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E.G. Coffman, JR
Columbia University

D. Matsypura
Faculty of Economics and Business
The University of Sydney

V.G. Timkovsky
Faculty of Economics and Business
The University of Sydney

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Portfolio Margining: Strategy vs Risk[*]

E. G. Coffman, Jr.[†] D. Matsypura[‡] V. G. Timkovsky[‡]

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Abstract

This paper presents the results of a novel mathematical and experimental analysis of two approaches to margining customer accounts, strategy-based and risk-based. Building combinatorial models of hedging mechanisms of these approaches, we show that the strategy-based approach is, at this point, the most appropriate one for margining security portfolios in customer margin accounts, while the risk-based approach can work efficiently for margining only index portfolios in customer margin accounts and inventory portfolios of brokers. We also show that the application of the risk-based approach to security portfolios in customer margin accounts is very risky and can result in the pyramid of debt in the bullish market and the pyramid of loss in the bearish market. The results of this paper support the thesis that the use of the risk-based approach to margining customer accounts with positions in stocks and stock options since April 2007 influenced and triggered the U.S. stock market crash in October 2008. We also provide recommendations on ways to set appropriate margin requirements to help avoid such failures in the future.

1 Introduction

> "We still have a 1930s regulatory system in place. We've got to update our institutions, our regulatory frameworks, . . . ". . . the banking system has been "dealt a heavy blow," the result of "lax regulation, massive overleverage, huge systematic risks taken by unregulated institutions, as well as regulated institutions." – *Barack Obama*[1]

Calculating margin requirements for *margin accounts*[2] is a critical intra-day and end-of-day risk management operation for any brokerage firm, clearing house or securities trader. In the margin accounts of investors, i.e., *customers* of brokers, margin payments are based on established minimum *margin requirements* which are subject of *margin regulations.* Margin rules exist for margining single positions, *offsets*[3] like trading *strategies* or larger groups of positions like *portfolios* comprising all positions with a common *underlying instrument*,[4] and entire accounts which are collections of portfolios.

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[†] Columbia University, NY 10027, USA, `coffman@cs.columbia.edu`.

[‡] University of Sydney, NSW 2006, Australia, `{dmytro.matsypura,vadim.timkovsky}@sydney.edu.au`.

Deducting the minimum margin requirement for an account from its market value we obtain its *loan value*, which is the maximum portion of the account's market value that the broker can lend. The amount that is actually lent to a customer by the broker constitute the *margin credit* given for all customer's margin accounts. The total margin credit provided by all brokers in a market constitutes the market's *margin debt*.

With adjustments in minimum margin requirements (or the maximum credit requirements), margin regulators keep margin debt at a level consistent with the canons of a healthy economy. High margin requirements can reduce investors' activity, lead to underpricing of securities, and cause economic slowdowns. Low margin requirements, in turn, lead to overpricing of securities, high levels of speculation, cash deficits, market crashes, and, again, economic slowdowns. The challenge is to find a proper approach to portfolio margining, a "golden mean" that keeps the growth of margin debt within tolerable limits. Such an approach is specified by the definition of offsets and *margin rates*, i.e., margin requirements per unit of security and per unit of offset.

Current margining practice uses two approaches, *strategy-based* and *risk-based*. The strategy-based approach uses offsets that imitate trading strategies while the risk-based approach considers the entire portfolio to be an offset. The strategy-based approach uses margin rates calculated without variations of the current market prices of the underlying instruments. In contrast, the risk-based approach uses margin rates calculated by artificial variations of the current market prices with the purpose to find a worst-case scenario in their moves. The goal of this paper is a comparative analysis of the approaches that, to the best of our knowledge, has never been done before.

1.1 State of the Art: Strategy

The strategy-based approach to margining customer accounts with equity derivatives such as stock or index options, futures and warrants have been used in the brokerage industry for more than four decades. By the end of the nineties, it was commonly recognized that this approach yields excessively high margin requirements.

The main reason for the high margin requirements obtained by the strategy-based approach is, as shown in Section 3, that the calculation of the minimum margin by using offsets with more than two *legs*[5] is an intractable combinatorial problem that is not well understood. Despite the fact that margin regulations have a 75-year history dating from Regulation T in the Securities Act of 1934, the literature on margin calculations is surprisingly small.[6] We can point to only two books [Geelan and Rittereiser, 1998; Curley, 2008] and two papers [Fortune, 2000, 2003] devoted to margining practice, two papers [Moore, 1966; Luckett, 1982] studying the influence of margin requirements on investor's equity ratio, and two papers [Rudd and Schroeder, 1982; Fiterman and Timkovsky, 2001] devoted to margining algorithms. The vast majority of publications on margining consists primarily of regulatory circulares and manuals. Margin calculations have never attracted the attention of mathematicians or computer scientists because they have been traditionally considered a prerogative of brokerage accounting, something that is enforced and interpreted by security market lawyers.

Consequently, margin calculation systems, developed and used in the brokerage industry up to 2005, ignore highly effective and broadly applicable discrete optimization methods, such as integer and dynamic programming. In particular, the reduction of the margin-minimization-by-pairing problem to the minimum-cost network-flow problem [Rudd and Schroeder, 1982] was seemingly forgotten for more than 20 years.

As a result, existing margin calculation technology, faced with the combinatorial complexity of the strategy-based approach, failed to take advantage of complex option spreads. The vast majority of margin calculation systems used in the brokerage industry, as our study shows, uses offsets with two legs only; and they are based on outdated heuristics proposed by brokers in the mid seventies [Cox and Rubinsein, 1985; Geelan and Rittereiser, 1998]. The most advanced margining systems recognize offsets with up to four legs by using heuristics that cannot guarantee the minimum margin. But the failure to use offsets with more that two legs, as we show in Section 5, can increase the margin requirement from zero to several thousands of dollars.

One of the goals of this paper is to rehabilitate the strategy-based approach and to show that it enables the calculation of much more appropriate margin requirements for security portfolios in comparison with those calculated by the risk-based approach.

1.2 The Regulatory Breakthrough of 2005: Strategy

High margin requirements obtained by the strategy-based approach can also be partially explained by the fact that margin rules prior to 2005 permitted the use of offsets with only few legs. However, the more legs an offset has the more margin reduction it gives. Thus, the reduction of minimum margin requirements can be achieved by designing new offsets with larger number of legs.

Offsets with two, three and four legs, imitating options trading strategies, such as calendar spreads, straddles, butterfly spreads, condor spreads, collars and box spreads were known and permitted in margin regulations since the mid seventies. Offsets with more than four legs, a very efficient means of achieving adequate margin reductions, did not appear until 30 years later in 2005.

Specifically, in August 2003 the CBOE[7] proposed new margin rules for offsets with 5, 6, 7, 8, 9, 10 and 12 legs that were called "complex option spreads."[8] After two revisions of this document,[9] the SEC[10] approved these rules[11] and added them to NYSE[12] Rule 431 in December 14, 2005. In August 2007, these rules were also recognized in Canada.[13]

The regulatory move of 2005 was a very important step in the development of the strategy-based approach because it demonstrated that four legs is not the final step. Moreover, as shown in [Matsypura et al., 2007], 12 legs is also not the final step, because there exists a method of designing offsets with any desired number of legs. In particular, it was shown that there exist offsets with a maximum of 134 legs if the number of different exercise prices of the options involved is at most four.

1.3 The Pilot Program of 2005–2008: Risk

The risk-based approach[14] was proposed in 1989 by the OCC[15] to calculate net capital requirements for brokers' proprietary portfolios of listed options.[16] It was implemented in 1996 in TIMS[17] and approved by the SEC[18] to be effective as of September 1, 1997. However, the approach was not used for margining customer accounts prior to 2005.[19]

Employing lower margin rates and larger offsets in contrast to the strategy-based approach, the risk-based approach produces substantially lower initial margin requirements. In the examples provided by the CBOE, the requirements for naked options and basic option spreads are at least two or three times lower.[20] (72 times lower for a long straddle!) After two NYSE proposals,[21] the SEC approved the use of the risk-based approach to margining customer accounts under a temporary *pilot program*.[22]

The pilot program can be divided into three phases: Phase I started on July 14, 2005 and permitted the use of the risk-based approach to margin accounts with only listed BBI[23] and ETF[24] derivatives such as options, warrants, futures, and future options. Phase II started on July 11, 2006 and included listed stock options and securities futures.[25] Phase III started on April 2, 2007 and included equities, equity options, unlisted derivatives and NBI[26] futures.[27] The pilot program was to expire on July 31, 2007. However, on July 19, 2007 it was extended for one more year, and the risk-based approach was finally approved to be used permanently on July 30, 2008.[28]

Monthly margin debt reports published by the NYSE provide clear evidence of the influence of the pilot program on the stock market; see Fig. 1. During the initial 20-month period of the pilot program (July 2005 - April 2007), the margin debt had increased by \$82.22 billion compared to the \$38.80 billion increase during the previous 20 months. Although the beginning of this period, between the starting points of Phases I and II, looks ordinary, the ending of this period, between the starting points of Phases II and III, is remarkable owing to the unusually high rate of increase in the margin debt and the trading volume volatility of the S&P 500 index.

During the subsequent four-month period (April 2007 - July 2007), the period when equities, equity options, unlisted derivatives and NBI futures joined the pilot program, the margin debt increased by another \$88.21 billion, i.e., at a rate at least five times higher. Thus, since April 2, 2007, the margin debt increased at a rate of more than \$22.05 billion per month. Historical records show that a fast growth of margin debt can be a sign of an approaching market crash. Such was the case in October 1929 and October 1987. Examining the two most recent examples, consider the assessments of the market crash of October 2000 in [Fleckenstein and Sheehan, 2008] (p. 87):

> "As of February 2000, total margin debt stood at \$265 billion. It had grown 45 percent since the previous October and had more than tripled since the end of 1995. Relative to GDP,[29] margin debt was the highest it had been since 1929, and over three times as high as it was in October 1987. It was an unmistakable sign of rampant speculation."

Bringing these assertions more up-to-date, we observe that, as of July 2007, total margin debt stood at \$381 billion. It had grown 30 percent since the previous March and had almost tripled since the end of 2002. Relative to GDP, margin debt was the highest it had been since February 2000; see Fig. 2.

