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Design and Reliability Performance Evaluation of Network Coding Schemes for Lossy Wireless Networks

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Abstract

Wireless communication between devices can be *lossy* in the sense that packet transmissions via a wireless link may fail (so that the packet is lost) due to a number of factors - channel fading, interference or mobility of devices. In some scenarios, the lossy characteristic of wire-less communication can be random, and thus better characterised from a stochastic perspective. This thesis investigates lossy wireless networks, which are wireless communication networks consisting of lossy wireless links, where the packet transmission via a lossy wireless link is successful with a certain value of probability.

In particular, this thesis analyses all-to-all broadcast in lossy wireless networks, where every node has a native packet to transmit to all other nodes in the network. A major challenge of all-to-all broadcast in lossy wireless networks is the *reliability*, which is defined as the probability that every node in the network successfully obtains a copy of the native packets of all other nodes.

The reliability of all-to-all broadcast in lossy wireless networks can be improved by network coding techniques. In this thesis, two novel network coding schemes are proposed:

- 1. the neighbour network coding scheme and
- 2. the random neighbour network coding scheme.

In the two proposed network coding schemes, a node may perform a bit-wise exclusive or (XOR) operation to combine the native packet of itself and the native packet of its neighbour, called the *coding neighbour*, into an XOR coded packet. By broadcasting the XOR coded packet to other nodes, the reliability of all-to-all broadcast can be improved compared with the corresponding non-coded network wherein a node only broadcasts its native packet. In the first proposed scheme, the coding neighbour is pre-designated; while in the second proposed

scheme, the coding neighbour is randomly chosen.

The reliability of all-to-all broadcast under both the proposed network coding schemes is investigated analytically using Markov chains and the theoretical analysis is validated by simulations. It is shown that the reliability of all-to-all broadcast can be improved considerably by employing the proposed network coding schemes, compared with non-coded networks with the same link conditions, i.e. same probabilities of successful packet transmission via wireless channels.

The research reveals that the gain in reliability brought by network coding can be significantly affected by channel conditions, which have however been largely overlooked in existing research in this area. The first proposed coding scheme takes the channel conditions of each node into account in the selection of coding neighbour and further proposes the optimal coding neighbour selection method that maximises the reliability of a given network employing the proposed neighbour network coding scheme.

Moreover, a tuning parameter is introduced in the second proposed coding scheme, which determines the probability that a node performs network coding at each transmission. Further, theoretical solutions to the optimal tuning parameter that maximises the reliability of all-to-all broadcast in networks employing the proposed random neighbour network coding scheme are provided. It is shown that the optimal value of the tuning parameter is also dependent on channel conditions. The observation that channel condition can have a significant impact on the performance of network coding schemes is expected to be applicable to other network coding schemes for lossy wireless networks.

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Statement of originality

I, Li Ma, hereby declare that this thesis, submitted in fulfilment of the requirements for the award of Master of Philosophy, in the School of Electrical and Information Engineering, the University of Sydney, is my own work unless otherwise referenced or acknowledged. The document has not been previously submitted for the award of any other qualification at any educational institution. The results presented in Chapter 4 has been accepted by ICC workshop 2013; and the results presented in Chapter 5 have been accepted by Globecom 2013.

Related publications

- [C1] L. Ma, Z. Lin, Z. Zhang, G. Mao, and B. Vucetic, "Improving reliability in lossy wireless networks using network coding," in *Proceedings of IEEE ICC workshop CoCoNet*, pp. 322-326, 2013.
- [C2] L. Ma, Z. Lin, Z. Zhang, G. Mao, and B. Vucetic, "Reliability of all-to-all broadcast with network coding," in *Proceedings of IEEE Globlecom*, 2013.

Table of contents

Abstract i								
Ac	Acknowledgements iii							
Sta	Statement of originality iv							
Re	Related publications v							
Ta	Table of contentsvi							
Li	st of f	ìgures		X				
Li	List of tables xi							
List of symbols xiii								
Li	st of a	abbrevia	ations	XV				
1	Intro	oductio	n	1				
	1.1	Backg	round	1				
		1.1.1	Wireless communication	1				
		1.1.2	Broadcast	2				
		1.1.3	Network coding	4				
	1.2	Resear	ch problems	5				
		1.2.1	Reliability	5				
		1.2.2	Coding design	6				
	1.3	Main c	contributions	7				

		1.3.1	Summary of the contributions	8
	1.4	Thesis	outline	9
2	Netv	vork Co	oding	11
	2.1	Backg	round	11
	2.2	Netwo	rk coding schemes	14
		2.2.1	XOR based network coding	14
		2.2.2	Linear network coding	15
		2.2.3	Random linear network coding	16
		2.2.4	Distributed random linear network coding	17
	2.3	Benefi	ts made by network coding	17
		2.3.1	Bandwidth efficiency	18
		2.3.2	Energy efficiency	20
		2.3.3	Delay performance	21
		2.3.4	Reliability	23
	2.4	The ch	allenges in code design	27
	2.5	Summ	ary	28
3	Syst	em Moo	lel	30
	3.1	The all	l-to-all model	30
	3.2	The no	de structure	31
		3.2.1	Encoder	31
		3.2.2	Decoder	33
		3.2.3	Buffer	33
		3.2.4	Packet structure	33
	3.3	Chann	el conditions	34
		3.3.1	Packet error rate	35
		3.3.2	End-to-end connection probability	35
		3.3.3	Probabilistic connectivity matrix	37
	3.4	Data fl	ow diagram	37
	3.5	Summ	ary	38

4	Neig	ghbour	Network Coding Scheme	40
	4.1	Propo	sed network coding scheme	40
	4.2	Theore	etical analysis	42
		4.2.1	Construction of the states	42
		4.2.2	Transition matrices	44
		4.2.3	The probability vectors	45
		4.2.4	Reliability	46
		4.2.5	Networks without network coding	47
	4.3	Bound	ls on the reliability	48
		4.3.1	The upper bound	48
		4.3.2	The lower bound	50
	4.4	Nume	rical results	52
		4.4.1	Validation of the theoretical analysis	52
		4.4.2	The examination of the reliability gain	54
		4.4.3	The bounds on the reliability	58
	4.5	Summ	ary	59
5	Random Neighbour Network Coding Scheme		60	
	5.1	Propo	sed network coding scheme	61
	5.2	Theore	etical analysis	61
		5.2.1	States	62
		5.2.2	Transition matrices	64
		5.2.3	Probability vector and the reliability	70
	5.3	Optim	isations	71
		5.3.1	Optimise the reliability at an individual round	71
		5.3.1 5.3.2	Optimise the reliability at an individual round	71 72
	5.4	5.3.1 5.3.2 Nume	Optimise the reliability at an individual round	717273
	5.4	5.3.1 5.3.2 Nume 5.4.1	Optimise the reliability at an individual round	71727373
	5.4	5.3.1 5.3.2 Nume 5.4.1 5.4.2	Optimise the reliability at an individual round	 71 72 73 73 74
	5.4	5.3.1 5.3.2 Nume 5.4.1 5.4.2 5.4.3	Optimise the reliability at an individual round	 71 72 73 73 74 77

6	Con	nparison of the Proposed Schemes 8			
	6.1	Comparison between two proposed network coding schemes	. 80		
		6.1.1 Similarities	. 80		
		6.1.2 Differences	. 81		
		6.1.3 Reliability performance	. 82		
	6.2 Comparison of the proposed schemes with the random linear network coding				
	scheme				
		6.2.1 Reliability performance	. 85		
		6.2.2 Delay performance	. 88		
	6.3	Summary	. 89		
7	Conclusions and Future Work		90		
	7.1	Conclusions	. 90		
	7.2	Future work	91		
Bi	Bibliography				

List of figures

1.1	Illustration of broadcast models.	3
1.2	An example of packets exchange between nodes N_1 and N_2 via N_3 by (a) tra-	
	ditional store and forward technique, and (b) network coding technique	4
2.1	The classic two-way relaying network applying XOR coding	14
2.2	Illustration of linear network coding, where the column vector assigned to each	
	channel is shown.	16
2.3	Butterfly networks where N_1 and N_2 transmit packets X_1 and X_2 to both N_4 and	
	N_6 with traditional store and forward scheme or network coding scheme	18
2.4	An example that network coding used for reduce the number of retransmissions	
	by combining the different lost packets.	24
2.5	The comparison of traditional method and the method combining network cod-	
	ing and multipath routing when a source transmits multiple packets	27
3.1	An example of the all-to-all model where the total number of nodes is n	31
3.2	The relay system for node-to-node transmissions.	32
3.3	An example of a packet in a network of four nodes	34
3.4	An illustration of a channel where the packet error rate is q_e	35
3.5	The paths between N_j and N_i	36
3.6	The system model of all to all communication in one time slot. Note that it is	
	assumed that N_j transmits in this time slot	38
4.1	Illustration of broadcast in lossy wireless network with <i>n</i> nodes applying neigh-	
	bour network coding	41
4.2	Illustration of two processes for packet from N_j to reach N_i	48

4.3	Simulation and theoretical results of the reliability of networks when $n = 3, 4, 5$.	53
4.4	Network reliabilities under different neighbour selections, where probabilistic	
	connectivity matrices for $n = 4$ and 5	55
4.5	The reliability gain of networks employing neighbour network coding scheme	
	over non-coded network when $n = 4$	56
4.6	The reliability gain of networks employing neighbour network coding scheme	
	over non-coded network when $n = 4$ and 6	57
4.7	Bounds on the probability that N_3 receives X_1 where $n = 3$	58
5.1	An illustration of the Markov chain of a network when $n = 3 $	65
5.2	The comparison between theoretical and simulation results of the network re-	
	liability applying random neighbour network coding when $n=3$ and 4	73
5.3	The reliability at the end of fourth round when the tuning parameter varies form	
	zero to one	75
5.4	The reliability at the sixth round when the tuning parameter varies form zero	
	to one	76
5.5	The expected number of rounds to reach the absorbing state when $n = 3$	77
5.6	The comparison between reliability of networks with and without network cod-	
	ing when $n = 3$ and 4	78
6.1	The performance comparison of two proposed schemes	82
6.1 6.2	The performance comparison of two proposed schemes	82 83
6.16.26.3	The performance comparison of two proposed schemes	82 83
6.16.26.3	The performance comparison of two proposed schemes. \dots \dots \dots \dots The performance comparison of two proposed schemes \dots \dots \dots \dots The performance comparison of two proposed schemes and random linear net- work coding scheme when the finite filed is $GF(2)$. \dots \dots \dots \dots	82 83 85
6.16.26.36.4	The performance comparison of two proposed schemes. \dots \dots \dots The performance comparison of two proposed schemes \dots \dots \dots The performance comparison of two proposed schemes and random linear net- work coding scheme when the finite filed is $GF(2)$. \dots \dots \dots \dots The performance comparison of two proposed schemes and random linear net-	82 83 85

List of tables

4.1	The states of N_1 for a network with three nodes	43
5.1	The values of the tuning parameter that maximise the reliability at $R = 3$ to 8.	76
6.1	The delay for reliability to reach 0.6, 0.7, 0.8 and 0.9 for a network of three	
	nodes	88
6.2	The delay for reliability to reach 0.6, 0.7, 0.8 and 0.9 for a network of four nodes.	88
6.3	The comparison of the proposed schemes and random linear network coding	
	scheme under $GF(2)$	89

List of symbols

n	the total number of nodes in a network
N_k, N_γ, N_θ	an arbitrary node
X_k	the original packet at N_k
Q	probabilistic connectivity matrix
p _{ji}	the probability that N_i receives a packet broadcast from N_j in one time slot
R	a round
t	a time slot
N_{j}	the transmitter
N_i	the receiver
ξ_k	the received packets at N_k
L	the total number of states
Т	the transition matrix at a node
$M^{\mu_1}_{ji},M^{\mu_2}_{ji}$	conditional transition matrices of a node
S	the probability vector
ψ	the reliability
U	the upper bound
L	the lower bound
ω	the tuning parameter, i.e., the probability to perform network coding
$\mu_{\gamma,k}$	the index of packet $X_{\gamma} \oplus X_k$
\mathcal{D}_j	the collection of packets that N_j has successfully decoded
m_j	the cardinality of \mathcal{D}_j
X_h	a packet from \mathcal{D}_j

V_A, V_B	an arbitrary state
<i>v_{kA}</i>	the packets of node N_k has when the network is in state V_A
М	the transition matrix of a network
$\Phi_j(V_A V_B)$	an entry in M

List of abbreviations

AP	Access Point
ARQ	Automatic Repeat reQuest
EM	electromagnetic
FEC	Forward Error Correction
FFNC	Finite Field Network Coding
GF	Galois Field
LNC	Linear Network Coding
LT	Luby Transform
MAC	Media Access Control
NNC	Neighbour Network Coding
RLNC	Random Linear Network Coding
RNNC	Random Neighbour Network Coding
QoS	Quality of Service
XOR	exclusive or

Chapter 1

Introduction

This chapter begins with a brief introduction to wireless communication in Section 1.1, including the lossy nature of wireless communication and a key mechanism in wireless communication networks - broadcast. Then, network coding techniques, which can be used to improve the performance of wireless communication, are briefly introduced. Then, Section 1.2 introduces some research problems of broadcast in wireless communication networks, followed by a summary of the main contributions of this thesis in Section 1.3. Lastly, the outline of the thesis is provided in Section 1.4.

1.1 Background

1.1.1 Wireless communication

Wireless communication refers to the data communication between devices over wireless channels [1]. Since its invention, wireless communication has created a great impact on everyday life: changing the communication and working habits of a huge number of people. Wireless communication can provide a large number of applications including Internet access, web browsing, short messaging, file transfer, video teleconferencing, entertainment, sensing, distributed control and health care [2, 3].

One feature of wireless communication that differs from wired communication is the lossy nature, in the sense that packet transmissions via a wireless link may fail and the packet may be lost during the transmission. The lossy characteristic of wireless communication can be caused by a number of issues, detailed thus:

- *Path loss* is the attenuation of signal power when a radio wave propagates through space: it is a major phenomenon affecting wireless connection between a transmitter and a receiver.
- *Shadowing* is the attenuation of signal power caused by large obstacles between a transmitter and a receiver; the variations of the received signal power caused by shadowing are usually modelled by a log-normal distribution.
- *Multipath propagation* is the phenomenon that a signal reaches the receiver by two or more paths. When a transmitted signal meets an interacting object, reflections and diffractions are created and signals created by reflections and diffractions may also reach the receiver. However, each of these signals may have different amplitudes, phases, delays and directions of arrivals; so there is a certain probability that the receiver cannot detect the transmitted signal because the main signal is not strong enough to be detected or because the reflected and diffracted signals may cancel with the main signal.
- *Interference* refers to the addition of unwanted signals to the useful signal. There are several different types of interference e.g., co-channel, adjacent channel, inter-symbol, inter-carrier, common mode and conducted interference [4]; they all affect the reception of the signal when the signal-to-interference ratio is small.

In summary, the characteristics of wireless channels cannot, usually, be uniquely determined, and are therefore better characterised from a stochastic perspective. In view of this, research has been conducted on *lossy wireless networks*, where the end-to-end communication is successful or unsuccessful with a certain probability [5, 6, 7]. This thesis investigates the performance of packet broadcast in lossy wireless networks.

1.1.2 Broadcast

Broadcast is a communication mechanism that disseminates identical data from a source to multiple receivers [1, 8]. There are several basic protocols for broadcast, such as flooding [9, 10] and gossip (epidemic) [11, 12, 13]. Flooding refers to the mechanism that a node sends a message to its neighbours, which in turn forward the message to all their neighbours except the message sender [9]. Using a flooding protocol, a transmitter keeps sending a message to all

its neighbours in the assigned time slots. Yet another broadcast protocol is gossip (epidemic). In this protocol, each node contacts only one or a few nodes in each round, and exchanges information with these nodes. Gossip protocol has been widely applied in ad hoc networks [13, 14].

There are two main types of broadcast models, which are the one-to-all model and the allto-all model. The *one-to-all* model represents the mechanism of transmitting information from one source to all other nodes in the network, as illustrated in Figure 1.1 (a). It is a model widely considered and referred to in the open literature. In an *all-to-all* model, every node is a source and transmits information to all other nodes in the network, as illustrated in Figure 1.1 (b).



Figure 1.1: Illustration of (a) a one-to-all model where a single node broadcasts to all other nodes; (b) an all-to-all model where every node broadcasts information to all other nodes.

Traditionally, the all-to-all broadcast is applied to routing algorithms, such as finding routes in adaptive routing mechanisms [15] and in ad-hoc routing mechanisms [16]. It is also applied to content distribution [17], where *content distribution* refers to the delivery of information to a large number of users in a network. In addition, it can be used for communication among users to update the status of all users [18]. More recently, it has become regarded as a key mechanism for data communications in intermittently connected ad hoc networks, e.g. an airborne network [19] or a delay tolerant network [20].

In this thesis, we investigate all-to-all broadcast in lossy wireless networks, where network coding is applied at each node to improve the reliability performance.

1.1.3 Network coding

Network coding [21] is a technique allowing nodes in a network to intelligently combine and compute independent incoming information flows into an outgoing information flow. In comparison, in traditional networks without network coding, nodes can only forward information without changing the content of the information using the store and forward technique.

In illustration of this, Figure 1.2 shows the packet exchange between nodes N_1 and N_2 : node N_1 has a native packet X_1 to send to N_2 , while N_2 has a native packet X_2 to send to N_1 .



Figure 1.2: An example of packets exchange between nodes N_1 and N_2 via N_3 by (a) traditional store and forward technique, and (b) network coding technique.

In Figure 1.2 (a), the network applies the traditional store and forward scheme: the relay node N_3 simply forwards packets X_1 and X_2 separately in different transmissions. The total number of time slots required for N_1 and N_2 to exchange their packets is four, and the total number of transmissions required is four. In the network, where exclusive or (XOR) network coding is applied, as shown in Figure 1.2 (b), the intermediate node N_3 is allowed to combine packets X_1 and X_2 to generate an encoded packet, e.g. $X_1 \oplus X_2$. Then, this coded packet is transmitted. Lastly, upon receiving the encoded packet, both nodes N_1 and N_2 can decode the required packet using their native packets. The required time slots and total number of transmissions for the exchange of packets are both three.

This demonstrates that the network coding technique can reduce the required number of time slots and total number of transmissions compared to the traditional store and forward technique; thus network coding can be seen to assist in the sharing of available network resources - e.g., bandwidth and energy [22]. A review of network coding techniques and the

benefits that network coding techniques bring to a network will be provided in Chapter 2.

1.2 Research problems

1.2.1 Reliability

There are a number of performance metrics for a communication network, such as delay, bandwidth efficiency, energy efficiency and reliability. Due to the lossy nature of wireless communication, the packet transmitted from a source node may not always be able to reach its intended destination(s). Therefore, reliability of information transmission is a key research problem in lossy wireless networks. In this thesis, the *reliability* is defined as the probability that a piece of information will reach its intended destination(s).

Numerous error control methods have been developed to attempt to ensure a high reliability communication in lossy networks. There are several error control approaches; the basic ideas of these methods are to intelligently retransmit a piece of information or to perform error correction. One common method is the Automatic Repeat reQuest (ARQ) [23], which enables receivers to send the receiving status (received/not received) back to the transmitter; then the transmitter decides whether or not to retransmit the same information, based on the feedback of the receiving status. However, this simple repetitive retransmission scheme results in delay and is a non-efficient use of bandwidth in the processes of sending the receiving status and retransmissions.

It has been shown that coding techniques can improve reliability [5], bandwidth efficiency [24] and reduce delay [25]. The challenge lies in finding suitable coding methods to improve reliability while taking into account delay and throughput.

There has been some research focusing on the expected number of retransmissions for information to reach its intended destinations in the one-to-all model [5]. However, the reliability in the all-to-all model is overlooked in the literature. In addition, the reliability after each retransmission is less understood; and it is useful to some real world applications - e.g., delay constrained transmissions.

1.2.2 Coding design

In wireless communication networks, coding operations are designed for data compression, cryptography and reliable transmission [26].

Source coding refers to the process of coding at the source of the data before the data is stored or transmitted. It involves encoding information using fewer bits than the original representation; therefore, it is popularly referred to as data compression [27].

Channel coding is the scheme that transmits redundancy information so that the noisy channel can behave like a noiseless channel [28]. The fundamental channel codes are block codes and convolutional codes. In particular, Turbo codes [29], which are a successful class of convolutional codes, introduce a way of designing codes with feasible decoding complexity.

Fountain codes have been studied extensively in the literature for their channel coding applications. Fountain codes are efficient and robust solutions for reliable transmission over erasure channels through which a transmitted packet can be either received without error or not received. [30]. Metaphorically speaking, the encoder is the source of a fountain providing an endless supply of water drops (encoded packets), and these encoded packets are a limitless and rateless sequence of encoded symbols from a set of symbols transmitted at the source. Then the receiver needs to collect a certain number of encoded packets to decode the whole set of original source symbols.

Among all fountain codes, the first practical and successful class are the Luby Transform (LT) codes [31]. Using LT codes, the source node generates an encoded symbol from a set of input packets as follows:

- The source node randomly chooses an integer d, which is called the *degree* of the encoding packet, according to a degree distribution $\rho(d)$. The optimal degree distributions in various systems have been investigated extensively in the literature [32].
- The source node chooses uniformly at random *d* distinct input packets; and encode these randomly chosen packets by performing an exclusive or (XOR) operation on them.

