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Margining Option Portfolios by Network Flows

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Abstract

As shown in [Rudd and Schroeder, 1982], the problem of margining option portfolios where option spreads with two legs are used for offsetting can be solved in polynomial time by network flow algorithms. However, spreads with only two legs do not provide sufficient accuracy in measuring risk. Therefore, margining practice also employs spreads with three and four legs. A polynomial time solution to the extension of the problem where option spreads with three and four legs are also used for offsetting is not known. In this paper we propose a heuristic network flow algorithm for this extension and present a computational study that proves high efficiency of this algorithm in margining practice.

1 Introduction

In brokerage business, a *margin* is a collateral that the holder of a margin account has to deposit to cover the credit risk of his\her broker. Since the margin has to stay above a regulatory minimum, the *margining*, i.e., the calculation of minimum margin requirements for margin accounts is a critical intra-day and end-of-day risk management operation for any brokerage firm.

There exist two approaches to margining portfolios, strategy-based and risk-based. Recently, Coffman et al. [2010b] have published an experimental analysis of the two approaches to margining. Their results suggest that the risk-based approach, recently adopted in the US, has serious shortcomings. Specifically, it significantly undermargins stock option portfolios and does not provide any exit strategy. Coffman et al. conclude that strategy-based approach to margining is more appropriate for portfolios of stock options, although it is computationally and analytically more challenging and lacks in depth academic research. These conclusions call for a more detailed and comprehensive analysis of the strategy-based approach which we offer in this paper.

The margining of accounts in accordance with the strategy-based approach [Curley, 2008; Coffman et al., 2010a] is a computationally complex problem because bullish and bearish positions in the account can be combined in numerous ways to offset each other and hence reduce the margin requirement. This brings a nontrivial combinatorial component to the calculation. The problem is not well studied even for option portfolios, i.e., when a margin account consists of only positions in options on the same underlying instrument.

Despite the fact that margin regulations have a 75-year history dating from Regulation T in the Securities Act of 1934, the literature on margin calculations is surprisingly small. We can point to only two books [Geelan and Rittereiser, 1998; Curley, 2008], three papers [Rudd and Schroeder, 1982; Fiterman and Timkovsky, 2001; Coffman et al., 2010b] devoted to margining algorithms and two papers [Fortune, 2000, 2003] devoted to margining practice. Literature on studying the influence of margin requirements on the market, such as for example [Moore, 1966; Luckett, 1982], is more representative; see the related survey in [Kupiec, 1998]. The vast majority of publications on margining consists primarily of regulatory circulares and manuals written by security market lawyers.

Rudd and Schroeder [Rudd and Schroeder, 1982] discovered that the problem of margining option portfolios by offsets based on two-leg[1] option spreads, such as bull and bear spreads, can be solved in polynomial time. They have shown that this problem reduces to the *minimum-cost-flow network problem* [Ford and Fulkerson, 1962] that has fast polynomial algorithms [Goldberg, 1997; Heineman et al., 2008]. Offsets based on three- and four-leg option spreads, such as butterfly spreads, condor spreads and box spreads represent substantially more efficient hedging mechanisms for margin reductions. However, the complexity status of the problem with offsets based on option spreads with more than two legs remains unknown. Therefore, existing margin calculation technology, faced with the combinatorial complexity of margining option portfolios, failed to take advantage of three- and four-leg option spreads.

In this paper we show that if the set of two-leg option spreads is already chosen, by using for example the reduction of Rudd and Schroeder, then the problem of margining option portfolios by offsets based on three- and four-leg option spreads can be solved in polynomial time. The solution follows from a reduction similar to that found by Rudd and Schroeder. In addition, if the spreads are margined by the *maximum-loss margin rule*, then there exist a reduction to a simpler problem, the *maximum-flow network problem* [Ford and Fulkerson, 1962], that has faster polynomial algorithms [Cherkassky and Goldberg, 1997; Heineman et al., 2008].

The main result of this paper thus implies the following two-step method of margining option portfolios: (1) margin a given portfolio of options by the reduction of Rudd and Schroeder and create the related subportfolio of two-leg option spreads; (2) margin the subportfolio by the reduction proposed in this paper.

Although this method does not guarantee the exact minimum margin for all option portfolios, it presents a reasonable and handy heuristic that uses only well developed minimum-cost-flow and maximum-flow network algorithms, usually available in one software package. Besides, our method finds an exact solution in the case when the

[1] A leg of an option spread or an offset based on this spread is a position in options with the same exercise price and expiry date.

option portfolio is preliminary structured into basic two-leg option spreads. This preliminary structure is usually available because the vast majority of option portfolios in customer margin accounts are formed as a result of trading two-leg option spreads.

The rest of the paper is organized as follows. In Section 2 we give a motivational example showing what advantage an additional leg in option spreads can give. Section 3 presents different types of margin requirements and explains that only the market risk component of margin requirements can be minimized. Section 4 describes a vector model of option spreads. A minimum-cost-flow network model of margining a portfolio of basic two-leg option spreads by offsets involving three- and four-leg spreads is considered in Sections 5. Section 6 presents a simplified model, a maximum-flow network model, for the case of the maximum-loss margin rule. A computational study that proves that the two-step method is highly efficient in practice is presented in Section 7. Section 8 gives conclusions and directions for future research.

2 Why Legs Matter

In this section, we show that the advantage of using even three-leg spreads over two-leg spreads in margin calculations can be significant.

Let us consider a margin account which consists of a long position in one call option A, a long position in one call option B and a short position in two call options C. The options' exercise prices and market prices are, respectively,

$$\begin{aligned} \mathsf{Ae} &= \$70.00, & \mathsf{Ap} &= \$55.90,\\ \mathsf{Be} &= \$90.00, & \mathsf{Bp} &= \$40.90,\\ \mathsf{Ce} &= \$80.00, & \mathsf{Cp} &= \$50.60. \end{aligned}$$

Each of the options expires by the end of day, January 15, 2010, and has the contract size of $\mathsf{100}$ shares. The market price of the underlying stock[2] is $\mathsf{Up} = \$123.62$.

In what follows, it will be convenient to denote a long or short position in an option, say, C, as $+\mathsf{C}$ or $-\mathsf{C}$, respectively; so, a set of positions then can be written as a formal sum of the positions. Following the definitions form NYSE Rule 431(f)(2)(C), we can conclude that, since $\mathsf{Ae} < \mathsf{Ce} < \mathsf{Be}$ and $\mathsf{Ce} - \mathsf{Ae} = \mathsf{Be} - \mathsf{Ce}$, the account represents a long butterfly spread $\mathsf{A} + \mathsf{B} - 2\mathsf{C}$ whose components are spreads $\mathsf{A} - \mathsf{C}$ and $\mathsf{B} - \mathsf{C}$.

Next, we show that the *regulatory minimum initial* and *maintenance* margin requirements for this account are $\$1000$ less if considered as the long butterfly spread, which has three legs, in comparison with the case where it is considered as a consolidation of the two two-leg spreads.

Indeed, in accordance with NYSE Rule 431(f)(2)(G)(v)(1), the initial margin requirement for $\mathsf{A} + \mathsf{B} - 2\mathsf{C}$ is the total market price of A and B, i.e.,

$$100 \cdot (\mathsf{Ap} + \mathsf{Bp}) = 100 \cdot (\$55.90 + \$40.90) = \$9680.$$

[2] The data is taken from `http://finance.yahoo.com` as of the end of day, May 21, 2008, at NYSE for the symbol IBM.

NYSE Rule 431(f)(2)(G)(i) states that the initial margin requirement for a two-leg spread is the market value of the option in the long position plus the lesser of the initial margin requirement for the option in the short position and the spread out-of-the-money amount.

