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Debajyoti Chakrabarty

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Growth and Business Cycles with Imperfect Credit Markets

Debajyoti Chakrabarty

School of Economics and Political Science
University of Sydney, Australia

Email: d.chakrabarty@econ.usyd.edu.au

ABSTRACT

We study the process of growth and business cycles in an open economy which has access to international financial markets. The financial market imperfection originates from costly state verification and a positive probability of default on loans. The degree of credit market imperfection is endogenously derived. The results show that developed economies are able to borrow on easier terms than emerging countries. The credit market imperfection may cause some economies to fall into a development trap if the initial endowment of capital is too low. The financial market frictions also generate interesting business cycle dynamics. Financial market imperfections help in replicating the empirical fact that output growth shows positive autocorrelation at short horizons. The model also predicts that a poorer economy will experience a more severe and persistent e.ect on investment and output due to an exogenous shock.

JEL CLASSIFICATION: G14, D82, G21, O16
KEYWORDS: Costly state verification, Credit markets, Growth, Business Cycles.

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Contents

1 Introduction

This paper develops a framework to study the role of credit market imperfections in the process of growth and business cycles in an economy. Economists have long regarded the credit market as key to understanding economic development and to transmitting cyclical shocks through modern industrial economies.

The literature on the connection between credit markets and the macroeconomy has developed in two directions. The growth branch of this literature started with Gurley and Shaw([14]) who noted that economic growth is almost always accompanied by financial deepening, i.e., by more extensive use of external finance in investment and the gradual easing of distortions of the credit market. Subsequent papers in this literature have focused on the role of credit markets as efficient allocators of savings into productive investment opportunities. Bencivenga and Smith([3]) studied the growth effects of financial intermediation in an overlapping generation economy with agents characterized by uncertain liquidity needs. Intermediation enhances growth because banks are efficient providers of liquidity which frees the individuals from the need to hold low yield liquid assets. Greenwood and Jovanovic([12]) also derive similar results. For a summary of the work in this area see Greenwood and Smith([13]).

The cyclical fluctuation branch of the literature focuses on the connection between credit markets and business cycles. The main focus of attention is how the credit market propagates and amplifies external shocks through the economy. This general sentiment dates back to Fisher and, Friedman and Schwartz, who argued that adverse conditions in financial markets may have worsened the effects of prewar recessions, including the Great Depression. Much of business cycle research investigates the informational role played by the credit markets. A seminal contribution in this line of research was made by Brenanke and Gertler([4]). They developed a general equilibrium model where agency costs arise endogenously. An important insight of their model is the theoretical possibility that agency costs will enhance the propagation of productivity shocks. Carlstrom and Fuerst([6]) built on the Bernanke-Gertler paper by constructing a computable general equilibrium model. They try to quantitatively capture the effect of agency costs on business cycles.

A related attempt to model credit market imperfection is provided by Keotaki and Moore([15]). They analyze the contracting problem between borrowers and lenders in an environment where value of a project cannot be extracted by the lender due to inalienability of human capital. The result is that borrowing is so tightly constrained by the value of collateral that default never occurs in equilibrium.

We follow Bernanke-Gertler approach and adopt costly state verification model of Townsend([17]). Our model differs from the earlier papers in the

literature in three important respects. Firstly, we study the effect of agency costs in an economy which has access to international financial markets. Unlike Bernanke & Gertler, and Carlstrom & Fuerst, the economy does not face an absolute borrowing constraint from domestic savings. Secondly, the agency cost problem exists in the production of final good and not in the production of intermediate goods alone. Finally, the inter-temporal preferences are determined endogenously. This makes the evolution of the economy dependant on its history. It also provides us with an insight as to why economies differ in terms of their credit market institutions over a long period of time.

The paper is organized as follows. The next section describes the production technology and the interaction between investment and international credit market. The inter-temporal optimization problem of the economy is presented in section 3. In section 4, we characterize the steady states and their stability properties. In section 5, we simulate the behavior of an economy whose characteristics are similar to a standard real business cycle economy and study the effect of exogenous shocks on it. Section 6 concludes.

2 The Model

A: The Production technology

Consider an economy with two production sectors. Both the production sectors produce the same homogeneous product, which can be used for consumption or investment. For tractability we will name the production sectors as household (H) and industrial (I). The household sector of the economy is characterized by a simple but primitive technology, which requires only labor for production. The production function in this sector is given by

$$Y^H = aL^H \ , \tag{1}$$

where L^H denotes the labor input and "a" denotes the marginal productivity of labor. The industrial sector uses capital (K) and labor(L) simultaneously. Production in the industrial sector can be thought of as a project or an endeavor to come up with a new technology. If the project is successful then production is high but if the project is unsuccessful then output is zero. The production function in the industrial sector is given by

$$Y^I = \begin{cases} F(L^I) & \text{with probability } \pi(K^I) \\ 0 & \text{with probability } 1 - \pi(K^I) \end{cases} \tag{2}$$

where L^I and K^I are the amount of labor and capital inputs respectively. The probability of success of the project is an increasing function of the amount of capital invested.