Fountain codes are near optimal for every erasure channel in the sense that they can be applied to an erasure channel regardless of the statistic of the erasure event.

However, fountain codes are not sufficient in distributed coding, and they are not always sufficient in the all-to-all model. This is because in fountain code, it is assumed that the complete set of packets to be broadcast is available in each transmission. However, the packets to be transmitted at a source node in an all-to-all model vary over time: they include not only the packet initially owned by the source node, but also the packets received and decoded. There-fore, in an all-to-all model, there may not be sufficient numbers of packets at a node to achieve a specific coding degree *d* required by fountain codes.

Network coding technique enables an intermediate node to combine and process received information. It is suitable for an all-to-all model where the encoding of the packets to be broadcast at a node involves the packets previously received by the node. However, network coding schemes are mostly designed and applied to one-to-one and one-to-all models in the open literature, as will be introduced in Chapter 2.

This thesis focuses on designing network coding scheme suitable for the all-to-all model, where the encoded packet is determined adaptively at a source depending on its received packets without the aid of feedback information. Moreover, these schemes should adapt to lossy networks with different channel conditions.

1.3 Main contributions

This thesis proposes two novel XOR based network coding schemes for all-to-all broadcast in lossy wireless networks, called the neighbour network coding scheme and the random neighbour network coding scheme, which will be introduced in detail in Chapters 4 and 5 respectively.

Both schemes are adapted to the time-varying status of the packets received at each node. More specifically, in both schemes network coding is performed at a source node between the native packets of the source node and another selected node, namely the *coding neighbour*, where the *native packet* of a node is the non-coded packet that the node initially has. It is worth mentioning that the selection of the coding neighbour does not rely on feedback information of the receiving status (received/not received), but it is affected by the end-to-end connection probabilities among nodes. The end-to-end connection probabilities of pair of nodes reflects the channel conditions, and the means to obtaining these probabilities will be introduced in Chapter 3.

Decoding processes for both schemes are of low complexity and the decoding delay is

small. Using the proposed coding algorithms, the successful decoding of a coded packet only requires that one of the two native packets forming the coded packet has already been successfully received or decoded. For example, X_z can be decoded from packets $X_z \oplus X_k$ and X_k by performing $(X_z \oplus X_k) \oplus X_k$, where $z \neq k$.

In the neighbour network coding scheme, each node selects another node as its coding neighbour. As soon as the native packet of the predetermined coding neighbour is received or decoded, the node begins to broadcast an encoded packet which is the bitwise XOR between the native packets of itself and that of its coding neighbour. On the other hand, in the random neighbour network coding scheme, each node randomly chooses 1) whether or not to perform coding according to a tuning parameter, which is described in detail in Chapter 5; and 2) with which packet to perform coding on-the-fly according to the packets that it has received and decoded. Therefore, no encoding delay is introduced in both schemes.

The reliability of all-to-all broadcast models applying the proposed schemes is investigated, where the *reliability* of all-to-all broadcast is defined as the probability that every node in the network receives or decodes the native packets of all other nodes. It is shown that the network reliability can be improved considerably by both proposed network coding schemes.

Further, optimisations are carried out to maximise the reliability in networks applying the proposed network coding schemes. For the first scheme, optimal neighbour selection rules are proposed; while for the second scheme, the optimal value of the tuning parameter is derived.

Lastly, the reliability performance of a network applying the proposed schemes are compared with each other and further compared with the performance of the random linear network coding scheme. It is shown that the proposed network coding schemes, when the coding neighbour selection or tuning parameter is optimised, can outperform the popular random linear network coding scheme where the randomly generated coefficients are chosen from the finite field GF(2).

1.3.1 Summary of the contributions

We proposed two novel network coding schemes which take advantage of the simplicity of the XOR-based coding. In the neighbour network coding scheme, each node selects another node as coding neighbour and will broadcast a coded packet as long as the native packet of the coding neighbour is received or decoded. This coded packet is the bitwise XOR between the native packet of the source node and the native packet of its coding neighbour. In the random neighbour network coding scheme, each node performs coding according to a tuning parameter, where the *tuning parameter* is the probability that all nodes perform network coding. Network coding is performed between the native packet of a source node and another native packet which is selected on-the-fly according to the received packets. It is observed that the key in the design of network coding schemes for lossy wireless networks is to take channel conditions into consideration. Both of the proposed schemes can be adjusted by selecting the optimal coding neighbours in the first scheme or the optimal value of the tuning parameter in the second scheme to maximise reliability in networks with different number of nodes and different channel conditions. The encoding and decoding processes of the proposed schemes are of low computational complexity. It is shown by theoretical analysis and simulations that the reliability of networks applying the proposed schemes is considerably improved compared with that in the networks without network coding and that in the networks employing random liner network coding with GF(2). For example, the proposed schemes are able to achieve reliability improvements of 272.34 percent over the non-coded networks, and 100.90 percent over the random linear network coding with GF(2). It is worth noting that the theoretical framework and analysis methods established in this thesis can be easily modified and extended to examine the reliability of networks applying other XOR-based network coding schemes

1.4 Thesis outline

The thesis proposes two novel network coding schemes, and each will be introduced in one individual chapter. The rest of the thesis is organised as follows:

- Chapter 2 reviews the fundamental network coding schemes, and evaluates their performance when applied in different network topologies under various traffic configurations. Then, the benefits made by network coding are summarised. Lastly, the challenges in code design are summarised.
- Chapter 3 introduces the system model. Firstly, it gives a description of the communication strategy in an all-to-all model. Secondly, the structure of a node is presented and the functions of some main parts are described. Thirdly, the metric used to measure channel

conditions from one node to another is introduced. Lastly, the data flow diagram of the system in one time slot is presented.

- Chapter 4 presents a novel neighbour network coding scheme. The network reliability under this coding scheme is evaluated analytically and the theoretical analysis is validated by simulations. It is shown that network reliability can be improved considerably by the proposed neighbour coding scheme compared to the reliability of the corresponding non-coded network. Further, closed-form upper and lower bounds on the network reliability are derived. Moreover, the optimal neighbour coding selection rules that maximise the reliability of a given network are introduced.
- Chapter 5 presents a novel random neighbour network coding scheme. The network reliability is analysed theoretically, the optimal value of the tuning parameter that maximises the reliability is derived and the theoretical analysis is validated by simulations. Furthermore, it is shown that the random neighbour network coding scheme can improve the network reliability significantly.
- Chapter 6 compares the proposed schemes with each other in regard to their similarities, differences and reliability performance. Further, both schemes are compared with random linear network coding scheme in relation to their reliability and delay performance.
- Chapter 7 concludes the thesis and proposes possible future work.

Chapter 2

Network Coding

Traditionally, a network delivers information using the store and forward technique. On the other hand, network coding technique breaks this tradition and allows nodes in a network not only to store and forward but also to intelligently combine and compute the independent incoming information flows into an outgoing flow [21].

A lot of research has been carried out on network coding theory. Among them, there are several successful categorises of network coding schemes. Section 2.2 reviews some basic network coding schemes, including XOR based network coding, linear network coding, random linear network coding and distributed random linear network coding.

Along with the development of network coding theory, the problem addressed by network coding is extended from the original multicast capacity to energy efficiency, delay, reliability, etc. Section 2.3 reviews the benefits brought by network coding techniques.

Further, the challenges in the design of network coding schemes are summarised in Section 2.4.

2.1 Background

Terminology. The topology of a communication network can be represented by a graph. A *graph* consists of vertices or nodes, and lines connecting the nodes are called edges, channels or links. A graph may be undirected, meaning that the links pointing from either one node to the other node within the same pair of nodes have no distinction. On the other hand, in a directed graph, the edge has a direction associated with it pointing from one node to another.

The concept of network coding is first proposed in 2000 [21] to solve the problem of defining admissible coding rate region, where the *admissible coding rate region* is the region represented by the points in space that can achieve an arbitrarily small decoding error probability by block codes. It reveals that it is in general not optimal to multicast information by storeand-forward. Rather, by employing network coding where the information can be coded and combined at a node, bandwidth efficiency can be improved. In [21], networks are represented by directed graphs, where the edges are error-free. This model can be used to represent a wired network where links are lossless, e.g., an Internet backbone.

Along the same direction, the network coding theory in error-free directed graphs is investigated. In [33], the linear network coding scheme and in [34], the random linear network coding scheme are proposed, showing that improvements in capacity can be made by network coding. These schemes lay the foundation of network coding theory and they will be reviewed in detail in Section 2.2.

On the other hand, the network coding technique is extended to and examined in other network scenarios. For example, it is applied to networks with transmission errors and to undirected networks.

When a packet is transmitted in lossy networks, errors may occur due to channel fading, interference, or mobility of devices. One possible solution for reliable transmission is to develop techniques that counteract the errors. The pioneering work employing network coding to solve this problem is the network error correction code which is proposed in [35] and later improved in [36, 37]. When information is transmitted over an individual link experiencing errors, network codes are applied for error correction. A network code is defined as *t-error correcting* if it can correct every one of a total of τ errors occurring in the network during transmission, where $\tau \leq t$, i.e., the total number of errors in the network is less than or equal to the maximum number of errors that can be corrected by the code, which is *t*. It is shown that the network error correction is a generalisation of point-to-point error correction and it has become an instrument for the effective use of network coding in lossy networks [38]. Following this idea, a lot of research has been conducted to examine the network error correction code of its performance bounds [39], code constructions [40] and decoding algorithms [41]. For example, the separating principle of channel coding and network coding is defined in [42]. By doing so, the information can be transmitted through a lossy link in a lossless way.

Note that the aforementioned research examines network coding in directed networks. The network coding technique is first applied to undirected networks in [43]. It is shown that network coding can also bring benefits in terms of throughput in undirected networks. However, unlike directed networks where the gain in throughput brought by network coding is not finitely bounded [44], the gain for undirected networks applying network coding over non-coded network is upper bounded by a constant factor of 2 [43]. Following this work, the bounds on the throughput are further analysed in [45, 46, 47]. In [47], it is shown that the upper bound derived in [43] is generally true in undirected network topologies with any link capacity configuration, any multicast group size, and any source information rate.

Despite the fact that network coding technique is proposed to be applied at network layer initially, its vast benefits draw attention of researchers to apply it at other protocol layers. Inspired by the combination of transmitted packets from different senders, in physical layer, the electromagnetic (EM) waves transmitted by different senders may be added at a relay node. Therefore, instead of processing one EM wave while treating other EM waves as interferences, multiple EM waves may be combined for computing an output signal [48]. This network coding scheme is referred to as physical layer network coding scheme and its publication opens a new research area. The performance of physical layer network coding scheme is examined extensively in terms of throughput [48], reliability [49] and delay [50]. For example, it is applied to a two-way relaying network in [51]. By examining and comparing the sum rate and sum bit error rate of different transmission schemes, the benefits of physical layer network coding are demonstrated.

Other problems relating to network coding techniques are widely examined in the open literature in regard to the memory size [44], number of nodes to perform encoding [52] and computation complexity [53] etc. Along with the development of network coding theory, its application is extended and shifted from the multicast throughput originally to other performance and other traffic configurations. For example, network coding is applied to storage networks, which are normally modelled by combination networks. The combination network $C_{n,k}$ refers to a network with one source, *n* relays and $\binom{k}{n}$ receivers. The capacity gain brought by network coding in these networks are examined in [54]. Further, a deterministic approach to achieve the multicast capacity limit in these networks is presented in [55].

2.2 Network coding schemes

This thesis considers network layer network coding, which concerns the data transmission in the packet level. There are several classes of network layer network coding schemes, which are explained in the following.

2.2.1 XOR based network coding

In the XOR based network coding schemes, bitwise XOR are performed among all or a certain set of packets at a node. Consider the example where nodes N_1 and N_2 exchange packets X_1 and X_2 via a relay N_3 , as shown in Figure 2.1. Assume that node N_1 has the native packet X_1 while node N_2 has the native packet X_2 initially.

Both nodes transmit their native packets to the relay node individually in the first stage. At the relay node, an XOR coding is conducted between the received packets X_1 and X_2 . Then, instead of transmitting two packets separately, N_3 transmits the coded packet $X_1 \oplus X_2$ in one transmission. Finally, at a destination node, the coded packet can be decoded with the assistance of its native packet. For example, at N_1 , the required packet X_2 can be decoded by performing $(X_1 \oplus X_2) \oplus X_1$.



Figure 2.1: The classic two-way relaying network applying XOR coding

XOR coding has drawn an increasing attention owing to its simplicity in both encoding and decoding processes. For example, COPE [56], which is the practical network coding scheme for wireless mesh networks, applies the XOR coding. In COPE, each encoded packet can be decoded upon arriving at a node. In order to achieve this, a router encodes packets with the assistance of MAC layer feedback information. By applying COPE, the throughput of the network can be improved.

This thesis focuses on the reliability benefit made by XOR based network coding when utilised in broadcasting without feedback information.

2.2.2 Linear network coding

Linear network coding has been proved to be able to achieve capacity limit from the source to each receiving node in multicast networks [33]. The capacity limit, given by max-flow min-cut bound [57], is a conclusion from graph theory. More specifically, the information from a source to a destination can be delivered through the network at a maximum rate equal to the min-cut between them [21].

The data transmission using linear network coding scheme is specified in a multicast model, where a source multicasts to multiple receivers. As shown in Figure 2.2, the source N_1 transmits information to destination nodes N_6 and N_7 .

Each link is assigned with a column vector of *d* dimensions, where *d* is the maximum value of the max-flow of every non-source node. The entries of these vectors are selected from a finite filed, say Galois field GF(q), where *q* is an arbitrary positive integer. In the example shown in Figure 2.2, d = 2. In addition, the selected finite field is GF(2). N_1N_2 is assigned with vector $[1 \ 0]^T$, where N_1N_2 stands for the channel from N_1 to N_2 .

Additionally, the vector assigned to an outgoing link from a node is the linear combination of the corresponding vectors assigned to the incoming links. As shown in Figure 2.2, the vector assigned for N_4N_5 is $[1 \ 1]^T$ which is equal to $[1 \ 0]^T + [0 \ 1]^T$, where $[1 \ 0]^T$ and $[0 \ 1]^T$ are the vectors assigned for two incoming links of N_4 , which are N_2N_4 and N_3N_4 respectively.

Further, the information to be transmitted is encoded into a *d*-dimensional row vector. The data flow on a channel is represented as the matrix product of the information row vector with the assigned column vector of the channel. In this case, the data sent on an outgoing channel of a node is a linear combination of the data sent on the incoming channels of the node. In Figure 2.2, suppose the data transmitted by N_1 is $[b_1 \ b_2]$. Then, the data sent on channel N_1N_2 is represented by $[b_1 \ b_2] \times [1 \ 0]^T$, which is b_1 . Similarly, at N_4 , the data transmitted is represented by $[b_1 \ b_2] \times [1 \ 1]^T$, which is $b_1 + b_2$.

Lastly, at a destination node, the information of the source can be retrieved from the received packets with the aid of the channel vectors [33].

Following [33], [58] solves an important problem of finding the encoding functions at intermediate nodes, where the encoding functions are used to determine the vectors assigned to each channel. The basic idea in [58] is to match the network coding solution to a solution of



Figure 2.2: Illustration of linear network coding, where the column vector assigned to each channel is shown.

a set of linear equations, because every solvable multicast network has a scalar linear solution over a sufficiently large finite-field alphabet. However, linear network coding is not always sufficient [59], because some networks do not have linear solutions in any field.

2.2.3 Random linear network coding

In order to apply linear network coding to networks with unknown or changing topologies, random linear network coding is proposed [34]. Unlike linear network coding where every link is assigned with a planned and deterministic vector, random linear network coding enables nodes to randomly select a coefficient for each incoming packet over a finite field.

Consider the same network topology as that in Figure 2.2. When random linear network coding is applied, the message transmitting on channel N_1N_2 can be represented by $\xi_1b_1 + \xi_2b_2$, where $\xi_1, \xi_2 \in GF(q)$, and q is an arbitrary positive integer. In fact, the message transmitting on every channel can be represented by the same expression. The reason is that in linear algebra, the result of arithmetic operations of two values from a finite filed falls into the same filed. For example, the transmitting message on channel N_4N_5 is $\xi_5(\xi_1b_1 + \xi_2b_2) + \xi_6(\xi_3b_1 + \xi_4b_2)$, which can be represented by $\xi_7b_1 + \xi_8b_2$, where $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6, \xi_7, \xi_8 \in GF(q)$. Therefore, in general, when a source transmits $X_1, X_2, ..., X_M$ and random linear network coding is applied, the data transmitting on a channel can be represented by $\sum_i^M \xi_i X_i$, where $\xi_i \in GF(q)$.

The encoding functions which determine the coefficients of the channels are required to be delivered to the destination nodes to allow them retrieve the coded packets. Then, at a receiver, decoding can be performed by Gaussian elimination when enough number of linear independent coded packets are received. It is worth noting that the probability to achieve the theoretical multicast capacity is exponentially approaching one with regard to the field size [34].

2.2.4 Distributed random linear network coding

In both [33] and [34], the global knowledge of coding coefficients is required for decoding. However, it can be difficult to achieve in reality. To solve this problem, a *distributed* network coding scheme is proposed [60]. In [60], a data-aided transmission scheme is applied, where each outgoing data packet flowing on an edge of the network includes a packet header containing the coefficients of the linear combination of the coded packets. With the cost of overhead, network coding can be utilised in a decentralized manner in networks. Moreover, a *generation tag* is introduced in [60] to mark the coded packets that are related to the same set of source information. This method solves the problem of synchronisation of incoming and outgoing packets belonging to the same set of packets.

However, these methods not only increase the time complexity of network coding-based approaches, but also increase the complexity to implement them practically.

2.3 Benefits made by network coding

Bandwidth and energy supply of nodes are primary resource constraints in wireless communication [61]. Network coding, by allowing different information flows sharing these resources, can improve bandwidth and energy efficiencies [22]. Besides, network coding has show benefit in reducing the delay of receiving a set of packets. Moreover, it has been utilised as an error control method for reliable transmission.

2.3.1 Bandwidth efficiency

The first paradigm where network coding shows its advantage is the improvement in capacity when it is applied to error-free multicast [21]. The coding at the intermediate node enables multiple information streams to be transmitted simultaneously, resulting in a substantial bandwidth improvement over networks applying transitional store and forward scheme. Since its publication, the bandwidth benefit made by network coding is examined extensively and network coding has been included in many bandwidth-efficient transmission schemes.

The basic idea

The following illustrates the idea behind the improvement in bandwidth efficiency brought by network coding. Figure 2.3 shows the packet transmissions on butterfly network where sources N_1 and N_2 transmit native packets X_1 and X_2 receptively to both destinations N_4 and N_6 . The network coding scheme utilised here is XOR coding. In addition, the store and forward scheme is drawn for comparison.



Figure 2.3: Butterfly networks where N_1 and N_2 transmit packets X_1 and X_2 to both N_4 and N_6 with traditional store and forward scheme or network coding scheme.

In Figure 2.3 (a), the network applies the traditional store and forward scheme. The central node N_3 can only forward one packet at one time. Therefore, it takes two transmissions for N_3 to broadcast both packets X_1 and X_2 . With the use of network coding, as shown in Figure 2.3 (b), intermediate node N_3 is allowed to combine packets X_1 and X_2 then to broadcast this network coded packet in one transmission. Finally, the coded packet can be decoded at nodes

 N_4 and N_6 . It can be seen that the number of transmissions for the destinations to have both packets is reduced. Therefore, the bandwidth efficiency is improved by network coding.

Following this idea, network coding has been applied in ad hoc networks [62, 63, 64, 65, 66, 67] and wireless mesh networks [68] to increase bandwidth efficiency by reducing the number of transmissions.

In [67], network coding is applied to one-to-all broadcast in multi-hop networks where a node predetermines and selects the neighbouring nodes to forward the broadcast packet. Network coding is performed according to the proposed algorithms which rely on two-hop topology information. It shows in networks applying the XOR based network coding algorithms that the number of transmissions can be reduced up to 45 percent compared to the non-coding approach. In [63], two-way relaying network, as shown in Figure 2.1, is examined. The information exchange rate is calculated when network coding is applied. It is shown that the exchange rate is improved compared to conventional solutions that separate the processing of two unicast sessions.

The average throughput has been analysed in a multicast model where there are *n* receivers and either linear network coding scheme or random linear network coding scheme is applied [69]. The average throughput is the averaged rate that individual receiver experiences. Using linear programming formulations, it is shown that network coding offers benefit in average throughput proportional to \sqrt{n} . Furthermore, when a network has a large number of nodes, the average throughput in the network with network coding at most doubles compared to that without network coding.

Other traffic configurations

The gain in bandwidth efficiency made by network coding is not constrained to the case of multicast [69], but also applicable to other traffic configurations, such as multiple unicast sessions [70].

COPE [56], as introduced in Section 2.1, is a technique that enhances the throughput of wireless multi-hop networks when the traffic is transmitting in unicast fashion. Based on that, a theoretical analysis of the throughput gain in multi-hop wireless networks which apply the COPE-type opportunistic network coding, is presented in [70]. Further, coding-aware routing is proposed in [70] and compared with interference-aware routing, where coding-aware
routing refers to finding the path that increases coding opportunity. Finally, an optimisation is conducted by linear programming to find the maximum unicast throughput. It is shown that a route selection strategy that is aware of network coding opportunities leads to higher end-to-end throughput when compared to coding-oblivious routing strategies.