Since $\mathsf{Ae} < \mathsf{Ce}$, the spread $\mathsf{A} - \mathsf{C}$ is in-the-money, therefore its out-of-the-money amount is zero, and hence the initial margin requirement for $\mathsf{A} - \mathsf{C}$ is $100 \cdot \mathsf{Ap} = \$5590$.

Since $\mathsf{Be} > \mathsf{Ce}$, the spread $\mathsf{B} - \mathsf{C}$ is out-of-the-money, and its out-of-the-money amount, $100 \cdot (\mathsf{Be} - \mathsf{Ce})$, is \$1000. In accordance with NYSE Rule 431(f)(2)(D)(i), the initial margin requirement for $-\mathsf{C}$ is $100 \cdot (\mathsf{Cp} + \mathsf{Cm}) = 100 \cdot (\$50.60 + \$24.724)$, where

$$\begin{aligned} \mathsf{Cm} &= \max\{0.2 \cdot \mathsf{Up} - \mathsf{Co}, 0.1 \cdot \mathsf{Up}\} \\ &= \max\{0.2 \cdot \$123.62 - \$0.00, 0.1 \cdot \$123.62\} = \$24.724, \\ \mathsf{Co} &= \max\{\mathsf{Ce} - \mathsf{Up}, 0\} \\ &= \max\{\$80.00 - \$123.62, 0\} = \$0.00. \end{aligned}$$

Note that Co here is the out-of-the-money amount of the call option C. Thus, the initial margin requirement for $\mathsf{B} - \mathsf{C}$ is $100 \cdot \mathsf{Bp} + \$1000 = \$5090$. Therefore, if the account is considered as a consolidation of the two two-leg spreads, the initial margin requirement for it is $\$5590 + \$5090 = \$10680$, which is \$1000 more.

Deducting the total market value of the options in the long positions A and B in both cases, we obtain the maintenance margin requirements, i.e., \$0 for $\mathsf{A} + \mathsf{B} - 2\mathsf{C}$ and \$1000 for the consolidation of $\mathsf{A} - \mathsf{C}$ and $\mathsf{B} - \mathsf{C}$. Thus, we have the advantage of \$1000 in the maintenance margin requirement as well.

We have demonstrated the advantage of using spreads with three legs over two legs for offsetting for margining a simple account with three positions. It is clear, however, that the more legs such option spreads have and the larger the account the more advantage we can obtain on margin.

3 Types of Margin Requirements

In the above example we calculated regulatory minimum initial and maintenance margin requirements, which are based on the estimation of the *current loss*, i.e, the loss that the account holder would experience if the account was liquidated at the moment of the calculation. Such a calculation uses *current market prices*. Any margin charge below this minimum is illegal. In margining practice, however, brokers and brokerage houses are allowed to use so called *house margin rules* with more stringent margin requirements. Although these rules may vary, more stringent margin requirements for option spreads are usually based on the estimation of the *maximum loss*.

To explain the difference between current loss and maximum loss margin requirements for option spreads, let us recall that the current loss initial margin requirement for $\mathsf{B} - \mathsf{C}$, see Section 2, is $100 \cdot (\mathsf{Bp} + \mathsf{Rm})$, where

$$\mathsf{Rm} = \min\{\mathsf{Cm}, \max\{\mathsf{Be} - \mathsf{Ce}, 0\}.$$

This margin consists of the following two components: $100 \cdot \mathsf{Bp}$, the *premium margin requirement*, and $100 \cdot \mathsf{Rm}$, the *market risk margin requirement*. Note that the latter remains the same in the calculation of maintenance margin requirements.

It is clear that we obtain a more stringent margin requirement if we replace the market risk component by the spread out-of-the-money amount

$$\mathsf{Rmax} = \max\{\mathsf{Be} - \mathsf{Ce}, 0\}$$

since $\mathsf{Rmax} \geq \mathsf{Rm}$. The new market risk component, i.e., $100 \cdot \mathsf{Rmax}$, ignores the fact that, if the current market price of the underlying stock falls, the current loss on the option in the short position can be less than the spread out-of-the-money amount and associates the market risk only with the worst case scenario in which the spread is out of the money and exercised. Note that the current loss on the spread $\mathsf{B} - \mathsf{C}$ in Section 2 is also the maximum loss because $\mathsf{Rm} < \mathsf{Cm}$, and hence $\mathsf{Rm} = \mathsf{Rmax}$.

Regardless of what margin requirement we are interested in, initial or maintenance, current loss or maximum loss, for a portfolio of option spreads, its premium component remains invariant to offsetting spreads in the portfolio. It is either the total market value of the options in long positions if we consider the initial margin requirement, or zero if we consider the maintenance margin requirement. Only the market risk component can be reduced by offsetting.

Therefore, in what follows, we will be dealing with only the *market risk margin requirements*. So, any margin formula for margining option spreads or portfolios of option spreads we obtain will represent either the maintenance margin requirement or the market risk component of the initial margin requirement. The initial margin requirement can be easily obtained from its market risk component by adding the total market value of all options in the long positions of the portfolio. We will also call a market risk margin requirement simply *market risk* to be short.

4 Vector Model of Option Spreads

The model presented in this section follows the regulatory definitions related to option spreads from NYSE Rule 431(f)(2) that can be found at http://rules.nyse.com/nyse/.

Option spreads of dimension h can be formally defined as integer vectors

$$\mathbf{v} = (\; c_1 \quad c_2 \quad \ldots \quad c_h \quad p_1 \quad p_2 \quad \ldots \quad p_h \;)$$

whose components are associated with positions in options in a margin account as follows. c_j, $1 \leq j \leq h$, is the number of option contracts in the jth call option series, with the exercise price e_j. Similarly, p_j is the number of option contracts in the jth put option series, with the same exercise price e_j. A positive, negative or zero component means that the related leg is long, short or absent, respectively. A *zero spread* is a spread without legs.

The exercise prices are assumed to be all different and placed in the increasing order, i.e., $e_1 < e_2 < \ldots < e_h$. The set $\{e_1, e_2, \ldots, e_h\}$ is called an *exercise domain*. If

Table 1: Basic spreads

spread	spread name	calls				puts				legs	net
a	1st bull call	1	-1							2	dr
b	2nd bull call		1	-1						2	dr
c	3rd bull call			1	-1					2	dr
e	1st bull put					1	-1			2	cr
f	2nd bull put						1	-1		2	cr
g	3rd bull put							1	-1	2	cr
-a	1st bear call	-1	1							2	cr
-b	2nd bear call		-1	1						2	cr
-c	3rd bear call			-1	1					2	cr
-e	1st bear put					-1	1			2	dr
-f	2nd bear put						-1	1		2	dr
-g	3rd bear put							-1	1	2	dr

the exercise prices are separated by the same price interval, then its length is

$$e_2 - e_1 = e_3 - e_2 = \ldots = e_h - e_{h-1} = \mathsf{D} = \textit{an exercise differential},$$

and the exercise domain is called *uniform*. Not all spreads within the uniform exercise domaine are uniform. We will only refer to a spread **v** as uniform if the exercise differential between its consecutive legs is either D or 0.