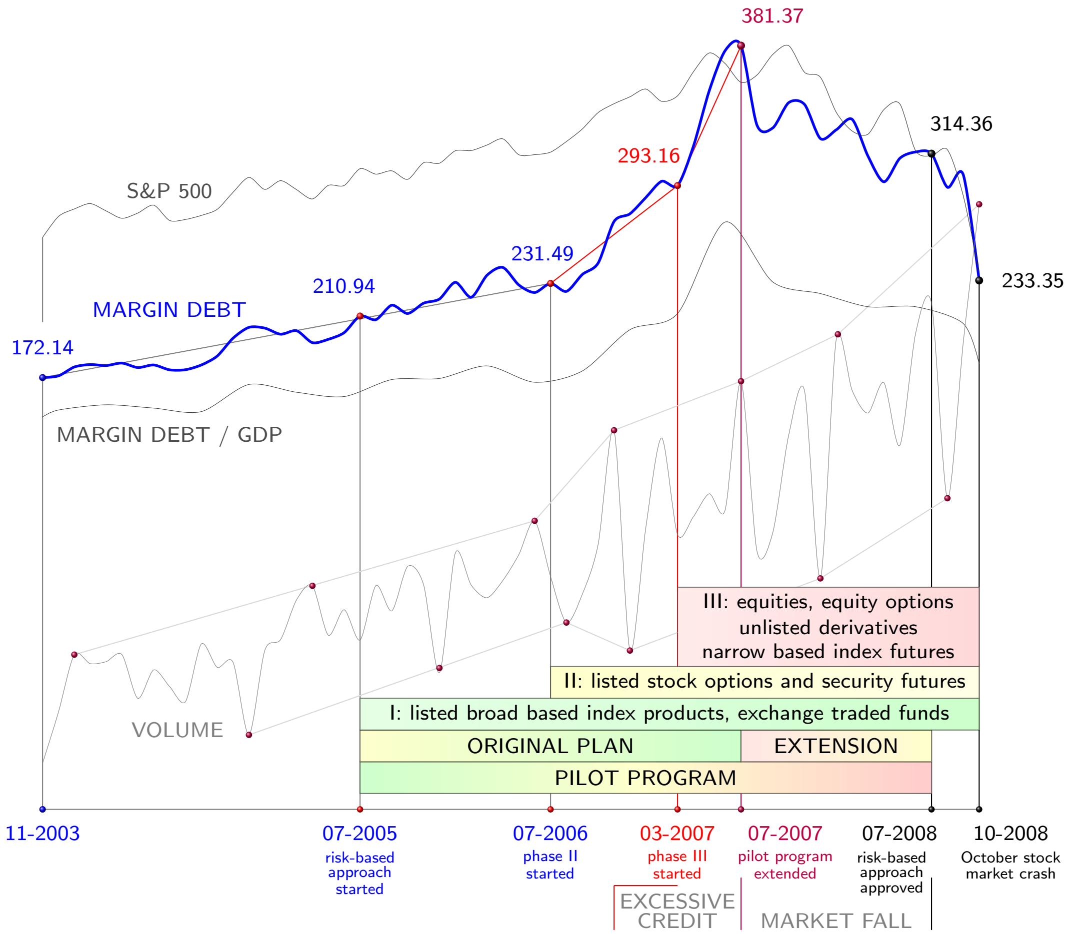

Figure 1: The margin debt in billions of dollars, the margin debt relative to GDP, and the level and trading volume of the S&P 500 index in the period from November 2003 through October 2008. The curves are scaled to fit on one graph and drawn by smoothing the end-of-the-month data from `http://www.nyse.com/` and `http://finance.yahoo.com/`.

Figures 1 and 2 clearly show that the market credit in the period from April through July 2007 was *excessive* in the extreme: 27% increase in Margin Debt/GDP. Even though the margin debt reached the level of $381.37 billion by the end of July 2007, the pilot program had, nevertheless, been extended for an additional year. In September 2007, it was clear that the growth in margin debt had lessened because in August 2007 it fell to $331.37 billion as a result, in particular, of numerous margin calls received by investors and associated forced sales from their undermargined accounts.[30] By July 2008, the time of the final approval of the risk-based approach, the margin debt had plunged to $314.36 billion, signalling an approaching stock market downfall. In September 2008 the margin debt had another plunge from $299.96 to $233.35 billion, and the stock market downfall was evident as indicated by the level of S&P 500.

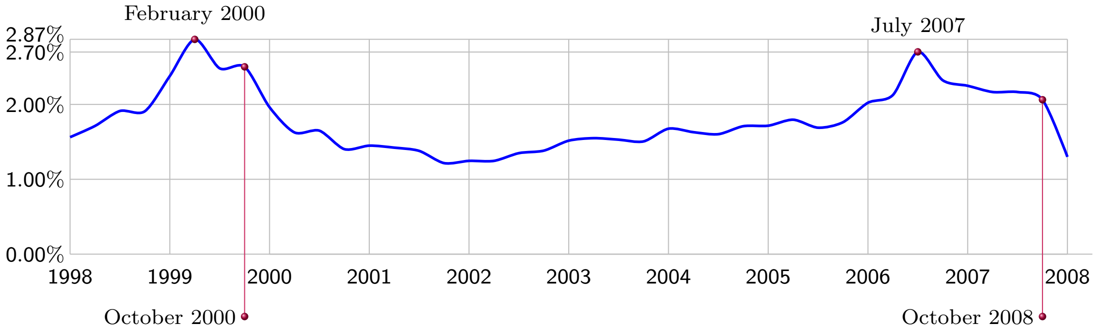

Figure 2: Margin Debt/GDP in the period from the end of 1998 through the end of 2008. Data from `http://www.nyse.com/` and `http://finance.yahoo.com/`

> "The task of the Board, as I see it, is to formulate regulations with two principal objectives. One is to permit adequate access to credit facilities for security markets to perform their basic economic functions. The other is to prevent the use of stock market credit from becoming excessive. The latter helps to minimize the danger of pyramiding credit in a rising market and also reduces the danger of forced sales of securities from undermargined accounts in a falling market." – *William McC. Martin, Jr.*[31]

In July 2007, it was clear that the market credit had been excessive during the preceding three months: it had been growing at an unprecedented pace and relative to GDP had reached its highest level since the market crash of October 2000. In July 2008, it was clear that the stock market had been falling for the previous twelve months. Yet the practice of using the risk-based approach, which evidently caused excessive market credit, was continuing.

We argue in this paper that the stock market crash of October 2008 has a direct link to the adoption of the risk-based approach for margining customer accounts in the U.S. stock market. As we show in Section 5, this approach provided a substantial margin reduction, especially for call portfolios and bear portfolios.

1.4 State of the Art: Risk

As we will see in Section 4.2, the main mathematical model of the risk-based approach is asset pricing. The literature on this topic is overwhelming; see [Duffie, 2003]. The main option pricing models are the Black-Sholes model [Black and Scholes, 1973] for European-style options[32] and the Brennan-Schwartz model [Brennan and Schwartz, 1977] for American-style options.[33] Although the SEC permits the use of any asset pricing algorithm or software approved by the DEA,[34] there exists only one approved asset pricing software system today: the pricing module of STANS[35] developed by the OCC. Thus, risk-based margining today is monopolized in the U.S. by the OCC.[36]

1.5 Contributions and Plan of the Remainder of the Paper

The contributions of this paper are (1) an argument showing that the risk-based approach is being misused in application to security portfolios, (2) an argument showing that the risk-based approach is appropriate to use in application to only index portfolios, and (3) an explanation of why the risk-based approach is advantageous for playing call or bear[37] and could have influenced the quick growth of the margin debt in the U.S. stock market in the period from July 2006 through July 2007.

To develop these contributions, we introduce a mathematical model of portfolio margining in Section 2. Within this model the analysis of the strategy and risk-based approaches is given in Sections 3 and 4, respectively. In Section 4.5, we show that the risk-based approach being applied to security portfolios can cause a pyramid of debt in a bull market and a pyramid of loss in a bear market. Computational experiments with both approaches are presented in Section 5. The results of the computational experiments are discussed in Section 6. Section 7 summarizes our findings.

2 A Model of Portfolio Margining

The existing practice of margin calculations uses the following four components that must be specified in any approach to margining portfolios:

- *Position Margining*, i.e., the method of calculating the margin requirement for an *individual* position, which is also called a *naked* position, in a margin account regardless of the presence of other positions. Position margining can also be considered as margining a portfolio that consists of only one position.
- *Offset Margining*, i.e., the method of calculating the margin requirement for a group of offsetting positions, which are simply called *offsets.* Since the positions offset each other, the margin requirement for an offset is lower than the total position margin in the offset. Therefore, offsets are the means of margin reductions. Offsets can be considered as portfolios of restricted sizes whose margin calculations are “manageable”, i.e., margin requirements for offsets can be easily calculated by a simple formula or procedure.
- *Portfolio Partitioning*, i.e., the method of partitioning a portfolio into blocks which are positions or offsets. Thus, if margin requirements for positions and offsets are defined, then the margin for the entire portfolio can be calculated as the total margin requirement of these blocks.
- *Portfolio Classification*, i.e., the method of defining portfolio types depending on what security types are in a portfolio’s positions. Position margining and offset margining are usually dependent on security types and portfolio types.

Definitions related to portfolio partitions and portfolio classifications are given in Sections 2.1 and 2.2.

2.1 Portfolio Partitions

Let S be the set of financial *securities* traded on the market. A *portfolio* on S can be formally defined as a triplet (P, i, q), where

- P is a nonempty *position set*, i.e., the set of positions, also called *components*, in securities from S;
- i is the *investment function* which defines an injection of the set P into S, i.e., $i(p)$ is the security in position p, and $p \neq p'$ implies $i(p) \neq i(p')$; and
- q is the *quantity function* on P, i.e., $q(p)$ is the number of units of $i(p)$.

The inequality constraint in the second bullet means that two or more positions in the same security are not allowed in portfolios for margining purposes; if such positions appear as a result of several buy transactions, they must be consolidated into one position with a netted quantity before margining.

In what follows, it will be convenient to associate portfolios with their position sets and trivial portfolios with positions. For the purposes of this paper, a *margin account* is just a portfolio. Let $|P|$ be the *size* of a portfolio P. The portfolio of size $|S|$ is the *market portfolio*. Portfolios of size one are *trivial*.

Let P_1 and P_2 be two portfolios on S with the investment functions i_1 and i_2 and the quantity functions q_1 and q_2. We say that the two portfolios are *similar* if $P_1 = P_2$ and $i_1 = i_2$. Thus, similar portfolios differ in position quantities only.

If P_1 and P_2 are similar portfolios and there exists an integer $n_1 > 1$ such that $n_1 q_1(p) = q_2(p)$ for all $p \in P_1$, then we say that n_1 is a *divisor* of P_2 and write $n_1 P_1 = P_2$. Portfolios without divisors are *prime*. A portfolio P has exactly one similar prime portfolio; it is called a *base* of P and denoted by $\dot{P}$. Thus, $P = n\dot{P}$ for some positive integer n, which we call the *multiplicity* of $\dot{P}$; the portfolio P thus is a *multiple* of $\dot{P}$.

