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Abstract

We examine innovation as a timing game with complete information and observable actions in which firms decide when to enter a market. We characterize all pure strategy subgame perfect equilibria for the two-player symmetric game. In particular, we describe all subgame perfect equilibria when both the leader's and the followers' payoff functions are multi-peaked, non-monotonic and discontinuous. We find that there are potentially multiple equilibria, which could involve: joint adoption by both firms, with and without rent equalization; and, alternatively, single-firm adoption with a second-mover advantage. Economic applications are discussed including process and product innovation and the timing of the sale of an asset.
Key words: timing games, entry, leader, follower, process innovation, product innovation.
JEL classifications: C72, L13, O31, O33.

1 Introduction

The availability of new products and processes underlies economic development and improvements in welfare (Romer, 1994). But new technology does not automatically equate to innovation in the market place. Rather, any innovation – be it market entry with a new product or adoption of a new production process – must be deliberately implemented as part of a firm's profit-maximizing strategy. In this paper, following the seminal contributions of Fudenberg and Tirole (1985), Dutta et al. (1995) and

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Hoppe and Lehmann-Grube (2005), we study an innovation-timing game in which two competing firms consider the optimal time to enter a market. Focusing on pure strategies, we solve for all subgame perfect equilibria, allowing for very general – possibly non-monotonic or multi-peaked – payoffs for both the leader and follower. We are also able to extend our solution method to situations where payoffs are not necessarily continuous in time.

When considering the optimal time to innovate, a monopolist must weigh up several different factors. If it moves early, it enjoys the benefits of innovation sooner and for longer. By waiting, a monopolist might be able to implement a better – and possibly more profitable – innovation. This is the case when the potential quality of the product that the monopolist can take to market improves over time, as in a product-innovation model. Similarly, waiting could allow the monopolist to implement the same innovation but at lower cost (a process-innovation model). In either situation, there is a benefit from delaying entry into the market. A monopolist will weigh up the benefits of early-versus-late entry and choose an entry time so as to maximize the net present value of innovating.[1]

Similar tradeoffs exist in an oligopoly, but the strategic interaction between firms also needs to be taken into account. For example, consider two firms contemplating the best time to launch a new phone, in the sort of dilemma Apple and Samsung face when launching a new handset. There could be a first-mover advantage in this market; a leader could develop a loyal customer base and a network of related products, helping secure its market dominance in the long run. But waiting and entering second could also be advantageous, allowing a firm to launch a better phone, with more features, memory, and so on, possibly even at lower cost.[2] Each firm will weigh up the relative advantage of early rather than late entry, taking into account the strategy of their rival.

Fudenberg and Tirole (1985) analyze the adoption of new technology by two rivals, neither of whom can pre-commit to their strategy.[3] They develop a method of solving the continuous-time game using subgame perfection that we adopt here. An important insight in their paper is that with a first-mover advantage there is a preemption equilibrium in which all the rents from entering the market as the leader are dissipated by 'excessively' early entry (that is, entry occurs at a time much earlier than a monopolist would choose). Furthermore, in this equilibrium: rents for the leader and follower are equalized; and entry times display diffusion (à la Reinganum (1981a)), in which one firm adopts early while the follower waits and enters the market later. In addition, Fudenberg and Tirole (1985) show there can be a continuum of joint-adoption equilibria that also involve rent equalization for the two firms.

Fudenberg and Tirole (1985) make several restrictive assumptions regarding payoffs. For instance, in their model profits at any point in time depend on whether a firm and its rival have entered, and not on how long either firm has been active

[1] Given it does not, in general, capture all of the surplus from innovation, a monopolist typically will not innovate at the socially optimal time; see Tirole (1988, Chapter 10).

[2] See Tellis and Golder (1996) for a study on second-mover advantages in a range of markets.

[3] Fudenberg and Tirole (1985) also consider a general setup with n players.

in the market. Using a vertical product-differentiation model, Dutta et al. (1995) extends Fudenberg and Tirole (1985) to study entry when the potential quality of the product improves over time. In contrast to Fudenberg and Tirole (1985), Dutta et al. (1995) assume that profits depend only on the difference of entry times. They show that in their model there still can be: a preemption equilibrium, in which rents are dissipated through excessively early entry; or an equilibrium in which no monopoly profit is dissipated but the follower makes higher profit than the market leader.

In modeling their timing game, Fudenberg and Tirole (1985) effectively assume that the follower's entry time is exogenously determined when the leader enters before a given time, after which the leader's and the follower's time of entry coincide. This implies that while Fudenberg and Tirole (1985) allow for multiple peaks in the leader's payoff, all but the first peak must perfectly coincide with the follower's payoff. Hoppe and Lehmann-Grube (2005) extend the analysis of Fudenberg and Tirole (1985) by allowing the leader's payoff curve to have multiple peaks (local maxima). Hoppe and Lehmann-Grube (2005), on the other hand, require that the follower's payoff is non-increasing in the leader's entry time; they solve for both the preemption and the second-mover advantage games.

In this paper we generalize these existing models in several dimensions. First, we solve for the pure strategy subgame perfect equilibria when payoffs for both firms can be non-monontonic or multi-peaked. Our framework allows us to solve a broader range of economic problems than was previously possible. In Section 2.3 we show that a product-innovation or a process-innovation model, augmented with an experience good or some switching cost, can generate a non-monotonic payoff for both the leader and the follower. The same point can be made for the profit derived from an asset; the revenue generated can vary non-monotonically depending on the time of sale.

Second, our model is also sufficiently general to accommodate discontinuities in the payoffs. Discontinuities arise in a variety of situations; at some point in time (in terms of the leader's entry time) a firm in a related complementary or substitute market could enter or decide to exit.[4] This decision could create a discontinuity in either the leader's or the followers' payoffs (or both). For instance, in the phone-handset example above, developers of apps could enter or exit, affecting discontinuously the payoff to either the leader or followers. Similarly, the product choices of firms selling substitute products, such as tablets, could also disrupt the phone handset sellers, generating discontinuities.

The model. Two firms decide when to make an irreversible and one-off decision to innovate. Like Fudenberg and Tirole (1985), Dutta et al. (1995) and Hoppe and Lehmann-Grube (2005), we assume that: there is complete information about the payoffs that could accrue from innovation, so that there is no uncertainty; and that the actions of a firm are observable to its rivals. Given this structure, we solve the

[4] As noted by Bobtcheff and Mariotti (2012), many factors that affect an entrant's profitability are exogenous, outside of the control of the firms themselves. These events could see a discontinuous jump in the payoffs of the leader and/or the followers. Fudenberg and Tirole (1985, Section 5) also discuss how three (or more) firms can generate a discontinuity in payoffs for the remaining firms in a timing game similar to the one we study here.

game focusing on the entry strategy of the leader, using subgame perfection.

Some of the results of the paper are as follows. Focusing on the two-player (leader and follower) game with symmetric firms, we characterize all subgame perfect equilibria. We find that there can be multiple equilibria. First, there could be a set of equilibria that exhibit rent equalization. The leader's entry times in these equilibria occur at times when the leader and follower payoff curves intersect and the leader's payoff is at a historic maximum for the game up until that time; they are equivalent to the joint-adoption equilibria in Fudenberg and Tirole (1985). In addition, equilibria can exist with the leader entering at points of discontinuity, provided the leader receives a higher payoff than the follower at this time, and that the expected payoff in equilibrium is higher than the payoff from entering as a leader at any earlier time. An example of this is immediate entry at the very start of the game when both firms prefer to be first into the market. Third, there could be equilibria with asymmetric payoffs, like the second-mover advantage equilibrium of Hoppe and Lehmann-Grube (2005) and the maturation equilibrium of Dutta et al. (1995). Finally, when there are multiple equilibria we are able to provide sufficient conditions to ensure that these equilibria can be pareto ranked. When pareto-ranking is feasible, it is possible to determine the superior subgame perfect equilibrium, in which both firms get a weakly higher payoff than they could receive in any other subgame perfect equilibria.[5]

Related literature. This paper draws on an extensive literature on innovation timing games.[6] Our analysis of an irreversible investment decision with complete information and observable actions (closed-loop equilibria) follows the seminal models of Fudenberg and Tirole (1985), Dutta et al. (1995) and Hoppe and Lehmann-Grube (2005).[7] Our paper extends this model, and synthesizes elements of the existing literature, by allowing for a multiple-peaked leader's and follower's payoff functions as well as discontinuities.

An alternative approach to study innovation is to assume players' actions are unobservable as in Reinganum (1981a) and Reinganum (1981b). In her models, unobservable actions are equivalent to each firm being able to pre-commit to its strategy at the start of the game. Reinganum shows that in the (open-loop) equilibria there will be diffusion in the sense that firms adopt the technology at different dates, even though all firms are ex ante identical. In our model firms use feedback rules to deter-

[5] Fudenberg and Tirole (1983) and Fudenberg and Tirole (1985) suggest that if one equilibrium pareto-dominates all others, it the most reasonable outcome to expect.

[6] See Hoppe (2002) or Van Long (2010, Chapter 5) for a survey of the literature. Fudenberg and Tirole (1991) also consider innovation when the firms make one irreversible decision (to enter) in a simple timing-game framework (see Chapter 4.5 and 4.12).

[7] Argenziano and Schmidt-Dengler (2012, 2013a,b) examine the order of market entry, clustering and delay using a similar model to Fudenberg and Tirole (1985). In particular, they show that with many firms the most efficient firm need not be the first to enter the market and that delays are non-monotonic with the number of firms. They also suggest a new justification for clustering of entry. Katz and Shapiro (1987) study an innovation game with heterogenous firms when there is licencing (by the leader) and imitation (by the follower). Dutta et al. (1993) consider a stochastic timing game with continuous payoffs. Gale (1995) shows that inefficient delays can occur when n players make a one-off investment decision in a dynamic coordination game.

mine their strategy at any particular point in time; this means that they are unable to commit to their strategy at the beginning of the game. Park and Smith (2005) develop an innovation game with unobservable actions that permits any firm (in terms of the order of entry) to receive the highest payoff. This allows for a war-of-attrition, with higher payoffs for late movers, a pre-emption game with higher payoffs for early movers, and a combination of both. They solve for the (open-loop) mixed-strategy equilibria.[8]

Finally, several other authors consider innovation when there is incomplete information. For example, Bobtcheff and Mariotti (2012), Hendricks (1992) and Hopenhayn and Squintani (2011) assume that a firm's capability to innovate is private information. In these models, delay allows a firm to get better information about the potential innovation (its costs, value, and so on), but waiting runs the risk that a rival will innovate first, capturing the lion's share of the returns.