The throughput of the network coded unicast sessions can be further improved by routing [71, 72, 7]. In [71, 72], random linear network coding is augmented upon existing routing protocols. It allows the source node to continuously send the random linear coded packets through multiple opportunistic paths until the destination collects a sufficient number of packets for decoding. This protocol is particularly suitable for delivering files of medium to large size [71, 72]. In [7], a protocol is proposed to optimise the multipath routing and code rate to maximise throughput when network coding is applied to lossy wireless networks.

Undirected networks

The throughput benefits are examined in undirected networks. In [73], the upper bound of throughput gain in a special network topology, i.e. the combination network topology, is studied. Linear network coding is applied to combination networks. By studying the cost of minimum multicast tree, it is shown that network coding can improve throughput of combination networks upper-bounded by a factor of 9/8.

2.3.2 Energy efficiency

Networks with network coding has shown advantage in energy efficiency [64, 74, 75, 17, 76], where energy efficiency is usually measured by the energy used to transmit one bit of information.

Multicast

In a conventional network where a node only store and forward incoming information, the energy efficient routing relies on finding the minimum-energy multicast tree [77]. There is some research focusing on computing the minimum-energy multicast tree [78, 79], which is, however, usually NP-hard.

An alternative method for minimum-energy multicast in a mobile ad hoc network is proposed in [64]. It considers a layered model of wireless networks, and then accordingly constructs a set of realisable graphs, where each edge of the graph is assigned with the energyper-bit for the corresponding channel. Then, the bit-rate demand on the edges is characterised by the edge-wise maximum of flow from the source to each destination. It is shown that the minimum energy multicast energy-per-bit can be obtained using linear programming, which can be attained by performing network coding, but not routing.

Other traffic configurations

The energy efficiency of a two-way relaying system is examined in [74], where a network coding scheme namely a hybrid automatic repeat request with incremental redundancy scheme is applied. It calculates the average energy consumption for signal transmission over block fading channel and concludes that network coding brings benefit in terms of energy efficiency to the two-way relaying networks.

The energy efficiency of broadcasting in an all-to-all model is examined in [75, 17, 76]. The network considered in [75] has a circular topology while Ref. [17] examines circular, rectangular grid and random network topologies. In [75], both linear network coding and random linear network coding schemes are applied to ad-hoc networks. The simulation results show that there are significant improvements in energy efficiency for these coded networks over networks applying the traditional store and forward approach. In [17], algorithms are proposed to improve energy efficiency by network coding. These algorithms aim at minimising the number of transmissions. Because it is assumed that each transmission consumes the same amount of energy, the total energy is proportional to the number of transmissions. In addition, the energy efficiency by a constant factor in fixed networks, and by a factor of log n in dynamically changed networks, where n is the number of nodes in the network [17].

2.3.3 Delay performance

Network coding technique does not always bring benefit in the delay point of view, because delay may suffer from the encoding and decoding procedures which need to collect enough packets to proceed. There is a trade-off between throughput efficiency and decoding delay: the larger throughput a system has, the more packets need to be coded together, which in turn results in longer delay, because the destination need to accumulate a large number of coded packets in order to decode a single packet [80]. In delay sensitive applications, the delay performance should be taken into consideration.

The average delay

Network coding can be used to reduce the delay in scenarios of large files transmissions. The system considered in [25] is a single-hop one-to-all model, where the source transmits files to multiple receives via time-varying lossy channels. Closed-form expression for the delay performance is given. It is shown that network coding can bring arbitrary gain in delay by scaling the system parameters. After comparing different network coding schemes, it is concluded that random linear network coding has the minimum mean completion time, where the completion time is the time that all receivers receive the complete file. However, these results are not feasible in the case of packet streaming.

In lossy networks, the mean time to complete the transmission of a block of packets from a source to all receivers is examined in [81]. Random linear network coding is used to encode the block of data, and the receiver can recover the information when enough coded packets have been collected. Therefore, it only needs to record the required number of packets to perform decoding rather than keeping track of which packets have been received. This required number of packets is included in the feedback information that is sent occasionally. A Markov chain is used to describe the transmission process where the state represents the number of packets required for decoding at each receiver. Then, the all-to-all model applying random linear network coding is analysed in [82]. The block of data to be encoded is ready in the first transmission. In comparison, the encoding is conducted on dynamically received packets in schemes proposed in this thesis.

In [83], the relation between the average delay and some factors, such as scheduling and the size of coding buffer is examined. The system model is that one transmitter transmits data stream to a set of one-hop receivers. In order to minimise the average delay per received packet at all receivers, an adaptive method is developed to find the maximum number of packets that can be encoded using random linear network coding. Moreover, a network coding scheme is proposed aiming at further reducing the delay by sending non-coded packets. The upper and lower bounds of the delay performance in multicast downlink transmission are derived [84]. The expected delay is also considered. The *expected delay* is the average time to receive

a block of packets that enables decoding. Additionally, the expected encoding delay and the expected transmission delay at the transmitting node are examined. It is shown that the network coding scheme proposed in [83] enables a network to achieve the theoretical lower bound of the expected delay.

Delay distribution

The design of some real-time applications requires not only the average decoding delay but also the worst-case delay, which can be inferred from the complete probability distribution of delay. It is worth to note that in [85], delay probability distribution is investigated. *Delay distribution* is the probability of successful decoding of all packets at individual delay, which is similar to the reliability after each transmission considered in this thesis. Ref. [85] considers a one-to-all model where a single source broadcasts packets encoded by random linear network coding scheme to all other nodes over erasure channels. The broadcast message is available prior to the first broadcast. Markov chain is used to analyse a network with three nodes only and a brute-force method is proposed for four nodes. In contrast, in the model considered in this thesis, the message to be broadcast by a source node at a given time slot depends on the packets that the source node has received in previous time slots. Consequently, the message to be broadcast by a source node does not vary over time. Moreover, this thesis considers all-to-all broadcast in networks with arbitrary number of nodes.

The exact probability to decode N linearly independent packets among K received packet at a receiver is examined in [86]. It is assumed that the transmitted packets are encoded by random linear network coding and the coefficients can be selected from a finite filed of arbitrary size. The number of received packets is proportional to the delay. Therefore, the methods introduced in this paper can be used to measure the delay distribution.

2.3.4 Reliability

Another important benefit that network coding technique is able to bring to a network is to improve the reliability. Conventionally, in a non-coded wireless network, if a transmission fails (lost or dropped due to unsuccessful error correction), then successful reception of a packet relies on multiple retransmissions of the same information from the source node. Lots of research has been conducted to reduce the number of retransmissions while maintaining network reliability. Most recently, network coding has been utilised as an error control method for loss recovery in lossy networks.

Combination of retransmission and network coding

Network coding is combined with retransmission techniques to improve the reliability of packets transmissions to multiple receivers. The XOR based network coding is applied to lossy networks to reduce the number of retransmissions.

When packets are broadcast in lossy wireless networks, many destinations may have disjoint lost packets. Instead of transmitting individual lost packet separately, the sender may encode the lost packets of different receivers and broadcast the coded packet to all receivers. Then at an individual receiver, the lost packets can be recovered with the knowledge of previously received packets. In this case, multiple receivers may recover their lost packets in one transmission and therefore, the total number of retransmissions can be reduced. In illustration of this, an example is given in Figure 2.4, where node N_1 transmits packets x, y and z to N_3 , N_4 and N_5 via a relay N_2 . After broadcasting, each node has different lost packets, which are y, z and z for N_3 , N_4 and N_5 respectively. If network coding is applied, the relay node N_2 can retransmit a coded packet $y \oplus z$, allowing every destination node to recovery their lost packets in one retransmission.



Figure 2.4: An example that network coding used for reducing the number of retransmissions by combining the different lost packets of different receivers.

The reliability of networks applying network coding are examined in [87, 24, 88] where a reception report mechanism is employed to inform the sender of lost packets of each receiver.

In this case, the encoded packets can be generated using the knowledge of lost packets at each receiver.

The network coding aided ARQ is applied to access point (AP) based networks in [87]. More specifically, a source node employs network coding to broadcast a selected combination of unsuccessfully received packets of different receivers. In [87], all users listen to all the packets, and intended users may decode the network-coded packet using the overheard packets. In this way, the number of retransmissions can be reduced.

Packet retransmission algorithms based on network coding for wireless broadcast in oneto-all model are proposed in [24]. An XOR coding is employed to combine the lost packets for different receivers. The expected number of retransmissions for a packet to reach every receiver successfully is calculated under different schemes. After comparison, it concludes that the scheme applying a dynamic network coding enjoys the smallest number of retransmissions.

The following work in [88] focuses on finding the optimal coding set of the lost packets so that the number of retransmissions is minimised while ensuring that receivers can decode the lost packets upon receiving a coded packet. This coding set can be obtained by a colouringbased heuristic algorithm. The encoding of the lost packets relies on a packet-loss table, which contains the packet loss information of every receiver. The maintaining of the packet-loss table requires feedback from every receiver.

However, in wireless communication, especially in broadcast scenario, the feedback is expensive in terms of bandwidth and energy efficiency. Therefore, the retransmission schemes without feedback information are proposed in [89, 90, 91]. Additionally, the network coding schemes proposed in this thesis do not rely on feedback information.

In [89, 90], the authors consider a two stage broadcast scheme where every node broadcasts its native packet in the first stage while an XOR coded packet in the second stage. They investigate the optimal numbers of packets to be encoded in the second stage to minimise the expected number of retransmissions where the connection probabilities of every channel are given. However, the coding scheme in [89] does not always outperform non-coded network in terms of the expected number of retransmissions. In comparison, the schemes proposed in this thesis allows a node to choose not to perform coding when decoding cannot be performed because of lacking of native packets; therefore, they can achieve *at least* the same performance as the non-coded networks, which are better than that proposed in [89].

Ref. [91] considers one-to-all model where one transmitter transmits multiple streams to multiple destinations over lossy channels. An XOR based retransmission scheme is proposed, aiming at providing fairness service to all users in terms of the service time and goodput. In the proposed scheme, the transmitter estimates the reception status of all receivers without extra overheads. Then after comparing all possible coding sets, the scheduler selects the frames which offer the best performance under fairness constraints. Multiple selected frames are encoded by XOR operation. This retransmission scheme is implemented in a real environment and its effectiveness is demonstrated.

Moreover, the reliability gain is characterised analytically in [92, 5], where network coding is compared with traditional error control protocols, such as ARQ and FEC. The considered systems have tree topologies where each multicast tree has equal number of children. The expected numbers of retransmissions by the source node under different error control protocols are computed. Based on numerical comparison, it is conjectured that the reliability gain made by network coding increases logarithmically with respect to the number of receivers in a multicast group compared with a simple ARQ scheme. This hypothesis is then proved in the latter work [5].

Combination of routing and network coding

Another approach to improve reliability is to combine routing with network coding [93, 94], where an individual node mixes different received packets heading towards the same destination [95]. An example is given in Figure 2.5.

Suppose that node N_1 needs to send three packets x, y and z to node N_5 . Figure 2.5 (a) illustrates the traditional method of sending these messages using three routes from node N_1 to node N_5 . After node N_1 sending three packets, node N_3 only receives x and y. Then, node N_3 forwards both packets x and y to node N_5 , whereas node N_5 only receives y. Similarly, node N_5 receives packets x and y from route N_1 - N_2 - N_5 and N_1 - N_4 - N_5 respectively. Finally, node N_5 has only packets x and y. Figure 2.5 (b) illustrates the case when both network coding and multipath routing techniques are applied. Consider the same scenario as Figure 2.5 (a) in terms of packet loss, node N_3 only receives packets x and y from node N_5 , whereas node N_1 . Then, node N_3 sends packet y and an encoded packet $x \oplus y$ to node N_5 , whereas node N_5 only receives the second packet similarly to the case in Figure 2.5 (a). At last, node N_5 receives packets x, $x \oplus y$ and $y \oplus z$, from



Figure 2.5: The comparison between (a) traditional routing and (b) combination of network coding and multipath routing when N_1 transmits three packets *x*, *y* and *z* to N_5 .

which all native packets x, y and z can be decoded. It can be seen that, with a proper coding design, network coding can improve the reliability of packet transmission.

Opportunistic routing improves the reliability of wireless mesh networks when channel quality is poor [72, 7]. Conventional opportunistic routing allows any node to overhear the transmission. Therefore, multiple nodes may hear the same packet and unnecessarily forward the same packet. In contrast, when combined with network coding, this problem can be solved. For example, in MORE [72], a node creates a random linear combination of the received packets. By doing this, multiple nodes broadcast coded packets employing different coding coefficients to avoid repetition. Consequently, the probability of a destination successfully receiving and decoding packets is raised and the reliability is improved.

2.4 The challenges in code design

There are some challenges existing in the design of network coding schemes, summarised as follows.

The XOR based coding schemes have the advantage of low computational complexity in both encoding and decoding processes [85] where both processes require only XOR operations. However, decodability of the coded packets should be taken into account in the XOR based network coding. An improper design of XOR coding may bring negative effect to a network. In some cases, a decoding may not be able to process because of lacking of native packets. For example, packets X_1 , X_2 and X_3 cannot be decoded from coded packets $X_1 \oplus X_2$, $X_2 \oplus X_3$ and $X_1 \oplus X_3$ without a native packet. Therefore, decoding delay may be introduced in order to collect a native packet or it may even fail if packets can only be sent in these coded version. In addition, the number of native packets to be encoded affects the encoding and decoding delay because enough packets are required to be obtained before coding can be proceed.

In order to apply the linear network coding in a network, several strict conditions have to be satisfied. First of all, the channels in the network have to be fixed and error free. Secondly, each channel is assigned with a vector, and the vector assigned to the outgoing channel of a node is a linear combination of the vectors assigned to its incoming channels. Thirdly, central knowledge of these vectors and network topology are required. However, real channels have errors and changing topology. Additionally, this coding scheme is ineffective in some cases (e.g., multi-source, multi-sink with arbitrary demands), because there may not exist a method to assign every channel with a coefficient satisfying the rule described above [59]. Therefore, liner network coding is mostly used for theoretical analysis and hardly applied to real networks.

The random linear network coding schemes [34] can work in a decentralised manner and in real channels with errors. However, there are several parameters should be carefully designed depending on the requirements of each system it is applied to. An important parameter is the number of packets to be encoded into a coded packet. It affects both throughput and the decoding delay. More specifically, the more packets being encoded into one coded packet, the larger throughput and longer delay the system will have. Therefore, in order to obtain higher throughput, more packets should be encoded into one coded packet. Yet another parameter is the size of the finite field for the coefficients. It affects the decoding probability and the size of the overhead. The larger the finite field is, the more likely that the coded packets are linearly independent and therefore, more likely to be decoded. However, the corresponding overhead and bandwidth used for transmitting these coefficients are larger.

2.5 Summary

This chapter reviews some basic network coding schemes: XOR based network coding, linear network coding, random linear network coding and distributed network coding. Moreover,

2.5. Summary

the benefits made by network coding are reviewed. Lastly, the challenges in code design are summarised.

Chapter 3

System Model

This chapter introduces the all-to-all model that the proposed schemes are applied to. In Section 3.1, the broadcast scheme is introduced. Then, we break down the system and examine the structure of a node and the packet to be transmitted in Section 3.2. Thirdly, we characterise the channel conditions between a pair of nodes in Section 3.3. Finally, the data flow diagram of the system model is given and described in Section 3.4.

3.1 The all-to-all model

The network model considered in this thesis is an all-to-all model, as illustrated in Figure 3.1. The network consists of *n* nodes, where the k^{th} node is denoted by N_k . Each node acts both as a source node that broadcasts a packet to all other nodes in the network, and as a sink that listens to the packet broadcast by other nodes. Define the non-coded packet that a node N_k has at time zero (t = 0) as its *native packet*, denoted by X_k , for $k \in \{1, 2, ..., n\}$.

Further, it is assumed that time is slotted. In a time slot only one source node (say N_j) broadcasts a single packet while all other nodes listen.

Assume that all nodes in the network transmit in a round robin manner. A *round* is defined as a sequence of time slots during which every source node broadcasts exactly once. In the case that a packet does not reach all nodes in one time slot, the source node has to broadcast more than once. The goal of all-to-all broadcast is that every node has the native packets of all other nodes.

In wireless broadcast with unreliable links, feedback is usually used to acknowledge a suc-



Figure 3.1: An example of the all-to-all model where the total number of nodes is *n*.

cessful transmission. For example, an acknowledgement is sent from the receiver to the sender to inform the sender of a successful transmission; and a negative-acknowledgement indicates a failure to receive the required packet. However feedback is inefficient and expensive because it introduces delay and extra consumption in bandwidth and energy. Therefore, feedback is not used in this thesis.

It is worth noting that in a conventional network without network coding, a source node can only re-broadcast its native packet. In this thesis, network coding is applied to improve the network performance. Therefore, a node is able to perform encoding and decoding. In the following section, the structure of a node is presented.

3.2 The node structure

In the system considered in this thesis, a node has many parts including a buffer, an encoder and a decoder. A buffer is equipped at each node to store the packets. Since network coding is introduced and every node has chances to be both a source and a sink in different time slots, every node is equipped with an encoder and a decoder. In the following, the functions of the encoder, decoder and buffer are introduced in Section 3.2.1, Section 3.2.2 and Section 3.2.3 respectively.

3.2.1 Encoder

At the encoder, the native packet of the coding neighbour of the source node will be checked. The *coding neighbour* is the node whose native packet is used for encoding at a source node. Before transmission, a source node selects another node as its coding neighbour. The selection of the coding neighbours can be either fixed or dynamically chosen according to the proposed network coding schemes, which will be introduced in detail in Chapter 4 and Chapter 5 respectively.

At the beginning of each time slot, encoding may be performed at the source node depending on the availability of the native packet of the coding neighbour (to be described in detail in Section 4.1 and Section 5.1). More specifically, if the native packet of the coding neighbour is available, encoding will be performed. On the other hand, if the required native packet is not available, no encoding is performed. The encoding is the bitwise XOR between two native packets (the native packets of the source and its coding neighbour).

The basic idea for encoding is that a node can employ network coding to assist other nodes to broadcast their native packets. That is to say, a node can act as a relay for its coding neighbour and help broadcast the native packet of its coding neighbour by transmitting a network coded packet. If we examine the packet transmission between a pair of nodes (say N_j to N_i), the system can be represented by a relay system as shown in Figure 3.2.



Figure 3.2: The relay system for node-to-node transmissions. Note that the number of relays range from 1 to n - 2, and in this figure, two relay nodes N_h , and N_d are drawn as an example.

In Figure 3.2, node N_j is the coding neighbour for nodes $N_h \dots N_d$ (the number of these nodes range from 1 to n - 2, two are shown here as an example). Therefore, nodes $N_h \dots N_d$ can act as relays for node N_j to transmit the packet X_j . For example, N_h may transmit $X_h \oplus X_j$ to node N_i .

3.2.2 Decoder

A decoder is equipped at each node. Whenever a packet is received, it goes through the decoder and decoding can be performed. A decoding is performed on the newly received packet using packets previously stored in the buffer.

The successful decoding of a network coded packet requires that one of the two native packets forming the coded packet has already been successfully received or decoded. For example, X_{γ} can be decoded from packets $X_{\gamma} \oplus X_k$ and X_k by performing $(X_{\gamma} \oplus X_k) \oplus X_k$, where $\gamma, k \in \{1, 2..., n\}$ and $\gamma \neq k$. It is worth noting that X_k can either be received directly or be decoded from another coded packet. After X_{γ} and X_k are decoded, they can be used to decode other coded packets stored in the buffer. The decoding process continues until no more network coded packets can be decoded.

3.2.3 Buffer

A buffer is used at each node for the storage of the received or decoded packets. Duplicated and corrupted packets are dropped in the consideration of buffer size.

There is a slight difference in the usage of buffers in the network coding schemes proposed in the following chapters. In Chapter 4, the buffer is used before the received packet goes into a decoder. For example, packets X_1 , $X_1 \oplus X_2$ and X_2 at a node will be stored as themselves. Therefore, the required size of a buffer at a node should be equal to the number of possible combinations of the received packets. In order to further reduce the size of the buffer, the buffer is used after the decoder in Chapter 5. Therefore, the packets given in the previous example will be stored as packets X_1 and X_2 . It can be seen that the number of packets to be stored is reduced. In this case, the size of a required buffer at a node should be equal to the number of possible combinations of the received packets after decoding.

3.2.4 Packet structure

A packet is a binary sequence including a header and its payload. The native packets of nodes do not need to be of the same length. If the encoding process involves packets of different length, zeros will be added to the end of the shorter packet.

In order to guarantee that the proposed schemes function in a distributed manner, the native

packets used for the encoding of the coded packet are required to be indicated in the encoded packet. This can be achieved by introducing n bits in the header, where each bit represents an individual native packet sequentially. If a native packet is used to form the network coded packet, the corresponding bit is set to 1; otherwise, it is set to 0.

For example, in a network of four nodes, four bits are added in the header. A coded packet of $X_1 \oplus X_3$ is represented by bits [1 0 1 0], as shown in Figure 3.3. In this way, there is no need for centralized knowledge of coding neighbour/s of each node.



Figure 3.3: An example of a packet in a network of four nodes, where the native packets forming the network coded packet $(X_1 \oplus X_3)$ are indicated by four bits $(1 \ 0 \ 1 \ 0)$ in the header.

3.3 Channel conditions

After examining the functions of a node, we examine the channel condition between pairs of nodes. The channels connecting a pair of nodes are lossy wireless channels. The characterisation of the channel conditions is introduced in the following.