Specifically, consider a uniform domain and let $v_1, v_2, \ldots, v_k$ be the sequence of legs of **v** such that $e_{v_1} \leq e_{v_2} \leq \ldots \leq e_{v_k}$ and $k > 1$. If $e_{v_{j+1}} - e_{v_j} = \mathsf{D}$ or 0 for all $j = 1, 2, \ldots, k-1$, then **v** is a *uniform* spread with the *exercise differential* D. Note that only uniform spreads are permitted for margining purposes [SEC, 2005], therefore we will consider further only uniform spreads. Simplest uniform spreads are basic and main spreads that can be defined as follows:

Definition 1 *A basic spread is a vector with two non-zero components,* 1 *and* -1 *such that (i) both non-zero components are on the same side, call or put; and (ii) non-zero components are consecutive.*

Definition 2 *A basic spread is a basic call spread if all non-zero components are on the call side, otherwise it is a basic put spread.*

Definition 3 *A basic spread is a basic bull spread if the first non-zero component is* 1*; otherwise it is a basic bear spread.*

Table 1 presents all basic spreads of a uniform domain of dimension 4. The abbreviations "dr" and "cr" mark *debit spreads* and *credit spreads*. They are called so because the net positions in these spreads are the results of net debit, respectively credit, trades, i.e. where the cost of the long options is more, respectively less, than the cost of the short options. Treating spreads as vectors we can add them, multiply by an integer scalar, cyclicly shift their components and take their *transpositions*, i.e., create the spreads $\bar{\mathbf{v}}$, where the components c_i and p_i are transposed for all $i = 1, 2, \ldots, h$.

Let a be a positive integer and $a > 1$. Then $a\mathbf{v}$ is a *multiple* of $\mathbf{v}$ and a is a *divisor* of $a\mathbf{v}$. A spread without divisors is *prime*.

Definition 4 *Let* $\mathbf{u}$ *and* $\mathbf{v}$ *be a bull spread and a bear spread respectively, and let* $\mathbf{u}+\mathbf{v}$ *be a uniform spread. Then* $\mathbf{u}+\mathbf{v}$ *is a three- or four-leg main spread.*

In what follows, we consider only main spreads and their multiples. Although our attention will be focused on the case of dimension four, all further results are valid for any dimension more than four. The set of all main spreads of dimension four is presented in Table 2. Note that the four is the minimum dimension that is required to model four-leg spreads.

All main spreads are well known except for *call iron condors*. In fact, the literature on the topic uses the term *iron condor* to denote a *put iron condor* (cf. [Cohen, 2005]) and does not mention call iron condors. Hence our model helps to discover two previously unknown spreads: *long* and *short call iron condors*. A long, respectively short, call iron condor is a combination of a bull, respectively bear, call spread and a higher exercise price bear, respectively bull put spread. The long call iron condor is a debit spread since it is a combination of two debit spreads. Similarly, the short call iron condor is a credit spread since it is a combination of two credit spreads. The maximum loss on a long call iron condor is the net debit; the maximum reward is two exercise differentials less the net debit. The maximum loss on a short call iron condor is two exercise differentials less net credit; the maximum reward is the net credit. Thus, call iron condors can be margined in the same way as known debit and credit spreads; cf. NYSE Rule 431.

It is not hard to verify that with the exception of the box spreads, three- and four-leg main spreads are long or short depending only on whether their leg with the lowest exercise price is long or short, respectively. The box spreads have both a long and a short leg with the lowest exercise price, therefore they cannot be classified in this way. We will say that the box spread is long if its call leg with the lowest exercise price is long and short otherwise.

Analogously, with the exception of the box spreads, three- and four-leg main spreads are call or put spreads depending only on whether their leg with the lowest exercise price is a call or a put leg, respectively. The box spreads have both a call and a put leg with the lowest exercise price, therefore they are call and put spreads simultaneously.

The described classification of main spreads has the following properties:

$$\begin{array}{rcl}
-\text{short call [name] spread} & = & \overline{\text{long put [name] spread}} \\
-\text{long call [name] spread} & = & \overline{\text{short put [name] spread}} \\
-\text{short put [name] spread} & = & \overline{\text{long call [name] spread}} \\
-\text{long put [name] spread} & = & \overline{\text{short call [name] spread}} \\
\text{debit/credit spread} & = & \overline{\text{credit/debit spread}} \\
-\text{debit/credit spread} & = & \text{credit/debit spread}
\end{array}$$

Table 2: Main spreads

spread	spread name	calls				puts				legs	net
a	1st bull call	1	-1							2	dr
b	2nd bull call		1	-1						2	dr
c	3rd bull call			1	-1					2	dr
e	1st bull put					1	-1			2	cr
f	2nd bull put						1	-1		2	cr
g	3rd bull put							1	-1	2	cr
-a	1st bear call	-1	1							2	cr
-b	2nd bear call		-1	1						2	cr
-c	3rd bear call			-1	1					2	cr
-e	1st bear put					-1	1			2	dr
-f	2nd bear put						-1	1		2	dr
-g	3rd bear put							-1	1	2	dr
a − b	1st long call butterfly	1	-2	1						3	dr
b − a	1st short call butterfly	-1	2	-1						3	cr
b − c	2nd long call butterfly		1	-2	1					3	dr
c − b	2nd short call butterfly		-1	2	-1					3	cr
e − f	1st long put butterfly					1	-2	1		3	cr
f − e	1st short put butterfly					-1	2	-1		3	dr
f − g	2nd long put butterfly						1	-2	1	3	cr
g − f	2nd short put butterfly						-1	2	-1	3	dr
a − c	long call condor	1	-1	-1	1					4	dr
c − a	short call condor	-1	1	1	-1					4	cr
e − g	long put condor					1	-1	-1	1	4	dr
g − e	short put condor					-1	1	1	-1	4	cr
a − f	1st long call iron butterfly	1	-1				-1	1		4	dr
f − a	1st short call iron butterfly	-1	1				1	-1		4	cr
b − g	2nd long call iron butterfly		1	-1				-1	1	4	dr
g − b	2nd short call iron butterfly		-1	1				1	-1	4	cr
e − b	1st long put iron butterfly		-1	1		1	-1			4	cr
b − e	1st short put iron butterfly		1	-1		-1	1			4	dr
f − c	2nd long put iron butterfly			-1	1		1	-1		4	cr
c − f	2nd short put iron butterfly			1	-1		-1	1		4	dr
a − e	1st long box	1	-1			-1	1			4	dr
e − a	1st short box	-1	1			1	-1			4	cr
b − f	2nd long box		1	-1			-1	1		4	dr
f − b	2nd short box		-1	1			1	-1		4	cr
c − g	3rd long box			1	-1			-1	1	4	dr
g − c	3rd short box			-1	1			1	-1	4	cr
e − c	long put iron condor			-1	1	1	-1			4	cr
c − e	short put iron condor			1	-1	-1	1			4	dr
a − g	long call iron condor	1	-1					-1	1	4	dr
g − a	short call iron condor	-1	1					1	-1	4	cr

Looking at Table 2 it is not hard to verify that the table of main spreads of dimension $h \geq 4$ contains $22h - 46$ spreads: $4h - 4$ basic spreads, $4h - 8$ butterfly spreads, $4h - 12$ condor spreads, $4h - 8$ iron butterfly spreads, $2h - 2$ box spreads and $4h - 12$ iron condor spreads. Thus, the number of main spreads of dimension five is 64.

The set of main spreads generates a bipartite graph on $4h - 4$ vertices with the vertex set $A \cup B$ and the edge set

$$A + B = \{\mathbf{u} + \mathbf{v} : \mathbf{u} \in A,\, \mathbf{v} \in B, \mathbf{u} + \mathbf{v} \text{ is a uniform spread}\},$$

where the parts A and B consist of the bear spreads and the bull spreads, respectively. Since $|A| = |B| = h - 1$, the bipartite graph is *balanced*. In the case of dimension four, i.e., where $h = 4$, we use the notation

$$A = \{-\mathbf{a}, -\mathbf{b}, -\mathbf{c}, -\mathbf{e}, -\mathbf{f}, -\mathbf{g}\} \quad \text{and} \quad B = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{e}, \mathbf{f}, \mathbf{g}\}.$$

It is easy to verify that if $h = 4$, then $\mathbf{u} + \mathbf{v}$ is a uniform spread if and only if $\mathbf{u} \neq -\mathbf{v}$. Therefore, if $h = 4$, then the bipartite graph of main spreads is a *crown graph* C_6 on 12 vertices; see Fig. 1.