2 The model

Assume two firms ($i = 1, 2$) are in a continuous-time stopping game starting at $t = 0$ until some terminating time $T > 0$.[9] Firm i's decision to stop (that is, 'enter' the market) at $t_i \geq 0$ can only be made once, and this decision is irreversible and observable immediately by the other firm. As outlined in Section 2.2, the focus in this paper is on pure strategy subgame perfect equilibria.

2.1 Setup and assumptions

We make the following standard assumptions.

Assumption 1. *Time is continuous in the sense that it is 'discrete but with a grid that is infinitely fine'.*

Assumption 2. *Firms always choose to stop earlier rather than later in payoff-equivalent situations.*

Assumption 3. *If more than one firm chooses to stop (enter) at exactly the same time, one of these firms is selected to stop (each with probability $\frac{1}{2}$ ex ante); the other firm is then able to reconsider its decision to stop at that time.*

Equivalent assumptions are adopted by Dutta et al. (1993), Dutta et al. (1995), Hoppe and Lehmann-Grube (2005) and Argenziano and Schmidt-Dengler (2013a). For example, Assumption 1 replicates A1 of Hoppe and Lehmann-Grube (2005). It invokes Simon and Stinchcombe (1989) who show that under certain conditions a continuous-time strategy profile is the limit of a discrete-time game with increasingly fine time grids. As our game satisfies these conditions, we use subgame perfection as

[8] They also briefly consider observable actions and show that there are multiple equilibria.
[9] It is possible that $T = \infty$.

our equilibrium concept.[10] Assumption 2, equivalent to A3 in Hoppe and Lehmann-Grube (2005), allows us to focus on just one (payoff equivalent) equilibrium in the case of indifference between early and late entry. This simplifies our analysis so as to focus on the timing of entry rather than on issues of equilibrium selection. Assumption 3 – part of A3 in Hoppe and Lehmann-Grube (2005) – avoids potential coordination failures with more than one firm stopping at the same time.

With two firms, the follower's entry is a single-firm decision problem, and its entry time will be its best response given the leader's choice. That is, we can write follower's entry time $t_2(t_1)$, where t_1 is the leader's time of entry. Consequently, the payoffs to both firms can be written as composite functions of the leader's entry time. Specifically, $L(t)$ and $F(t)$ are the payoffs to the leader i and the follower $j \neq i$ respectively, given that i is the first firm to stop at t. Let us now outline the next Assumption.

Assumption 4. *$L(t)$ attains a global maximum at finite $t^{max} < T$.*

A similar assumption is adopted by others in the literature; it ensures that the leader stops in finite time. For example, this is equivalent to Assumption 3 in Dutta et al. (1995) and Assumption 2(ii) in Fudenberg and Tirole (1985).

Finally, our last assumption ensures that both firms innovate at $t_i \leq T \ \forall \ i$ as entering provides a higher payoff than its outside option of zero. This means that our analysis is not unnecessarily complicated by having to consider the case when one or both firms never enter the market.

Assumption 5. *Each firm's outside (non-entry) payoff is normalized to 0, and $L(t) \geq 0$ and $F(t) \geq 0$.*

This Assumption plays a similar role to Assumption 4 in Dutta et al. (1995) and Assumption 2(ii) in Fudenberg and Tirole (1985).

2.2 Strategies and equilibrium

Let us now comment on the concept of equilibrium we use in this paper. Given this is a game of complete information, we solve for the pure strategy subgame perfect equilibria (SPE). With two firms, after entry by the leader at a given time, the follower must make its decision regarding entry. Given there is only one remaining firm that has not entered, the outcome in this subgame is always unique.[11] This allows us to solve for all pure-strategy SPE in the two-player symmetric games.

A pure strategy for firm i, σ_i, denotes the decision at any time t whether to 'enter' or 'not enter', provided that it has not already entered the market, for $i = 1, 2$. Specifically, $\sigma_i(t) = 1$ if a firm enters at time t and $\sigma_i(t) = 0$ if it opts to not enter

[10] Also see the discussion in Fudenberg and Tirole (1985).

[11] The follower finds the optimal stopping time, which gives the maximum to his payoff function. If there is more than one entry time that gives the maximum payoff, the earliest time is the one selected for our equilibrium.

at t (given it has not already done so). This decision to enter or not depends on the history of the game h_t (that is, the entry decision of the other firm up until that time). There is complete information between both of the firms regarding this history.

Define the payoff for firm i to be $\pi_i(h_t, \sigma_i, \sigma_j)$, where σ_j is the strategy profile of firm $j \neq i$ and $i, j = 1, 2$. An SPE is defined as a strategy profile $\sigma^* = (\sigma_1^*, \sigma_2^*)$ where $\pi_i(h_t, \sigma^*) \geq \pi_i(h_t, \sigma_i, \sigma_j^*)$ for all feasible strategies σ_i and all feasible histories h_t, $\forall\ i = 1, 2$, $j \neq i$.

As we will show, there is the possibility of multiple SPE in our game. When this is the case, sometimes it is possible to pareto rank the equilibria and determine the superior subgame perfect equilibrium (SSPE) – that is, the SPE that pareto-dominates all other SPE. Fudenberg and Tirole (1985) argue that this equilibrium would be a natural 'focal point' for firms in the game. Explicitly, we define the SSPE to be the equilibrium in which all firms receive a payoff at least as high as they could have received in any other SPE. Specifically, an SPE strategy profile $\hat{\sigma}^*$ is the SSPE of the game if, for any other SPE strategy profile σ^*, it must be the case that $\pi_i(h_t, \hat{\sigma}^*) \geq \pi_i(h_t, \sigma^*)$, $\forall\ i \in 1, 2$. In our model, we give sufficient conditions when it is possible to pareto rank the SPE and determine the SSPE of the game.

In addition, for convenience, we label the SPE that provides the leader with the highest possible payoff as the the leader's preferred subgame perfect equilibria (LSPE). That is, an SPE strategy profile $\hat{\sigma}^*$ is the LSPE of the game if, for any other SPE strategy profile σ^*, it must be the case that $\pi_1(h_t, \hat{\sigma}^*) \geq \pi_1(h_t, \sigma^*)$, where π_1 denotes the payoff to the leader.

2.3 Process, product innovation and asset sales: three examples

To provide some intuition, and allow for a closer comparison to the previous literature, we construct three examples. The first two are modifications of the process- and product-innovation timing examples of Hoppe and Lehmann-Grube (2005). Essentially, we augment their examples to allow for an experience effect or switching cost for consumers. This alters consumers' incentive to switch supplier when there is entry; this setup can generate non-monotonic payoff functions for both the leader and the follower. Our third asset-market example is adapted from Dutta et al. (1993).

Consider the following setup for the first two examples. Two firms are contemplating entering a market at some time $t_i \in [0, T]$ for $i = 1, 2$. The first firm that enters gets an instantaneous flow of monopoly profit R_m until the time when the second firm enters, which is optimally chosen by the second firm. After entry by the second firm, they share the market in proportions (R_1, R_2). We assume that the market exists for a finite period of time T. We also assume, for simplicity, that each firm's R&D costs per unit of time are zero. The payoffs are discounted by a common discount factor $e^{-\tau}$, so that the net-present value of profits for the leader entering at t_1 and follower

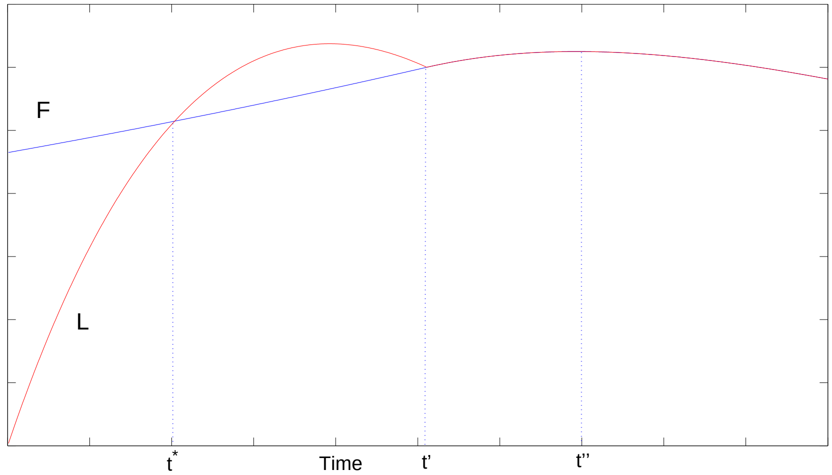

Figure 1: Non-monotonic $L(t)$ and $F(t)$ payoff functions in a process-innovation game

entering at t_2 are:

$$\pi_1(t_1,t_2) = \int_{t_1}^{t_2} e^{-\tau} R_M(t_1) d\tau + \int_{t_2}^{T} e^{-\tau} R_1(t_1,t_2) d\tau; \tag{1}$$

and

$$\pi_2(t_1,t_2) = \int_{t_2}^{T} e^{-\tau} R_2(t_1,t_2) d\tau. \tag{2}$$

Process innovation with an experience effect. In the process-innovation game, the production technology a firm can use when it enters the market improves over time, allowing for a lower marginal cost with later entry. We assume that a firm adopts the best technology available when it enters, and that it uses this technology until the end of the game (at time T). This means that a firm entering later has a lower cost. Specifically, marginal costs decrease overtime according to the cost function $c_i(t) = e^{-t_i}$. The market demand in each period is 1 unit at a constant price of 1. Given these assumptions, the per-period monopoly profit is $R_M = 1 - c_1$.

