allowing cardinality-dependent operations such as taking expected values. This approach will generate errors, even where each of the underlying x -values falls at the centre of the range associated with the relevant utility-value, but may serve as a useful heuristic.

The second, more sophisticated, method of recardinalising ordinal utility data applies only in situations where evolution is ‘aware’ of the distribution of payoff values.¹² In such cases, evolution is also ‘aware’ of the expected (cardinal) gap, as measured in payoff values, between any two utility values. On the basis of this knowledge, evolution can cause the human agent to mentally remap ordinal utility values to a cardinal scale designed to correct the nonlinearity of the original payoff-to-utility mapping. Such a transformed utility scale will not be truly cardinal, both because each value represents a range of outcomes and because the remapped scale will lack intermediate values between individual utility values, but it will preserve, in expectation, cardinal differences between outcomes, and permit relatively accurate cardinal utility calculations. Another way of thinking about this approach is to note that there is no necessary reason why the human agent should treat the fixed number of available utility-values as lying on a linear scale; the gaps between individual utility values are not specified and can be set so as to mirror the expected gaps between the underlying payoff values at each utility level.

An example may help clarify these concepts. We will assume that the underlying payoffs, x_t^i follow a nonlinear distribution between 0 and 100, and that the utility function takes four values, which we will initially label 0, 1, 2 and 3. We will further assume that the frequency distribution of x is such that optimising according to one the models analysed in 2.3.2, above, yields the following, nonlinear mapping:

Utility 0 maps to $0 \leq x < 30$

Utility 1 maps to $30 \leq x < 50$

Utility 2 maps to $50 \leq x < 70$

¹²We repeat our earlier caution that evolution is being so personified only to ground this discussion, its role is in fact played by a combination of increased genetic fitness and inheritance of the relevant genes and these forces actually construct a neural system which is capable of responding to inputs on a less-than-evolutionary timescale, in much the same way as our visual cortex has been ‘designed’ to respond to motion.

Utility 3 maps to $70 \leq x \leq 100$

Obviously, the initial utility values, 0–3, are not usable cardinally. However, if we remap the utility scale such that:

$$U_1 = E[x|0 \leq x < 30] \quad U_2 = E[x|30 \leq x < 50] \quad U_3 = E[x|50 \leq x < 70] \quad U_4 = E[x|70 \leq x \leq 100]$$

Then for purposes of performing ordinal operations, we would evaluate the three U -values as being equal to the median value of x in the relevant range.

If this model held, we would expect to observe discontinuities in the gaps (as measured in underlying x -payoffs) between neighbouring utility states for two reasons. First, because each utility value captures a range of x -values, and, secondly, because utility values are mentally treated as lying along a nonlinear scale.

While the above material is admittedly somewhat speculative, its real value will become apparent when we invert the problem of evolution generating a utility mapping for individuals and consider the analagous problem faced by individuals reporting utility on a bounded, granular subjective well-being scale. We will return to this material in section 2.5.1, below.

2.4 Variable Mappings and Mean Reversion

Given the results derived above, there are a number of circumstances in which we would expect to observe mean reversion in experienced utility. In this section we will consider the positive and normative effects of mean reversion, and consider whether these consequences should cause us to reevaluate the evolutionary models derived above.

We will begin by clarifying what, precisely, we intend by ‘mean reversion’. ‘Mean reversion’ – which we will use interchangeably with the idea of ‘set-points’ in utility and which is also known as habituation and as hedonic or affective adaptation – is here used to refer to the tendency of the level of utility associated with continued exposure to a particular set of circumstances to return to, or at least move towards, some fixed, ‘mean’ value. That is: an individual whose utility function displays mean reversion can be expected to return gradually towards some constant utility value provided that they are in the presence of an unchanging stimulus. As such, depending on the rate at which reversion occurs relative to the rate at which an individual’s circumstances change, a strong set-point hypothesis is consistent

with the observation of continually changing levels of utility. In fact, we formalise the likelihood of observing exactly this combination of circumstances in 2.4.5, below.

So if an individual who is initially at ‘equilibrium’, that is, deriving utility equal to their set-point level, is faced with a permanent increase in, for instance, income, they will initially undergo an increase in experienced utility. Thereafter, in the case of the strong form of the mean reversion hypothesis and provided income remains constant at the new, higher level, the individual’s utility will revert, over time, to its pre-shock level. A weaker form of mean reversion would simply predict that the initial increase in utility associated with a permanent rise in income (or any other variable in which utility is increasing) would be larger than the eventual equilibrium level to which utility would return, *ceteris paribus*.

There is significant practical evidence of mean reversion at least in its weaker form, a general survey of which is provided by Easterlin (2006), who concludes that, while strong set-point theory does not appear to be supported by the available evidence, some level of reversion is obviously present in response to certain classes of stimuli. It is also worth noting that, in addition to the difficulty of holding stimuli constant to judge whether reversion ought to have occurred, the observation of incomplete reversion is consistent with a full reversion process taking place over a time scale longer than that covered by the available data.

2.4.1 Modelling Mean Reversion from Restricted Utility

We now consider the structure of mean reversion implied by the kinds of evolutionary mapping from signals to utility derived in the Becker/Robson models outlined above.

The material in this section draws directly on the original models, and assumes, as do those approaches, that the optimal mapping from nature’s signals to utility can be conditioned on the cardinal values of the signals relative to the thresholds of the utility function.

We will derive a model for mean reversion based on changes in optimal signal-utility mapping in discrete-time, and consider the consequences predicted by this model for the subjective experience of changes in objective payoffs.

Initially we consider an individual with a two-value utility function like that proposed by Robson (2001), whereby an individual can distinguish only two hedonic states, which are associated via a mapping function with ‘good’ or ‘bad’ outcomes in nature. In any given period, this utility function can

be described according to a single cutoff value, c_t , above which that period's outcome from nature, x_t , will generate a positive hedonic state and below which leads to a negative hedonic state.

As Robson demonstrates, the mapping which minimises the expected errors from failing to properly distinguish between unlike payoffs calls for c to be set equal to the mean of the probability distribution of the payoffs provided by nature, $f(X)$.

Since the optimal mapping from nature to utility, summarised by the value of c , is a function of the probability distribution of the signals themselves, any change to this probability distribution will change the optimal value of c . Note, however, that while we assume the individual (or at least the cognitive machinery responsible for maintaining an optimal mapping) observes the signals from nature as they are realised each period (that is, the values of x_t for each period t), they are not able to directly observe the underlying distribution from which the x_t values are drawn, meaning that, contrary to our assumption in relation to recardinalisation, above, nature cannot condition the payoff function on the underlying distribution. This inability to observe the distribution on which the optimal threshold of the utility function, c , is conditioned requires evolution to adopt a heuristic to respond to possible changes to the underlying distribution of signals from nature on a period-by-period basis. The optimal structure of this heuristic by which c is updated in response to signals from nature will prescribe the structure of mean reversion experienced by an individual with a Robson two-value utility function.¹³

Consider an individual with the two-value utility function described above, initially in an equilibrium state with the threshold value of their utility function, c , set equal to the mean of the distribution of signals received from nature each period, \bar{x}_t . Now, on receiving a new signal in period t , the individual's utility function will initially return a high or low value dependent on whether x_t is greater than or less than the initial value of the utility threshold, c_t . Having done this, the evolutionary process responsible for the mapping now needs to consider whether the existing mapping, described by the value of c_t , remains optimal given the additional information provided by the observed value of x_t . That is, evolution must consider whether x_t implies that the underlying distribution of x -values, $f(X)$, has shifted.

Assuming no underlying knowledge as to the structure of likelihood of distributional change – that is, evolution can condition c on the distribution

¹³Here we are using 'mean reversion' in the same sense as set-out in section 2.4, above, to describe a mapping from states of nature to hedonic states which changes in response to past states of nature.

of X but lacks information as to when and how a new distribution will arise in response to a change in the fundamental natural conditions – the probability of a given x_t signaling a new distribution is simply the inverse of the probability of observing a value as extreme as that particular x_t under the existing distribution.

In the formal model that follows we will assume that $f(X)$ is a uniform distribution with a range which we will normalise to 1, with the minimum value of the initial distribution normalised to 0, that changes to this distribution always take the form of a change to the mean of the distribution, and that ‘evolution’ accurately assumes these condition will hold. This assumption is made for mathematical convenience, but it is trivial to replicate the analysis which follows for other distributions, most obviously the normal distribution, of $f(X)$ about c . The situation where evolution is unable to make assumptions about the structure or range of potential future distributions resists detailed analysis, but somewhat resembles the model set-out in section 2.3.1, above.

Recalling that we have initially set c_t to its equilibrium value of \bar{x}_t , the mean of the preexisting distribution of X , and normalised the range of that distribution to 1, and the minimum value to 0, then the likelihood of observing a given signal from nature, x_t , is as follows:

If $x_t < (c_t - 0.5)$ then $p(x_t|c_t = \bar{x}_t) = 0$

If $(c_t - 0.5) \leq x_t < c_t$ then $p(x_t|c_t = \bar{x}_t, x_t < c_t) = \frac{c_t - x_t}{2}$

If $(c_t + 0.5) > x_t \geq c_t$ then $p(x_t|c_t = \bar{x}_t, x_t > c_t) = \frac{x_t - c_t}{2}$

If $x_t > (c_t + 0.5)$ then $p(x_t|c_t = \bar{x}_t) = 0$

Note that this structure, by using the probability that an underlying distribution could produce a particular value conditional on the value lying above or below the distribution’s mean, assumes a more conservative ‘one tailed test’ approach to determining whether the observed x value is inconsistent with the distribution. This accurately reflects a situation whereby evolution is separately weighing the possibilities of an upward or downward shift in the distribution of payoffs from nature.

Now, where there *has* been a shift in the mean of the payoff distribution, the best available estimate of the updated optimal value of c , c_{t+1} , is the sole observed value of the new distribution, x_t . Where no shift in the underlying distribution has occurred, the optimal value of c_{t+1} remains equal to c .

As a result, the expected optimal value for c_{t+1} for values of x_t such that $c_t + 0.5 > x_t > c_t$ is given by:

$$c_{t+1} = E[\bar{x}_t | c_t, x_t] = c_t + \frac{c_t^2}{2} + \frac{x_t^2}{2} - c_t x_t \quad (2.1)$$

and for values of x_t such that $c_t - 0.5 < x_t < c_t$:

$$c_{t+1} = E[\bar{x}_t | c_t, x_t] = c_t - \frac{c_t^2}{2} - \frac{x_t^2}{2} + c_t x_t \quad (2.2)$$

This difference equation responds to the possibility of a shift in the mean value of x within the range of the previous distribution by incompletely moving the value of the utility function cutoff, c , towards the latest observed value of x . Note that:

$$\frac{\partial c_{t+1}}{\partial x_t} = x_t - c_t \quad (2.3)$$

which, recalling our normalising assumptions which imply $0 \leq c \leq 1$, indicates that, unless the change to the mean of the underlying distribution is larger than the distribution's (*ex hypothesi* constant) range, the value of c will approach but never reach the new value of \bar{x} even as t becomes arbitrarily large.

In the next section we consider the practical subjective consequences of the incomplete, gradual nature of the mean reversion implied by the two state utility models for small changes in the underlying distribution of natural payoffs.

2.4.2 Consequences of Restricted Utility for Mean Reversion

The mean reversion structure we derive above possess a number of implications for how individuals possessing a mapping from payoffs to utility of this type will experience changes to the distribution of goods through time. While the particular model and difference equation arrived at above are specific to the two-value utility case and assume uniform distribution of payoffs, we can nonetheless make a number of extrapolations as to the consequences of restricted utility for the existence and structure of mean reversion.

Consider initially a situation in which an individual whose mapping from payoffs to utility, which can be completely described by the value of the cutoff between high and low utility, c_t , is at 'equilibrium' with $c_t = \bar{x}_t$. When faced with a permanent, positive shock to the mean of the payoff distribution, such that the new mean payoff $\bar{x}_t + 1$ lies above its previous value but

below the maximum value possible under the prior distribution (which is equal to $c_t + 0.5$ under the normalisation assumptions adopted above), the individual will experience high utility as the value of the cutoff, c , gradually adjusts towards the new mean of the payoff distribution. Based on the timepath for c_{t+1} derived below, this adjustment will occur slowly, and c will only approach, without ever reaching, the new value of \bar{x} . Extrapolating to a multi-value utility model, whereby the individual is able to obtain some sense of the cardinal differences between payoffs received and the mean payoff implied by their current mapping, a permanent increase in the mean payoff will ultimately lead to increased utility aggregated across time which is a large multiple (but a nonlinear function) of the initial shift in mean payoffs. In addition, aggregation is sufficiently slow that an individual who is capable of discerning and recalling the size of the differences between their expected mean and actual payoff (as we have assumed here) is unlikely to closely approach complete adaptation across a realistic timescale.

Conversely, consider the case of a temporary increase in observed payoff from nature which does *not* result from a change to the underlying distribution or, equivalently, results from a temporary, one period shift in the distribution of payoffs. In this instance, the individual will initially experience high utility (or a significant increase in utility, were we to extrapolate these results to the multi-value utility case as discussed above) in the period for which the increased payoff is experienced, followed by low utility (though lowered by less than the equivalent of the first period positive change) for *every period thereafter* until there is another change in the distribution of payoffs. That is: a one-off high payoff leads to a long term reduction in utility in subsequent periods, and the aggregate effect of these reductions is to greatly outweigh any additional utility provided by the initial improvement in circumstances.

These consequences do not arise, it should be noted, in circumstances where the change to payoffs is large enough, in either direction, to lie outside the potential range of the previous distribution. In this instance the optimal evolutionary response is to update the predicted mean value of the payoff distribution immediately and completely to match the most recent observed payoff value.

In 2.4.3, immediately below, we introduce a more general model of mean reversion, and discuss its implications for the ‘principal-agent’ style incentive design problem faced by evolution, without explicitly relying on the structure of the restricted utility models considered above. What we can learn from the results in this section, though, is that the optimal level of mean reversion directly implied by restricted utility functions is slow and incomplete – an observation which is consistent with the mean reversion observed in the literature on subjective wellbeing. Secondly, since temporary

increases in payoffs can generate, ultimately and in aggregate, significant decreases in experienced utility as a result of overcorrection on the part of the evolutionary mapping function, forward-looking individuals may develop a preference for smoothness of payoffs through time, even at the expense of accepting payoffs with a lower mean value. This has implication for our discussion and modelling of just such a preference for consistency of payoffs through time in chapter 3, below.

2.4.3 A General Model of Mean Reversion

The following, stylised, discrete-time model represents a more general approach to modelling mean reversion, not reliant on the specific predictions of the Becker/Robson restricted underlying utility hypotheses, or indeed on any restriction to underlying utility. It can be motivated by any one of a number of explanations for nonconstant mappings from objective states to subjective utility and captures the relevant features of those mean reversion hypothesis.

We show that, in the presence of complete, linear mean reversion utility maximisation does not motivate an individual to pursue *any* impermanent increase in utility. We further show that the existence of a non-zero pure rate of time preference is sufficient to motivate utility maximising behaviour. In 2.4.4, below, we demonstrate that there is empirical evidence that mean reversion has this kind of linear structure.

We begin with the following relatively general model of complete, linear mean reversion:¹⁴

An individual subjectively experiences utility of U_t in period t , where U_t is given by the equation:

$$U_t = U_t^R(C_t) - \bar{U}_t \quad (2.4)$$

and

$$\bar{U}_t = \bar{U}_{t-1} + \alpha(U_{t-1} - \bar{U}_{t-1}) \quad (2.5)$$

where $U_t^R(C)$ is the ‘raw’ utility at time t as a function of consumption at time t , C_t , $\frac{dU_t^R}{dC_t} > 0 \forall t \in \mathbf{R}_+$, calculated according to a standard (reversion-free) utility function, and $0 \leq \alpha \leq 1$ is a constant representing the speed of

¹⁴The mean reversion modelled here is complete in the sense that it will return an individual exactly to their set-point experienced utility level in finite time following a temporary or permanent shock. It is linear in the sense that the speed of reversion is a linear function of shock size, which is consistent with the empirical evidence: we provide a qualitative analysis of nonlinear reversion below.

adaptation. If $\alpha = 0$ then there is no adaptation, and experienced utility becomes a simple affine transformation of $U_t^R \forall t \in \mathbf{R}_+$. If $\alpha=1$ then complete reversion occurs between any time t and time $t + 1$, and a permanent shift in income will provide only a one period increase in experienced utility.

While we could imagine a reversion mechanism which calculates \bar{U}_t on the basis of some more complicated weighing of several previous periods, this simple model captures the key elements of the process.

Now, given this setup, the individual maximises experienced utility by solving:

$$U^* = \text{Max} \sum_{t=0}^{\infty} U_t(C_t) \quad (2.6)$$

Now, we consider the effect on total, lifetime utility at an optimised equilibrium of a one-unit increase in ‘raw’ utility at time t , $\frac{\partial U^*}{\partial U_t^R}$. The initial, period t , effect of this shock is, *ex hypothesi*, a one-unit increase in utility. In period $t + 1$, the shock has vanished from U^R , which returns to its optimum value, but \bar{U}_{t+1} has adjusted to the positive shock, increasing by α units. As a consequence, experienced utility, U_{t+1} is α below optimum in period $t+1$. In period $t + 2$, \bar{U}_{t+2} adjusts upwards in response to the fall in experienced utility in period $t + 1$, rising by α^2 units, so that experienced utility in period $t = 2$ is $\alpha - \alpha^2$ below optimum.

Following this process of reversion through to its conclusion, the total sum of the changes to experienced utility as a result of a one-period unit shock to raw utility, setting the timing of the shock to $t = 0$, is given by:

$$1 - \sum_{t=0}^{\infty} \alpha(1 - \alpha)^t \quad (2.7)$$

Where the sum is a geometric series with $r=1 - \alpha$ and $A=\alpha$, giving:

$$1 - \frac{\alpha}{1 - (1 - \alpha)} = 0 \quad (2.8)$$

This result indicates that for *any* temporary shock, and *any* nonzero constant of reversion, α , the negative utility effects of habituating to the temporary increase in utility will, eventually, exactly offset the initial increase in utility.

We note, however, that we have treated the individual as having a zero

rate of time preference. If were instead to assume that the individual displayed preferences of the following form:

$$U^* = \text{Max} \sum_{t=0}^{\infty} (1 - \delta)^t U_t(C_t) \quad (2.9)$$

Where $0 < \delta \leq 1$, is the pure rate of time preference experienced by the individual.

Then we would have, as the result of a one period unit shock to optimised equilibrium utility:

$$1 - \sum_{t=0}^{\infty} (1 - \delta)^t \alpha (1 - \alpha)^t \quad (2.10)$$

Giving:

$$1 - \frac{\alpha}{1 - (1 - \alpha)(1 - \delta)} = 1 - \frac{\alpha}{\alpha + \delta - \alpha\delta} > 0 \quad (2.11)$$

Given the conditions placed on α and δ .

So provided the individual has any positive rate of time preference, temporary gains in utility will lead to (smaller) increases in total utility for any coefficient of reversion, α .

We will now consider the effects of a permanent, unit increase in raw utility, U^R , starting from an optimised equilibrium, initially in the absence of time preference. The ultimate total change to utility from such a shock, starting at $t = 0$ is given by:

$$1 + \sum_{t=0}^{\infty} (1 - \alpha)^{t+1} \quad (2.12)$$

Which is a geometric series with $A = r = 1 - \alpha$, such that change in utility is given by:

$$1 + \frac{1 - \alpha}{1 - (1 - \alpha)} = \frac{1}{\alpha} \geq 1 \quad (2.13)$$

Meaning that the increase in total experienced utility from a permanent unit increase in raw utility is, after taking into account mean reversion, equal to the inverse of the rate of reversion, α . By extension, a permanent shock

to raw utility of size S will yield an overall increase in experienced utility of $\frac{S}{\alpha}$.

Finally, for completeness, we consider the case of a permanent shock to utility in the presence of a positive rate of time preference, δ . In that case we would have a change to lifetime experience utility given by:

$$1 + \sum_{t=0}^{\infty} (1 - \delta)^{t+1} (1 - \alpha)^{t+1} \quad (2.14)$$

With a limiting sum equal to:

$$1 + \frac{(1 - \alpha)(1 - \delta)}{1 - (1 - \alpha)(1 - \delta)} = \frac{1}{\alpha + \delta - \alpha\delta} \quad (2.15)$$

We note that the (unsightly) multiplier on permanent shocks to raw utility in the presence of positive time preference is less than $\frac{1}{\alpha}$, the multiplier in the no-time preference case, because the rate of time preference, δ , imposes an additional discount on the value of future increases in utility. In fact, it can be seen from equation 2.15, above, that α and δ play identical roles, so that, if a given rate of reversion is replaced with an equal rate of time preference, the effect of a shock to utility will be unchanged. We discuss the practical consequences of this observation below.

2.4.4 An Empirical Estimate of Mean Reversion

We have presented a model of utility which reverts based on a linear function of the gap between current and set-point utility captured by the parameter α . We have also demonstrated, above, that the existence and functional form of mean reversion has significant consequences for individual behaviour (if anticipated) and demonstrate below that it influences optimal social policy (if unanticipated). We now set out to arrive at an empirical estimate of the mean reversion process characterised above.

In this section we present empirical estimates of the speed and structure of mean reversion derived from an analysis of measured response to SWB shocks taken from the British Household Panel Survey. We demonstrate evidence for a linear mean reversion structure with an α value of approximately 0.5 and consider an apparent asymmetry in response to positive and negative utility shocks.

The current version of the British Household Panel Survey gathers data annually from a mostly fixed panel of approximately 15,000 carefully selected

representative individuals across the United Kingdom. The average age of the panel is 37.6 year, and it is 51.7% female. Starting in the survey's sixth year it has included a series of questions designed to elicit subjective wellbeing on a 1 to 7 scale, with 1 equating to "Not satisfied at all" and 7 equating to "Completely satisfied". While a number of different SWB domains are examined, we have, in keeping with the focus of this thesis, chosen to focus on responses to the overall satisfaction question: "*Using the same scale how dissatisfied or satisfied are you with your life overall?*"

Since we are interested in mean reversion in the form of lagged response to SWB shocks, we have restricted our consideration to the seven most recent surveys, waves L through R of the BHPS running through to 2008, which provide an unbroken series of overall satisfaction responses. It would be possible to replicate this analysis for the five waves F through J, until the availability of time series SWB data is interrupted by the failure of wave K to include a section on life satisfaction, but we have not done so here.

Restricting our analysis to individuals who responded to the BHPS in each of the seven years to 2008 and then excluding any individuals who failed to provide an answer on the 1 to 7 scale for any one of the seven waves, to avoid problems with unidentifiable lagged responses, left 8,844 sets of seven SWB estimates.

In order to investigate mean reversion in response to shocks to utility we decided to restrict our analysis to situations in which an individual's overall satisfaction changed by more than one point on the 1 to 7 scale between one year and the next. One point shifts were excluded from this analysis since they could be generated by relatively small shifts in underlying utility, where the individual moved from the cusp of one value to the cusp of another (see the discussion on the individual's optimal mapping to a given SWB scale in 2.5.1, below) and are more likely to result from short term random variations in circumstances. This left a total of 5,758 individual year on year shifts of two or more points on the 1 to 7 SWB scale which were analysed on a panel basis as described below.

