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Bayesian Forecasting for Financial Risk Management, Pre and Post the Global Financial Crisis

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ABSTRACT

Value-at-Risk (VaR) forecasting via a computational Bayesian framework is considered. A range of parametric models are compared, including standard, threshold nonlinear and Markov switching GARCH specifications, plus standard and nonlinear stochastic volatility models, most considering four error probability distributions: Gaussian, Student-t, skewed-t and generalized error distribution. Adaptive Markov chain Monte Carlo methods are employed in estimation and forecasting. A portfolio of four Asia-Pacific stock markets is considered. Two forecasting periods are evaluated in light of the recent global financial crisis. Results reveal that: (i) GARCH models out-performed stochastic volatility models in almost all cases; (ii) asymmetric volatility models were clearly favoured pre-crisis; while at the 1% level during and post-crisis, for a 1 day horizon, models with skewed-t errors ranked best, while IGARCH models were favoured at the 5% level; (iii) all models forecasted VaR less accurately and anti-conservatively post-crisis.

KEY WORDS: EGARCH model; generalized error distribution; Markov chain Monte Carlo method; Value-at-Risk; Skewed Student-t; market risk charge; global financial crisis.

INTRODUCTION

Financial risk management has undergone much change and greater regulation in the last twenty years following, and in many ways in response to, the major stock-market crash ("Black Monday")

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of October, 1987. Now, another major market incident, the global financial crisis (GFC) in 2008-09, has prompted calls for more and different financial regulation. In order to better control the risk of financial institutions and to protect them against large unexpected losses, the group of G-10 countries agreed in 1988 to sponsor and subsequently form the original Basel Capital Accord. In the last two decades, however, large unexpected losses have continued to occur with regularity: e.g. in December 1994, Orange County (US) announced a loss of $1.6 billion in its' investment portfolio; in 1995, Nick Leeson, of Barings Bank (UK), lost $1.4 billion in speculation, primarily on futures contracts; in 1997, the Asian financial crisis began, which started in Thailand with the financial collapse of the Thai baht; among others, and finally the very recent GFC. Financial markets and the products traded on them are continuing to become more complicated and difficult to properly understand and assess by existing risk management tools and regulations. Such methods and rules clearly need to evolve as well.

Value-at-Risk (VaR) was pioneered in 1993, as a part of the "Weatherstone 4:15pm" daily risk assessment report, in the RiskMetrics model at J.P. Morgan. By 1996, amendments to the Basel Accord (Basel Accord II) allowed banks to use an 'appropriate model' to calculate their VaR thresholds. Jorion (1997) defines VaR as a measure of the highest expected loss, over a given time interval, under normal market conditions, at a given confidence level: VaR is thus a conditional quantile of the asset return loss distribution. Following Basel II, VaR has become more popular and is widely used in practice for risk management and capital allocation. The recommended back-testing guideline proposed by the Basel Committee on Banking Supervision (1996) is to evaluate a one percent (1%) VaR model over a 12 month test period (250 trading days). VaR has been criticised for not measuring the magnitude of a loss in case of an extreme event. As such, and following McAleer and da Veiga (2008), we also consider various criteria measuring the loss magnitude given a violation, such as mean and maximum absolute deviation. These measures go beyond assessing violation rates and allow risk management to incorporate loss magnitude. Further, the different measures of model performance allow financial institutions to select different combinations of alternative risk models to forecast VaR using selection or combining strategies to suit their purpose.

The GFC came to the forefront of the business world and global media in September 2008, with the failure and merging of several American financial companies, e.g. the federal takeover of Fannie Mae and Freddie Mac, Lehman Brothers filing for bankruptcy after being denied support by the Federal Reserve Bank. However, the "credit-crunch" became apparent in January, 2008 and it has been suggested the whole GFC was pre-empted by house prices falling in June, 2007. In late 2008, a number of indicators suggested that the major stock indexes were in a downward spiral globally. Consequently, how to forecast market risk, via VaR, during such extreme periods, becomes a crucial issue in risk management and investment. To shed light on this issue, this study examines a sample of four major Asia-Pacific Economic Cooperation (APEC) stock markets, being the daily stock indices: Nikkei 225 Index (Japan), HANG SENG Index (Hong Kong), the Korea Composite (KOSPI) Index; and the US S&P 500 Index. To test a range of competing models in varying market conditions, the forecast period was split up into two segments: the first finishes at 29 February 2008, well before the effects of the GFC on world markets were clear. The second validation sample starts in August 2008 and includes the worst of the GFC period and some "post-crisis" period as well.

There are many approaches to forecasting VaR: these include non-parametric methods, e.g. historical simulation (using past or in-sample quantiles); semi-parametric approaches, e.g. extreme value theory and the dynamic quantile regression CAViaR model (Engle and Manganelli, 2004); and parametric statistical approaches that fully specify model dynamics and distributional assumptions e.g. RiskMetrics^{TM} (J.P. Morgan, 1996) and GARCH models (see Engle, 1982 and Bollerslev, 1986). The aim of this paper is to compare a range of well-known, modern and fully parametric econometric models to forecast VaR, under a Bayesian framework, before, during and after the GFC. Each model includes a specification for the volatility dynamics and most consider four specifications for the conditional asset return distribution: Gaussian, Student-t, generalized error distributions (GED) and the skewed Student-t of Hansen (1994). When forecasting VaR thresholds, our goal is to find the optimal combination of volatility dynamics and error distribution in terms of the observed violation rates and the magnitude of the deviation of violating returns, both pre and during/after the GFC.

The focus here is on parametric models and Monte Carlo simulation. However, many of the models are flexible, with quite pliable error distributions and differing specifications for volatility dynamics, that can capture the main empirical or stylized facts observed for financial asset return data: fat tails (lepto-kurtosis), volatility clustering and asymmetric volatility (Poon and Granger, 2003). We consider popular variants and extensions of the GARCH model family as follows: RiskMetrics; symmetric GARCH; integrated GARCH (IGARCH), Engle and Bollerslev (1986); asymmetric GJR-GARCH, Glosten, Jaganathan, and Runkle (1993); asymmetric exponential GARCH (EGARCH), Nelson (1991); threshold nonlinear GARCH (TGARCH), Zakoian (1994) and the Markov switching GARCH, Chen, So and Lin (2009). Further, we consider two stochastic volatility (SV) models: the symmetric SV and the threshold nonlinear SV model of Chen, Liu and So (2008).

Bayesian Markov chain Monte Carlo (MCMC) methods have a number of advantages in estimation, inference and forecasting, including: (i) accounting for parameter uncertainty in both probabilistic and point forecasting; (ii) exact inference for finite samples; (iii) efficient and flexible handling of complex models and non-standard parameters, e.g. threshold and degrees of freedom parameters, which can be validly infinite; (iv) efficient and valid inference under parameter constraints. As such MCMC methods were generally used to forecast VaR thresholds for each model in this paper. We follow Chen and So (2006) and design an efficient, adaptive MCMC sampling scheme for estimation and quantile forecasting.

Section 2 reviews the list of heteroscedastic models considered, whose details are given in an Appendix, and details the Bayesian MCMC methods used for estimation and forecasting. The amendment to the Basel Accord was designed to reward institutions with superior risk management systems and suggested back-testing procedures, whereby actual (past) returns were compared with forecasts of VaR, be used to assess the quality of 'internal' models; we favour this approach. Seven different criteria are used to compare the forecasting performance of the various conditional volatility models considered in Section 4, namely: (1) violation rates; (2) mean market risk charge (MRC); (3) maximum absolute deviation (AD) of violations; (4) mean AD; (5)observed penalty factor; (6) the conditional coverage test; and (7) the unconditional coverage test. The last two criteria are the standard back-testing procedures. Section 5 presents a simulation study of EGARCH with three error distributions showing the estimation performance of the methods in Section 3. Section 6 presents the empirical results and forecasting study. Concluding remarks are given in Section 7.

MODELS

We investigate general Bayesian VaR forecasting from a list of nine popular parametric volatility models with specified dynamics and four error distributions. The volatility dynamics for each model are specified in detail in Appendix A. Each model has the general mean equation and error specification:

$$r_t = a_t, \quad a_t = \sqrt{h_t}\varepsilon_t, \quad \varepsilon_t \sim D(0,1),$$

where r_t is the return observation at time t; ε_t is a sequence of i.i.d. random variables, with mean zero (0), variance one (1) and distribution D; and h_t is the conditional variance of r_t. Each model has a dynamic specification for h_t, as in Appendix A. The common names for the models are: GARCH, IGARCH, RiskMetrics, GJR-GARCH, EGARCH, Threshold GARCH (TGARCH), Markov switching GARCH, stochastic volatility (SV) and threshold SV.

Four error distributions are used for the i.i.d. disturbances in each GARCH-type model ε_t. The choice $D(0,1) \equiv N(0,1)$ is standard, and labeled as (a). The Student-t (b), GED (c), and skewed Student-t (d) distributions need to be standardized to have unit variance, as specified in Appendix A.

BAYESIAN APPROACH

Bayesian methods usually require the specification of a likelihood function and a prior distribution on model parameters. This section presents the general likelihood functional form for all models considered in the paper and then presents specific details for two of the nonlinear models, together with details of the prior distributions employed under GED and skewed Student-t errors. We give details in the case of the EGARCH model with GED errors and GJR with skewed Student-t errors. Details of the likelihoods for the other models can either be deduced from the model forms above, or found in the papers referenced above.

Let $\boldsymbol{\Theta}$ denote the full parameter vector for any of the combinations of model and error distribution considered. The conditional likelihood can be written as:

$$L(\mathbf{r}|\boldsymbol{\Theta}) = \prod_{t=1}^{n} \frac{1}{\sqrt{h_t}} p_{\varepsilon}\left(\frac{r_t}{\sqrt{h_t}}\right), \tag{1}$$

where h_t is given by the relevant volatility equation and $p_{\varepsilon}(\cdot)$ is the relevant error density function for ε_t.

Exponential GARCH model and prior

Let $\boldsymbol{\Theta}$ denote the vector $(\alpha_1, \alpha_2, \gamma, \beta, \lambda)$. The conditional likelihood for the EGARCH-GED error model is thus:

$$L(\mathbf{r}|\boldsymbol{\Theta}) = \left[\frac{\lambda}{2\sigma\Gamma(\frac{1}{\lambda})}\right]^n \prod_{t=1}^{n} \frac{1}{\sqrt{h_t}} \exp\left\{-\sum_{t=1}^{n}\left|\frac{r_t}{\sqrt{h_t}\sigma}\right|^{\lambda}\right\}, \tag{2}$$

where $\mathbf{r} = (r_1, \ldots, r_n)$ and $\sigma = [\Gamma(\frac{1}{\lambda})/\Gamma(\frac{3}{\lambda})]^{0.5}$.

The prior distribution is chosen to be reasonably uninformative so that the likelihood dominates inference. The prior for $\boldsymbol{\alpha}$=$(\alpha_1, \alpha_2, \gamma)$ is chosen as a Gaussian: $\boldsymbol{\alpha} \sim N(\mathbf{0}, \boldsymbol{V})$, with the diagonal variance-covariance matrix ($\boldsymbol{V}$) chosen to have 'large' diagonal elements; e.g. 1. This prior makes sense since these parameters are unrestricted, but empirical studies show they are usually estimated to be close to, though still significantly different from, 0. This prior is reasonably diffuse in the region close to 0, and well beyond, where empirical parameter estimates usually lie.