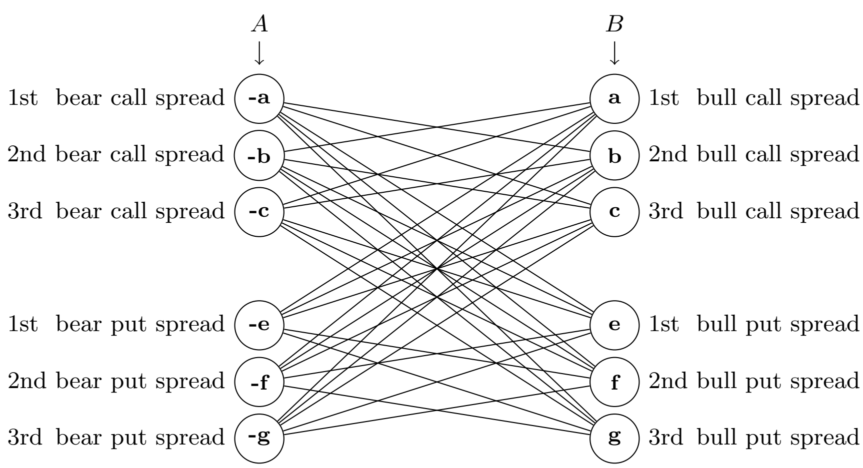

Figure 1: The crown graph C_6 of main spreads

5 Margining Portfolios of Basic Spreads

A *position in a spread* $\mathbf{x}$ is the pair $[\mathbf{x}, q(\mathbf{x})]$, where $q(\mathbf{x})$ is a nonnegative integer indicating how many spreads $\mathbf{x}$ are involved in the position. Thus, $q(\mathbf{x})$ is the largest divisor of the spread $q(\mathbf{x})\mathbf{x}$. A *portfolio of basic spreads* is a set of positions in basic spreads with the same underlying instrument and exercise differential.

To calculate a margin requirement for a portfolio of basic spreads we can find the total margin requirement for all the basic spreads in it. However, it will not be the minimum requirement because basic spreads in the portfolio offset each other in many ways providing different margin reductions. Thus, we have the problem of minimizing the margin requirement for a portfolio of basic spreads. Note that some of the basic spreads in the portfolio can have zero quantities. The crown graph on Fig. 1 whose vertices are marked by nonnegative integers is a graph model of the portfolio.

Let us calculate the margin requirements $m(\mathbf{v})$ for all basic spreads $\mathbf{v} \in A \cup B$, then $M = \sum_{\mathbf{v} \in A \cup B} m(\mathbf{v}) q(\mathbf{v})$ will be a regulatory margin requirement for the portfolio.

However, the requirement M can be reduced by *offsetting* $x(\mathbf{u}_1 + \mathbf{v}_1)$ units taken from a position in a bear spread $\mathbf{u}_1$ and the same number of units taken from a position in a bull spread $\mathbf{v}_1$ such that $\mathbf{u}_1 + \mathbf{v}_1 \in A + B$ and $0 \leq x(\mathbf{u}_1 + \mathbf{v}_1) \leq \min\{q(\mathbf{u}_1), q(\mathbf{v}_1)\}$. The result of this operation is an *offset* $O(\mathbf{u}_1 + \mathbf{v}_1)$, i.e., a position in the spread $\mathbf{u}_1 + \mathbf{v}_1$ with quantity $x(\mathbf{u}_1 + \mathbf{v}_1)$. If $m(\mathbf{u}_1) + m(\mathbf{v}_1) \geq m(\mathbf{u}_1 + \mathbf{v}_1)$, the offset $O(\mathbf{u}_1 + \mathbf{v}_1)$ reduces M by the amount $[m(\mathbf{u}_1) + m(\mathbf{v}_1) - m(\mathbf{u}_1 + \mathbf{v}_1)] \cdot x(\mathbf{u}_1 + \mathbf{v}_1)$.

Deducting quantity $x(\mathbf{u}_1 + \mathbf{v}_1)$ from $q(\mathbf{u}_1)$ and $q(\mathbf{v}_1)$ we can apply the same operation to the residual portfolio and create an offset $O(\mathbf{u}_2 + \mathbf{v}_2)$, etc. It is clear that there exist an integer $k \geq 1$ such that after the creation of an offset $O(\mathbf{u}_k + \mathbf{v}_k)$ the residual portfolio will not contain offsets because the uncovered position quantities, i.e., not covered by the crated k offsets, remain on only one side, bullish or bearish.

Since we allow zero quantities of the offsets, then we can assume that the spreads $\mathbf{u}_1 + \mathbf{v}_1, \mathbf{u}_2 + \mathbf{v}_2, \ldots, \mathbf{u}_k + \mathbf{v}_k$ are all spreads in $A + B$. Therefore, the problem of minimizing the margin requirement for a portfolio of basic spreads is equivalent to finding nonnegative integer quantities $x(\mathbf{u} + \mathbf{v})$ of spreads $\mathbf{u} + \mathbf{v}$ in $A + B$ to maximize the total margin reduction provided by the offsets with these quantities. This problem can be efficiently solved by a reduction to the following minimum-cost-flow network problem.

Definition 5 (the Hitchcock problem [Hitchcock, 1941; Ford and Fulkerson, 1962]) *Given a bipartite network with the demand part D and the supply part S, the set of edges E connecting D and S, demands $d(u)$ for all demand nodes $u \in D$, supplies $s(v)$ for all supply nodes $v \in S$ such that $\sum_{u \in D} d(u) = \sum_{v \in S} s(v)$ (balance condition), and costs $c(u, v)$ of running a unit of flow through the edges (u, v), find a minimum-cost flow through the edges to satisfy the demands by the available supplies.*

To show the reduction, we create the following network N: Introduce a dummy bear spread $-\mathbf{d}$ and a dummy bull spread $\mathbf{d}$ with margin requirements and quantities

$$m(-\mathbf{d}) = m(\mathbf{d}) = 0, \quad q(-\mathbf{d}) = \sum_{\mathbf{v} \in B} q(\mathbf{v}), \quad q(\mathbf{d}) = \sum_{\mathbf{u} \in A} q(\mathbf{u}),$$

respectively, set $D = A \cup \{-\mathbf{d}\}$, $S = B \cup \{\mathbf{d}\}$, $E = D + S$, see Fig. 2, and set

$$\begin{array}{rrll} & c(\mathbf{u}, \mathbf{v}) = m(\mathbf{u}, \mathbf{v}) & \text{for all} & \mathbf{u} + \mathbf{v} \in A + B, \\ d(\mathbf{u}) = q(\mathbf{u}), & c(\mathbf{u}, \mathbf{d}) = m(\mathbf{u}) & \text{for all} & \mathbf{u} \in D, \\ s(\mathbf{v}) = q(\mathbf{v}), & c(-\mathbf{d}, \mathbf{v}) = m(\mathbf{v}) & \text{for all} & \mathbf{v} \in S. \end{array}$$

It is easy to verify that the balance condition is observed. Note that the edges $(\mathbf{u}, \mathbf{d})$ for all $\mathbf{u} \in D$ and the edges $(-\mathbf{d}, \mathbf{v})$ for all $\mathbf{v} \in S$ present the dummy spreads $\mathbf{u} + \mathbf{d}$ and $\mathbf{v} - \mathbf{d}$, respectively. The edge $(-\mathbf{d}, \mathbf{d})$ presents a dummy zero spread.