We then undertook a number of steps to estimate the relationship between a two-or-more point shift in a given year and the size and direction of changes in reported satisfaction in subsequent years. For the purposes of this exercise, which is to model rather than to demonstrate the existence of mean reversion, we have looked only at reported SWB and have *not* attempted to estimate later responses while holding constant covariates which might serve as the objective determinants of utility, though of course the use of panel data ensures that the individuals in the sample and their immutable characteristics remain constant. As such, the identified responses to a shock are evidence of mean reversion only to the extent that changes

in the arguments of the utility function are either uncorrelated or positively correlated. That is: if most shocks to utility tend to predict further shocks in the same direction, as health continues to decline or income continues to rise, for example, then any observed mean reversion in the year following a shock will understate the degree of change to the utility function itself. In situations where the objective consequences of shocks are permanent, such as the death of a relative, incurring a permanent disability or the separation of a marriage, any measured systematic change in utility in subsequent years will be entirely due to mean reversion. If, however, most shocks to utility are transitory – like a brief period of unemployment or becoming a random victim of crime for instance – so that the individual’s material circumstances tend to return to pre-shock levels, then observed reversion in those instances will reflect only a change in the determinants of utility, not a change to the utility function itself.

As a result of this ambiguity, we do not set out here to make the case for the *existence* of mean reversion, an argument which has been made elsewhere in great detail.¹⁵ In citing observed subsequent changes in utility as “reversion” we are implicitly assuming that the arguments of utility follow something close to a random walk, so that an individual’s utility in a given year is, absent the effects of mean reversion, the best predictor of their utility the following year. Readers who have strong views as to the likely positive or negative correlations between utility shocks will wish to adjust the stated estimates for the speed of reversion up or down accordingly. Alternatively, readers may simply choose to view the estimates presented below as an objective summary of how experienced utility changes through time following a shock, without seeking to ascribe the causes of that response to either mean reversion or negatively correlated shocks.

Results

We observed a strong one year mean reversion effect in response to both positive and negative utility shocks of two units or greater. In the year following a positive shock the average individual lost 35.05% (SE 1.04%) of the increase in the following year. In the case of negative shocks, the average regained was 53.82% (SE 1.06%) of the initial loss. Overall, an average 44.55% (SE 0.75%) of the absolute value of the initial shock disappeared during the subsequent year. Subject to the caveats regarding potentially correlated shocks above, this overall average reversion rate of approximately 45% equates to a value of α of 0.45 in the model of mean reversion outlined in 2.4.3, above, for values of t equal to one year.

¹⁵See, for example, Layard and Layard (2005) and Easterlin (2005) for a general survey of the literature on this subject.

We then examined the evidence for a longer lag structure by looking at the average change in utility between the first and second year following a two-or-more unit utility shock. This revealed no statistically significant evidence for further mean reversion after the first year: the average response to a shock was 1.53% (SE 0.88%). While this estimate is signed consistent with further mean reversion it is not significantly different from zero. There is insufficient evidence to conclude that further mean reversion is occurring more than one year after the initial shock. This may mean that most or all of the reversion from a utility shock occurs during the first year following the shock, or it may mean that the 1 to 7 scale used to capture overall satisfaction in the data set is insufficiently fine-grained to detect the smaller amount of reversion predicted after the first year.

Strictly, this evidence is not consistent with the model presented in 2.4.3, which assumes a geometric decline in rate of reversion. This may reflect mean reversion over a time scale shorter than that captured by the available data, it may demonstrate that mean reversion occurs incompletely, or it may simply be the random changes in utility render smaller reversion undetectable.

Given the absence of mean reversion at the two year mark we did not attempt to estimate deeper lag structures.

Having found evidence of significant mean reversion, we then repeated this analysis after excluding shocks immediately following another shock – that is, shocks which themselves were, *ex hypothesi*, likely to be the result of mean reversion. The adjusted results were similar but, as would have been predicted, found slightly higher levels of mean reversion:

Ignoring mean reversion shocks, in the year following a shock the average individual reverted 51.67% (SE 0.89%) of the absolute value of the initial shock during the subsequent year. As with the unadjusted data, no significant two year lagged effects were detected in this data set.

We also considered the applicability of non-linear reversion models to the observed data. We examined the correlation between the one year reversion and shock size, square of shock size and square root of shock size, and performed a linear regression of reversion against each transformation of shock size and a Ramsey reset test on the linear specification.

The highest level of correlation (0.6948) was found between reversion and the linear measure of shock size. A linear regression of reversion against shocks yielded an R^2 of 0.4828, and the following estimated relationship (t values in brackets):

$$Reversion = -0.0047(3.88) - 0.54(390)shock \quad (2.16)$$

Which is broadly consistent with the adjusted average level of reversion of 52% calculated above. A Ramsey reset test showed no omitted variables, meaning that we did not estimate a function of multiple powers of the shock term simultaneously.

A regression of reversion against the square of the shock yields a lower R^2 of 0.402 and:

$$Reversion = -0.0397(31.04) - 0.138(331.39)shock^2 \quad (2.17)$$

While regressing reversion against square root of the shock gives R^2 of 0.4828, identical to the regression against untransformed shock, and:

$$Reversion = 0.002(-1.73) - 0.923(390)shock^{\frac{1}{2}} \quad (2.18)$$

Analysis of Results

We find significant evidence of a strong, linear relationship between large shocks to utility and the degree of reversion towards pre-shock utility the following year. If shocks to utility are assumed to be uncorrelated on average, this is evidence of individuals experiencing mean reversion in experienced utility consistent with the models derived above.

One curious feature of these results is the significant asymmetry between reversion arising from positive and negative utility shocks. We found significantly higher average reversion for negative shocks, an outcome which is not predicted by the stylised models considered above. While this asymmetry may arise only because negative shocks are more likely to be negatively correlated than positive shocks, there may also be value in additional modelling of mean reversion which is designed to provide an evolutionary justification for differential rates of reversion to positive and negative changes in the individuals' circumstances.

We note that throughout this section we have ignored our own admissions from chapter one and treated SWB reports obtained by the BHPS as interval scaled data capable of generating arithmetic means.¹⁶ If there are significant non-linearities in how individuals distribute utilities across the seven point scale the results above will be misleading. In Section 2.5.5,

¹⁶Ordered probit regressions were also conducted and provided results of consistent sign and significance to the OLS results reported here.

below, we re-derive the estimates of reversion speed based on a rescaling of the BHPS designed to remove the estimated effects of non-linear mapping from utility to reported SWB.

2.4.5 Evolution and Mean Reversion

In this section will undertake a qualitative analysis of the preference structures identified in the BHPS data, and their likelihood of generating evolutionarily efficient behaviour.

The consequences of the above analysis for the evolutionary approach to deriving utility functions depend on whether individuals are able to foresee the consequences of mean reversion when optimising their behaviour. In the language of the formal model above, it turns on whether individuals maximise the total value of U , or of U^R . For the purposes of this section we will assume that evolution has the ability to “dictate” which of these individuals maximise, but that there may be constraints on evolution’s ability to require agents to forget their recent past without imposing other, undesirable cognitive impairments on them.

Assuming that individuals are ‘reversion-aware’, we find that individuals who lack a positive pure rate of time preference will have no incentive to pursue increases in utility in circumstances where the source of the increase is potentially transient. That is, a reversion-aware individual will be indifferent between improving any aspect of their utility function and remaining at equilibrium, unless their positive rate of time preference causes them to reduce the weight placed on future periods in which the mean-reversion function causes their utility to ‘overshoot’.

Obviously, a utility function which fails to motivate its subject to pursue even short-term improvements in health and wealth is not likely to be fitness-maximising. On this view, we can see a pure rate of time preference as a means of removing the distorting effects of reversion from an individual’s utilitarian calculus. By weighing the future less heavily – a workaround which introduces distortions of its own – evolution is able to retain the advantage of a mean-reverting utility function (as demonstrated in 2.3.1, above) while retaining in its agents the motivation to pursue short-term increases in utility.

If we assume, on the other hand, that individuals are ‘reversion-blind’ – that they attempt to maximise U^R while experiencing U – we require an alternative explanation for pure time preference. Individuals who ignore the effects of reversion on their future utility will act optimally, from an evolutionary point of view, in pursuing temporary gains in utility, even

in the absence of a positive rate of time preference. However, from the individual's own point of view, they will, in a certain sense (outlined in 2.4.6, below) 'overinvest' in increasing facets of utility which will experience mean reversion. A pure rate of time preference, though, allows a reversion-blind individual to simulate the effect of reversion-awareness, by correctly predicting that future increases in utility will be less valuable than their associated increase in raw utility would suggest.

These two observations suggest that time preference arises as part of a struggle between evolution, which requires a flexible, mean-reverting utility function to respond to changing payoff distributions in nature, and rational, forward-looking humans. Nature, optimally, would seek to design a mean-reverting utility function but to ensure that the individuals influenced by that function are not aware of its future effects. But the reason evolution has adopted an incentive structure rather than simply programming humans to choose optimal courses of action is that it also wishes to take advantage of humans' ability to make rational judgements, particularly in relation to future states of the world.

It seems reasonable to suggest that sufficiently rapid mean reversion, whereby experienced utility returned to its constant baseline value days or even hours after a change in circumstances, would be inconsistent with reversion-blindness for any sufficiently rational and forward-looking individual. People might be persuaded to chase fortunes or avoid injuries to which they will ultimately become habituated, but it seems unlikely that this motivation would persist if they were constantly and immediately confronted with the failure of their achievements to deliver any net increase in utility.¹⁷

We might imagine, then, that evolution is constrained as to the maximum rate of reversion (α , in the argot of the formal model, above) it can adopt consistent with retaining reversion blindness in its otherwise rational agents. We might further suggest that reversion blindness and reversion awareness represent a continuum, rather than a dichotomy; to the extent that the rate of reversion represents a compromise between the rate of change in nature's payoffs and the need to conceal the process from individuals who would be demotivated by it, the optimum may be an 'interior solution' where individuals are partially but incompletely aware of the level of mean reversion they will experience.

In treating the rate of mean reversion and individuals' ability to foresee it as the result of an evolutionary optimisation, weighing time horizon against adaptability, the pure rate of time preference arises as filling a key balancing role. As demonstrated above, a positive time preference tends to

¹⁷Though the more cynical among the proponents of SWB suggest that exactly this kind of self-deception does occur, albeit according to a slightly extended timetable.

counterbalance the effects of reversion awareness, by causing individuals to undervalue the future periods in which reversion will have occurred relative to the present. Similarly, in the presence of reversion blindness, positive time preference will tend to simulate the effect of habituation awareness in weighing future utility profiles. In fact, as noted above, reversion blind individuals with a rate of time preference equal to their rate of (unobserved) reversion will predict permanent changes in raw utility consistently with how they experience them.

What we have, then, is perhaps analogous to an inflationary expectations model: mean reversion is a valuable optimising tool from the point of view of the central body, in this instance evolution, but, like inflation, it remains effective only to the extent that agents do not fully anticipate it. Time preference, then, enhances the credibility of future promises of zero reversion. First, by ensuring that individuals with nonzero reversion expectations behave consistently with evolution's preferences, and second, by ensuring that, to the extent reversion is unanticipated, individuals experience less expectation-increasing dissonance between their choices and their outcomes.

Levinson (2013), provides an empirical analysis of the tradeoff between mean reversion (which the author refers to as 'habituation') and a particular form of myopia (focussing on 'projection bias, the tendency to incorrectly believe that transitory circumstances will remain permanent'). Levinson argues that the existence of this 'projection bias' prevents survey respondents from properly predicting the future effect of mean reversion on changes in their circumstances, enabling SWB surveys to detect the effect on utility of habituable changes.

We will conclude by pointing out that these conjectures link to the practical difficulties in detecting and measuring mean reversion, discussed above in 2.4; if the rate of reversion has been judged so that full reversion occurs, on average, less frequently than changes in circumstances, then we would not expect the optimum rate of reversion to yield constant reported utility in practice.

2.4.6 Normative Consequences of Mean Reversion

We have argued above, in 1.5, that a benevolent social planner should attempt to maximise experienced utility, rather than the measure of utility used by individuals to weigh their decisions. Accepting, for the moment, that this is the case, the existence of set-points in utility, and the linked role played by time preference in the model derived above, have significant consequences for normative measurement of utility.

The first point to note is that a reversion blind individual will systematically make choices which yield less experienced utility than predicted and, under the specific model examined above, any transitory change in utility will in fact be exactly counterbalanced by future reversion.

Now, provided that reversion is a universal phenomenon, both in the sense that it applies to all persons and that it applies to all determinants of utility, this is dispiriting, but not normatively-significant news. If all peoples' gains tend to disappear, then the role of a social planner is simply to maximise those transient gains, which is equivalent to maximising non-reverting utility, U^R , as it is referred-to above.

There is, on this view, no basis for the argument traditionally made by some SWB advocates, that mean reversion justifies a tax on labour income to discourage natural overinvestment in consumption goods, unless it can be shown that the substitutes for consumption, most obviously leisure, are not themselves subject to habituation.¹⁸ A strong set-point theory actually makes a social planner's life much easier, as any set of policies is consistent with a roughly equal long-term level of social welfare.¹⁹

There is, however, at least one significant aspect of utility which we do not consciously experience for sufficiently long to become habituated to it; namely, death. When we spoke of 'overinvestment' in consumption, above, at least one variable we can irrationally trade out of is expected years of life. Because we incorrectly predict that the increased consumption associated with a higher wage will be permanently rewarding, we will be willing to forgo, in expectation, a larger number of years of life in order to achieve it; but we do not, of course, habituate to being dead.²⁰

This effect will ensure that, *ceteris paribus*, wage premia for dangerous jobs are too low, from the point of view of an experienced-utility-maximising social planner. Likewise, we will tend to be too willing to adopt life-shortening unhealthy behaviours, the pleasure of which we will rapidly become acclimated to.

We can note, however, that, to the extent reversion blind individuals

¹⁸Layard and Layard (2005) provide a survey of their work on this area, and make the distinct argument that leisure, unlike income, is not subject to peer-comparison effects. We do not address that argument here.

¹⁹We are assuming, for the purposes of this argument, that the social planner is strictly utilitarian and does not include the interests of non-humans in the utilitarian calculus except in as much as they are valued by humans themselves. It may otherwise be the case that the existence of reversion calls for the expenditure of additional resources on preserving the welfare of plants and/or animals.

²⁰This assumes that equilibrium utility is greater than zero – that is, that expected total lifetime utility is increasing in life expectancy. This appears to be a reasonable assumption.

tend to project into the future increases in utility which will not persist, this effect will cause them to incorrectly estimate the level of utility associated with additional years of life. Where the shocks incorporated in estimating end-of-life utility are on average positive (rising income, for instance) this effect will tend to counteract the bias against extending expected lifetime established above. The net effect will depend on the precise size of the shock in question and the structure of the individual's utility function. Conversely, if the shocks considered are negative in nature (ill health, for example), then reversion blindness will cause individuals to further underestimate the relative value of extending their lifespan, as they fail to correctly anticipate their ability to habituate to sickness and disability.

This effect reopens the role for the social planner, who may be able to productively intervene to correct this undervaluing of inhabitable experiences. The available evidence suggests²¹ that it is not only death which resists habituation. Certain serious emotional traumas, like the death of an immediate family member, on the downside, and increases in relative status, on the up, appear to be source of lasting sorrow and joy, respectively.

We might speculate, in the context of the evolutionary principal/agent model set out above, that these kinds of shocks are excluded from the normal operation of mean reversion because they are either particularly significant or remain easy to objectively value regardless of circumstances. That is, while evolution may not be able to dictate an acceptable number of calories or sexual partners relevant to all states of the world, it may be comfortable dictating that the death of a close genetic relative is an *absolute* bad, to be avoided at all costs in all states of nature. Similarly, since status is inherently a relative ranking, very high status can be viewed as an objective good in any state of the world.

Regardless of whether we accept the above speculation as to the causes of certain outcomes failing to revert, provided we accept their existence we can view any such outcomes as potentially permitting utility-increasing intervention by a social planner so as to increase their perceived relative value.

Finally we will consider the alternative argument that a benevolent social planner ought to maximise evolutionary, rather than experienced utility. On this view, mean reversion is a necessary evil, required in order for the 'principal' to calibrate an 'agent's' incentive function to the quality of the times. A social planner who attempted to step into evolution's shoes, rather than those of the individuals themselves, would therefore seek to maximise raw utility, $\sum_{t=0}^{\infty} U_t^R$, rather than experienced utility $\sum_{t=0}^{\infty} U_t$. This is equivalent to attempting to incentivise the population to act as if they were

²¹See Easterlin (2006).

entirely reversion-blind. In any case, the notion of an abstract concept like evolution, rather than individuals themselves, as the sole subject of the utilitarian calculus smacks of an unpleasant form of social Darwinism, and we will not pursue it beyond this illustrative example.

2.4.7 Normative Consequences of Time Preference

We turn now to the question of how a benevolent social planner should treat the influence of positive time preference on individuals' choices.

In the model described above, a positive time preference arises as an evolutionary workaround for:

1. the tendency of reversion-aware individuals to fail to pursue opportunities for temporary increases in utility; and
2. the tendency of reversion-blind individuals to become reversion-aware when expected utility gains fail to eventuate.

Depending on the optimal level of reversion awareness, based on some tradeoff between the costs of suboptimal behaviour created by reversion awareness and the cognitive limits on foresight required to maintain reversion blindness, one of these motives will predominate.

If we set aside the small level of positive time preference generated rationally in response to the possibility of death prior to realising the predicted benefit, time preference, in this model is no more 'real' than mean-reversion. The desire to experience a benefit today, rather than tomorrow, setting aside the tiny possibility that one will die tonight, is simply a cognitive quirk designed as a response to another, larger such quirk.

On this basis, a benevolent social planner should treat a reduced weighing on future utility identically to the related phenomenon of overvaluing future benefits likely, by then, to have been habituated to. Such a social planner, provided they have been persuaded (perhaps by 1.5, above) that their role is to maximise experienced utility, even where that measure differs from the predicted utility used by individuals to rank choices, should ignore time preference when calculating the present value of utility flows through time.

2.4.8 Nonlinear Mean Reversion and Utility Flows

The above discussion of normative consequences of mean reversion is based on the model specified in 2.4.3, above. In that model, the rate of

reversion is given by

$$\bar{U}_t = \bar{U}_{t-1} + \alpha(U_{t-1} - \bar{U}_{t-1}) \quad (2.19)$$

where α is a constant between 0 and 1, so that the rate of reversion is a linear function of the difference between set-point utility and raw utility. In 2.4.4, above, we found that this specification is broadly consistent with empirical estimates derived from the British Household Panel Survey.

It is a consequence of this, precise, specification that individuals are indifferent as between equally-sized total changes to utility distributed differently through time.

Without attempting to replicate the analysis in 2.4.3 for an entirely new specification, we will note that, were the value of α itself an increasing function of the gap between equilibrium and raw utility (in a manner consistent with the geometric growth of reversion speed predicted in 2.3.1), we would experience nonlinear rates of reversion. This would mean that the ultimate returns to equally-sized changes to utility would depend on their distribution through time, in a manner not properly anticipated by a reversion blind, or partially reversion blind individual.

Consider, for instance, a one-period 100 unit increase in raw utility, as against a 10-period, 10 unit increase – which, assuming constant marginal utility of income, we could imagine as a decision as to the consumption of a bequest. In the case where α is a constant, these two strategies lead to identical experienced outcomes. However, when α is increasing in the deviation from the set-point, the 100-unit increase will lead to faster proportional reversion, and a larger ‘overshoot’ and subsequent period of below-equilibrium utility than the 10 by 10 case, while a reversion blind individual will be indifferent between the two strategies.²²

In the presence of non-linear mean reversion,²³ then, there will be welfare gains available from smoothing changes in utility through time, to an even greater extent than dictated by either diminishing marginal utility of income or the ‘peak-aversion’ introduced in chapter three, below – recall that this was exactly the relationship between mean reversion and time preference derived from the Becker/Robson model in section 2.4.1, above. The policy mechanisms for encouraging such smoothing are probably similar to those used to combat myopic under-saving – such as subsidies to saving and restrictions on or disincentives to the early withdrawal of funds – but directed at smoothing changes to utility rather than delaying them.

²²The reverse argument holds where α is decreasing in the size of the utility gap, but such a model holds less intuitive appeal.

²³Which is positively correlated with the size of the utility gap.

2.5 Application of Restricted Underlying Utility Models to SWB

There are obvious similarities between the problem faced by evolution in the restricted utility models considered above and the problem faced by an individual who is asked to report their subjective wellbeing according to a predetermined scale. In each case, a continuous, unbounded distribution is being mapped to a discrete, bounded scale, requiring the generation of a mapping such that, when unlike circumstances are reported identically, such reporting minimises some measure of ‘error’.

We will use this similarity as the basis for a formal consideration of the individual’s mapping problem in reporting their subjective wellbeing.

We note that we will not directly attempt to incorporate the predicted results from the evolutionary utility models – that utility is in fact likely to be discontinuous and bounded due to limited neurological resources – into the modelling in this section. While the above results strongly suggest that the number of discernable utility states is finite, it remains likely that the number is very, very large relative to the numbers of distinct states captured by SWB surveys. For the purposes of this analysis we will treat utility as functionally continuous, while recognising that this is a slight simplification of reality.

2.5.1 The Individual’s Mapping Problem

At this point, we move away from our discussion as to the nature of underlying utility, and turn more directly to the question of how individuals map from their utility to reported subjective wellbeing - how they allocate a particular level of utility to a particular point on the SWB scale.

Traditionally, studies of subjective wellbeing have either assumed that reported subjective wellbeing, whether solicited according to a qualitative or quantitative scale, followed a linear, cardinal scale. More recently, practitioners have begun using maximum likelihood methods to avoid the question entirely. Oswald (2005) directly raises the issue of the linearity of mapping from utility to subjective wellbeing, and Layard, Mayraz, and Nickell (2008) attempt to determine whether there is “compression of the reported happiness scale” by assuming that utility is linear in income and then comparing the income-to-SWB mapping predicted under this assumption to that disclosed by the data. The linearity assumption is an obvious difficulty with this approach, and one that is unavoidable when attempting to determine the mapping from an unobservable variable to an observed one.

In Blanchflower and Oswald (2004) the authors propose a relationship between reported ordinal SWB, r and utility, u based on the equation:

$$r = h(u(y, z, t)) + e \tag{2.20}$$

Where utility is determined by y , z and t , being income, personal characteristics and time, respectively, h is a continuous, non-differentiable function mapping utility to reported wellbeing and e is an error term reflecting imperfect individual knowledge of utility or imperfect application of the mapping function. Modelling data using this approach, without direct knowledge of the mapping function, h , requires that reported SWB values be treated ordinally.

While it is preferable to simply assuming a linear, cardinal scale, ordinal approaches remains imperfect; to the extent that differences in reported SWB values encode cardinal utility information from the individual, treating them as ordered categories leads researchers to discard valuable data. We demonstrate below that there is clear evidence that individuals do not encode SWB values ordinally, which means that there is value in attempting to model what they *do* mean by particular values.

We will take a different approach to this purely ordinal one, attempting to determine the optimal structure of h from the individual's point of view – essentially treating utility revelation via reported SWB as a kind of cooperative signalling game. We will draw on the technical similarities between an individual constructing a SWB mapping and evolution's task, in the restricted utility models outlined above, of mapping from evolutionary payoffs to utility.

The focus of our analysis is an individual who is asked to report their subjective wellbeing according to a bounded, discrete, quantitative scale.²⁴ Initially, and contrary to the restricted utility models discussed above, we will assume that underlying utility is both continuous and unbounded.

We will further assume, consistent with the evidence as to the robustness of SWB data in chapter one, above, that the individual surveyed treats the request to reveal their subjective wellbeing seriously and wishes to provide an accurate, error-minimising assessment of their true underlying hedonic state – that they generate a coherent mapping scheme, h .