After both firms enter they share the market in proportions $(s(t_2 - t_1), 1 - s(t_2 - t_1))$, where $s(t) = 1 - 0.5e^{-t/2}$. This functional form allows for an *experience effect*; the longer the first firm operates alone the larger the share it has of the market after the entry of the second firm. Given this, with both firms in the market, the duopoly profits are

$$R_1 = (1 - c_1)s(t_2 - t_1), \;\; R_2 = (1 - c_2)(1 - s(t_2 - t_1)).$$

Herein lies the tradeoff for the firms when deciding their optimal entry times. Early entry – if they manage to do so before their rival – allows a firm to develop a captive customer base. Later entry, on the other hand, allows a firm to enter the market with lower production costs.

Figure 1 shows that in this case both payoff functions $L(t)$ and $F(t)$ are non-monotonic, with both curves increasing and decreasing overtime.[12]

Product innovation with switching costs. In this model, the potential quality of the product a firm can take to market improves over time; for example, the quality of a phone handset will typically improve the longer a firm waits to launch it. In a similar way as to the process-innovation model above, when a firm enters the market, they sell a product of the highest quality available at the time. Note that, this is a one-off decision – firms sell the same quality product from their time of entry, until the end of the game. As a result, waiting is advantageous as it allows a firm to sell a better quality product.

Following Dutta et al. (1995) and Hoppe and Lehmann-Grube (2005), consumers value quality in a vertically-differentiated goods model. We assume that the quality of a product, denoted by s, is increasing monotonically over time according to the function $s = t^2$. For simplicity, it is assumed that R&D and production costs are zero and independent of quality.

Like in Tirole (1988), preferences differ according to a taste parameter θ, where θ is uniformly distributed between $[0, 1]$. Each consumer has a unit demand for the good and has utility of $U = s_i\theta - p_i$, where s_i and p_i are the quality and price offered by firm i. A consumer will buy at most one unit from a firm provided that $U \geq 0$ for that product and, if there are more than one firm in the market, the consumer will buy one unit from the firm that provides her with the highest net utility (again provided that $U \geq 0$).[13]

To this framework, we introduce a switching cost for consumers that have had experience with a particular good. Specifically, a consumer that has been serviced by firm 1 for the period of time τ will require an additional utility of at least $E(\tau) = \tau^2/2$ if he is to have an incentive to switch to firm 2. Consequently, taking each entry time as given at t_1 and t_2, respectively, there will be a consumer with a taste parameter $\theta = \theta_2$ who is just indifferent between switching from buying the leader's product to changing over to buy the second entrant's offering. That is, θ_2 solves

$$\theta_2 t_1^2 - p_1 + E(t_2 - t_1) = \theta_2 t_2^2 - p_2.$$

There will also be a consumer with a taste parameter θ_1 who is just indifferent between buying from the leader and not buying at all. In other words, for this indifferent consumer, θ_1 solves

$$\theta_1 t_1^2 - p_1 = 0.$$

Furthermore, if switching between providers is to occur, it will happen only at the point in time at which the second firm enters, and not at a later date.

For this model, the instantaneous monopoly profit is $R_M = t_1^2/4$, while the in-

[12]It is evident in this example that for $t \geq t'$, $L(t)$ and $F(t)$ coincide, in a similar manner to Fudenberg and Tirole (1985). In the following product-innovation and asset-market examples, the two curves do not coincide (other than their intersection points).

[13]See Hoppe and Lehmann-Grube (2001) and Hoppe and Lehmann-Grube (2005) for more details.

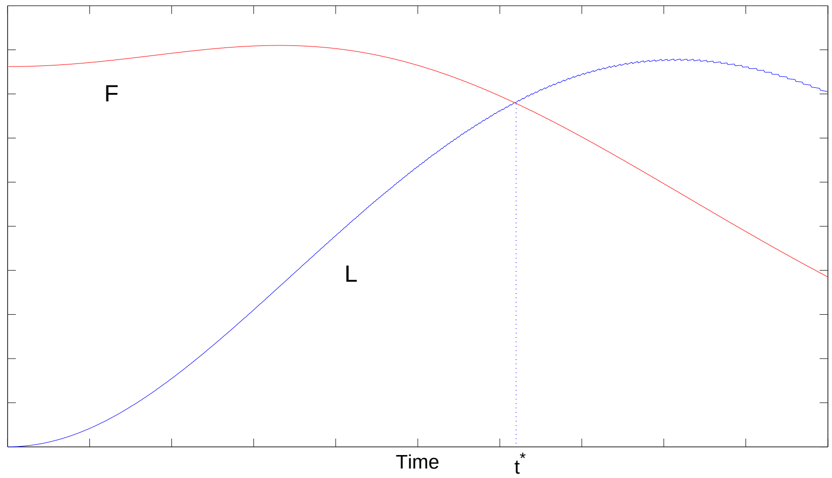

Figure 2: Non-monotonic payoff functions for $F(t)$ and $L(t)$ in a product-innovation game

stantaneous duopoly profits are

$$R_1 = \frac{(t_2^2 - t_1^2 + E(t_2 - t_1))^2 t_1^2 t_2^2}{(4t_2^2 - t_1^2)^2 (t_2^2 - t_1^2)}, \quad R_2 = \frac{(2(t_2^2 - t_1^2)t_2^2 + E(t_2 - t_1)(t_1^2 - 2t_2^2))^2}{(4t_2^2 - t_1^2)^2 (t_2^2 - t_1^2)},$$

given the entry times are t_1 and t_2, respectively.

Figure 2 shows that in this case the payoff functions of both the $L(t)$ and $F(t)$ are non-monotonic. Moreover, other than their intersection point, these functions do not coincide.

Asset sales. When should a trader sell an asset? A vendor making this decision will have to take into account the actions of other sellers. Following Dutta et al. (1993), we consider two potential sellers of an asset in a market with the following features. First, the price of the asset is appreciating, perhaps representing the case when the market demand for the asset increases over time. Second, the follower's sale price is negatively affected if the other party sells their asset first. A possible example of the payoffs to the first seller, shown by $L(t)$, and the second seller, $F(t)$, is illustrated in Figure 3. As before, both payoffs are functions of the leader's time of sale t. In this example, as both payoffs are increasing, the techniques of Hoppe and Lehmann-Grube (2005) cannot be used to determine the SPE.

3 The model with continuous payoffs

Let us first consider two symmetric firms with continuous payoff functions $L(t)$ and $F(t)$. While this setup is similar to Hoppe and Lehmann-Grube (2005), an important departure is that in our paper $F(t)$ can be a non-monotonic function.

Here, we develop a method to determine the leader's time of the entry in all SPE.

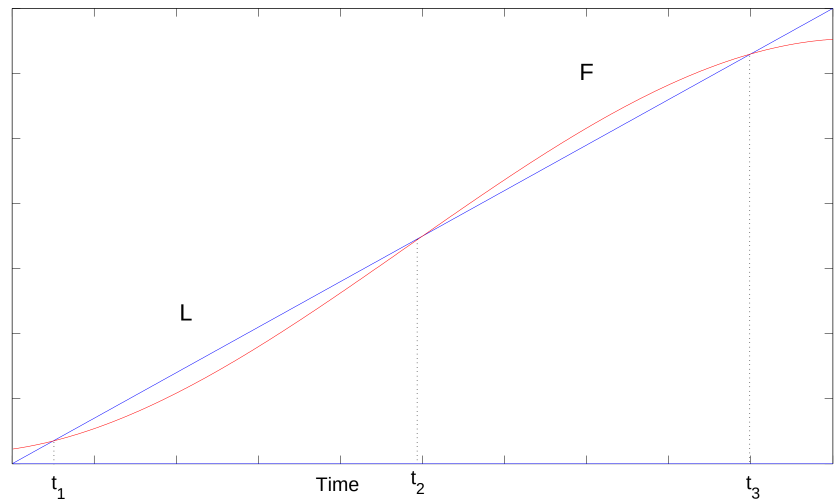

Figure 3: Asset sales with increasing potential sale prices

To do this we first construct a set $A(t')$, defined as

$$A(t') = \{\ t \geq t' \ \mid \ F(t) \geq L(t) > L(\tau) \ \ \forall \ \tau \in [t', t)\}. \tag{3}$$

Let us consider conditions specifying set $A(0)$.[14] As we now show, any SPE with the leader's entry time $t > 0$ must belong to set $A(0)$. In any of these SPE it must be the case that the condition $L(t) > L(\tau) \ \forall \ \tau \in [0, t)$ be satisfied at the time of the leader's entry. If not, the leader will have incentive to enter earlier. It is also the case that $F(t) \geq L(t)$ must be satisfied in equilibrium, otherwise the follower will have an incentive to preempt the leader and enter earlier, as in Fudenberg and Tirole (1985).

Now we are in a position to characterize the set of all SPE of the game in terms of the time of entry by the leader. We start by considering the leader's preferred SPE (LSPE), as presented in the following lemma.

Lemma 1. *The first firm's stopping time in the LSPE is:*

$$t^* = \begin{cases} \arg\max_t A(0) & \textit{if} \quad A(0) \neq \emptyset, \\ 0 & \textit{otherwise.} \end{cases} \tag{4}$$

The strategies firms adopt in the LSPE are:

$$\sigma_1(t) = \begin{cases} 1 & \textit{if} \ \ \nexists \ t' > t, \ \ t' \in A(t), \\ 0 & \textit{otherwise}; \end{cases}$$

[14]Note that for $t = t'$, the condition $L(t) > L(\tau) \ \forall \ \tau \in [t', t)$ is not applicable; rather, only condition $F(t) \geq L(t)$ is required.

$$\sigma_2(t) = \begin{cases} 1 & \textit{if} \quad L(t) \geq F(t) \ \& \ \nexists \ t' > t, \ t' \in A(t), \\ 0 & \textit{otherwise.} \end{cases}$$

Proof: See Appendix A.