We say that P_1 is a *subportfolio* of P_2 or that P_2 *contains* P_1 if $P_1 \subseteq P_2$ and i_1 is a restriction of i_2 to P_1, i.e., $i_1(p) = i_2(p)$, and $q_1(p) \leq q_2(p)$ for all $p \in P_1$.

Let $P_1, \ldots, P_k$ be mutually disjoint subportfolios of some portfolio P with quantity functions $q_1, \ldots, q_k$, respectively. Then we say that these subportfolios generate a *partition* of P and write $P = P_1 + \ldots + P_k$ if

$$q(p) = \sum_{i=1}^{k} \sum_{p \in P_i} q_i(p)$$

for any position $p \in P$. If $n_1, \ldots, n_k$ are multiplicities of $\dot{P}_1, \ldots, \dot{P}_k$, respectively, then the sum $P = n_1\dot{P}_1 + \ldots + n_k\dot{P}_k$ is called a *prime partition* of P on $\rho = \{\dot{P}_1, \ldots, \dot{P}_k\}$.

Let ρ be a collection of prime portfolios on S including all $|S|$ trivial prime portfolios, so that $|S| \leq k$. Then it is clear that a prime partition of any portfolio P on ρ always exists. If the maximum position quantity in the portfolio P is n then the number of all possible prime partitions of P on ρ is at most $(n+1)^k$.

2.2 Portfolio Classification

Let I and S denote the sets of *indices* and *securities* in the market, respectively. We distinguish these *financial instruments* because, unlike indices, securities (including ETFs based on indices) are traded on the market, i.e., they have trading units. The market itself can be defined as the union $M = I + S$.

Let $u : S \to M$ be the *underlying function* on S, i.e., $u(s)$ is the *underlying instrument* for the security $s \in S$, and s is a *derivative* from $u(s)$. Let U denote the set of *underlying securities*, i.e., $U = \{s \in S : \exists s' : u(s') = s\}$. Then the sum $B = I + U$ is the set of *underlying instruments* of M, which we call the *base* of the market M. Let $u^{-1}(b)$ be the set of derivatives from the underlying instrument $b \in B$. Then the set

$$S_b = \begin{cases} \{b\} + u^{-1}(b) & \text{if } b \text{ is a security or} \\ u^{-1}(b) & \text{if } b \text{ is an index} \end{cases}$$

is a *security class* associated with the underlying instrument $b \in B$. We say that P is a *class portfolio* based on the underlying instrument b if all securities in P belong to the class S_b. If not stated otherwise, the term “portfolio” refers to a class portfolio. Margin regulations distinguish the following three types of underlying instruments:

1: a high-capitalization BBI and an ETF based on it,
2: a low-capitalization BBI and an ETF based on it,
3: an NBI and a margin eligible security.

The type of an underlying instrument b defines the type of a portfolio P based on b.

3 Strategy-Based Approach

The combinatorial essence of the strategy-based approach arises from the ability to partition a margin account in many different ways in accordance with a large variety of offsets given in the margin rule book. Each securities market follows its own margin rule book, for example, NYSE Rule 431 in the U.S. or Regulation 100 in Canada. The strategy-based offsets are of fixed size and imitate trading strategies.

In brokerage jargon, offsets of sizes 1, 2, 3 and 4 are *singles*, *pairs*, *tripos* and *quads*; the positions of the offsets are *legs*. Offsets of sizes at least two are *proper*. Proper offsets such as *bull* or *bear spreads*, *straddles* or *option-security combinations* are pairs, *butterfly spreads* or *collars* are tripos, and *iron butterfly spreads*, *condor spreads* or *box spreads* are quads; cf. [Cohen, 2005]. Complex offsets[38] can have up to 12 legs.[39]

3.1 Position Margining: Strategy

The margin requirement for an individual position in the strategy-based approach is a certain percentage of the current security market value contained in this position. This percentage is called a *margin rate*. The higher the market price of the security, the more reliable it is considered; therefore, a smaller margin rate is assigned to this security in strategy-based margin regulations. The margin rate is also dependent on security type, rating, expiry date, the market where it is traded, whether the position is long or short, and other factors. Margin rules for positions in derivatives are not simple at times, and the related margin requirements can have nontrivial margin formulas.[40]

Fixed-income securities like bonds, debentures and notes have much lower margin requirements than equity securities because they are much less volatile. Stocks are the most volatile equity securities and have the most stringent margin requirements. For example, U.S. initial and maintenance margin rates for stocks in long positions are 50% and 25%, respectively.[41] Note that the Regulation T initial rate of 50% has remained unchanged since 1974, and it has never been lower than 40% since 1940. In Section 4.1, we will see that the risk-based approach assigns both initial and maintenance margin rates for stocks as low as 15%.

3.2 Hedging Mechanism of Strategy-Based Offsets

Trading strategies, called *strategies* for short, are special portfolios with known "exits". The positions of a strategy appear in a margin account as a result of simultaneous buy and sell trades. Strategies are designed not only to maximize the profit in favorable scenarios but also to provide the possibility to *liquidate* all or some of their positions in unfavorable scenarios; see e.g. [Cohen, 2005].

Offsets in the strategy-based approach *imitate* strategies. A single exit is also associated with an algorithm for liquidating (closing) all or some of the positions of the offset with a *minimum loss.* In accordance with the strategy-based approach, margin regulators choose the *worst exit* and calculate the *maximum minimum loss*, i.e., the minimum loss in the *worst case scenario.* This loss constitutes the minimum regulatory margin requirement for the offset.[42]

We can formally define an offset as a portfolio with the associated collection of liquidation algorithms that we call a *hedging mechanism* for the offset. Each liquidation algorithm is usually associated with a subset of positions in *exercisable securities*[43] in the offset (such as options, futures, warrants, convertible bonds) and represents a sequence of *trades* (*buy* and *sell* transactions) which must be triggered once all exercisable securities in the subset are exercised.[44] Thus, offsets in the strategy-based approach are supported by hedging mechanisms for exiting from dangerous situations. Since every liquidation algorithm is a sequence of trades, the hedging mechanism defines necessary and sufficient *cash flows* and *security flows* that must be run to satisfy all *parties* involved in the offset liquidation.

It is important to observe that the hedging mechanism for a prime offset $\dot{O}$ generates the hedging mechanisms for all its multiples because the hedging mechanism for $m\dot{O}$ is just the hedging mechanism for $\dot{O}$ applied m times to the offset O. Margin regulations therefore describe only prime offsets. Any margin rule represents actually a prime offset and the margin requirement for it, and any margin *rule book* is a collection of margin rules. In what follows, we will associate a rule book with a collection of prime offsets ρ that includes all $|S|$ trivial prime offsets and keep in mind that the margin requirements for the prime offsets are given. The more offsets there are in the rule book, the more opportunities exist for margin reductions. It is important to emphasize that offsets make margin reductions possible not at the expense of increasing the risk but, in contrast, through hedging against the worst case scenario, using the *hedging mechanisms* provided for that purpose.

3.3 Hedging Mechanism of a Call Spread

Let us consider, for example, the hedging mechanism of a *call spread* involving a short position in a call option C[45] and a long position in another call option D on the same underlying security U, at the same expiry date, exercise price e and contract size s. The mechanism consists of only one liquidation algorithm which must be triggered once the holder of C exercises the option:

- the *buy* of s units of U by the holder of D from the writer[46] of D at the price e;
- the *sell* of s units of U by the holder of D to the holder of C at the price e.

Thus, the holder of the spread is protected by this hedging mechanism against the underlying security price fluctuations. The maximum loss here is the cost of the spread paid at the time when the position in this spread was entered.

Each of these trades implies the cash flow of quantity se from a buyer to a seller and the underlying security flow of quantity s from a seller to a buyer. It is also important to notice that this pair of trades involves (apart from the brokers) the following *three* parties:

- the writer of D;
- the holder of the call spread, i.e., the holder of D who is also the writer of C; and
- the holder of C.

Thus, the call spread holder can trigger the hedging mechanism in order to avoid a loss larger than the spread cost in the case when the holder of C decides to exercise the option. The call spread holder in this case plays the role of an *intermediary* (third party) between the holder of C who buys s units of the underlying security at the price e and the writer of D who sells them at the same price. The hedging mechanism, therefore, determines who receives the money in the amount of se, who receives s units of the underlying security and from where, when the call spread must be closed.

We have considered the hedging mechanism of the simplest offset. In general, the hedging mechanism of an offset with many positions involves a series of trades (cf. [Cohen, 2005]), but the main point is that it makes clear the *money distribution* (money side) and the underlying *security distribution* (security side) among the parties involved in the related set of trades, when the offset must be closed. That is why the offsets in the strategy-based approach are two-sided offsets.

In the above example we considered *security options*, i.e., options on a security, such as an equity, ETF, bond, currency, or commodity. Exercising security options triggers a cash flow and a security flow. Exercising *index options*, i.e., options on an index, triggers a cash flow only. For example, exercising an index call option brings to the option holder the difference between the index price and the option exercise price. Thus, the hedging mechanism of an index call spread is much simpler because it entails only the movement of money.

3.4 Cross Margining: Strategy

If an offset is a class portfolio (see the definition in Section 2.2), i.e., all its positions have the same underlying instrument, then we call it a *class offset.* The vast majority of offsets created for margin calculations are actually class offsets. There also exist *cross offsets* whose positions are based on different (crossing) underlying instruments, which do not necessarily belong to the same securities market. This explains the term *cross margining* when cross offsets are permitted. Cross offsets are usually permitted if they are underlain by highly correlated market indices or ETFs based on these indices. Therefore, cross offsets are cross-index products.

Cross offsets can be easily designed from class offsets by allowing some positions to have different underlying instruments. Margin requirements for cross offsets are usually more stringent and take into consideration the correlation between the crossing underlying instruments. Although class offsets are often more advantageous than cross offsets, there is no priority to class offsets in the strategy-based approach. If cross offsets turn out to be more efficient than class offsets in margin reductions then they are allowed to be used first. As we will see in Section 4.2, the situation is different in the risk-based approach because it gives priority to class offsets.