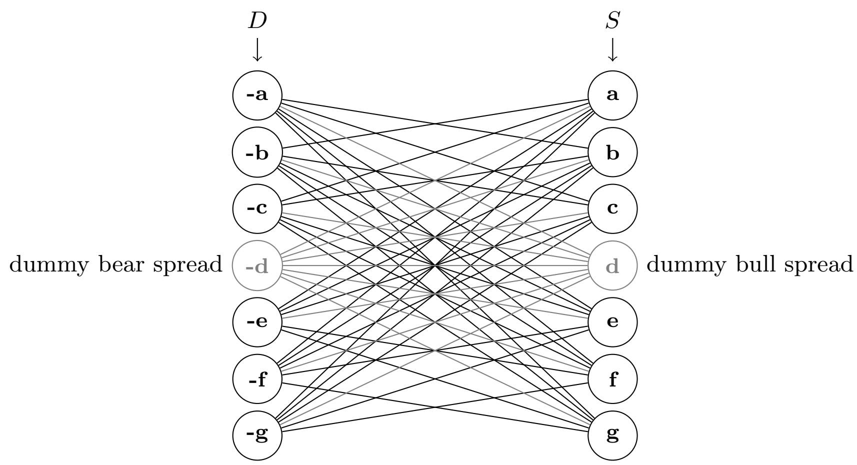

Figure 2: The network N of main spreads with two dummy spreads.

Feasible integer flows in the network N model offsets as follows. Let $\mathbf{u}+\mathbf{v} \in A+B$. Then the flow quantities through the edges $(\mathbf{u}, \mathbf{v})$ present the quantities $x(\mathbf{u}+\mathbf{v})$ of the offsets $O(\mathbf{u}+\mathbf{v})$; the flow quantities through the edges $(\mathbf{u}, \mathbf{d})$ and $(-\mathbf{d}, \mathbf{v})$ present the uncovered quantities of the bear spreads $\mathbf{u}$ and the bull spreads $\mathbf{v}$, respectively; the flow quantity through the edge $(-\mathbf{d}, \mathbf{d})$ presents the total offset quantity.

The cost of running a flow is exactly the margin requirement for the portfolio, hence the minimum margin requirement and related offsets can be found as a minimum cost flow in the network N. It is well know that a minimum cost flow is integer if all supplies and demands are integer[Ford and Fulkerson, 1962].

Note that the described reduction will turn into a modification of the reduction in [Rudd and Schroeder, 1982] if we interpret A as a set of bear positions in options (short calls and long puts), B as a set of bull positions in options (long calls and short puts), and $A+B$ as the the related set of two-leg option spreads.

The Goldberg-Tarjan algorithm [Goldberg and Tarjan, 1990] is one of the most efficient algorithms for solving the minimum-cost-flow network problem [Bünnagel et al., 1998]. For a network with n nodes and m edges, its theoretical time complexity is $O(nm\log(n^2/m))$. Since the network N has $O(h)$ nodes and $O(h)$ edges, the algorithm margins a portfolio of basic spreads in time $O(h^2 \log h)$.

6 Margining by the Maximum-Loss Margin Rule

It is important to observe that the reduction described in Section 5 solves the problem of margining portfolios of basic spreads for any margin rules for main spreads because the reduction works for any given margin requirements for main spreads. In this section we show that the problem can be solved more efficiently if main spreads are margined by the maximum-loss margin rule. This improvement becomes possible because this rule assigns the same margin requirement for all credit spreads that are useful for margin reductions. As before, the term "market risk" will mean the maximum-loss market risk margin requirement; see Section 3.

It is well known, cf. [McMillan, 2002], that debit spreads are free of market risk, i.e., they have no loss associated with underlying instrument price changes. Credit spreads, in contrast, are not free of market risk. The maximum loss on a credit spread associated with underlying instrument price changes is the exercise differential D in all cases except for a *short call iron butterfly* and a *short call iron condor* for which the maximum loss is $2\mathsf{D}$. Therefore, the market risk $m(\mathbf{x})$ for an option spread $\mathbf{x}$ is

$$m(\mathbf{x}) = \begin{cases} 0 & \text{if } \mathbf{x} \text{ is a debit spread,} \\ 2\mathsf{D} & \text{if } \mathbf{x} \text{ is a short call iron butterfly or} \\ & \qquad \text{a short call iron condor spread,} \\ \mathsf{D} & \text{otherwise.} \end{cases} \tag{1}$$

Note that a *short call iron butterfly* spread and a *short call iron condor* spread do not give any advantage in the market risk in comparison with the pairs of their basic components, and as such they are not used as trading strategies. Their "put" counterparts, however, are commonly used as trading strategies, and the word "put" in their names is usually omitted; see e.g. [Cohen, 2005].

In this section we use the following modification of the offsetting concept. An option spread an *offset* if its market risk is strictly lower than the total market risk of its components. Thus, a main spread $\mathbf{u}+\mathbf{v}$ is an offset only if $m(\mathbf{u})+m(\mathbf{v}) > m(\mathbf{u}+\mathbf{v})$. Among main spreads only offsets are advantageous for margin reductions.

As we mentioned in Section 4, a short call iron butterfly and a short call iron condor are not offsets because $m(\mathbf{u}+\mathbf{v}) = 2\mathsf{D}$ while $m(\mathbf{u}) = \mathsf{D}$ and $m(\mathbf{v}) = \mathsf{D}$. The following lemma generalizes this result.

Lemma 1 *Among main spreads only long butterfly, long condor, short box, long put iron butterfly and long put iron condor are offsets that reduce the market risk by* D*. The rest of the main spreads are not offsets.*

Proof Formula (1) and Table 2 imply that $m(\mathbf{u}) + m(\mathbf{v}) - m(\mathbf{u}+\mathbf{v}) = \mathsf{D}$ if $\mathbf{u}+\mathbf{v}$ is a long butterfly, long condor, short box, long put iron butterfly or long put iron condor spread; and 0 if $\mathbf{u}+\mathbf{v}$ is another main spread. □

Figure 3 depicts the *offset graph* in the case of dimension four, i.e., the subgraph of C_6 with only those edges that represent offsets. Note that it does not contain the 3rd bull call spread $\mathbf{c}$ and the 1st bear put spread $-\mathbf{e}$ because they are not components of offsets, therefore they can be margined separately.

Let $A' \subset A$, respectively $B' \subset B$, be the set of basic bear, respectively bull, spreads that can be used as components of offsets, O be the set of offsets, and let us calculate the total market risk of the spreads $\mathbf{x} \in A' \cup B'$, i.e.,

$$\mathsf{M} = \sum_{\mathbf{x} \in A' \cup B'} q(\mathbf{x}) m(\mathbf{x}).$$

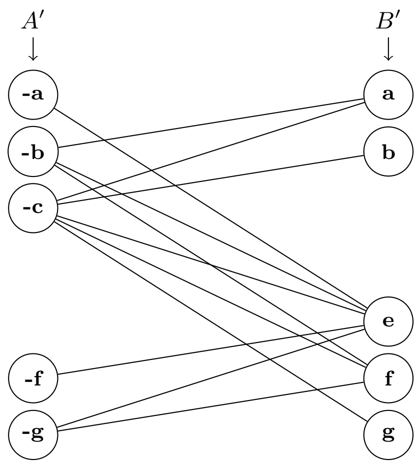

Figure 3: The offset graph.