In information theory, the channel condition is measured by different criteria. For example, it is measured in terms of channel capacity, spectral bandwidth, symbol rate, bit error rate, packet error rate, etc.

Channel capacity is defined as the maximum information rate (in units of information per unit time) that can be achieved with arbitrarily small error probability [26]. *Spectral bandwidth* refers to the difference between the upper and lower frequencies in a continuous set of frequencies used by a wireless channel [1]. *Symbol rate* is the number of symbol changes (waveform changes or signalling events) made to the transmission medium per second using a digitally modulated signal or a line code.

The above criteria reflect the amount of information that a wireless channel can process, while the following criteria show the Quality of Service (QoS) that a channel can provide. More specifically, *bit error rate* is the number of bit errors divided by the total number of transmitted

bits during a given time interval [96]. *Packet error rate* is the number of error packets divided by the total number of transferred packets during a given time interval.

3.3.1 Packet error rate

In the network layer, the packet error rate is usually used as a metric to measure channel conditions. It is usually assumed that the packet error rate for a channel is q_e , i.e., with probability $1 - q_e$, the packet is transmitted over the real channel successfully and correctly; on the other hand, with probability q_e , the transmitted packet is corrupted [97]. Figure 3.4 shows the mapping of output packet C_y when an input packet C_x is transmitting over a channel where the packet error rate is q_e .



Figure 3.4: An illustration of a channel where the packet error rate is q_e .

Although channels can be characterised by different models, the packet error rate can usually be calculated. There are numerous references relating to obtaining the packet error rate under various channel models [98, 99, 100, 101]. For example, Ref. [100] investigates the packet error rate in non-frequency selective Rayleigh fading channels. Analytical methods to calculate the packet error rate are provided. It is shown that the packet error rate can be derived from several parameters: fading bandwidth, signal-to-noise ratio and the rate of error correction.

3.3.2 End-to-end connection probability

The all-to-all models refer to multi-hop ad hoc networks, where the transmitted packet may reach a destination node via a single hop path (i.e., a channel), a multi-hop path or multiple paths. Therefore, the complement of the packet error rate of a channel (say N_jN_i), which is the direct connection probability from the transmitter N_j to the receiver N_i via a channel, may not truly represent the end-to-end connection probability from node N_j to node N_i .

In this thesis, we use the end-to-end connection probability to characterise the channel conditions between a pair of nodes. Denote p_{ji} , where $p_{ji} \in (0, 1]$, $i \neq j$ and $i, j \in \{1, 2, ..., n\}$, as the *end-to-end connection probability* that N_i receives a packet broadcast from N_j successfully and correctly in one time slot. Further, let $p_{jj} \triangleq 1$. Then, $1 - p_{ji}$ is the probability that N_i fails to receive the packet from N_j in one time slot, which includes the probability that a packet from N_j gets lost, experiences a time-out or there are some uncorrectable errors in the received packet.

The end-to-end connection probability is affected by many factors, such as routing protocols and multiple paths existing between the pair of nodes. It can be calculated if the packet error rates of every channel and the knowledge of the possible routes are given. In illustration of this, Figure 3.5 gives a simple example where N_j has two paths connecting to N_i , which are $N_j \rightarrow N_i$ and $N_j \rightarrow N_h \rightarrow N_i$. Suppose that the packet error rates for channel N_jN_h , N_jN_i and N_hN_i are q_e^{jh} , q_e^{ji} and q_e^{hi} respectively.



Figure 3.5: The paths between N_i and N_i .

If both paths are of equal chance to be used, the end-to-end connection probability from N_j to N_i , denoted by p_{ji} , is:

$$p_{ji} = 1 - (q_e^{jh} + q_e^{hi} - q_e^{jh} q_e^{hi}) q_e^{ji}$$

= $1 - q_e^{jh} q_e^{ji} + q_e^{hi} q_e^{ji} - q_e^{jh} q_e^{hi} q_e^{ji},$ (3.1)

where $q_e^{jh} + q_e^{hi} - q_e^{jh}q_e^{hi}$ is the probability that the packet experiences at least one error transmitting via path $N_j \rightarrow N_h \rightarrow N_i$.

It is obvious that this result, considering multipath, is different from the complement of the

packet error rate of the channel N_jN_i , which is $1 - q_e^{ji}$. Moreover, in scenarios with different routing protocols, such as greedy forwarding, the packet error rate between these two nodes may be different, where using a *greedy forwarding* algorithm, every node tries to forward the packet to the node within its transmission range which is closest to the destination [102].

There is some research in regard to finding these probabilities in the open literature. For example, in [103], these probabilities are calculated by algorithms that use the knowledge of all possible paths and the associated channel packet error rates.

3.3.3 Probabilistic connectivity matrix

These end-to-end connection probabilities can be written into a matrix form, known as the *probabilistic connectivity matrix* [104]. Denote the probabilistic connectivity matrix as Q, it is a $n \times n$ square matrix:

$$Q = \begin{vmatrix} p_{11} & \cdots & p_{in} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{vmatrix},$$
(3.2)

where $(j, i)^{th}$ entry is p_{ji} , representing the end-to-end connection probability from node N_j to node N_i .

This thesis focuses on the application of probabilistic connectivity matrix in the analysis of network performance in lossy wireless networks employing network coding. Therefore, we assume the probabilistic connectivity matrix for a network is given.

3.4 Data flow diagram

In this section, the data flow diagram of the all-to-all model in one time slot is presented. Recall that in one time slot, only one node broadcasts and all other nodes listen. Therefore, the system model is equivalent to an one-to-all model introduced in Section 1.1.2 in one time slot. Assume that node N_j transmits in this time slot, the system model can be represented by Figure 3.6. Then, in the next time slot, node $N_{(j+1) \mod n}$ transmits and all other nodes listen.

In the first time slot, node N_1 acts as the source. It bears the native packet of itself, which is the output of the first block (Source *j*) in Figure 3.6. This packet then passes through a network coding encoder. Because there is only one packet, no encoding can be performed and

3.5. Summary



Figure 3.6: The system model of all to all communication in one time slot. Note that it is assumed that N_i transmits in this time slot.

this non-coded packet will be sent through different lossy wireless channels towards the whole network and may reach several receiver(s). At a receiving end, this non-coded packet passes through a decoder but no decoding will be performed.

In the following time slots, another node acts as a source and the packet to be transmitted at the source may be updated. This occurs because the source in the current time slot may have received the packets transmitted by other nodes in the previous time slots. Therefore, the outputs of the first block (Source j) in Figure 3.6 are both its native packet and the packets received previously. Then, in the network coding encoder, network coding may be performed depending on the proposed network coding schemes and the output packets of the source. In the following, the packet is transmitted through different lossy wireless channels and may reach certain receivers. Finally, at each receiving end, decoding may be performed before the received packet is stored in each node's buffer.

3.5 Summary

This chapter first introduces the broadcast scheme in all-to-all models. Secondly, the structure of a node is examined and the function of a buffer, encoder and decoder are explained. Thirdly, the metric used to measure channel conditions in this thesis is introduced. Lastly, the data flow

diagram of the system in one time slot is presented.

In the following chapters, the network coding schemes designed for all-to-all model are introduced. Moreover, reliability is used as a key metric to evaluate the performance of all-to-all broadcast with the proposed network coding schemes. The *reliability* of the network at round R is defined as the probability that every node in the network has a copy of the native packets of all other nodes at the end of R.

Chapter 4

Neighbour Network Coding Scheme

This chapter introduces a novel *neighbour network coding scheme* for reliable broadcast in all-to-all models. More specifically, a node can assist its neighbour by broadcasting a network coded packet consisting of the native packets of its own and its coding neighbour. The detailed encoding process of the proposed network coding scheme is described in Section 4.1.

Then, in Section 4.2, a Markov chain is established, using which, exact results of the reliability after each round of transmission are obtained. Recall that the *reliability* refers to the probability that every node has the native packets of all other nodes. The reliability after each transmission considered in this thesis is more general applicable compared with previous work on reliability that only considers the *expected* or *average* number of retransmissions for information to reach its intended destinations.

In Section 4.3, upper and lower bounds on the reliability of networks employing the proposed neighbour network coding scheme are provided in closed-form formulas, shedding insight to the relation between factors determining the reliability, e.g. end-to-end connection probabilities and the selection of coding neighbours.

Lastly, in Section 4.4, simulations are conducted to validate the analysis. Further, the optimal neighbour coding scheme which maximises the reliability of a given network is proposed.

4.1 Proposed network coding scheme

Every node (say N_j) selects another node (say N_h), namely *coding neighbour* to encode a broadcast packet according to the following rules.

Encoding

The network coding can only be conducted between the native packets of a code and its coding neighbour. Note that the only constraints on the selection of coding neighbour are:

- The coding neighbour of a node cannot be the node itself, i.e., $j \neq h$;
- A pair of nodes cannot mutually select each other.

Therefore, the analysis is generally applicable to arbitrary neighbour selection rules, where the optimal rule is proposed in Section 4.4. Figure 4.1 is an example of broadcast in lossy wireless network applying neighbour network coding scheme.



Figure 4.1: Illustration of broadcast in lossy wireless network with *n* nodes applying neighbour network coding. p_{ji} is the end-to-end connection probability between node N_j and N_i ; $X_j \oplus X_h$ is the coded packet that N_j broadcasts. Note that every pair of nodes is connected with certain probability, while some paths are not shown in the figure.

In the assigned time slot, the transmitting node checks its buffer to decide whether to broadcast a coded packet or not. Note that the buffering model is as that stated in Chapter 3. If N_j has X_h in its buffer, it broadcasts $X_j \oplus X_h$; otherwise, it broadcasts X_j . Therefore, the packet that node N_j broadcasts at time *t* depends on the packets received by N_j from other nodes up to time *t*. A method that keeps track on individual types of packets should be established. This creates challenge to the theoretical analysis, as shown in the next section.

4.2 Theoretical analysis

In this section, we analyse the reliability by examining the packets received by a node (say N_i) from an arbitrary node (say N_i).

Suppose that the coding neighbour of N_j is N_h . Then, N_j may broadcast either X_j or $X_j \oplus X_h$, depending on the packets that N_j has. It follows that the random process which governs received packets by node N_i in a future time slot, depend only on the packets that N_i and N_j have currently *and* the packet reception in this time slot, *but* not on the processes in the previous time slots. Therefore, this process can be modeled by a first order Markov chain.

In the following, we first construct the states of the Markov chain in sub-section 4.2.1, followed by the derivation of the transition matrices in sub-section 4.2.2. Then, after obtaining the probability vectors in sub-section 4.2.3, network reliability is studied in sub-section 4.2.4.

4.2.1 Construction of the states

States representation

Let the *state* of a node (say N_i) be the combination of packets it has. More specifically, a state is expressed by a $1 \times n$ vector, $[\xi_1, \ldots, \xi_n]$, where an entry ξ_k indicates the packets received and stored from node N_k . There are four possible values for each ξ_k where $k \neq i$, which are: $\xi_k = 0$, 1, 2 and 3 representing the cases that the node N_i has received no packet, native packet, XORed packet, and both native and XORed packets from N_k respectively. Note that the entry $\xi_i = 1$ in every state of N_i , because N_i always has its own packet. Therefore, the total number of states L for each node is equal to 4^{n-1} .

There are some reasons to construct states keeping track on the reception of each individual packet rather than considering the total number of decoded packets as that in the literature [82, 85]. First of all, every packet is headed towards all other nodes, it is interested to know which packets have been received and which have not. Secondly, the coding of the broadcast packet at a source node depends on the packet of its selected coding neighbour. Then, it is essential to know whether or not the packet form the coding neighbour is available for coding.

Index	1	2	3	4
State	[100]	[101]	[102]	[103]
Packets	X_1	X_1, X_3	$X_1, X_3 \oplus X_2$	$X_1, X_3, X_3 \oplus X_2$
Index	5	6	7	8
State	[110]	[111]	[112]	[113]
Packets	X_1, X_2	X_1, X_2	X_1, X_2	X_1, X_2
		X_3	$X_3 \oplus X_2$	$X_3, X_3 \oplus X_2$
Index	9	10	11	12
State	[120]	[121]	[122]	[123]
Packets	$X_1, X_2 \oplus X_1$			
		X_3	$X_3 \oplus X_2$	$X_3, X_3 \oplus X_2$
Index	13	14	15	16
State	[130]	[131]	[132]	[133]
	X_1	X_1	X_1	<i>X</i> ₁
Packets	X_2	$X_2, X_2 \oplus X_1$	$X_2, X_2 \oplus X_1$	$X_2, X_2 \oplus X_1$
	$X_2 \oplus X_1$	X_3	$X_3 \oplus X_2$	$X_3, X_3 \oplus X_2$

Table 4.1: The states of N_1 and corresponding packets for a network with three nodes, where the coding neighbour for N_1 , N_2 and N_3 are N_3 , N_1 and N_2 respectively. For example, the 5th state is [110], which represents that N_1 has packets X_1 and X_2 .

The initial state and absorbing states

The *initial state* of a node is defined as the state at which the node stays at time 0, which indicates the native packet that the node has initially. That is to say, a node N_i contains the packet X_i only in the initial state.

Once a source node has the packet of its designated coding neighbour, it starts to broadcast the XORed packet. Consequently, it is impossible for a node to receive the native packet (X_j) from a source node (N_j) after the node has received the XORed packet $(X_j \oplus X_h)$ from the source node, indicating that there are some absorbing states which cannot exit after entering. More specifically, if a node falls in an *absorbing state*, it means that the node has received a XORed packet from each of other nodes. The absorbing states of node N_i have the characteristic that $\xi_k = 2$ or 3 for all $k \in \{1, 2, ..., n\} \setminus \{i\}$.

Take a network with three nodes as an example. Suppose that the coding neighbours for N_1 , N_2 and N_3 are N_3 , N_1 and N_2 respectively. There are $L = 4^{(3-1)} = 16$ states for each node. The states of N_1 and their corresponding packets are listed in Table 4.1. In this example, the initial state is the 1st state and the absorbing states are the 11th, 12th, 15th, and 16th states.

4.2.2 Transition matrices

At a destination node, a state transition takes place at the end of each time slot. The state transition at a receiving node, say N_i is examined in the following.

Consider that in time slot t, N_j broadcasts. Denote a as the state of N_i in time slot t and b as the state of N_i in time slot t + 1. Denote by $T_{ji}(t)$ the *transition matrix* governing the transitions of the states of N_i when N_j broadcasts. The size of $T_{ji}(t)$ is $L \times L$, where L is the total number of states of node N_j .

A entry $P_{T,t}(b|a)$ is the transition probability that N_i is in state *b* by the end of time slot t + 1, given that N_i is in state *a* by the end of time slot *t*. This transition probability depends on the events that 1) the packet that N_j transmit and 2) the packet reception in time slot t + 1. These two questions will be discussed in detail in the following.

It is worth noting that $T_{ji}(t)$ is time varying and depends on the packet N_j broadcasts, which can be either its native packet or the XORed packet. Consequently, denote $M_{ji}^{\mu_1}$ and $M_{ji}^{\mu_2}$ as the conditional transition matrices representing the transition matrices of the state of N_i conditioned on the event that N_j broadcasts its native packet and the XORed packet respectively. $M_{ji}^{\mu_1}$ and $M_{ji}^{\mu_2}$ are $L \times L$ matrices.

Each element of $M_{ji}^{\mu_1}$ and $M_{ji}^{\mu_2}$, denoted by $P_M^{\mu_1}(b|a)$ and $P_M^{\mu_2}(b|a)$ respectively, is the probability that the state of N_i changes from a to b during time slot t conditioned on the event that N_j broadcasts its native packet and the XORed packet respectively. By doing this categorisation, the first question mentioned above is solved.

Then, $T_{ii}(t)$ can be computed as follows according to total probability theory:

$$T_{ji}(t) = \mu_1(t)M_{ji}^{\mu_1} + \mu_2(t)M_{ji}^{\mu_2}, \qquad (4.1)$$

where $\mu_1(t)$ (resp. $\mu_2(t)$) is the probability that N_j transmits its native packet (resp. the XORed packet) in time slot *t*.

The probabilities $\mu_1(t)$ and $\mu_2(t)$ will be discussed in the next sub-section. The conditional transition matrices $M_{ji}^{\mu_1}$ and $M_{ji}^{\mu_2}$ are time-invariant and can be constructed according to the following algorithms. The event that whether the destination node receives a packet from the source node or not depends on the probability of successful transmission. Therefore, an element of the conditional transition matrices can be expressed by an element from the collection

 $\{0, 1, p_{ji}, 1 - p_{ji}\}$. Consequently, the second problem states above is solved.

More specifically, each element of $M_{ji}^{\mu_1}$, say $P_M^{\mu_1}(b|a)$, can be constructed by comparing states *a* and *b*, according to Algorithm 1. In the algorithm, $a\{k\}$ denotes the k^{th} element of state *a* and we say a = b if $a\{k\} = b\{k\}$ for all $k \in \{1, 2, ..., n\}$. Similarly, each element of $M_{ji}^{\mu_2}$, say $P_M^{\mu_2}(b|a)$, can be constructed by comparing the states *a* and *b*, according to Algorithm 2.

Algorithm 1 Construct <i>N</i>	${\it I}^{\mu_1}_{ji}$
--------------------------------	------------------------

Algorithm 2 Construct $M_{ii}^{\mu_2}$

for each $P_M^{\mu_2}(b|a)$ in $M_{ji}^{\mu_2}$ do if a = b and $a\{j\} = b\{j\} = 0$ or 1 then N_i does not receive the packet from N_j , which happens with probability $P_M^{\mu_2}(b|a) = 1 - p_{ji}$; else if $a\{j\} = 0$ and $b\{j\} = 2$ or $a\{j\} = 1$ and $b\{j\} = 3$, while $a\{k\} = b\{k\}$ for all $k \in \{1, 2, ..., n\} \setminus \{j\}$ then N_i receives the packet from N_j , which happens with probability $P_M^{\mu_2}(b|a) = p_{ji}$; else if a = b and $a\{j\} = 2$ or 3 then the state transition does not depend on whether or not N_i receives the packet from N_j , hence $P_M^{\mu_2}(b|a) = 1$; else let $P_M^{\mu_2}(b|a) = 0$. end if end for

4.2.3 The probability vectors

Denote the *probability vector* of node N_i in time slot t as $S_i(t)$. A probability vector is a $1 \times L$ row vector whose l^{th} entry represents the probability that N_i is at the l^{th} state in time slot t. Suppose that N_j broadcasts in time slot t, then using equation (4.1), the probability vector of N_i in time slot t + 1 can be calculated by:

 $S_i(t+1) = S_i(t)T_{ji}(t)$

$$= S_{i}(t) \left(\mu_{1}(t) M_{ji}^{\mu_{1}} + \mu_{2}(t) M_{ji}^{\mu_{2}} \right).$$
(4.2)

Next, we need to obtain $\mu_1(t)$ and $\mu_2(t)$. Denote B_j as a $L \times 1$ indicator vector, whose l^{th} entry is set to one if N_j broadcasts the XORed packet in the l^{th} state; otherwise it is set to zero. On the other hand, let A_j be a $L \times 1$ indicator vector, whose l^{th} entry is set to one if N_j broadcasts its native packet in the l^{th} state; otherwise it is set to zero. Then we have:

$$\mu_{1}(t) = S_{j}(t)A_{j},$$

$$\mu_{2}(t) = S_{j}(t)B_{j}.$$
(4.3)

Consequently, the probability vector of N_i in time slot t + 1 can be generated by a recursive formula including the probability vectors of N_i and N_j in time slot t:

$$S_{i}(t+1) = S_{i}(t) \left(S_{j}(t) \times A_{j} \times M_{ji}^{\mu_{1}} + S_{j}(t) \times B_{j} \times M_{ji}^{\mu_{2}} \right).$$
(4.4)

In the initial probability vector $S_i(0)$, the initial state is assigned with probability one and all other states are with probability zero. For example, if the states of N_1 are arranged as shown in Table 4.1, the initial state is [100]. Therefore, $S_1(0)$ is a 1 × 16 vector whose first entry is one and all other entries are zero.

4.2.4 Reliability

Denote by $\psi_{t,i}(t)$ the probability that N_i has packets of all other nodes in time slot t. Then, it can be calculated by:

$$\psi_{t,i}(t) = \sum_{x \in \chi} S_i^x(t), \tag{4.5}$$

where $S_i^x(t)$ is the x^{th} entry of $S_i(t)$, the set χ includes the indexes of states in which N_i has the packets from all other nodes. Take i = 1 as an example, as shown in Table 4.1, we have $\chi = \{4, 6, 7, 8, 10, 11, 12, 14, 15, 16\}$.

Denote by $\psi_t(t)$ the reliability of the network in time slot *t*. It is the probability that every node receives the native packets of all other nodes. We assume that the event that a node N_{γ} receives the native packets of all other nodes is independent with the event that another node N_{θ} receives the packets of all other nodes, where $\gamma, \theta \in \{1, 2..., n\}$ and $\gamma \neq \theta$. The reliability of the network can be expressed by:

$$\psi_t(t) = \prod_{i=1}^n \psi_{t,i}(t).$$
(4.6)

Finally, denote the reliability of the network at round *R* as $\psi(R)$. It can be calculated by:

$$\psi(R) = \prod_{t=1}^{n} \psi_t \left(\frac{t}{n}\right). \tag{4.7}$$

4.2.5 Networks without network coding

For comparison, we also calculate the reliability of the networks that do not employ network coding. All nodes transmit in a round robin manner and a node can only broadcast its native packet in a time slot. The reliability can be computed using the same technique as that described previously.