3.5 Account Offsetting and Margining: Strategy

The essence of the strategy-based approach is to find a hedging mechanism for a margin account by the consolidation of the hedging mechanisms of the offsets contained in the account. Let A be a margin account, let $\rho = \{\dot{O}_1, \ldots, \dot{O}_k\}$ be a rule book, and let

$$A = x_1\dot{O}_1 + \ldots + x_k\dot{O}_k$$

be a prime partition of A on ρ; see Fig. 3 for an example. Since the hedging mechanisms of the prime offsets $\dot{O}_j$ generate the hedging mechanisms of the multiples $x_j\dot{O}_j$, the hedging mechanism of A is just a consolidation of the hedging mechanisms for the multiples. Thus, the hedging mechanisms of the prime offsets $\dot{O}_j$ must be applied x_j times to the positions of A that are covered by these offsets and, since the multiples $x_j\dot{O}_j$ cover all position quantities in the account A, the entire account becomes hedged.

If m_j is the margin requirement for the prime offset $\dot{O}_j$ then,in accordance with the strategy-based approach, the margin requirement for the margin account A is

$$m_1x_1 + \ldots + m_kx_k$$

Thus, a margin requirement for A is associated with a prime partition of A, and every prime partition of A gives a margin requirement for A. Hence, finding a minimum

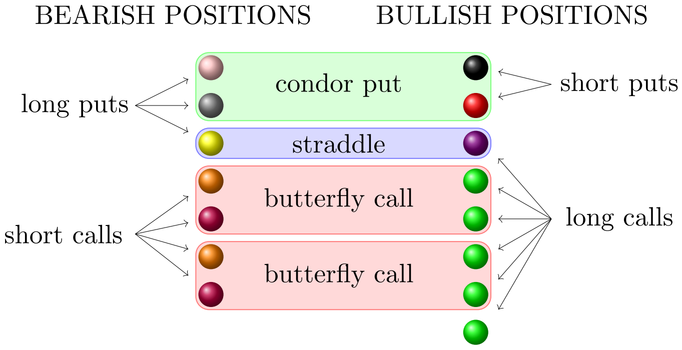

Figure 3: An account A that consists of only one class portfolio with nine positions in options which are marked by nine different colors. Each option contract is depicted by a ball. The account A thus has total of fifteen option contracts. The figure shows the prime partition $A = \dot{O}_1 + \dot{O}_2 + 2\dot{O}_3 + \dot{O}_4$ on NYSE Rule 431 that involves the prime offsets $\dot{O}_1, \dot{O}_2, \dot{O}_3$, and $\dot{O}_4$ which are a condor put spread, a long straddle, a butterfly call spread, and a naked long position in one call option contract, respectively.

regulatory margin requirement for an account is finding its prime partition, i.e., multiplicities $x_1, \ldots, x_k$, such that the latter sum achieves the minimum. We will refer to this problem as the *account margin minimization* (AMM) problem. If the maximum offset size in the rule book ρ is d then we will say that the AMM problem is of dimension d. Consequently, the AMM problem of dimension two is the AMM problem by pairing.

Thus, finding the margin requirements for margin accounts in the strategy-based approach means solving the AMM problem. Achieving the minimum is important for both the brokers and the account owners because the knowledge of the regulatory minimum against the real margin requirement gives the exact estimate of the overcharge. On the other hand, the minimum represents an adequate measure of risk.

3.6 Mixed Integer Program for Strategy-Based Margining

We show next that the AMM problem reduces to a *mixed integer program* (MIP). This reduction will allow us to use standard optimization software for the calculation of strategy-based margins in our computational experiments presented in Section 5.

Let a margin account have m positions $i = 1, 2, \ldots, m$ with quantities q_i and n prime offsets $j = 1, 2, \ldots, n$ from ρ. Prime offset j can be represented by the column vector $\mathbf{o}_j$ whose ith component o_{ij} is the quantity of the ith components of prime offset j.[47] Note that $o_{ij} = 0$ if and only if position i is not involved in prime offset j. We assume that $m \leq n$ and that prime offsets $j = 1, 2, \ldots, m$ are trivial, i.e., they represent the positions. Thus, the column vectors have $o_{jj} = 1$ as the only nonzero elements.

Let m_j be the margin requirement for prime offset j. Thus, m_j is the margin requirement for a single unit of the security in position j if $1 \leq j \leq m$, or the margin for prime offset j of size more than one if $m+1 \leq j \leq n$. Let x_j denote the multiplicity of prime offset j. If we introduce the vectors[48]

$$\mathbf{m} = \|m_1, \dots, m_n\|^\top \qquad \mathbf{q} = \|q_1, \dots, q_m\|^\top \qquad \mathbf{x} = \|x_1, \dots, x_n\|^\top$$

which we call a *margin*, *quantity* and *variable vectors*, respectively, and the *offset matrix* $\mathbf{O} = \|\mathbf{o}_1, \mathbf{o}_2, \dots, \mathbf{o}_n\|$, then the AMM problem can be posed as finding an $\mathbf{x}$ which solves

$$\min \left\{ \mathbf{m}^\top \mathbf{x} : \; \mathbf{O}\mathbf{x} = \mathbf{q}, \; \mathbf{x} \geq \mathbf{0} \right\} \tag{1}$$

where $\mathbf{0}$ is a zero column vector. Note that the size of $\mathbf{O}$ is $m \times n$. In this MIP, components of the variable vector $\mathbf{x}$ from $m+1$ to n must be integers because they define offset multiplicities of offsets with sizes more than one, while components from 1 to m can be real. They are integers if they are quantities of positions in such securities like stocks (shares), options, warrants, but they can be real if they are, for example, bond quantities. The quantity vector $\mathbf{q}$ is also integer but the margin vector $\mathbf{m}$ is real.

Without loss of generality, we assume that $\mathbf{O} = \|\mathbf{I}, \mathbf{P}\|$, where $\mathbf{I}$ and $\mathbf{P}$ are the *identity matrix*, i.e., the matrix of trivial prime offsets, and the matrix of *proper prime offsets*, i.e., the matrix of prime offsets of size at least two, respectively.

MIP (1) can be viewed as an extension of the transportation model introduced in [Rudd and Schroeder, 1982] for the calculation of the minimum margin by pairing, i.e., in the case where the matrix of proper prime offsets is a matrix of prime pairs.

3.7 Margining Practice: Strategy

Surprisingly, current margin calculation practice still uses heuristics designed in the mid seventies based on brokers' intuition and taste, even for margining by pairing. In the general case, the most advanced heuristics take advantage of the result from [Rudd and Schroeder, 1982] by using minimum-cost network-flow algorithms. There exist, however, counterexamples showing that these heuristics can double regulatory minimum margin requirements or even raise them from zero to catastrophic margin calls. Millions of margin accounts maintained by prime brokers with the use of the strategy-based approach therefore are substantially over-margined, just because there has been no serious attempt to find efficient optimization algorithms.

Despite the fact that the AMM problem was posed (originally as a problem of margining accounts with options only) more than thirty five years ago, it has not been well studied and remains one of the most intractable problems in the brokerage industry. Neither useful theoretical results nor solution techniques with reasonable computing times are known. The only exception is the paper [Rudd and Schroeder, 1982] devoted to the AMM problem by pairing. It was shown that this case reduces to the well-studied bipartite minimum-cost network-flow problem.

An analysis of the literature suggests that the AMM problem has never been considered in the MIP form, one of the more natural and popular ways of solving discrete optimization problems. However, in Section 5, we show that MIP algorithms yield very good results for the AMM problem.

4 Risk-Based Approach

The risk-based approach has a much lighter optimization component (for cross margining only), but depends on artificial variations of current prices of underlying instruments and the use of probabilistic models of asset pricing theory for pricing derivatives related to these variations. To decide what portfolios qualify for cross margining, the method looks through historical data of the correlation of the instruments underlying the portfolios.

As we will see, the combinatorial complexity of the strategy-based approach is replaced in the risk-based approach by the complexity of asset pricing models related to the difficulties in obtaining a high-level of statistical significance. In addition to rejecting the entire rule book of *fixed-size offsets* based on trading strategies, it uses much looser *variable-size offsets* based on grouping positions by underlying securities. Details of the risk-based approach are as follows.

4.1 Position Margining: Risk

Similarly to the strategy-based approach, the risk-based approach uses the current market price of a security for margin calculations, and in addition, it considers price variations within certain ranges in an attempt to catch the worst-case price movement for the entire portfolio. This technique is called *portfolio choking.*

According to the risk-based approach, in order to calculate the margin requirement for each unit of a security s in a position whose underlying security u has the market price c, it is necessary to consider in addition to c, ten more *valuation points*

$$c(1-ai) \quad \text{and} \quad c(1+bi)$$

where $i = 1, 2, 3, 4, 5$, and the constants a and b depend on the type of the underlying instrument as follows:

- $a = 0.016$, $b = 0.012$ for type 1,
- $a = 0.020$, $b = 0.020$ for type 2,
- $a = 0.030$, $b = 0.030$ for type 3,

where the types are defined in Section 2.2. Thus, the lowest (highest) valuation points appear to be 8%(6%), 10%(10%), 15%(15%) lower (higher) than the current market price c for types 1, 2, and 3, respectively.[49]

It is important to observe that, in accordance with this rule, the risk-based margin rate for stocks and margin eligible equities in customer margin accounts is only 15%, the lowest margin rate since 1929. Before the stock market crash in October 1929 it was 10%. The current strategy-based margin rates are 50% (initial) and 25% (maintenance).

In our opinion, this significant margin reduction in margining individual stock positions together with the faulty hedging mechanism of risk-based offsets for stocks were the main reasons for the stock market crash on October 2008, as noted in Section 1.3. We consider the hedging mechanism of risk-based offsets in Sections 4.4 and 4.5.

Let c_v, $1 \leq v \leq 11$, be one of the eleven valuation points including c. If $s = u$, i.e., s is the underlying security, then the difference $o_v = c_v - c$ or $c - c_v$ shows the outcome (*gain* if positive, and *loss* if negative) associated with point c_v for long or short positions in s, respectively, for each security unit.[50]

If s is a derivative, then the outcome associated with the valuation point c_v should be calculated in accordance with the mechanism of the derivative. In most cases, o_v is a function of c_v, e (the exercise price of the derivative) and p_v (the market price of s estimated at the valuation point c_v).[51] The estimated price p_v must be calculated using a qualified *theoretical pricing model.*

4.2 Account Offsetting and Margining: Risk

Unlike the strategy-based approach, the risk-based approach uses much more aggressive offsets. Their margin requirements are simply net losses of the involved positions. To calculate the margin requirement for an account A we need to perform the following six steps, where Steps 2 through 5 should be repeated for each valuation point.

Step 1. *Account Partition:* Find the partition of A into the portfolios $P_1, \ldots, P_k$ associated with the underlying instruments involved in A. Note that this operation is free of any combinatorial element because it is equivalent to the enumeration of all positions in the account.