Lemma 1 describes the solution for the SPE that provides the leader with its highest possible payoff, allowing for any continuous $L(t)$ and $F(t)$ payoff functions. The equilibrium strategy of firm 1 is to enter whenever there is no additional gain from delaying entry – this is represented here by the condition $\nexists \ t' > t, \ t' \in A(t)$. On the other hand, the equilibrium strategy of firm 2 is to wait unless they are (weakly) better off being a leader at a given time t. This is represented by two conditions: $L(t) \geq F(t)$; and $\nexists \ t' > t, \ t' \in A(t)$. The first condition means that they are (weakly) better off being a leader rather than a follower at a given time t, while the second condition means they prefer being a leader at t rather than at some later time. Note that firms have different strategies to allow for asymmetries in equilibria. Given each firm is otherwise identical, to avoid coordination failures in which both firms enter at the same time we assume, for convenience, that firm 1 has a slightly weaker bargaining position in comparison with firm 2 so that it receives (or is willing to 'accept') the lower payoff available in this LSPE. With these somewhat 'predetermined' roles, firm 1 becomes the leader when both firms prefer to be the follower.[15]

In equilibrium, we observe the leader enter the market immediately when either $A(0) = \emptyset$ or $A(0) = \{0\}$. In the first case, $L(0) > F(0)$ and there is no benefit from waiting because $A(0)$ is empty. In the second case, $L(0) \leq F(0)$, and as $A(0) = \{0\}$, again, there is no advantage in delaying entry in equilibrium. Alternatively, entry by the leader occurs after a delay when $t^* = \arg\max_t A(0) > 0$. In this case, there is an advantage of waiting until t^*.

Having outlined the LSPE, we are able to describe all the equilibria of the game (which also include the LSPE). First let us consider the equilibria that occur when the leader and follower curves coincide or intersect; note that there are similar joint-adoption equilibria in Fudenberg and Tirole (1985). To do this we introduce the following set B, where

$$B = \{ \ t \ | \ F(t) = L(t) > L(\tau) \ \ \forall \ \tau \in [0, t)\}. \tag{5}$$

Using this set we can present a lemma that describes all SPE of the game in which there is rent equalization (RE). The players' strategies in these SPE are also outlined.

Lemma 2. *For any $t^* \in B$ there is a corresponding SPE with rent equalization in*

[15] Note an equivalent assumption is made in the previous literature in order to avoid the coordination issues; see for example, Fudenberg and Tirole (1985), Dutta et al. (1995) and Hoppe and Lehmann-Grube (2005).

which both firms enter at t^*. The strategies firms adopt in this SPE are:

$$\sigma_1(t) = \begin{cases} 1 \quad \textit{if} \quad t = t^* \;\; \textit{or} \;\; t > t^* \;\; \& \;\; \nexists \; t' > t, \;\; t' \in A(t), \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \textit{otherwise}; \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 \quad \textit{if} \quad t = t^* \;\; \textit{or} \;\; t > t^* \;\; \& \;\; L(t) \geq F(t) \;\; \& \;\; \nexists \; t' > t, \;\; t' \in A(t), \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \textit{otherwise}. \end{cases}$$

Proof: See Appendix A.

Here, if both firms are entering at t^*, there is no gain from a unilateral deviation to enter earlier, as the payoff to a leader at t^* is greater than a leader's payoff from entry at any earlier date; this follows from the way set B is constructed – specifically, $L(t) > L(\tau) \;\; \forall \;\; \tau \in [0, t)$. Similarly, there is no gain from deviating and entering later; the set B is constructed so that the payoffs to the leader and the follower are equal, $F(t) = L(t)$. The equilibrium strategies of both firms are to wait before t^*, enter at t^*, and for both firms to adopt the strategies specified for the LSPE off-the-equilibrium path (that is, for $t > t^*$). It is worth noting that if $L(t^*) = F(t^*)$ in the LSPE of the game, set B also includes the leader's preferred subgame perfect equilibrium.

Next, there is another potential SPE at $t = 0$.

Lemma 3. *If $L(0) > F(0)$, there is an SPE in which both firms enter at $t^* = 0$. The strategies firms adopt in this SPE are:*

$$\sigma_1(t) = \begin{cases} 1 \quad \textit{if} \quad t = 0 \;\; \textit{or} \;\; t > 0 \;\; \& \;\; \nexists \; t' > t, \;\; t' \in A(t), \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \textit{otherwise}; \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 \quad \textit{if} \quad t = 0 \;\; \textit{or} \;\; t > 0 \;\; \& \;\; L(t) \geq F(t) \;\; \& \;\; \nexists \; t' > t, \;\; t' \in A(t), \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \textit{otherwise}. \end{cases}$$

Proof: See Appendix A.

In a similar manner to the case in Lemma 2, there is no gain from unilaterally deviating and entering later as $L(0) > F(0)$. The equilibrium strategies of both firms are to enter at $t = 0$ and adopt the strategies specified for the LSPE off-the-equilibrium path (that is, for $t > 0$). Note that this potential equilibrium is explicitly ruled out by Fudenberg and Tirole (1985), as they assume that the follower's payoff is greater than the leader's at the start of the game. Furthermore, this equilibrium can be the LSPE if $A(0)$ is an empty set. Finally, as a point of clarification, there is an equilibrium when $L(0) = F(0)$. This equilibrium is not captured here; rather, it is included in set B, as described in Lemma 2.

Next, let us discuss equilibria with a second-mover advantage. To do this, we disregard the areas in which $L(t) > F(t)$. This will naturally divide $[0, t^{max}]$ into k regions where $L(t) \leq F(t)$, labeled as $U_s, \;\; s = 1, \ldots, k$.[16] Now we are in a position

[16] There is no equilibria with leader entry at $t > t^{max}$ because the leader will have an incentive to enter at t^{max}. Also note that as both $L(t)$ and $F(t)$ are continuous $\forall \;\; t \in [0, t^{max}]$, k must be finite.

to present a lemma that characterizes all SPE of the game with a second-mover advantage.

Lemma 4. *For any region U_s, $s = 1, \ldots, k$, apply Lemma 1 to find the LSPE of the region with the leader's entry time at t^*. If $t^* \in A(0)$ and the equilibrium is not a RE ($t^* \notin B$) or zero ($t^* \neq 0$) equilibrium, it is a second-mover advantage equilibrium for the entire game. The strategies firms adopt in this SPE are:*

$$\sigma_1(t) = \begin{cases} 1 & \textit{if} \quad t = t^* \; \textit{or} \; t > t^* \; \& \; L(t) \geq F(t) \; \textit{or} \; t > t^* \; \& \; \nexists \; t' > t, \; t' \in A(t), \\ 0 & \textit{otherwise}; \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 & \textit{if} \quad t > t^* \; \& \; L(t) \geq F(t), \\ 0 & \textit{otherwise}. \end{cases}$$

Proof: See Appendix A.

In the equilibrium described in this lemma, the leader invests at t^* as there is no gain from investing earlier because $t^* \in A(0)$. There is also no gain from investing later as: the equilibrium is the LSPE for a given region U_s; and both firms enter whenever $L(t) \geq F(t)$ for $t > t^*$, ensuring entry cannot be postponed until after this region. Here, in a similar manner to Lemma 1, the firms have different strategies to allow for asymmetries in the equilibria. In these equilibria the leader receives a lower payoff than the follower. As a result, the follower also has no incentive to deviate. Second-mover advantage equilibria are present in Dutta et al. (1995) and Hoppe and Lehmann-Grube (2005).

Using the lemmas presented above, we are now in the position to summarize all the SPE of the game, and this is done in the following proposition.

Proposition 1. *Lemmas 2, 3 and 4 characterize all of the SPE in the continuous-payoff timing game.*

Proof: See Appendix A.

As described in Proposition 1, our technique allows for the characterization of all SPE in the continuous entry game with two symmetric firms. Lemma 2 outlines the equilibria in which there is rent equalization between the firms. There is a first-mover advantage in the SPE described in Lemma 3; there will be immediate joint adoption at $t = 0$. Finally, Lemma 4 describes equilibria in which there is a second-mover advantage. Note that the equilibria detailed in Lemmas 2, 3 and 4 are mutually exclusive. However, the LSPE, described by Lemma 1, is covered in either Lemmas 2, 3 or 4.

Consider now the possibility that one of the equilibria is an SSPE. In an SSPE both firms receive a higher payoff than in any other SPE. For the SSPE to exist it must be feasible to pareto rank all equilibria. This will be possible, for sure, when there are no second-mover advantage equilibria (as described in Lemma 4) or when there is a unique second-mover advantage equilibrium and it is also the LSPE of the game (detailed in Lemma 1). If only rent equalization or immediate-entry equilibria exist, they are directly comparable. This is not necessarily true with a second-mover advantage equilibrium; one firm could be better off while the other is worse off compared to alternative SPE. Only when the second-mover advantage equilibrium is unique and provides the leader with its highest possible payoff can we be sure that a pareto ranking is feasible. Note, it is also possible to determine that the SSPE, provided it exists, is the LSPE of the game. No other SPE can be the SSPE of the game as the LSPE provides the leader with their highest payoff. This is summarized in the following corollary.

Corollary 1. *If the equilibria can be ranked, the SSPE is the LSPE. A sufficient condition for the SSPE to exist is that: (i) there are no second-mover advantage equilibria; or (ii) there is a unique second-mover advantage equilibrium that is also the LSPE.*

Proof: Follows from the discussion above.

Let us now apply our technique to the entry model of Hoppe and Lehmann-Grube (2005). Our method can be simplified for the case in which $F(0) \geq L(0)$ and $F(t)$ is non-increasing.

Corollary 2. *If $F(0) \geq L(0)$ and $F(t)$ is non-increasing, there is a unique SPE of the timing game in which the time of the leader's entry t^* is given by*

$$t^* = \min \arg \max_t \min[L(t), F(t)]. \tag{6}$$

The strategies firms adopt are:

$$\sigma_1(t) = \begin{cases} 1 & \text{if} \quad \nexists \ t' > t, \quad \min[L(t'), F(t')] > \min[L(t), F(t)], \\ 0 & \text{otherwise}; \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 & \text{if} \quad L(t) \geq F(t) \ \ \& \ \nexists \ t' > t, \quad \min[L(t'), F(t')] > \min[L(t), F(t)], \\ 0 & \text{otherwise.} \end{cases}$$

Proof: See Appendix A.