Similarly to the construction of states in sub-section 4.2.1, the state of a node can also be represented by a $1 \times n$ row vector, where the k^{th} element takes value either 0 or 1, representing the event that the node does or does not have packet X_k respectively. Then, the probability vector of a node (say N_i) can be generated by equation (4.2).

Further, the transition matrix T_{ji} is time-invariant because each node transmits its native packet only. Define $T_i \triangleq \prod_{j=1}^{n} T_{ji}$. Then, the probability vector of N_i in time slot t = nR can be calculated by:

$$S_i(t) = S_i(0)T_i^R, (4.8)$$

where $S_i(0)$ is the initial probability vector of N_i at time 0, as introduced in sub-section 4.2.3.

Similarly, the probability vector of all other nodes in the network can be generated by equation (4.8). Then, the reliability of the network can be calculated by equations (4.5), (4.6) and (4.7).

The final expression for the network reliability at round R is simple when no coding is applied, which is:

$$\psi(R) = \prod_{j=1}^{n} \prod_{i=1}^{n} \left(1 - (1 - p_{ji})^{R} \right), \tag{4.9}$$

where p_{ji} is an entry in the probabilistic connectivity matrix of the network.

4.3 Bounds on the reliability

The theoretical results presented in the previous section are exact results but the computation can be complicated. To shed more insights into the impact of fundamental network parameters, e.g. the connection probability between nodes p_{ij} and the selection of coding neighbour, on the network reliability, we present closed-form results of upper and lower bounds on the network reliability in this section.

The analysis starts with the reception of a single packet X_j at a node N_i . Assume that node N_j selects N_h as its coding neighbour and N_j is selected by N_d as coding neighbour, as shown in Figure 4.2. Then, there are two possible *processes* for the packet X_j to reach N_i . The first one is through the path N_j to N_i , via the reception of packets X_j or $X_j \oplus X_h$; and the second one is through the path N_j to N_d and then through the path N_d to N_i , via the reception of packet X_j .



Figure 4.2: Illustration of two processes for packet from N_i to reach N_i

Denote $F_{ji}(R)$ as the probability that N_i receives and decodes X_j by round R and denote $f_{ji}(R)$ as the probability that N_i receives and decodes X_j at round R.

4.3.1 The upper bound

Theorem 4.1. Suppose that N_d selects N_j as coding neighbour. The probability that node N_i receives X_j in first R rounds satisfies:

$$F_{ji}(R) \leq (1 - p_{ji})^{R} \sum_{\alpha=1}^{R} \left(1 - (1 - p_{di})^{R - \alpha} \right) (1 - p_{jd})^{\alpha - 1} p_{jd} + \left(1 - (1 - p_{ji})^{R} \right) \triangleq U_{ji}(R).$$
(4.10)

Proof. To obtain an upper bound on the probability $F_{ji}(R)$, we consider that N_i can decode X_j upon receiving any packet from N_j , regardless of whether the packet is the native packet X_j or

the XORed packet. Note that even if a packet broadcast from N_j has been received by N_i , it still may not be able to decode X_j . This is because N_j may broadcast $N_j \oplus N_h$ and the successful decoding of X_j relies on whether or not N_i has X_h in its buffer.

Denote by Ξ_R (resp. Γ_R) the event that a packet containing X_j (either X_j or an XORed packet containing X_j) reaches N_i by round R via the first (resp. the second) process. Then, it is evident that:

$$F_{ji}(R) \leq \Pr(\Xi_R \cup \Gamma_R)$$

= $\Pr(\Xi_R) + (1 - \Pr(\Xi_R)) \Pr(\Gamma_R).$ (4.11)

Further, it is straightforward that:

$$\Pr(\Xi_R) = 1 - (1 - p_{ji})^R$$

$$\Pr(\Gamma_R) = \sum_{\alpha=1}^R \left(1 - (1 - p_{di})^{R - \alpha}\right) f_{jd}(\alpha),$$
(4.12)

where α is the round at which the packet broadcast by N_j reaches N_d for the first time. It is evident that $f_{jd}(\alpha)$ follows a geometric distribution with success probability p_{jd} . Therefore, equation (4.12) becomes:

$$\Pr(\Gamma_R) = \sum_{\alpha=1}^R \left(1 - (1 - p_{di})^{R - \alpha} \right) (1 - p_{jd})^{\alpha - 1} p_{jd}.$$
(4.13)

Then, equation(4.10) can be readily obtained.

Finally, the upper bound of the reliability of the network at the R^{th} round, denoted by U(R), can be calculated by:

$$U(R) = \prod_{i,j \in \{1,2,\dots,n\}} U_{ji}(R),$$
(4.14)

where $U_{ji}(R)$ is given by Theorem 4.1.

4.3.2 The lower bound

Theorem 4.2. Suppose that the coding neighbours of N_j , N_d and N_h are N_h , N_j and N_g respectively. The probability that node N_i has packet X_j by the R^{th} round satisfies:

$$F_{ji}(R) \geq \sum_{\beta=1}^{R} \left(\Pr(\Omega_{R}|\beta) + \Pr(\Psi_{R}|\beta) - \Pr(\Omega_{R}|\beta) \Pr(\Psi_{R}|\beta) \right) \\ \times f_{hj}^{L}(\beta) \triangleq L_{ji}(R),$$
(4.15)

where $\Pr(\Omega_R|\beta)$, $\Pr(\Psi_R|\beta)$ and $f_{hj}^L(\beta)$ are given by equation (4.18), equation (4.19) and equation (4.21) respectively.

Proof. Similarly to the proof of Theorem 4.1, we separately investigate the two processes described at the beginning of this section. Denote α as the round at which N_d first has X_j and begins to broadcast $X_d \oplus X_j$. Further, denote β as the round at which N_j first receives X_h from N_h and begins to broadcast $X_j \oplus X_h$. To obtain a lower bound on the network reliability, we consider only the cases when N_h broadcasts its native packet in the first β rounds and omits the probability that N_h broadcasts coded packets.

Regarding the first process, it is obvious that the XORed packet broadcast by N_j , i.e., $X_j \oplus X_h$, can be decoded by N_i if N_i has packet X_h . Denote by Ω_R^A the event that N_i receives the packet X_j via the first process by round R. Further, denote by Ω_R^B the event that N_i receives the packet $X_j \oplus X_h$ via the first process by round R but N_i only stores the packets received from N_h in the first β rounds.

Denote by $Pr(\Omega_R^A|\beta)$ the probability that event Ω_R^A occurs conditioned on the event that β is the round at which node N_i receives X_h for the first time. It is straightforward that:

$$\Pr(\Omega_{R}^{A}|\beta) = 1 - (1 - p_{ji})^{\beta}$$

$$\Pr(\Omega_{R}^{B}|\beta) = \left(1 - (1 - p_{ji})^{R - \beta}\right) \left(1 - (1 - p_{hi})^{\beta} + (1 - p_{hi})^{\beta} \left(1 - (1 - p_{ji})^{\beta}\right)\right). \quad (4.16)$$

where the first multiplication term is the probability that N_i receives $X_j \oplus X_h$ and the second multiplication term is the probability that N_i receives either X_j or X_h , so that the XORed packet can be decoded. Further, because events Ω_R^A and Ω_R^B are correlated, there is:

$$\Pr(\Omega_R^A \cap \Omega_R^B) = \Pr(\Omega_R^B | \Omega_R^A) \Pr(\Omega_R^A)$$

$$= \left(1 - (1 - p_{ji})^{R-\beta}\right) \left(1 - (1 - p_{ji})^{\beta}\right).$$
(4.17)

Finally, it is evident that the probability $Pr(\Omega_R|\beta)$, defined as:

$$\Pr(\Omega_{R}|\beta) \triangleq \Pr(\Omega_{R}^{A} \cup \Omega_{R}^{B}|\beta)$$

$$= \Pr(\Omega_{R}^{A}|\beta) + \Pr(\Omega_{R}^{B}|\beta) - \Pr(\Omega_{R}^{A} \cap \Omega_{R}^{B}|\beta),$$
(4.18)

provides a lower bound on the probability that N_i receives and decodes X_j by round R via the first process.

Regarding the second process, both $X_d \oplus X_j$ and X_d are required for N_i to decode X_j . Denote by Ψ_R the event that N_i receives X_j via the second process by round *R* but N_i only receives X_d from N_d when N_d broadcasts its native packet. Then, the probability that event Ψ_R occurs conditioned on the event that N_j receives X_h for the first time at round β is:

$$\Pr(\Psi_{R}|\beta) = \sum_{\alpha=1}^{\beta} (1 - (1 - p_{di})^{\alpha}) (1 - (1 - p_{di})^{R-\alpha}) f_{jd}(\alpha)$$

$$= \sum_{\alpha=1}^{\beta} (1 - (1 - p_{di})^{\alpha}) (1 - (1 - p_{di})^{R-\alpha}) ((1 - p_{jd})^{\alpha-1} p_{jd}),$$
(4.19)

where the first term in the summation is the probability that N_i receives X_d ; the second term is the probability that N_i receives the XORed packet $X_d \oplus X_j$; and the third term is the geometric distribution with success probability p_{jd} as introduced in the proof of Theorem 4.1.

Therefore, the probability that node N_i receives X_j by R^{th} round satisfies:

$$F_{ji}(R) \ge \sum_{\beta=1}^{R} \phi_{hj}(\beta) \operatorname{Pr}(\Omega_R \cup \Psi_R | \beta)$$

$$= \sum_{\beta=1}^{R} \phi_{hj}(\beta) \left(\operatorname{Pr}(\Omega_R | \beta) + \operatorname{Pr}(\Psi_R | \beta) - \operatorname{Pr}(\Omega_R | \beta) \operatorname{Pr}(\Psi_R | \beta) \right),$$
(4.20)

where $\phi_{hj}(\beta)$ is the probability that N_j receives X_h from N_h at round β for the first time, which satisfies:

$$\phi_{hj}(\beta) \ge (1 - p_{hj})^{\beta - 1} p_{hj}(1 - F_{gh}(\beta)) \triangleq f_{hj}^L(\beta), \tag{4.21}$$

where the first term is the probability that N_j does not receive X_h by round $\beta - 1$; the second term is the probability that N_j receives a packet from N_h at round β ; and the third term is the

probability that N_h broadcasts its native packet at round β . Note that $F_{gh}(.)$ can be calculated using Theorem 4.1.

Similarly to equation (4.14), the lower bound of the network reliability at R is readily obtained:

$$L(R) = \prod_{i,j \in \{1,2,\dots,n\}} L_{ji}(R),$$
(4.22)

where $L_{ji}(R)$ is given by Theorem 4.2.

4.4 Numerical results

In this section, simulations are conducted to validate our theoretical analysis. The benefits in reliability of networks applying the neighbour network coding scheme over non-coded networks are shown, followed by examination on the impact of the coding neighbour selections on the network reliability.

Simulations are conducted by MATLAB. They focus on the packet transmission in network layer, whereas other effects, such as modulation and routing, are not considered. Therefore, we can examine the impact of network coding on the reliability of lossy networks, where the connection between every pair of nodes is successful with a certain probability.

The probabilistic connectivity matrix of a network can be computed with the knowledge of the network topology. That is to say, different networks have different probabilistic connectivity matrices. In a random network topology, the probabilistic connectivity matrix is random [104]. Therefore, in this section, the entries are chosen uniformly from (0, 1] to generate numerical results.

4.4.1 Validation of the theoretical analysis

The reliability of networks with an arbitrary number of nodes at an arbitrary round can be calculated using equation (4.7). Figure 4.3 is the comparison of theoretical and simulation results for networks applying the proposed neighbour network coding scheme from round R = 1 to round R = 10.

In Figure 4.3, the solid lines are the theoretical results of reliability for networks with 3, 4 and 5 nodes utilizing neighbour network coding schemes. The probabilistic connectivity



Figure 4.3: Simulation and theoretical results of the reliability of networks when n = 3, 4, 5, where the probabilistic connectivity matrices are [1 0.3 0.6; 0.4 1 0.5; 0.7 0.4 1], [1 0.3 0.6 0.5; 0.4 1 0.5 0.7; 0.7 0.4 1 0.3; 0.3 0.6 0.4 1] and [1 0.3 0.6 0.5 0.4; 0.4 1 0.5 0.7 0.3; 0.7 0.4 1 0.3; 0.3 0.6 0.4 1] and [1 0.3 0.6 0.5 0.4; 0.4 1 0.5 0.7 0.3; 0.7 0.4 1 0.3; 0.3 0.6 0.5 0.3 0.4 1] respectively.

matrices, denoted by Q, for networks of n = 3, 4 and 5 are listed as follows:

$$Q(\text{Figure 4.3}): \begin{bmatrix} 1 & 0.3 & 0.6 \\ 0.4 & 1 & 0.5 \\ 0.7 & 0.4 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.3 & 0.6 & 0.5 \\ 0.4 & 1 & 0.5 & 0.7 \\ 0.7 & 0.4 & 1 & 0.3 \\ 0.3 & 0.6 & 0.4 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.3 & 0.6 & 0.5 & 0.4 \\ 0.4 & 1 & 0.5 & 0.7 & 0.3 \\ 0.7 & 0.4 & 1 & 0.3 & 0.5 \\ 0.3 & 0.6 & 0.4 & 1 \end{bmatrix}, (4.23)$$

The coding neighbour for each node is set to its index neighbour, i.e. N_k chooses $N_{(k+1) \mod n}$ as coding neighbour.

The simulations are conducted under the same configurations, and the results are plotted for comparison, which are the dotted lines in Figure 4.3. The simulation results are the averaged values for 10^4 runs. Recall that in the analysis, we assumed that the events that respectively govern a node receiving the native packets of all other nodes are independent for the simplicity of calculation. In fact, these events are correlated because the packets received by a node determine the packets that the node broadcasts, which consequently affect the received packets of other nodes. It can be seen in Fig. 4.3 that the analytical results match with the simulation

results. More specifically, the differences between the analytical and the simulation results are less than 5 percent in the networks of 3, 4 and 5 nodes. Therefore, the analysis is accurate enough for practical purposes.

4.4.2 The examination of the reliability gain

In the simulations, the networks applying the proposed neighbour network coding scheme are shown to have improvement in the reliability over the non-coded networks. However, the gain varies in different scenarios. In order to examine the factors that affect the reliability benefits, the proposed neighbour network coding schemes are applied to networks with different configurations.

The effects on the neighbour selections

First, we examine the reliabilities of coded networks under all combinations of coding neighbours. The examined networks have 4 and 5 nodes, and the probabilistic connectivity matrices are:

$$Q(\text{Figure 4.4}) : \begin{bmatrix} 1 & 0.1 & 0.5 & 0.4 \\ 0.6 & 1 & 0.2 & 0.6 \\ 0.7 & 0.3 & 1 & 0.1 \\ 0.1 & 0.3 & 0.2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.3 & 0.6 & 0.5 & 0.4 \\ 0.4 & 1 & 0.5 & 0.7 & 0.3 \\ 0.7 & 0.4 & 1 & 0.3 & 0.5 \\ 0.3 & 0.6 & 0.4 & 1 & 0.6 \\ 0.6 & 0.5 & 0.3 & 0.4 & 1 \end{bmatrix}.$$
(4.24)

The best and the worst reliabilities can be achieved are shown in Figure 4.4. In the best scenario, the coding neighbours for nodes N_1 , N_2 , N_3 and N_4 are nodes N_2 , N_3 , N_4 and N_1 respectively when n = 4; and the ones for nodes N_1 , N_2 , N_3 , N_4 and N_5 are nodes N_2 , N_5 , N_4 , N_1 and N_3 respectively when n = 5. In the worst case, the coding neighbours are N_4 , N_1 , N_2 , N_3 and N_4 , N_1 , N_5 , N_3 , N_2 for nodes N_1 , N_2 , N_3 , N_4 and nodes N_1 , N_2 , N_3 , N_4 , N_5 respectively. It shows that the coded networks have higher reliability than that of the corresponding non-coded networks in every case, and the reliability gain can be considerable in some scenarios. For example, in the network of four nodes, the neighbour network coding scheme brings a reliability gain of more than 200 percent over the corresponding non-coded network at R = 10.

Additionally, the selection of coding neighbours affects the network reliability. Based on numerous simulations, it is conjectured that if every node selects the node to which the connec-



Figure 4.4: Network reliabilities under different neighbour selections, where the probabilistic connectivity matrices for n = 4 and 5 are [1 0.1 0.5 0.4; 0.6 1 0.2 0.6; 0.7 0.3 1 0.1; 0.1 0.3 0.2 1] and [1 0.3 0.6 0.5 0.4; 0.4 1 0.5 0.7 0.3; 0.7 0.4 1 0.3 0.5; 0.3 0.6 0.4 1 0.6; 0.6 0.5 0.3 0.4 1] respectively.

tion probability is the lowest as coding neighbour, the network reliability can be maximised. For example, in the network of n = 4, node N_1 is connected to nodes N_2 , N_3 and N_4 with probabilities 0.1, 0.5 and 0.4 respectively. According to the conjecture, N_1 should select N_2 as coding neighbour, because the probability connecting to N_2 is the lowest. Then, the coded packet of N_1 , i.e., $X_1 \oplus X_2$, has useful information of at most one native packet X_1 for N_2 with successful transmission probability 0.1. Additionally, the coded packet $X_1 \oplus X_2$ includes the useful information of at most two native packets, X_1 and X_2 , to N_3 and N_4 with probabilities 0.5 and 0.4 respectively. On the other hand, if N_1 selects another node, say N_3 , as coding neighbour, then the probabilities of transmitting useful information to other nodes are: two useful native packets of $X_1 \oplus X_3$ to N_2 and N_4 with probabilities 0.1 and 0.4 respectively and one useful packet to N_3 with probability 0.5. Apparently, the first neighbour selection scheme transmits the useful information of two native packets with a larger probability in total compared to other neighbour selection schemes.

In summary, the instinct behind this conjecture is to send out more useful information through good links and to send out less useful information though bad links. This interesting observation might have the potential of being applicable to other network coding schemes,
such as a scheme selects more than one coding neighbours. The proof is complicated due to the recursive expression of network reliability and left as future work.

The effect of channel conditions and size of the networks

There are other factors affecting the reliability gain, such as channel conditions and the size of networks. In Figure 4.5, the entries in the probabilistic connectivity matrices are set to be equal to three different sets of values, as follows:

$$Q(\text{Figure 4.5}): \begin{bmatrix} 1 & 0.2 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 & 0.2 \\ 0.2 & 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.35 & 0.35 & 0.35 \\ 0.35 & 1 & 0.35 & 0.35 \\ 0.35 & 0.35 & 1 & 0.35 \\ 0.35 & 0.35 & 0.35 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}.$$

$$(4.25)$$

For comparison, the reliability of the non-coded networks with the same probabilistic connectivity matrices are plotted in Figure 4.5.



Figure 4.5: The reliability gain of networks employing neighbour network coding scheme over non-coded network when n = 4, where the probabilistic connectivity matrices have three sets of values: 0.2, 0.5 and 0.35.

It can be seen that the gain brought by the proposed network coding scheme is moderate in these networks. It is because the instinct behind the benefit of the neighbour network coding scheme is that a source node can assist a neighbour by transmitting a coded packet consisting of the packet of its neighbour's. The benefits would not be significant if the channel conditions are of the same.

Moreover, the neighbour network coding scheme is applied to networks of different sizes and channel conditions. The reliabilities for networks of 4 and 6 nodes with and without the proposed scheme are shown in Figure 4.6.



Figure 4.6: The reliability gain of networks employing neighbour network coding scheme over non-coded network when n = 4 and 6, where the probabilistic connectivity matrix for n = 6 is [1 0.6 0.3 0.5 0.4 0.4; 0.7 1 0.5 0.2 0.3 0.4; 0.5 0.4 1 0.6 0.3 0.2; 0.3 0.6 0.4 1 0.5 0.4; 0.4 0.2 0.5 0.3 1 0.4; 0.4 0.3 0.6 0.4 0.2 1]; and is part of it when n = 4. The selected neighbours for nodes 1...n are nodes {3,4,2,1} and {3,4,6,1,2,5} receptively.

The probabilistic connectivity matrices for networks of n = 4 and 6 are:

$$Q(\text{Figure 4.6}): \begin{bmatrix} 1 & 0.6 & 0.3 & 0.5 \\ 0.7 & 1 & 0.5 & 0.2 \\ 0.5 & 0.4 & 1 & 0.6 \\ 0.3 & 0.6 & 0.4 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.6 & 0.3 & 0.5 & 0.4 & 0.4 \\ 0.7 & 1 & 0.5 & 0.2 & 0.3 & 0.4 \\ 0.5 & 0.4 & 1 & 0.6 & 0.3 & 0.2 \\ 0.3 & 0.6 & 0.4 & 1 & 0.5 & 0.4 \\ 0.4 & 0.2 & 0.5 & 0.3 & 1 & 0.4 \\ 0.4 & 0.3 & 0.6 & 0.4 & 0.2 & 1 \end{bmatrix}.$$
(4.26)

In addition, the optimal neighbours that maximise the reliability gain are selected according to the previous rule. The coding neighbours for nodes N_1 , N_2 , N_3 , N_4 and nodes N_1 , N_2 , N_3 ,

 N_4 , N_5 , N_6 are nodes N_3 , N_4 , N_2 , N_1 and nodes N_3 , N_4 , N_6 , N_1 , N_2 , N_5 respectively. It shows in Figure 4.6 that the neighbour network coding scheme, when best coding neighbours are applied, is able to bring reliability benefit to networks of different sizes and channel conditions.