B: Industrial Sector Investment and the International credit market

The international credit market revolves around a risk free asset which yields a

gross return $R^* \geq 1$ which we will call the world interest rate. The international credit market consists of a large number of potential lenders so all lenders on average earn R^* on their loans.

Let us begin with the case when an industrial sector firm decides to borrow capital from the international market. Suppose the firm is endowed with K units of capital. The firm has the option of investing in its own industrial sector project or the risk free asset. In addition he can also borrow capital from international market at a lending rate of interest R^l. The firm can invest K^I units of capital out of its endowment in the industrial sector project and earn R^* on the remaining capital. The amount K^I is the owners equity in the project and is observed by everyone. Suppose the firm borrows B from the international credit market. The firm could in theory use B^I for the industrial project and invest the remaining amount in the risk free asset. This allocation of borrowed funds is not ex-ante observed by the international lender.

The optimal contract between the firm and the international lender is a standard debt contract where the firm repays the lender a gross interest rate of R^l on each unit of borrowed capital if the firm announces that the project was successful. If the agent announces that the project was unsuccessful the lender must take over the project and verify that the firms's announcement was truthful. The takeover of project and subsequent verification of the status of the project is essential to prevent strategic defaulting. Let the verification cost to the lender on a loan size of B be mB[1]. On takeover of the project the lender is able to retrieve the amount of borrowed capital, which the firm had diverted into the risk-free asset $R^*(B - B^I)$. The limited liability clause in the debt contract prevents the lender from attaching other sources the agent's income on the takeover of the project. Hence the zero profit condition for the lenders in the international market can be written as

$$\pi(K^I + B^I)\ R^l B + [1 - \pi(K^I + B^I)]\ [R^*(B - B^I) - mB] = R^* B\ . \qquad (3)$$

Simple manipulation of equation (3) yields

$$R^l = R^* \left[1 + \frac{1 - \pi(K^I + B^I)}{\pi(K_t^I + B^I)} \left(\frac{B_t^I}{B_t}\right)\right] + \left[\frac{1 - \pi(K_t^I + B_t^I)}{\pi(K_t^I + B_t^I)}\right] m\ . \qquad (4)$$

Thus the economy can borrow capital from the international credit market as long as they pay the break-even interest rate to the lenders $R^l > R^*$. Given the contractual setup the only situation when the agent is going to declare bankruptcy is when the project has been unsuccessful. The difference between the lending rate and the risk-free interest rate is the interest premium. Let us now study the borrowing and investment decision of the domestic agent.

Assumption 1: The functions F(.) and π(.) are increasing and strictly concave in their arguments. In addition $\lim_{L\to 0} F'(.) = \infty$ *, and* $\lim_{K\to 0} \pi'(.) = \infty$*; where* $F'(.)$ *and* $\pi'(.)$ *denote the derivatives of functions* $F(.)$ *and* $\pi(.)$ *respectively.*

[1]This means that monitoring cost is constant for every unit of loan. We make this assumption as this is the usual assumption in the literature. Relaxing this assumption however, will not alter our results significantly.

Assumption 1 ensures some amount of investment in the industrial sector firm is worthwhile in the sense that it yields positive expected return.

Proposition 1 *If the industrial sector firm borrows in the international credit market then the entire borrowing is invested in the industrial project.*

Proof: See the appendix. ■

Once the lenders internalize this fact they will be willing to lend at a rate given by

$$R^l = \frac{R^*}{\pi(K^I + B)} + \left[\frac{1 - \pi(K^I + B)}{\pi(K^I + B)}\right] m. \qquad (5)$$

Proposition 2 *The firm will never borrow and lend simultaneously in the international credit market. All debt contracts exhibit maximum equity participation i.e.,* $K^I = K$*.*

Proof: See the appendix. ■

The intuition behind the above result is that the benefit to the firm from investing the entire capital in the project in terms of being able to borrow at a lower interest rate outweighs the gain in income from the risk-free asset. Thus proposition 2 establishes that the economy will save in the risk-free asset if it does not need to borrow from the international credit market. The borrowing interest rate in the international credit market is given by

$$R^l = \frac{R^*}{\pi(K + B)} + \left[\frac{1 - \pi(K + B)}{\pi(K + B)}\right] m\ . \qquad (6)$$