Similar to Lemma 1, the equilibrium strategy of firm 1 is to enter whenever there is no additional gain from delaying entry – that is, $\nexists \ t' > t$ where $\min[L(t'), F(t')] > \min[L(t), F(t)]$. On the other hand, the equilibrium strategy of firm 2 is to wait unless they are (weakly) better off being a leader at a given time t; that is, $L(t) \geq F(t)$, and $\nexists \ t' > t$ where $\min[L(t'), F(t')] > \min[L(t), F(t)]$. Effectively the strategy the leader adopts is to maximize $L(t)$ while $L(t) \leq F(t)$. Note, the assumption of a non-increasing $F(t)$ is critical here, as we illustrate in the asset-market example below.

Furthermore, the equilibrium described in Corollary 2 is unique. The intuition is similar to that outlined in Hoppe and Lehmann-Grube (2005). First, the potential equilibrium detailed in Lemma 3 is explicitly ruled out as $F(0) \geq L(0)$. Second, because the non-increasing $F(t)$ and the non-decreasing envelope of $L(t)$ intersect only once (or more precisely, at one level), there can only be at most one SPE with rent equalization arising from set B (Lemma 2). Third, because of non-increasing $F(t)$, the region with $L(t) \leq F(t)$ is effectively unique. This means that there is at

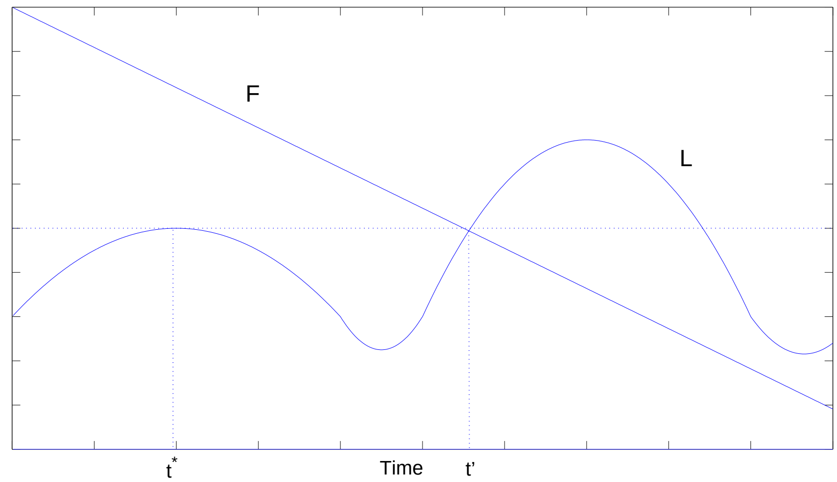

Figure 4: Example of optimal entry when $F(t)$ is non-increasing

most one second-mover advantage equilibrium. Finally, the unique SPE will involve either: rent equalization if there is a time where $L(t)$ and $F(t)$ intersect and $L(t)$ is at its historic maximum for the game up until that time; or, alternatively, a second-mover advantage.

We now apply our solution algorithm to the case studied by Hoppe and Lehmann-Grube (2005), as illustrated in Figure 4. Applying our method, we derive $A(0) = [0, t^*]$. The LSPE is t^* because it is the largest t belonging to $A(0)$. Note that this SPE is unique because $B = \emptyset$ and $L(0) < F(0)$. In particular, note that the point at which $F(t)$ and $L(t)$ intersect, t', does not belong to either $A(0)$ or B – even though $L(t)$ reaches the same level as at t^*, it is not strictly greater than $L(\tau)\ \forall\ \tau < t$. By Assumption 2, we consider t^*, the shortest time required to reach this maximum. The equilibrium strategies that support this SPE are for firm 1 to enter at t^*, and for both firms to enter at any $t \geq t'$ (and not to enter otherwise).

In addition, because $F(t)$ is monotonically decreasing, Corollary 2 applies. First, we construct a new curve that is the minimum of $F(t)$ and $L(t)$. We then find points for which this new curve reaches maximum. In this example there are two points – t^* and t'. Finally, we choose the earliest of these two possible times, t^*, as the leader's entry time in the SPE. The strategies of each firm that support this SPE are outlined in the previous paragraph.

Second, we can apply our algorithm to the examples outlined in Subsection 2.3 (see Figures 1, 2 and 3). We completely characterize all SPE of these games, something that could not be done by the previous literature. One can see that in the process-innovation example in Figure 1, $A(0) = [0, t^*]$ and $B = \{t^*\}$. This is because for all times between 0 and t^* the payoff to the leader is increasing, while it is still less than the payoff to the follower. At t^*, the leader and follower's payoff coincide – at this point there is a pre-emption equilibrium, *à* la Fudenberg and Tirole (1985).[17] This equilibrium is unique, and the strategies are for both firms to enter at any $t \geq t^*$.

Similarly, in the product-innovation example, illustrated in Figure 2, $A(0) = [0, t^*]$ and $B = \{t^*\}$. In this case, the $L(t)$ and $F(t)$ curves intersect once at t^*. Consequently, the unique SPE involves the leader entering at t^*. The equilibrium strategies are for both firms to enter at any $t \geq t^*$.

In the asset-market example illustrated in Figure 3, $A(0) = [0, t_1] \cup [t_2, t_3]$. Set B contains $\{t_1\}$, $\{t_2\}$, and $\{t_3\}$; consequently, the rent-equalization equilibria involve entry at t_1, t_2 or t_3, with the last of these equilibria being the pareto preferred SSPE. These are the only pure strategy equilibria of the game. The equilibrium strategies that support leader entry at t_3^* are for firm 1 to enter at any $t \geq t_3^*$ and for firm 2 to never enter as a leader. The equilibrium strategies that support t_i^* where $i = 1, 2$ are for both firms to enter at t_i^* and for the first firm to enter at any $t \geq t_3^*$. Finally, the algorithm outlined in Corollary 2, which assumed $F(t)$ is non-increasing, cannot be applied here. In fact, the method in Corollary 2 would suggest an entry time of T, which is not an equilibrium because $L(T) > F(T)$.

4 Discontinuous payoffs

As noted in the introduction, discontinuities in payoffs arise in many economic situations. We turn our attention to this issue now. To do this we need to make an assumption regarding the nature of these discontinuities. To this end, we assume that all functions are right-continuous; that is, all the functions have no break when the limit point is approached from the right. Given the sort of structural breaks that are likely to arise in timing games, this seems like the most natural assumption to make; for example, an action by a third party in a related market could result in a discontinuous jump (up or down) in the payoff from innovating in the market of interest. Similarly, when selling an asset, a sale by one party could have a discontinuous effect on the potential sale price for the second vendor. Moreover, right-continuous functions are consistent with the (always present) discontinuity at $t = 0$.

We assume that there is a finite number of discontinuities and introduce the following set D that contains all times at which either the leader's or follower's payoff function is discontinuous. Specifically,

$$D = \{ \ t \ | \ \lim_{\tau \to t^-} L(\tau) \neq L(t) \ \text{ or } \ \lim_{\tau \to t^-} F(\tau) \neq F(t) \ \text{ or } \ t = 0 \ \}. \tag{7}$$

It is worth noting that $t = 0$ is also included in this set D as it has similar properties to other elements of this set, in that limits with $\tau \to 0^-$ are not defined.

To find the set of SPE we adapt the technique developed in Section 3. A crucial proviso here is that we need to ensure that an equilibrium exists; for example non-existence could be an issue if the set $A(0)$ does not contain its supremum. To explore

[17] Note that even though the two curves coincide from t' on, these times are not part of B because $L(t)$ is larger at some earlier time $t < t'$ – this is similar to Case A in Fudenberg and Tirole (1985).

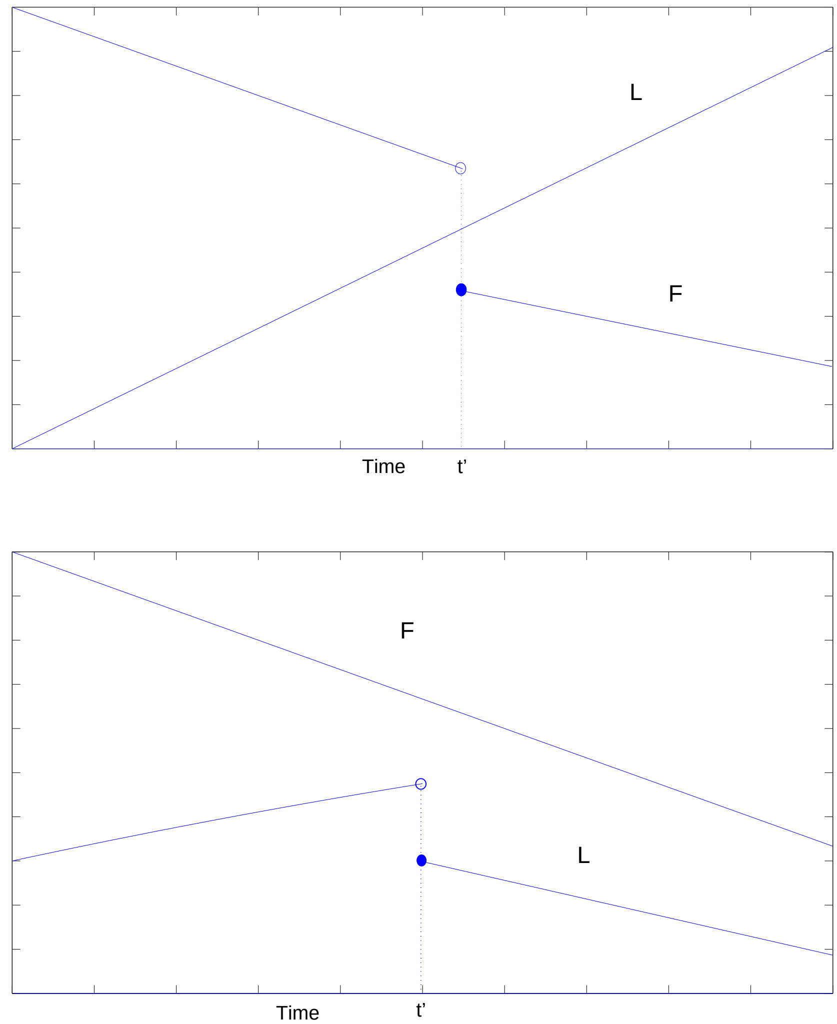

Figure 5: Discontinuities in the payoff functions

this, first consider the case when $F(t)$ is discontinuous but the supremum is not in set $A(0)$. This situation is illustrated in Figure 5a. Note that in this case $A(0) = [0, t')$. Consequently, the leader wants to enter before t' but as close to this time as possible; no pure-strategy equilibrium exists.