The parameter β is restricted for stationarity, via $|\beta_1| < 1$. The prior for this parameter is chosen to be uniform over this region. For the shape parameter λ, Vrontos, Dellaportas, and Politis (2000) set a log-normal prior with mean $1.04 \cdot 10^{22}$ and variance $2.93 \cdot 10^{87}$. Such a choice seems excessively diffuse. Instead we employed a half-normal distribution, $\lambda \sim N_c(0, 1)$, which is a standard normal truncated to lie on the positive real line, i.e. $\lambda \in (0, \infty)$.

The prior for $(\boldsymbol{\alpha}, \beta, \lambda)$ is assumed independent in the three groupings, so that:

$$p(\boldsymbol{\alpha}, \beta, \lambda) \propto \exp\{-\frac{1}{2}(\boldsymbol{\alpha})^T V^{-1}(\boldsymbol{\alpha}) - \frac{1}{2}\lambda^2\} \cdot I(\beta \in (-1, 1)). \tag{3}$$

GJR-GARCH model and prior

The GJR-GARCH model specification is given in (16), here considered with a skewed Student-t error distribution. The likelihood for the GJR-GARCH-st model is thus:

$$L(\boldsymbol{r}|\boldsymbol{\alpha}, \nu, \eta) = \prod_{t=1}^{n} \frac{bc}{\sqrt{h_t}} \left\{ \left[\left[1 + \frac{1}{h_t(\nu-2)} \left(\frac{br_t + a\sqrt{h_t}}{1-\eta} \right)^2 \right]^{\frac{-(\nu+1)}{2}} I_{1,r_t} + \left[1 + \frac{1}{h_t(\nu-2)} \left(\frac{br_t + a\sqrt{h_t}}{1+\eta} \right)^2 \right]^{\frac{-(\nu+1)}{2}} I_{2,r_t} \right] \right\}$$

where $\boldsymbol{\alpha} = (\alpha_0^{,}\alpha_1, \beta_1, \gamma_1)'$; $I_{1,r_t} = I\left(r_t < \frac{-a\sqrt{h_t}}{b}\right)$, and $I_{2r_t} = 1 - I_{1r_t}$.

Again priors are set that are mostly uninformative over the restricted parameter region in (17). That is, the prior for $\boldsymbol{\alpha}$ is flat over (17); while for the degrees of freedom, we re-parameterize via $\tau = \nu^{-1}$ and set the prior for τ as U$(0, 0.25)$. This ensures that $\nu > 4$ and that the first four moments of the error distribution are finite. Finally, we set a flat prior over $\eta \in (-1, 1)$.

These settings for the EGARCH and GJR-GARCH models are indicative of the prior settings used for the other models. Further details for the other models may be found in Chen and So (2006). The joint posterior distribution for each model is formed by multiplying the likelihood by the joint prior for that model. The posteriors for each model are not in the form of a standard or known distribution in the parameters. As such we turn to computational MCMC methods to obtain estimation and inference from each posterior.

MCMC methods

MCMC methods have proven successful for nonlinear time series in general, e.g. see Chen and Lee (1995); Vrontos, Dellaportas, and Politis (2000); Chen and So (2006) and others. MCMC methods simulate iteratively from the conditional posteriors of groups of model parameters. We discuss general details here, as well as some specific details for the EGARCH model and for the GED and skewed Student-t distributions. This is the first time a skewed Student-t error GARCH-type model has been estimated by MCMC methods in the literature, to the best of our knowledge.

The typical parameter groupings are: $\boldsymbol{\alpha}$, plus any thresholds or parameters in the error distribution. For the EGARCH-GED model, parameter groupings are: (i) $(\boldsymbol{\alpha}, \beta)$ and (ii) λ. For the TGARCH model with skewed Student-t errors, parameter groupings would be: $\boldsymbol{\alpha}$, w, d, ν, η. The posterior for each parameter group, conditional upon the other parameters, is formed separately by multiplying the likelihood (1) by the prior for that parameter group. None of these conditional posterior distributions are in a standard form to facilitate direct simulation from, for all models here, as such we turn to Metropolis-Hastings type methods; see Metropolis *et al.* (1953) and Hastings (1970).

To speed convergence and to allow optimal mixing properties, we employ an adaptive MCMC algorithm that combines a random walk Metropolis (RW-M) and an independent kernel (IK-)MH algorithm, following Chen and So (2006). For the burn-in period, a Gaussian proposal distribution is employed in a RW-M algorithm. The variance-covariance matrix of this proposal is subsequently tuned to achieve optimal acceptance rates, as in Gelman *et al.* (1996). After the burn-in period, the sample mean vector and sample variance-covariance matrix of the iterates are formed. These are employed in the sampling period as the proposal mean and proposal variance-covariance matrix for a Gaussian proposal in an IK-MH algorithm. Such an adaptive proposal updating procedure will be highly efficient, as long as the burn-in period has 'covered' the posterior distribution. See Chen and So (2006) for more details. We extensively examine trace plots and autocorrelation function (ACF) plots from multiple runs of the MCMC sampler, for each model parameter and from different starting positions, to confirm convergence and infer adequate coverage. Details for the sampling scheme for the MS-GARCH model can be found in Chen, So and Lin (2009).

FORECASTING RETURNS, VOLATILITY AND VaR

Forecasting utilizing MCMC methods can efficiently incorporate parameter uncertainty in a straightforward fashion. The steps below outline how to generate l-step-ahead l-day return and volatility forecasts, from the models and error distributions considered, using forecast origin $t = n$. These steps are performed at each MCMC iteration in the MCMC sampling period, using the current iterate (j) for each model's full parameter set, denoted $\mathbf{\Theta}^{[j]}$:

1. Calculate h_{n+1} using the in-sample data up until time $t = n$, $\boldsymbol{r}$, the relevant volatility equation from (1)-(9) and $\mathbf{\Theta}^{[j]}$. Set $i = 1$.
2. Simulation step: draw $\varepsilon_{n+i} \sim D(0, 1)$ where D is one of the four standardized error distributions. Calculate $r_{n+i} = a_{n+i} = \sqrt{h_{n+i}}\varepsilon_{n+i}$.
3. Evaluation step: evaluate h_{n+i+1} using h_{n+i}, the simulated a_{n+i} from 2., the in-sample data $\boldsymbol{r}$, the relevant volatility equation from (1)-(9) and $\mathbf{\Theta}^{[j]}$.
4. Set $i = i + 1$ and go to 2.

The process is continued up to the simulation of ε_{n+l} and calculation of r_{n+l}. These steps generate one realization from the joint distribution of $r_{n+1}, \ldots, r_{n+l}|\boldsymbol{r}, \boldsymbol{\Theta}^{[j]}$. Repeating this process for $j = 1, \ldots, J$ while also simulating $\boldsymbol{\Theta}^{[j]}$ from the relevant model's posterior distribution, numerically integrates out $\boldsymbol{\Theta}$ and obtains a Monte Carlo sample from the forecast distribution $r_{n+1}, \ldots, r_{n+l}|\boldsymbol{r}$. Summing each l-day vector of returns, i.e. $\sum_{i=1}^{l} r_{n+i}$ gives one sample from the l-day forecast return distribution, conditional upon $\boldsymbol{r}$, as required.

The main purpose of this paper is to forecast VaR thresholds. VaR at level α can be defined as:

$$P_r\left(\Delta V(l) \leq -\text{VaR}\right) = \alpha, \tag{4}$$

where $\Delta V(l)$ is the change in the asset value over l time periods. As standard, we consider $\alpha = 0.05, 0.01$.

A one-step-ahead VaR is simply the α-level quantile of the $l = 1$-step conditional distribution $r_{n+1}|\mathcal{F}_n \sim D(0, h_{n+1})$. Here h_{n+1} is given by one of the models (1)-(7), and D is the relevant error distribution in (a)-(d). This predictive distribution is estimated via the MCMC simulation using the steps above: i.e. the MCMC samples give $\boldsymbol{\Theta}^{[j]}, h_{n+1}^{[j]}$ for iterates $j = M + 1, \ldots, N$, which is the MCMC sampling period. Then, the quantile VaR is given by:

$$\text{VaR}_{n+1}^{[j]} = -\left[D_{\alpha}^{-1}(\boldsymbol{\Theta}^{[j]})\sqrt{h_{n+1}^{[j]}}\right], \tag{5}$$

where D^{-1} is the inverse CDF for the distribution D. For errors (b), (c) and (d) the CDF depends on some unknown parameters, which explains the notation. Then, the final forecasted one-step-ahead VaR is the Monte Carlo posterior mean estimate:

$$\text{VaR}_{n+1} = \frac{1}{N - M}\sum_{j=N-M}^{N} \text{VaR}_{n+1}^{[j]}, \tag{6}$$

The l-day VaR is the α-level quantile of the l-day return distribution $A_n(l) = \sum_{i=1}^{l} r_{n+i}|\mathcal{F}_n$. The steps detailed above simulate a Monte Carlo sample $A_n^{[j]}(l) = a_{n+1}^{[j]} + ... + a_{n+l}^{[j]}$; $j = 1, \ldots, J$ from this forecast distribution. The l-day VaR is:

$$\text{VaR}_n(l) = -G_{\alpha}^{-1}(A_n(l)|\mathcal{F}_n). \tag{7}$$

where in general the l-day CDF G is not D. As such, we take the empirical or sample quantile estimate from the Monte Carlo sample $A_n^{[j]}(l)$; $j = 1, \ldots, J$ at the required level α to estimate $\text{VaR}_n(l)$.

One exception is under the RiskMetrics^{TM} model. Here, the square root of time rule is implied by the model so that:

$$\text{VaR}_n(l) = \sqrt{l} \times \text{VaR}_{n+1}. \tag{8}$$

Table I. Basel Accord Penalty Zones

Zone	Number of Violation	Plus factor k
Green	$0 \sim 4$	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	10	1.00

The number of violations is calculated on the basis of 250 trading days.

Testing and comparing VaR models

Here details for the criteria employed, to compare and test the competing VaR forecast models are given, including forecast accuracy, minimum loss and hypothesis testing criteria. These are measured by observed violation rates, market risk charges (Jorion 2002), absolute deviations given a violation (see McAleer, 2008) and two standard back-testing criteria.

A simple method to compare VaR forecasts is the violation rate (VRate):

$$\text{VRate} = \frac{\Sigma_{t=n}^{n+m} I(r_t < -\text{VaR}_t)}{m}, \tag{9}$$

where n is the number of in-sample observations and m is the forecast sample size. Naturally, VRates close to α are desirable. Further, under the Basel Accord, models that over-estimate risk (VRate $< \alpha$) are preferable to those that under-estimate risk levels.

The Basel II Accord of 1996 gives guiding principles to help financial institutions better assess the violations from VaR models. Table I reproduces Table VII from McAleer and da Veiga (2008), which categorizes zones based on $\alpha = 0.01$ and 250 forecast trading days: Green indicates a good model, Yellow indicates possibly, but less, acceptable models and Red indicates an unacceptable model.