This assumption means that we reject the possibility that the initial SWB assessment provided by an individual is defined in arbitrary relation to underlying utility. That is: we ignore the possibility that an individual's

²⁴This approach can easily be generalised to include ordinal qualitative scales such as “Not at all happy, happy, very happy”.

initial wellbeing is placed at an arbitrary point on the proffered scale and that mapping between utility and reported SWB is generated on an ad hoc basis only in response to any subsequent enquiries. We also assume that the individual understands a question about their overall satisfaction or subjective wellbeing as a question about utility, as a benevolent social planner would conceive of it. See chapter one for a detailed discussion as to whether this assumption is likely to hold in practice.

As such, we will proceed from the assumption that an individual asked to report their SWB on an arbitrary scale will respond to the question by first constructing a complete mapping between underlying utility and reported SWB on the appropriate scale.

The individual's goal in constructing such a mapping is, like that of evolution in the similar model considered above, to display an unbounded, continuous variable on a bounded, discrete scale in such a way as to minimise the conflation of cardinaly unlike states. This is, in effect, a cooperative signalling game, whereby one player, the individual surveyed, tries to signal to another, the surveyor, the true value of a variable using a restricted-value proxy.

A mapping will be said to generate error, then, when it would be expected to lead the individual, when surveyed multiple times, to report identical SWB values for differing levels of utility, and, as with the evolutionary model above, the size of those errors will depend linearly on the underlying cardinal differences between the states assigned the same SWB value.

2.5.2 Mapping from an Unbounded Scale

The first, general observation we can make is that, where utility is unbounded in at least one direction, a linear mapping from utility to SWB is impossible.²⁵ We can be certain that, in the presence of unbounded utility, the level of reported SWB closest to the bound must cover an infinite range of utility values, unless part of the potential utility range is being excluded from the mapping.

It may be the case that, while potential utility is unbounded (or unbounded above), the individual assigns a probability of zero to utilities above a certain value – it may always be possible to make one happier than one is today, but at a certain point one can confidently predict that the required resources will not be made available. This links to our discussion of whether a mapping from utility to SWB can be expected to remain constant, in 2.5.8

²⁵Except for the special case where there are only two SWB categories and utility is unbounded at both ends.

below.

2.5.3 Mapping Where Utility is Ordinal

The second preliminary observation we can make deals with cases where underlying utility is, itself, perceived ordinally by the individual constructing the mapping. In this case, the problem faced by the individual constructing the mapping is relatively simple – with no cardinal distinctions between utility values to preserve, so that equal error is generated by the conflation of any two neighboring utility values, they will simply generate a mapping so as to maximise the number of pairwise comparisons possible between utility values. This is equivalent to minimising the maximum expected number of SWB observations in any single category. In the case where future utility follows a distribution known to the individual, the mapping will simply divide the distribution function into n equal intervals, where n is the number of SWB values provided by the survey in question.

We can reverse the above logic and note that, if individuals *did* experience utility ordinally, their mappings from utility to subjective wellbeing should show each available response with equal frequency. Since equal rates of each response on the reported scale are not, in fact, observed in practice,²⁶ we can conclude either that utility is not perceived ordinally by most respondents – they are trying to capture something of the cardinal degree of difference between utility states through their mapping function.

2.5.4 Mapping Where Utility is Cardinal

Having concluded that utility is not being perceived ordinally, we now consider the cardinal utility case. In this instance, the individual is not trying simply to ensure the maximum possible number of pairwise distinctions, as in the ordinal case, above. Rather, the individual will try to minimise the total size of the errors generated when unlike utility values are reported as equal levels of subjective wellbeing.

Taking, as an example, the case of a three unit SWB response scale, letting x_1 and x_2 represent two randomly selected utility values, and letting c_1 and c_2 represent the cutoffs between SWB of 1 and 2 and SWB of 2 and 3 respectively, then, assuming we aim to minimise the sum of errors, rather than the sum of their squares, we have an error function equal to:

²⁶We show below that the BHPS satisfaction data shows significant left skew, for instance

$$\begin{aligned}
e = & p(x < c_1)^2 |E[x_1 - x_2 | x_1, x_2 < c_1]| \\
& + (p(x < c_2) - p(x < c_1))^2 |E[x_1 - x_2 | c_2 > x_1, x_2 > c_1]| \\
& + (1 - p(x < c_2))^2 |E[x_1 - x_2 | x_1, x_2 > c_2]|
\end{aligned}$$

which, in words, represents the possibility of two utility values, randomly selected from their distribution, lying in the same SWB category weighed by the absolute value of the expected difference between the two utility values, given that they lie in the same SWB range.

This minimisation is mathematically identical to the problem faced by evolution in setting the optimal mapping from payoffs to utility in the restricted utility models considered above.²⁷ Consistent with result derived by Robson (2001), the optimal position for the cutoffs will be given by dividing the cumulative distribution function, F , weighed by the x -values, by the number of available thresholds.

To see how this would work in practice, consider the observed low frequency of SWB reported at the extremes of any scale. In the British Household Panel Survey data considered in section 2.4.4, above, for example, only 1.41% of reported subjective wellbeing values are 1 on the 1 to 7 scale. Smith (1979) notes that this is a general phenomenon, particularly in relation to values at the bottom of the scale, and that even adding an additional negative value to the bottom of the scale does not tend to reduce the mean value of responses, as individuals try to ‘reserve’ the bottom value for (cardinally) exceptional circumstances. It is not clear why this effect is not as strong at the top of the scale (though it is still present) as at the bottom. It is possible that utility is viewed as being unbounded below but not above.

In a cooperative signaling game, mathematically analogous to the principle agent model solved by Robson, the signalling party’s decision to reserve a portion of their signal spectrum for a rarely encountered phenomenon must imply that that phenomenon, when it does occur, is of great cardinal significance, so that minimising the expected *size* of errors requires that it be assigned to a distinct signal, even though this increases the expected *probability* of errors, relative to dividing utility into equal frequency buckets, by condensing the remaining utility values into fewer points on the SWB scale.

Counterintuitively, then, in order to minimise the size of expected signalling errors, the SWB categories least often reported in surveys must occupy more cardinal utility space than those reported more frequently. In

²⁷The key difference is that, where, in the evolutionary case, errors only arise in case of an equal ranking with a probability of $\frac{1}{2}$, as the correct option may still be chosen, in the SWB case any conflation of unlike states is automatically treated as harmful.

fact, consistent with the optimisation result borrowed from Robson, values on an individual’s SWB mapping must cover cardinal utility space *in exact inverse proportion to the frequency with which they appear*. So, returning to the example of the BHPS data, if the response “1” occurs 23 times less frequently than the most common response (“6”) then, in order for the utility mapping to be optimal, the range of utility values covered by “1” must be 23 times as large as that covered by “6”. In general, for any two neighbouring reported SWB values, A and B , optimally assigned to an individual’s range of utilities with a cut-point of c it must be the case that:

$$P(U \in A) \cdot E[c - U \mid U \in A] = P(U \in B) \cdot E[U - c \mid U \in B] \quad (2.21)$$

As a result, we can determine the proportion of cardinal utility space covered by SWB values simply by taking the inverse of their frequency. This enables us to test the assumption that SWB values are reported in a linear scale – this will only be the case if the frequency of different values is approximately equal over a large sample – and, more importantly, it enables us to rescale the reported values to achieve a ratio scale recreation of an individual’s underlying utility. As long as individuals across the sample are (or can be assumed to be, see chapter one) attempting to accurately depict similar underlying utility functions in an error-minimising fashion, we can directly determine the true interval between any two SWB values based on their relative frequencies in the data set.

We demonstrate an empirical application of this technique below.

2.5.5 Empirical Examples of Linearising Wellbeing Data

Example one: Reexamining mean reversion in the BHPS data

We return to the British Household Panel Survey data analysed in relation to mean reversion in section 2.4.4, above. As noted above, this estimation was conducted on the assumption that utility values were linearly distributed in expectation across the seven-point SWB scale – that is, that a move from a score of “2” to a score of “4” was exactly as large as a move from “4” to “6”. We now have the means to progress beyond this assumption, and to rescale the reported data to reflect the actual estimated interval utility gaps between individual scores, subject to the (strong) assumption that observed frequency distributions across the entire sample are a suitable proxy for individual frequency distributions. Ideally we would undertake this exercise using an individual’s own observed response frequencies, but

this would require either a much longer timescale or much more frequent sampling than the available data permit.

Drawing on the technique outlined immediately above, we first measure the relative frequency of each response across the data set (in this instance we have not excluded values provided by individuals who did not provide overall satisfaction estimates in all seven years, so our sample is slightly different to the one used to estimate mean reversion, but we do not expect there to be any significant bias):

1. is reported 1.41% of the time
2. is reported 2.11% of the time
3. is reported 6.05% of the time
4. is reported 13.65% of the time
5. is reported 29.67% of the time
6. is reported 33.32% of the time
7. is reported 13.79% of the time

Normalising the range of underlying utility to match the 1 to 7 scale of the data and calculating the portion of this scale covered by each value based on its inverse frequency gives:

1. covers utility values between 1 and 3.73
2. covers utility values between 3.73 and 5.55
3. covers utility values between 5.55 and 6.19
4. covers utility values between 6.19 and 6.47
5. covers utility values between 6.47 and 6.60
6. covers utility values between 6.60 and 6.72
7. covers utility values between 6.72 and 7

With expected utility values given that score is reported lying at the mid point of each range.

Having converted these SWB values to ordinal data we can now repeat the analysis from 2.4.4, above, using the differences between the estimated

true utility values associated with each SWB score, rather than assuming linearity. So the gap in expected utility between a satisfaction score of 1 and a score of 2 becomes 2.275, while the gap between 5 and 6 is 0.123.

We then repeated the analyses conducted above using the cardinalised estimates shocks. In order to retain a comparable sample size, the arbitrary cutoff for a significant utility shock was changed from 1 in the previous model to 0.32, reflecting the greater compression of values on the ordinalised scale. The results for the one year speed of linear regression, α , were slightly different from those derived under the linearity assumption: the overall estimate of α for all shocks other than those immediately following an earlier shock fell from 51.67% to 48.513% (SE 1.61%), while the comparable figures for positive only shocks were 35.05% in the original and 51.87% (SE 2.67%) in the updated model, with negative shock adjustment changing from 53.82% to 47.02% (SE 2.19%). Second year reversion remained insignificant for both positive and negative shocks.

We note, then, that the initially identified bias toward recovery from negative shocks has been reversed with the corrected data, though the corrected difference between positive and negative recovery rates is not statistically significant at the 5% level. The apparent increased recovery rate was simply an artifact of ignoring the left skew of the optimal mapping of utility to the SWB scale and in fact we appear to have some weak evidence of the opposite relationship – speedier adjustment to positive shocks than to negative ones – once we take into account the actual expected utility gaps between different satisfaction scores.

Example two: Income effects using cardinalised SWB data

Powdthavee (2008) considers the effect of differences in annual income on reported subjective wellbeing in the British Household Panel Survey. While this analysis, being intended as a proof of concept rather than the final word on income and SWB, does not attempt to replicate every portion of the author’s approach here,²⁸ we do hope to demonstrate the potential value of having access to cardinal rather than ordinal SWB measures via rescaling the BHPS data to reflect expected cardinal differences between neighbouring SWB values.

We note initially that cross-sectional observation of the relationship between income and SWB in the presence of mean reversion will tend to understate the short term effect on an individual of a change in their income. See Powdthavee (2010) for a useful survey of the potential biases in SWB-

²⁸In particular, we do not control for known determinants of SWB which may also correlate with income, particularly gender and health.

based estimates of the marginal utility of income, including the potential for reverse causation, where high SWB leads to high income.

For our purposes we will consider monthly total net income from all sources from wave L of the BHPS.

Treating reported overall life satisfaction as an ordinal variable, we performed an ordered logit regression of life satisfaction against the natural log of income. The result is a statistically significant negative relationship between income and overall satisfaction (the coefficient on the log of income was -0.074 with a standard error of 0.14), which fails to accord with our expectation that income will relate positively to SWB.

Once we rescale the overall satisfaction data, based on the observed frequencies of the various points on the scale, we can legitimately treat the resulting values as lying along an interval scale. This enables us to perform an ordinary least squares regression of rescaled satisfaction against income in thousands of pounds.²⁹ This regression returned a small but positive and statistically significant relationship between net earnings and overall satisfaction, with an extra thousand pounds of income increasing expected subjective wellbeing by 0.0216 on a 1 to 7 scale (SE 0.00041).

We can conclude, then, that recovering information about the cardinal relationships between points on a reported SWB scale improves our ability to correctly identify the determinants of subjective wellbeing.

2.5.6 Recovering cardinal SWB data: conclusions

If SWB data is being encoded to reflect cardinal differences in utility by individuals, then recovering and rescaling SWB data using an estimate of those cardinal differences is unambiguously superior to treating the data ordinally or assuming it reflects a linear, cardinal mapping from utility.

The fact that populations report different levels of SWB with different frequency demonstrates cardinal differences in what they intend to signal via each point on the scale. The inverse frequency technique demonstrated above should be used to rescale subjective wellbeing data prior to conducting statistical analysis.

²⁹We also tested the relationship between SWB and log of income, but found a better fit with the linear relationship.

2.5.7 Mapping in the Absence of a Distribution Function

In the discussion above we have assumed that an individual tasked with constructing an optimal, forward-looking mapping has perfect knowledge of the distribution describing their utility levels through time. This assumption requires that an individual infer a distribution from a finite number of observed signals, a difficult task, particularly when that distribution will frequently shift in response to changed circumstances. That is, when utility is viewed time-separably, many utility influencing events, such as a pay rise, will permanently shift the expected future distribution of utility values. We could instead treat these events as random draws from a single distribution of lifetime utility distributions, in which case individuals would be attempting to infer a distribution from a single, partially observed outcome. As such, the known distribution assumption is likely, in most cases, to be unreasonably stringent.

In the absence of (formal) distributional information, how might an individual go about predicting their future utility in order to construct an optimal mapping? We suggest a number of potential options:

1. assume future utility will be (normally, uniformly) randomly distributed about current utility;
2. assume the distribution of future utility will mirror the discrete probability distribution implied by the finite number of remembered past utility values;
3. assume the distribution of future utility will follow the (linear) trend implied by the finite number of remembered past utility values; and
4. assume the distribution of future utility will follow the (linear) trend or discrete probability distribution implied by the finite number of remembered past utility values, weighed so as to prioritise more recent events.

Consider, by way of example, an individual who has experienced sequential time-separable instant utility values of 5, 10 and 20, and is asked to construct a mapping onto a two-value SWB scale.

In case 1, the individual assumes that utility will follow a random-walk about its current value, 20, and will assign the cutoff between SWB=1 and SBW=2 at utility=20 (and randomising to determine current SWB).

In case 2, the individual assumes that the distribution of remembered past utility reflects its expected future distribution and that each of the three

values is equally likely to reoccur in the future. Expected error is minimised by setting the cutoff at the mean of the discrete distribution, 11.67.

In case 3, the individual treats the utility values as a continuing trend. They will predict that utility will continue to increase by 7.5 units per period, and set the threshold at some value >20 based on period length and time horizon.

In case 4, the individual weighs the current value more heavily than the past value in constructing either a trend or discrete distribution. If we arbitrarily set this weighing so that more recent information is twice as important as less recent information, we will have a linear trend equal to 8.33, raising time horizon and period length questions as above, or a distribution with a mean of 15.

The individual will be able to refine their predictions as to the future behaviour of utility, and hence their mapping, as they experience new utility values. To the extent these utility values continue to follow a fixed distribution, mappings constructed according to case 2 will dominate. To the extent the distribution is subjected to correlated shocks, cases 3 and 4 will produce more accurate results. Which of these scenarios is more accurate will depend heavily on the time-horizon over which utility is evaluated. That is, multiple utility values observed over the course of a single day are much more likely to describe a random walk or fixed distribution, while annual utility values will tend to follow a trend.

We can, based on the available data, discard case 1 as a commonly adopted approach. The assumption that future utility will be randomly distributed about its current value implies that current utility should always be placed in the midpoint of the provided scale, and this is not what is observed in practice.

There is, on the other hand, no simple way to determine which of cases 2–4 are being employed on the basis of a single data point or even multiple data points unless we assume that the heuristic being used is accurate. If we do assume accuracy, or approximate accuracy, on the basis that the individual has adopted an approach after closely observing their utility values through time, then time series data ought to reflect either a continuing trend (for case 3) or returns to previous values (for case 2).

Note however, that we can only infer information about how a mapping was constructed from a sequence of reported SWB values through time, if we are confident that the mapping in question has remained static. We turn to the possibility of changes to mapping between periods below.

2.5.8 Dynamic Mapping

Prior to 2.5.7, we had assumed that an individual is able to think of their future utility in terms of its distribution function. For the purposes of this section, we will continue to assume that individuals construct their mapping from utility to subjective wellbeing on the basis of a finite number of remembered past utility values, and do not undertake the intermediate step of deriving the distribution function implied by utility's past behaviour.

Considered this way, the individual's mapping problem becomes one of predicting future utility values on the basis of those they have experienced in the past. Therefore, as new information, in the form of the utility provided by new experiences, is acquired, their optimal mapping will shift in response. If an individual participates in multiple SWB surveys through time, the answers they would have provided in previous surveys may change in response to new their experiences in the interim. Mathematically, this is analogous to evolution's problem of updating the mapping from payoffs to utility in response to each new observed payoff, as set out in section 2.4.1, above.

To capture this phenomenon, we introduce the concept of an 'optimal remapping', which is a modification to an individual's mapping from utility to subjective wellbeing, such that the error introduced in the answers provided up to that point is outweighed by the expected improvement in future SWB estimates under the updated mapping.³⁰ Two obvious features of optimal remapping behaviour are that the costs of remapping are increasing in the number of periods an individual has been surveyed, and that these costs may be zero where a new mapping does not change the SWB value explicitly allocated to any past utility value.

Consider the case of an individual who has provided a single SWB estimate, but who has, *ex hypothesi*, constructed a complete mapping from utility to SWB. We will assume that this individual's utility is drawn from a uniform distribution and that the individual has attempted to infer the structure of this distribution based on a finite number of utility values.

On this basis, we can conclude that the individual will have constructed an initial mapping based on the discrete distribution implied by the (finite number of) past utility values know to them, so that the SWB cutpoints will be placed such that $UF(U)$ is equal for each interval. If we consider, for illustrative purposes, the case of a two-value SWB function and r remembered utility values, including the one at $t = 0$, then we have initially:

³⁰Optimal remapping could also be thought of it terms of changes to the observed distribution of utility values, but, since this distribution is not directly observed, we will not adopt this interpretation.

$$c_0 = \frac{\sum_{s=1}^r U_s}{r} \quad (2.22)$$

Such that the initial value of the cutpoint, c_0 , is equal to the mean of the remembered utility values. This is equal, in expectation, to the mean of the unobserved underlying distribution. The individual reports their initial SWB as either 0 or 1, based on whether U_0 is above or below c_0

Now, the individual observes a new value of utility, U_1 at $t = 1$, and, in the absence of a precommitment to a particular mapping, would automatically update the value of c to reflect the new mean of the remembered distribution.³¹ In this case, however, the individual has an announced SWB value at $t = 0$, and, if the new mapping implied by including U_1 in the calculation of c renders the reported SWB value associated with U_0 incorrect, this will introduce error into the individual's stated mapping.

There is a tension, then, between minimising errors in past statements of SWB and dynamically remapping as new information becomes available. This tradeoff can be measured as a function of the error induced in the existing reported SWB values minus the predicted future errors avoided by optimally remapping.

In the specific case considered here, we have the following possibilities:

$$\begin{aligned} c_0 < U_0 \text{ and } c_1 &= \frac{\sum_{s=1}^r U_s + U_1}{r + 1} < U_0 \\ c_0 > U_0 \text{ and } c_1 &= \frac{\sum_{s=1}^r U_s + U_1}{r + 1} > U_0 \\ c_0 < U_0 \text{ and } c_1 &= \frac{\sum_{s=1}^r U_s + U_1}{r + 1} < U_0 \\ c_0 > U_0 \text{ and } c_1 &= \frac{\sum_{s=1}^r U_s + U_1}{r + 1} > U_0 \end{aligned}$$

In the first two cases, the implied SWB value associated with U_0 is unchanged by the observation of U_1 . In the second two cases, adopting the new mapping implied by considering U_1 along with the previous r utility values would cause a reassessment of SWB at $t = 0$.

Now we consider the error function generated in cases 3 and 4. The individual has a choice between optimally remapping, such that $c_1 = \frac{\sum_{s=1}^r U_s + U_1}{r+1}$,

³¹We will not model the possibility of old utility values being forgotten as new ones are added, but the conclusions reached are not particularly sensitive to this assumption.

and inducing an *ex post* 1-unit error in reported SWB, or changing c_1 by the maximum amount implied by U_1 consistent with retaining the existing value of SWB for U_0 – which would involve setting $c_1 = U_0$. In this case, the error generated is the expected value of the future errors caused by misreporting SWB relative to the optimal mapping implied by U_1 .

When will a remapping cause violations of ordinal ranking? This will depend firstly on how many remembered utility values an individual uses to construct their mapping from utility to SWB, and on the size and frequency of shocks to the distribution of utility. A mapping constructed on the basis of a large sample size should be relatively stable through time until there is a significant shift in the distribution of utility. In order to generate a violation of ordinal ranking, we would require that an observed shift in the distribution of utility generate a new optimal mapping of the sort described in the following example:

Consider an individual who, at time $t = 0$ reports an SWB of 5 on a 10-point scale. Now, between time $t = 0$ and $t = 1$, the individual observes a change in the distribution of utility, such that their error-minimising mapping from utility to SWB changes. We will assume, without loss of generality, an upward shift in the utility distribution. Assuming the individual adopts a new mapping, we will have a violation of ordinal ranking only where the new mapping, associated with an on-average high utility distribution, causes them to report SWB of 4 or less. That is, we would require an upward shift in utility sufficiently large that the resulting downward shift in the SWB-value associated with a given level of utility outweighed the observed increase in utility from which the higher distribution was inferred. Intuitively, observed utility must rise by so much that the new, higher, utility-level belongs on a lower point on the new scale than the old, lower, utility-level.

In a stylised model where only a single utility value is experienced and reported as SWB each period, this is categorically impossible given any sensible method of inferring utility distribution from its observed values – it is not possible for your happiness to increase so much that you realise that you are, relatively speaking, worse off than you were before.

Even in a more general model, whereby multiple instant utilities are observed within survey periods, remappings which induce violations of ordinal rankings should be extremely rare.

Unfortunately, because the surveyor only ever observes SWB-values, they will be unable to determine when there has been a shock to the utility distribution with assuming a constant mapping – a series of correlated shocks to the distribution of utility could lead the individual surveyed to report constant SWB with a new underlying mapping from utility to SWB in each period.

Conversely, where an individual has retained a single mapping across a series of SWB reports through time, it may become very difficult for a shift in the distribution of utility to be large enough in any one period to motivate an optimal remapping, even where the collective effect of the distributional change over a series of periods calls for a remapping. That is, the decision to remap is a one-shot decision in any given period, and where a remapping is not undertaken, the incorporation of a new value under the existing mapping scheme increases the inferred change in distribution necessary to cause a remapping in subsequent periods – effectively generating an escalating commitment to a potentially increasingly inaccurate mapping.

In a time series context, we will not wish to encourage such an ‘anchoring’ to the mapping on which previously announced values were based where the mapping in question has become suboptimal. As such, it would be useful to allow individuals to restate past SWB-values as well as providing a new SWB-value when they are resurveyed.³² This would give the individuals surveyed an opportunity to signal that a remapping had taken place, and therefore lower the threshold at which an optimal remapping would take place.