Having derived some basic results concerning the working of the credit market let us now see how the aggregate economy behaves. Suppose the economy consists of a continuum of identical industrial sector firms of unit measure. The economy consists of a representative agent who is endowed with $\overline{L}$ units of labor in every period. The agent invests his capital equally among the industrial sector firms and at the end of each period receives dividends from the successful firms. In each period $\pi(.)$ proportion of the firms will be successful. The proportion of the successful firms in each period depends on the amount of investment in the industrial sector project. Depending on the amount of capital the agent brings into period t the economy will either borrow or lend in the credit market. This decision will in turn determine the inter-temporal budget constraint of the agent and then determine the long run output of the economy. Let K_t denote the amount of capital brought into period t by the agent. The aggregate production function of the economy is

$$Y(K_t) = \pi(K_t^I + B_t)[F(L_t^I) - R_t^l B_t] + a(\overline{L} - L_t^I) + R^*(K_t - K_t^I).$$

Proposition 3 *Let K^* be solution to $\pi'(K)F(L^I) = R^*$. If $K_t \in [0, K^*)$ then $K_t^I = K_t$ and $B_t > 0$. If $K_t \in [\ K^*, \infty)$ then $K_t^I \leq K_t$ and $B_t = 0$.*

Proof: See the appendix. ■

When the capital brought into period t by the agent is sufficiently small the agent will borrow from the credit market. Also, from equation (6) we can see that if the capital endowment of the agent is high the interest premium is

lower. This corresponds well with the fact that industrialized economies find it easier to borrow funds in comparison to emerging economies. Once the return from investment in the industrial sector project falls below the rate of return on the risk free asset the agent will become a lender in the international credit market.

The domestic agent when faced with a lending rate R_t^l will borrow until

$$\frac{\pi'(K_t + B_t)F(L_t^I)}{\pi(K_t + B_t) + \pi'(K_t + B_t)B_t} = R_t^l. \quad (7)$$

Equation (7)[2]implies the level of investment in the industrial project is decreasing in the lending rate. The amount of investment in the industrial project when the agent has to borrow from the international credit market is less than the first-best[3].

In any period t, the agent also decides on the amount of labor and the capital to employ in the industrial sector, and weather to borrow or lend in the international credit market. From assumption 1, the optimal allocation of labor across the industrial and household sector will satisfy

$$\pi(K_t^I + B_t)F'(L_t^I) = a$$

[2]For detailed derivation of this condition see Appendix: Proof of Proposition 3.

[3]The first best level of investment will be given by the solution to $\pi'(K_t + B_t)F(L_t^I) = R^*$. According to equation (7) the demand for credit from the agent satisfies $\frac{\pi'(K_t+B_t)F(L_t^I)}{\pi(K_t+B_t)+\pi'(K_t+B_t)B_t} = R_t^l$. $R_t^l > R^*$ and $\frac{\pi'(K_t+B_t)F(L_t^I)}{\pi(K_t+B_t)+\pi'(K_t+B_t)B_t} < \pi'(K_t + B_t)F(L_t^I)$, hence the level of investment is less than first best.

if $L_t^I < \overline{L}$. Hence, L_t^I is an increasing function of the amount of investment in the industrial sector $(K_t^I + B_t)$. Let us write the labor employed in the industrial sector as $L^I(K_t+B_t)$. From Proposition 3 we know that the economy will borrow if $K_t < K^*$ and the entire capital stock will be invested in the industrial sector. Therefore from equation (7) we can write B_t as a function of K_t. Once the economy attains K^* units of capital it will stop borrowing from the international credit market and invest any additional amount of capital i.e., $(K_t - K^*)$ in the risk free asset. To save on notation, in the subsequent analysis L_t^I and B_t will refer to the optimal choice of labor and borrowing given the level of capital.

Proposition 4 *If* $K_t \in [0, K^*)$ *then total investment in the industrial sector is increasing in* K_t.

Proof: See the appendix. ■

When an economy is a borrower in the international credit market, the lending rate of interest will decrease as the capital stock of the economy increases. As a result the level of investment in the industrial sector increases with the level of capital.

3 Inter-temporal optimization

Now we study the inter-temporal problem faced by the economy. In order to ensure the existence of at least one steady state equilibrium we assume the agent's preferences are characterized by endogenous rate of time preference. One way of interpreting these preferences is to view the discount factors as an agents probability of surviving to the next period[4]. The agent given the initial endowment of capital, has to decide his consumption and savings. The discount factor between periods t and $t+1 (\rho_{t,t+1})$ is a continuous function of consumption at time t in the following way:

$$\rho_{t,t+1} = \beta(C_t) \tag{8}$$

where $0 < \beta(C_t) < 1$.