The Accord recommends the use of 'market risk charge' to further assess model performance. The market risk charge is set at the maximum of the previous day's VaR and the average VaR over the last 60 days multiplied by a penalty weight. As in Jorion (2002) the equation is:

$$\text{MRC}_t = \sup \left\{ \text{VaR}_{t-1}, \overline{\text{VaR}}_{60} \cdot (3+k) \right\}, \tag{10}$$

where VaR_{t-1} is the previous day's VaR estimate, $\overline{(VaR)}_{60}$ is the 60 day average VaR, and k is the penalty factor, as shown in Table I, which penalizes anti-conservative market risk projections. Under the Accord MRC is defined for a horizon of 10 trading days when $\alpha = 0.01$ and it must be based on at least a year of historical in-sample data. Models with lower MRCs are considered better in terms of risk measurement.

The magnitude of violating returns, not just their VRate, is also important, i.e. the expected loss given a violation. Thus, measures of loss magnitude are also considered here. The AD of violating returns, considered by McAleer and da Veiga (2008), is:

$$AD_t = |r_t - (-(VaR)_t)| \, , \tag{11}$$

defined only when r_t is a violation. The mean and maximum AD are calculated here to compare competing VaR models: models with lower mean and/or maximum ADs are preferred.

SOME MONTE CARLO RESULTS

Simulation studies are performed to examine the effectiveness of the MCMC sampling scheme. The error distributions were chosen as: (i) the GED with parameters $\lambda = 1, 1.5$ and $\lambda = 2$ and (ii) the skewed Student-t $St(7, \eta)$ with $\eta = -0.05, -0.5, -0.99$. Specifically, the models we consider are:

Model 1: The true model is an EGARCH-GED model.

$$\begin{aligned} r_t &= a_t, \\ a_t &= \sqrt{h_t}\varepsilon_t, \qquad \varepsilon_t \overset{\text{i.i.d.}}{\sim} \text{GED}(0, \lambda), \\ \ln(h_t) &= -0.2 + 0.2\frac{|a_{t-1}| - 0.26a_{t-1}}{\sqrt{h_{t-1}}} + 0.93\ln(h_{t-1}), \end{aligned}$$

where ε_t follows the standardized $\text{GED}(0, \lambda)$ distribution. The form of Model 2 is the same as Model 1, but the distribution of ε_t is set as the skewed Student-t, $\text{St}(7, \eta)$.

For each model we simulated 100 replicated data sets, repeating this over sample sizes of n=2,000 and 4,000. For each dataset we used a total of 20,000 MCMC iterations, with a burn-in period of M=8,000 iterations. We choose initial values for the EGARCH parameters as $\boldsymbol{\alpha} = \mathbf{0}$ and tail-thickness parameter $\lambda = 0.1$ in Model 1, while the degrees of freedom ν was set at 200 and η was set as 0 in Model 2. These are generally quite poor starting values and our results are not sensitive to different choices.

Table II about here

Table II shows the estimation results for the simulated datasets, including true parameter values, means, standard deviations, 2.5 and 97.5 percentiles for the 100 posterior mean estimates, over the replicated data sets, at each sample size. All of the means of the estimates are close to their respective true values, with reasonable standard errors that reduce with increasing sample size. For the GED errors, $\lambda = 2$ causes no problem at all, despite the low prior weight attached to this value from the half-standard normal prior. For the skewed Student-t error model, $\eta = -0.99$ also causes minimal problems, despite being close to the boundary value of $\eta = -0.99$, the bias in estimation being practically negligible.

EMPIRICAL STUDY

For the empirical study, an asset portfolio of four major Asia-Pacific Economic Cooperation (APEC) stock markets is considered. Four daily stock price indices, including three major Asian markets: the Nikkei 225 Index (Japan), HANG SENG Index (Hong Kong) and the Korea Composite (KOSPI) Index; as well as the US S&P 500 Index. The data were obtained from Datastream International over a 12-year time period, from October 1, 1997 to December 30, 2009.

For each market, the returns are the logarithmic difference of the daily price index, as a percentage:

$$r_t = (\log(P_t) - \log(P_{t-1})) \times 100,$$

where P_t is the closing index value on day t. We consider a single equally weighted portfolio of these assets, with return:

$$r_{p,t} = \sum_{i=1}^{4} w_i \times r_{i,t},$$

where $r_{p,t}$ is the portfolio return at time t, $r_{i,t}$ is the return of asset $i = 1, \ldots, 4$ at time t and $w_i = 0.25$ is the weight on each market's return. This portfolio return series is now analyzed.

To examine the performance of the models under highly varied market conditions, this study examines two distinct forecasting periods. The first complete data set is divided into two: an in-sample period of October 1, 1997 to July 8, 2005, and a forecast or validation period, containing the $m = 588$ observations: July 9, 2005 to February 29, 2008. This is a period before the effects of the GFC hit the markets.

To examine how the models perform during the 2008-09 GFC, and evaluate how the crisis affects risk management, a second time span in considered: a learning period of October 4, 2000 to July 31, 2008, of similar sample size to the pre-crisis learning sample, and a 2nd validation or forecast period of 316 trading days: August 1, 2008 to December 30, 2009. This covers the worst effects of the GFC on markets.

A rolling window approach is used to produce 1 and 10-day forecasts of the 1% VaR and 1-day forecasts of the 5% VaR thresholds in both forecast samples. The models were: RiskMetrics^{TM}, six GARCH-type models: IGARCH, GARCH, TGARCH, GJR-GARCH, EGARCH, and MS-GARCH, respectively, where the GARCH-type models all employed each of the four error distributions; and two SV models: the symmetric SV and THSV models, specified in equations (22) − (23), with Gaussian and Student-t distributions only. The threshold value $r = 0$ and the delay lag $d = 1$ were used, in accord with general assumptions in the literature. Thus, 29 risk models in total are considered. The first n return observations, i.e. each in-sample period, were initially used to estimate each model and then to forecast the returns $r_{n+1}, \ldots, r_{n+l}$, as detailed in Section 5, for $l = 1, 10$. The in-sample period was then rolled forward by one observation, so that it ranged from r_2 to r_{n+1}, whereby the returns $r_{n+2}, \ldots, r_{n+l+1}$ are forecasted. This roll-forward process was repeated until each day in the forecast sample was forecast. To strike a balance between estimation efficiency and a feasible number of forecasts, a rolling window size of approximately $n = 1700$ observations was chosen, leaving $m = 588$ observations to be forecasted in the first sample period, and $m = 316$ in the second.

For illustration, time series plots of the one-day-ahead forecasts of h_t based on the GJR-GARCH-t, GJR-GARCH-st, EGARCH-GED, and EGARCH-t models are presented in Figure 1. This illustrates the similarity among well-specified volatility models, but also highlights that differences can occur, especially in periods of high volatility.

Back-testing

Two back-testing criteria (unconditional, UC and conditional coverage, CC) for examining the accuracy of the models for VaR are employed. The simplest method tests the hypothesis that the VRate is equal to α. Kupiec (1995) examines whether VaR estimates, on average, provide correct UC of the lower α percent tails of the forecasted return distributions. Christoffersen (1998) developed a CC test that simultaneously examines unconditional coverage and independence of violations: it is a joint test that the true violation rate equals α and that the violations are independent.

Several criteria are used to compare the forecasting performance of the various conditional volatility models considered, namely: (1) VRate; (2) mean MRC; (3) maximum AD of violations; (4) mean AD; (5)observed penalty factor; (6) the CC test; and (7) the UC test.

1 day forecasting results: pre-crisis period

We first discuss the pre-crisis forecast period: July 9, 2005 to February 29, 2008. Table VII shows p-values for the UC and CC tests for the one-day VaR forecast models at the 99% confidence level

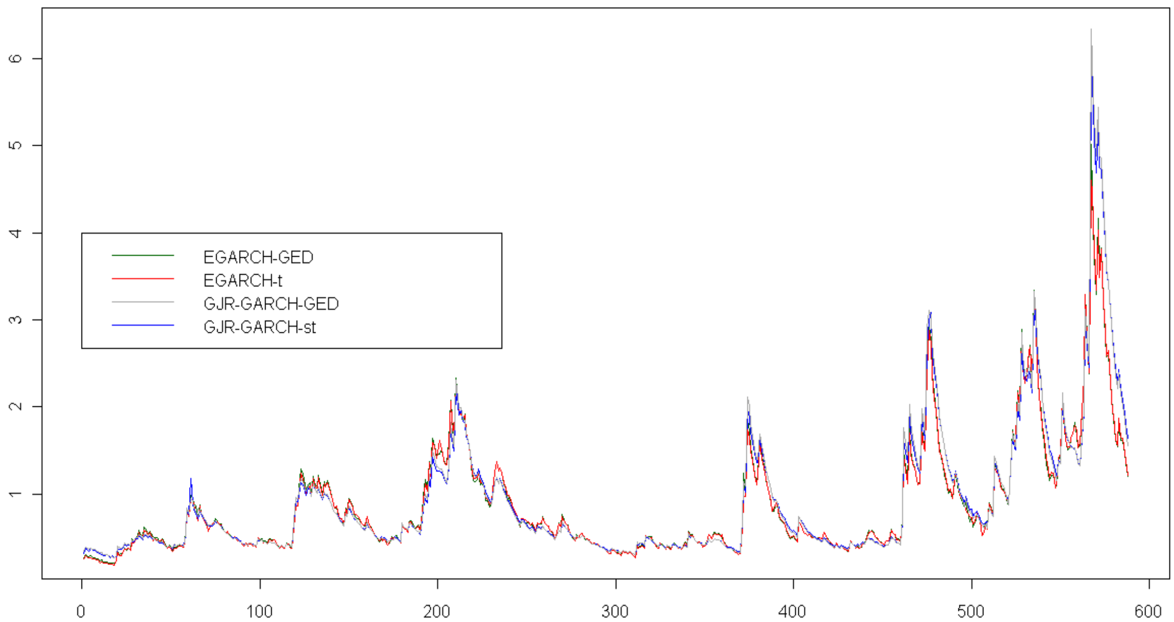


Figure 1. The one-step-ahead volatility forecasts for portfolio return.

for the pre-crisis period in columns 2 and 3. Only the RiskMetrics^{TM} and THSV-n models, which fail both tests at the 5% level, can be rejected among the 29 forecast models. As usual, these tests have revealed that most models cannot be formally rejected as accurate VaR forecasters under quiet market conditions. However, during the financial crisis period, many of the models can be rejected when $l = 1$

Table III presents the first five criteria for each model (for 99% and 95% one-day VaR). In order to evaluate overall performance, we rank the 29 forecast models for each criteria and each VaR level in Table III. For each model and given α, the closest VRate ratio to one is ranked 1, the next closest ratio ranked 2 and so on. These ranks are not given to save space. For $\alpha = 1\%$ and 1-day-ahead forecasting, there are ten best models in terms of VRate: the GARCH-GED, GARCH-st, GJR-GARCH-t, GJR-GARCH-GED, GJR-GARCH-st, EGARCH-GED, MS-t,MS-GED, MS-st and SV-t models, all with $\hat{\alpha} = 1.02\%$. The next best two models are the EGARCH-st and IGARCH-st with $\hat{\alpha} = 0.85\%$. This is a mix of symmetric, asymmetric and nonlinear volatility models. However, five of the top 12 ranked models for VRate have skewed Student-t errors, four have GED and three have Student-t errors: clearly fat tails are required in this dataset. Further five of these twelve are asymmetric volatility models.