Generally speaking, the possibility of individuals altering their mapping from utility to SWB to response to new utility observations represents a significant challenge for the drawing of cardinal inferences from time-series SWB data. Because there is no way of signalling a change to an individual’s mapping to the surveyor, we cannot necessarily assume that SWB results through time are even ordinally consistent with earlier data measured on the same scale (though this will usually be the case) and certainly cardinal relationships are unlikely to hold through time.

We might, as a rule of thumb, associate with any shift in reported SWB a positive probability that a remapping has occurred, exponentially increasing in the size of that shift. That is, we might treat a one-unit rise in SWB as disclosing a small, say 2%, possibility that the preexisting mapping has been modified, while a three-unit shift might suggest a 2³% possibility. This would allow us to construct an additional threshold for cardinal claims about time-series SWB to cross before they are treated as persuasive.

³²Note, however, that this recommendation assumes away persistently-demonstrated problems with recall bias – see, for example Suh, Diener, and Fujita (1996). The information associated with restated past SWB values would need to be weighed to try to extract remapping “signal” from recall bias “noise”.

2.5.9 Dynamic Mapping and Mean Reversion

We will now consider the interaction between the model of mean reversion, laid out in 2.4, above and the individual's mapping problem.

Initially, assume that we have a reversion blind individual³³ who otherwise rationally projects their future utility and attempts to map it to SWB in an error-minimising fashion.

Such a reversion blind individual who attempts to determine their future utility values based on trends or on the assumption of a random walk about current utility³⁴ will tend to overestimate the possibility of reaching the extremes of their utility function, and will, incorrectly, reserve the highest and lowest values of their SWB scale for levels of utility which, as a result of mean reversion, are observed much less frequently in practice than would be implied by an optimal mapping.³⁵

Consider, for instance, an individual who has concluded that their utility follows a linear trend, increasing by one unit each period, constructing a mapping to a ten point SWB scale. Over a sufficiently long time horizon, any constant trend mapping will predict arbitrarily high values of future utility, and will result in a mapping whereby present utility values are 'bunched' at the bottom of the SWB scale. We will counteract this tendency by assuming the individual places a lower weight on future errors in constructing their mapping.

Now, depending on the strength of the discount rate on future error, the individual will arrive at some optimal mapping in which current utility-values are focussed more or less tightly on the lower end of the reported SWB. We can, at a minimum, be confident that the initially reported SWB value will lie in the bottom-half of the scale. However, as we collect additional SWB reports through time, the individual's utility will tend to rise, but also to revert to its mean level, so that, depending on the relative rates at which these phenomena occur, experienced utility may tend to rise or fall.

In this situation, the original mapping, constructed on the basis of steadily increasing utility, will prove inaccurate and, in particular, too large a portion of the SWB scale will have been reserved for high utility values

³³That is, an individual who does not foresee the effects of mean reversion on their future utility.

³⁴Cases 1 and 3 in 2.5.7, above.

³⁵Note, however, that this phenomenon only arises where the remembered utility values on which the individual bases their mapping occur over a sufficiently short time period so as not to have experienced adaptation, and where the future time-horizon taken into account when constructing a mapping is greater than this remembered period.

which will never be experienced in the presence of mean reversion.

2.5.10 Dynamic Mapping and Restricted Utility Models

We have, up to this point, assumed that the underlying utility being mapped to subjective wellbeing was cardinal, continuous and unbounded. We now turn to the case where, as modeled in 2.2, above, underlying utility is, itself discrete and bounded as the result of an evolutionary compromise.

In this case, there is a much clearer resemblance between the SWB scale and the underlying utility scale mapping to it. We have a mapping from one discrete, bounded scale to another, whereby the restricted utility function serves as a filter between reported subjective wellbeing and the original (and continuous and unbounded) payoffs generated by nature.

In the limit case, where the utility function takes the same number of values as the SWB scale, the optimal mapping will simply assign each SWB-value to its corresponding utility value and the mapping from utility will be linear and free from error.

In the more realistic case, where the utility function is less granular and takes a wider range of possible values than the SWB function, the individual will still be faced with a mapping problem, similar to those discussed above.

We can begin by noting that, to the extent we have already been able to reject the possibility (in 2.5.3, above) that the mapping to SWB is based on ordinally-perceived utility, we can conclude that underlying utility, even if it is restricted, is not perceived subjectively as ordinal. This suggests the existence of a quasi-cardinal remapping by a mechanism such as that suggested in 2.3.4, above.

The next point to make is that, where utility is generated accorded to a restricted discrimination of states of nature those states are, *ex hypothesi*, unobservable to individuals subject to the utility function so generated. As such, an individual will not be aware of any of the distortions imposed by their mapping from evolutionary payoffs to utility, and, where a recardinalisation like that described above is in place, will experience utility as if it was truly cardinal. Because of this ability to linearise a nonlinear mapping, while distinctions in utility levels will display clustering around the mean of the distribution of evolutionary payoffs, the *perceived value* of the these utility levels will not.

In this case, the individual's mapping problem folds-back to the unrestricted utility case, discussed above.

Note, however, that when considering restricted utility functions, there

is a distinction between experienced utility and the actual payoffs provided by nature. Even where restricted utility accurately maps to SWB, any distortions imposed by the mapping from payoffs to utility will remain in the final reported SWB value. This is of concern, though, only if we are interested in maximising actual evolutionary outcomes rather than experienced utility. As we have argued in 2.4.6, above, this ought not to be the case.

2.6 Consequences for Subjective Wellbeing Data

In this section we present recommendations for the modification of raw subjective wellbeing data based on the above analysis of the individual's SWB mapping problem.

1. The relationship between utility and SWB will depend on the distribution of the underlying utility function. Where the distribution of utility is not uniform, an error minimising mapping will distribute the cutpoints of the SWB scale at the mean-points of the utility distribution. For normally distributed underlying utility, this will imply that extreme SWB-values occupy each occupy a larger portion of the underlying utility scale than more central values. Cardinal use of SWB-values will therefore require a convex transformation of reported SWB.
2. The optimal mapping strategy from utility to reported SWB will result in SWB values being reported with a frequency inverse to the cardinal 'width' of the utility scale they cover. This observation allows the calculation of expected differences between any two points on an SWB scale, enabling the recovery of cardinal utility estimates. This approach is preferable both to treating SWB values and cardinal and linear and to treating them as purely ordinal.
3. The expected error induced by boundedness of scale will be increasing in the frequency with which observations are taken. Extreme utility-values are unlikely to persist over long periods, and brief above-scale values will be averaged out when reporting aggregate utility over a longer period. As such, the expected minimum and maximum instant utility values over a period will be decreasing in the length of the period captured by the 'instant' measures, and the level of discrimination between more moderate utility-values on a reported SWB scale will likewise rise as the expected range falls.
4. While remappings which lead to later reported SWB-values being incorrectly ordinally ranked relative to earlier values are likely to be

extremely rare, remappings which induce changes to the cardinal relationship between time series data are potentially common. Sensitivity testing, based on the possibility of an inter-period remapping as a function of the size of the change in SWB, should be conducted prior to conducting cardinal-level analysis of such data. A first-best solution would be allowing subjects to make *ex post* revisions to earlier SWB estimates as part of the time series process.

5. Underlying utility is experienced cardinally, though this may result partially from a subjective recardinalisation of ordinal, restricted underlying utility, data.
6. In the presence of mean reversion, mappings will tend to assign more weight to extreme utility values than the optimal mapping would suggest. As with point 1, above, this calls for a convex transformation of reported SWB data.

2.7 Conclusions

In this chapter we have presented a new model for deriving a utility function from restricted observations. This model, which conditions the allocation of states of nature to utility payoffs only on the observed distribution of final utility, predicts the allocation of available utility states to mean points in the distribution of natural payoffs. In the presence of shifts in the distribution of payoffs, this model, consistent with earlier restricted utility models, predicts reversion of utility to its mean level following a permanent change in circumstances.

We then discussed the nature of the utility outputs generated by restricted utility models such as the one presented here, concluding that, while restricted utility was in some sense ordinal for all non-uniform distributions of payoffs in nature, it could be remapped in such a way as to produce partially-cardinal utility.

We showed that utility functions with a restricted range of values require mean reversion, and demonstrated the optimal structure for that mean reversion in the two-value limit case.

In response to the general mean reversion predicted by the restricted utility function models, new and old, we have presented a detailed model of mean reversion in discrete time. This model predicts that anticipated mean reversion will exactly cancel-out any gains to utility from temporary increases in consumption in individuals with a zero pure rate of time preference regardless of the rate at which reversion occurs. We further showed

that permanent increases in consumption with unanticipated mean reversion and a pure rate of time preference equal to the rate of reversion is equivalent to anticipated mean reversion at the same rate.

Taking data from the British Household Panel Survey, we considered the empirical evidence for the mean reversion structures modelled in this chapter. We demonstrated that utility shocks show significant mean reversion, and that the evidence suggests that the degree of reversion is a linear function of the size of the shock.

We then discussed the consequences of this model of mean reversion from an evolutionary perspective, for subjective wellbeing and from the point of view of a benevolent social planner.

In relation to evolution, we posited the existence of evolutionary limits to the rate of mean reversion consistent with otherwise forward-looking rational behaviour. We further suggested that the interaction between time preference and mean reversion in our model explained time preference as an evolutionary mechanism for optimising behaviour under partially anticipated mean reversion.

Our conclusions for subjective wellbeing were that mean reversion of the type predicted by this model would lead to mappings between utility and SWB which were inconsistent through time and which inaccurately reserved space on the reported SWB scale for unachievable utility values.

And, in relation to social choice under mean reversion, we concluded that a social planner could potentially generate Pareto gains by intervening to correct the underpricing of unhabituable risks, particularly the risk of early death, and that time preference should be ignored in the social choice calculus.

Finally, we considered the cooperative signalling problem faced by an individual trying to construct an optimal mapping from utility to subjective wellbeing by analogy to the role of evolution in the restrictive utility models discussed at the outset of this chapter.

We concluded that the observed data were inconsistent with individuals generating a mapping based on ordinally experienced underlying utility. We then showed that, in the presence of a known utility distribution, the individual's problem is solved by assigning cutoffs in the SWB scale to the mean points of the utility distribution, analogously to the optimal strategy for evolution in the restricted utility models considered earlier in the chapter. As this is equivalent to assigning a portion of underlying utility space to SWB values inverse to their expected frequency of occurrence, this enables us to recover cardinal estimates of expected differences in utility between

any two SWB values and to rescale reported data accordingly.

We demonstrated the application of this recardinalisation approach using the BHPS data, examining both mean reversion and the influence of income on subjective wellbeing.

We then considered the situation in which an individual attempts to infer the distribution of future utility in the absence of a known formal distribution. We demonstrated that the data are inconsistent with individuals modeling future utility as a random walk about its current value, and considered the role of dynamic remapping in response to subsequently observed utility values.

We concluded that, under most reasonable assumptions, the subjective wellbeing scale would be a concave transformation of underlying utility, generating non-cardinal data, and that inconsistencies in time-series data could arise as a result of optimal remapping on the part of those surveyed.

On this basis, we propose modifications to the treatment of subjective wellbeing data which take into account the results derived in this chapter. We suggest that an optimal mapping will be nonlinear in the underlying utility values and that a remapping of reported SWB based on observation frequency is necessary prior to treating data as defined up to an interval scale. We further suggest sensitivity analysis to take into account the possibility of changes to an individual's mapping, and a modification to time series survey methodology to permit explicit re-estimation of past SWB values in response to observed changes in the distribution of utility.

Chapter 3

Aversion to Peaks in Utility

3.1 Introduction

In this chapter we initially turn away from our discussion of subjective wellbeing, in favour of presenting a modification to traditional models of risk aversion. For those unpersuaded by the defence of SWB research presented in chapter one, the bulk of the material discussed herein can be considered independently of one's views on the value of self-reported happiness data.

While the aversion to peaks in utility can be considered on its own, it intersects with the preceding subjective wellbeing material in three ways.

First, and most directly, we go on show in section 3.6 that, contrary to existing approaches, such an aversion should be taken into account in aggregating individual SWB data through time.

Second, section 3.12 deals with the social choice consequences of interpersonal aggregation of both utility and wellbeing and argues for the use of a strictly concave social welfare function in comparing aggregate SWB and utility across different states of the world.

Third, section 3.5 considers the interaction between pure rates of time preference and preference for smoothness of utility distribution through time. We contrast the normative significance of such a preference for smoothness with the role of time preference, as suggested in chapter two, in offsetting the effects of future predicted mean reversion.

3.2 Introducing Peak Aversion

In the late 1950's and early 1960's, the town of Vernon, Florida, drew the attention of the United States' insurance industry. For a period of several years, more than two thirds of the United States' loss-of-limb accident claims originated from Vernon and the nearby portion of the Florida panhandle. According to Lake (2007), citing a local insurance inspector, victims included:

“... a man who sawed off his left hand at work, a man who shot off his foot while protecting chickens, a man who lost his hand while trying to shoot a hawk, a man who somehow lost two limbs in an accident involving a rifle and a tractor, and a man who bought a policy and then, less than 12 hours later, shot off his foot while aiming at a squirrel. . . [Another man] was a farmer and ordinarily drove around the farm in his stick shift pickup. This day - the day of the accident - he drove his wife's automatic transmission car and he lost his left foot. If he'd been driving his pickup, he'd have had to use that foot for the clutch. He also had a tourniquet in his pocket. We asked why he had it and he said, 'Snakes. In case of snake bite.'”

So prevalent did this accidental dismemberment become that Vernon was nicknamed 'nub city' and the aforementioned insurance inspector was sent by the major provider to investigate.

Though none of those who lost their limbs in Vernon was ever convicted of fraud, we have strong circumstantial evidence that at least some of its residents were deliberately severing their extremities in pursuit of the resulting insurance payout.

What does Vernon have to tell economists about preferences and in particular, attitudes to risk? We might dismiss the particular mania which struck Vernon as pathological, or we might conclude that its residents faced an unusually high marginal utility of income, relative to their marginal utility of arm, leg, hand and foot. But the overarching question arising from the incidents in Vernon is this: *why did a market exist for the residents of Vernon to insure their limbs at a value far above the fall in income resulting from their loss?*

While Vernon is an unusual case, the phenomenon of insurance for non-pecuniary health risks is not. A typical London corporate law firm, for instance, offers its lawyers, as part of its standard insurance package £500,000 for the loss of a single eye. Depth-perception, while useful in general life,

is not a critical skill for a corporate lawyer, and it is difficult to reconcile this sum with the size of any reduction in future earning potential, even as a result of related disfigurement, or with the costs of a prosthesis.

Traditionally, economists have treated insurance markets as a consequence of risk aversion, in the specific sense arising from diminishing marginal utility of income. They are used, theory suggests, to transfer money from states of the world where one is relatively wealthy, and marginal utility is low, to states where one is relatively poor, and marginal utility is high.

This risk aversion-based motivation cannot explain the insurance contracts taken out by the residents of Vernon, though, or the unusual focus on compensation for loss of individual eyes. In these instances, loss of income is, as the actions of Vernon's residents demonstrate, far below the sum of treatment costs and projected loss of income. So we have an insurance contract which serves to concentrate wealth in one state of the world, rather than smoothing it between all states.

However, the puzzle of nonpecuniary medical insurance goes beyond the failure to equalise marginal utility of income, as theory predicts. A one-armed Floridan, or a one-eyed lawyer, have, at the margin, somewhat fewer opportunities for pleasure than their able-bodied equivalents. Even once income is equalised and their surroundings modified, an individual with one leg will tend to forgo dancing, or motorcycle riding, or hiking in the wilderness. On average, then, we expect the prospects of increasing one's utility to be *lower* for the victims of a permanent injury and consequently, again on average, we expect them to experience reduced marginal utility of income at any given income level relative to their able-bodied doppelgänger.

In fact, Finkelstein, Luttmer, and Notowidigdo (2013) calculate that a one standard deviation increase in the measure of chronic illness is associated with, on average, an 11% reduction in the marginal utility of income and that, as a consequence, optimal actuarially fairly priced insurance should cover only two thirds of *medical expenses*, let alone non-pecuniary utility loss.

So these observed levels of medical insurance run counter to the traditional explanations in two ways. Not only is wealth not being smoothed across states, people are shifting money towards states of the world which exhibit *lower* marginal utility of income. We would expect, based on standard models of risk aversion, to see the existence of contracts whereby individuals transferred small amounts of wealth from states in which they predicted they would be less able to enjoy it, but instead we see the reverse.

While these kinds of insurance present a challenge for economists, the general public appears to consider them unremarkable. Insurance markets

exist, not only for loss of limbs and eyes, but for irreplaceable antiques and family heirlooms, for the sentimental value of one's engagement ring, and for the loss of a child. Travel insurance contracts exist to provide compensation for cancelled holidays even in circumstances where a replacement holiday will not be taken, and Chinese insurance markets now offer contracts which pay off when the moon festival is ruined due to poor weather. Fans of a sporting team will occasionally bet against their team in order to hedge against the possibility of a loss. In each of these cases, the standard, diminishing marginal utility-driven explanation fails to cover the phenomenon in question. Some people, it appears, insure not only to avoid unequal marginal utilities, but also *because they don't like risk*.

Risk aversion, as economists think of it, is not an 'aversion to risk', at least not as those terms are commonly understood – rather, it is an emergent property of the phenomenon of diminishing marginal utility. A risk averse individual will be indifferent when faced with a fair gamble offering equal changes to utility across states – if a risk averse individual values their grandmother's vase and another person's mint, out-of-print baseball card equally, they will be willing to wager one for the other on the flip of a fair coin.

If we wish to explain the behaviour of individuals who insure their inessential body parts, unobstructed views and their irreplaceable heirlooms, then, we need to invoke something more than traditional risk aversion. In this chapter, we introduce the concept of 'peak aversion', a preference for equality of utility across states, to bridge this gap. To capture peak aversion, we suggest a relaxation of expected utility maximisation, such that individuals place more weight on states where their utility is low than those where it is high, leading to a generalised aversion to risk across pecuniary and nonpecuniary outcomes.

In section 3.3, we present a simple mathematical model of peak aversion, and demonstrate that it is able to explain observed patterns of behaviour.

In section 3.4, we compare peak aversion to standard risk aversion models and consider its relationship to and potential for integration with existing modifications to choice under uncertainty.

Section 3.5 introduces a second, related notion of peak aversion, this time in relation to utility flows through time. This model is given a technical specification in 3.5.1, and its consequences for the aggregation of SWB explored in 3.6.

Section 3.7 provides a series of shorthand summary measures of peak aversion, based on existing risk aversion measures.

Sections 3.9 and 3.10 consider the practical application of peak aversion in calculating the value of Quality Adjusted Life Years and tortious damages for negligence. We also consider a common error in discounting nonpecuniary changes in utility through time.

Section 3.11 draws on an existing data set which captures peak averse behaviour in order to arrive at an estimate of the typical size of the peak aversion effect.

Finally, section 3.12 deals with the normative consequences of peak aversion, both from the point of view of social choice theory and its proper treatment when measuring welfare.

Section 3.13 summarises our conclusions.

3.2.1 Observed Differences in Group Risk Preference

Even if one rejects the specific idea of peak aversion as a psychologically accurate account of individual decision making, a simple preference structure which allows for differential average preferences for risk between different groups, independent of consumption levels and their associated utilities, is nonetheless a useful tool.

There is a significant literature on systematic group and circumstantial difference in risk taking behaviour – see, for instance, Nelson (2012) and Apicella, Dreber, Campbell, Gray, Hoffman, and Little (2008) – in circumstances not readily explained by differences in the marginal utility of consumption. We believe this literature both represents an argument for the existence of peak aversion in the general form outlined above as a free-standing characteristic which can vary between individuals and, alternatively, represents a use for peak aversion as a *predictive* rather than descriptive theory, on the basis that it can, at a group level, be demonstrated that certain subpopulations act as-if they possessed a non-expected utility maximising utility function.

Similarly, when modelling the behaviour of policy makers, or other decision makers whose financial payoffs are not linearly related to the outcomes of their decisions it is useful to have a simple numerical tool, and terminology with which to describe a potential preference for consistency in outcomes between states – an ‘aversion to risk’ which is not, of course ‘risk aversion’ in the technical economic sense. In situations where a policy maker has a non-financial preference for certainty¹ it is unhelpful and misleading to press into service the unrelated vocabulary of diminishing marginal utility risk

¹As we demonstrate below, this is distinct from the separate phenomenon of uncertainty aversion.

aversion. This is particularly true where both ‘true’ (diminishing marginal utility) risk aversion and a preference for certainty of outcome are at work in the same situation.

Having terminology and simple model for the distinct phenomenon of peak aversion is therefore unquestionably superior to using ‘risk aversion’ to describe both the rate at which marginal utility of consumption diminishes in individuals who are risk neutral across nonpecuniary outcomes *and* a more general attitude to unequal payoffs between states.

3.3 A Model of Peak Aversion

We propose a functional-form for utility such that an individual preference for equality or ‘smoothing’ of utility between states can be accommodated in a similar manner to traditional measures of diminishing marginal utility of income-based risk aversion.

Traditionally, an expected utility maximiser possessing a von Neumann-Morgenstern utility function will be indifferent between distributions of utility across states having an equal expected value. This is a strong assumption to make in general and it is, in particular, seemingly inconsistent with the common behaviours outlined in 3.1, above.

How, then, can we modify the expected utility maximising decision-rule to capture a preference for smoothed utility between states?

Consider, to begin with, a standard expected value maximiser with a utility function $U(C, H)$, where C is consumption and H is a proxy for overall level of health.² The individual is faced with a choice between:

1. a lottery across two states, 1 and 2, with those states occurring with probabilities p and $1-p$ respectively; or
2. a degenerate lottery leading to state 0 with a probability of 1.

The individual will choose to take the gamble iff:

$$U(C_0, H_0) < pU(C_1, H_1) + (1 - p)U(C_2, H_2) \quad (3.1)$$

Which is linear in both the utilities and the probabilities associated with each state.

²Equally, H can be viewed as representing an aggregate of all non-pecuniary variables influencing utility.

If we wish to capture a preference for equal utility across states we need a functional form that reduces the marginal value of increased *utility*, rather than consumption, in high-utility states. Consider the following alternative specification for the decision-rule employed by an individual faced with the choice described above:

$$fU(C_0, H_0) < pfU(C_1, H_1) + (1 - p)fU(C_2, H_2) \quad (3.2)$$

where $\frac{\partial f}{\partial U} > 0$ and $\frac{\partial^2 f}{\partial U^2} < 0 \forall U \in \mathbf{R}_+$, making total utility non-linear in the utilities associated with individual states of the world. In this case it is possible, depending on the exact U and p values, and on the structure of f , for an individual to refuse an actuarially fair gamble in situations where the payoffs, as measured in utility, are skewed heavily towards one state of the world.

Preferences where utility across states is aggregated in this way allow us to capture a preference for smoothness in utility – ‘peak aversion’ – with a relatively minor modification to the underlying utility function and to expected utility maximisation. An individual who is peak averse will wish to insure against losses in utility, regardless of whether those losses can be corrected with money or whether they reduce effective wealth.

An alternative way to conceive of peak aversion is as a ‘diminishing marginal utility of utility’ in any given state. That is, the marginal utilities of all determinants of utility are, *ceteris paribus*, decreasing in total utility, such that the cross partial derivatives of the utility function would tend to be negative.