4.4.3 The bounds on the reliability

Lastly, the bounds on the probability that X_1 is received by N_3 , given by Theorem 4.1 and Theorem 4.2, are shown in Figure 4.7. The coding scheme is the same as that in Table 4.1 and the probabilistic connectivity matrix is:

$$Q(\text{Figure 4.7}): \begin{vmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.1 \\ 0.7 & 0.4 & 1 \end{vmatrix}.$$
(4.27)

It can be seen that the bounds are valid. Moreover, the bounds can be further improved and be used to characterise the reliability gain. Additionally, they can facilitate the proof of the aforementioned conjecture on the optimal neighbour selection rules in future work.



Figure 4.7: Bounds on the probability that N_3 receives X_1 where n = 3 and the probabilistic connectivity matrix is $[1\ 0.2\ 0.3;\ 0.4\ 1\ 0.1;\ 0.7\ 0.4\ 1]$.

4.5 Summary

In this chapter, a neighbour network coding scheme is proposed for broadcast in all-to-all models. Network reliability is investigated analytically and it has been shown that the proposed neighbour coding scheme can improve network reliability significantly. The factors affecting the reliability performance are examined. Further, the optimal neighbour coding scheme that maximises the reliability is proposed. Closed-form bounds on the reliability of a network applying the proposed coding scheme are given.

Note that in this neighbour network coding scheme, a node keeps transmitting the *same* coded packet after the reception or the decoding of the native packet of its designated coding neighbour. This scheme can improve the reliability in general, but the reliability gain (compared to non-coded networks) can be marginal sometimes. For example, in the case that the probability of receiving a packet successfully from its coding neighbour is very low, the chance for the source node to transmit a coded packet is small, resulting in a small coding gain.

In the next chapter, we will investigate the performance of a random neighbour coding scheme that allows a node to randomly choose a coding neighbour in each round of transmission. Moreover, a node can also choose to transmit its native packet even after the reception of the native packet of its coding neighbour.

Chapter 5

Random Neighbour Network Coding Scheme

A large number of existing network coding schemes are *fixed* network coding schemes, where network coding is performed on all or certain predetermined packets [34, 5, 85]. Since practical wireless networks usually contain lossy channels, the packets that a node receives are usually random and the predetermined packets required for encoding by the fixed network coding schemes may not be available, hence *random* network coding schemes receive increasingly interest recently [105, 90].

This chapter proposes a novel *random neighbour network coding scheme* for all-to-all broadcast in lossy wireless networks. Specifically, each node randomly chooses 1) whether or not to perform coding, according to a tuning parameter, which is described in detail in Section 5.1; and 2) with which packet to perform coding on-the-fly, according to the packets that the node has received and decoded. In Section 5.2 theoretical analysis is conducted to characterise the reliability of networks using the proposed random neighbour network coding scheme. Furthermore, the optimal value of the tuning parameter that maximises the reliability of all-to-all broadcast in a network with a given probabilistic connectivity matrix is derived in Section 5.3. Numerical results are given in Section 5.4. It is shown that network reliability can be increased significantly by utilising the proposed random neighbour network coding scheme.

5.1 Proposed network coding scheme

In each time slot, a source node (say N_j) encodes a broadcast packet according to the following rules.

Encoding

Denote the collection of native packets that N_j possesses as \mathcal{D}_j . It includes the native packet of itself, the native packets of other nodes received directly and the native packets decoded from coded packets. If $\mathcal{D}_j = \{X_j\}$, then N_j broadcasts X_j . If $\mathcal{D}_j \setminus \{X_j\} \neq \emptyset$, then N_j does not employ network coding and broadcasts the native packet with probability $1 - \omega$; and with probability ω , N_j randomly selects a packet from $\mathcal{D}_j \setminus \{X_j\}$ with equal probabilities and performs bitwise XOR between the selected packet and its native packet X_j . The node whose native packet is selected for encoding at a source node in a round is the *coding neighbour* for the source node in this round.

The parameter ω is introduced to overcome the cases when network coding may have negative impact on reliability. For example, when a node receives a number of coded packets, it may not able to decode them due to lacking of native packets, as described in [90]. Therefore, ω is used to allow a node choosing to broadcast its native packet with a certain probability even if there are other packets available for coding. For example, if $\mathcal{D}_2 = \{X_1, X_2, X_3\}$, then N_2 broadcasts X_2 , $X_1 \oplus X_2$ and $X_3 \oplus X_2$ with probabilities $1 - \omega$, $\omega/2$ and $\omega/2$ respectively. Additionally, the optimal ω that maximises the reliability will be studied in Section 5.3.

The reason to select only one neighbour for encoding is that this coding scheme enjoys simple decoding process, as described in Chapter 3.

5.2 Theoretical analysis

In each time slot, a source node broadcasts a packet. According to the encoding rules, this packet may contain different information depending on the packets stored in the source node. Furthermore, the packet may be received by different destination nodes with different probabilities. Therefore, it is challenging to track the packets that each node receives and stores after each transmission.

Denote the packets that N_k has at the end of the time slot t as $v_k(t)$. Consequently, denote the packets stored at all the nodes individually in the network at the end of time slot t as a matrix $V(t) = [v_1(t); v_2(t); ...; v_n(t)]$. Let the random process Z(t) represents the packets that are individually stored at every node in the network at the end of time slot t. The received packets of every node in time slot t include the packets received and stored in the previous 0 to t - 1 time slots. Using the definition of Z(t), it can be shown that:

$$\Pr\left(\left|\mathcal{Z}(t+1) = V(t+1)\right| | \mathcal{Z}(1) = V(1), \mathcal{Z}(2) = V(2), \dots, \mathcal{Z}(t) = V(t)\right)$$

=
$$\Pr\left(\mathcal{Z}(t+1) = V(t+1) | \mathcal{Z}(t) = V(t)\right).$$
 (5.1)

It is clear that this process has memory one and it is a first order Markov process. Therefore, a Markov chain governs the random process Z can be established by examining the change of packets at every individual node in the network after each node broadcasts. Using the Markov chain, network reliability can be investigated.

The method to establish and reduce states of the network is introduced in Subsection 5.2.1, followed by the calculation of the transition matrices in Subsection 5.2.2. Finally, the reliability of a network is given in Subsection 5.2.3.

5.2.1 States

Due to the property of the first order Markov chain, the state of a network can be defined without respect to the time slot t. Denoted by matrix V_a the a^{th} state of a network, where the state means the status of stored packets at all nodes and $a \in \{1, 2, ..., L\}$. L is the total number of states, which is examined in Remark 5.1. Further, let the k^{th} row of V_a , denote by v_{ka} , be the packets that node N_k has when the network is in state V_a .

There are two categories of packets in the network. The first category is the native packet of each source node, while the second category is the XORed packet of a pair of native packets. Each packet is assigned with a unique index. Specifically, the native packet X_k is assigned with index k; and an XORed packet, say $X_{\gamma} \oplus X_k$ (it is assumed $1 \le \gamma < k \le n$ without losing generality) is assigned with index:

$$\mu_{\gamma,k} \triangleq n + [(n-1) + (n-\gamma+1)](\gamma-1)/2 + (k-\gamma)$$

$$= n\gamma - \gamma^2/2 - \gamma/2 + k.$$
 (5.2)

Then, the total number of distinct packets is:

$$n + \binom{n}{2} = \frac{(n^2 + n)}{2},\tag{5.3}$$

where the first term *n* is the total number of distinct native packets and the second term $\binom{n}{2}$ is the total number of distinct XORed packets. Consequently, v_{ka} can be represented by a row vector which is composed of $(n^2 + n)/2$ elements where each element represents a packet. More specifically, the first *n* elements of v_{ka} represent native packets of *n* nodes respectively, and the following elements represent XORed packets. The possession of a packet by node N_k is represented by assigning the corresponding element in v_{ka} to one; otherwise, it is set to zero. For example, in a network with three nodes, if node N_1 has packets X_1 and $X_2 \oplus X_3$, then $v_{1a} = [1, 0, 0, 0, 0, 1]$.

Initial state

Denote by V_1 the initial state of the network. It is the state at time 0. Initially, every source node only has the native packet of itself. Therefore initial state is the combination of an $n \times n$ identity matrix and a $n \times \frac{n^2 - n}{2}$ zero matrix. An example of the initial state for a network with three nodes is given in Figure 5.1.

Absorbing state

Among all states, there is one and only one absorbing state representing the event that all nodes in the network have successfully received or decoded the native packets of all other nodes. When this occurs, the status of packet receptions no longer matters. This state is represented by the state V_L , in which the first *n* elements of every row are one and every other element is zero. The adsorbing state for a network of three nodes is given as an example in Figure 5.1.

States reduction

To reduce the complexity of analysis, two methods are introduced to reduce the number of states. Firstly, the number of states can be reduced by taking the decoding process into account,

i.e., the states including the XORed packets whose corresponding native packets have already been received/decoded, are merged.

Secondly, the number of states can be further reduced by discarding invalid states. The invalid states are the states that can never be entered. For example, when neither N_{γ} nor N_k has both X_{γ} and X_k , it is not possible for a third node N_{θ} where $\theta \in \{1, 2, ..., n\} \setminus \{\gamma, k\}$ to have encoded packet $X_{\gamma} \oplus X_k$. Therefore the associated states are invalid.

The states reduction procedure is not applied to the neighbour network coding scheme studied in Chapter 4; this is because, in Chapter 4, the states represent the received packets of a single node and the number of states is relatively small.

Remark 5.1. Before reduction, the total number of states for a network with *n* nodes can be calculated by:

$$2^{(\frac{n^2+n}{2}-1)\times n} = 2^{\frac{n^3+n^2-2n}{2}}$$
(5.4)

where the base 2 represents that there are $n^2/2 + n/2 - 1$ bits for each v_{ka} (for $k \in \{1, 2, ..., n\}$) taking values of either 0 or 1. The *minus one* in the expression represents that there is one bit that has a fixed value. This corresponds to the fact that node N_k always has X_k and the k^{th} bit of v_{ka} is always equal to one.

The state reduction can significantly reduce the number of states. For example, after the state reduction, the total number of states for networks with three and four nodes are 103 and 30519 respectively (outputs from Matlab), compared with 32768 and 68719476736 respectively before the state reduction.

Even after reduction, the total number of states still increases significantly with the additional number of nodes of the network. Fortunately, the states can be constructed by programming in Matlab following the simple rules described above.

5.2.2 Transition matrices

At the end of each time slot, the packet broadcast from a source node may be received by destination nodes with different probabilities resulting in update of the status of packets stored in some nodes. This can be reflected by a transition of states in the Markov chain that is used to describe the packets stored in every node of the network.

In illustration of this, Figure 5.1 shows some states of the Markov chain of a network when

n = 3. In Figure 5.1, V_1 represents the initial state, V_L represents the absorbing state, and $\Phi_j(V_A | V_B)$ represents the transition probability that the network change from state V_B to state V_A when N_j broadcasts.



Figure 5.1: An illustration of the Markov chain of a network when n = 3, where V_1 represents the initial state, V_L represents the absorbing state and $\Phi_j(V_A | V_B)$ is the transition probability that the state of the network change from state V_B to state V_A when N_j broadcasts a packet. Some states are omitted.

In this subsection, the *transition matrices* are constructed separately when each node broadcasts. Denote \mathcal{M}_j as the transition matrix when N_j broadcasts. It is an $L \times L$ matrix, defined as:

$$\mathcal{M}_{j} = \begin{bmatrix} \Phi_{j}(V_{1}|V_{1}) & \cdots & \Phi_{j}(V_{L}|V_{1}) \\ \vdots & \ddots & \vdots \\ \Phi_{j}(V_{1}|V_{L}) & \cdots & \Phi_{j}(V_{L}|V_{L}) \end{bmatrix},$$
(5.5)

where the entry in row B and column A, denoted by $\Phi_j(V_A|V_B)$, is the probability that the state of the network is in V_A in the time slot t + 1, given that the state of the network is in V_B in a time slot t. The following content illustrates the procedures to calculate an entry $\Phi_j(V_A|V_B)$.

The first step is to examine the packets stored in the transmitting node N_j in these two time slots, which are v_{jA} and v_{jB} . If $v_{jA} \neq v_{jB}$, then the transition probability $\Phi_j(V_A | V_B) = 0$. This is because, the transmitting node N_j only broadcasts packets it already has and this will not lead to a variation of the packets it owned.

Next, we consider the case that $v_{jA} = v_{jB}$. Recall that the native packets that N_j has are \mathcal{D}_j . Further, denote the cardinality of \mathcal{D}_j as m_j . If $m_j = 1$, then the source node has the native

packet of itself only and no encoding can be performed. Therefore N_i transmits X_i .

In the case that $m_j > 1$, the source node may broadcast m_j different types of packets, among which $m_j - 1$ are XORed packets and one is its native packet. Denote π_{jh} , where $h \in \mathcal{D}_j \setminus \{j\}$ as the event that N_j broadcasts an XOR coded packet $X_j \oplus X_h$; and denote π_{jj} as the event that N_j broadcasts its native packet X_j . According to encoding rules of random neighbour network coding scheme introduced in Section 5.1, we have:

$$Pr(\pi_{jh}) = \frac{\omega}{m_j - 1},$$

$$Pr(\pi_{jj}) = 1 - \omega.$$
(5.6)

Further, denote $\Pr(V_A | V_B, \pi_{jj})$ and $\Pr(V_A | V_B, \pi_{jh})$ as the conditional probabilities that the state of the network transforms from V_B to V_A conditioned on the events that the source node broadcasts packets X_j and $X_j \oplus X_h$ respectively. Then according to total probability theorem [106],

$$\Phi_{j}(\boldsymbol{V}_{\boldsymbol{A}} | \boldsymbol{V}_{\boldsymbol{B}}) = \Pr\left(\boldsymbol{V}_{\boldsymbol{A}} | \boldsymbol{V}_{\boldsymbol{B}}, \pi_{jj}\right) \Pr(\pi_{jj}) + \sum_{h \in \mathcal{D}_{j} \setminus \{j\}} \Pr\left(\boldsymbol{V}_{\boldsymbol{A}} | \boldsymbol{V}_{\boldsymbol{B}}, \pi_{jh}\right) \Pr(\pi_{jh})$$
$$= \Pr\left(\boldsymbol{V}_{\boldsymbol{A}} | \boldsymbol{V}_{\boldsymbol{B}}, \pi_{jj}\right) (1 - \omega) + \sum_{h \in \mathcal{D}_{j} \setminus \{j\}} \Pr\left(\boldsymbol{V}_{\boldsymbol{A}} | \boldsymbol{V}_{\boldsymbol{B}}, \pi_{jh}\right) \frac{\omega}{m_{j} - 1}, \quad (5.7)$$

where $m_j > 1$.

In the case that $m_j = 1$, N_j only has its native packet X_j . Therefore, the probability for N_j to transmit the native packet is one, i.e., $Pr(\pi_{jj}) = 1$, and the probability of N_j transmitting a network coded packet is 0, i.e., $Pr(\pi_{jh}) = 0$. We have:

$$\Phi_j(\mathbf{V}_A | \mathbf{V}_B) = \Pr\left(\mathbf{V}_A | \mathbf{V}_B, \pi_{jj}\right).$$
(5.8)

The transition of state from V_B to V_A , i.e. the event that the status of the packets in the buffers of all nodes in the network changes from state V_B to V_A , can be divided into *n* mutually independent events - the events that the status of the packets in the buffer of each individual node N_k changes from v_{kB} to v_{kA} for all $k \in \{1, 2, ..., n\}$.

Denote by $P_{ji}^h(v_{iA}|v_{iB})$ the probability that the status of the packets in the buffer of node N_i changes from v_{iB} to v_{iA} for all $i \in \{1, 2, ..., n\}$ when node N_j transmits $X_j \oplus X_h$. Then, denote

by $P_{ji}^{j}(v_{iA}|v_{iB})$ the probability that the status of packets in node N_i change from v_{iB} to v_{iA} for all $i \in \{1, 2, ..., n\}$ when node N_j transmits X_j .

It is obvious that the conditional probability $\Pr(V_A | V_B, \pi_{jh})$ is the product of *n* probabilities $P_{ji}^h(\mathbf{v}_{iA} | \mathbf{v}_{iB})$. Similarly, the conditional probability $\Pr(V_A | V_B, \pi_{jj})$ can be calculated by the product of *n* probabilities $P_{ji}^j(\mathbf{v}_{iA} | \mathbf{v}_{iB})$ for all $i \in \{1, 2, ..., n\}$. We have:

$$\Pr\left(\mathbf{V}_{A} \middle| \mathbf{V}_{B}, \pi_{jj}\right) = \prod_{i=1}^{n} P_{ji}^{j} \left(\mathbf{v}_{iA} \middle| \mathbf{v}_{iB}\right),$$

$$\Pr\left(\mathbf{V}_{A} \middle| \mathbf{V}_{B}, \pi_{jh}\right) = \prod_{i=1}^{n} P_{ji}^{h} \left(\mathbf{v}_{iA} \middle| \mathbf{v}_{iB}\right).$$
(5.9)

Probabilities $P_{ji}^{j}(v_{iA}|v_{iB})$ and $P_{ji}^{h}(v_{iA}|v_{iB})$ depend on whether or not the reception of the packet from N_{j} leads to the status of the packets stored in N_{i} changing from v_{iB} to v_{iA} . These probabilities can be either one value from the collection $\{0, 1, p_{ji}, 1 - p_{ji}\}$, and can be obtained by comparing the packets stored at the receiving node N_{i} in these time slots, which are v_{iB} and v_{iA} . Algorithm 3 is used to find the value of $P_{ji}^{j}(v_{iA}|v_{iB})$ while Algorithm 4 is used to find $P_{ii}^{h}(v_{iA}|v_{iB})$.

In the algorithms, denote by $v_{iA}\{\lambda\}$ (resp. $v_{iB}\{\lambda\}$) the λ^{th} element of v_{iA} (resp. v_{iB}), and we say that $v_{iA} = v_{iB}$ if $v_{iA}\{\lambda\} = v_{iB}\{\lambda\}$ for all $\lambda \in \{1, 2, ..., (n^2 + n)/2\}$.

Further, in Algorithm 3 denote by \mathcal{H}_{ji} the collection of indices of the native packets that node N_i is able to decode upon receiving packet X_j under the condition that the packets stored in N_i is v_{iB} . The collection \mathcal{H}_{ji} can be obtained recursively by adding in the corresponding indices of native packets (e.g., γ and k) of an XORed packet $(X_{\gamma} \oplus X_k)$, if the following two conditions are both satisfied: 1) N_i has the XORed packet $(X_{\gamma} \oplus X_k)$; and 2) \mathcal{H}_{ji} has the index of one of the native packets (either γ or k). More specifically, the iteration begins with $\mathcal{H}_{ji} = \{j\}$. Then, if N_i has $X_j \oplus X_k$, \mathcal{H}_{ji} is updated to $\mathcal{H}_{ji} = \{j, k\}$. The iteration finishes when no more packets can be decoded. Denote by \mathcal{G}_{ji} the collection of every index of the XORed packet that can be decoded at node N_i when the packet X_j is received, where the index of an XORed packet can be calculated by equation (5.2) given in Section 5.2.1.

Similarly, in Algorithm 4, denote by \mathcal{H}_{jhi} the collection of indices of the native packets that node N_i is able to decode upon receiving packet $X_j \oplus X_h$ under the condition that the packets stored in N_i is v_{iB} . It can be obtained from a recursive method similar to that for obtaining \mathcal{H}_{ji} . The iteration begins with $\mathcal{H}_{ji} = \{j, h\}$ when N_i either has X_j or X_h , i.e., the packet $X_j \oplus X_h$ can be decoded at N_i . Moreover, $\mathcal{H}_{ji} = \emptyset$ when N_i has neither X_j nor X_h , viz. the packet $X_j \oplus X_h$ cannot be decoded at N_i . Denote by \mathcal{G}_{jhi} the collection of indices of all XORed packets that can be decoded at node N_i upon receiving packet $X_j \oplus X_h$.

Algorithm 3 when N_i transmits X_i to N_i

if $v_{iB} \{j\} = 1$ **then** if $v_{iB} = v_{iA}$ then $P_{ji}^{j}(v_{iA} | v_{iB}) = 1;$ else $P_{ji}^{j}(\mathbf{v}_{iA}|\mathbf{v}_{iB}) = 0$; end if else if $v_{iB} \{\mu_{j,\lambda}\} = 1$ for any $\lambda \in \mathcal{N} \setminus \{i, j\}$ then if $v_{iA}\{x\} = 1$ and $v_{iA}\{y\} = 0$ for all $x \in \mathcal{H}_{ji}, y \in \mathcal{G}_{ji}$; and $v_{iB}\{\lambda\} = v_{iA}\{\lambda\}$ for all $\lambda \in \{1, 2, \dots (n^2 + n)/2\} \setminus \{\mathcal{H}_{ji}, \mathcal{G}_{ji}\} \text{ then } P^j_{ii}(\mathbf{v}_{iA} | \mathbf{v}_{iB}) = p_{ji};$ else if $v_{iB} = v_{iA}$ then $P_{ji}^{j}(v_{iA}|v_{iB}) = 1 - p_{ji};$ else $P_{ji}^{j}(\mathbf{v}_{iA}|\mathbf{v}_{iB}) = 0;$ end if else if $v_{iA}{j} = 1$ and $v_{iB}{\lambda} = v_{iA}{\lambda}$ for all $\lambda \in {1, 2, ..., (n^2 + n)/2} \setminus {i}$ then $P_{ji}^j(v_{iA})$ $\boldsymbol{v_{iB}} = p_{ji};$ else if $v_{iB} = v_{iA}$ then $P_{ji}^{j}(v_{iA} | v_{iB}) = 1 - p_{ji}$; else $P_{ji}^{j}(\mathbf{v}_{iA}|\mathbf{v}_{iB}) = 0$. end if end if end if

Remark 5.2 (Explanation of Algorithm 3). If the destination node N_i already has packet X_j (i.e. $v_{iB} \{j\} = 1$), the state of node N_i does not change regardless of whether or not N_i receives X_j in this transmission. Therefore, $P_{ii}^j(v_{iA}|v_{iB}) = 1$ if $v_{iB} = v_{iA}$.