Let the period utility function of the agent be $U(C_t)$. The maximization problem faced by the agent is

$$\max E_0 \sum_{t=0}^{\infty} \rho_{0,t}\, U(C_t) \ ,$$

subject to

$$K_{t+1} = Y(K_t) - C_t \ , \tag{9}$$

[4]For a detaled discussion of the preferences used below see Chakrabarty([7]).

$$Y(K_t) = \max_{K_t^I, L_t^I} \pi(K_t^I + B_t)[F(L_t^I) - R_t^l B_t] + a(\overline{L} - L_t^I) + R^*(K_t - K_t^I), \quad (10)$$

and a transversality condition

$$\lim_{t \longrightarrow \infty} \rho_{0,t} K_t \geq 0 \ . \quad \text{(TC)}$$

At this point we make some assumptions concerning the functions $U(.)$ and $\beta(.)$ to ensure that the necessary conditions for maximum are also sufficient.

Assumption 2: $U(C_t) > 0$, $U'(C_t) > 0$, $U''(C_t) < 0$, and $\beta''(C_t) < 0 < \beta'(C_t)$ for all C_t.

Let R_t denote the rate of return on capital in period t. From Proposition 3, we have

$$R_t = \begin{cases} R^* + [1 - \pi(K_t + B_t)]m & \text{if } K_t \in [0, K^h), \\ R^* & \text{if } K_t \in [K^h, \infty). \end{cases}$$

The solution to the economy's optimization problem will satisfy the following difference equations[5].

$$\frac{U'(C_t) + \beta'(C_t)\ \phi_{t+1}}{\beta(C_t)[U'(C_{t+1}) + \beta'(C_{t+1})\ \phi_{t+2}]} = \beta(C_t) R_{t+1}, \quad (11)$$

$$\phi_t = U(C_t) + \beta(C_t)\phi_{t+1} \qquad \text{for all } t \geq 1, \quad (12)$$

[5]See the appendix for a detailed derivation.

and (9). The variable ϕ_{t+1} is the present discounted value of future consumption from period $t+1$ onwards[6].

Definition 1 *A rational expectation equilibrium(REE) of this economy are sequences $\{C_t\}_{t=0}^{\infty}$, $\{K_{t+1}\}_{t=0}^{\infty}$, $\{K_t^I\}_{t=0}^{\infty}$, $\{L_t^I\}_{t=1}^{\infty}$, $\{\phi_t\}_{t=1}^{\infty}$, $\{B_t\}_{t=1}^{\infty}$ such that (9), (11), (12) and TC hold for a given K_0.*

Equation (9) is the inter-temporal budget constraint of the agent. Equation (11) tells us that the loss in welfare due to foregoing consumption in period t has to equal the discounted value of gain in welfare from period $t+1$ onwards. This condition is commonly referred to as the Fisher equation.

4 Steady-State Equilibria and Stability

Let us first study the steady state solutions to the difference equations (9), (11), (12) . In a steady state, equations (9), (11) and (12)reduce to

$$C = Y(K) - K \ , \quad \text{(BC)}$$

$$\beta(C) R(K) = 1, \quad \text{(RR)}$$

and

[6]$\phi_{t+1} = \sum_{s=t+1}^{\infty} \rho_{t+1,s} U(C_s)$

$$\phi = \frac{U(C)}{1-\beta(C)}, \tag{13}$$

where $R(K) = \max\{R^* + [1-\pi(K+B)]m,\, R^*\}$. Equation (RR) is the steady state counterpart of the Fisher's inter-temporal optimum. Equation (BC) gives us the locus of points along which the agent's consumption and capital stock are constant and satisfy the budget constraint. The (BC) curve is increasing in the level of capital[7]. The slope of RR curve in consumption-capital plane is given by $-\frac{R'(K)\beta(C)^2}{\beta'(C)}$ which is positive when $K \in [0, K^*)$. When $K \in [K^*, \infty)$ the (RR) curve is a horizontal line. Now we characterize the steady states and their stability properties.

Proposition 5 *If* $\beta^{-1}(\frac{1}{R(0)}) < Y(0)$ *and* $\beta^{-1}(\frac{1}{R^*}) > Y(K^*) - K^*$ *then there exists at least one steady state level of capital stock* $K \in [0, K^*)$ *which is locally unique.*

Proof: See the appendix. ■

The proposition above establishes a sufficient condition for the existence of at least one stable steady state equilibrium. The condition in the proposition means that when an economy has no capital the discount factor should be sufficiently high to induce the economy to save and accumulate capital. Note that it is possible that there may be more than one stable steady state equilibria.