Note that the function which aggregates across states, f , which we will refer to as the ‘aggregator function’, is symmetrical across all states of the world, including single states in the case of degenerate lotteries. This might seem to suggest that the peak averse utility function might be replicable simply by respecifying the utility functions to be used for traditional expected utility maximisation. From a notational point of view this is correct, and an advantage of peak averse utility over state dependent utility³, but it is important to remember that a peak averse individual is actually willing to sacrifice expected utility in order to achieve a smoothing of utility between states, and as such it is not correct to view the $f(U)$ values as being actual utility in any state any more than, for instance, discounted utility values, $(1 - \delta)^t U_t$, are treated as actual future values of utility.⁴

By way of example, consider an individual with state-specific utility

³See 3.4.2, below.

⁴See 3.5 for more discussion on this comparison between time preference and peak aversion.

given by:⁵ $U_s(C, H) = C_s^{\frac{1}{2}} + H_s^{\frac{1}{2}}$ and who aggregates utility across states according to the rule:

$$U_T(C, H) = \sum_{s=1}^S p_s \cdot \sqrt{U(C_s, H_s)} \quad (3.3)$$

where U_T is the total utility associated with a lottery, S is the number of possible states and C_s , H_s and p_s are, respectively, consumption in state s , health-level in state s and the probability of state s occurring.

So, for the two state case, we have:

$$U_T(C, H) = p_1 \cdot \sqrt{U(C_1, H_1)} + (1 - p_1) \cdot \sqrt{U(C_2, H_2)} \quad (3.4)$$

and

$$\frac{\partial U_T(C, H)}{\partial C_s} = \frac{p_s}{4(C_s + \sqrt{C_s H_s})} \quad (3.5)$$

or

$$\frac{\partial U_T(C, H)}{\partial C_s} = \frac{p_s}{4\sqrt{C_s}(\sqrt{C_s} + \sqrt{H_s})} \quad (3.6)$$

which is equal to

$$\frac{\partial U_T(C, H)}{\partial C_s} = \frac{p_s}{4\sqrt{C_s}(U_s)} \quad (3.7)$$

Note that marginal utility of consumption in any state is decreasing in the level of utility in that state, even if consumption is held constant.

Now, for ease of exposition, we will assume that the individual in question can purchase health insurance at actuarially fair prices, and that the lottery they face, prior to purchasing insurance, is as follows:

- in state 1, which occurs with probability $p_1 = 0.50$, the individual remains uninjured and receives $C = 100$ and $H = 100$; and

⁵Note that, for simplicity, this particular utility function does not imply that the marginal utility of income depends positively on health, as suggested above.

- in state 2, which occurs with probability $p_2 = 1 - p_1 = 0.50$, the individual suffers a nonpecuniary injury⁶ and receives $C = 100$ and $H = 81$.

In the absence of insurance, the individual's marginal utility of consumption in state 1 is $\frac{1}{1600}$ and in state 2 it is $\frac{1}{1520}$. Given the possibility of insuring so as to exchange consumption in state 1 for consumption in state 2 on a dollar-for-dollar basis, the individual will purchase insurance up to the point where the marginal total utility of consumption in each state is equal. This condition is solved by setting C_1 such that:

$$C_1 + 10C_1^{\frac{1}{2}} = 200 - C_1 + 9(200 - C_1)^{\frac{1}{2}} \quad (3.8)$$

Which reduces to a quartic polynomial suggesting that the optimal level of C_1 , C_1^* is approximately 96.61. This implies an optimal purchase of health insurance equal to approximately 3.39% of income. After the purchase of insurance, the individual will experience utility of 19.83 in state 1, and 19.17 in state 2.

We observe that, while the individual in question does purchase health insurance as predicted by the peak aversion model, they do not fully insure, due to the separate effects of diminishing marginal utility of income in state 2. An individual who was peak averse, but risk neutral in the traditional sense, would completely insure in the presence of actuarially fair insurance contracts, whereas even a risk averse individual would not purchase any insurance in the absence of a positive level of peak aversion.

Were we to further complicate our analysis by using a utility function whereby marginal utility of consumption was increasing in health-level,⁷ as we suggest above, we would find, *ceteris paribus*, that the optimal level of insurance would fall, and possibly become negative, depending on the relative strengths of the influence of health on marginal utility of consumption and the peak aversion embodied by the aggregator function.

Similar analysis can be conducted for utility functions based around consumption and other nonpecuniary variables, such as the ownership of an heirloom or engagement ring. We will, as in the case above, expect to observe purchases of a positive but incomplete level of actuarially fair insurance.

⁶That is, an injury for which neither results in nor is able to be offset by a reduction in consumption

⁷A constant returns to scale Cobb-Douglas function in consumption and health, for instance.

Note that this approach can be generalised, so as to describe utility functions with no preference for smoothness ('peak neutral' preferences) and those for individuals who affirmatively desire an unequal distribution of utility between states ('peak loving'). By simply altering the aggregator function, f , such that $\frac{\partial^2 f}{\partial U^2} = 0$ (in the peak neutral case) or so that $\frac{\partial^2 f}{\partial U^2} > 0$ (in the peak loving case), the peak averse utility function presented in 3.26 becomes a general descriptor for a variety of attitudes towards smoothness of utility distribution, including the traditional, peak neutral case, depending on the concavity, convexity or linearity of the aggregator function.

For the remainder of this chapter, we will use the term 'peak sensitive' to describe preferences which are either peak averse or peak loving – that is, preferences where $\frac{\partial^2 f}{\partial U^2} \neq 0$.

In 3.5, below, we will go on to apply this notion of peak sensitivity to preferences across streams of utility through time.

3.4 Peak Aversion and Existing Risk Models

In this section we distinguish peak aversion from existing models of choice under uncertainty, including other non-standard models, indicating differences in approach and considering possibilities for integrating peak aversion with the alternative specifications.

3.4.1 Peak Aversion and Risk Aversion

As demonstrated in the worked example provided in 3.3, above, peak aversion is perfectly compatible with risk aversion arising from diminishing marginal utility of consumption.

In general terms, increasing levels of risk aversion and peak aversion will both tend to generate an increasingly smoothed consumption profile across states and increase the required risk premium associated with purely pecuniary gambles. However, in the case of nonpecuniary gambles, like that considered in the example above, increasing risk aversion will tend to reduce the optimal level of insurance, while increasing peak aversion will tend to increase it.

We further note that, when observing the behaviour of an individual who is both peak and risk averse in response to pecuniary gambles, we will tend to overestimate the rate at which their marginal utility is decreasing if we incorrectly assume them to be peak neutral. Estimates of marginal utility of income, such as those provided by Layard, Mayraz, and Nickell

(2008), will tend to be too low if they are incorrectly conditioned on peak neutrality, and this inaccuracy has potential consequences for cost benefit analysis and social welfare theory.

Technically speaking, existing observational measures of marginal utility based on pecuniary risk averse behaviour are in fact measuring peak aversion and diminishing marginal utility of consumption simultaneously. That is, the behaviour observed is based not on $\frac{dU}{dC_s}$, but on

$$\frac{df(U(C_s))}{dC_s} = \frac{dU}{dC_s} \cdot \frac{df(U)}{dU} \quad (3.9)$$

We can view this hybrid measure as a single, summary measure of aversion to pecuniary risks based on both peak and risk aversion.

Finally, we observe that, under a pure risk aversion model, there is no distinction between an individual's attitude to risk and the rate at which their marginal utility of income decreases. This dual role for diminishing marginal utility can be difficult to reconcile with certain observed behaviours, not only in response to nonpecuniary gambles like those discussed above, but also in relation to pecuniary risks, as with the behaviours which generate the equity premium paradox. We believe there is practical value in being able to model a consumer who values purchases just beyond their means only slightly below their marginal purchase but who also has a free-standing dislike of risk across states, since it seems psychologically plausible that an individual might possess both of these traits.

3.4.2 Peak Aversion and State-Dependent Utility

We will begin by distinguishing our notion of peak aversion from more general models of state-dependent utility.

Karni (1983), describes state-dependent utility as appropriate for situations "where the utility assigned to any given level of wealth varies with the state of nature". Similarly, Kelsey and Nordquist (1991), frame the kind of problems requiring a generalised state-dependent utility framework as follows:

"There are a number of practical problems in which the individual's valuation of wealth will depend upon how uncertainty is resolved, for instance in the choice of life or health insurance"

Given each of these descriptions, the above forms of state-dependent utility are perhaps better thought of simply as multivariate utility. That

is: any useful utility function will of course produce utility values which “[vary] with the state of nature” for some nonpecuniary values of ‘nature’, and state-dependent utility therefore arises only where the changes in state being considered affect a variable not normally treated as an element of the utility function. This, conversely, implies that state-independent utility is simply utility which is additively separable in its arguments.

As should be obvious from the worked example of peak aversion operating on an additively separable utility function in 3.3, above, degree of peak sensitivity can alter preferences even where the non-consumption arguments of the utility function have no influence on marginal utility of consumption, making the applicability of peak sensitivity independent of the structure of the state-specific utility function.

So, unlike generalised state dependent utility functions, peak sensitive utility functions calculate multivariate utility in each state of the world according to an identical utility function, and simply transform those values using a non-linear aggregation function⁸ As such, peak sensitivity can be viewed as lying on the spectrum between traditional state-independent utility and generalised state-dependent utility.

It is, of course, possible to duplicate the preferences generated by peak sensitive utility functions under state-dependent utility, simply by specifying a higher marginal utility of income in low-utility states. While this approach can be generalised to reflect other state-dependent features, it has the disadvantage of being far less tractable and less easily integrated into state-independent analysis than peak sensitivity modeled using a state-independent utility function and a state-independent aggregator function.

Similarly, one could integrate a peak sensitive aggregator function across states with separate, state-dependent utility functions in each state, to capture state-based phenomena other than peak aversion. An individual with preferences structured in this way would attempt to equalise⁹ state-dependent utility across states. We will not further formalise such generalisation of the peak sensitivity architecture at this time.

Peak sensitivity is therefore best viewed as a slight relaxation of the assumptions of state-independent utility theory, rather than as a very specific form of state-dependent model. Or, put another way, peak sensitivity makes a specific prediction as to the way in which changes to secondary utility arguments will effect consumption utility and the results of this prediction can be captured using an aggregator function which is, itself, uniform across states.

⁸Or with a linear aggregation function in the peak neutral limit case.

⁹In the peak averse case.

3.4.3 Peak Aversion and Uncertainty Aversion

Our survey of the relationship between peak aversion and existing alternative choice-under-uncertainty frameworks continues with a consideration of uncertainty aversion.

Preferences which exhibit uncertainty aversion, as it is explained by Schmeidler (1989) and Epstein (1999), display a bias towards lotteries with known probabilities over those with unknown probabilities. The classic illustration of uncertainty aversion is the Ellsberg Paradox, which is as follows:

Subjects are offered two chances to draw a single ball from an urn which they know to contain 30 red balls and a total of 60 balls which are either black or yellow. On the first draw they are offered a choice between two lotteries. The first of these ('lottery A') pays \$100 where a red ball is drawn and \$0 otherwise, while the second ('lottery B') pays \$100 where a black ball is drawn, but \$0 otherwise.

On the second draw, a choice is offered between a further two lotteries. The first ('lottery C') pays \$100 if the ball drawn is red or yellow, and \$0 otherwise, while the second ('lottery D') pays \$100 in the case of a black or yellow ball, and \$0 otherwise.

Surveyed individuals typically reveal a preference for lottery A over lottery B, indicating a belief that drawing a red ball is more likely than drawing a black ball, while also preferring lottery D to lottery C, which is consistent with a belief that black balls are more common than red.

This apparent paradox can be resolved by postulating that, where they would otherwise be indifferent between two lotteries, individuals will prefer the lottery with known probabilities to one where the probabilities are unknown – lottery A, in the first case, where the number of red balls is known to be exactly 30, and lottery D in the second case, where the total number of black and yellow balls is known to be exactly 60.

The existence of uncertainty aversion is consistent with, but distinct from, peak aversion. Peak aversion represents a preference for equal final utility values in all states of the world, whereas uncertainty aversion is a preference for known probabilities for any given state of the world occurring. It is, therefore, possible to be both peak averse and uncertainty averse, and there is little practical relationship between the two concepts.

3.4.4 Peak Aversion and Rank-Dependent Utility

Quiggin (1993), sets out a model of rank-dependent utility theory, as

a generalisation of standard expected utility theory in response to certain observed paradoxes. Under rank dependent utility, individuals faced with uncertainty continue to maximise a weighed sum of utilities in each state, but the weights attached to each state are no longer the raw probabilities that that state will occur, as in standard expected utility maximisation.

Instead, rank-dependent utility maximisers weigh state based utilities according to a transformed probability distribution, which reweighs the possibility of each state, while maintaining the ranking of the probability of each state relative to any other.

Under peak aversion, on the other hand, an individual calculates the expected utility of a lottery using linear probability weights but with a nonlinear function of the state-based utility values. As such, peak aversion and rank-dependent utility operate on completely different elements of the choice-under-uncertainty problem; rank-dependent utility modifies the objective probabilities associated with a state to capture inaccurate beliefs about the relative likelihood of unlikely events, while peak aversion modifies the objective utility values associated with a state to capture a belief about the desirability of smoothed utility.

It is therefore possible to be both peak averse and to calculate expected values according to rank-dependent utility, but we do not attempt to set out such a model here.

3.4.5 Peak Aversion and Loss Aversion

We now turn to the relationship between peak aversion and the notion of loss aversion proposed by Kahneman, Knetsch, and Thaler (1991).

Loss averse preferences cause individuals to assess risks based on a systematic misestimation of the probabilities associated with different states of the world. Individuals will, Kahneman et al. demonstrate, overestimate the objective likelihood of unlikely events and underestimate the likelihood of relatively more common occurrences when estimating expected utility. This element of the loss aversion thesis can be viewed as a restricted form of generalised rank dependent utility, as discussed above in 3.4.4, and can be distinguished from peak aversion on similar grounds – peak aversion deals with a rescaling of expected values based on the final utility associated with the payoffs, rather than the probability of their occurring.

In addition to the predicted nonlinear rescaling of probabilities in computing expected utility values, a loss averse individual, according to Kahneman et al., measure utility relative to their existing endowment of goods and weighs losses relative to this reference point more heavily than gains, in a

manner inconsistent with traditional diminishing marginal utility of income models. A loss averse individual will, therefore, value goods¹⁰ with which they consider themselves already endowed more highly than goods they have the opportunity to add to their endowment.

On this basis, loss aversion generates risk aversion-like behaviour – a refusal to engage in actuarially fair gambles for instance – while remaining observationally distinguishable in many cases, such as risks involving as-yet unendowed assets.

Can loss aversion explain the kinds of behaviour which motivated the creation of the peak aversion model? We will begin to answer this question by considering how we might distinguish peak averse behaviour from that of a loss averse individual.

Take the case of an individual with a utility function, $U(C)$, based only on consumption, and an initial consumption endowment C_0 . We will assume, initially, that the individual is loss averse, but neither risk nor peak averse. This individual will reject a lottery which pays x or $-x$ with a probability of 0.5, as a consequence of the greater weight placed on the loss of x from their original endowment. But the same individual will be indifferent between two lotteries, one of which pays $2x$ half the time and 0 the other half, and the other paying x with certainty. This result arises because none of the payoffs in the second set of alternatives lead to a reduction in the individual's endowment, and as such the individual simply acts as an unweighed-expected utility maximiser.

A peak averse, but loss and risk neutral, individual, on the other hand, will reject the first lottery, and will choose x with certainty over the lottery across $2x$ and 0. In fact, in the single-variable utility case, a risk neutral but peak averse individual behaves identically to a risk averse but peak neutral individual with a marginal utility of consumption equal to the derivative of the aggregator function with respect to single-state utility, $\frac{df}{dU_s}$.

Turning, then, to the health insurance case modeled in 3.3, above, we can show that, while a peak averse and risk averse individual with marginal utility of consumption independent of health will purchase a positive amount of insurance against a nonpecuniary health risk, a loss averse individual will not.

Consider a risk neutral, loss averse individual with utility determined by health, H and consumption, C , with an initial endowment of $H = 10$ and $C = 10$, and an initial utility level, U_0 , of 20. This individual calculates changes in utility based on the rule:

¹⁰With goods being broadly defined to include, for instance, health and political rights.

$$\Delta U = \Delta C + \Delta H \quad (3.10)$$

where $\Delta C \geq 0$ and $\Delta H \geq 0$, and

$$\Delta U = 2(\Delta C + \Delta H) \quad (3.11)$$

where $\Delta C < 0$ and $\Delta H < 0$.

So that losses relative to the endowment are weighted twice as heavily as gains.

Now, in the case where the signs on ΔC and ΔH do not match we have two possibilities, each consistent with the general model of loss aversion. The first is:

$$\Delta U = \Delta C + \Delta H \quad (3.12)$$

where $\Delta C + \Delta H \geq 0$, and

$$\Delta U = 2(\Delta C + \Delta H) \quad (3.13)$$

where $\Delta C + \Delta H < 0$.

Showing the case where losses are measured relative to a particular reference point measured in final utility. The second is:

$$\Delta U = \Delta C + 2\Delta H \quad (3.14)$$

where $\Delta C \geq 0$ and $\Delta H < 0$, and

$$\Delta U = 2\Delta C + \Delta H \quad (3.15)$$

where $\Delta C < 0$ and $\Delta H \geq 0$.

Showing the case where each argument of utility is measured relative to its own reference point. This second, stronger sense of loss aversion is probably closer to the concept presented in Kahneman, Knetsch, and Thaler (1991).

We assume, as in 3.3, above, that the individual faces a lottery such that:

- in state 1, which occurs with probability $p_1 = 0.50$, the individual remains uninjured and receives $C = 100$ and $H = 100$; and
- in state 2, which occurs with probability $p_2 = 1 - p_1 = 0.50$, the individual suffers a nonpecuniary injury and receives $C = 100$ and $H = 81$

and that insurance can be purchased at an actuarially fair price.

In the case of the second, individual good reference point, form of loss aversion described above, the individual's marginal utility of consumption for transfers *into* either state is one, and their marginal utility for transfers *from* either state is two, for transfers of any size. This means that no insurance will be purchased.

In the first described, weaker loss aversion case, where the reference point is a function of final utility rather than the component goods, marginal utility of consumption for transfers from state 1 to state 2 is two in both states for transfers less than or equal to one, with marginal utility in state 2 falling to one for transfers greater than one. This means that the individual will be indifferent between not purchasing actuarially fair insurance and purchasing any level of insurance up to one. If insurance is priced even marginally above the actuarially fair rate, or if the individual is risk averse in addition to being loss averse, no insurance will be purchased.

We can see, then, that neither version of loss aversion provides an affirmative motivation for insuring nonpecuniary risks. We might attempt to link this behaviour to loss aversion by changing the way we define our reference points, so that, for instance, future consumption endowments, but not future health endowments, are exempt from the endowment effect. If we accept the psychological plausibility of such changes to the loss aversion model, peak aversion nonetheless remains an effective, tractable means of representing the effect of loss aversion in such a context. We suggest that any model of loss aversion which is sufficiently general to capture the types of behaviour explained here with reference to peak aversion will be capable of explaining almost any imaginable pattern of responses to risk on the basis of *post hoc* manipulation of the baseline from which endowments are measured. In situations where we are attempting to explain an observed aversion to risks, in the sense of unequal distributions of utility across states, peak aversion represents a more parsimonious, more quantifiable and more falsifiable alternative to an extremely generalised loss aversion model.

Finally, we note in passing that to the extent asymmetric measurement of utility relative to some objectively detectable reference point explains behaviour not predicted by the peak aversion model, it is possible to incor-

porate a reference point framework within the peak sensitivity model set out above, but we do not attempt to do so here.

3.5 Time Based Peak Aversion

In this section we will consider an alternative application of the peak aversion concept defined above. Up to this point, we have considered peak aversion as an aspect of choice under uncertainty, reflecting a desire to achieve a relative smoothness in utility across different states of the world. We motivated this discussion, in 3.1, above, by reference to the existence of health insurance for nonpecuniary injuries (or for amounts in excess of the pecuniary value of a loss) and irreplaceable goods.

Here, we will suggest that a similar desire for smoothness in one's utility, not across states of the world but through time, is both intuitively appealing and consistent with observed behaviour.

The intuitive appeal of a preference for smoothness in consumption through time is perhaps captured by the suggestion of Nozick (1989)¹¹ that a life of misery, ending with a single moment of incredible happiness, while it might deliver higher average utility than a normal life, would be unlikely to be judged as better. This claim captures our sense that, much as most of us would rather not risk terrifying lows for the chance at dizzying highs, we would rather not experience both crammed into a single lifetime where a middle course is available.¹²

As explained in relation to state based peak aversion in 3.4.1, above, in a univariate consumption-based utility model, the behaviour induced by peak aversion is identical to that suggested by traditional, diminishing marginal utility-based risk aversion. That is, both peak averse and risk averse individuals will, *ceteris paribus*, seek to equalise consumption across states and through time. However, once we introduce multivariate utility we can expect to observe deviations from equal consumption expenditure in time and state space in peak averse individuals. This is demonstrated in relation to nonpecuniary health insurance across states in 3.3, above, but it is equally true of time based peak aversion.

A peak averse individual, when faced with predicted changes to non-pecuniary sources of utility, would tend to alter consumption expenditure

¹¹At pp. 100–102.

¹²We might, of course, imagine the opposite preference, for ups and downs rather than consistency in life's rewards, at least across some utility ranges. Our measures of peak aversion easily accommodate these kinds of 'peak-loving' preferences, though we suggest they are likely to be the exception rather than the rule.

inversely to the size and direction of those changes, so as to equalise their instant utility through time. On this basis, we would tend to observe unequal consumption expenditure through time as individuals spend ‘counter-cyclically’ in response to nonpecuniary changes in their utility.

If we wish to confirm the existence of this time based peak aversion observationally, we might note the failure of most individuals, even those in rich nations with deep capital markets, to equalise consumption across their life-cycle as predicted by diminishing marginal utility. In fact, if SWB data is to be believed, periods of highest consumption correlate with periods of lower reported utility. This behaviour is inconsistent with utility maximisation under diminishing marginal utility of in-period consumption, but consistent with an attempt to minimise exogenously-caused variations in utility through time using consumption expenditure.

The low consumption expenditure of university students, to take one commonly observed example of this phenomenon, is traditionally explained with reference to their inability to access capital markets. We note, however, that a nontrivial proportion of Australian university students prepay their interest-free university loans, graduate with positive savings and/or have access to informal lines of credit through family or credit cards. Further, were there a widespread demand for consumption-smoothing personal loans from university students, the oft-cited difficulty of borrowing against human capital (which has not prevented the existence of fertile markets in loans towards fees in many jurisdictions) would be insufficient to prevent some sort of financial product whereby students borrow against future wages becoming common, if not ubiquitous.

However, if we view low student expenditure levels through the lens of time based peak aversion, it can easily be explained as a result of high non-pecuniary utility during one’s student years. As a rational response to these high levels of leisure and/or learning, peak averse students may rationally delay consumption expenditure so as to smooth their utility through time.

Similarly, we might view individuals whose consumption expenditure rises as they age as peak aversively responding to declining health through time, so that increases in expenditure offset falling health so as to generate a relatively smoothed lifetime utility profile.

In any case, whether or not one views the above intuition and examples and generally applicable, a time based peak sensitivity model, like the state based peak sensitivity outlined above in 3.3, is not conditioned on the universality of peak aversion. Rather, such a model simply allows a minor modification to the aggregation of utility through time so as to admit peak averse, peak neutral and peak loving preferences, rather than assuming universal peak neutrality. This procedure is, in many ways, comparable to the

existing practice of discounting future utility to reflect positive pure rates of time preference.