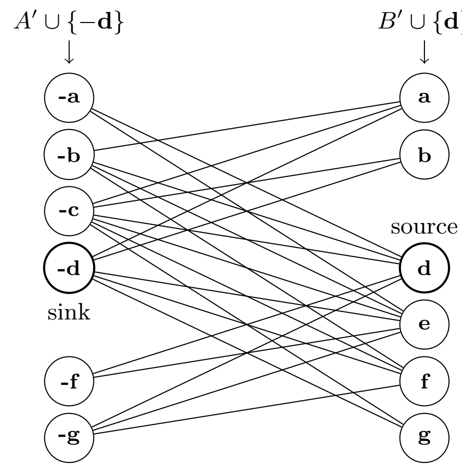

Figure 4: The offset network.

Note that $A' = A \setminus \{-\mathbf{e}\}$ and $B' = B \setminus \{\mathbf{c}\}$ in the case of dimension four.

It is clear that only credit spreads $\mathbf{x}$ with positive quantities $q(\mathbf{x})$ contribute to this sum because market risk of debit spreads is zero. The total market risk M overestimates the portfolio market risk if the portfolio has a bear spread $\mathbf{u}$ and a bull spread $\mathbf{v}$ such that $\mathbf{u} + \mathbf{v}$ is an offset. By Lemma 1, an offset reduces market risk by D. Therefore, if the portfolio has an offset $\mathbf{u} + \mathbf{v}$ its market risk is at most $\mathsf{M} - \mathsf{D}$.

Decreasing $q(\mathbf{u})$ and $q(\mathbf{v})$ by one we can apply the described above offsetting operation to the residual portfolio, choose the next offset, and show that the portfolio market risk is at most $\mathsf{M} - 2\mathsf{D}$, etc. It is clear that on a certain step the residual portfolio will not contain any offsets because the *uncovered quantities* of the positions, i.e., not covered by the created offsets, remain only on one side, bullish or bearish. Let us define the *total offset quantity* to be the number of offsets created during this procedure.

Theorem 1 *Let x be the total offset quantity for a portfolio of basic spreads with the total market risk* M *and the common exercise differential* D*. Then the market risk of this portfolio is* $\mathsf{M} - x\mathsf{D}$.

Proof Directly follows from Lemma 1 and the definition of the offsetting operation. □

Thus, the problem of minimizing the margin requirement for a portfolio of basic spreads by the maximum-loss margin rule is equivalent to the problem of maximizing the total offset quantity. The problem of maximizing the total offset quantity, in turn, can be reduced to the following network problem.

Definition 6 (the maximum-flow network problem [Ford and Fulkerson, 1962]) *Given a network with the set of nodes N, a* source *$s \in N$, a* sink *$t \in N$, $s \neq t$, and the set of edges E with* capacities *$c(e)$ of running a flow through $e \in E$, find a maximum flow from s to t.*

To show the reduction let us set $N = A' \cup B' \cup \{s, t\}$,

$$E = \{(\mathbf{u}, t) : \mathbf{u} \in A'\} \cup \{(s, \mathbf{v}) : \mathbf{v} \in B'\} \cup \{(\mathbf{u}, \mathbf{v}) : \mathbf{u} + \mathbf{v} \in O\},$$

$c(\mathbf{u}, t) = q(\mathbf{u})$ for all $\mathbf{u} \in A'$, $c(s, \mathbf{v}) = q(\mathbf{v})$ for all $\mathbf{v} \in B'$ and make the capacities $c(\mathbf{u}, \mathbf{v})$ unrestricted for all $\mathbf{u} + \mathbf{v} \in O$; see Fig. 4.

Feasible integer flows in this network can be interpreted as follows: The amount of flow through the edges $(\mathbf{u}, \mathbf{v})$ represents the quantities $x(\mathbf{u}+\mathbf{v})$ of the offsets $o(\mathbf{u}+\mathbf{v})$; the differences

$$q(\mathbf{u}) - \sum_{\mathbf{u}+\mathbf{v} \in O} x(\mathbf{u} + \mathbf{v}) \quad \text{and} \quad q(\mathbf{v}) - \sum_{\mathbf{u}+\mathbf{v} \in O} x(\mathbf{u} + \mathbf{v})$$

represent uncovered position quantities of the bear spread $\mathbf{u}$ and the bull spread $\mathbf{v}$, respectively; the flow from s to t represents the total offset quantity.

The Goldberg-Rao algorithm [Goldberg and Rao, 1998] finds a maximum-flow in a network with n nodes and m edges of maximum capacity U with the record theoretical time complexity $O(m \min\{n^{2/3}, \sqrt{m}\} \log(n^2/m) \log U)$. Since the offset network has $O(h)$ nodes and $O(h)$ edges whose maximum capacity U is the maximum position quantity $q_{\max}$, the algorithm margins a portfolio of basic spreads by the maximum-loss margin rule in time $O(h^{3/2} \log h \log q_{\max})$.

We can now combine the reductions to network flow problems presented in Sections 5 and 6 with the results of Rudd and Schroeder to propose the following two-step method of margining option portfolios:

Heuristic H

1. Margin a given portfolio of options by the reduction of Rudd and Schroeder and create the related subportfolio of two-leg option spreads;

2. Margin the subportfolio by the reduction proposed in this paper.

Although this method does not guarantee the exact minimum margin for all option portfolios, it presents a reasonable and handy heuristic that uses only well developed minimum-cost-flow and maximum-flow network algorithms, usually available in one software package. Besides, our method finds an exact solution in the case when the option portfolio is preliminary structured into basic two-leg option spreads. This preliminary structure is usually available because the vast majority of option portfolios in customer margin accounts are formed as a result of trading two-leg option spreads.

7 Computational Study

The goal of our computational study is to compare margin requirements produced by the two-step heuristic described above with the requirements produced by method of Rudd and Schroeder [1982] and an algorithm of dimension four proposed by Coffman et al. [2010b]. To achieve this, we compare the behaviour of the portfolio *maintenance* margin requirement as a function of the portfolio size that is calculated by all three methods in different scenarios. Note that the method of Rudd and Schroeder is an LP that uses offsets with 2 legs only, whereas the algorithm of dimension four of Coffman et al. [2010b] is an MIP that uses offsets with 2, 3, and 4 legs. All three methods were implemented using CPLEX.[3] As before, we follow NYSE Rule 431.

7.1 Design of the Experiments

The main idea of portfolio variation is to first build a *maximal portfolio* and then randomly remove positions to provide a monotonic reduction of its size. The margin requirement was computed for each generated portfolio by using the strategy-based algorithms of dimensions two and four, and the two-step algorithm. The portfolios were generated by performing the following steps:

Step 1. A group of 16 call options and a group of 16 put options were selected[4] such that exactly 8 options inside each group were in the money; see Table 3, where each row presents a call option and a put option with the same exercise price.

Step 2. The maximal portfolio with 32 positions was built by creating 8 long positions in randomly chosen 8 call options and 8 short positions in the remaining 8 call options; the other 16 positions in put options were created in the same way. This step was repeated 10 times and resulted in 10 unique maximal portfolios. The next steps were repeated for each maximal portfolio.

Step 3. The number of option contracts in each position was randomly generated in the range from 1 to 10. This step was repeated 50 times.

Step 4. A randomly selected position from one side (bearish or bullish) was removed to get a portfolio of the smaller size.

Step 5. The number of option contracts in each remaining position was randomly generated in the range from 1 to 10. This step was repeated 50 times.

Step 6. Steps 4-5 were repeated 29 times to get a total of 30 sets of 50 randomly generated portfolios with sizes monotonically decreasing from 32 to 3. The side from which a position was to be removed in Step 4 was alternated to maintain a balance between the number of bearish and bullish positions.

Step 7. Steps 3-6 were repeated 25 times alternating the starting side in Step 4. Each time the random number generator was restarted to avoid repeated patterns.