[7]Using the envelope theorem, the slope of BC curve is $\pi'(K+B)R^l - 1$ if $K \in [0, K^*)$ and $R^* - 1$ if $K \in [K^*, \infty)$. Hence the slope of the BC curve is always positive.

5 Simulation

In this section we study the behavior of a simulated economy whose characteristics are similar to a standard international business cycle model except for the credit market imperfection. We assume that the period utility and discount factor are of the following functional forms:

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma}, \qquad \sigma > 1,$$

$$\beta(C) = \overline{\beta} - (\overline{\beta} - \underline{\beta})e^{-\delta C}, \quad \text{where } \overline{\beta} > \underline{\beta} > 0 \text{ and } \delta > 0.$$

The production functions in the industrial and household sectors are

$$F(L) = AL^{\theta} \text{ and } Y^H = aL$$

respectively. The probability of success of an industrial sector project is

$$\pi(K+B) = \begin{cases} \tau(K+B)^{\alpha} & \text{if } 0 < K+B < \tau^{-\frac{1}{\alpha}} \\ 1 & \text{if } \quad K+B > \tau^{-\frac{1}{\alpha}} \end{cases}$$

where $\tau < 1$. From the first-order condition for optimal allocation of labor, the amount of labor employed in the industrial sector is given by

$$L^I = \begin{cases} (\frac{A\tau\theta}{a})(K+B)^{\frac{\alpha}{1-\theta}} & \text{if} \quad K < \overline{L}, \\ \overline{L} & \text{otherwise.} \end{cases}$$

The level of capital when the economy starts lending in the international credit market $K^* = (\frac{c}{R^*})^{\frac{1-\theta}{1-\alpha-\theta}}$, where $c = \alpha\tau A(\frac{A\tau\theta}{a})^{\frac{\theta}{1-\theta}}$. Note that as long as $K^* < \tau^{-\frac{1}{\alpha}}$ the economy which borrows from the international credit market will have to pay an interest premium. These functional forms yield us a Cobb-Douglas production function for the industrial sector. We now simulate the economy using certain parameter values which are presented in Table 1. The relationship between capital stock of the economy and the lending rate is given in Figure 1a. The corresponding optimal level of borrowing is shown in Figure 1b. The solution to inter-temporal maximization problem of the economy shows the possibility of two stable steady state equilibria. The function $g(K) = \beta(Y(K) - K)R(K) - 1$ is shown in figure 2. If the initial endowment of capital of the economy is less than K_u the economy converges to the poverty trap steady state K_p. If the initial endowment of capital exceeds K_u the economy converges to the high level steady state K_h. Some characteristics of the two steady states are summarized in Table 2. Understandably the poverty trap steady state shows a lower share of industrial sector in GDP. The debt-equity ratio(B/K) is higher for K_p. We now study the impact of a small productivity and interest rate shock on these economies. These shocks are assumed to follow the following processes:

$$\widehat{R}_t^* = 0.9\widehat{R}_{t-1}^* + \upsilon_R,$$

$$\widehat{A}_t = 0.95\widehat{A}_{t-1} + \upsilon_A,$$

around their steady state values, where υ_R and υ_A are serially uncorrelated shocks to world interest rate and productivity respectively.

The impulse response functions for capital and output due to a 1% productivity shock are shown in Figures 3a and 3b. Both the steady state economies experience amplification and persistence. This is due to the credit market friction. Figures 4a and 4b are the deviation in capital and output from their steady state values due to a 1% shock to the world interest rate. Interestingly the effect on the poverty trap steady state is more severe and persistent.

This behavior of the impulse response functions is similar to Carlstrom and Fuerst([6]). However, in their model an economic expansion is associated with an increase in risk premium and bankruptcy rates. Our model predicts a lowering of risk premium and bankruptcy rates during periods of expansion. This is due to the fact that in our model economies can borrow from the foreign markets and are not constrained my domestic savings.

6 Conclusion

We develop a model of imperfect credit markets where an economy has access to foreign capital markets and potentially can borrow unlimited amount of funds. On a purely theoretical level this adds to the present literature by allowing the economy to borrow beyond domestic savings. In conjunction with endogenous rate of time preference this paper explains why history of an economy matters in the process of development. The model also predicts hump shaped impulse response functions: a well established empirical fact(see for example Cogley and Nason([8])).