3.5.1 Modeling Time Based Peak Aversion

We will now present a simple model of time based peak aversion in discrete time. It is, as will become obvious, closely related to the state-based model presented above, in 3.3, and the two are easily integrated.

Consider an individual with an instant utility function, capturing utility at time t , of $U_t = U(C_t, H_t)$. Initially assuming a zero rate of time preference, the individual's total (aggregate) utility, U_A , from the present at $t = 0$ to some future point time $t = T$ is, in the absence of peak aversion, given by:

$$U_A(C, H) = \sum_{t=0}^T U(C_t, H_t) \quad (3.16)$$

so that total utility is simply the unweighed sum of the instant utility values experienced over the period. In the presence of time based peak aversion, though, total utility becomes:

$$U_A(C, H) = \sum_{t=0}^T f(U(C_t, H_t)) \quad (3.17)$$

where $\frac{\partial f}{\partial U} > 0$ and $\frac{\partial^2 f}{\partial U^2} < 0 \forall U \in \mathbf{R}_+$ for the peak averse case, and $\frac{\partial f}{\partial U} > 0$ and $\frac{\partial^2 f}{\partial U^2} > 0 \forall U \in \mathbf{R}_+$ for the peak loving case.

This concave transformation of U -values (in the peak averse case) leads the individual, where possible, to attempt to equalise utility between states. In the presence of a nonzero rate of time preference, δ , this simply becomes:

$$U_A(C, H) = \sum_{t=0}^T f((1 - \delta)^t U(C_t, H_t)) \quad (3.18)$$

where we assume that smoothing is applied to the discounted stream of utilities, rather than discounting being applied to the smoothed stream. This gives us the standard result that, at an optimum, the marginal total utility of consumption at time t , $\frac{\partial U_A}{\partial C_t}$, is equal to the marginal total utility of the following period weighed by the rate of time preference, $(1 - \delta) \frac{\partial U_A}{\partial C_{t+1}}$. This

leads to the somewhat surprising outcome that the degree of front-loading of consumption implied by the pure rate of time preference is independent of the degree of peak aversion, f'' .

It is, likewise, relatively trivial to incorporate both time and state based peak aversion into a single model, and this incorporation, suggesting as it does a common attitude to smoothness across both states and times, is intuitively appealing. As such, for the purposes of this chapter, we will assume that a common rate of peak aversion, determined by the functional form of the aggregator function f , governs both time and state based peak aversion, though of course it would be a relatively trivial matter to allow these two phenomena to be characterised by separate functional forms.

An individual with utility governed by consumption and health and who experiences generalised peak aversion will have a total utility function, U_A across S_t states at any time t from $t = 0$ to $t = T$, given by:

$$U_A = \sum_{t=0}^T \sum_{s=1}^{S_t} p_s f((1 - \delta)^t U(C_{s_t}, H_{s_t})) \quad (3.19)$$

such that total utility is simply the discounted sum of the instant utilities of all potential states through time, weighed according to their probability and aggregated according to the peak averse aggregate function, f . This equation can also be rewritten to capture lotteries which provide a stream of T utility values through time as:

$$U_A = \sum_{s=1}^S p_s \sum_{t=0}^T f((1 - \delta)^t U(C_{t_s}, H_{t_s})) \quad (3.20)$$

We explore the normative parallels between state based and time based peak aversion in 3.12, below.

3.6 Time Based Peak Aversion and Subjective Well-being

The reader might, at this stage, question the relevance of the above discussion of peak aversion to the specific subject of subjective wellbeing, which is allegedly the focus of this thesis. In this section, we will explore the links between time series subjective wellbeing data and time based peak aversion.

Generally, time series wellbeing data (or panel data from multiple time periods) is aggregated on the basis of arithmetic mean or, equivalently, according to total subjective wellbeing over a period. Not only does this raise issues as to linearity of scale, as discussed in 1.6.5 and 2.5, above, it also, in the language of this chapter, presupposes peak neutrality of the surveyed individuals' attitude to utility through time.

We suggest, consistent with the view outlined in 3.5, above, that many individuals are likely to have a preferences across streams of utility, and therefore streams of subjective wellbeing, even where those streams yield identical arithmetic means, and that any such preference should be properly reflected when aggregating an individual's subjective wellbeing through time.

This is, in some sense, the inverse of the distinction between revealed preference behaviour and optimal behaviour we drew in chapter one. Much as an individual might chose a given choice of action based on decision utility, and then regret that choice based on the experienced utility it yields, a peak averse individual draws a distinction between the utility *experienced* through a period of time and how *decision* utility should rank those time profiles. But, contra chapter one, in this instance we suggest that the decision utility approach is the correct one. So a peak averse individual *actually experiences* the extremes of utility associated with an unbalanced utility time profile, and in no sense is that experience actually based on the concave transformation of instant utility suggested by the peak averse aggregator function. Rather, the aggregator function, f , is a decision rule, used to scale utility profiles *for the purpose of ranking them*, according to a higher-order preference as to how instant utility is best distributed through time.

As such, we can not expect that an individual will report their subjective wellbeing taking into account their level of peak aversion, any more than they will report instant utility values discounted based on their rate of time preference measured from some point in the past. It is, therefore the role of the surveyor the modify that data so gathered to take into account the individual's higher-order preferences for smoothness in utility through time.

On this basis, we propose that time series subjective wellbeing data be subject to a concave transformation, similar to that proposed for utility in 3.5, above. In the absence of sensible estimates of the values of the coefficients of peak aversion, we propose sensitivity testing the results of SWB research for a range of plausible functional forms for the aggregator function, in much the same way as existing cost benefit analysis is sometimes tested for a range of discount rates.

We note that the consequence of performing a concave transformation on subjective wellbeing data would be to decrease the relative importance

of high outlying reported SWB-values, while increasing the weight placed on low outliers. The former change would tend, somewhat, to counteract the effect of nonlinear mapping as discussed in 2.6, above, while the latter change would tend to increase the changes required due to nonlinearity in mapping from utility to subjective wellbeing.

These consequences – high SWB-values becoming less significant while low values become more so – link to our discussion on peak aversion and interpersonal equity in 3.12, below.

3.6.1 Kahneman’s Integral Utility Approach

Kahneman, Wakker, and Sarin (1997) present a detailed model of subjective wellbeing aggregation in continuous time. While this model deals explicitly only with intraday reported wellbeing, we will contrast it to the general time based aggregation framework under peak aversion suggested above.

Kahneman et al. demonstrate that, under certain assumptions, an individual’s aggregate utility is given by the integral of a transformation of their ordinal instant utilities through time. Under this approach individuals are indifferent between a period at a zero level of subjective wellbeing (what Kahneman et al. referred to as ‘an hedonically neutral state’) and an equal period of time divided evenly between utilities of x and $-x \forall x \in \mathbf{R}_+$. This is equivalent to conditioning preferences as peak neutral.

This approach turns on the use of the hedonically neutral state as a ‘true’ zero value, about which other utility values are ordinally distributed. This state also retains its zero value in any mapping from ordinal to cardinal utilities. The key to the significance of the hedonically neutral state is that:

“The theory assumes that the instant utilities are ordinal except for the zero [hedonically neutral] point... the transformation [of nonzero ordinal utility values] yields a ratio scale”

which is a difficult claim to interpret. Ordinality and cardinality describe relationships between values, rather than values themselves, so the notion of a set of ordinal data containing a single cardinal value presumably implies that the value in question is accurate according to some external scale - that is, the hedonically neutral state is a ‘zero’ according to some preexisting scale. And it is also this ‘true’ zero value which satisfies the prerequisite for generating a ratio scale. In our view, however, it is not accurate to view an hedonically neutral state as capable of representing a zero-point capable

of founding a ratio scale. There is no sense in which this ‘zero utility’ measure represents a zero ‘of’ any countable concept, even if we treat it as meaningfully cardinal, it is surely closer to degrees Celsius than degrees Kelvin, and as such it cannot be used as the basis for a mapping of ordinal data to a ratio scale.

On this basis, we suggest that the approach of Kahneman et al. falls short of convincingly rejecting a peak averse aggregation function through time.

3.7 Summary Measures of Peak Sensitivity

Given that peak aversion bears the same relationship to state-specific utility as risk aversion does to state-specific consumption, we can coopt much of the architecture developed for describing levels of risk aversion into service as summary measures of peak aversion.¹³

We therefore suggest a measure of the absolute level of peak aversion by analogy to the Arrow-Pratt measure of absolute risk-aversion,¹⁴ so that the coefficient of absolute peak aversion (‘APA’) is given by:

$$p_U = -\frac{f(U)''}{f(U)'} \quad (3.21)$$

where U is evaluated using the bundle consumed at the point in question.

Similarly, a relative measure of peak aversion can be derived on the basis of the comparable Arrow-Pratt measure, so that the coefficient of relative peak aversion, P_U is given by:

$$P_U = -\frac{U f(U)''}{f(U)'} \quad (3.22)$$

where, as above, utility is evaluated according to the relevant consumption bundle.

We can, finally, construct a peak aversion measure comparable to the certainty equivalent and risk premium associated with a gamble by measuring the expected utility premium necessary to render an individual indifferent

¹³We note at this point that existing observational measures of pecuniary risk aversion are, in fact capturing the joint effect of peak aversion and risk aversion, as discussed in 3.4.1, above.

¹⁴See Pratt (1964).

between a given gamble and a certain payoff. This would give us a peak aversion-based certainty equivalent CE_p for a lottery giving instant utility U_1 in state $s = 1$ with probability p and U_2 in state $s = 2$ with probability $1 - p$ equal to:

$$CE_p = f^{-1}(pf(U_1) + (1 - p)f(U_2)) \quad (3.23)$$

with the peak premium, P_P , equal to the expected utility of the gamble minus the peak certainty equivalent, i.e.:

$$P_P = pU_1 + (1 - p)U_2 - f^{-1}(pf(U_1) + (1 - p)f(U_2)) \quad (3.24)$$

both for the two state case. This can easily be generalised for S states each occurring with probability p_s .

Note that these values are denominated in utility, rather than in units of consumption as the case with marginal utility of consumption-based measures of risk aversion. They can be redenominated using the monetary value associated with a given change in utility or utility level (holding nonpecuniary variables constant), at the cost of thereby incorporating risk aversion as well as peak aversion into our measure. For most pecuniary gambles, this composite measure of total aversion to imbalance in consumption between states of the world is probably the most appropriate, and it can be broken down between the portion explained by risk aversion and that explained by peak aversion as necessary.

The above measures can also be generalised to provide an indication of the strength of time based peak aversion. In the case of absolute and relative peak aversion measures, the derivatives of the aggregator function, f' and f'' remain unchanged and we simply evaluate them based on utility at a point in time rather than a particular state.

In the case of the peak certainty equivalent and peak risk premium, we evaluate the total utility provided by a variable stream of instant utility values through time against the constant level of instant utility necessary to provide the same utility. So the time based measure of the peak certainty equivalent, CE_t , for a utility profile from $t = 0$ to $t = T$ and a zero rate of pure time preference is given by:

$$CE_t = \frac{f^{-1}(\sum_{t=0}^T f(U_t))}{T + 1} \quad (3.25)$$

while the time based peak premium, P_t is:

$$P_t = \frac{\sum_{t=0}^T U_t - f^{-1}(\sum_{t=0}^T f(U_t))}{T + 1} \quad (3.26)$$

on a per-period basis. Which can be modified to take account of nonzero rates of time preference, or to include both forms of peak aversion, as in equation 3.19, above.

The above measures can be used to provide a sense of the strength of an individual's peak aversion (or peak love) and the calculated premia can be measured observationally, based on revealed preference across utility profiles through time and across states of the world. Observed levels of health insurance, for example, given the measured utility consequences of a health state, the predicted reduction in marginal utility of consumption while in ill health and the premium levied by the insurance company, enable us to determine not only that peak aversion exists but to estimate its strength. We consider another means of empirically estimating the strength of peak aversion in 3.11, below.

Finally, note that the aggregate total utility of a peak averse or peak loving individual is a decision utility rather than experienced utility measure. As such, differences in total utility produced by the aggregator function, f , whether across states or through time, do not have cardinal significance. We can say that lottery A is preferred to lottery B, but not that is preferred by a particular number of utils, as total utility is, *ex hypothesi*, not linear in underlying instant utility.

3.8 Peak Aversion and Bounded Utility

A standard argument for the upper-boundedness of utility involves a modified version of the St Petersburg gamble.¹⁵ The classic version of the gamble involves determining willingness to pay to engage in a gamble whereby the return is given by $\$2^n$, where n is the number of consecutive times a fair coin comes up heads. It is easy to show that the expected return from this gamble, and hence the willingness to pay for an expected income maximiser, is infinite. However, once diminishing marginal utility of income is taken into account, willingness to pay becomes finite.

Cowen and High (1988) discuss a modified version of the St Petersburg gamble and note that, with a sufficiently high rate of increase in the payoff (such as $\$2^{g(n)}$, $g(n) > n$ for instance) willingness to pay once again becomes unbounded even in the case of an expected utility maximiser.

¹⁵Formally known as the St. Petersburg-Menger Paradox, and described in detail in Cowen and High (1988).

A peak averse total utility maximiser, when faced with a modified St Petersburg gamble of the general form suggested by Cowen and High, will, at a minimum, require a higher rate of payoff growth ($g(n)$) in order for their expected total utility to become unbounded. In addition, though, certain functional forms of the peak averse aggregator function, $f(U)$, such as $f(U) = \log(U)$, are consistent with utility which is bounded in relation to increases in utility associated with a particular state, while remaining unbounded with respect to changes in other states and, where the set of possible states is infinite, unbounded overall.

This preference structure is consistent with a finite willingness to pay for any version of the St Petersburg gamble, while remaining responsive to changes in utility in states in which instant utility is not at its upper bound. A particularly clear example of this possibility is provided by a peak averse aggregator function of the form:

$$U_A = \sum_{s=1}^S \min(U(C_s), \bar{U}) \quad (3.27)$$

where \bar{U} represents the maximum utility value, for purposes of preference-ranking lotteries, which can be associated with any one state. Peak aversion in this form neatly solves any form of the St Petersburg paradox, while retaining flexibility in states where utility lies below \bar{U} .

3.9 Peak Aversion and Quality-Adjusted Life Years

In this section we will consider the calculation of the value of a Quality Adjusted Life Year ('QALY') as a case study for the practical application of the peak aversion model set out above. Since QALYs assume expected utility maximisation, failure of this condition to hold will lead to systematic biases in valuation of health states and predictable differences between competing approaches to valuation.

A Quality Adjusted Life Year is a unit used in performing cost benefit analysis¹⁶ on market interventions intended to extend life or reduce impairment. A value is attached, through one of the methods outlined below, to a year of fully healthy life, and this figure is scaled proportionately in response to the level of disability experienced by the individual whose lifespan has been altered, or in response to changes in the level of disability.

¹⁶Strictly, QALYs are more often applied using cost utility or cost effectiveness methodology. For a general discussion of these and other terms in this section see Drummond and McGuire (2001) at pp 22-46.

Interventions will be beneficial where their cost lies below the value of the extra QALYs (including changes to the quality of existing years) they are projected to produce, or, in the presence of a binding budget constraint on overall health care expenditure, where the cost per additional QALY lies below the cost of the least efficient treatment currently being funded.

The value of a life year itself is calculated, as outlined by Hirth, Chernew, Miller, Fendrick, and Weissert (2000), on the basis of either a contingent valuation, where an individual is asked to estimate the price at which they would accept a fixed increase in their risk of death, or using revealed preferences in relation to occupational risk, non-occupational safety and human capital investment.

The key method for calculating the proportional degree of impairment associated with a given condition is the ‘standard gamble’. An individual is asked to assess a choice between life with the disease or condition in question and a lottery between full health and instant death, and to provide the probability distribution between the two payoffs which would leave them indifferent between the lottery and the status quo. This probability value is then used as the percentage weighting for setting the value of a year of life with that condition relative to a year of full health. Alternatively, under the ‘time tradeoff’ approach to valuation, patients can be asked to assess the proportion of life years with the condition they would sacrifice in return for the condition’s removal. We consider the predicted distinction between the standard gamble and time tradeoff in the presence of peak aversive preferences in 3.11, below.

We note that the structure of utility implicit in the QALY calculation is additive utility in discrete time, which bears comparison with our discussion of peak aversion through time, above. We will therefore consider the role of both time and state based peak aversion in assessing the accuracy of both contingent valuation estimates of the value of a life year and of the standard gamble approach to determining degree of impairment.

Pliskin, Shepard, and Weinstein (1980) show that, in addition to the standard axioms of expected utility theory, this QALY model requires independence of utility between health and life years, risk neutrality over years of life and that tradeoffs in lifespan are constant with respect to the proportion of remaining life gambled.

This second condition, risk neutrality across gambles denominated in life years, is inconsistent with the presence of either state or time based peak aversion.¹⁷

¹⁷In fact, this thought experiment represents a useful conceptual means of linking the two forms of peak aversion – how should an individual who prefers consistency in utility both across states and through time value gambles across life extension?

3.9.1 Peak Aversion and the Value of a Life Year

We begin by considering the contingent valuation approach to valuing a (non-quality adjusted) life year. Consider, initially, an individual who is faced with a choice between their status quo with certainty and a lottery offering, with equal probabilities, an increase in wealth, $\$X$, and immediate death. In order for a peak averse individual to be indifferent between accepting and rejecting this lottery, it must be the case that their total aggregate utility through time and across the two states is equal to their existing estimated total lifetime utility. That is, if we initially assume a zero rate of time preference, that $f(0) = 0$ and normalise the instant utility of being dead to zero,¹⁸ and the status quo endowment of consumption and health is constant at C_0 and H_0 respectively, then we have:

$$\begin{aligned}
 U_A(C_0, H_0) &= \sum_{t=1}^T f(U(C_0, H_0)) \\
 &= T f(U(C_0, H_0)) \\
 &= 0.5 \sum_{t=1}^T f(0) + 0.5 \sum_{t=1}^T f(U(C_0 + x, H_0)) \\
 &= 0.5 \sum_{t=1}^T f(U(C_0 + x, H_0)) \\
 &= 0.5T f(U(C_0 + x, H_0)) \\
 f(U(C_0, H_0)) &= 0.5f(U(C_0 + x, H_0))
 \end{aligned}$$

assuming that the lump sum compensation, X , is allocated optimally across all T periods such that $x = \frac{X}{T}$ and neglecting, for the purposes of this discussion, the possibility of market interest.¹⁹ Given that, for a peak averse individual, $f''(U) < 0$, and $\frac{\partial^2 U}{\partial C^2} \leq 0$ we can conclude that $\$x > C_0$, that is, an individual, even one who is risk neutral in consumption, will demand compensation which more than doubles their per-period consumption in return for accepting a lottery with a 50% chance of death, which gives an

¹⁸Can assigning a zero utility value to death serve to found a ratio scale? We concluded earlier that an arbitrarily identified “hedonically neutral state” is insufficient to generate ratio scale measures around it. If death can meaningfully be said to generate no utility at all then ratio scale measurement will be possible but, in any case, in this section we rely only on the interval scale properties of utility measures.

¹⁹We return to the question of optimal allocation of lump sum compensation in 3.10, below. The results shown here could equally be achieved by assuming a rate of time preference equal to the market rate of interest.

estimate for an unimpaired QALY which is greater than annual income.²⁰

Looking at this figure, we note firstly that the normative question implicitly underlying QALY analysis based on the statistical value of a life year is “under what circumstances will adding a year to this individual’s life increase aggregate societal welfare?”,²¹ but that the question being answered by contingent valuation surveys is “for what level of compensation would *you* risk sacrificing a year of *your* life”. These questions differ in how the expense associated with the hypothetical procedure is allocated.

Consider the following example of a society comprised of two individuals, organised according to a pure utilitarian social welfare function. We will assume the individuals have identical, peak neutral utility functions with a zero rate of time preference, $U = C_1^{\frac{1}{2}}$ and that society’s initial per-period resource endowment of 200 is allocated optimally between them such that $C_1 = C_2 = 100$ giving $U_1 = U_2 = 10$.

Now, we imagine that person 2 falls ill such that their life expectancy will be reduced by one year, and that they can be cured, with certainty, for an expenditure of $\$Y$ in the final year of their life. For what range of values for Y is investing in a cure socially optimal?

In the absence of a cure, utility for the (pre-illness) final year of person 2’s life is 10 for person 1 and 0 for person 2, making total social utility equal to 10. If a cure is purchased, total utility will be equal to $2U(\frac{200-Y}{2}) = 2\sqrt{100 - \frac{Y}{2}}$. The cure should be purchased as long as this value is greater than 10, the level of social utility in the absence of a cure. Solving for Y yields the result that a cure should be purchased provided that $Y < 150$.²²

Now, consider the contingent valuation method. One of the individuals, prior to the illness eventuating, is asked what level of compensation would make them indifferent to a gamble between the payment of compensation and the loss of a year of life. The individual (who we have assumed is peak neutral) will require compensation sufficient to leave their expected utility equal to its status quo value of 10. Given a 50% chance of zero utility, this will require utility in the compensated state of 20 and therefore compensation of 300 in addition to their existing consumption bundle of 100 units. According to this method of valuation, then, the society should be willing to purchase the cure provided that $Y < 300$.

²⁰This is consistent with contingent valuation based survey estimates, which suggest that the median value of a QALY is in the order of US\$100,000-200,000. See, for example, Hirth, Chernew, Miller, Fendrick, and Weissert (2000).

²¹As noted above, the analysis being conducted where there is a binding budget constraint on health spending is somewhat different.

²²We assume throughout this discussion, for ease of notation, that a strict increase in utility is necessary to justify investment in a cure.

We can incorporate peak aversion into this analysis in one of two ways. Firstly, by noting that we would receive the same results in relation to risk neutral individuals who had a peak averse aggregator function given by $f(U) = U^{\frac{1}{2}}$. Secondly, we can point out that peak averse individuals would display the same inconsistency between social welfare optimising and contingent valuation based evaluations in relation to compensation denominated in final utility rather than consumption – leading to an even greater divergence between the two methods where individuals are both peak averse and risk averse.

What accounts for this difference between the socially optimal value of a life year and that produced by contingent valuation? In the first, social optimum, calculation, the costs of the proposed intervention are distributed optimally, while in the second, contingent valuation case, compensation is being used to equalise individual, rather than social utility. In situations where returns to compensation are diminishing, whether due to risk aversion, peak aversion, or both, this will lead to an overestimation of the value of a life-year.²³

3.9.2 Peak Aversion and Quality Adjustment

We will now turn to procedure for determining the severity of an impairment. As noted above, this is often undertaken by finding the probability weighting on a lottery between full health and immediate death which leaves a patient indifferent between the lottery and continuing to experience the condition in question. Taking the case of a peak averse individual whose aggregator function $f(U)$ yields $f(0) = 0$, if we let utility while suffering from the disability equal U_d and healthy utility equal U_h , while normalising the utility associated with death to 0, we have:

$$\begin{aligned} f(U_d) &= p^* f(U_h) - (1 - p^*) f(0) \\ &= p^* f(U_h) \\ U_d &= f^{-1}(p^* f(U_h)) \\ &= f^{-1}(p^*) U_h \end{aligned}$$

where p^* is the minimum acceptable probability of a successful cure. A positive level of peak aversion, then, renders lotteries across states nonlinear

²³Bleichrodt and Quiggin (1997), consider the consequences for QALY-values of a rank dependent approach to choice under uncertainty, whereby individuals' behaviour is nonlinear in probabilities rather than payoffs. As with rank dependence and peak aversion generally, this approach is distinct from, and potentially compatible with, the modifications suggested here.

in the utility payoffs, meaning that the value of p^* does not indicate the proportional relationship between the utility of being healthy and the utility of living with the disability. Given that we assume $f' > 0$ and $f'' < 0$, p^* -values obtained via standard gamble from peak averse individuals will understate the degree of disability and undervalue the QALY loss associated with the condition in question. Intuitively, individuals' relative unwillingness to accept a lottery between the wildly divergent states of complete health and sudden death biases downwards their willingness to accept a large risk of death and leads to an inaccurate estimate of their disability in situations where no such gamble is under consideration.