In terms of mean market risk charge (MRC), 6 of the top 7 ranked models had Gaussian errors. Since the Gaussian error models all under-estimated risk levels at 1%, it is not surprising they show the smallest MRC, which depends on the average VaR over 60 days. Under the maximum and mean ADs the asymmetric models dominate the top rankings, with 6 of the top 9 ranked models, for AD Max, and 10 of the top 12 for AD Mean. Further, under AD fat-tailed errors occupy the top 8 rankings for AD max and 9 of the top 12 for AD mean.

The overall best models are the GJR-GARCH models: with GED, Student-t, skewed Student-t and Gaussian errors. The four EGARCH models are next best overall took. Thus asymmetric volatility models did best here, while among these models, those with fat-tailed errors did best, especially those with GED and skewed-t errors. The RiskMetrics^{TM} model performed the worst in two of the five measures, including VRate (with a large 2.72%), had the largest penalty factor and was overall close to the THSV-n model in performance.

It is clear that volatility asymmetry is highly important at $\alpha = 0.01$ and $l = 1$ while the choice of error distribution was less important prior to the GFC. Further, GARCH-type models mostly finished well ahead of the SV-type models; only the SV-t was competitive with any GARCH model here, with the other three SV models ranking close to the bottom across all measures. The results suggest that, prior to the crisis, at the 1% quantile of the distribution, the asymmetric volatility effect, is strong and important and capturing this feature allowed better predictability for extreme returns in this portfolio, far more so than the shape of the (error) distribution and any associated properties like skewness, kurtosis, etc did.

Table III about here

For $\alpha = 5\%$, the overall best models are the GARCH-st, GJR-GARCH-st model (which ranked 1st for both mean and max AD), the EGARCH-st and the IGARCH-st models, so the first four overall best models had skewed Student-t errors. The RiskMetrics^{TM} model ranked last for VRate and close to last overall, the THSV models overall being marginally worse. Clearly, skewed errors are highly important at $\alpha = 0.05$ when $l = 1$ and a GARCH or GJR specification seems best under that choice. The results suggest that at the 5% quantile of the distribution, the shape of the (error) distribution, especially whether it is skewed, is very important when $l = 1$, and capturing this feature allowed better predictability for the 5th percentile of returns in this portfolio. The asymmetric volatility effect was also still important, but was secondary in this respect.

Figure 2 exhibits one-day ahead VaR forecasts and realized returns for the best four models considered, in the forecast sample, at $\alpha = 0.01$. The four GJR-GARCH models' VaR forecast thresholds are violated six to eight times in 588 returns. In summary for one-day ahead VaR forecasting in this sample, asymmetric models have dominated the overall rankings at $\alpha = 0.01$, while still featuring prominently at $\alpha = 0.05$; while skewed Student-t errors were only strongly favoured when $\alpha = 0.05$. The best combined choice of model was the GJR-GARCH with skewed Student-t errors. The RiskMetrics^{TM}, symmetric SV with Gaussian errors and both THSV models tended to be at or near the bottom of the rankings for this sample of data under these measures.

1 day forecasting results: GFC period

We now discuss the results at the 1% risk level for 1-day-ahead forecasting in the period that contains the GFC: August 1, 2008 to December 30, 2009. Table VII shows p-values for the UC, CC tests at the 99% confidence level for this period in columns 4 and 5. The RiskMetrics^{TM}, GJR-GARCH-n, EGARCH-n, EGARCH-t, EGARCH-GED and all four SV-type models are rejected by both tests at the 5% level. Further, models rejected by UC only include the GJR-GARCH-t, GJR-GARCH-GED and MS-n. These models are excluded from the discussion to follow.

Results for the other five criteria are shown in Table IV. For models surviving the UC, CC tests, there are nine best models in terms of VRate: the GARCH-t, GARCH-GED, GARCH-st, IGARCH-t, IGARCH-GED, IGARCH-st, MS-t, MS-GED and MS-t, all with $\hat{\alpha} = 1.58\%$, i.e. risk under-estimated by 58%, with 5 observed violations compared to the expected 3.16; all models under-estimated risk levels in this GFC dominated period. This is a mix of symmetric and nonlinear volatility models, with asymmetric volatility and Gaussian error distributions not represented. Six of the nine models have non-stationary volatility equations, indicating the enormous and quickly changing effects from the GFC.

In terms of MRC, again MS, GARCH and EGARCH models occupied the top 7 ranks, again all with fat-tailed errors. Under the maximum AD the rejected EGARCH model takes the first three rankings, while four of the top 6 ranked models have skewed-t errors; for both max AD and mean AD IGARCH-st ranks best among surviving models, followed by GJR-st and GARCH-st: clearly skewed errors are important for ADmax and mean. The overall best models are GARCH-st model and IGARCH-st. Most of the best overall models have skewed-t errors, while models with Gaussian errors all did poorly on most criteria. The RiskMetrics^{TM} model ranked worst of all the GARCH-type models, but ahead of the four SV models that again occupied the bottom, now including the SV-t.

Clearly, during and following the GFC, a skewed error distribution with fat tails is very important to capture risk dynamics and level, at 1%, at a one day horizon. Gaussian errors were least favoured, while nonlinear and symmetric volatility models were favoured over asymmetric ones.

For $\alpha = 5\%$, again all models under-estimated risk, s.t. $\hat{\alpha} > 5\%$. The overall best models are the IGARCH-st (top ranked in VRate, with 5.38%), GJR-GARCH-st, IGARCH-GED and IGARCH-n models. The RiskMetrics^{TM} model did much better here ranking 6th overall and equal 4th for violation rate. The SV models, however, again performed the worst of all models.

The results suggest that at the 5% quantile of the return distribution in the GFC period, the shape of the (error) distribution, and whether asymmetry is included, are not the most important aspects when $l = 1$. Instead, non-stationary IGARCH models seem to do best, even including the RiskMetrics approach.

At both 1, 5% levels during the crisis IGARCH models performed comparably among the best during and after the GFC. At the 1% level fat tails and skewness were important in the error distribution; while at 5% the error distribution was not that important.

Table IV about here

10 day forecasting results: pre-crisis

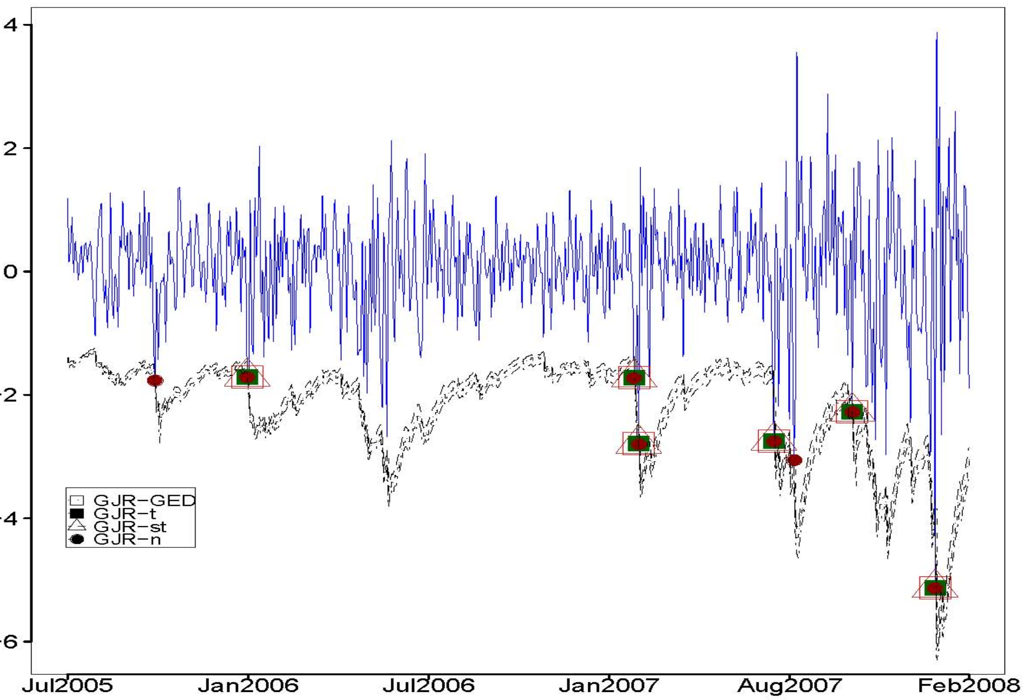
When considering 10-day VaR, it is not appropriate here to conduct either the UC or CC tests since we considered over-lapping ten-day returns. We would expect these returns to cluster, as would our VaR forecasts and hence the observed violations. Violations should thus not be independent when over-lapping returns are used, nor should the iid assumption in the UC test be valid.

The empirical results of the 10-day VaR forecasting ($l = 10$) for the pre-crisis forecast period are in Table V. The amendments to Basel II allow banks freedom to use 'appropriate' internal models to measure their exposure to market risks, requiring this to be summarized as a 1% Value-at-Risk over a 10 day horizon. Thus only $\alpha = 1\%$ was considered. Here, the stand-out best model in terms of VRate was the EGARCH-n, with $\hat{\alpha} = 1.04\%$, followed by the EGARCH-st with $\hat{\alpha} = 1.21\%$. The GJR-GARCH models ranked poorly here with 1.55% (skewed Student-t error) and 1.73% for the other error distributions, while the RiskMetrics^{TM} was worst with 2.94%. In terms of mean MRC the TGARCH-t model ranked best, with the GJR models all ranking in the bottom places. For both maximum and mean ADs, the EGARCH and GJR-GARCH obtained all the top 8 rankings, with EGARCH best. The overall top ranked models were the four EGARCH models, with EGARCH-n and EGARCH-st the best two. The RiskMetrics^{TM} model was again the last ranked model overall and regarding VRate.

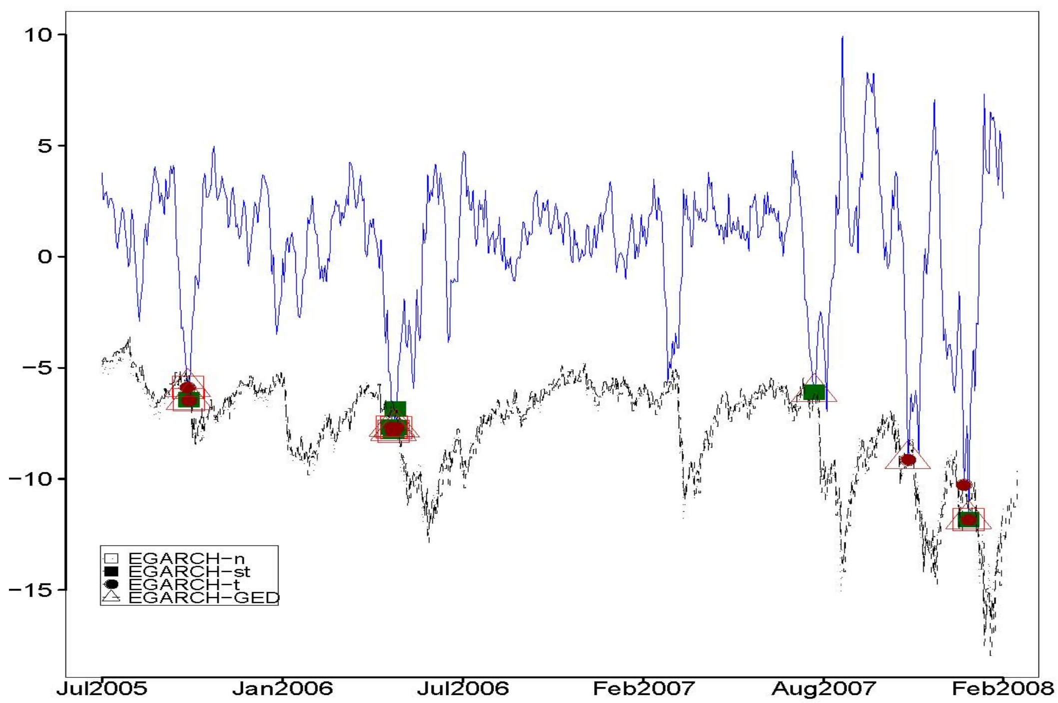
In summary for ten-day return VaR forecasting, at $\alpha = 0.01$, in this pre-crisis sample, asymmetric models have dominated, with the E-GARCH dominating the high rankings; the EGARCH-n ranking first or second for 3 forecast risk measures. The RiskMetrics^{TM} model was again at the bottom of the rankings for this sample of data under these measures.