In the case that N_i does not have X_j but has $X_j \oplus X_k$, the reception of X_j leads to every native packets in set \mathcal{H}_{ji} decoded from XOR coded packets whose indices are included in \mathcal{G}_{ji} . Recall that in v_{ka} , the possession of a packet is represented by setting its corresponding index to 1; and 0 otherwise. Therefore, the indices of XORed packets that are decoded are set to 0. In the variation of status from v_{iB} to v_{iA} , if the x^{th} bit for all $x \in \mathcal{H}_{ji}$ changes from 0 to 1 and y^{th} bit for all $y \in \mathcal{G}_{ji}$ changes from 1 to 0; while other bits stay the same, then X_j reaches N_i successfully and the corresponding probability is equal to p_{ji} . On the other hand, if $v_{iB} = v_{iA}$, then no packet is received by N_i in this time slot and the corresponding probability is $1 - p_{ji}$.

Lastly, in the case that N_i does not have X_j nor any coded packet consisting of X_j , the

reception of X_j only changes the j^{th} element of v_{iB} from 0 to 1. That is because no packet can be decoded upon receiving packet X_j . If $v_{iB}\{j\}=0$ and $v_{iA}\{j\}=1$, then the packet X_j reaches N_i successfully in this time slot, which happens with probability p_{ji} . On the other hand, the probability for the status of packets stored in N_i to stay the same is equal to $1 - p_{ji}$.

Algorithm 4 when N_j transmits $X_j \oplus X_h$ to N_i for all $h \in \mathcal{D}_i$

if $v_{iB} \{\mu_{j,h}\} = 1$ or $(v_{iB} \{j\} = 1$ and $v_{iB} \{h\} = 1)$ then if $v_{iB} = v_{iA}$ then $P_{ji}^{h}(v_{iA} | v_{iB}) = 1$; else $P_{ji}^{h}(v_{iA} | v_{iB}) = 0$; end if else if $v_{iB}\{j\} = 0$ and $v_{iB} \{h\} = 0$ then if $v_{iA}\{\mu_{j,h}\} = 1$ and $v_{iB} \{\lambda\} = v_{iA}\{\lambda\}$, for all $\lambda \in \{1, 2, ..., (n^{2} + n)/2\} \setminus \{\mu_{j,\lambda}\}$ then $P_{ji}^{h}(v_{iA} | v_{iB}) = p_{ji}$; else if $v_{iB} = v_{iA}$ then $P_{ji}^{h}(v_{iA} | v_{iB}) = 1 - p_{ji}$; else $P_{ji}^{h}(v_{iA} | v_{iB}) = 0$; end if else if $v_{iA}\{x\} = 1$ and $v_{iA}\{y\} = 0$ for all $x \in \mathcal{H}_{jhi}, y \in \mathcal{G}_{jhi}$; and $v_{iB} \{\lambda\} = v_{iA}\{\lambda\}$ for all $\lambda \in \{1, 2, ..., (n^{2} + n)/2\} \setminus \{\mathcal{H}_{jhi}, \mathcal{G}_{jhi}\}$ then $P_{ji}^{h}(v_{iA} | v_{iB}) = p_{ji}$; else if $v_{iB} = v_{iA}$ then $P_{ji}^{h}(v_{iA} | v_{iB}) = 1 - p_{ji}$; else if $v_{iB} = v_{iA}$ then $P_{ji}^{h}(v_{iA} | v_{iB}) = 1 - p_{ji}$; else if $v_{iB} = v_{iA}$ then $P_{ji}^{h}(v_{iA} | v_{iB}) = 1 - p_{ji}$; else $P_{ji}^{h}(v_{iA} | v_{iB}) = 0$; end if end if

Remark 5.3 (Explanation of Algorithm 4). If N_i already has either packet $X_j \oplus X_h$ or both packets X_j and X_h , there will be no variation in the packets stored at N_i regardless of whether or not N_i receives $X_j \oplus X_h$. If $v_{iB} = v_{iA}$, the probability $P_{ii}^j(v_{iA} | v_{iB})$ is 1.

Similarly to Algorithm 3, in the case that there is neither X_j nor X_h in v_{iB} , the reception of $X_j \oplus X_h$ cannot lead to the decoding of any packet. Consequently, if the only difference between v_{iB} and v_{iA} is that v_{iB} does not have $X_j \oplus X_h$ but v_{iA} does, then the packet $X_j \oplus X_h$ reaches N_i successfully in this time slot, which happens with probability p_{ji} . On the other hand, if $v_{iB}=v_{iA}$, then no packet arrives at N_i in this time slot, whose corresponding probability is $1 - p_{ji}$.

Otherwise, if N_i has either X_j or X_h , then $X_j \oplus X_h$ can be decoded upon arriving at N_i . In addition, X_j and X_h can be used to decode other XORed packets. Therefore, in the variation of status from v_{iB} to v_{iA} , if x^{th} bit for all $x \in \mathcal{H}_{jhi}$ changes from 0 to 1 and y^{th} bit for all $y \in \mathcal{G}_{jhi}$ changes from 1 to 0; while all other bits stay the same, then $X_i \oplus X_h$ reaches N_i successfully

and the corresponding probability is equal to p_{ji} . On the other hand, if $v_{iB} = v_{iA}$, it indicates that no packet arrives at N_i in this time slot, the corresponding probability is $1 - p_{ji}$.

After obtaining every $P_{ji}^{j}(\mathbf{v}_{iA}|\mathbf{v}_{iB})$ and $P_{ji}^{h}(\mathbf{v}_{iA}|\mathbf{v}_{iB})$, substitute equation (5.9) to equation (5.7). The entry $\Phi_{j}(\mathbf{V}_{A}|\mathbf{V}_{B})$ when $m_{j} > 1$ is readily obtained, which is:

$$\Phi_{j}(\boldsymbol{V}_{\boldsymbol{A}}|\boldsymbol{V}_{\boldsymbol{B}}) = \Pr\left(\boldsymbol{V}_{\boldsymbol{A}}|\boldsymbol{V}_{\boldsymbol{B}},\pi_{jj}\right)(1-\omega) + \sum_{h\in\mathcal{D}_{j}\setminus\{j\}}\Pr\left(\boldsymbol{V}_{\boldsymbol{A}}|\boldsymbol{V}_{\boldsymbol{B}},\pi_{jh}\right)\frac{\omega}{m_{j}-1}$$
$$= \prod_{i=1}^{n} P_{ji}^{j}\left(\boldsymbol{v}_{i\boldsymbol{A}}|\boldsymbol{v}_{i\boldsymbol{B}}\right)(1-\omega) + \sum_{h\in\mathcal{D}_{j}\setminus\{j\}}\prod_{i=1}^{n} P_{ji}^{h}\left(\boldsymbol{v}_{i\boldsymbol{A}}|\boldsymbol{v}_{i\boldsymbol{B}}\right)\frac{\omega}{m_{j}-1}.$$
(5.10)

Finally, $\Phi_j(V_A | V_B)$ is equal to:

$$\begin{cases} \prod_{i=1}^{n} P_{ji}^{j} (\boldsymbol{v}_{iA} | \boldsymbol{v}_{iB}) (1 - \omega) + \sum_{h \in \mathcal{D}_{j} \setminus \{j\}} \prod_{i=1}^{n} P_{ji}^{h} (\boldsymbol{v}_{iA} | \boldsymbol{v}_{iB}) \frac{\omega}{m_{j} - 1} & \text{if } m_{j} > 1; \\ \prod_{i=1}^{n} P_{ji}^{j} (\boldsymbol{v}_{iA} | \boldsymbol{v}_{iB}) & \text{if } m_{j} = 1. \end{cases}$$

$$(5.11)$$

By setting the state V_B to V_1 and varying the state V_A from V_1 to V_L , the first row of transition matrix \mathcal{M}_j is obtained. Then, repeat the procedure for every V_B , where $B \in \{2, 3, ..., L\}$, the transition matrix \mathcal{M}_j which governs the state transition of the network when N_j transmits is obtained. Similarly, the transition matrices \mathcal{M}_j for every $j \in \{1, 2, ..., n\}$ can be calculated. Accordingly, define $\mathcal{M} \triangleq \prod_{j=1}^n \mathcal{M}_j$ as the transition matrix for a round during which every node broadcasts once.

It is worth noting that the entries of the transition matrices are found by either 0, 1 or the product of the tuning parameter and a series of p_{ji} and $1 - p_{ji}$ where $i, j \in \{1, 2, ..., n\}$. The value of p_{ji} is given in a probabilistic connectivity matrix. Therefore, if the probabilistic connectivity matrix is the same during the time slots when N_j transmits, the \mathcal{M}_j in these time slots are equal.

5.2.3 Probability vector and the reliability

The *probability vector* indicates the probabilities of the network at every possible state. It is a row vector of size $1 \times L$, where the l^{th} element is the probability that the network is at state V_l . Denote the probability vector at the end of each round R as S(R). In the initial probability vector, denoted by S(0), the element of the initial state is of probability one and all other elements are zero. Assume the initial state is represented by the first element in the probability vector. According to Markov theory, the probability vector at the end of round R is equal to:

$$S(R) = S(R-1) \prod_{j=1}^{n} \mathcal{M}_{j}$$
$$= S(0) (\prod_{j=1}^{n} \mathcal{M}_{j})^{R}$$
$$= S(0) \mathcal{M}^{R}.$$
(5.12)

Finally, we calculate the reliability at the end of round *R*, denoted by $\psi(R)$. It is the probability that the network falls into the absorbing state by the end of round *R*.

$$\psi(R) = S(R)\{L\},$$
(5.13)

where the absorbing state is indicated by the L^{th} bit in the probability vector.

5.3 Optimisations

In a network applying random neighbour network coding scheme, the encoding at a source node N_j is performed on the randomly selected packet from set $\mathcal{D}_j \setminus \{X_j\}$, where \mathcal{D}_j is defined in Section 5.1. Further, the received packets at a node depend on the probabilistic connectivity matrix, which is a property of the network and can not be controlled. On the other hand, the tuning parameter ω , which is the probability for a source node to perform network coding and can be controlled.

The value range of ω is [0, 1], where 0 represents the case that a node does not perform coding and 1 represents the case that a node always performs coding. Therefore, this parameter determines the impact of coding on reliability. By tuning ω , we are tuning the reliability of a network ranging from non-coding to certainly coding. It is important to obtain the optimal ω that maximises the network reliability when the probabilistic connectivity matrix of a network is given.

5.3.1 Optimise the reliability at an individual round

The network reliability $\psi(R)$ at round *R*, given in equation (5.13), is a function of variables ω , p_{ji} and $1 - p_{ji}$ where $i, j \in \{1, 2, ..., n\}$. In the case that the probabilistic connectivity matrix is

given, the expression $\psi(R)$ can be reduced to a single-variable polynomial by substituting the values of the entries in the given probabilistic connectivity matrix. There is:

$$\psi(R) = f(\omega), \tag{5.14}$$

where $\omega \in [0, 1]$. It is a constrained nonlinear optimisation problem. Detailed steps to find the optimal value of the tuning parameter is given through an example in Section 5.4.

5.3.2 Optimise the expected round to absorb

Recall that there is one absorbing state in the Markov chain. Rearrange the transition matrix into the canonical form:

$$\mathcal{M} = \begin{pmatrix} W & Y \\ 0 & 1 \end{pmatrix}, \tag{5.15}$$

where W is the matrix governs the transitions among all transient states, and Y is the matrix governs the transitions from transient states to the absorbing state. 0 and 1 are scalars. Then, the fundamental matrix, denoted by N, is:

$$N = (I - W)^{-1}, (5.16)$$

where the (A, B)th entry describes the expected number of rounds to reach a transient state B starting from a transient state A. Finally, the expected number of rounds to reach the absorbing state, i.e., every node receives or decodes the native packets of all other nodes, can be calculated by:

$$E = Nc, (5.17)$$

where c is a column vector of size $L \times 1$, whose entries are all equal to one. Therefore, the expected number of rounds to reach the absorbing state V_L (assume it is the last state) from the initial state V_1 (assume it is the first state) is the entry $E\{1\} \triangleq E_{exp}$, which is a function of ω , p_{ji} and $1 - p_{ji}$. Substitute the values of entries of a given probabilistic connectivity matrix, then E_{exp} becomes a single variable polynomial of ω . Finally, the minimum E_{exp} and the corresponding ω can readily be found by solving a constrained nonlinear optimisation problem.

5.4 Numerical results

This section provides numerical evaluations of the analytical results of network reliability derived in this chapter. Moreover, examples are given showing the procedures to optimise the random neighbour network coding scheme in networks with given probabilistic connectivity matrix. Then the benefit made by the proposed scheme in reliability is examined in networks with different parameters.

Recall that the analysis allows arbitrary values in the probabilistic connectivity matrices indicating the end-to-end connection probabilities of pairs of nodes. In this section, their entries are generated uniformly from (0, 1] for numerical evaluations. Other simulation settings are the same as that in Chapter 4.

5.4.1 Validation of the theoretical analysis

The random neighbour network coding is applied to networks with 3 and 4 nodes.



Figure 5.2: The comparison between theoretical and simulation results of the network reliability applying random neighbour network coding when n=3 and 4. The probabilistic connectivity matrix for n = 4 is [1, 0.2, 0.3, 0.6; 0.4, 1, 0.5, 0.3; 0.6, 0.7, 1, 0.2; 0.3, 0.4, 0.5, 1], and for n = 3 is [1, 0.2, 0.3; 0.4, 1, 0.5; 0.6, 0.7, 1]; $\omega = 0.6$.

The analytical results of the reliability after each round of transmissions, which are derived by the methods introduced in Section 5.2, are shown in Figure 5.2 as the dotted lines. The probabilistic connectivity matrices utilised in Figure 5.2 for n = 3 and 4 are:

$$Q(\text{Figure 5.2}): \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.5 \\ 0.6 & 0.7 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.2 & 0.3 & 0.6 \\ 0.4 & 1 & 0.5 & 0.3 \\ 0.6 & 0.7 & 1 & 0.2 \\ 0.3 & 0.4 & 0.5 & 1 \end{bmatrix}, (5.18)$$

and the tuning parameter ω is set to 0.6 (note that the optimal value of ω is derived in the next sub-section). Additionally, the simulation results under the same network configurations are plotted for comparison. The Monte Carlo simulation results are the averaged values over 10⁵ runs and they are shown in Figure 5.2 as the solid lines. As can be seen in Figure 5.2, the theoretical results match the simulation results tightly, which validates the theoretical analysis.

5.4.2 Optimal selection of the tuning parameter

In the following, two optimisations are conducted on 1) maximise the reliability at a certain round, or 2) minimise the expected number of rounds to reach the absorbing state.

Consider a network consisting of three nodes, where the probabilistic connectivity matrix is:

$$Q(\text{Figure 5.3, 5.4, 5.5}): \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.5 \\ 0.6 & 0.7 & 1 \end{bmatrix}.$$
(5.19)

The reliability of the network at round R = 4 can be calculated by $\psi(4)$ given by equation (5.13). Then, substituting the corresponding entries of the probabilistic connectivity matrix into $\psi(4)$, the expression for the reliability is simplified into a polynomial of a single variable ω , which is (rounded to four decimal places):

$$\psi(4) = - 0.0003\omega^{6} + 0.0083\omega^{5} - 0.0468\omega^{4} + 0.1362\omega^{3} - 0.4154\omega^{2} + 0.1285\omega + 0.5432,$$
(5.20)

where $\omega \in [0, 1]$.

Then, the differentiation is calculated:

$$\frac{d(\psi(4))}{d\omega} = - 0.0018\omega^5 + 0.0415\omega^4 - 0.1872\omega^3 + 0.4086\omega^2 - 0.8308\omega + 0.1285.$$
(5.21)

Let $\frac{d(\psi(4))}{d\omega} = 0$, there exists a solution for ω , subject to the constraint that $\omega \in [0, 1]$. This solution is the optimal value of the tuning parameter by the end of round R = 4, which is $\omega = 0.8325$. The corresponding $\psi(4)$ is the maximum value for the reliability by the end of round R = 4, where $\psi(4) = 0.5537$.



Figure 5.3: The reliability of a network at the end of the fourth round when ω varies form zero to one, where the network has three nodes and the probabilistic connectivity matrix is [1, 0.2, 0.3; 0.4, 1, 0.5; 0.6, 0.7, 1].

In Figure 5.3, the reliability at the end of round R = 4 is plotted against ω . The optimal value of ω which maximises the network reliability is marked. By applying the same method, the network reliability at the end of round R = 6 can be obtained. It is shown in Figure 5.4. There is a maximum value for $\psi(6)$, which is 0.8243 and it occurs when $\omega = 0.8349$.

By comparing Figure 5.3 and Figure 5.4, it shows that the line that indicates $\psi(6)$ has a similar tendency with $\psi(4)$. Further, in these figures, the minimum values for $\psi(4)$ and $\psi(6)$ both take place when $\omega = 0$. Since $\omega = 0$ corresponds to the event that all nodes transmit their native packets only, it suggests that the networks using the proposed coding scheme always



Figure 5.4: The reliability of a network at the end of the sixth round when ω varies form zero to one, where the network has three nodes and the probabilistic connectivity matrix is [1, 0.2, 0.3; 0.4, 1, 0.5; 0.6, 0.7, 1].

outperform the non-coded networks with the same setting when $\omega > 0$. This conclusion can be readily proved from a theoretical point of view.

The optimal value of the tuning parameters which maximise the network reliability at round R = 3 to 8 are examined by the same method. The results are shown in table 5.1.

Round	3	4	5	6	7	8
optimal ω	0.8476	0.8325	0.8310	0.8349	0.8408	0.8472

Table 5.1: The values of the tuning parameter that maximise the reliability at R = 3 to 8, when the network has three nodes and the probabilistic connectivity matrix is [1, 0.2, 0.3; 0.4, 1, 0.5; 0.6, 0,7 1].

It can be seen that the optimal ω for the expected number of rounds to reach the absorbing state may not be optimal for the reliability at each individual round. On the other hand, these optimal values of ω are close to each other: the expected value for the optimal ω for Round R = 3 to R = 8 is 0.8390 and the variance is 0.0164.

Next, using the methods given in Section 5.3.2, optimisation is conducted on minimising the expected number of rounds to reach the absorbing state. Similarly, after substituting in values from Q, E_{exp} is converted to a single variable polynomial of ω . Then, the minimum value of E_{exp} and the corresponding ω can be found by solving a constrained nonlinear optimisation problem, which is similar to that in the previous optimisation problem hence its details are omitted here. The expected number of rounds to reach the absorbing state is plotted against ω in Figure 5.5.



Figure 5.5: The expected number of rounds to reach the absorbing state when ω varies from zero to one; the network has three nodes and the probabilistic connectivity matrix is [1, 0.2, 0.3; 0.4, 1, 0.5; 0.6, 0.7, 1].

It can be seen that E_{exp} reaches its minimum value 4.7147 when $\omega = 0.8460$. This optimal value of tuning parameter is close to those used to optimise the reliability by the end of a particular round.

Therefore, the network designer can choose the optimal value of ω for a particular round, without seriously affecting the optimality of reliability at other rounds.

5.4.3 Examination on the reliability gain

In order to examine the gain in reliability that random neighbour network coding brings, the proposed scheme is applied to networks under various configurations. Figure 5.6 includes networks with three and four nodes applying the proposed scheme. The tuning parameters are set to the optimal values that maximise the reliability at the fourth round, which are 0.833 and

0.965 for n = 3 and 4 respectively. The probabilistic matrices for n = 3 and 4 are:

$$Q(\text{Figure 5.6}): \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.5 \\ 0.6 & 0.7 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.1 & 0.5 & 0.4 \\ 0.6 & 1 & 0.2 & 0.6 \\ 0.7 & 0.3 & 1 & 0.1 \\ 0.1 & 0.3 & 0.2 & 1 \end{bmatrix}.$$
(5.22)

The reliabilities of non-coded networks under the same network configurations are plotted for comparison.



Figure 5.6: The comparison between reliability of networks with and without network coding when n = 3 and 4. The connectivity matrices for n = 3 and n = 4 are [1, 0.2, 0.3; 0.4, 1, 0.5; 0.6, 0.7, 1] and [1,0.1,0.5,0.4; 0.6,1,0.2,0.6;0.7,0.3,1,0.1;0.1,0.3,0.2,1] respectively. $\omega = 0.833$ and 0.965 for n = 3 and 4 respectively.