As such, where individuals are peak averse, existing methods will tend to overestimate the social value of a healthy life year ²⁴ (in that the rate at which the average individual would trade consumption for life years is overstated) and to overestimate the utility associated with a life year in less than full health. When dealing with interventions which will deliver additional life years at less-than-perfect health, these errors may (but will not necessarily) cancel each other out. An additional consequence of the bias in these standard gamble-based estimators is that health care interventions which improve quality of life will be unambiguously undervalued relative to those which delay mortality, as peak aversion causes individuals to overvalue life years and to undervalue improvements in quality of life.

3.9.3 Individual QALY Valuations and Social Choice

We turn now to issues of social choice underlying the QALY calculation, and show that use of QALY estimates biased as shown above can be consistent with a range of social welfare functions.

Bleichrodt, Diecidue, and Quiggin (2004) consider the equity implications for the distribution of QALYs based on each individual's existing expected lifetime allocation of QALYs, assigning weights to improvements to an individual's QALY endowment based on where that endowment ranks in society. We suggest, however, that a more holistic ethical approach be taken, whereby lifetime utility, rather than lifetime QALYs, serves as the basis of individual weights. This links to our discussion of peak aversion's relationship with social welfare theory in 3.12, below.

Our starting point is to note that an unweighted QALY approach, which simply seeks to maximise the sum of individual utilities,²⁵ implicitly applies

²⁴the value at which interventions total utility increasing in expectation.

²⁵Strictly, QALY maximisation undertaken from an extra-welfarist perspective – see for example Brouwer, Culyer, Van Exel, and Rutten (2008) and the general review provided by Tsuchiya and Williams (2001) – is indifferent to the total utility implications of this

a pure utilitarian social welfare function. If our preferred means of aggregating social utility is inequality averse, in the sense of calling for some level of strictly concave transformation of individual utility values prior to their aggregation (or, equivalently, if it calls for strictly convex social indifference curves), then raw QALY estimates will incorrectly estimate the degree to which we should value changes to life expectancy and disability.

In particular, if we assume, reasonably, that individuals suffering from treatable disabilities or who are likely to benefit from life-extending interventions have, on average, lower utility levels than those of the general population, then unweighed QALYs will systematically understate the benefits to a Rawlsian or inequality averse social planner of expenditure to reduce disability.²⁶

However, as we have demonstrated above, peak aversion at the individual level leads to an overestimate of the social value of a year of life and, *ceteris paribus*, of the value of a QALY. In fact, as we discuss in more detail in 3.12, below, peak aversion causes an individual to have preferences for their own welfare across states which mirror those of an inequality averse social planner in relation to individuals across society. In this sense, we can view the peak averse overstatement of the value of life-extension as preemptively incorporating a degree of concave transformation of individual utility values, potentially leading to QALYs which accurately reflect a weak form of Rawls' maxmin principle.

3.10 Peak Aversion and Tortious Damages

The question of optimal compensatory damages, particularly under the tort of negligence, is closely linked to the points we have made in relation to Quality Adjusted Life Years in 3.9, above. The underlying principle in calculating damages for negligence is that the individual or entity harmed must be restored to the position they would have occupied but for the negligent act. This is, of course, equivalent to suggesting that the level of compensation should be gauged so as to return the individual to the utility level they experienced prior to the negligent harm occurring. This distinguishes tortious compensation from optimal health insurance, which seeks only to equalise *financial* assets across states of the world.

approach. Provided we believe that total utility is linearly related to total QALYs, this difference in motivation is irrelevant for our purposes.

²⁶We might argue for the reverse conclusion on the basis that medical resources are likely to be allocated, for reasons of political economy, to conditions disproportionately affecting the relatively well-off. This effect would have to be very strong indeed to outweigh the utility-reduction associated with the illness itself.

A related question when considering the appropriate level of compensation is the optimal deterrence of negligent conduct. If the damages paid by a negligent individual are equal to the level of social harm caused by their actions, then individuals will engage in potentially negligent behaviour only where the personal benefit outweighs its social risk. We assume for the purposes of this discussion that transaction costs prevent efficient Coasian bargains from arising in the absence of optimal legal rules.²⁷

We will consider these two questions, of optimal compensation and optimal deterrence, from the point of view of the theory of peak aversion presented above.

To begin with, we will consider the situation of an individual who has suffered a nonpecuniary injury which will reduce both their long term health and their lifespan and who will receive a lump sum payment intended to fully compensate them for this disability.

Calculating the correct value of this payment will require the aggregation of a series of reductions in instant utility through time, in addition to determining the present value of the future forgone years of life. We will assume, for the purposes of this discussion, the existence of perfect capital markets and a market interest rate equal to the individual's rate of time preference across monetary flows.

The reduction in utility through injury can be characterised as a stream of negative utility values, each of which can be measured in terms of the monetary compensation required in that period to return the individual to their pre-injury utility level.

We further note that loss aversion, as outlined in 3.4.5, above, predicts a large distinction (over and above the predicted gap between the equivalent and compensating variations) between the amount of compensation an individual would require *ex ante* to accept the injury and the amount required *ex post* to compensate them for it. This difference arises as a result of endowment effects over existing allocations of health, and has led to a ban, in United States courtrooms, on lawyers couching compensation enquiries in *ex ante* terms.²⁸

If we assume, as US courts appear to, that compensation is best viewed as an *ex post* restorative measure rather than an *ex ante* willingness to pay, we note that the contingent valuation approach to lost expectation of life, as outlined at 3.9, above, is therefore inappropriate. Rather, we wish to determine what level of compensation, optimally allocated across the victim's remaining years of life, would yield total utility equal to that

²⁷Calabresi (1975) provides a considerably more detailed outline of this view.

²⁸See Sunstein (2000).

which would have been experienced had the accident not occurred. Even in the presence of peak aversion, this will lead to per-annum compensation equal to the injury-related fall in utility divided by the marginal utility of consumption.

In the absence of a reduction in expected lifespan, this calculation would involve equal utility in each remaining period, and as such would not be influenced by the degree of peak aversion. Once we consider the extra years of life for which utility will be zero, though, compensation will need to generate a utility profile which condenses pre-injury utility into a shorter period of time. As such, compensation will be rising in the degree of peak aversion. To the extent that negligence law typically functions on the basis that a negligent individual must “take their victim as they find them”, that is, to include the effect of their idiosyncracies in determining proper compensation, this seems like a reasonable outcome, subject to the discussion in 3.12.3, below, as to the proper normative role for peak averse preferences.²⁹

A final note on this point: while the calculation of tortious compensation involves equalising utility across states, it does not imply equality of marginal utilities. In fact, because consumption will be higher in the compensated state, we would expect marginal utility of consumption to be lower, and that the victim’s utility would respond very differently to shocks to consumption pre-and-post accident. Depending on the structure of the shocks, and on the sign on $\frac{\partial^3 U}{\partial C^3}$, these differences could increase or decrease expected post-injury utility.

Turning to optimal deterrence, the analysis of the gap between the contingent valuation approach and the socially optimal expenditure outlined in 3.9, above, is equally relevant here. Where an individual experiences peak or risk aversion, fully compensating them will require more expenditure than the true social cost of the negligent act. As such, damages awards will lie above the optimally deterring level. This may suggest a damages award based on the mean of these two divergent measures, or that a state-run compensation scheme enable a distinction between damages levied on the perpetrator and compensation paid to the victim.

3.11 Estimating Peak Aversion

In the material above we have pointed to a number of common behaviours and market phenomena which, intuitively, seem to be best ex-

²⁹Note that we suggest in 2.4.6, above, that years of life are likely to be undervalued in the presence of mean reversion. The effect of peak aversion here may be to correct for this undervaluation.

plained by a preference for smoothness in the distribution of utility between different states of the world – by a genuine aversion to risk rather than one arising solely from the diminishing marginal utility of consumption. In this section we take advantage of survey data which elicits utility estimates for situations between which expected utility maximisers ought to be indifferent, but which involve unequal payoffs across states. By considering the size of the premium associated with the certain outcome over and above an uncertain outcome which delivers equal expected utility we can arrive at an estimate of the size and structure of the average level of peak aversion in the surveyed population and, by extension, in the population generally.

The instruments we consider in estimating the degree of peak aversion are the Quality Adjustments estimated using Standard Gamble methodology (‘SG QALY’), described above in 3.9, and the Quality Adjustment calculated via Time Tradeoff (‘TTO QALY’),³⁰ which is currently the quality adjustment elicitation method preferred by the United Kingdom’s National Institute for Health and Care Excellence.³¹

As outlined above, the QALY embodies an estimate of the proportion of the aggregate annual full-health utility associated with a year spent in a particular health state. The SG QALY elicits this estimate by asking the subject to choose a probability distribution for a lottery between full health and instant death at which they would be indifferent between that lottery and living with the reduced health state in question.

The TTO QALY arrives at a theoretically identical estimate of the proportional reduction in utility associated with a health state by requesting that the individual surveyed choose a proportional distribution of *time* (rather than of risk as in the SG case) between full health and death which at which they would be indifferent between accepting the proportionally shortened lifespan or living the full original lifespan in the reduced health state.

Imagine, by way of example, a health state which we have presupposed reduces expected utility of the sufferer by half. We would predict that an expected utility maximising sufferer (or hypothetical sufferer) of this condition would be willing to risk (strictly: be indifferent to risking) a 50% chance of death in order to achieve a 50% of being completely cured. We would similarly expect a sufferer who had, say, 2 years of life left in the reduced health state to be willing to sacrifice (strictly: be indifferent between sacrificing and not sacrificing) half of those years if the reduced lifespan could

³⁰Note that throughout this section we use ‘QALY’ to refer to the quality adjustment associated with a particular health state, rather than the value of a life year itself, which we have discussed above.

³¹Strictly, NICE calls for quality adjustment assessment via EQ-5D methodology, which itself prefers TTO-based estimates. See NICE (2013).

be experienced in full health.³²

When comparable surveys using TTO and SG QALYs are carried out, we do not, in fact, observe equivalence in the expected number of life years people are willing to trade off to avoid a particular condition. Rather, we find that the adjustment estimates produced via the standard gamble method are systematically higher than those produced under time tradeoff – that is, the average individual is willing to sacrifice a larger portion of their remaining life years than the equivalent proportional risk of death in order to remove the same condition.³³

This difference can not be adequately explained on the basis of even arbitrarily high discount rates being applied by people performing the time tradeoff, as the gap between the two estimates persists, and remains relative constant, even when the total period over which the proportional time tradeoff is being contemplated is radically truncated, so that the final period being valued is a matter of weeks rather than years away.

This failure of individuals to equally rank apparently identical distributions of utility is perfectly consistent with the existence of an on-average positive level of peak aversion among the surveyed population: Because standard gamble QALYs involve an unequal distribution of utility across states – utility is obviously heavily concentrated in the ‘alive and healthy’ scenario relative to the ‘instant death’ scenario – while the utility flow in the TTO is constant across states, we would predict that a peak averse population would systematically prefer the certain utility distribution over the uncertain proposition and, equivalently, that they would be willing to sacrifice some level of expected utility in order to guarantee a particular level of aggregate utility.

We might also imagine that this failure of TTO and SG QALYS to produce identical estimates is a result of some other aspect of risk preference: Bleichrodt (2002), for instance suggests that the difference can be accounted for on the basis of loss aversion. As we argue above in 3.4.5, treating these phenomena as an example of loss aversion requires us to tinker with the original definition of the term: there is no evidence that survey recipients are incorrectly scaling probabilities, and since both sides of a standard gamble involve states with utility values below that of the healthy respondent, we

³²Note that the SWB literature, as discussed in chapters one and two, above, predicts and documents significant differences between estimates of the disutility of health states elicited from sufferers of those states, who are likely to have habituated to the condition, and purely hypothetical sufferers, who have not – see Headey, Muffels, and Wagner (2010), for example. The convention towards using hypothetical rather than experienced assessments of health states thus generates a downward bias on QALY estimates, which warrants further examination at a later date.

³³See Brazier, Deverill, and Green (1999) for a survey of the studies on this subject.

need to choose an arbitrary baseline based on the unhealthy state in order to generate endowment effect-driven loss weightings. As we suggest above, loss aversion can explain such results only *ex post* as a descriptive theory, making the more general and more parsimonious model of peak aversion more useful as a predictive tool.

Similarly, we could suggest that SG QALYs are influenced instead by the failure of constant utility of remaining life years condition to hold. Much of the analysis that follows can be interpreted on this basis, much as the examples of peak aversion above can be explained away via a panoply of different diminishing marginal utilities. We suggest that, once one has a theory of “the diminishing marginal utility of almost everything”, a unitary theory of diminishing marginal utility is more appealing. Ultimately, though, any attempt to demonstrate peak aversion in a particular domain is vulnerable to the critique that the results are potentially specific to that domain.

By examining a data set showing the extent to which SG QALYs exceed TTO QALYs for identical conditions, we are in a position to estimate the size of this ‘certainty premium’ across equal expected utility distributions. Just such a data set underlies Tsuchiya, Brazier, and Roberts (2006), which draws on a survey of 101 healthy members of the public in order to address an unrelated question as to the correct method of administering surveys in order to elicit valuations for competing sets of health state summary descriptors.³⁴

Relevantly for our purposes, Tsuchiya, Brazier, and Roberts (2006), each of the 101 individuals surveyed was asked to estimate both a TTO and SG QALY value for a series of health states, with each health state described according to one of two widely-used summary descriptors, the EQ5D-3L or the SF6-D.³⁵ Full details of the sample selection and the treatment of unresponsive participants are set-out in Tsuchiya, Brazier, and Roberts (2006) at pp337-340.

The overall arithmetic mean SG quality of life estimate provided by the sample population across the eight health states considered was 0.474, while

³⁴This summary does not really do justice to the content of Tsuchiya, Brazier, and Roberts (2006), but a more detailed explanation of its implications requires more space than is appropriate here. We will, however, take this opportunity to thank the authors of this paper for providing us with access to the raw data gathered in the course of their research and for their assistance in its interpretation.

³⁵Each of these descriptive systems summarise a health state by assigning its objective impact on a series of ‘dimensions’ – which can be thought of as a subset of the arguments of the sufferer’s utility function – to one of a fixed number of qualitatively described levels – see Walters and Brazier (2005) for a more detailed discussion. As outlined in NICE (2013), this approach is viewed as a more accurate means of describing a health state to a survey population than simply relying on clinical descriptions of the disease or disability, but is not otherwise significant in relation to estimating the degree of peak aversion.

the comparable estimate for the proportion of full health utility associated with the same eight health states elicited using the (risk-free) TTO method was 0.416. The higher QALY rating for the set of health states using the SG methodology is consistent both with our expectations based on a positive average level of peak aversion in the survey population and with prior observations in the QALY estimation literature.³⁶

Given the relatively small sample size, we should be cautious about placing too much weight on these exact estimates, but we will make use of them here in demonstrating how a larger sample based on the same methodology could be used to estimate summary measures of peak aversion and, in 3.12, below, to choose optimal settings for social and health policy.

The differences in evaluation of sets of equal-expected-utility states implies a particular level of peak aversion in the survey population and, by extension, in the wider population from which the survey group was drawn. Arbitrarily adopting the functional form for peak aversion suggested in 3.3, above, and normalising the utility from being dead to zero, we have evidence that for the sample group on average:

$$0.474 \cdot U(\overline{H})^{\frac{1}{n}} = (0.416 \cdot U(\overline{H}))^{\frac{1}{n}} \quad (3.28)$$

Where n is the coefficient in the function for aggregating utility across states and is increasing in the degree of peak aversion, and $U(\overline{H})$ is the utility associated with full health.

Solving for the level of peak aversion necessary to bring about this equality we have:

$$n = \frac{\ln(0.416)}{\ln(0.474)} = 1.16 \quad (3.29)$$

That is, the survey population will, on average, assess lotteries across s states according to:

$$\text{Max } U(C_s, H_s) = \sum_{s=1}^S p_s \cdot U(C_s, H_s)^{\frac{1}{1.16}} \quad (3.30)$$

Which suggests a starting point for modelling peak aversion in other real-world circumstances. We note, again, that the existence of peak aversion at this, or any, level would render existing estimates of risk aversion, and the

³⁶This approach assumes that the TTO approach is not disproportionately influenced by discounting relative to the aggregation of utility through time implicit in the SG approach.

diminishing marginal utility of consumption generally entirely inaccurate, and invalidate any inferences drawn from such estimates uncorrected for a free-standing dislike of risk.

In the section below we examine the general implications of peak aversion for questions of social choice, and then use the estimate derived here to suggest the optimal social welfare function. The social welfare function thus estimated is then used to suggest the optimal policy rule for allocation of QALYs based on patients' pre-treatment quality of life.

3.12 Peak Aversion and Social Choice

We have discussed, in relation to Quality Adjusted Life Years and compensation for negligence, the link between peak aversion's role in determining an individual's optimal utility distribution across states, and the social choice question as to the optimal distribution of utility between different individuals. In this section we will further clarify those links and suggest that they provide a justification for the adoption of an inequality averse social welfare function in a society where individuals possess peak averse preferences.

Harsanyi (1953), draws on elements of Rawls' theory of justice,³⁷ to propose a formal model whereby individuals, while behind a veil of ignorance with respect to their final positions in society, are aware that they may occupy each available position with equal probability. Based on these assumptions, he shows that individual expected utility maximising behaviour will lead individuals to choose to maximise the sum of individual utilities across all members of society.

In this chapter, however, we have proposed and estimated a modification to expected utility maximising behaviour consistent with observed patterns of behaviour, on the basis that individuals are, in response to states and time-profiles providing equal expected utility, averse to peaks in their distributions of utility through time and across states. How would these, peak averse, individuals choose to structure society from behind Harsanyi's account of the veil of ignorance?

A peak averse individual, faced with an equal possibility of being placed in any position in society, would seek to maximise their total utility across all S possible states, according to the equation:

³⁷See Rawls (1999).

$$\text{Max } U_A = \sum_{s=1}^S f(U(C_s)) \quad (3.31)$$

which departs from expected utility maximising behaviour to the extent that $\frac{df^2}{d^2U} \neq 0$. A peak averse individual, then, for whom $\frac{df^2}{d^2U} > 0$, will tend, relative to the peak neutral expected utility maximiser, to prefer forms of social organisation which deliver relatively equal distributions of utility across different positions in society.

In technical terms, a peak averse individual operating from behind Harsanyi's veil of ignorance (and otherwise granting Harsanyi's arguments as to the relationship between individual and social choice), will choose a strictly concave (inequality averse rather than pure utilitarian) social welfare function and, in particular, the degree of concavity of their chosen SWF will be exactly defined by $\frac{d^2f}{dU^2}$, which also governs the convexity of their indifference curves in expected utility space. For sufficiently large values of $\frac{d^2f}{dU^2}$, the optimal SWF approaches the Rawlsian maxmin rule.

This observation provides a defence of inequality averse social welfare functions rooted solely in individual preferences; we, as a society, should place an additional weight on the welfare of the less fortunate no less than the additional weight we place on our own welfare in our own less fortunate times and circumstances. Or, conversely, we can view peak averse individuals as acting as inequality averse social planners in relation to their own future or alternate selves, focussing disproportionately on the worst-off among them.

As discussed above in 3.9 and 3.10, where peak aversion tends to generate a demand for excess compensation in bad states of the world, this demand can be seen as introducing a degree of inequality averse social welfare aggregation into otherwise purely utilitarian calculations. If the degree of peak aversion is identical to our collective level of inequality aversion, however it is arrived at, QALYs calculated in response to peak averse preferences will be identical to those determined according to peak neutrality and the corresponding inequality averse social welfare function.

Conversely, we can suggest that the observed preference for relatively more equal rather than utility-maximising distributions (see, among many others, Nord, Pinto, Richardson, Menzel, Ubel, et al. (1999)) is inconsistent with rational expected utility maximisation on the part of the general public, but entirely consistent with the positive average degree of peak aversion derived above.

3.12.1 Estimating the Social Welfare Function

Taken together with the empirical estimate of the average degree of peak aversion derived in 3.11, above, the arguments in this section imply a specific value for the social welfare function based on the preferences of rational non-expected utility maximisers positioned behind the veil of ignorance.

We show in 3.11 that the best estimate of the individual utility function across S states based on the readily available data is:

$$\text{Max } U(C_s, H_s) = \sum_{s=1}^S p_s \cdot U(C_s, H_s)^{\frac{1}{1.16}} \quad (3.32)$$

which in turn implies that where there are N individuals in the population, the inequality averse Social Welfare Function is given by:

$$\text{Max } U = \sum_{n=1}^N \frac{1}{N} \cdot U_n^{\frac{1}{1.16}} \quad (3.33)$$

where U_n is the utility of the n th member of the population. This gives a socially optimal marginal rate of interpersonal substitution between any two individuals, x and y , equal to:

$$\text{MRS}_{xy} = \frac{U_x^{0.14}}{U_y^{0.14}} \quad (3.34)$$

which allows us to determine the slope of, and to sketch the social indifference map arising from the optimal social welfare function.

3.12.2 The Peak Averse SWF and QALYs

One of the issues faced in allocating a limited health budget across the set of available health interventions is the appropriate weighting of QALY increases which accrue to individuals with different pre-treatment endowments. In the past, the UK National Institute for Health and Care Excellence (NICE UK) has adopted the principle that “a QALY is a QALY” and that any and all distributional concerns should be set-aside in favour of maximising total expected QALYs.³⁸ For this approach to be optimal in

³⁸See, for example, Weinstein (1988). Strictly, current NICE methods incorporate a QALY weighting for end of life which might be thought of as a crude proxy for health equity weights – see NICE (2013) s6.2.10.

the presence of useful proxies for the pre-treatment allocation of utility, we would need to endorse a pure utilitarian social welfare function.

As a result of critiques along these general lines, there is currently consideration being given to modifying the valuation of QALY gains in order to explicitly take into account the distribution of those gains as well as their total value.³⁹ The proposed model does not contemplate use of all available proxies for utility, such as income, race or gender, but does consider the weighting of QALYs to take into account pre-treatment QALY endowment, which also serves as a proxy for lifetime utility. While there remains significant debate as to the most appropriate measure of pre-treatment shortfall,⁴⁰ a SWF derived on the basis of a veil of ignorance approach, that is one which takes seriously the arguments for optimising without knowledge of ones position in life as set out above, clearly calls for such a weighting to be carried out on the basis of differences in lifetime expected QALYs, since, *ex hypothesi*, individuals are imagining receiving each others' lifetime utilities, rather than some arbitrary subset. This is consistent with the "fair-innings" approach suggested by Williams et al. (1997), though Williams does not rely on a veil of ignorance construction.

We will, for the moment, take pre-treatment lifetime expected QALYs as the relevant proxy from which to calculate optimal QALY weights based on the SWF derived above, though we note that the approach shown here can easily be modified to capture alternative measures of health-related disadvantage.

Taking an individual with 82 expected lifetime QALYs as our (relatively arbitrary) numeraire, and assigning a weight of one to QALYs allocated to these individuals, we can calculate the appropriate comparative multiplicative weighting for QALYs accruing to individuals with lower expected lifetime health according to:

$$\frac{82^{0.14}}{Q_i^{0.14}} \tag{3.35}$$

where Q_i is the expected lifetime QALY endowment of the average recipient of the proposed treatment.