10 day forecasting results: GFC period

The 10-day VaR forecasting results and rankings for the 2nd forecast sample period are given in Table VI. Here, all models under-estimate risk levels substantially. The best models in terms of VRate were the IGARCH-st and the GJR-GARCH-n, each having $\hat{\alpha} \approx 3$, indicating that observed



(a)



(b)

Figure 2. VaR forecasts for the period before the global financial crisis (a) 1-day-ahead and (b) and ten-day ahead VaR forecasts at 1% level.

VRate was 3 times higher than nominal. Oevrall, the IGARCH-st, IGARCH-GED and IGARCH-n were best overall. In terms of mean MRC the EGARCH models ranked best, followed by the TGARCH models, with the IGARCH and RM models ranking in the bottom places. For both maximum and mean ADs, the IGARCH and GARCH obtained most of the top 10 rankings. The overall best models were the IGARCH-st, IGARCH-GED and IGARCH-n models. The SV-type and RiskMetrics^{TM} model were the last ranked models overall.

In summary for ten-day return VaR forecasting, at $\alpha = 0.01$, in this GFC dominated sample, all the model struggled and substantially under-estimate risk levels. The IGARCH model did comparatively better, but no model does well at all. The RiskMetrics^{TM} model was again at the bottom of the rankings for this sample of data under these measures.

Tables V-VI about here

For these two forecast periods, it seems that completely different models have dominated for $l = 1, 10$ and during pre-crisis and crisis periods. No overall single model can be recommended in both quiet and highly volatile market conditions. Instead, the best model depends on the forecast horizon l and quantile level α and overall market conditions. For one ($l = 1$) and ten ($l = 10$) day VaR forecasting pre-crisis, modeling asymmetry is very important, but in different ways. For one-day forecasting the GJR-GARCH with skewed Student-t errors did best overall, while the GJR models as a group occupied the top 4 placings at $\alpha = 0.01$ and 3 of the top 6 at $\alpha = 0.05$. The EGARCH models tended to rank just below the GJR models for $l = 1$. The choice of error distribution for $l = 1$ seemed slightly less important than ensuring that asymmetry was effectively captured, though skewed Student-t error models dominated at $\alpha = 0.05$. However, during the GFC period, asymmetry was far less important; instead employing a skewed error distribution with fat tails was critical to capturing risk dynamics and level, while non-stationary IGARCH and MS models did comparatively best, though all models did under-estimate risk levels during this period.

For ten-day VaR forecasting, in the pre-crisis period the EGARCH model with Gaussian errors did best for $\alpha = 0.01$, followed by the other three EGARCH models. In this case Gaussian errors seemed to be quite adequate and to even do better than the fat-tailed distributions; this result might be influenced by the aggregation of 10 single day returns being closer to normality, as expected statistically, than a single day's return distribution. We further note that the simplest and most parsimonious asymmetric models (i.e. not the TGARCH) dominated at both $l = 1$ and $l = 10$ days.

In the GFC forecast period, however, all models significantly under-estimated risk levels at a 10-day horizon and no model could be recommended as accurate. To better understand this outcome, the bottom panel of Figure 2 exhibits ten-day ahead VaR forecasts and realized returns for the best four models during the pre-crisis period, which are EGARCH with various errors. The VaR forecasts violate the thresholds six to eight times from 579 forecasts in the pre-crisis period: the violations are few and spread out without clustering. Figure 3 shows the equivalent results in the GFC-dominated 2nd forecast period, for the best two models. Now, there is a large number of clustered violations all occurring in quick succession in October, 2008, at the start of the most dramatic effects of the GFC on daily returns. The dates for the clustered violations are the 9th, 10th, 14th, 15th, 20th, 21st and 23rd October, 2008. Clearly and logically, the 1-day ahead VaR forecasts can adjust to the global financial crisis effects and subsequent extreme returns far more quickly (9 days more quickly in fact) than the 10-day ahead VaR forecasts. This result is clearly heavily influenced by our use of 10 day periods that overlap by 9 days; the 10 day forecasting results results may have been better if we analysed non-overlapping 10 day periods.

This study considered a range of well-known, modern and popular, fully parametric econometric models to estimate and forecast VaR under a Bayesian framework. Each model includes a specification for the volatility dynamics and further, most models consider four specifications for the asset return error distribution. We observed from the empirical study that a conservative risk model often yielded a lower violation rate and correspondingly higher mean market risk charge and that different models were required depending on length of forecast horizon and quantile level, as well as for different market conditions. Also, while the 1-day forecasts, especially for non-stationary models, adapted reasonably well to the recent GFC, no model could be recommended for the recent GFC dominated period for 10-day ahead forecasting. McAleer, Jimenez-Martin, and Pérez-Amaral (2009) illustrate two useful variations to the standard mechanism for choosing forecasts, namely: (i) combining different forecast models for each period, such as a daily model that forecasts the supremum or infinum value for the VaR; (ii) alternatively, select a single model to forecast VaR, and then modify the daily forecast, depending on the recent history of violations under the Basel II Accord. Our study can provide valuable information for Deposit-taking Institutions (ADIs) to help choose risk models for predicting their VaR. Further, ADIs could employ combinations of prominent models based on our findings as a management strategy for forecasting VaR.

Table VII about here

CONCLUSIONS

This paper assesses the possibility of general Bayesian forecasting for carrying out one to ten day ahead VaR forecasting across a range of competing parametric heteroskedastic models. Nine popular volatility models are compared, most with four separate error distributions. For one and ten-day VaR forecasting, the well-known RiskMetrics^{TM} model ranked last in most measures and was rejected in all cases by the diagnostic tests. No model did consistently well across the different forecast horizons or quantile levels or market conditions. For one day ahead forecasting prior to the financial crisis, the GJR-GARCH with skewed Student-t errors ranked best, followed by other asymmetric volatility models. Volatility asymmetry is most important to capture, with skewed errors also prominent, especially at $\alpha = 0.05$. During and after the crisis, asymmetry is not important, instead skewness and fat tails dominate at the 1% level, with non-stationary models doing best at 5%. For ten-day ahead forecasting prior to the crisis, the EGARCH models had the best performance, with volatility asymmetry again an important feature, while normality seemed the best choice of error distribution. In both 1 and 10 day forecasting, all models under-estimated risk levels during the crisis, in fact all 10-day forecasting models were rejected for risk coverage during and after the crisis. Further, generally, GARCH models dominated the SV models in forecast performance. We observed from the empirical study that a conservative risk model often yielded a lower violation rate and correspondingly higher mean market risk charge. Therefore, we suggest employing combinations of prominent models as a management strategy for forecasting VaRs. We will focus on the Bayesian method helping to forecast the VaR under different investment strategies in the future.

Acknowledgement

We thank Professor Ruey S. Tsay and the anonymous referees for their insightful comments that helped improve the paper. Cathy Chen is supported by National Science Council (NSC) of Taiwan grant NSC96-2118-M-035-002-MY3.

Appendix A

The nine models considered are now given in detail:

1. **Symmetric GARCH**

   Bollerslev (1986) introduced a parsimonious extension to Engle's ARCH model:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}. \tag{12}$$

   Positivity and stationary dynamics are ensured via the standard restrictions:

$$\alpha_0 > 0;\ \ \alpha_i \geq 0,\ \ \beta_i \geq 0 \qquad \text{and} \qquad \sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1. \tag{13}$$

Based on Bollerslev, Chou and Kroner (1992) we set $p = q = 1$. The unknown parameters are: $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \beta_1)$, plus any unknown parameters in D.

2. **IGARCH:**
   The IGARCH model of Engle and Bollerslev (1986) is a special case of a GARCH(1,1) with $\alpha_1 + \beta_1 = 1$, i.e.:

$$h_t = \alpha_0 + \alpha_1 a_{t-1}^2 + (1 - \alpha_1) h_{t-1}, \tag{14}$$

   where it is common to enforce $\alpha_0 \geq 0$ and $0 < \alpha_1 < 1$. The volatility dynamics here are akin to those of a random walk.

3. **RiskMetrics**
   RiskMetrics^{TM} was developed by J.P. Morgan (1996), specifically for VaR calculation and is apparently still a popular method. It is a special case of the IGARCH, where $\alpha_0 = 0$, and is thus an exponentially weighted moving average (EWMA) of squared shocks; further the restriction $D(0,1) \equiv N(0,1)$ is used. The model form is:

$$h_t = \delta h_{t-1} + (1 - \delta) a_{t-1}^2, \tag{15}$$

   where a decay factor of 0.94 is recommended by J.P. Morgan for computing daily volatility.

4. **GJR-GARCH**
   The GJR-GARCH model by Glosten, Jaganathan, and Runkle (1993) captures asymmetric volatility via an indicator term in the GARCH equation:

$$h_t = \alpha_0 + \sum_{i=1}^{p} (\alpha_i + \gamma_i S_{t-i}^{-}) a_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{16}$$

$$\text{where} \quad S_{t-i}^{-} = \begin{cases} 1 & \text{if} \quad a_{t-i} \leq 0, \\ 0 & \text{if} \quad a_{t-i} > 0, \end{cases}$$

   Stationarity and positive volatility are ensured via:

$$\alpha_0 > 0, \ \alpha_i, \beta_i \geq 0, \ \sum_{i=1}^{p} \alpha_i + \gamma_i \geq 0 \text{ and } \sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i + 0.5 \sum_{i=1}^{p} \gamma_i < 1. \tag{17}$$

   The usual asymmetric volatility effect, i.e. falling markets increase volatility, implies that negative shocks at time t lead to a larger rate of increase in conditional volatility, of $\alpha_i + \gamma_i$ at time $t + 1$ (assuming $\gamma_i > 0$), whereas the positive shocks at time t lead to an increase in rate of conditional volatility of α_i at time $t + 1$.