Second, with a discontinuous $L(t)$ it is also possible that the supremum does not belong to set $A(0)$ itself. We illustrate this situation in Figure 5b. One can see that $A(0) = [0, t')$. The equilibrium does not exist in this example because the leader would like to enter before t', but as close as possible to this time.

To proceed, utilizing Assumption 1, let there be a small length of time ε just prior to the discontinuity in payoffs that represents the last time before the discontinuity that a firm can enter, as outlined below.

Assumption 6. *The minimum time before a discontinuity that a firm can opt to* *enter the market is ε, where $\varepsilon > 0$.*

This minimum time ε before the discontinuity is effectively the last 'period' in which a firm can enter prior to the break in payoffs. This assumption effectively ensures that the presence of a discontinuity does not result in the non-existence of equilibria.

Now we can modify the method developed in Section 3 to accommodate for payoff functions with discontinuities. To do this we introduce the following set $C(t')$, where

$$C(t') = \{t \geq t', t \in D | \ L(t) > F(t) \ \& \ (L(t) + F(t))/2 > L(\tau) \ \forall \ \tau \in [t', t)\}. \quad (8)$$

Note that some discontinuities could be included in $A(0)$, provided the conditions in (3) are satisfied. By introducing set $C(0)$ we are able to consider potential entry times where there are discontinuities and $L(t) > F(t)$ in situations that are not described by $A(0)$. Furthermore, as pointed out earlier when $A(t')$ was defined, the second condition does not apply if $t = t'$. In particular, this means that for $t = 0$ to be contained in $C(0)$ only condition $L(0) > F(0)$ is required.

We are now in a position to present a lemma, that modifies Lemma 1 to accommodate discontinuous payoff functions.

Lemma 5. *The first firm's stopping time in the LSPE is:*

$$t^* = \arg\max_t [A(0) \cup C(0)] \quad (9)$$

The strategies firms adopt in the LSPE are:

$$\sigma_1(t) = \begin{cases} 1 & \text{if} \ \nexists \ t' > t, \ t' \in [A(t) \cup C(t)], \\ 0 & \text{otherwise}; \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 & \text{if} \ L(t) \geq F(t) \ \& \ \nexists \ t' > t, \ t' \in [A(t) \cup C(t)], \\ 0 & \text{otherwise}. \end{cases}$$

Now, the LSPE could occur at a point of discontinuity; all points with $L(t) \leq F(t)$ – including discontinuities – are covered by $A(0)$, whereas discontinuities with $L(t) > F(t)$ are covered by $C(0)$. Consequently, the set $A(0) \cup C(0)$ covers all possible SPE arising both at continuous and discontinuous points. In contrast to Lemma 1, the time $t = 0$ will necessarily belong to either set $A(0)$ or $C(0)$, meaning that non-existence when $A(0) = \emptyset$ is no longer an issue.

The following lemma modifies Lemma 2 to accommodate discontinuous payoff functions in the rent-equalization equilibria. As illustrated, there are very few changes from Lemma 2, except for the firms' off-equilibrium strategies.

Lemma 6. *For any $t^* \in B$ there is a corresponding SPE in which both firms enter at t^*. The strategies firms adopt in this SPE are:*

$$\sigma_1(t) = \begin{cases} 1 & \text{if} \ t = t^* \ \text{or} \ t > t^* \ \& \ \nexists \ t' > t, \ t' \in [A(t) \cup C(t)], \\ 0 & \text{otherwise}; \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 & if \;\; t = t^* \;\; or \;\; t > t^* \;\; \& \;\; L(t) \geq F(t) \;\; \& \;\; \nexists \; t' > t, \;\; t' \in [A(t) \cup C(t)], \\ 0 & otherwise. \end{cases}$$

Proof: See Appendix A.

Lemma 3 details the equilibrium and strategies when there is immediate entry. Below, we generalize this result to any point of discontinuity that could be an SPE.

Lemma 7. *For any $t^* \in C(0)$ there is a corresponding SPE in which both firms enter at t^*. The strategies firms adopt in this SPE are:*

$$\sigma_1(t) = \begin{cases} 1 & if \;\; t = t^* \;\; or \;\; t > t^* \;\; \& \;\; \nexists \; t' > t, \;\; t' \in [A(t) \cup C(t)], \\ 0 & otherwise; \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 & if \;\; t = t^* \;\; or \;\; t > t^* \;\; \& \;\; L(t) \geq F(t) \;\; \& \;\; \nexists \; t' > t, \;\; t' \in [A(t) \cup C(t)], \\ 0 & otherwise. \end{cases}$$

Proof: See Appendix A.

Next, we consider equilibria with a second-mover advantage. As in the previous section, we disregard the areas in which $L(t) > F(t)$. This will naturally divide $[0, t^{max}]$ into k regions where $L(t) \leq F(t)$, labeled as $U_s, \;\; s = 1, \ldots, k$.[18] We now present our next lemma.

Lemma 8. *For any region U_s, $s = 1, \ldots, k$, apply Lemma 5 to find the LSPE of the region, denoted as t^*. If $t^* \in A(0)$ and this equilibrium is not a RE ($t^* \notin B$) or a discontinuity equilibrium ($t^* \notin C(0)$), it is a second-mover advantage equilibrium for the entire game. The strategies firms adopt in this SPE are:*

$$\sigma_1(t) = \begin{cases} 1 & if \;\; t = t^* \;\; or \;\; t > t^* \& L(t) \geq F(t) \;\; or \;\; t > t^* \& \; \nexists \; t' > t, t' \in [A(t) \cup C(t)], \\ 0 & otherwise; \end{cases}$$

$$\sigma_2(t) = \begin{cases} 1 & if \;\; t > t^* \;\; \& \;\; L(t) \geq F(t), \\ 0 & otherwise. \end{cases}$$

Proof: See Appendix A.

Now we generalize Proposition 1 to accommodate discontinuities.

Proposition 2. *Lemmas 6, 7 and 8 characterize all of the SPE when payoffs can be discontinuous.*

Proof: See Appendix A.

Proposition 2 characterizes all of the SPE in the entry game with two symmetric firms, allowing for the possibility that payoffs are discontinuous. Lemma 6 outlines

[18] Set D divides $[0, t^{max}]$ into a finite number of areas separated by discontinuities in $L(t)$ and $F(t)$. Within each area, because $L(t)$ and $F(t)$ are continuous there are a finite number of regions where $L(t) \leq F(t)$. Consequently, over $[0, t^{max}]$ there are a finite number of regions where $L(t) \leq F(t)$; hence k must be finite.

the equilibria in which there is rent equalization between the firms. There is a first-mover advantage in the SPE described in Lemma 7. Finally, Lemma 8 describes equilibria in which there is a second-mover advantage. As before the equilibria detailed in Lemmas 6, 7 and 8 are mutually exclusive. However, the LSPE, described by Lemma 5, is covered in either Lemmas 6, 7 or 8.

Consider, now, the possibility that one of the equilibria is an SSPE. One can generalize Corollary 1 to the case when payoffs are discontinuous.

Corollary 3. *If the equilibria can be ranked in the discontinuous game, the SSPE is the LSPE. A sufficient condition for the SSPE to exist is that: (i) there are no second-mover advantage equilibria; or (ii) there is a unique second-mover advantage equilibrium that is also the LSPE.*

Proof: Follows from the discussion above.

Fudenberg and Tirole (1985, Section 5) discuss the possibility of discontinuities in the oligopoly case with three or more entrants. Similarly, Hoppe and Lehmann-Grube (2005) consider an example with a discontinuous $L(t)$ payoff function. Here, we characterize all SPE for any finite number of discontinuities in both the leader's and the follower's payoff functions.

The techniques developed here can also be applied when $F(t)$ and $L(t)$ have discontinuities at the same time. This is a conceivable scenario, given the sort of event that produces a discontinuity – such as entry or exit in a related market – will potentially affect both the leader's and the follower's payoff. The ability to be able to handle joint discontinuities demonstrates both the generality and the usefulness of the solution algorithm outlined in Proposition 2.

5 Concluding comments

The decision when to launch a new product is a critical question for many firms; it can determine profit, firm survival and the shape of markets. More generally, it drives economic development. Given its importance, innovation has received a great deal of attention from economists. We follow in this tradition by studying a market-entry game with complete information, when firm's actions are observable to all and in which there is no uncertainty. Our focus is on situations in which firms cannot commit to their strategy ex ante; given this, we use the equilibrium concept of subgame perfection.

We characterize all of the pure strategy subgame perfect equilibria for a two-player innovation game when the payoffs can potentially be non-monotonic, multiple-peaked and discontinuous. This new method is relevant in a variety of economic situations; for example, our algorithm can be applied to a process-innovation game with switching costs, to product innovation when there is an experience good, and to the timing of the sale of an asset. There can be non-standard payoffs in each of these examples, making them beyond the scope of existing techniques.