In future work we would like to carefully calibrate the parameters of the model to quantitatively evaluate the predictions of the model. In this paper we have played down the role of the banking system. Explicit modeling of the banking system will not alter our results as there is no aggregate uncertainty in the economy. Introduction of aggregate uncertainty in the production process may lead to phenomenon such as banking crisis and self fulfilling expectations. The role of banking system would become important in such a scenario. Such issues are left for future research.

Appendix

Proof of Proposition 1:

We want to show that if $B > 0$ then $B^I = B$. If the firm decides to borrow funds from the international capital market it maximizes expected income i.e.,

$$\max_{B,B^I,K^I} \pi(K^I + B^I)[F(L^I) + R^*(B - B^I) - R^l B] + R^*(K - K^I),$$

with respect to B and B^I. If $B > 0$ and $B^I < B$, then it is possible to increase the expected income of the agent by reducing B since $R^l > R^*$ which is a contradiction. ■

Proof of Proposition 2:

We want to show that if $B > 0$ then $K^I = K$. Incorporating the previous

proposition we can re-write the expected income of the agent as

$$\max_{B,B^I,K^I} \pi(K^I + B)[F(L^I) - R^l B] + R^*(K - K^I).$$

If $B > 0$ then $\pi'(K^I + B)[F(L^I) - R^l B] - \pi(K^I + B)\ R^l \geq 0$. Suppose $K^I < K$ then $\pi'(K^I + B)F(L^I) = R^*$. Substituting this in the previous inequality we get that $R^* - R^l B - \pi(K^I + B)R^l \geq 0$. Substituting (6) and carrying out simple manipulation we get that for the $B > 0$ and $K^I < K$ to hold simultaneously $-R^l B - [1 - \pi(K^I + B)]m$ has to be non-negative which is a contradiction. ■

Proof of Proposition 3:

The expected income of the domestic agent in period t is

$$Y(K_t) = \pi(K_t^I + B_t)[F(L_t^I) - R_t^l B_t] + a(\overline{L} - L_t^I) + R^*(K_t - K_t^I).$$

First order conditions for optimum with respect to L_t^I yields

$$\pi(K_t^I + B_t)F'(L_t^I) \geq a.$$

$L_t^I > 0$ from our assumptions and is increasing in $(K_t^I + B_t)$ as long as $L_t^I < \overline{L}$. Therefore we can write $L_t^I = L(K_t^I + B_t)$; where $L'(.) > 0$ if $L_t^I < \overline{L}$ and $L'(.) = 0$ otherwise. Let K^* be solution to $\pi'(K)F(L^I) = R^*$. We want to show if $K_t \geq K^*$ then $B_t = 0$. If $K_t \geq K^*$ then $\pi'(K_t + B_t)F(L_t^I) - \pi'(K_t + B_t)B_t - \pi(K_t + B_t)R_t^l \leq R^* - \pi'(K_t + B_t)B_t - \pi(K_t + B_t)R_t^l < 0$ implying borrowing will lower expected income. Hence $B_t = 0$ when $K_t \geq K^*$. Using Proposition 2, optimal choice of B_t after some manipulations can be written as

$$\frac{\pi'(K_t + B_t)F(L_t^I)}{\pi(K_t + B_t) + \pi'(K_t + B_t)B_t} - R_t^l \geq 0 \text{ if } B_t > 0.$$

Notice the first order condition for maximum is also sufficient since $\pi''(K_t + B_t)[F(L_t^I) - R_t^l] - 2\pi'(K_t + B_t)R_t^l < 0$. Define $z(B, K_t) = \frac{\pi'(K_t+B)F(L_t^I)}{\pi(K_t+B)+\pi'(K_t+B)B} - R_t^l$. If $K_t < K^*$ then $z(0, K_t) = \frac{\pi'(K_t)F(L_t^I)}{\pi(K_t)} - R^* > 0$. Hence optimal $B_t > 0$. ■

Proof of Proposition 4:

The optimal level of borrowing by an economy with capital less than K^* solves

$$\pi'(K_t + B_t)F(L_t^I) - \pi'(K_t + B_t)B_t - \pi(K_t + B_t)R_t^l = 0.$$

The condition above gives the borrowing function $B_t = B(K_t)$. $B^{'}(K) = \frac{dB}{dK}$ $= \frac{\pi^{''}(K+B)[F(L^I)-R^l B]-\pi'(K_t+B_t)R_t^l}{\pi''(K+B)[F(L^I)-R^l B]-2\pi'(K_t+B_t)R_t^l} > -1$. Therefore total investment $K_t + B_t$ is increasing in K_t. ■

Inter-temporal Optimization problem of the economy:

The Lagrangian for the agent's problem can be written as

$$\pounds = \sum_{t=0}^{\infty}\{\rho_{0,t}\ U(C_t) + \widetilde{\lambda}_t[Y_t - C_t - K_{t+1}]\}\ .$$