5. **Exponential GARCH**

Nelson (1991) proposed the first asymmetric volatility model, to capture asymmetric volatility: EGARCH. The general EGARCH(p,q) form is:

$$\ln(h_t) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(\frac{|a_{t-i}| + \gamma_i a_{t-i}}{\sqrt{h_{t-i}}} \right) + \sum_{j=1}^{q} \beta_i \ln(h_{t-j}), \tag{18}$$

where again we consider $p = q = 1$. Here the logarithm of volatility is modeled, allowing the usual positivity restrictions on GARCH parameters to be relaxed. We expect the asymmetric effect $\gamma_1 < 0$, so that $\varepsilon_{t-1} < 0$ increases the volatility h_t, where $\varepsilon_{t-1} = a_{t-1}/\sqrt{h_{t-1}}$, but did not enforce this. For stationary dynamics (see Nelson, 1991) it is natural to assume $|\beta_1| < 1$. The original specification of this model used the GED for the distribution of ε_t.

6. **Threshold GARCH**

A standard deviation TGARCH model was first proposed by Zakoian (1994). Instead, we consider the dynamic variance TGARCH specification:

$$h_t = \begin{cases} \alpha_0^{(1)} + \Sigma_{i=1}^{p} \alpha_i^{(1)} a_{t-i}^2 + \Sigma_{j=1}^{q} \beta_j^{(1)} h_{t-j} \ r_{t-d} \le w \\ \alpha_0^{(2)} + \Sigma_{i=1}^{p} \alpha_i^{(2)} a_{t-i}^2 + \Sigma_{j=1}^{q} \beta_j^{(2)} h_{t-j} \ r_{t-d} > w, \end{cases} \tag{19}$$

where d is threshold lag and w is the threshold value. Here each parameter can change in response to lagged returns, at unknown lag d. We again set $p = q = 1$. The unknown model parameters are $(\alpha_0^{(1)}, \alpha_1^{(1)}, \beta_1^{(1)}, \alpha_0^{(2)}, \alpha_1^{(2)}, \beta_1^{(2)}, w, d)$.

7. **Markov switching GARCH models**

Gray (1996) and Tsay (2005) proposed simple two-state Markov switching models, with different risk premium and different GARCH dynamics in each regime. Chen, So and Lin (2009) proposed the double Markov switching GARCH model, where here we focus on the volatility only. The Markov switching GARCH (MS-GARCH) is specified as:

$$h_t = \alpha_0^{(s_t+1)} + \Sigma_{i=1}^{p} \alpha_i^{(s_t+1)} a_{t-i}^2 + \Sigma_{j=1}^{q} \beta_j^{(s_t+1)} h_{t-j}, \tag{20}$$

where s_t is an unobserved discrete Markov process indicator. A two-regime model is employed, with $p = q = 1$, and a Markov transition matrix $P = p_{(i,j)}$, where:

$$p_{(i,j)} = Pr(s_t = j | s_{t-1} = i) \ \ i, j = 1, 2.$$

The unknown parameters are $(\alpha_0^{(1)}, \alpha_1^{(1)}, \beta_1^{(1)}, \alpha_0^{(2)}, \alpha_1^{(2)}, \beta_1^{(2)}, p_{1,1}, p_{2,2})$, state vector $\boldsymbol{s}$, plus any parameters in D.

8. **Stochastic volatility models**

SV models are considered as an alternative approach to GARCH-type processes. Here, volatility has a specific source of randomness and is thus stochastic, as proposed by Taylor (1982, 1986). The discrete-time symmetric SV model is:

$$a_t = \sqrt{h_t}\varepsilon_t, \quad \log h_{t+1} = \alpha_0 + \alpha_1 \log h_t + u_t, \tag{21}$$

where u_t is a Gaussian innovation with zero mean and variance σ_u^2. We restrict $|\alpha_1| < 1$ for stationarity.

9. **Threshold SV models**

   There are quite a few papers presenting or considering a nonlinear SV model framework: e.g. So, Li and Lam (2002) presented the threshold SV (THSV) model to describe both mean and volatility asymmetry, while Chen, Liu and So (2008) generalized the THSV model and incorporated a heavy-tailed error distribution, plus estimation of the unobserved threshold value and time delay parameter. We consider nonlinear SV models in asymmetric volatility but without a mean equation. Therefore the THSV model is:

$$a_t = \sqrt{h_t}\varepsilon_t, \quad \log h_{t+1} = (\alpha_0 + \beta_0 s_t) + (\alpha_1 + \beta_1 s_t)\log h_t + u_t, \tag{22}$$

   where the state variable s_t is defined by

$$s_t = \begin{cases} 0 & \text{if} \quad r_{t-d} < r, \\ 1 & \text{if} \quad r_{t-d} \geq r, \end{cases}$$

   with the delay d and threshold value r.

Apart from the Riskmetrics model, all the GARCH-type volatility models are estimated under the following distributional assumptions of the unconditional shocks (a) standard normal, (b) the Student-t, (c) GED, and (d) skewed Student-t distributions, where:

(c) **Generalised Error Distribution**: The density function for ε_t a standardized GED with scale parameter σ is:

$$p_\varepsilon(\varepsilon_t) = \frac{\lambda}{2\sigma\Gamma(1/\lambda)} \exp\left\{-\left|\frac{\varepsilon_t}{\sigma}\right|^\lambda\right\}, \tag{23}$$

   where $\sigma = [\Gamma(\frac{1}{\lambda})/\Gamma(\frac{3}{\lambda})]^{0.5}$. $\lambda \in (0, \infty)$ is the tail-behaviour determining parameter. When $\lambda > 2$, the distribution has thinner tails than the normal; when $\lambda = 2$, it is exactly a normal distribution with mean 0 and standard error σ; while for $\lambda < 2$, the distribution has excess kurtosis relative to the normal. For real asset return data, we expect $\lambda < 2$.

(d) **Skewed Student-t Distribution**: To allow for skewness in the shape of the conditional return density, the skewed Student-t distribution was defined by Hansen (1994) as:

$$p_\varepsilon(\varepsilon_t|\nu,\eta) = \begin{cases} bc\left[1 + \frac{1}{\nu-2}\left(\frac{b\varepsilon_t + a}{1-\eta}\right)^2\right]^{-(\nu+1)/2} & \text{if} \quad \varepsilon_t < -\frac{a}{b} \\ bc\left[1 + \frac{1}{\nu-2}\left(\frac{b\varepsilon_t + a}{1+\eta}\right)^2\right]^{-(\nu+1)/2} & \text{if} \quad \varepsilon_t \geq -\frac{a}{b} \end{cases} \tag{24}$$

where degrees of freedom ν and skewness parameter η satisfy $2 < \nu < \infty$, and $-1 < \eta < 1$, respectively. The constants a, b, and c are fixed as:
$a = 4\eta c\left(\frac{\nu-2}{\nu-1}\right)$; $\quad b^2 = 1 + 3\eta^2 - a^2$; $\quad c = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)}$.
This distribution already has zero mean and unit variance. We use the notation $\mathrm{St}(\nu, \eta)$. The standardized Student-t distribution is a special case of this skewed Student-t, when $\eta = 0$. The Gaussian is thus the limiting distribution as $\nu \to \infty$, also when $\eta = 0$.

The symmetric and skewed Student-t and the GED all allow fat-tailed error distributions, compared to the Gaussian, while each contains the Gaussian as a special case.

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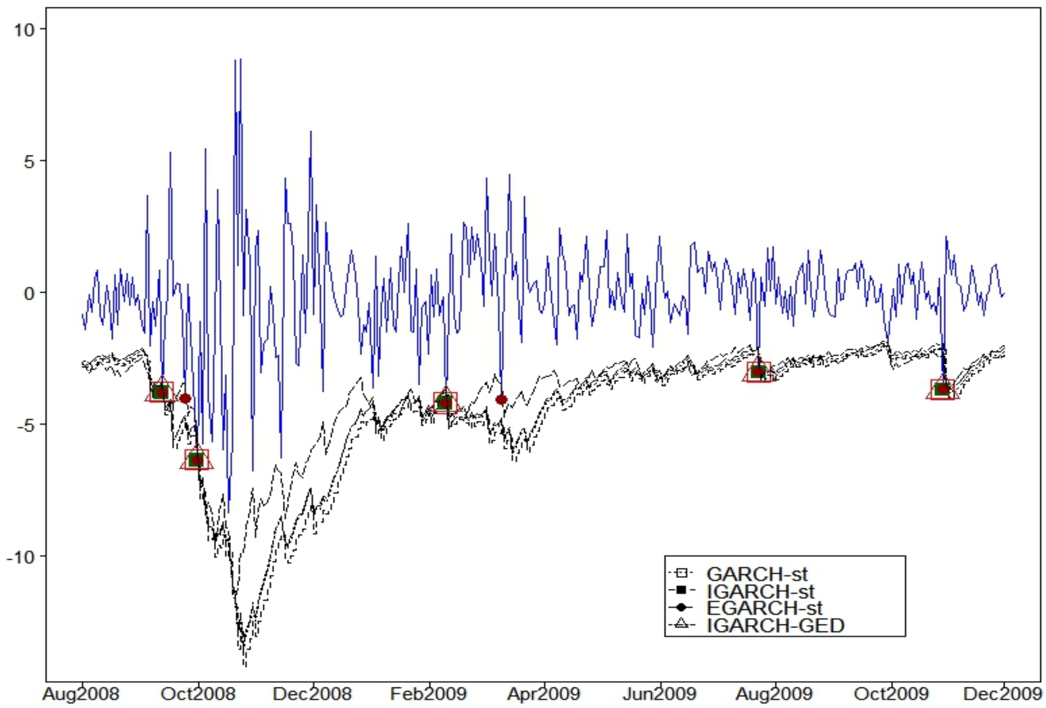
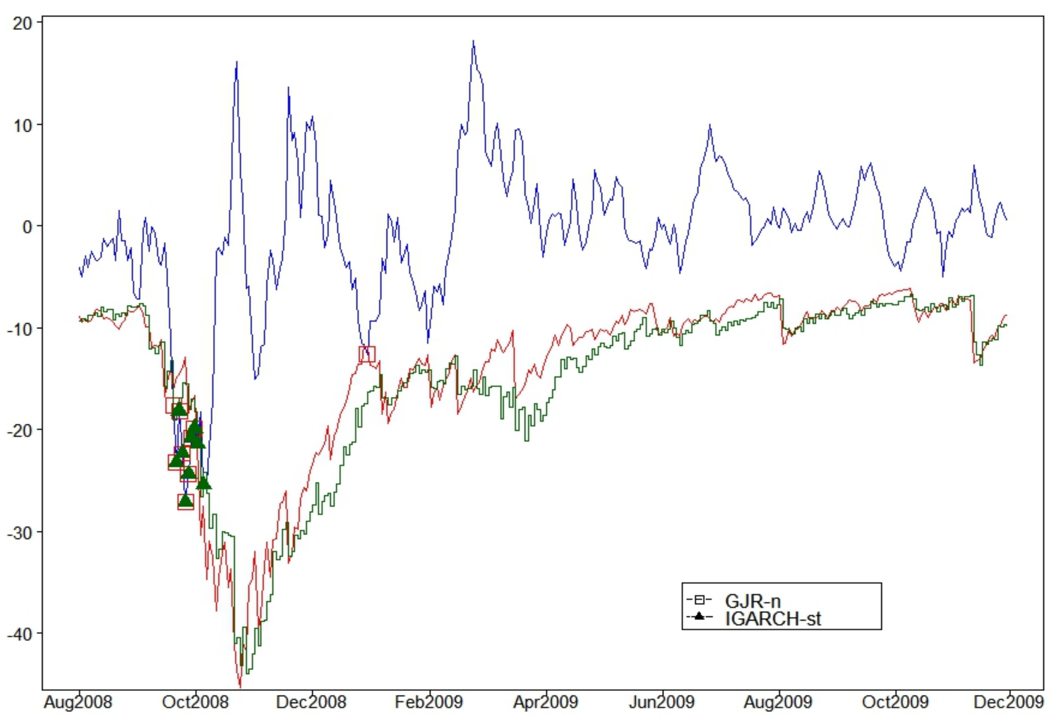


(a)



(b)

Figure 3. VaR forecasts for the global financial crisis period (a) 1-day-ahead and (b) and ten-day ahead VaR forecasts at 1% level.