It is shown in Figure 5.6 that the reliability of the networks applying the proposed random neighbour network coding scheme outperforms the corresponding non-coded networks. Moreover, in some cases, the gain in reliability is considerable. For example, as is shown in Figure 5.6 the reliability of coded network with four nodes at round R = 10 is 0.7685, which is an improvement of 272.34 percent compared with the reliability of 0.2064 in the corresponding non-coded network with the same probabilistic connectivity matrix.

There are some other factors affecting the reliability gain, such as number of nodes in the network and the probabilistic connectivity matrix. Based on numerous simulations, it is observed that 1) in the networks where the probabilistic connectivity matrices have the same mean and variance, the reliability gain increases with the addition number of nodes; 2) in networks with the same number of nodes, the larger the variance is, the larger reliability gain that the random neighbour network coding scheme brings. However, the proof of these conjectures requires a comprehensive and non-trivial analysis and left as future work.

5.5 Summary

This chapter proposes a random neighbour network coding scheme allowing a source node to perform network coding on-the-fly according to its received packets. The reliability of networks applying the proposed coding scheme is analysed and the improvement in reliability has been shown. Further, the optimal tuning parameter that maximises the reliability at given round or minimises the expected number of rounds to reach the absorbing state has been derived.

Chapter 6

Comparison of the Proposed Schemes

In the previous chapters, two novel network coding schemes are proposed. They share some similarities and differences. In Section 6.1, the proposed schemes are compared with each other. Additionally, the proposed schemes are compared with the well-known random linear network coding scheme in Section 6.2.

6.1 Comparison between two proposed network coding schemes

The two proposed schemes share some similarities but have differences in some aspects. The following compares the two schemes in terms of encoding and decoding methods, the procedures for obtaining network reliability, and reliability performance.

6.1.1 Similarities

Both schemes are applied to all-to-all models where the communications between any pair of nodes are lossy. In a transmitting node, the bitwise XOR is conducted between the native packet of the source node and the native packet of one coding neighbour node. The encoding is conducted opportunistically based on the packets that have already been received and decoded. That is to say, as long as the native packet of the coding neighbour is available at the transmitting node, encoding is performed; otherwise, no coding is performed and a non-coded packet is transmitted. Therefore, no significant encoding delay is introduced. This is in contrast with the schemes where encoding is not performed until enough packets are collected.

In a destination node, decoding is performed whenever a packet is received. The decoding

rules are the same for both schemes as introduced in Chapter 3.

Additionally, in both schemes, the packets in the buffer of a node in a time slot only depend on the packets in the buffer in the previous time slot, but not on the time slots before that. Therefore, Markov chains can be used to examine the change of packets at each node in a network using both schemes.

6.1.2 Differences

Firstly, the selections of coding neighbours are different. In the first scheme, each node selects one other node in the network as its coding neighbour. Once selected, the coding neighbour of a node stays the same in all future rounds of transmissions. Therefore, the coding neighbour for each node is fixed and predetermined. On the other hand, in the second scheme, the coding neighbour for each node is dynamic. It is selected on-the-fly among the decoded packets in each round of transmission. Therefore, in the second scheme, there is a higher opportunity for a node to perform network coding.

Secondly, the encoding schemes are different. In the first scheme, a source node broadcasts a coded packet as long as it receives or decodes the packet of its designated coding neighbour. However, in the second scheme, a tuning parameter is introduced allowing a node to broadcast a non-coded packet even if it has the chance to perform encoding. It is shown that it may not be optimal for a node to take every chance to perform coding.

Thirdly, the representations of states in the Markov chains are different. In the first scheme, an individual Markov chains is established for each node, where the state transition of a destination node depends on the state of the source node. The advantage of establishing an individual Markov chain for each node is that the number of states in each Markov chain is relatively small. However, it includes iteration to analyse the reliability of the network. Conversely, in the analysis of reliability in the second scheme, a Markov chain is used and a state can indicate the received packets of every node in the whole network. Moreover, the transition matrix stays the same in every round of transmission. The disadvantage is that the number of the possible states is large and it increases significantly with the addition of number of the nodes.

6.1.3 Reliability performance

In this subsection, the two proposed schemes are applied to the same networks where their reliability performance can be compared. In Figure 6.1, both schemes are applied to a network with three nodes, where the probabilistic connectivity matrix is:

$$Q(\text{Figure 6.1}): \begin{vmatrix} 1 & 0.5 & 0.2 \\ 0.3 & 1 & 0.4 \\ 0.6 & 0.4 & 1 \end{vmatrix}.$$
(6.1)

Both schemes are optimised to fit in the network. In the first scheme, optimal neighbours are selected according to the optimal neighbour selection rules given in Section 4.4. The neighbours that maximise the network reliability are N_3 , N_1 , N_2 for nodes N_1 , N_2 , N_3 respectively. On the other hand, in the second scheme, the tuning parameter is used to optimise the reliability at round R = 5 based on the method introduced in Section 5.3. The resultant optimal value is $\omega = 0.8310$.



Figure 6.1: The reliability gain of two proposed schemes when n = 3. The probabilistic connectivity matrix is [1, 0.5, 0.2; 0.3, 1, 0.4; 0.6, 0.4, 1]. In the neighbour network coding scheme, the optimal neighbours for nodes N_1 , N_2 , N_3 are N_3 , N_1 , N_2 respectively, while in the random neighbour network coding scheme, $\omega = 0.8310$ which optimises the reliability of the network at the end of round R = 5.

As shown in Figure 6.1, the reliability is increased by both proposed schemes when compared to the non-coded network. However, the extents of the reliability gain are different: the random neighbour network coding scheme brings a larger gain than the neighbour network coding scheme.

These schemes have been applied to other networks to compare the reliability performance. Figure 6.2 shows another example:



Figure 6.2: The performance comparison of a network employing the two proposed schemes and the non-coded network. The probabilistic connectivity matrix is [1, 0.3, 0.5, 0.4; 0.6, 1, 0.2, 0,6; 0.7, 0.3, 1, 0.4; 0.5, 0.3, 0.2, 1]. In the neighbour network coding scheme, the optimal neighbour scheme that maximises the reliability gain is applied, where the neighbours for nodes N_1, N_2, N_3, N_4 are nodes N_4, N_3, N_1, N_2 respectively. In the random neighbour network coding scheme, the tuning parameter is set to the optimal value that maximises the network reliability at round R = 4, where $\omega = 0.8435$.

In Figure 6.2, the network has four nodes, and the probabilistic connectivity matrix of the network is:

$$Q(\text{Figure 6.2}): \begin{vmatrix} 1 & 0.3 & 0.5 & 0.4 \\ 0.6 & 1 & 0.2 & 0.6 \\ 0.7 & 0.3 & 1 & 0.4 \\ 0.5 & 0.3 & 0.2 & 1 \end{vmatrix}.$$
(6.2)

In addition, in the neighbour network coding scheme, optimal coding neighbour selections

are applied where the neighbours for nodes N_1 , N_2 , N_3 , N_4 are N_4 , N_3 , N_1 , N_2 respectively. In the random neighbour network coding scheme, the tuning parameter is set to the optimal value that maximises the network reliability at round R = 4, where $\omega = 0.8435$.

It can be seen that the reliability of a network applying either proposed scheme is improved when compared to the reliability of the network without coding. Furthermore, the reliability is improved even more by the random neighbour network coding scheme than the neighbour network coding scheme.

The reason is that in the random neighbour network coding scheme, the selections of the coding neighbours are not fixed. Therefore, a node is able to assist more than one neighbour with the transmission of their native packets in different rounds. Moreover, the tuning parameter enables a node to choose not to perform coding when coding may have negative impact on reliability, as introduced in Section 5.1.

6.2 Comparison of the proposed schemes with the random linear network coding scheme

The random linear network coding scheme [34], introduced in Chapter 2, is chosen as the benchmark, as it is one of the most popular network coding schemes. In order to achieve a fair comparison, the finite field for generating coefficients is set to GF(2). In this way, its computational complexity is similar to the proposed network coding schemes. Other performances such as reliability and delay among these schemes will be compared in the following.

In the all-to-all model, the random linear network coding under GF(2) is the linear combination of randomly selected packets including the received packets of a source node and its native packet. In this thesis, simulations are used to examine the reliability performance of random linear network coding. In the simulations, the random linear network coding at node N_j is the linear combination of the packets in its buffer and its native packet, which is represented by:

$$Y_j = \sum_{s \in \mathcal{S}} \xi_s \Upsilon_{j,s} + \xi_j X_j, \tag{6.3}$$

where Y_j is the network coded packet that node N_j transmits; ξ_s and ξ_j are the coefficients, which are random numbers from GF(2); S is the number of packets in the buffer of node N_j ; $\Upsilon_{j,s}$ is the *s*th packet in the buffer of N_j .

There is a constraint for the chosen coefficients, which is: in one encoding procedure, the randomly-generated coefficients for each received packet cannot all be equal to zero at the same time, i.e., $\sum_{s \in S} \xi_s + \xi_j \neq 0$. A zero coefficient for a packet indicates not including this packet into the coded packet. Therefore, the meaning of the constraint is that each node has to broadcast a packet rather than nothing in each transmission.

6.2.1 Reliability performance

In this subsection, the reliability performance of the random linear network coding scheme is compared with the proposed schemes. The following contents show some simulation results when applying three different network coding schemes in two different networks.



Figure 6.3: The performance comparison of a network employing the two proposed schemes and the random linear network coding scheme. The probabilistic connectivity matrix of the network is [1, 0.5, 0.2; 0.3, 1, 0.4; 0,6, 0.4, 1]. In the neighbour network coding scheme, optimal neighbours that maximise the reliability are applied where the coding neighbours for nodes N_1 , N_2 , N_3 are N_3 , N_1 , N_2 respectively. In the random neighbour network coding scheme, the tuning parameter is set to the optimal value that maximises the network reliability at the end of round R = 4, where $\omega = 0.8310$. In the random linear network coding scheme, the finite field for the coefficients is GF(2).

Figure 6.3 shows the comparison of the reliability performance of the proposed schemes and the random linear network coding scheme (GF(2)) when they are applied to a network with

three node, where the probabilistic connectivity matrix is:

$$Q(\text{Figure 6.3}): \begin{vmatrix} 1 & 0.5 & 0.2 \\ 0.3 & 1 & 0.4 \\ 0.6 & 0.4 & 1 \end{vmatrix}.$$
(6.4)

In Figure 6.3, the optimal neighbour network coding scheme is used, where the coding neighbours for nodes N_1 , N_2 and N_3 are nodes N_3 , N_1 and N_2 respectively. Additionally, the optimal random neighbour network coding scheme at the end of round R = 4 is applied, where the tuning parameter $\omega = 0.8310$.

It is shown that both schemes outperform the random linear network coding scheme in terms of reliability in this network. For example, in the fifth round, the reliability for the network applying the optimal random neighbour network coding scheme, optimal neighbour network coding scheme and random linear network coding scheme is 0.7159, 0.5949 and 0.3563 respectively. There are improvements of 100.90 % and 66.97 % in the reliability performance for the proposed schemes over the random linear network coding scheme under GF(2).

These schemes are then applied to a four-node network, and the results are shown in Figure 6.4. In Figure 6.4, the probabilistic connectivity matrix for the network is:

$$Q(\text{Figure 6.4}): \begin{vmatrix} 1 & 0.1 & 0.5 & 0.4 \\ 0.6 & 1 & 0.2 & 0.6 \\ 0.7 & 0.3 & 1 & 0.1 \\ 0.1 & 0.3 & 0.2 & 1 \end{vmatrix}.$$
(6.5)

Additionally, in the neighbour network coding scheme, optimal coding neighbours are chosen, where the neighbours for nodes N_1 , N_2 , N_3 , N_4 are N_2 , N_3 , N_4 , N_1 respectively. In the random neighbour network coding scheme, the tuning parameter is set to the optimal value that maximises the network reliability at the end of round R = 4, which is $\omega = 0.9650$.

Similar results can be found in Figure 6.4 showing that the proposed schemes can outperform the random linear network coding scheme under GF(2) in terms of reliability.

In the random linear network coding scheme, the encoding is purely random, i.e., it is not possible to control which packet/s and how many packets are used for encoding. In some cases,



Figure 6.4: The performance comparison of a four-node network employing the two proposed schemes and the random linear network coding scheme. The probabilistic connectivity matrix is [1, 0.1, 0.5, 0.4; 0.6, 1, 0.2, 0.6; 0.7, 0.3, 1, 0.1; 0.1, 0.3, 0.2, 1]. In the neighbour network coding scheme, the optimal neighbour scheme that maximises the reliability gain is applied, where the neighbours for nodes N_1 , N_2 , N_3 , N_4 are N_2 , N_3 , N_4 , N_1 respectively. In the random neighbour network coding scheme, the tuning parameter is set to the optimal value that maximises the network reliability at the end of round R = 4, where $\omega = 0.9650$. In the random linear network coding scheme, the finite field for the coefficients is GF(2).

the randomly generated coefficient for the native packet of a source node may be zero, meaning that the native packet of the source not is not included in the coded packet. Therefore, no other nodes can receive this native packet. There are other cases where most of the packets are sent in coded version and decoding may hardly be performed due to a lack of native packets. To sum up, random linear network coding adopting GF(2) has limited ability of improving the reliability performance. However, with the increasing of the field size, the random linear network coding is able to achieve higher reliability at the cost of higher complexity and bandwidth consumption.

On the other hand, in the proposed schemes, the native packet of a source node is included in every transmitted packet. In addition, in the second proposed scheme, a non-coded packet can still be sent out even if there is chance to perform encoding. Moreover, optimisations are conducted in both of the proposed schemes, and this helps to fit the network coding scheme to networks with given probabilistic connectivity matrices.

6.2.2 Delay performance

In this subsection, the delay performance of the proposed schemes is compared with the random linear network coding scheme when its coefficients are chosen from GF(2). The delay performance is measured by delay $D(\psi)$, which is defined as the minimum number of rounds of transmissions required for the network reliability to reach $\psi \in [0, 1]$. Simulations are used to evaluate the delay of these schemes.

In the network with three nodes where the probabilistic connectivity matrix is given by equation (6.4). The applied network coding schemes are the same as that in Figure 6.3. The delay $D(\psi)$ is shown in Table 6.1. Another example is given for a network of four nodes, where the probabilistic connectivity matrix is given by equation (6.5) and the applied network coding schemes are the same as that in Figure 6.4. The corresponding $D(\psi)$ are shown in Table 6.2.

Delay	RNNC	NNC	RLNC $GF(2)$
D(0.6)	5	6	8
D(0.7)	5	6	9
D(0.8)	6	8	10
<i>D</i> (0.9)	8	10	12

Table 6.1: The delay for reliability to reach 0.6, 0.7, 0.8 and 0.9 for a network of three nodes, where the probabilistic connectivity matrix is [1, 0.5, 0.2; 0.3, 1, 0.4; 0.6, 0.4, 1] and optimal neighbour network coding scheme, optimal random neighbour network coding scheme under GF(2) are applied.

Delay	RNNC	NNC	RLNC $GF(2)$
D(0.6)	9	9	10
D(0.7)	10	10	11
D(0.8)	11	12	13
D(0.9)	13	14	15

Table 6.2: The delay for reliability to reach 0.6, 0.7, 0.8 and 0.9 for a network of four nodes, where the probabilistic connectivity matrix is [1, 0.1, 0.5, 0.4; 0.6, 1, 0.2, 0.6; 0.7, 0.3, 1, 0.1; 0.1, 0.3, 0.2, 1] and optimal neighbour network coding scheme, optimal random neighbour network coding scheme under GF(2) are applied.

In Tables 6.1 and 6.2, the contents in the first column are the delay $D(\psi)$ for the random neighbour network coding scheme. The second and the third are those for the neighbour network coding scheme and the random neighbour network coding scheme respectively. It can be seen that in order to achieve the same reliability (same row), the proposed schemes experi-

ence a shorter delay compared to the random linear network coding scheme when coefficients are from GF(2). The reason behind the improvement in $D(\psi)$ is similar to that introduced in Section 6.2.1, i.e., the random linear network coding scheme is purely random and the delay performance is relatively poor when compared with the proposed schemes where optimisations can be conducted.

6.3 Summary

In this chapter, the two proposed schemes are compared with each other in terms of their similarities and differences, and reliability performance. Then, they are compared to the random linear network coding scheme, whose coding coefficients are generated randomly from the finite field GF(2) so that the computational complexity of coding is similar to the proposed network coding schemes. Furthermore, simulations are conducted to compare the reliability and delay performance among three network coding schemes. The major comparison results are summarised in Table 6.3.

	RNNC	NNC	RLNC $GF(2)$
packets for encoding	X_j and	X_j and	random
	random one	fix one	all
opportunity for encoding	high	low	high
probability for encoding			
(when enough packets are	ω	1	1
available for encoding)			
complexity	low	low	low
reliability	high	medium	low
delay	small	medium	high

Table 6.3: The comparison of the proposed schemes and random linear network coding scheme under GF(2), where X_j is the native packet of the source node.

In Table 6.3, the first three rows are the comparison of the encoding processes of the three schemes, which refers to Section 6.1 and Section 6.2. More specifically, they show the packets selected by each scheme for encoding, the opportunity that each scheme has for encoding, and the probability that each scheme performs encoding when enough packets are available for encoding. The last two rows are the comparison of the reliability and delay performance of the three schemes, which can be found in Section 6.2.1 and Section 6.2.2 respectively.

Chapter 7

Conclusions and Future Work

This thesis examines all-to-all broadcast in wireless lossy networks where each pair of nodes is connected with a certain probability. Two novel network coding schemes have been proposed in the thesis to improve the reliability of all-to-all broadcast, where reliability is measured by the probability that every node has a copy of the native packets of all other nodes. Further, theoretical analysis has been established to characterise the reliability. The main results of the thesis are summarised in the following.

7.1 Conclusions

- Two novel network coding schemes are proposed for all-to-all broadcasting in lossy wireless networks, called the *neighbour network coding scheme* and the *random neighbour network coding scheme*. In both schemes, a source node utilises simple XOR coding to combine native packet of itself and its coding neighbour. In the first scheme, each source node has a predetermined neighbour, while in the second scheme, each source node randomly selects a neighbour according to the packets stored in the buffer.
- The reliability of the networks applying the proposed schemes is examined analytically using Markov chains. It is shown that network reliability can be improved significantly using the proposed network coding schemes in networks with various configurations, such as different numbers of nodes, and different channel conditions.
- · Optimisations are conducted in both proposed schemes when the probabilistic connectiv-

ity matrix of a network is given. In the first scheme, the criteria on the coding neighbour selection is proposed. It reveals that each node should select the node to which the connection probability is the lowest as its coding neighbour, then the reliability can be maximised. Additionally, in the second scheme, optimisation is carried out on the tuning parameter, i.e., the probability to perform network coding. It suggests that a node should spend a small amount of time sending non-coded packets even it has plenty of packets to perform network coding. The optimal tuning parameter is derived by solving a constrained nonlinear optimisation problem.

• By comparing the two proposed schemes with each other and with an existing network coding scheme, interesting results are observed. Specifically, it is shown that the benefit in reliability that network coding brings can be affected largely by channel conditions. Therefore, an optimal design of network coding schemes should take channel conditions into account, which have been largely overlooked in this field. Moreover, it may not be optimal for a node to take every opportunity to perform coding. These observations are novel in the sense that previous research usually ignores the effects of channel conditions, where some network coding schemes tend to perform coding as much as possible. These results provide insight into the design of network coding schemes for reliable packet transmission in lossy wireless networks.

7.2 Future work

The framework of analysing network reliability established in this paper can be applied to examine the delay and reliability of broadcasting in networks applying different XOR based coding schemes. For example, the technique developed in this work can be readily combined and extended to the schemes, where a source node chooses 1) whether or not to perform coding, 2) the number of packets to be encoded and 3) with which packets to perform coding according the channel conditions in a lossy wireless network.

Further optimisation can be conducted in a network where the channel condition, such as a probabilistic connectivity matrix, is given. In this thesis, the tuning parameter ω is kept the same for every node. In the future, each node can be assigned with an individual tuning parameter, ω_i where $j \in \{1, 2, ..., n\}$ so that each node can choose its own tuning parameter
according to the channel condition of its neighbours, to achieve a higher reliability in packet transmission in the network. Moreover, in the random neighbour network coding scheme, the packet used for encoding is uniformly selected from the set of already decoded packets. The challenge in future work is to find the optimal probability distribution for individual packets according to the channel conditions from the source node to each individual node. The channel condition information can be made available by an indicator included in the packet header. In the case where multiple packets can be used for encoding, the optimal degree distribution to achieve better reliability could be considered in the future, where the *degree distribution* is the probability distribution of the number of packets that are used by a randomly chosen node in a randomly chosen time slot for generating an encoded packet. Another possible future work is to extend the XOR network coding scheme to finite-field network coding (FFNC) schemes to further improve the reliability and reduce delay.

Networks with dynamic channel conditions have a large number of interesting yet open research problems. This thesis assumes that the probabilistic connectivity matrix remains the same in each round of transmission. As technology develops, nowadays there are increasing numbers of mobile wireless communication devices such as mobile phones, tablet PCs and intelligent vehicles. The channel conditions in these networks can vary dramatically over time. Therefore, it is important to study network coding schemes that can improve the reliability of packet transmission in networks with time-varying channel conditions. Another research challenge lies in finding the optimal time scale of averaging/reporting the channel condition information in a dynamic network where channel conditions can change rapidly over time.

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