Step 8. Three margin requirements were calculated for each portfolio and averaged for portfolios of the same size. Hence, we calculated 30 averaged margin requirements for each algorithm.

Steps 1 through 8 create a *symmetric scenario* because position quantities chosen at Steps 3 and 5 are distributed uniformly between 1 and 10. The symmetric scenario

[3] We used ILOG CPLEX 12.1 on Dell Precision T7400 with two 3.5 GHz Quad-Core Intel Xeon CPUs, 32 GB RAM running Windows XP 64-bit.

[4] These 32 options were on the IBM stock at the market price of $84.92 and expired on April 17, 2009. The data was taken as of the end of the day of January 16, 2009, from `http://finance.yahoo.com/`. Note that we could have chosen any other stock with sufficient number of options.

generates *balanced portfolios* where the numbers of call and put options, in-the-money and out-of-the-money options, long and short positions are approximately the same. Thus, to obtain each point in the graph, we computed and averaged margin requirements for 12,500 portfolios, with the total of $12{,}500 \cdot 30 = 375{,}000$ unique and randomly generated balanced portfolios.

We also performed this experiment in six *asymmetric scenarios* to model *unbalanced portfolios* with different kinds of asymmetry. We performed the same steps as in the above algorithm except for Steps 3 and 5, where the quantities of options in the positions were ranging according to the following three scenarios, where quantities A and B were random integers in the intervals $[7, 10]$ and $[1, 4]$, respectively:

Long Portfolio : $A \backslash B$ option contracts for each long\short, position;
Call Portfolio : $A \backslash B$ option contracts for each position in call\put options;
Bull Portfolio : $A \backslash B$ option contracts for each bullish\bearish, position.

The other three asymmetric scenarios, Short Portfolio, Put Portfolio, Bear Portfolio, respectively, were obtained by transposing A and B in the above three definitions.[5]

#	ex price	call price	put price
1	45	39.70	0.45
2	50	35.50	0.67
3	55	31.90	1.00
4	60	25.30	1.45
5	65	21.50	1.90
6	70	17.30	2.70
7	75	13.50	3.90
8	80	10.10	5.34
9	85	7.10	7.38
10	90	4.63	10.00
11	95	2.85	14.83
12	100	1.75	17.02
13	105	0.95	21.50
14	110	0.50	26.03
15	115	0.20	28.40
16	120	0.15	32.90

Table 3: Selected options and their prices

7.2 Results of the Experiment

The results of the experiment are presented in Figs. 5 through 11, where margin requirements are given in thousands of dollars for portfolio sizes $3, 4, 5, \ldots, 31, 32$. The

[5] Recall that long positions in call options and short positions in put options are bullish, long positions in put options and short positions in call options are bearish.

	balanced	long	short	call	put	bull	bear
average	70.23	61.04	73.10	71.50	77.50	78.52	61.63
min	36.41	0.00	8.40	31.51	62.04	58.33	1.20
max	77.43	83.63	83.82	76.94	82.07	100.00	75.42

Table 4: The average, minimum and maximum relative error provided by two-step heuristic H for different scenarios for all portfolios (in %). Margin computed using S4 was used as a baseline and S2-S4 as the maximum error (100%).

	balanced	long	short	call	put	bull	bear
average	56.32	37.18	50.01	61.88	70.35	73.70	37.53
min	36.41	0.00	8.40	31.51	62.04	58.33	1.20
max	67.32	57.62	70.69	71.03	76.16	100.00	59.25

Table 5: The average, minimum and maximum relative error provided by heuristic H for different scenarios for portfolios with sizes ≤ 10 (in %). Margin computed using S4 was used as a baseline and S2-S4 as the maximum error (100%).

notation S2, S4, H stands for margin requirements obtained by the strategy-based algorithms of dimensions two [Rudd and Schroeder, 1982], four [Coffman et al., 2010b], and heuristic H, respectively.

To compare the performance of heuristic H to the other two algorithms we have computed the relative error: (H-S4)/(S2-S4). Our main conclusion is that the two-step heuristic H performs well in all scenarios. It allows us to improve the results obtained by the algorithm of Rudd and Schroeder by 21-39% on average, depending on the scenario (see Table 4). Heuristic H is especially effective in the case of long and bear portfolios. The average relative error in this cases is 61.4% and 61.63% respectively.

The majority of portfolios have 10 or less positions. For such portfolios heuristic H performs even better. It has a relative average error of 56% or less for all scenarios except call, put, and bull (see Table 5).

8 Conclusions and Future Research

In this paper we have proposed a vector model of option spreads that allows for a full characterization of main option spreads. We have also proposed a reduction of the minimum margin requirement for a portfolio of basic option spreads to a minimum-cost-flow network problem and hence, shown that it can be efficiently solved in polynomial time by minimum-cost-flow algorithms. In the case of margining option spreads by the maximum-loss margin rule there exist an even more efficient model, a maximum-flow network problem, that has even faster algorithms. These results have strong practical implications. They allow to upgrade margin calculation systems to not only solve the

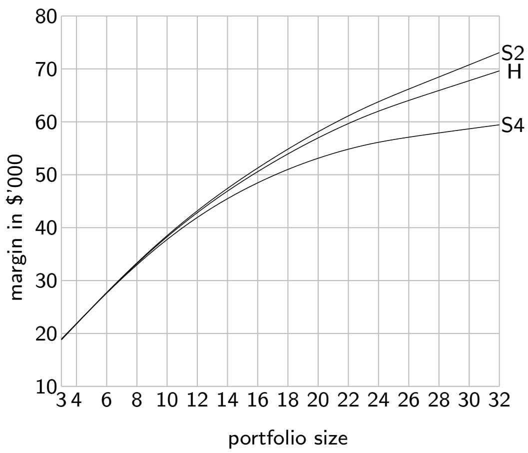

Figure 5: Balanced Portfolios

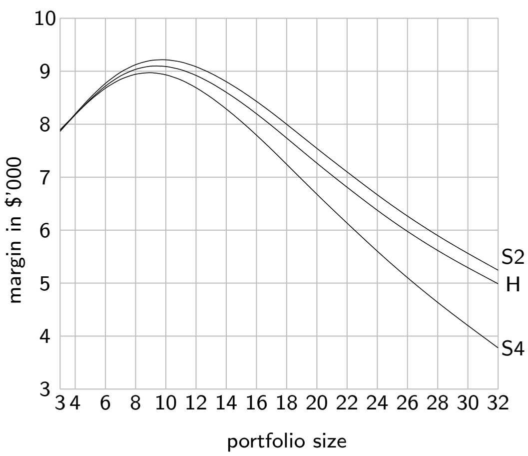

Figure 6: Long Portfolios

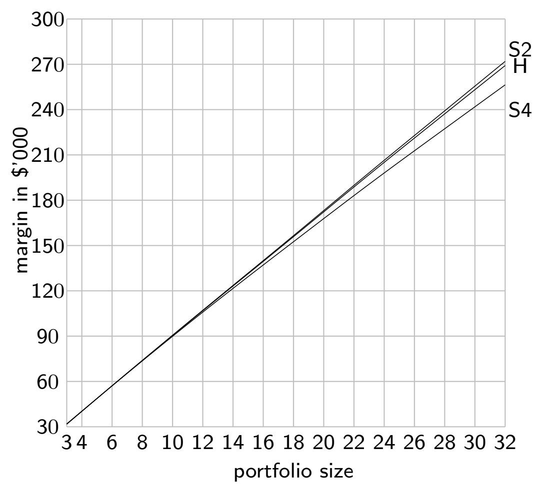

Figure 7: Short Portfolios

problem of margining option portfolios in polynomial time but to solve it fast using network flow algorithms that have efficient implementations.