Table II. Summary statistics for parameter estimates from 100 simulated data sets from the EGARCH(1,1) model.

		$n = 2000$				$n = 4000$			
	True	Mean	Std	Lower	Upper	Mean	Std	Lower	Upper
GED									
α_0	-0.20	-0.232	0.043	-0.322	-0.158	-0.210	0.028	-0.269	-0.158
α_1	0.20	0.220	0.038	0.151	0.299	0.204	0.026	0.155	0.259
γ	-0.26	-0.259	0.121	-0.504	-0.038	-0.269	0.085	-0.441	-0.110
β	0.93	0.907	0.028	0.844	0.952	0.922	0.017	0.885	0.951
λ	1.00	1.003	0.042	0.924	1.087	1.001	0.029	0.944	1.059
GED									
α_0	-0.20	-0.226	0.038	-0.307	-0.157	-0.213	0.024	-0.263	-0.169
α_1	0.20	0.213	0.035	0.150	0.285	0.208	0.023	0.165	0.255
γ	-0.26	-0.273	0.103	-0.482	-0.084	-0.260	0.067	-0.391	-0.136
β	0.93	0.905	0.027	0.844	0.949	0.920	0.015	0.888	0.947
λ	1.50	1.505	0.069	1.374	1.645	1.496	0.049	1.403	1.593
GED									
α_0	-0.20	-0.222	0.034	-0.294	-0.161	-0.204	0.023	-0.251	-0.162
α_1	0.20	0.212	0.032	0.154	0.279	0.200	0.022	0.158	0.245
γ	-0.26	-0.262	0.090	-0.448	-0.097	-0.277	0.065	-0.409	-0.154
β	0.93	0.909	0.023	0.856	0.947	0.923	0.014	0.892	0.948
λ	2.00	2.015	0.101	1.823	2.221	2.001	0.071	1.866	2.143
t									
α_0	-0.20	-0.227	0.040	-0.311	-0.156	-0.211	0.026	-0.264	-0.163
α_1	0.20	0.212	0.036	0.147	0.287	0.206	0.025	0.160	0.256
γ	-0.26	-0.252	0.109	-0.474	-0.049	-0.270	0.071	-0.412	-0.140
β	0.93	0.901	0.030	0.834	0.948	0.920	0.016	0.885	0.948
ν	7.00	7.231	1.180	5.386	9.974	7.304	0.815	5.928	9.117
st									
α_0	-0.20	-0.225	0.040	-0.309	-0.155	-0.211	0.026	-0.264	-0.164
α_1	0.20	0.215	0.037	0.148	0.291	0.207	0.025	0.160	0.257
γ	-0.26	-0.273	0.112	-0.502	-0.065	-0.279	0.077	-0.438	-0.136
β	0.93	0.909	0.026	0.850	0.952	0.921	0.016	0.887	0.948
ν	7.00	7.073	1.101	5.326	9.621	7.194	0.784	5.859	8.923
η	-0.05	-0.047	0.030	-0.106	0.012	-0.051	0.021	-0.093	-0.009
st									
α_0	-0.20	-0.208	0.028	-0.266	-0.158	-0.202	0.019	-0.240	-0.167
α_1	0.20	0.205	0.029	0.153	0.266	0.200	0.020	0.164	0.240
γ	-0.26	-0.276	0.105	-0.490	-0.088	-0.281	0.073	-0.431	-0.147
β	0.93	0.923	0.016	0.889	0.949	0.926	0.010	0.905	0.945
ν	7.00	7.245	1.173	5.417	9.955	7.151	0.780	5.831	8.883
η	-0.50	-0.500	0.027	-0.551	-0.446	-0.498	0.019	-0.534	-0.460
st									
α_0	-0.20	-0.201	0.006	-0.213	-0.190	-0.200	0.004	-0.208	-0.194
α_1	0.20	0.200	0.007	0.188	0.213	0.200	0.004	0.192	0.209
γ	-0.26	-0.259	0.022	-0.301	-0.214	-0.259	0.014	-0.287	-0.232
β	0.93	0.930	0.003	0.924	0.935	0.930	0.002	0.927	0.933
ν	7.00	7.441	0.843	6.185	9.247	7.184	0.524	6.366	8.220
η	-0.99	-0.980	0.006	-0.988	-0.965	-0.985	0.003	-0.989	-0.977

(1): t and st refer to Student-t and skewed Student-t errors, respectively.

Table III. Summary statistics for 1-day VaR forecast over the time period from July 2005 to February 2008 at 1% and 5% level.

	Models	Violation rates %	Mean daily capital charge	AD of violation Max	AD of violation Mean	Penalty	Violation number	Zone
$\alpha = 1\%$	RiskMetrics	2.72	6.9410	1.9939	0.4138	0.6284	16	Yellow
	GARCH-n	1.53	6.1738	1.9952	0.5269	0.0	9	
	GARCH-t	1.19	6.5115	1.8209	0.5370	0.0	7	
	GARCH-GED	1.02	6.5396	1.8053	0.6127	0.0	6	
	GARCH-st	1.02	6.9327	1.6227	0.4714	0.0	6	
	GJR-GARCH-n	1.36	6.1262	1.4457	0.4401	0.0	6	
	GJR-GARCH-t	1.02	6.4372	1.2765	0.4854	0.0	8	
	GJR-GARCH-GED	1.02	6.4556	1.2732	0.4745	0.0	6	
	GJR-GARCH-st	1.02	6.8147	1.0981	0.3549	0.0	6	
	EGARCH-n	1.53	6.1024	1.5276	0.4229	0.0	9	
	EGARCH-t	1.36	6.3696	1.3857	0.4060	0.0	8	
	EGARCH-GED	1.02	6.4004	1.3869	0.5201	0.0	6	
	EGARCH-st	0.85	6.7258	1.1990	0.4911	0.0	5	
	TGARCH-n	1.70	6.5370	1.9050	0.5104	0.2921	10	Yellow
	TGARCH-t	1.36	6.3350	1.7288	0.4949	0.0	8	
	TGARCH-GED	1.36	6.3413	1.7149	0.4994	0.0	8	
	TGARCH-st	1.36	6.3529	1.7222	0.4958	0.0	8	
	IGARCH-n	1.53	6.1865	1.8442	0.5031	0.0	9	
	IGARCH-t	1.19	6.5776	1.6668	0.5113	0.0	7	
	IGARCH-GED	1.19	6.5702	1.6579	0.5241	0.0	7	
	IGARCH-st	0.85	7.0254	1.4346	0.5373	0.0	5	
	MS-n	1.53	6.2370	2.0112	0.5034	0.0	9	
	MS-t	1.02	6.5736	1.8466	0.5659	0.0	6	
	MS-GED	1.02	6.6150	1.8346	0.5854	0.0	6	
	MS-st	1.02	6.5911	1.8320	0.5951	0.0	6	
	SV-n	1.53	6.0501	2.0343	0.5343	0.0	9	
	SV-t	1.02	6.4846	1.1786	0.6450	0.0	6	
	THSV-n	2.04	6.6032	2.2179	0.5230	0.4121	12	Yellow
	THSV-t	1.53	6.1971	1.9638	0.6160	0.0	9	
$\alpha = 5\%$	RiskMetrics	6.29	-	2.9119	0.5670	-		
	GARCH-n	5.10	-	2.9128	0.5868	-		
	GARCH-t	5.10	-	2.9370	0.6204	-		
	GARCH-GED	5.10	-	2.8992	0.5859	-		
	GARCH-st	4.93	-	1.6227	0.4714	-		
	GJR-GARCH-n	5.27	-	2.5243	0.5065	-		
	GJR-GARCH-t	5.44	-	2.5469	0.5256	-		
	GJR-GARCH-GED	5.27	-	2.5156	0.5059	-		
	GJR-GARCH-st	5.10	-	1.0981	0.3549	-		
	EGARCH-n	5.27	-	2.5822	0.5187	-		
	EGARCH-t	5.61	-	2.6083	0.5074	-		
	EGARCH-GED	5.27	-	2.5859	0.5072	-		
	EGARCH-st	5.27	-	1.1990	0.4911	-		
	TGARCH-n	5.44	-	2.8490	0.5765	-		
	TGARCH-t	5.78	-	2.8738	0.5617	-		
	TGARCH-GED	5.27	-	2.8370	0.5884	-		
	TGARCH-st	5.78	-	1.7222	0.4958	-		
	IGARCH-n	4.93	-	2.8061	0.5931	-		
	IGARCH-t	5.44	-	2.8433	0.5712	-		
	IGARCH-GED	5.10	-	2.8057	0.5762	-		
	IGARCH-st	4.76	-	1.4346	0.5373	-		
	MS-n	4.93	-	2.9241	0.5918	-		
	MS-t	5.10	-	2.9512	0.5996	-		
	MS-GED	4.93	-	2.9173	0.5897	-		
	MS-st	5.10	-	1.8320	0.5951	-		
	SV-n	5.44	-	2.9554	0.5704	-		
	SV-t	3.92	-	1.8046	0.5566	-		
	THSV-n	5.78	-	3.0843	0.5986	-		
	THSV-t	5.78	-	3.0640	0.6291	-		

Table IV. Summary statistics for 1-day VaR forecast over the time period from August 2008 to December 2009 at 1% and 5% levels.