Step 2. *Finding Positions' Theoretical Gains/Losses:* Calculate the outcome o_{pv} for each position $p \in A$.[52]

Step 3. *Netting of Positions:* Calculate the outcome n_{uv} for each portfolio P_u, $u = 1, \ldots, k$, using the formula $n_{uv} = \sum_{p \in P_u} o_{pv}$. Thus, n_{uv} is the net outcome of all positions in P_u for the evaluation point c_v.

Step 4. *Finding Gain and Loss Portfolios:* At this point, each portfolio P_u is either a *loss portfolio* with $n_{uv} < 0$ or a *gain portfolio* with $n_{uv} > 0$ or a *null portfolio* with $n_{uv} = 0$. In what follows, the notation $L_{1v}, \ldots, L_{mv}$ and $G_{1v}, \ldots, G_{nv}$ will stand for the loss and gain portfolios, respectively. Their lower case counterparts, i.e., $l_{1v}, \ldots, l_{mv}$ and $g_{1v}, \ldots, g_{nv}$ will stand for their losses and gains.

Step 5. *Cross Margining:* Find the *guarantee-guarantor deficit* d_v for the valuation point c_v (see Section 4.3) and assign it as the margin requirement for c_v. Note that, if cross margining was not allowed, the guarantee-guarantor deficit would be the total loss through all loss portfolios, i.e., $d_v = l_{1v} + \ldots + l_{mv}$ for each $v = 1, \ldots, 11$.

Step 6. *Account Margining:* Assign the margin requirement for the account A to be $\max\{d_0, d_1, \ldots, d_{11}\}$ with

$$d_0 = \sum_{i=1}^{l} \max\{0.375, p_i\} \cdot q_i r_i + 0.375 \cdot \sum_{i=l+1}^{k} q_i r_i$$

where $1, \ldots, l$ and $l+1, \ldots, k$ are, respectively, long and short positions in A in derivatives, and where p_i, q_i and r_i are the position i unit price (per underlying unit), quantity and derivative contract size, respectively.

4.3 Cross Margining: Risk

As is evident from Step 3 of the above algorithm, the risk-based approach allows us to fully offset position losses by gains of another position inside each portfolio. In addition, it allows us to offset, to a certain extent, losses of index portfolios of type i by gains of index portfolios of type j if the pair (i, j) is either $(1, 1)$, $(2, 2)$ or $(3, 3)$ (see the definition of types in Section 2.2). Since these offsets connect different security classes based on different indices, this technique is called *cross margining*.

It is important to note that cross margining offsets between portfolios are permitted only if the OCC determines that the *crossing* indices associated with types 1, 2 and 3 are sufficiently correlated.[53] Furthermore, the offset of an index by an ETF based on it is permitted only if the ETF holds securities in the same proportion as the index.[54]

- 90% of the gain of a type 1 can offset the loss of type 1,
- 75% of the gain of a type 2 can offset the loss of type 2,
- 90% of the gain of a type 3 can offset the loss of type 3,

where the types are defined in Section 2.2. These percentages follow the *haircut* rule[55] that must be applied before netting. Thus, prior to solving the guarantee-guarantor deficit problem, one needs to reduce the gains g_j by 10% for all gain portfolios G_j of type 1 or 3, and by 25% for all gain portfolios G_j of type 2. We consider at this point that the haircut has been applied.[56]

The problem of the distribution of gains to cover losses is not trivial. We now show that a solution can be found by formulating it as a linear program. Let R denote the guarantee-guarantor relation between the loss portfolios and the gain portfolios of the same type defined by the OCC. Specifically, let $1 \leq i \leq m$ and $1 \leq j \leq n$, then

$$\begin{aligned} (i, j) \in R \text{ if and only if the loss portfolio } L_i \text{ (guarantee)} \\ \text{can be offset by the gain portfolio } G_j \text{ (guarantor)} \end{aligned}$$

Let x_{ij} denote the portion of the loss l_i in the loss portfolio L_i that is assigned to be offset by the gain portfolio G_j. Then the guarantee-guarantor deficit is

$$d = \sum_{i=1}^{m} \left(l_i - \sum_{j=1}^{n} x_{ij} \right) = \sum_{i=1}^{m} l_i - \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}$$

Since the gain portfolio G_j can offset at most the g_j loss of the loss portfolios,

$$\sum_{i=1}^{m} x_{ij} \leq g_j \tag{2}$$

for all j such that $(i, j) \in R$; and, since the loss portfolio L_i needs at most l_i to be offset by the gain portfolios,

$$\sum_{j=1}^{n} x_{ij} \leq l_i \tag{3}$$

for all i such that $(i, j) \in R$. If we are interested in reducing the guarantee-guarantor deficit as much as possible, we need to solve the linear program that maximizes

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}$$

with nonnegative variables x_{ij} under Conditions 2 and 3.

With the exception of the aggressive offsetting[57] at Step 3 of the algorithm described in Section 4.2, nothing contradicts the strategy-based approach. Portfolio choking, however, has never been used in the strategy-based approach – although it could be. Without portfolio choking, the risk-based approach is just a simplified case of the strategy-based approach with substantially lower margin requirements for singles and offsets. Further, for each class portfolio, the risk-based approach squeezes the entire rule book of the strategy-based offsets into only one offset, which is the class portfolio itself; see Fig. 4 for an example. If Step 3 were replaced by the procedure for creating strategy-based offsets, the risk-based approach would turn into a hybrid approach. It would have been natural to use this approach in the pilot program for a "cushioned" transition.

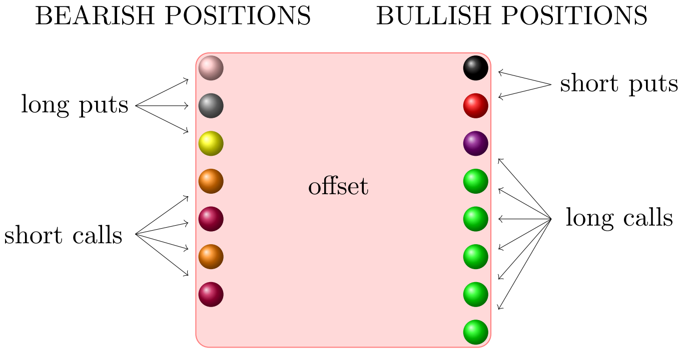

Figure 4: The account from Fig. 3. It has only one risk-based offset which is the union of all four strategy-based offsets shown in Fig. 3.

4.4 Hedging Mechanism of Risk-Based Offsets

As can be seen from the algorithm in Section 4.2, a risk-based offset is the entire portfolio. Its hedging mechanism is just netting position losses and gains. It is important to observe that, if the underlying instrument of the portfolio is a security, the netting does not show security movements in the case when the portfolio must be liquidated.

The risk-based approach, however, works well for index portfolios where exercises of index options do not trigger any security movements.[58] To confirm this fact we can observe that the margin debt growth was quite moderate in the period between Phases I and II when only portfolios with index products were involved in the pilot program, see Fig. 1. However, once portfolios with security options and futures joined the program, the debt started growing very fast. The following example shows a situation which, if occurs, can cause such a fast growth of debt.

Consider a portfolio P with the following three positions: a long position in a call option C, a short position in a call option C_0 and a short position in a call option C_1. Thus, P is a *call tripo*. In what follows, it will be convenient to denote a long (short) position in an option C as $+C$ $(-C)$, respectively, and a portfolio as a formal sum of positions. Also, it will be natural to omit the + symbol, as accepted in formal sums. Thus,

$$P = C - C_0 - C_1$$

All three options are on the same underlying stock and have the same expiry date and contract size of 100 shares. Suppose that the expiry date is distant enough, so that the options' current prices are much smaller than the current price u of the underlying stock, and therefore can be disregarded in the calculation of gains and losses. Suppose further that at this point in time the options C, C_0, and C_1 are in the money and their exercise prices have the following relationship with u:

$$u - u/2, \quad u - u/4 \;\text{ and }\; u - u/4,$$

respectively. Thus, the holder of P experiences gain in the amount of

$$50u = 100(u - u + u/2)$$

on C and loss in the amount of

$$50u = 100(u - u + u/4) + 100(u - u + u/4)$$

on C_0 and C_1. Following the risk-based approach we should net the losses and the gain and conclude that the margin requirement for P is zero.

Suppose now that the holders of C_0 and C_1 decide to exercise the options and the holder of P decides to liquidate the portfolio exercising C. Upon exercising these options, the holder of P has zero cash balance, because $50u - 25u - 25u = 0$, 100 shares of the stock from exercising the option C and the obligation to deliver $100 + 100$ shares to the holders of options C_0 and C_1. Thus, the holder of P is 100 shares short, and these shares must be purchased to liquidate the portfolio. Since the risk-based approach allows us to do this on 85% credit (the margin requirement is just 15%), the portfolio liquidation requires borrowing $85u$.

Thus, risk-based offsets do not provide a real hedge and encourage borrowing money if P must be liquidated. Note that the strategy-based approach provides a complete hedge for this portfolio, as it catches the spread $C - C_0$ or $C - C_1$ and requires a proper margin charge for the naked short position $-C_1$ or $-C_0$, respectively.

4.5 Pyramid of Debt

Now we show that the call tripos considered in the previous section can be used as blocks for building a pyramid where each block would require borrowing money if all options of pyramid's bottom were exercised.

Suppose that the holder of the call option C_0 is short in two call options C_{00} and C_{01}, and the holder of the call option C_1 is short in two call options C_{10} and C_{11}. Each of these four options has the exercise price $u - u/8$. Thus, these holders are actually the holders of the portfolios

$$P_0 = C_0 - C_{00} - C_{01} \quad \text{and} \quad P_1 = C_1 - C_{10} - C_{11}$$

Each of the portfolio holders experiences a gain in the amount of

$$25u = 100(u - u + u/4)$$

on the long position and a loss in the amount of

$$25u = 100(u - u + u/8) + 100(u - u + u/8)$$

on the short positions. Again, each of the portfolio holders is in the same situation as the portfolio P holder and must purchase 100 shares of the stock to liquidate the portfolio. Together they should borrow $2 \cdot 85u$.

The short options C_{00}, C_{01}, C_{10}, C_{11} in their turn can be long options in the next layer of portfolios P_{00}, P_{01}, P_{10}, P_{11}, in which options in short positions have exercise price $u - u/16$; see Fig. 5.