We believe that the concept of uniform option spreads came into margining practice from the desire to simplify margin rules for credit spreads. Margining of non-uniform option spreads is more complex because it involves the current price of the underlying instrument. Nevertheless, our model also works for non-uniform option spreads because margin requirements for option spreads are external parameters.

Our vector model of option spreads suggests that together with main spreads there exist other spreads with a more complex structure. Our future research will be devoted to the study of complex option spreads that have more than four legs.

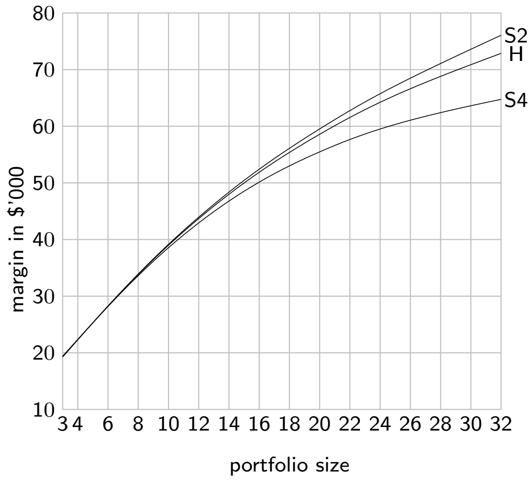

Figure 8: Call Portfolios

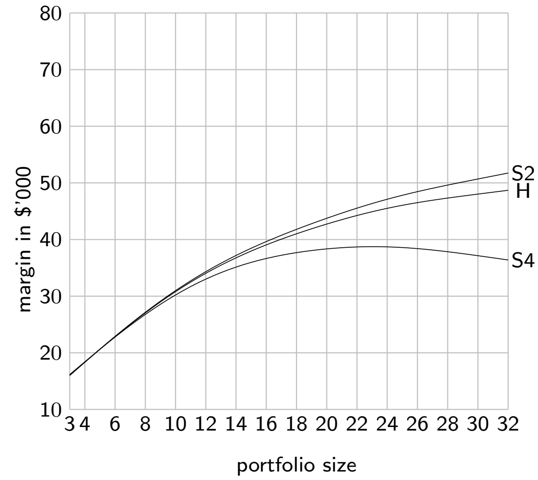

Figure 9: Put Portfolios

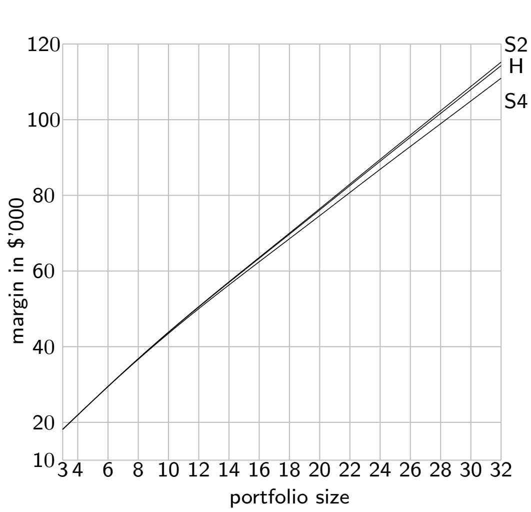

Figure 10: Bull Portfolios

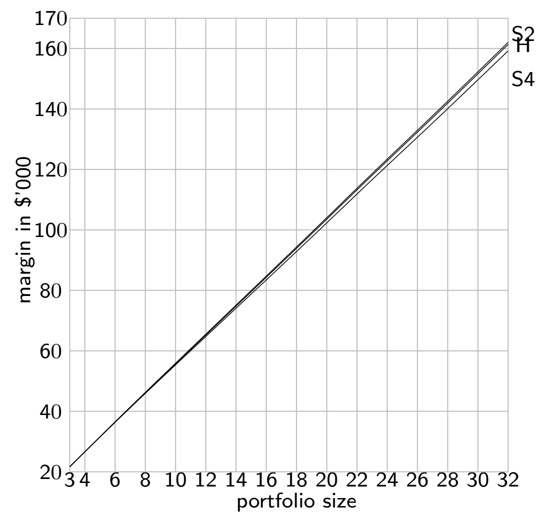

Figure 11: Bear Portfolios

References

Bünnagel, U., Korte, B., and Vygen, J. (1998). Efficient emplementation of the Goldberg-Tarjan minimum-cost flow algorithm. *Optim. Methods Softw.*, 10(2):157–174.

Cherkassky, B. V. and Goldberg, A. V. (1997). On implementing the push-relabel method for the maximum flow problem. *Algorithmica*, 19:390–410.

Coffman, E. G., J., Matsypura, D., and Timkovsky, V. G. (2010a). A computational study of margining portfolio of options by two approaches. In Prasad, S. L., Vin, H. M., Sahni, S., Jaiswal, M. P., and Thipakorn, B., editors, *Information Systems, Technology and Management*, volume 54 of *Communications in Computer and Information Science*, pages 325–332. Springer-Verlag, Berlin, Heidelberg, 1 edition.

Coffman, E. G., J., Matsypura, D., and Timkovsky, V. G. (2010b). Strategy vs risk in margining portfolios of options. *4OR: A Quarterly Journal of Operations Research*, Online First:1–12.

Cohen, G. (2005). *The bible of options strategies: the definitive guide for practical trading strategies*. Financial Times Prentice Hall. Pearson, New Jersey.

Curley, M. T. (2008). *Margin trading from A to Z: A complete guide to borrowing, investing and regulation*. John Wiley & Sons, New Jersey.

Fiterman, A. E. and Timkovsky, V. G. (2001). Basket problems in margin calculations: modelling and algorithms. *European J. Oper. Res.*, 129:209–223.

Ford, L. R. and Fulkerson, D. R. (1962). *Flows in networks*. Princeton Univ. Press.

Fortune, P. (2000). Margin requirements, margin loans, and margin rates: practice and principles. *New England Econ. Rev.*, pages 19–44.

Fortune, P. (2003). Margin requirements across equity-related instruments: how level is the playing field? *New England Econ. Rev.*, pages 29–50.

Geelan, J. P. and Rittereiser, R. P. (1998). *Margin regulations and practices*. NYIF Corporation, Division of Simon & Schuster, Inc., New York, 4 edition.

Goldberg, A. V. (1997). Implementation of a scaling minimum-cost flow algorithm. *J. Algorithms*, 22:1–29.

Goldberg, A. V. and Rao, S. (1998). Beyond the flow decomposition barrier. *J. ACM*, 45:753–782.

Goldberg, A. V. and Tarjan, R. E. (1990). Finding minimum-cost circulations by successive approximation. *Math. Oper. Res.*, 15:430–466.

Heineman, G. T., Pollice, G., and S., S. (2008). *Algorithms in a nutshell*, chapter 8: Network flow algorithms, pages 226–250. O'Reily Media.

Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. *J. Math. Phys. Camb.*, 20:224–230.

Kupiec, P. H. (1998). Margin requirements, volatility and market integrity: what have we learned since the crash? *J. Financ. Serv. Res.*, 13(3):231–255.

Luckett, D. G. (1982). On the effectiveness of the federal reserve's margin requirements. *J. Financ.*, 37(3):783–795.

McMillan, L. G. (2002). *Options as a strategic investment.* Prentice Hall, 4 edition.

Moore, T. G. (1966). Stock market margin requirements. *J. Polit. Economy*, 74(2):158–167.

Rudd, A. and Schroeder, M. (1982). The calculation of minimum margin. *Manage. Sci.*, 28(12):1368–1379.

SEC (2005). Release 34-52738.