In the case of an individual who faces death at the age of one and will live in perfect health for that year in the absence of the health intervention under consideration, we would consider QALYs accruing to that individual as being

³⁹See NICE (2014) and the surrounding documentation for a summary of these proposals as at this date.

⁴⁰See Nord (2012) for a discussion of the arguments as between expected lifetime QALYs, pre-treatment expected QALYs and proportional pre-treatment QALY shortfall.

1.85 times as valuable as those accruing to an individual who otherwise expects to live a full, healthy life. It is trivial to extend this analysis so as to assign weights to all potential QALY values, and to use it to estimate values for the kind of non-marginal changes to QALY values that are the typical result of health interventions.

We note that the structure of the social indifference curves being used to calculate this optimal multiplier implies that the relative value of a treatment approaches infinity as the patient being treated comes arbitrarily close to dying as an infant, or, if the model is re-denominated to accommodate Nord's favoured approach of using remaining lifetime QALYs rather than overall lifetime QALYs, as any patient's lifespan without treatment approaches zero.

This extremely high weighting on treatments delivered to individuals faced with imminent death is consistent with the independently-observed 'rule of rescue'⁴¹ which suggests a strong societal preference for delivering care to patients faced with an immediate prospect of death, almost regardless of the cost.

3.12.3 Normative Significance of Peak Aversion

Finally, we will consider the normative significance of preferences generated by peak aversion as against the linear sum of the underlying utility values. When peak aversion causes us to state a preference for consistency in utility through time and across states, does that preference distort, or merely describe, the utility values a benevolent social planner ought to maximise?

There is a clear analogy, in calculating QALYs and for welfare analysis generally, between the role of peak aversion and the role of time preference. In each case the preferences in question are modified only in relation to decision utility and have no influence on experienced utility. That is, an individual with a positive rate of time preference *values* future utility less than equal current utility, but they will not, when that future utility arrives, *enjoy* it any less. Similarly, a peak averse individual discounts very high utility times or states when ranking lotteries or time paths across utility, but this discounting does not reflect a reduction in the level of utility ultimately experienced in those states or times.

There is a tension, then, in relation to both peak aversion and pure time preference, between giving weight to an individual's genuinely-held decision-rules for ranking future and alternate events, and seeking to maximise the level of utility they actually experience. We are sympathetic, for instance, towards policy interventions designed to encourage deferred gratification,

⁴¹See, for instance McKie and Richardson (2003).

and we ought, perhaps, to be similarly enthusiastic about overruling an aversion to peaks which tends to reduce our overall expected utility. Whether peak aversion occupies a similar normative position to pure time preference probably turns on whether choices influenced by peak aversion are optimal *ex post* as well as *ex ante*. That is, an impatient individual who organises their affairs accordingly, will look back and wish that their earlier self had placed as much weight on today as on yesterday. It is not immediately clear whether the same is true of the peak averse individual who sacrifices total utility in order to ensure a smoother progression through time.

On this basis, we suggest that peak aversion deserves to be taken into account by social planners, even in circumstances where positive pure rates of time preference would be ignored.

3.13 Conclusions

In this chapter we have suggested that certain aspects of observed human behaviour, and certain intuitively appealing philosophical beliefs are best explained by the existence of a preference for smoothness in utility across states and through time.

We have suggested that this preference is distinct from, and a useful addition to, traditional diminishing marginal utility models of risk aversion in capturing behaviour in response to multiple-argument shocks to utility, without requiring the adoption of a generalised state dependent utility measure. While we have distinguished between the state based and time based versions of the peak aversion phenomena, so that the reader could accept the existence of the one without the other, we believe that they are best seen as twin manifestations of a single phenomenon – perhaps a sense that individuals view their future and alternate selves with a degree of Rawlsian concern for the worst off among them.

We have presented a simple mathematical modification to standard utility aggregation across states to capture this phenomenon of peak aversion, specified so as to generalise to cover both peak neutral and peak loving preferences. Using this model, we demonstrated that a peak aversion structure of this general form can generate behaviour consistent with observed violations of pure diminishing marginal utility-based risk aversion. Given its structural similarities to standard diminishing marginal utility risk aversion, we showed that the key summary statistics used to describe degrees of risk aversion can easily be repurposed to provide analogous indicators with respect to peak aversion.

We have argued that peak aversion deserves a place in the economist's

choice-under-uncertainty and time preference toolkit, alongside traditional risk aversion, and a series of other non-standard models of choice under uncertainty: state dependent utility, uncertainty aversion, rank dependent utility and loss aversion. While some of these are superficially related to peak aversion, they each serve a different role and, in most cases, can be combined with a peak averse preference structure.

In relation to subjective wellbeing, we show that the existence of time based peak aversion calls for an aggregation rule for time series SWB data different from the unweighed linear models presently in use. We considered a particular alternative to peak averse aggregation, in the form of Kahneman et al.'s integration about an hedonically neutral state, and suggested that its underlying assumptions were unduly stringent.

Finally, we have considered the practical application of peak aversion in three areas.

First, we considered the role of contingent valuation gambles in calculating both year-value and disability-level in quality adjusted life years. We concluded that peak aversion will generate opposing biases at each stage of this calculation, and that such bias may be desirable where a prevailing inequality averse attitude to social choice is not reflected in equity weights used for QALY calculation.

Second, we discussed optimal compensation and optimal deterrence in the context of tortious damages for negligence. We concluded that, as with the QALY case, peak aversion would tend to lead to overstatement of the value of lost life years, relative to their value calculated at a socially optimal distribution. We also suggested that this estimate would, where applied as a damages award, tend to lead to greater than optimal deterrence of potentially negligent conduct.

Finally, we considered the normative implications of peak aversion relative to those of time preference, and looked at the role of peak aversion in social choice. We showed that a peak averse individual would, according to a modification of Harsanyi's veil of ignorance argument, endorse an inequality averse social welfare function with a degree of concavity equal to that of their peak averse aggregator function.

Drawing this material together, we have used a data set demonstrating empirical evidence of peak aversion to estimate the strength of the preference and used that estimate to calculate the optimal curvature of the social welfare function and the optimal QALY-weighting for burden of illness.

Chapter 4

Aggregation of subjective wellbeing, mean reversion and risk

As befits a topic as broad as subjective wellbeing, the material presented in this thesis ranges widely. In this brief final chapter we will draw together the core argument presented across the preceding three chapters: how SWB data can best be aggregated through time, across states and interpersonally, to serve as a guide to policy.

In addition, we will present the distinct contributions offered by this thesis to the analysis of mean reversion – the tendency of SWB and therefore utility values to return towards some baseline level – and peak aversion – a non-expected utility maximising preference for smoothness in utility across states and through time – and the policy implications of each.

4.1 Using subjective wellbeing data to set policy

In chapter one we argued that SWB surveys can, in practice, be used to produce a stable, consistent measure for what people consider to be their level of happiness.¹ We further suggested that, provided it is clear to all concerned that “happiness” is intended to convey a strength of preference for a given state of the world, rather than some other more trivial or more transitory meaning, happiness is a useful proxy for utility.

¹We cite, among many others, the detailed review provided by Kahneman and Krueger (2006).

How *will* happiness² be understood in practice? We suggest that an instrumental definition of happiness – “I vote for or against this state of the world with the following intensity” – will arise organically from the publicised use of SWB as a policy tool. Once people understand that SWB will be taken as a broad judgement on their view of the state of their life, any temptation to define it narrowly (“this is fun but I am unsatisfied at a deeper level”) will disappear.

There are, of course, a variety of philosophical objections to policy use of SWB data. On review, though, we concluded that if SWB is a coherent and consistent proxy for utility then these objections are either non-utilitarian in nature and beyond the scope of this discussion (“we should prioritise ‘higher’ pleasures for some non-utilitarian definition of ‘higher’.”) or involve an assertion of interpersonal incomparability (“who is to say that my ‘7’ is not more intense an emotion than your ‘9’?”). Leaving aside the argument from burden of proof (“who is to say it is?”), we have suggested that comparability of sensations arising from almost entirely shared neural architecture is an implicit or explicit assumption in a range of personal and policy fields (nuisance law has no interest in idiosyncratic subjective experience of loudness, to pick an example at random) and that even if interpersonal compatibility of SWB is merely a useful policy fiction rather than a provable truth, we are comfortable with any small harms this may inflict on the bon vivant who asserts a greater capacity for joy than the rest of us.

But why compromise at all? Economists are able to answer a wide range of policy questions using revealed preference methods, which involve no assumption as to interpersonal comparability, the meaning of “satisfied”, or the many other technical issues baked-in the SWB analysis. And when those methods fail, we can simply ask people directly to dollar-denominate the utility they derive from parks, pollution and pandas. We have argued that revealed preference methods are often useful but potentially tautological when applied without an outside measure of preference with which they can be falsified. “Consumers engage in [seemingly irrational behaviour] because they receive more utility from doing so than if they did not, therefore the behaviour is not irrational” is a useful way to end discussion, but only if we are satisfied with excluding even the possibility of utility-reducing choices. A good deal of experimental behavioural economics literature suggests that this assumption is not justified.³ As for stated valuation surveys: the salience given to pandas when asked to think about them and then spend imaginary money on them makes it hard to rely on a conscious monetary estimate of panda value. These surveys, too, are an imperfect tool for measuring utility.

²Or satisfaction, joy, wellbeing or any other related or synonymous term.

³See Schwartz (2004) for a review of one class of examples.

So SWB gives us a means to check whether our choices really do systematically reveal our preferences, and an alternative means of evaluating changes to the world which do not arise from our behaviour but nonetheless alter our utility. It can also serve as a useful proxy for *strength* of preference, which is absent from binary revealed preference data and untrustworthy in costless stated value signals.

So we believe that subjective wellbeing is worth collecting and worth considering. If a policy is confidently expected to increase aggregate SWB then it is probably a good policy, and *visa versa*. But what do we mean by “aggregate SWB”? This issue of aggregation, between people, across time and across states of the world, is the first key technical issue addressed by this thesis.

4.1.1 Interpersonal Aggregation of SWB

Even assuming interpersonal comparability of wellbeing, in order to compare states of the world using SWB data we ideally require interval scale measures of differences in subjective wellbeing. Unfortunately, there is no reason to suppose that individual responses are distributed across any given SWB survey scale in a manner which preserves cardinal differences in underlying utility.

This means that the arithmetic means of SWB data are misleading and that cardinal aggregates of differences in SWB through time or between states potentially have the opposite sign to changes in aggregate utility. We can respond to this difficulty by falling back to the Pareto criterion, or by treating SWB values as ordinal state rankings and regressing them using maximum likelihood techniques. Neither solution, though, is optimal, since each involves discarding information about cardinal preference strengths which *should* be embedded in individuals’ SWB responses.⁴

Ideally, then, we need to recover the underlying mapping by which the individual has assigned (continuous, unbounded) utility values to the finite set of SWB responses – we need the relationship between unobservable utility and reported SWB. Clearly, since one half of this relationship is inherently unobservable (or else we would not require SWB data in the first place) observational methods will struggle to establish what this mapping is.⁵

Instead, we proceed by asking “what relationship between utility and

⁴In fact, using the techniques described below we show that the observed distribution of SWB estimates is entirely inconsistent with individuals experiencing or perceiving their own utility ordinally.

⁵It is this difficulty which undermines the attempt to directly recover the mapping in Layard, Mayraz, and Nickell (2008).

SWB would explain the observed distribution of responses?”. That is: we model the individuals’ assignment of utility values to SWB as a cooperative signalling game, where the individual attempts to send the surveyor an error-minimising signal about the value of her underlying utility using the range of SWB values available to them. We have shown that, mathematically, this problem is analogous to the problem faced by an evolutionary “principal” attempting to design a finite-value utility function for its human “agent”, a problem which has been solved in closely related form by Rayo and Becker (2007).

In general terms, we show that, if one assumes an expected error-minimising respondent, the cardinal utility distribution expected by an individual will be distributed across the SWB scale in directly inverse proportion to its observed frequency. So, for example, if a particular value on the SWB scale is reported ten times less often than another, then the amount of “space” on the individual’s utility function mapped to that value is ten times as large: infrequently reported SWB values (typically values at the ends of the scale and especially the lowest available signal) are infrequent precisely because they are reserved to report particularly significant events.

We argue that this means of recovering mapping between SWB and utility improves significantly on the alternative methodologies (assuming linearity or treating data ordinally), is consistent with the intuition that there is likely to be compression of utility at the ends of SWB scales⁶ and, we show, can be used to produce usable cardinal results from differences in appropriately rescaled SWB.

This, then, is how we aggregate SWB interpersonally: we simply rescale the cardinal differences in SWB to reflect the mapping which would be consistent with the observed response distribution minimising expected error.

4.1.2 Cross State and Intertemporal aggregation of SWB

Even once SWB data has been cardinalised, we are left with the challenge of how to add them through time and across states. Provided SWB values are seen as an acceptable proxy for utility, this is a problem which economics views as having been firmly resolved: valuation of a stream of utility through time is simply the sum of utilities discounted by the pure rate of time preference, while values across states can be aggregated using their expected value – rational individuals are inherently, definitionally, risk neutral across utility payoffs.

We begin by suggesting that this conclusion seems to be inconsistent with

⁶See Oswald (2005).

certain observed behaviours – non-pecuniary insurance aimed at smoothing utility, not money, across states, like the sports fan who bets against her own team so as to not feel as bad should they lose, or the couple who insure their engagement ring because of its sentimental value.

We will not rehash these arguments in too much detail here. In short: some people seem to have an attitude to risk, as opposed to the universal risk neutrality perturbed only by non-linearity in underlying marginal valuations which is assumed by all existing economic models.

All economic models? Yes. We go on to show that the existing advances on traditional risk aversion are either looking at something else entirely (rank dependent utility, uncertainty aversion) or making much less clearly defined, less tractable claims (loss aversion, generalised state dependent utility). If we concede that some people have preferences across risk, we need to propose a new, distinct concept to capture and model this phenomenon. We label this new concept “peak aversion”, and show that it can be operationalised, incorporated into existing risk attitude measures and calculated with only relatively minor changes to existing notation and procedure.

Using a small data set, we find objective evidence of peak aversion – individuals treat gambles across life years systematically differently to how they treat time tradeoffs – and, importantly, are able to arrive at a preliminary estimate as to the degree of aversion. That is: we provide an estimate for the degree of average curvature of utility functions in state space – previously assumed to be zero – albeit for a small, not globally representative sample.

In doing so, we propose a new approach to aggregating SWB, and utility, across states. We ought not to assume risk neutrality across utility payoffs (“peak neutrality”) and, in the absence of better estimates, should assume that individuals weigh risks according to the estimated peak aversion parameter observed in their aversion to gambles over certain tradeoffs which ought to be linear in utility.

This argument also applies, we argue, to aggregation across time. Traditional models assume, and Kahneman, Wakker, and Sarin (1997) purport to show, that individuals inherently lack any preference for the smoothness or peakedness of their experience of utility through time. We are unpersuaded that this is universally the case, and show that Kahneman et al rely on an inaccurate definition of ratio scale in their proof to that effect.

Instead, we argue, aggregation through time requires, in addition to existing notions of discount and pure time preference, a parameter to admit a preference for smoothed (or, alternatively, jagged) utility through time, at least at certain margins. We show that the mathematical infrastructure

for state based peak aversion is perfectly adapted for this role, and that we can draw analogies between how an individual aggregates across their future selves and how they aggregate across their potential selves.

When aggregating SWB and utility through time, in addition to the recognised pure preference for utility now rather than later, individuals may have a preference for smooth rather than jagged utility through time. We lack an estimate for the ubiquity or strength of this phenomenon, but suggest that it may be equal to the measure provided for smoothness of utility across states.

4.1.3 Peak Aversion and the Policy Maker

A pure utilitarian policy maker, believing that SWB represents the best available proxy for utility, will simply seek to maximise aggregate SWB. A prioritarian policy maker, or, at the extreme, a Rawlsian policy maker, might prefer to maximise some weighted measure of SWB, giving precedence to the welfare of the worst off.

We suggest that the existence of peak aversion – a preference for smoothness of utility across states – also has direct implications for this choice. Rawls’ argument⁷ for a max-min social welfare function is that it is the SWF we, collectively, would agree to if we lacked knowledge as to how uncertainty as to the allocation of individuals to utility profiles would be resolved. Harsanyi (1953) shows that, provided we are rational utility maximisers, this would not be the case, even on Rawls’ terms: we would simply seek to maximise the average utility we received from our assigned role, meaning that we would not be disproportionately exercised about the possibility of becoming the worst off in society.

As a model of preferences across unresolved utility profile risk, peak aversion has a clear conclusion as to how the representative peak averse individual would weigh a choice of lotteries across utility profiles: they would place a disproportionate weight on states of the world where they ended up badly off. That is, they would endorse a prioritarian welfare function, with the curvature of the social indifference curves directly proportional to the extent they prioritise smoothness in utility across states. In fact, based on our preliminary estimate of peak aversion, we can estimate the curvature of the SWF that would be endorsed by the members of that sample from behind the veil of ignorance.

So far we have spoken only about policy makers charged with maximis-

⁷Most recently updated in Rawls (1999).

ing some universal measure of wellbeing. As extra-welfarists⁸ point out in the context of health care, this is not always the case. A decision maker may be, for sound democratic reasons, charged with maximising some subset of the social welfare function. We consider the application of the preference aggregation results derived here to two such cases: Quality-Adjusted Life Year maximising Health Technology Assessment committees and compensation-setting judges.

In each case we show that existing measures fail to fully take into account even currently accepted features of preferences across states, and that recognising the existence of peak aversion further complicates the calculation of optimal decision in each sphere.

In particular, we show that, if health care decision makers wish to reflect social preferences in prioritising patients with the highest measure of some definition of “burden”,⁹ peak aversion provides an estimate of the degree of priority which is consistent with societal preference.

4.2 Mean Reversion: Mapping States to Utilities

While resolving the interpersonal aggregation problem, we rely on results from the restricted utility function literature, which we show are analogous to the individual’s SWB mapping problem. But this literature is also of more direct relevance to SWB research and policy setting. We explore this relevance through the lens of mean reversion:¹⁰ are mappings between SWB and utility likely to be constant through time? (it depends on changes in utility distribution, more data are needed in order to draw firm conclusions) And are mappings between states of the world and utility likely to be constant through time? The answer to the later question, if we accept the restricted utility literature’s contention that unbounded continuous utility functions are likely to be prohibitively demanding on evolutionarily-limited cognitive resources, is no. The level of utility we associate with a given set of circumstances is likely to change based on the degree of prosperity (or, endogenously, utility) we enjoy through time.

We consider two models for how a utility function might change how it assigns utilities to states of the world in response to the frequency distribution of those states. The first of these models attempts a generalisation of the models provided by Robson (2001) and Rayo and Becker (2007) for the

⁸See, for example, Culyer (1989).

⁹As is proposed in England and Wales, NICE (2014) for example.

¹⁰Also variously known as “adaptation”, “habituation” and the “hedonic treadmill” and a form of the “habit formation” class of utility functions – see Abel (1990).

plausible situation in which an evolution designed utility function which cannot discern fine differences in state of the world cannot condition its mapping on those differences. As a result of the assumptions about restricted inputs into the model, its conclusions are, of necessity, extremely general: in the presence of a new payoff distribution, the mapping from real world payoffs to utility we display an escalating series of shocks until it “overshoots” the shock to payoff distribution, after which it will undergo a decreasing series of shocks until it converges on a new mapping for which the mean of the utility distribution matches the mean of the payoff distribution.

A second, more specific, model assumed similar underlying dynamics but allows evolution to condition the utility-payoff mapping on the prior distribution of payoffs. This model assumes that evolution will update utility payoffs in a Bayesian fashion, inferring the likelihood that the payoff distribution has shifted based on the likelihood of observing the actual payoff under the existing distribution. This model predicts that, counterintuitively, individuals will adjust their expectations more quickly in response to very large shocks (which are clearly inconsistent with the prior distribution) than to smaller shocks (to which reversion never fully adjusts). A small amount of good news can lead to permanent disappointment in the future.

Our third model of mean reversion discards the evolutionarily restricted utility framework altogether. Instead we consider a general habit formation utility function, whereby individual experience utility based on its value relative to some baseline, with the baseline updating in discrete time based on the difference between experienced and baseline utility and a linear speed-of-adaptation parameter, “ α ”.

Under this model, we show, the aggregate utility associated with temporary shocks to utility is always zero – the effect of changed expectations exactly offsets the temporary change to experienced utility in every case. We argue that this represents a significant problem from the point of view of the “principal-agent” model considered above: how can evolution motivate a rational, forward looking individual to undertake utility maximising actions if aggregate utility is constant? We show that the existence of any positive pure rate of time preference is sufficient to restore utility maximising behaviour, and argue that this may explain observed phenomenon of positive pure time preference.

4.2.1 Measuring Mean Reversion

Are any of the models considered above a reasonable approximation of how mean reversion occurs in practice? Using data on shock to subjective wellbeing from the British Household Panel Survey, we show that there

is significant evidence of either mean reversion or negative intertemporal correlation in utility arguments, and that the value of reversion speed, α , over one year periods appears to be approximately 0.5.

This estimate is offered as a contribution to the large existing literature on existence and rates of habituation to utility states – see, for example, Diener, Lucas, and Scollon (2006). We also incorporate the cardinalisation procedure, based on inverse response frequency, outline above to rescale the cardinal time series differences in SWB used for this exercise.

4.2.2 Mean Reversion and Policy Setting

We argue that the existence of mean reversion has significant implications for optimal policy. If experienced utility adapts to all changes in circumstance, then the appropriate policy for distributing improvements in utility arguments through time will depend entirely on the linearity or the relationship between shock size and reversion speed. With linear reversion, which is the relationship for which we find preliminary evidence in the BHPS data, appropriate tradeoffs between habituable goals will be given by the traditional utility maximising rule, for permanent changes, or be undefined for temporary changes (any policy rule both maximises and minimises aggregate utility, which is fixed in relation to temporary changes in inputs).

The key suggestion offered by this analysis is that unforeseen habituation is likely to lead individuals to overvalue gains to the arguments of utility at the expense of life extension – we will probably be, for example, too willing to take on dangerous jobs which provide additional income to which we adapt at the cost of an increased risk of death, to which one never really adjusts.

We also argue that similar questions arise in relation to peak aversion preferences: should policy makers maximise expected experienced utility, or individuals' decision utility, which potentially reflects a preference for smoothness across states and through time. This is a complex philosophical question, but we suggest that, by contrast with time preference, a benevolent social planner should recognise societal peak aversion provided that peak aversion choices are considered optimal by individuals *ex post* as well as *ex ante*.

4.3 On Wellbeing, Mean Reversion and Risk

Subjective wellbeing is a valid instrument, practically and theoretically, for informing policy choices. There are a number of difficulties with putting

this advice into practice, some of which relate to problems with aggregating individual SWB values between persons, across states and through time. We suggest solutions and preliminary parameter values for all three of these problems and provide examples of how these solutions could be applied to policy questions both broad (calculating the social welfare function) and narrow (setting compensation and measuring the value of health).

Subjective wellbeing also tells us about how the relationship between utility and its determinants can change. Models of restricted utility tell us to expect these changes, and help us predict the relationship between changes in the world and changes in our utility mapping. We can look at shocks in reported SWB and use them to measure the speed and structure of this mean reversion, and consider the relevance of an “iron law of happiness” to setting policy and to predicting utility and SWB from their arguments.

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