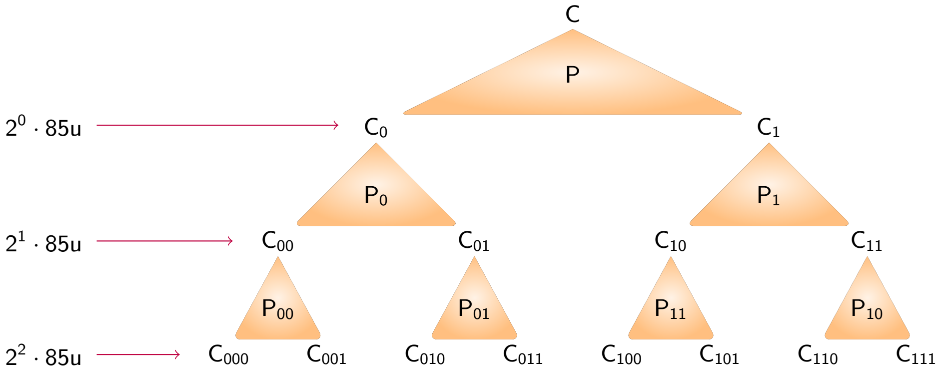

Figure 5: pyramid of debt with three layers and total credit requirement $(2^3 - 1) \cdot 85u = 595u$. The arrows point to layers $k = 1, 2, 3$ with credit requirements $2^{k-1} \cdot 85u$.

Continuing by induction we can conclude that layer k contains 2^k blocks, whose options in short positions have exercise price $u - u/(2^{k+1})$ and require the amount $2^{k-1} \cdot 85u$ for closing these positions. The blocks thus build a pyramid with the total debt

$$\left(2^0 + 2^1 + \ldots + 2^{k-1}\right) \cdot 85u = \left(2^k - 1\right) \cdot 85u$$

If layer k is the base of the pyramid then exercising all 2^k options in layer k requires borrowing not only the amount $2^{k-1} \cdot 85u$ to close short positions in these options, but an additional amount

$$\left(2^k - 1\right) \cdot 85u - 2^{k-1} \cdot 85u = \left(2^{k-1} - 1\right) \cdot 85u$$

to close all of the other short positions of this pyramid. Thus, the borrowed amount is almost doubled. Note that the underlying security moves from the top to the bottom of the pyramid. Also note that the pyramid "works" only in the bull market when the market price of the underlying security exceeds the exercise price of the call options at the bottom of the pyramid; otherwise the holders of these options would not be interested in exercising them.

We should emphasize that our example involves only the simplest blocks requiring credit of $85u$. In practice, the blocks could be more credit demanding. For example, the quad with a long position in a call option with exercise price $u - u/2$ and three short positions in call options with the same exercise price $u - u/6$ has zero risk-based margin but requires the credit $170u$ to be liquidated. The base of the exponential growth associated with the pyramid of debt built from these blocks then becomes three.

4.6 Pyramid of Loss

Replacing call options with put options everywhere in Sections 4.4 and 4.5 and keeping the same exercise prices, we can construct blocks that are *put tripos* with zero risk-based margin requirement and build a similar pyramid. In comparison with a call tripo, a put tripo works in the opposite way. Suppose now that the market is bear. When the price of the underlying security drops below the exercise price of the two put options in the short positions in the tripo, its holder will experience a loss due to the obligation to buy 200 units of the underlying security at the price higher than the current market price. Since the holder is long in a single put option, a half of the loss can be eliminated by exercising this option. However, after exercising this option, the holder of the put tripo will be left with 100 units of the underlying security. Thus, the original loss is not completely hedged.

The pyramid built from put tripos is a pyramid of loss because every block in it brings loss. Obviously, the pyramid of loss does not work in the bull market. However, it may be triggered as soon as the market becomes bearish. Once triggered, the pyramid will continue to pull the market down due to numerous forced sales similar to those that happened in July-August of 2007.

5 Computational Experiments

The strategy-based approach cannot compete with the risk-based approach in the speed of computations because the latter is free of any combinatorial quantities. Nor does it give an advantage in *initial* margin requirements, which are substantially higher than those obtained by the risk-based approach; see Sections 1.3 and 4.1. But an answer to the question of whether the strategy-based approach always yields higher *maintenance* margin requirements in comparison with the risk-based approach is not obvious.

The results of our experiments show indeed the opposite. To demonstrate this, we compare the behavior of the portfolio *maintenance* margin requirement as a function of the portfolio size that is calculated by both the strategy-based approach and the risk-based approach in different scenarios. As before, we follow NYSE Rule 431.

Another goal of our experiments is to contrast different strategy-based algorithms and clarify to what extent offsets with numbers of legs more than two can reduce maintenance margin requirements and increase computing time in comparison with two-leg offsets. Note that the strategy-based approach accommodates many margining algorithms, while the risk-based approach has only one hard-coded margining algorithm.

5.1 Design of the Experiments

As we discussed in Section 3, the strategy-based approach uses offsets of fixed sizes. We say that a strategy-based algorithm is of *dimension d* if it solves the PMM problem of dimension d, i.e., it takes into consideration all regulatory offsets of sizes $1, 2, \ldots, d$. We consider dimensions two and four only.[59] Strategy-based algorithms of dimensions two and four were CPLEX algorithms[60] solving MIP (1) from Section 3.6, where only two-leg offsets and two-, three- or four-leg offsets were used, respectively.

The main idea of portfolio variation is to first build a *maximal portfolio* and then randomly remove positions to provide a monotonic reduction of its size. The margin requirement was computed for each generated portfolio by using the strategy-based algorithms of dimensions two and four, and the risk-based algorithm. The portfolios were generated by performing the following steps:

Step 1. A group of 16 call options and a group of 16 put options were selected[61] such that exactly 8 options inside each group were in the money; see the table on Fig. 6, where each row presents a call option and a put option with the same exercise price.

Step 2. The maximal portfolio with 32 positions was built by creating 8 long positions in randomly chosen 8 call options and 8 short positions in the remaining 8 call options; the other 16 positions in put options were created in the same way. This step was repeated 10 times and resulted in 10 unique maximal portfolios. The next steps were repeated for each maximal portfolio.

Step 3. The number of option contracts in each position was randomly generated in the range from 1 to 10. This step was repeated 50 times.

Step 4. A randomly selected position from one side (bearish or bullish) was removed to get a portfolio of the smaller size.

Step 5. The number of option contracts in each remaining position was randomly generated in the range from 1 to 10. This step was repeated 50 times.

Step 6. Steps 4-5 were repeated 29 times to get a total of 30 sets of 50 randomly generated portfolios with sizes monotonically decreasing from 32 to 3. The side

#	ex price	call price	put price
1	45	39.70	0.45
2	50	35.50	0.67
3	55	31.90	1.00
4	60	25.30	1.45
5	65	21.50	1.90
6	70	17.30	2.70
7	75	13.50	3.90
8	80	10.10	5.34
9	85	7.10	7.38
10	90	4.63	10.00
11	95	2.85	14.83
12	100	1.75	17.02
13	105	0.95	21.50
14	110	0.50	26.03
15	115	0.20	28.40
16	120	0.15	32.90

Figure 6: Selected Options

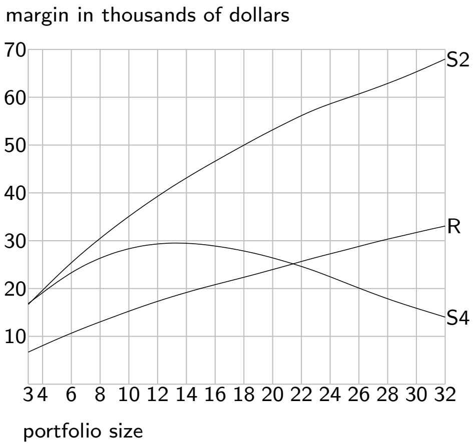

Figure 7: Balanced Portfolios

from which a position was to be removed in Step 4 was alternated to maintain a balance between the number of bearish and bullish positions.

Step 7. Steps 3-6 were repeated 25 times alternating the starting side in Step 4. Each time the random number generator was restarted to avoid repeated patterns.

Step 8. The margin requirements were averaged for portfolios of the same size. Hence, we calculated 30 averaged margin requirements for each algorithm.

Steps 1 through 8 create a *symmetric scenario* because position quantities chosen at Steps 3 and 5 are distributed uniformly between 1 and 10. The symmetric scenario generates *balanced portfolios* where the numbers of call and put options, in-the-money and out-of-the-money options, long and short positions are approximately the same. Thus, to obtain each point in the graph, we computed and averaged margin requirements for 12,500 portfolios, with the total of $12{,}500 \cdot 30 = 375{,}000$ unique and randomly generated balanced portfolios.

We also performed this experiment in six *asymmetric scenarios* to model *unbalanced portfolios* with different kinds of asymmetry. We performed the same steps as in the above algorithm except for Steps 3 and 5, where the quantities of options in the positions were ranging according to the following three scenarios, where quantities A and B were random integers in the intervals $[7, 10]$ and $[1, 4]$, respectively:

Long Portfolio : A/B option contracts for each long/short position,
Call Portfolio : A/B option contracts for each position in call/put options,
Bull Portfolio : A/B option contracts for each bullish/bearish position.

The other three asymmetric scenarios, Short Portfolio, Put Portfolio, Bear Portfolio, respectively, were obtained by transposing A and B in the above three definitions.[62]

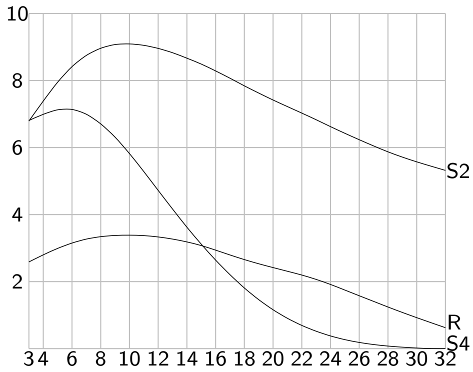

Figure 8: Long Portfolios

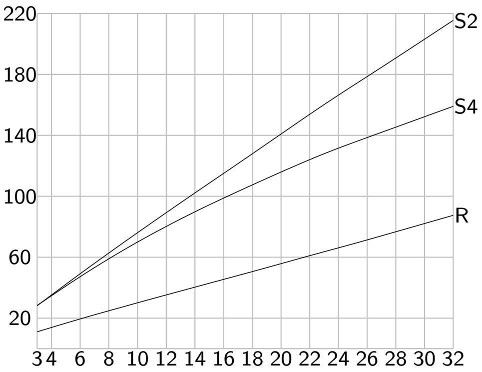

Figure 9: Short Portfolios

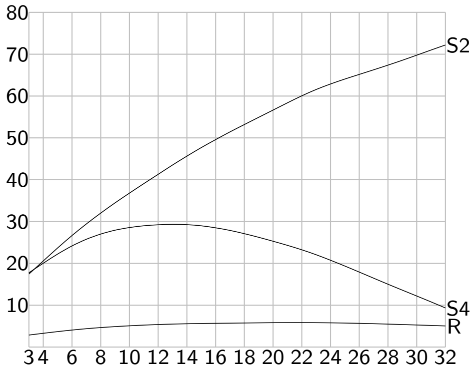

Figure 10: Call Portfolios

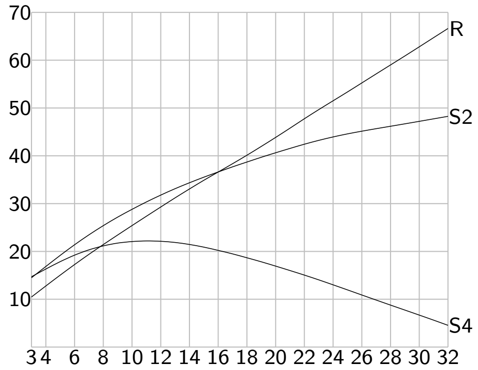

Figure 11: Put Portfolios

6 Results of the Experiment

The results of the experiment is presented in Figs. 7 through 13, where margin requirements are given in thousands of dollars for portfolio sizes $3, 4, 5, \ldots, 31, 32$. The notation S2, S4, R stands for margin requirements obtained by the strategy-based algorithms of dimensions two, four, and the risk-based algorithm, respectively.