It is worth making a comparison of our results to those in the literature. First, our framework is sufficiently general to allow for non-monotonic or discontinuous payoffs for both the leader and the follower. Second, our algorithm allows us to distinguish between different types of equilibria in this general framework. The equilibria can display rent-dissipation with joint adoption or single-firm adoption with a second-mover advantage. It also allows for a first-mover advantage with immediate entry, when $L(0) > F(0)$, or at points of discontinuity. Third, we provide conditions that ensure that equilibria can be pareto ranked. If all equilibria involve rent equalization or a first-mover advantage, they are pareto comparable and it is possible to determine the superior subgame perfect equilibrium. Similarly, if there is a unique second-mover advantage equilibrium that is also the leader's preferred subgame perfect equilibrium, pareto ranking is feasible.

6 Appendix A

Proof of Lemma 1

This proof consists of four parts: A), B), C) and D). In part A) we show that all SPE with positive entry times must belong to $A(0)$. In part B) we prove that there exists a unique t^*, given by (4), at which $L(t)$ is maximized over $A(0)$. Part C) shows that t^* delivers the highest possible equilibrium payoff to the leader, while part D) proves that t^* is an SPE.

A) As a preliminary step, let us prove all SPE with entry time $t^* > 0$ must belong to $A(0)$. Assume, on the contrary, that there is an SPE with a positive entry time $t^* \notin A(0)$. It must be the case that either the condition $L(t) > L(\tau),\ \forall\ \tau \in [0, t^*)$, or the condition $F(t^*) \geq L(t^*)$ is not satisfied. If for some $\tau < t^*$ it is the case that $L(\tau) \geq L(t^*)$, the leader will have an incentive to enter earlier at τ. On the other hand, if $F(t^*) < L(t^*)$, the follower will have an incentive to preempt the leader and enter slightly earlier, as in Fudenberg and Tirole (1985). Neither of these situations are possible in equilibrium. Consequently, there is a contradiction and the statement that all SPE with positive entry times must belong to $A(0)$ is proved.

B) Next, let us prove that there exists a unique t^* at which $L(t)$ is maximized over $A(0)$, given by

$$t^* = \begin{cases} \arg\max\limits_t A(0) & \text{when} \quad A(0) \neq \emptyset, \\ 0 & \text{when} \quad A(0) = \emptyset. \end{cases} \tag{4}$$

Note that when $A(0) = \emptyset$, $L(t)$ is maximized over an empty set and the problem is not well defined. We assign $t^* = 0$ in this situation; we show that this is part of an equilibrium strategy in Part D).

Let us prove the existence of the solution to this problem of maximizing $L(t)$ over $A(0)$ when $A(0) \neq \emptyset$. Note that set $A(0)$ is bounded because t^{max} is finite, where t^{max} is the time t at which $L(t)$ reaches its global maximum (Assumption 4). We need to show that set $A(0)$ always contains its supremum. Assume that it does not. This means that there is a sequence $\{t_k\}$ contained in $A(0)$ and convergent to some limit t^* that is not contained in set $A(0)$. This requires that either: there is $t' < t^*$

such that $L(t') \geq L(t^*)$; or that $F(t^*) < L(t^*)$. On the other hand, any $\tau \in [t', t^*)$ belongs to $A(0)$, which means that $L(\tau) > L(t')$ and $F(t^*) \geq L(t^*)$. This leads to a contradiction given $L(t)$ and $F(t)$ are continuous functions, proving existence.

The uniqueness follows immediately from the way set $A(0)$ is constructed. If two points were to maximize $L(t)$ over $A(0)$ then the one with the later time would not belong to $A(0)$.

Next, let us show that if $t^* = \arg\max_{t \in A(0)} L(t)$ it is also the case that $t^* = \arg\max_t A(0)$ when $A(0) \neq \emptyset$. Assume the opposite that $t^* \neq \arg\max_t A(0)$. If $t^* < \arg\max_t A(0)$, then t^* does not maximize the leader's payoff over $A(0)$. If $t^* > \arg\max_t A(0)$, t^* does not belong to $A(0)$. Both situations lead to a contradiction. We have now shown that $t^* = \arg\max_t A(0)$, concluding the proof of part B).

C) Next, we prove that t^* given by (4) delivers the highest possible payoff to the leader. Given that in A) we proved that all SPE with positive entry times must belong to $A(0)$, this point follows immediately.

D) Let us prove that the proposed equilibrium with t^* defined in (4) is an SPE. When $A(0) \neq \emptyset$ there are three cases to consider for possible profitable deviations.

(1) If $L(t^*) = F(t^*)$, the strategies specified in the lemma result in both firms entering at t^*, generating a payoff of $(L(t^*) + F(t^*))/2 = L(t^*)$ for both firms. If either of the firms enters earlier at $\tau < t^*$, it will get a payoff of $L(\tau)$. From the construction of set $A(0)$ in (3) it follows that $L(\tau) < L(t^*)$. On the other hand, if either firm enters later, that firm will get a payoff of $F(t^*)$, which is equal to $(L(t^*) + F(t^*))/2$. Consequently, if $L(t^*) = F(t^*)$ there is no profitable deviation for either firm.

(2) If $L(t^*) < F(t^*)$, one needs to consider deviations of the two firms separately. Without loss of generality, the first firm is the leader; it enters at t^* and gets a payoff of $L(t^*)$. The second firm is the follower; it gets a payoff of $F(t^*)$. If the follower deviates by entering earlier at some time $\tau < t^*$, it will get a payoff of $L(\tau) < L(t^*) < F(t^*)$. If it deviates by entering at t^*, it will get a payoff of $(L(t^*) + F(t^*))/2$, which is less than $F(t^*)$. If the follower enters at $t > t^*$, there will be no change to the equilibrium outcome. Consequently, there is no profitable deviation for the follower.

If the leader deviates by entering earlier at some time $\tau < t^*$, it will get a payoff of $L(\tau) < L(t^*)$. If the leader deviates by entering later, it will get a smaller payoff because, as proved previously, t^* given by (4) delivers the highest possible payoff to the leader; see part C) of the proof.

(3) If $L(t^*) > F(t^*)$, an equilibrium with the leader entering at a positive time is not feasible. If this were the case, each firm would have an incentive to enter slightly earlier; consequently, the only possible equilibrium involves leader entry at $t^* = 0$. Note that in this case $t^* \notin A(0)$, meaning that $A(0) = \emptyset$.

Let us next consider the general case with $A(0) = \emptyset$. If $A(0) = \emptyset$, $L(0) > F(0)$. The strategies specified in the lemma result in both firms entering at $t^* = 0$. This generates a payoff of $(L(0) + F(0))/2$ for both firms. If either firm decides to enter later; it will get a payoff of $F(0)$, which is less than $(L(0) + F(0))/2$. Consequently, there is no profitable deviation for either firm and $t^* = 0$ is a unique SPE. This proves part D), and concludes the proof of the lemma. □

Proof of Lemma 2

Let us prove that if $x^* \in B$, x^* is an SPE. Given $L(t^*) = F(t^*)$, the strategies specified in the lemma result in both firms entering at t^*, generating a payoff of $(L(t^*) + F(t^*))/2 = L(t^*)$ for both firms. There are two cases to consider for possible profitable deviations. If either firm enters earlier at $\tau < t^*$ it will get a payoff of $L(\tau)$. From the definition of set B in (5), it follows that $L(\tau) < L(t^*)$. On the other hand, if either firms enters later it will get a payoff of $F(t^*)$, which is equal to $(L(t^*) + F(t^*))/2$. Consequently, there is no profitable deviation for either firm if $L(t^*) = F(t^*)$. This proves the lemma. □

Proof of Lemma 3

Let us prove that if $L(0) > F(0)$, then $t^* = 0$ is the leader's entry time in the SPE. Given $L(0) > F(0)$, the strategies specified in the lemma result in both firms entering at $t^* = 0$. This generates a payoff of $(L(0) + F(0))/2$ for both firms. If either firm decides to enter later it will get a payoff of $F(0)$, which is less than $(L(0) + F(0))/2$. Consequently, there is no profitable deviation for either firm from entering at $t^* = 0$ if $L(0) > F(0)$. The lemma is proved. □

Proof of Lemma 4

Let us prove that if $t^* \in A(0)$, $t^* \notin [B \cup \{0\}]$ and the equilibrium is the LSPE of a given region U_s, then this equilibrium is a second-mover advantage equilibrium of the entire game.

First, note that all possible equilibria with RE, when $L(t^*) = F(t^*)$, are covered by set B. Similarly, a possible first-mover advantage equilibrium, if $L(0) > F(0)$, is covered by $t^* = 0$. The only other possibility not already covered is when $F(t^*) > L(t^*)$.

Second, given $F(t^*) > L(t^*)$, one needs to consider deviations of the two firms separately. Without loss of generality, the first firm is the leader; it enters at t^* and gets a payoff of $L(t^*)$. The second firm is the follower; it gets a payoff of $F(t^*)$. Given $t^* \in A(0)$, if the follower deviates by entering at some time $\tau < t^*$, it will get a payoff of $L(\tau) < L(t^*) < F(t^*)$. If it deviates by entering at t^*, it will get a payoff of $(L(t^*) + F(t^*))/2$, which is less than $F(t^*)$. If the follower enters at $t > t^*$, there will be no change to the equilibrium outcome. Consequently, there is no profitable deviation for the follower.