The first-order conditions for maximum are

$$\rho_{0,t}U'(C_t) + \sum_{s=t+1}^{\infty}\frac{\partial \rho_{0,s}}{\partial C_t}U(C_s) = \widetilde{\lambda}_t\ ,$$

$$\widetilde{\lambda}_t = \widetilde{\lambda}_{t+1}R_{t+1},$$

and the transversality condition holding with equality. Let $\left(\frac{\widetilde{\lambda}_t}{\rho_{0,t}}\right) = \lambda_t$ and $\sum_{s=t+1}^{\infty}\rho_{t+1,s}U(C_s) = \phi_{t+1}$, where ϕ_{t+1} is the present discounted value of future consumption from period $t+1$ onwards. The first order conditions can now be re-written as

$$U'(C_t) + \beta'(C_t)\ \phi_{t+1} = \ \lambda_t\ , \tag{14}$$

$$\lambda_t = \lambda_{t+1}\beta(C_t)R_{t+1}, \tag{15}$$

and

$$K_{t+1} = Y_t - C_t\ . \tag{16}$$

Substituting (14) in (15) we get,

$$\frac{U'(C_t) + \beta'(C_t)\ \phi_{t+1}}{\beta(C_t)[U'(C_{t+1}) + \beta'(C_{t+1})\ \phi_{t+2}]} = \beta(C_t)R_{t+1} \tag{17}$$

Notice that the variable ϕ_t, the present discounted value of utilities from period t onwards, evolves in the following fashion:

$$\phi_t = U(C_t)\ + \beta(C_t)\ \phi_{t+1} \qquad \text{for all } t \geq 1\ . \tag{18}$$

■

Proof of Proposition 5:

Define $g(K) = \beta(C)R(K) - 1$. Substituting (BC) for C, $g(0) = \beta(Y(0))R(0) - 1 > 0$ if $\beta^{-1}(\frac{1}{R(0)}) > Y(0)$ and $g(K^*) = \beta(Y(K^*) - K^*)R^* - 1 < 0$ if $\beta^{-1}(\frac{1}{R^*}) < Y(K^*) - K^*$. Then there must exist a $K_s \in [0, K^*)$ such that $g(K_s) = 0$. Moreover the derivative of the function $g(.)$ along that steady state is $g'(K_s) = \beta'(C)R(K_s)(R(K_s) - 1) + \beta(C)R'(K_s) < 0$. To study the stability properties of the steady state, we first log-linearize equation (9), (12) and (11) around steady state which yields

$$\widehat{K}_{t+1} = \beta^{-1}\widehat{K}_t + \ s_1\ \widehat{C}_t, \tag{19}$$

where '^' denotes percentage deviation of the variable from its steady state value and $s_1 = -C/K$ at steady state. Log-linearization of (12) gives us

$$\widehat{\phi}_{t+1} = - - \beta^{-1}\Delta_2\ \widehat{C}_t + \ -\beta^{-1}\widehat{\phi}_t \tag{20}$$

where $\epsilon_\beta(C) = \left(\frac{\beta'(C)\ C}{\beta(C)}\right) > 0$, $\epsilon_U(C) = \left(\frac{U'(C)\ C}{U(C)}\right) > 0$ and $\Delta_2 = (1 - \beta)\epsilon_U(C) + \beta\epsilon_\beta(C) > 0$.We rewrite the above equation as

$$\widehat{\phi}_{t+2} - \widehat{\phi}_{t+1} = -\beta^{-1}\Delta_2\ \widehat{C}_{t+1} + (\ \beta^{-1} - 1)\ \widehat{\phi}_{t+1}, \tag{21}$$

to simplify our analysis in future. From (14), we have

$$[s_2\sigma(C) + s_3\eta_\beta(C)\phi]\ \widehat{C}_t + \ s_3\ \widehat{\phi}_{t+1} = \widehat{\lambda}_t,$$

where $\sigma(C) = \left(\frac{U''(C)\ C}{U'(C)}\right) < 0$ and $\eta_\beta(C) = \left(\frac{\beta''(C)\ C}{\beta'(C)}\right) < 0$. $s_2 = \left(\frac{U'(C)}{\lambda}\right)$ and $s_3 = 1 - s_2$. We write the above equation more compactly as

$$\Delta_1\ \widehat{C}_t + \ s_3\ \widehat{\phi}_{t+1} = \widehat{\lambda}_t, \tag{22}$$

where $\Delta_1 = [s_2\sigma(C) + s_3\eta_\beta(C)\phi] < 0$, from our previous assumptions. From (15) we have

$$\widehat{\lambda}_t - \ \widehat{\lambda}_{t+1} = \epsilon_\beta(C)\ \widehat{C}_t + R'(K)\widehat{K}_{t+1} \tag{23}$$