Our main conclusion is that the strategy-based approach does not always yield a higher margin requirement than the risk-based approach. Comparing the curves S4 and R, we can conclude that the strategy-based approach yields lower margin requirements than the risk-based approach for balanced (Fig. 7), long (Fig. 8) and put (Fig. 11) portfolios of sizes more than 21, 15 and 8, respectively; and they are substantially lower for portfolio sizes closer to 32.

In the other four scenarios, i.e., for short (Fig. 9), call (Fig. 10), bull (Fig. 12) and bear (Fig. 13) portfolios, the risk-based approach is always more advantageous for all portfolio sizes. It is especially advantageous for investors playing call or bear since the related curves R are almost flat for call (Fig. 10) and bear (Fig. 13) portfolios.

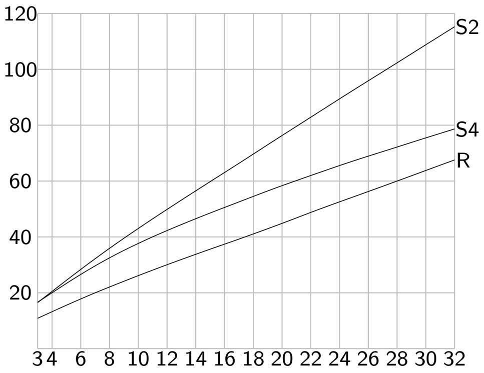

Figure 12: Bull Portfolios

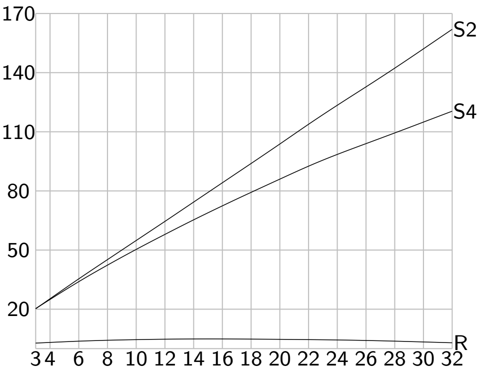

Figure 13: Bear Portfolios

With regard to the strategy-based approach, we can conclude that using tripos and quads along with pairs yields a substantial margin reduction in comparison with using only pairs; it is especially noticeable for balanced (Fig. 7), long (Fig. 8), short (Fig. 9), call (Fig. 10) and put (Fig. 11) portfolios.

Except for scenarios with short (Fig. 9), bull (Fig. 12) and bear (Fig. 13) portfolios, the trend of the strategy-based requirement is to be *smaller* for *larger* portfolios. Only long portfolios (Fig. 8) reveal the same but weaker trend for the risk-based approach. Thus, all margin requirements are decreasing with the growth of long portfolio size.

It is interesting to notice that the strategy-based approach remains almost insensitive to switching the asymmetry type from call (Fig. 10) to put (Fig. 11) portfolios, while the risk-based approach changes almost flat margin requirement curve into linearly increasing. Thus, the risk-based approach signals about risk associated with put options but remains neutral to call options. Switching the asymmetry type from bull (Fig. 12) to bear (Fig. 13) portfolios reveals the same behavior of the strategy-based approach but an opposite behavior of the risk-based approach. This phenomenon can be explained only by the portfolio choking technique introduced in the risk-based approach but not used in the strategy-based approach.

Obtaining significant margin reduction owing to offsets with three and four legs in the strategy-based approach is computationally time consuming. CPU time for portfolios of sizes between 3 and 20 (our study shows that the average size of customers' margin portfolios in the U.S. is smaller than 12) is approximately 0.25 sec. If we multiply this time by a million (this is a good estimate of customers' margin accounts under the custody of a large prime brokerage firm), then we get slightly more than 69 hours of CPU time. Note that this does not include data retrieval time.

Hence, in order to use the CPLEX optimization package, a large brokerage firm on NYSE will require a very significant amount of computational resources because there are only 17.5 hours available to produce margin account status reports by the morning (9:30am) of the next business day, from the end (4pm) of the previous business day.

6.1 Margining Component in the October 2008 Stock Market Crash

As shown in Section 4.1, the risk-based approach requires significantly lower initial margin than the strategy-based approach. Hence, the start of Phase III of the pilot program, when equities, equity options, unlisted derivatives and NBI futures became eligible for risk-based margining, was widely anticipated by investors. Note that Phase III was approved by the SEC in December 2006 to be effective on April 2, 2007. With the start of this phase, investors received an additional incentive to enter the stock market due to significantly lower initial margin requirement offered by the risk-based approach. Consequently, during April-June 2007 numerous investors entered the market stimulating demand for stocks and their derivatives and driving the market up. Growing market, in turn, made the entrance even more attractive. During this period margin debt jumped more than $82.22 billion; see Fig. 1.

At the same time the housing market started to see an increasing number of defaults on mortgage loans. As a result, by July 2007 hedge-funds that invested in subprime mortgages ran into trouble and were "forced to do a lot of selling."[63] Due to the large scale of forced sales and a variety of other reasons the market became bearish.

Investors, who entered the market in April-June 2007 attracted by low initial margin requirements of the risk-based approach, were faced with large margin calls in August-September 2007. In order to fulfil their obligations they were forced to sell equity, which drove the market down even more. Thus, the risk-based approach effectively played a role of a trap with an easy entrance and a hard exit.

Investors, who observed that the market became bearish in August 2007, were given a big incentive to enter bearish positions with low initial and almost zero maintenance margin requirements thanks to the risk-based approach (Fig. 13). This drove the market even further down leading to a subsequent market crash in October 2008.

7 Concluding Remarks

The strategy-based approach is, at this point, the most appropriate one for margining security portfolios in customer margin accounts because it provides an exit strategy. Our computational experiments show that over-margining can be eliminated by the use of offsets of higher sizes and related strategy-based algorithms of higher dimensions.

In contrast, the risk-based approach does not provide any exit strategy for security portfolios. In addition, it can be misleading with respect to the level of risk exposure of the account as shown in Section 4.4. Hence it should not be used for margining security portfolios in customer margin accounts.

However, the risk-based approach can work efficiently for margining index portfolios in customer margin accounts and inventory portfolios of brokers, because the liquidations of such portfolios do not involve any security movements. It is possible in the former case because the underlying instrument is an index, which does not have trading units, and in the latter case because the security movements are needed only inside the inventory of a single broker. The application of the risk-based approach to security portfolios in customer margin accounts is in fact very risky and can result in the pyramid of debt in the bullish market and the pyramid of loss in the bearish market. We suggest that Phases II and III of the pilot program of using the risk-based approach for margining security portfolios in customer margin accounts were major contributors to the growth of margin debt in the period from December 2006 through July 2007 and the subsequent stock market crash in October 2008.

We also have shown that the margin minimization problem with complex offsets can be efficiently solved by optimization packages such as CPLEX. In our opinion, the strategy-based approach was unjustly discredited by the belief that the combinatorial problem stemming from the use of complex offsets could not be efficiently solved with the help of standard optimization packages. Naturally, the development of special optimization algorithms that can take into consideration particularities of the margining can be expected to bring much better results.

Our general observation concerns margin rules for naked positions in underlying securities. In both the strategy and risk-based approaches, the rules are based on assigning a margin rate, i.e., a fixed percentage of the current security market value. This percentage usually depends on the security type (in all rules known today), market price bands (Canadian rule), and expiry date (bonds, debentures, and notes).

For example, in accordance with NYSE Rule 431, the margin rate for a naked long stock position is 50%/25% (initial/maintenance) in the strategy-based approach and 15% in the risk-based approach; see Sections 4.1 and 3.1.

The main question regarding margin rates is how they are assigned. This question is valid for any approach to portfolio margining because it is always possible to have a trivial portfolio with only one security position. Why choose rates of 50%, 20%, 15% or, say, 100% as was done in 1946? This fundamental issue must be addressed in further research on margining. To the best of our knowledge, a model, method, or justifiable technique for assigning securities margin rates does not exist today.[64] So far, this issue has been a prerogative of margin regulators, their intuition and experience. Margin rates are being assigned now by "gut feeling", the same way that option prices were assigned before the appearance of the Black-Scholes formula.

The practice of margin calculations has a long history. However, the science and art of margin calculations has only just begun to evolve. We hope that this paper will attract the attention of margin regulators and academic researchers who are involved in studying efficient exits from the current economic crisis.

8 Acknowledgments

The authors are grateful to Patrick Nobel for his help on pricing options and Eddie Anderson whose critical comments helped to improve this paper.

Notes

[1] From “Obama, Brown call for global changes, say financial regulations need to be revamped” by Roger Runnigen and Robert Hutton, Bloomberg News, p. 4A · MARCH 4, 2009 · USA TODAY.

[2] Entering a security market an investor opens whether a cash account, where the market value of purchased securities must be paid in full and short sales are not permitted, or a margin account, where only a certain portion, i.e., a margin, of the market value of purchased securities can be paid and short sales are permitted. The portion of the market value, complimentary to a margin, is a loan value that is lent to the investor by the broker. Short sales mean that the broker lends to the investor the sold securities. Purchases and short sales create long and short positions in a margin account.

[3] An offset is a group of positions where the risk associated with certain positions is hedged (offset) by the other positions in the group.