Third, given $t^* \in A(0)$, if the leader deviates by entering earlier at some time $\tau < t^*$, it will get a payoff of $L(\tau) < L(t^*)$. If the leader deviates by entering later, it will get a smaller payoff because entering at t^* occurs in the LSPE for a given region U_s. Moreover, the strategies of both firms to enter whenever $L(t) \geq F(t)$ for $t > t^*$ ensure that entry cannot be postponed until after this region. This proves the lemma. □

Proof of Proposition 1

Let us prove that there is no other SPE with the leader entering at t^*, that is not characterized in Lemmas 2, 3 and 4. There are three cases to consider.
(1) The case $L(t^*) = F(t^*)$ is covered by Lemma 2. Both firms entering at t^* is an SPE only if condition $L(t^*) > L(\tau)\ \forall\ \tau \in [0, t^*)$ is satisfied. Otherwise firms will have an incentive to deviate by entering earlier.
(2) The case $L(t^*) > F(t^*)$ is covered by Lemma 3. Both firms entering at $t^* = 0$ is an SPE only if condition $L(0) > F(0)$ is satisfied. No other equilibria are possible in this case because in any candidate equilibrium with positive entry time both firms will have an incentive to deviate by entering earlier.
(3) The case with $L(t^*) < F(t^*)$ is covered by Lemma 4. The equilibrium is a second-mover advantage SPE only if $t^* \in A(0)$, $t^* \notin [B \cup \{0\}]$ and the equilibrium is the LSPE of a given region U_s. If $t^* \notin A(0)$, firms will have an incentive to deviate by entering earlier. If the equilibrium is not the LSPE of a given region U_s the leader will have an incentive to enter at a different time. Furthermore, the condition that $t^* \notin [B \cup \{0\}]$ guarantees that it is a second-mover advantage SPE. This completes the proof. □

Proof of Corollary 2

Let us prove that the equilibrium described in Corollary 2 is unique. First, the potential equilibrium detailed in Lemma 3 is explicitly ruled out as $F(0) \geq L(0)$. Second, because the non-increasing $F(t)$ and the non-decreasing envelope of $L(t)$ intersect only once (or more precisely, at one level), there can only be at maximum one SPE with rent equalization arising from set B (Lemma 2).

Third, the assumption about non-increasing $F(t)$ allows us to divide the time line into two zones: $t \leq T_1$ and $t > T_1$, where T_1 is defined as the earliest time where $F(t)$ and the envelope of $L(t)$ intersect. Importantly, there is no SPE with a leader entering in the second zone where $t^* > T_1$. The reason is that the follower at $t^* > T_1$ will receive a weakly smaller payoff than from entering as a leader at $t_1^* = \arg \max\limits_{\tau \in [0, T_1]} L(\tau)$, the time that maximizes $L(t)$ in the first zone.[19] In the first zone it will be the case that $L(t) \leq F(t)$. Consequently, there can only be at most one second-mover advantage equilibrium arising from this zone.

Now we show that there is a unique equilibrium, with either rent equalization or a second-mover advantage. There are two cases to consider:

(1) If there is an intersection of $L(t)$ and $F(t)$ such that the leader's payoff is at its historic maximum for the game up until that time, the equilibrium with rent equalization is the unique SPE. There is no equilibrium with earlier entry as leader's payoff is at its historic maximum. Later entry by the leader is not feasible, as already argued.

[19]Note that we are using the same notation T_1 and t_1^* as Hoppe and Lehmann-Grube (2005).

(2) If there is no intersection of $L(t)$ and $F(t)$ such that leader's payoff is at its historic maximum for the game up until that time, then there is no equilibria with rent equalization and the SPE with a second-mover advantage is the unique equilibrium.

Next let us prove that the time of entry given by (6) is the same as the time of entry given by (4). In the first zone with $t \leq T_1$, the criteria in (6) and (4) are equivalent to maximizing $L(t)$, while the second zone, where $t > T_1$, is not relevant (as discussed previously, entry will occur in the first zone). The corollary therefore is proved. $\square$

Proof of Lemma 5

In a similar manner to the proof of Lemma 1, this proof consists of four parts: A), B), C) and D). In part A) we show that all SPE must belong to $A(0) \cup C(0)$. In part B) we prove that there exists a unique t^* given by (9), at which $L(t)$ is maximized over $A(0) \cup C(0)$. Part C) shows that t^* delivers the highest possible equilibrium payoff to the leader, while part D) proves that t^* is an SPE.

A) As a preliminary step, let us prove all SPE must belong to $A(0) \cup C(0)$. Assume, on the contrary, that there is an SPE with entry time $t^* \notin [A(0) \cup C(0)]$. This requires us to consider two possible situations, one in which the candidate entry time occurs when payoffs are continuous and, secondly, when entry occurs at a point of discontinuity. In the case of continuous payoffs we apply the same arguments as outlined in the proof of Lemma 1. In the case of entry at a point of discontinuity, there two scenarios to consider. Firstly, if $L(t^*) \leq F(t^*)$, the condition that $L(t) > L(\tau),\ \forall\ \tau \in [0, t^*)$ must hold, otherwise the leader will prefer to enter earlier at τ. This means that $t^* \in A(0)$. Secondly, if $L(t^*) > F(t^*)$ then $(L(t) + F(t))/2 > L(\tau)\ \forall\ \tau \in [0, t)$ must hold to rule out possible preemption by the leader. This means that $t^* \in C(0)$. Consequently, t^* must belong to either $A(0)$ or $C(0)$.

B) Next, let us prove that there exists a unique t^* at which $L(t)$ is maximized over $A(0) \cup C(0)$, given by

$$t^* = \arg\max_t [A(0) \cup C(0)]. \tag{9}$$

Let us prove existence of the solution to this problem of maximizing $L(t)$ over $A(0) \cup C(0)$. Note that set $A(0)$ is bounded because t^{max} is finite, where t^{max} is the time t at which $L(t)$ reaches its global maximum (Assumption 4). In addition, set $C(0)$ is both closed and bounded because there is a finite number of discontinuities. If the supremum occurs at a point at which the payoffs are continuous, the arguments in Lemma 1 apply to show that set $A(0)$ always contains its supremum. When the supremum occurs at a point of discontinuity, we make use of Assumption 6 so as to ensure that set $A(0)$ contains its supremum. This proves existence.

The uniqueness follows immediately from the way set $A(0) \cup C(0)$ is constructed. If two points were to maximize $L(t)$ over $A(0) \cup C(0)$ then the one with the later time would not belong to $A(0) \cup C(0)$.

Next, using the same arguments as presented in the proof of Lemma 1,

$t^* = \arg\max_{t\in A(0)\cup C(0)} L(t) = \arg\max_t[A(0)\cup C(0)]$. This concludes the proof of part B).

C) Let us prove that t^* given by (9) delivers the highest possible payoff to the leader. Given that in A) we proved that all SPE must belong to $A(0)\cup C(0)$, this point follows immediately.

D) Now we show that the proposed equilibrium with t^* defined in (9) is an SPE. When $L(t^*) = F(t^*)$ and $L(t^*) < F(t^*)$, the same arguments utilized in the proof of Lemma 1 apply. Consequently, let us concentrate on the case when $L(t^*) > F(t^*)$. If this is true, an equilibrium with the leader entering at a positive time at which the payoffs are continuous is not feasible because each firm would have an incentive to enter slightly earlier. As a result, the only possible equilibrium involves joint entry at $t^* = 0$ or at points of discontinuity. Note that as $t^* \in C(0)$, entering at t^* generates a payoff of $(L(t^*) + F(t^*))/2$ for both firms, which dominates any payoff from entering earlier. Entering later leads to a payoff of $F(t^*)$, which is less than $(L(t^*)+F(t^*))/2$. Consequently, there is no profitable deviation for either firm and t^* is an SPE. This proves part D), and concludes the proof of the lemma. □

Proof of Lemma 6

The same argument can be applied as in the case of Lemma 2. Note that the fact that there are discontinuities does not affect the argument. The lemma therefore is proved. □

Proof of Lemma 7

Let us prove that if $t^* \in C(0)$, then t^* is an SPE. With $L(t^*) > F(t^*)$, the strategies specified in the lemma result in both firms entering at t^*, generating a payoff of $(L(t^*) + F(t^*))/2$ for both firms. If either firm enters earlier at $\tau < t^*$ it will get a payoff of $L(\tau)$. From the definition of set $C(0)$ in (8) it follows that $L(\tau) < L(t^*)$. On the other hand, if either firms decides to enter later it will get a payoff of $F(t^*)$, which is less than $(L(t^*) + F(t^*))/2$. Consequently, there is no profitable deviation for either firm if $L(t^*) > F(t^*)$. The lemma therefore is proved. □

Proof of Lemma 8

Let us prove that if $t^* \in A(0)$, $t^* \notin [B \cup C(0)]$ and the equilibrium is the LSPE of a given region U_s, that equilibrium is a second-mover advantage equilibrium of the entire game.

Note that all possible equilibria with RE, when $L(t^*) = F(t^*)$, are covered by set B. Similarly, all possible first-mover advantage equilibria, if $L(t^*) > F(t^*)$ are covered by $C(0)$. The only other possibility not already covered is when $F(t^*) > L(t^*)$.

The remaining arguments in Lemma 4, that there are no profitable deviations, apply in the case here despite the presence of discontinuities. This proves the lemma. □

Proof of Proposition 2

Let us prove that there is no other SPE with the leader entering at t^* not characterized in Lemmas 6, 7 and 8. The proof closely follows the arguments made in the proof of Proposition 1. There are three cases to consider.

(1) The case $L(t^*) = F(t^*)$ is covered by Lemma 6. Both firms entering at t^* is an SPE only if condition $L(t^*) > L(\tau)\ \forall\ \tau \in [0, t^*)$ is satisfied. Otherwise firms will have an incentive to deviate by entering earlier.

(2) Lemma 7 focuses on the case when $L(t^*) > F(t^*)$. Both firms entering at $t^* \in C(0)$ is an SPE only if conditions $L(t^*) > F(t^*)$ and $(L(t)+F(t))/2 > L(\tau)\ \forall\ \tau \in [0, t)$ are satisfied. No other equilibria are possible in this case, because both firms will have an incentive to deviate by entering earlier in any candidate equilibrium that involves entry at a time other than points of discontinuity.

(3) Lemma 8 covers the case when $L(t^*) < F(t^*)$. The equilibrium is a second-mover advantage SPE only if $t^* \in A(0)$, $t^* \notin [B \cup C(0)]$ and the equilibrium is the LSPE of a given region U_s. If $t^* \notin A(0)$, firms will have an incentive to deviate by entering earlier. If the equilibrium is not the LSPE of a given region U_s the leader will have an incentive to enter at a different time. Furthermore, the condition that $t^* \notin [B \cup C(0)]$ guarantees that it is a second-mover advantage SPE. This completes the proof. □

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