	Models	Violation rates %	Mean daily capital charge	AD of violation		Penalty	Violation number	Zone
				Max	Mean			
$\alpha = 1\%$	RiskMetrics	2.85	16.0731	1.8785	0.6485	0.6662	9	Yellow
	GARCH-n	2.22	14.3194	1.8719	0.7963	0.4701	7	Yellow
	GARCH-t	1.58	13.0971	1.7594	0.9461	0.0	5	Green
	GARCH-GED	1.58	13.0876	1.7702	0.9300	0.0	5	Green
	GARCH-st	1.58	13.9422	1.5949	0.7281	0.0	5	Green
	GJR-GARCH-n	3.16	14.8051	1.8134	0.6631	0.7580	10	Yellow
	GJR-GARCH-t	2.53	14.7487	1.7184	0.6935	0.5706	8	Yellow
	GJR-GARCH-GED	2.53	14.7377	1.7316	0.6800	0.5706	8	Yellow
	GJR-GARCH-st	1.90	14.7298	1.5661	0.7063	0.3632	6	Yellow
	EGARCH-n	4.11	14.5129	1.6121	0.5983	1.0	13	Red
	EGARCH-t	2.53	13.6267	1.4990	0.8194	0.5706	8	Yellow
	EGARCH-GED	2.53	13.5988	1.5195	0.8017	0.5706	8	Yellow
	EGARCH-st	2.22	13.9888	1.3186	0.7026	0.4701	7	Yellow
	TGARCH-n	1.90	13.8305	1.8613	0.9585	0.3632	6	Yellow
	TGARCH-t	1.90	14.6257	1.7547	0.7708	0.3632	6	Yellow
	TGARCH-GED	1.90	14.6817	1.7878	0.7561	0.3632	6	Yellow
	TGARCH-st	1.90	14.6806	1.7476	0.7621	0.3632	6	Yellow
	IGARCH-n	1.90	14.4787	1.8534	0.8203	0.3632	6	Yellow
	IGARCH-t	1.58	13.7040	1.7249	0.8046	0.0	5	Green
	IGARCH-GED	1.58	13.6782	1.7460	0.7743	0.0	5	Green
	IGARCH-st	1.58	14.7171	1.5414	0.5613	0.0	5	Green
	MS-n	2.53	14.5145	1.8755	0.7306	0.5706	8	Yellow
	MS-t	1.58	12.8460	1.7673	0.9710	0.0	5	Green
	MS-GED	1.58	12.8570	1.7756	0.9538	0.0	5	Green
	MS-st	1.58	12.9057	1.7617	0.9552	0.0	5	Green
	SV-n	3.80	15.3247	1.8177	0.7905	1.0	12	Red
	SV-t	3.80	15.5351	1.8859	0.7766	1.0	12	Red
	THSV-n	4.11	15.0285	1.9473	0.7461	1.0	13	Red
	THSV-t	2.85	14.8220	1.9787	0.9578	0.6662	9	Yellow
$\alpha = 5\%$	RiskMetrics	5.70	-	2.8836	1.1481	-	18	
	GARCH-n	6.33	-	2.5240	1.0811	-	20	
	GARCH-t	6.65	-	2.6710	1.0637	-	21	
	GARCH-GED	6.33	-	2.5601	1.0685	-	20	
	GARCH-st	6.01	-	2.5288	1.0490	-	19	
	GJR-GARCH-n	6.65	-	2.3643	0.9913	-	21	
	GJR-GARCH-t	6.65	-	2.3694	1.0054	-	21	
	GJR-GARCH-GED	6.65	-	2.3588	0.9705	-	21	
	GJR-GARCH-st	5.70	-	2.2991	1.0628	-	18	
	EGARCH-n	7.91	-	2.6377	1.1202	-	25	
	EGARCH-t	7.59	-	2.7201	1.1769	-	24	
	EGARCH-GED	7.28	-	2.6604	1.1973	-	23	
	EGARCH-st	6.96	-	2.6315	1.1924	-	22	
	TGARCH-n	6.33	-	2.4742	1.1005	-	20	
	TGARCH-t	6.33	-	2.6080	1.1314	-	20	
	TGARCH-GED	6.33	-	2.5451	1.0833	-	20	
	TGARCH-st	6.33	-	2.5871	1.1215	-	20	
	IGARCH-n	5.70	-	2.3926	1.0611	-	18	
	IGARCH-t	5.70	-	2.5165	1.1056	-	18	
	IGARCH-GED	5.70	-	2.3844	1.0458	-	18	
	IGARCH-st	5.38	-	2.3541	1.0231	-	17	
	MS-n	6.33	-	2.5323	1.1190	-	20	
	MS-t	6.65	-	2.6741	1.1023	-	21	
	MS-GED	6.33	-	2.5654	1.1149	-	20	
	MS-st	6.33	-	2.6500	1.1438	-	20	
	SV-n	6.96	-	3.4082	1.3281	-	22	
	SV-t	7.28	-	3.4188	1.3105	-	23	
	THSV-n	7.59	-	3.3565	1.2375	-	24	
	THSV-t	6.65	-	3.5129	1.4970	-	21	

Table V. Summary statistics for 10-day VaR forecast over the time period from July 2005 to February 2008 at 1% level.

	Models	Violation rates %	Mean MRC	Max AD	Mean AD	Penalty	Violation number
$\alpha = 1\%$	RiskMetrics	2.94	23.8549	2.8712	1.3516	1.0	17
	GARCH-n	1.55	20.5372	3.1377	1.3509	0.0	9
	GARCH-t	1.73	22.6950	2.1331	0.9699	0.3018	10
	GARCH-GED	1.38	20.6476	2.6871	1.2621	0.0	8
	GARCH-st	1.73	22.7209	3.0982	1.2371	0.3018	10
	GJR-GARCH-n	1.73	24.4863	1.7257	0.7946	0.3018	10
	GJR-GARCH-t	1.73	24.3045	1.7436	0.6610	0.3018	10
	GJR-GARCH-GED	1.73	24.5265	1.5587	0.7595	0.3018	10
	GJR-GARCH-st	1.55	22.9909	1.5799	0.7104	0.0	9
	EGARCH-n	1.04	22.8345	1.0846	0.6005	0.0	6
	EGARCH-t	1.38	22.4090	1.1944	0.6784	0.0	8
	EGARCH-GED	1.38	22.6314	1.4140	0.6635	0.0	8
	EGARCH-st	1.21	22.9848	1.0897	0.5954	0.0	7
	TGARCH-n	2.07	22.2895	3.1077	1.1523	0.4228	12
	TGARCH-t	1.55	19.9576	2.9191	1.3728	0.0	9
	TGARCH-GED	1.90	22.2917	2.8059	1.1400	0.3636	11
	TGARCH-st	1.73	22.0230	2.8814	1.2719	0.3018	10
	IGARCH-n	1.90	23.1623	2.3457	1.0436	0.3636	11
	IGARCH-t	1.55	20.8814	1.9460	1.1676	0.0	9
	IGARCH-GED	1.38	20.7364	2.3458	1.3886	0.0	8
	IGARCH-st	1.38	21.9034	1.7871	1.0428	0.0	8
	MS-n	1.38	20.9498	2.5762	1.1561	0.0	8
	MS-t	1.55	20.9517	3.4019	1.1469	0.0	9
	MS-GED	1.38	21.1400	2.9161	1.1839	0.0	8
	MS-st	1.38	21.0313	2.6669	1.1512	0.0	8
	SV-n	1.55	21.4229	2.2366	0.8763	0.0	9
	SV-t	1.55	21.1760	6.2874	1.8754	0.0	9
	THSV-n	1.73	21.9426	2.4967	1.1147	0.3018	10
	THSV-t	2.07	22.4779	2.0721	0.9318	0.4228	12

(1): Ranking is based on the rank sum - min(rank) +1.

Table VI. Summary statistics for 10-day VaR forecast over the time period from August 2008 to December 2009 at 1% level.

	Models	Violation rates %	Mean MRC	Max AD	Mean AD	Penalty	Violation number
$\alpha = 1\%$	RiskMetrics	4.56	56.4562	14.3576	5.9010	1.0	14
	GARCH-n	4.56	55.7485	13.5591	4.2880	1.0	14
	GARCH-t	3.91	56.3347	13.9064	5.0868	1.0	12
	GARCH-GED	4.23	56.4016	12.7615	4.5084	1.0	13
	GARCH-st	4.23	56.7473	13.0827	4.4584	1.0	13
	GJR-GARCH-n	2.93	53.5776	14.3099	5.4996	0.6908	9
	GJR-GARCH-t	3.26	56.2045	14.0292	5.4960	0.7842	10
	GJR-GARCH-GED	3.26	55.7099	14.2936	5.4079	0.7842	10
	GJR-GARCH-st	3.26	57.4997	13.0436	4.9579	0.7842	10
	EGARCH-n	3.91	51.9789	15.7044	6.7057	1.0	12
	EGARCH-t	4.23	52.5195	15.5255	6.2060	1.0	13
	EGARCH-GED	4.23	52.1868	16.2898	6.2683	1.0	13
	EGARCH-st	4.23	53.7964	15.2139	6.0211	1.0	13
	TGARCH-n	3.58	55.3203	13.6326	5.2069	1.0	11
	TGARCH-t	3.58	56.4543	12.9199	5.0923	1.0	11
	TGARCH-GED	3.58	56.1658	13.2295	5.0925	1.0	11
	TGARCH-st	4.23	56.3909	13.1970	4.5041	1.0	13
	IGARCH-n	3.26	56.1735	11.4370	4.4251	0.7842	10
	IGARCH-t	3.58	59.8153	13.1993	4.4120	1.0	11
	IGARCH-GED	3.26	56.6589	12.9220	5.0194	0.7842	10
	IGARCH-st	2.93	58.6673	11.6024	4.3722	0.6908	9
	MS-n	4.23	54.7926	13.3201	4.5804	1.0	13
	MS-t	4.23	55.1816	13.1817	4.6993	1.0	13
	MS-GED	4.89	54.9963	14.0789	4.0494	1.0	15
	MS-st	3.58	55.4635	13.1504	5.4578	1.0	11
	SV-n	4.89	54.4702	14.5713	5.7060	1.0	15
	SV-t	4.23	54.0484	14.3183	7.1610	1.0	13
	THSV-n	4.89	51.7328	15.1607	5.8934	1.0	15
	THSV-t	4.89	52.9577	15.8645	7.3028	1.0	15

Table VII. P-values of unconditional and conditional coverage tests for each model

Validation period	July 2005 to February 2008			August 2008 to December 2009	
Models	LR_{uc}	LR_{cc}		LR_{uc}	LR_{cc}
RiskMetrics	0.0005	0.0005	0.0070	0.0134	
GARCH-n	0.2303	0.1458	0.0613	0.1481	
GARCH-t	0.6522	0.1659	0.3376	0.5825	
GARCH-GED	0.9605	0.9388	0.3376	0.5825	
GARCH-st	0.9605	0.9388	0.3376	0.5825	
GJR-GARCH-n	0.4048	0.6329	0.0020	0.0062	
GJR-GARCH-t	0.9605	0.9388	0.0219	0.0586	
GJR-GARCH-GED	0.9605	0.9388	0.0219	0.0586	
GJR-GARCH-st	0.9605	0.9388	0.1532	0.3209	
EGARCH-n	0.2303	0.1458	0.0000	0.0001	
EGARCH-t	0.4048	0.6329	0.0219	0.0299	
EGARCH-GED	0.9605	0.9388	0.0219	0.0299	
EGARCH-st	0.7081	0.8931	0.0613	0.1481	
TGARCH-n	0.1206	0.1088	0.1532	0.3209	
TGARCH-t	0.4048	0.1691	0.1532	0.3209	
TGARCH-GED	0.4048	0.1691	0.1532	0.3209	
TGARCH-st	0.4048	0.1691	0.1532	0.3209	
IGARCH-n	0.2303	0.1458	0.1532	0.3209	
IGARCH-t	0.6522	0.1659	0.3376	0.5825	
IGARCH-GED	0.6522	0.1659	0.3376	0.5825	
IGARCH-st	0.7081	0.8931	0.3376	0.5825	
MS-n	0.2303	0.1458	0.0219	0.0586	
MS-t	0.9605	0.9388	0.3376	0.5825	
MS-GED	0.9605	0.9388	0.3376	0.5825	
MS-st	0.9605	0.9388	0.3376	0.5825	
SV-n	0.2303	0.1458	0.0001	0.0005	
SV-t	0.9605	0.1338	0.0001	0.0005	
THSV-n	0.0262	0.0419	0.0000	0.0001	
THSV-t	0.2303	0.1458	0.0070	0.0134	