[4] If D is a derivative from a security or index U, then U is the underlying instrument for D.

[5] This brokerage term stands for a number of positions in an offset.

[6] This remark does not refer to the literature devoted to studying the relationship between margin requirements and market volatility. A survey of this literature can be found in [Kupiec, 1998].

[7] Chicago Board Options Exchange.

[8] CBOE Regulatory Circular RG03-066, August 13, 2003.

[9] CBOE Regulatory Circulares RG04-90, August 16, 2004, and RG05-37, April 6, 2005.

[10] The U.S. Securities and Exchange Commission.

[11] SEC Release 34-52738, December 14, 2005.

[12] New York Stock Exchange.

[13] IDA Bulletin 3654, August 13, 2007.

[14] The concept also appears in margin regulations and business-related literature under the names “portfolio margining approach” and “risk-based portfolio margining approach”. (In what follows, we omit the adjective “margining” for compactness.) The argument standing behind the term “portfolio” is based on the *misrepresentation* of the strategy-based approach as a treatment of a margin account by considering individual positions only, while the “portfolio approach” treats a margin account as a whole. The strategy-based approach also treats a margin account as a whole; although it does so in a different way, it is still a portfolio approach. So we consider the term “risk-based approach” to be most appropriate. For the same reason, we do not think that the term “rule-based approach,” frequently used on the Internet as synonymous with strategy-based approach, is suitable because the risk-based approach is also based on certain rules, such as Rule 15c3-1a or rules from Regulation T.

[15] The Options Clearing Corporation.

[16] SEC Release 34-27394, October 26, 1989.

[17] Theoretical Intermarket Margining System.

[18] SEC Releases 34-38248, February 6, 1997, see also [GAO/GGD-98-153, 1998]

[19] The SEC published the related NYSE proposal for public comments in SEC Releases 34-46576, October 1, 2002 and 34-50885, December 20, 2004, before approving the approach in July 2005.

[20] www.cboe.com/margin, CBOE Rules 12.4, 9.15(c), 13.5 and 15.8A.

[21] SEC Releases 34-46576, October 1, 2002, and 34-50885, December 20, 2004.

[22] SEC Release 34-52031, July 14, 2005.

[23] Broad Based Index.

[24] Exchange Traded Fund.

[25] SEC Release 34-54125, July 11, 2006.

[26] Narrow Based Index.

[27] However, it was approved on December 12, 2006, SEC Release 34-54918.

[28] Exchange Act Release No. 58251, July 30, 2008, 73 FR 45506, August 5, 2008.

[29] Growth Domestic Product.

[30] See, for example, The Wall Street Journal, July-August 2007, for numerous reports on margin calls and associated forced sales.

[31] From `http://fraser.stlouisfed.org/docs/historical/martin/martin55_0314.pdf`, the speech of William McC. Martin, Jr., Chairman of the Board of Governors of the Federal Reserve System from

April 2, 1951, through January 31, 1970, at the hearing on the study of the stock market before the U.S. Senate Committee on Banking and Currency on Monday, March 14, 1955.

[32] European-style options can be exercised during a short period of time, usually one day, prior to the expiry date.

[33] American-style options can be exercised at any time prior to the expiry date.

[34] Designated Examining Authority, Federal Register, Vol 73, no. 29, February 12, 2008.

[35] Recently the OCC replaced TIMS (Theoretical Intermarket Margining System) with STANS (The System for Theoretical Analysis and Numerical Simulations), which was approved by SEC Release 34-53322, February 15, 2006, on the basis "that the amount of margin it will collect under STANS will be significantly less than the amount of margin it currently collects under TIMS" and that "STANS identifies a more realistic correlative relationship among underlying assets than TIMS."

[36] SEC Release No. 34-57270, February 5, 2008. Also, see footnotes in Section 1.3.

[37] The brokerage term "to play bull, bear, long, short, call or put" means to have a margin account whose positions are primarily bull, bear, long, short, positions in call or put options, respectively.

[38] SEC Release 34-52738, December 14, 2005.

[39] We should mention that it is possible to design *offsets-centipedes* with as many legs as desired. A method in [Matsypura et al., 2007] generates centipedes with up to 134 legs.

[40] See NYSE Rule 431 for U.S., Regulation 100 for Canada, web sites of particular securities markets.

[41] Examples of margin rates in different countries can be found in [Roll, 1989].

[42] The worst case scenarios for the offset holder and the offset holder's broker can be different. That is one of the reasons why margin rules for brokers and their customers are usually not the same.

[43] Exercisable securities, such as options, futures, warrants, convertible bonds, can be exercised, i.e., converted under certain conditions into the underlying security.

[44] Note that the exercises are not to be performed during margin calculations, they are virtual only, as in a simulation of reality.

[45] Here and in the rest of the examples we consider American-style options.

[46] An option writer is an investor who sells an option short and therefore has a short position in this option in his/her margin account.

[47] Component quantities of prime offsets are integers, unlike the quantities of convertible securities with non-integer conversion ratios.

[48] In what follows, $A^\top$ denotes the transpose of matrix A. Thus, the transpose of a row vector is a column vector and vice versa.

[49] These percentages follow Rule 15c3-1a, section (b)(1)(i)(B).

[50] These gains and losses are called "theoretical gains and losses" in SEC Release 34-53577.

[51] For example, if the security s is an option then, after calculating its in-the-money amount, $i_v = \max\{c_v - e, 0\}$ if s is a call option or $\max\{e - c_v, 0\}$ if s is a put option, its outcome in long positions and short positions can be calculated as i_v and $p_v - i_v$, respectively, multiplied by the option contract size for each option.

[52] Note that since we add the position index p to o_v to distinguish the outcomes from different positions, the same index must be added to the parameters c_v, e and p_v in Section 4.1.

[53] The recent OCC's methodology implemented in STANS even disregards portfolio types associated with product groups; see SEC Release 34-53322, February 15, 2006.

[54] SEC Release 34-53577, March 30, 2006.

[55] The haircuts are used as cushions against market falls; see SEC Rule 15c3-1a (b)(1)(v)(B).

[56] In this section we remove the subscript v from the notation related to gains and losses bearing in mind that the guarantee-guarantor deficit must be found for each of the eleven valuation points c_v.

[57] Note that netting is the most radical way of offsetting because it brings the maximum advantage to covering losses. On the other hand, netting "cares" about money movement only. As we show in Sections 4.4 and 4.5, netting is very risky for security portfolios.

[58] Upon the exercise of an index call or put option, its holder receives, respectively, $\max\{u - e, 0\}$ or $\max\{e - u, 0\}$, where u is the underlying index price and e is the option exercise price.

[59] Note that an algorithm of dimension one just calculates the total positions' margin. Offsets of sizes more than four were not permitted up to December 2005 and are currently not being used in margining

practice. The popularity of the risk-based approach, which appeared in the U.S. market in July 2005, stalled further developments of strategy-based margining systems.

[60] We used ILOG CPLEX 12.1 on Dell Precision T7400 with two 3.5 GHz Quad-Core Intel Xeon CPUs, 32 GB RAM running Windows XP 64-bit.

[61] These 32 options were on the IBM stock at the market price of $84.92 and expired on April 17, 2009. The data was taken as of the end of the day of January 16, 2009, from `http://finance.yahoo.com/`. For pricing options we used interest rate `0.3%` and historical volatility `15%` (since 2000). Note that we could have chosen any other stock with sufficient number of options.

[62] Recall that long positions in call options and short positions in put options are bullish, long positions in put options and short positions in call options are bearish.

[63] Gregory Zuckerman and Ianthe Dugan. 2007. Moving the Market: Funds Accelerate Subprime Exit Strategy; Two Firms Move To Stem Bleeding; 'Unraveling' Market. Wall Street Journal, July 5, Eastern Edition. http://www.proquest.com/ (accessed October 1, 2009).

[64] A theory of the investor's optimal margin account based on a model of optimal borrowing for the entire margin account can be found in [Luckett, 1982].

References

Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *J. Pol. Econ.*, 81:637–654.

Brennan, M. J. and Schwartz, E. S. (1977). The valuation of American put options. *J. Financ.*, 32(2):449–462.

Cohen, G. (2005). *The bible of options strategies: the definitive guide for practical trading strategies.* Financial Times Prentice Hall. Pearson, New Jersey.

Cox, J. and Rubinsein, M. (1985). *Options Markets.* Prentice-Hall.

Curley, M. T. (2008). *Margin trading from A to Z: A complete guide to borrowing, investing and regulation.* John Wiley & Sons, New Jersey.

Duffie, D. (2003). Intertemporal asset pricing theory. In Constantinides, G. M., Harris, M., and Stulz, R. M., editors, *Handbook of the Economics of Finance*, volume 1B of *Handbooks in Economics*, chapter 11, pages 632–742. Elsevier, 1 edition.

Fiterman, A. E. and Timkovsky, V. G. (2001). Basket problems in margin calculations: modelling and algorithms. *European J. Oper. Res.*, 129:209–223.

Fleckenstein, W. A. and Sheehan, F. (2008). *Greenspan's Bubbles: The Age of Ignorance at Federal Reserve.* McGraw Hill Professional, New York.

Fortune, P. (2000). Margin requirements, margin loans, and margin rates: practice and principles. *New England Econ. Rev.*, pages 19–44.

Fortune, P. (2003). Margin requirements across equity-related instruments: how level is the playing field? *New England Econ. Rev.*, pages 29–50.

GAO/GGD-98-153 (July 1998). *Risk-based capital: regulatory and industry approaches to capital and risk.* U.S. General Accounting Office, Washington D.C. 20548.

Geelan, J. P. and Rittereiser, R. P. (1998). *Margin regulations and practices.* NYIF Corporation, Division of Simon & Schuster, Inc., New York, 4 edition.

Kupiec, P. H. (1998). Margin requirements, volatility and market integrity: what have we learned since the crash? *J. Financ. Serv. Res.*, 13(3):231–255.

Luckett, D. G. (1982). On the effectiveness of the federal reserve's margin requirements. *J. Financ.*, 37(3):783–795.

Matsypura, D., Oron, D., and Timkovsky, V. G. (2007). Option spreads: centipedes that cannot have more than 134 legs. Technical report, The University of Sydney.

Moore, T. G. (1966). Stock market margin requirements. *J. Polit. Economy*, 74(2):158–167.

Roll, R. (1989). Price volatility, international market links, and their implications for regulatory policies. *J. Financ. Serv. Res.*, 3:211–246.

Rudd, A. and Schroeder, M. (1982). The calculation of minimum margin. *Manage. Sci.*, 28(12):1368–1379.