We can now use equations (19), (20), (21), (22) and (23) to write a system of difference equations in $\widehat{C}_t$, $\widehat{\phi}_t$ and $\widehat{K}_t$ where the dynamical system can be expressed as

$$\begin{bmatrix} \widehat{C}_{t+1} \\ \widehat{\phi}_{t+1} \\ \widehat{K}_{t+1} \end{bmatrix} = M \begin{bmatrix} \widehat{C}_t \\ \widehat{\phi}_t \\ \widehat{K}_t \end{bmatrix}, \tag{24}$$

where

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ -\beta^{-1}\Delta_2 & \beta^{-1} & 0 \\ s_1 & 0 & \beta^{-1} \end{bmatrix},$$

$M_{11} = \left(\frac{\epsilon_\beta(2 - \beta^{-1}) + s_1R'(K)K - \Delta_1}{\epsilon_\beta - \Delta_1}\right)$, $M_{12} = \left(\frac{\epsilon_\beta(\beta^{-1} - 1)\Delta_2^{-1}}{\epsilon_\beta - \Delta_1}\right)$ and

$M_{13} = \left(\frac{R'(K)K\beta^{-1}}{\epsilon_\beta - \Delta_1}\right)$. The eigenvalues of matrix M are going to determine the local behavior of the system.

The roots of the polynomial $\det[M - \mu I] = 0$, will determine the behavior of the above system.

$$\det[M - \mu I] = (\beta^{-1} - \mu)[(M_{11} - \mu)(\beta^{-1} - \mu) + M_{12}\beta^{-1}\Delta_2 - s_1 M_{13}] \ .$$

Therefore $\mu_1 = \beta^{-1}$ is one of the roots of the polynomial. The other two roots of $\det[M - \mu I]$ are the roots of the polynomial,

$$P(\mu) = \mu^2 - (M_{11} + \beta^{-1})\mu + (M_{11}\beta^{-1} + M_{12}\beta^{-1}\Delta_2 - s_1 M_{13}).$$

Consider $(M_{11}\beta^{-1} + M_{12}\beta^{-1}\Delta_2 - s_1 M_{13}) = \beta^{-1}$ $(\frac{\epsilon_\beta + \epsilon_\beta(1-\beta^{-1}) + s_1 R'(K)K - \Delta_1 + \epsilon_\beta(\beta^{-1}-1) - s_1 R'(K)K}{\epsilon_\beta - \Delta_1}) = \beta^{-1}$. Therefore $P(0) = \beta^{-1} > 1$. $P(1) = 1 - M_{11} - \beta^{-1} + \beta^{-1} = 1 - M_{11} = \frac{\epsilon_\beta(\beta^{-1}-1) - s_1 R'(K)K}{\epsilon_\beta - \Delta_1} < 0$ if $\epsilon_\beta(\beta^{-1}-1) - s_1 R'(K)K < 0$. Note that if $g'(K_s) = \beta'(C)R(K_s)(R(K_s)-1) + \beta(C)R'(K_s) < 0$ it implies that $\epsilon_\beta(\beta^{-1} - 1) - s_1 R'(K)K < 0$. Hence there exists at least one root of the polynomial $\det[M - \mu I]$ which is less than one. Its easy to show that the third root is strictly greater than unity in absolute value. Hence the steady state is a saddle path and locally unique. ■

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Table 1: Simulation Parameters

Parameter	Value	Description
R^*	1.025	World interest rate(per annum)
m	0.005	Average monitoring Cost
τ	0.1	Shift parameter for probability of success of industrial sector projects
α	0.53	Elasticity of "probability of success" with respect to investment in the industrial sector
θ	0.26	Elasticity of output with respect to labor in the industrial sector
A	34	Shift parameter in the industrial sector production function
a	1	Labor productivity in household sector
$\underline{\beta}$	0.40	Lower bound of the discount factor
$\bar{\beta}$	0.99	Upper bound of the discount factor
δ	0.03	$-[\beta''(C)/\beta'(C)]$
σ	1.5	Elasticity of marginal utility

Table 2: Some steady state characteristics

	B/K	Y^I/Y	$R^l B/F(L^I)$	C/Y	$\beta(C)$	R^l
K_p	70.22	75.93	17.64	77.39	0.89	1.10
K_h	21.99	87.04	8.56	63.80	0.90	1.05

Figure 1a: Lending Rate

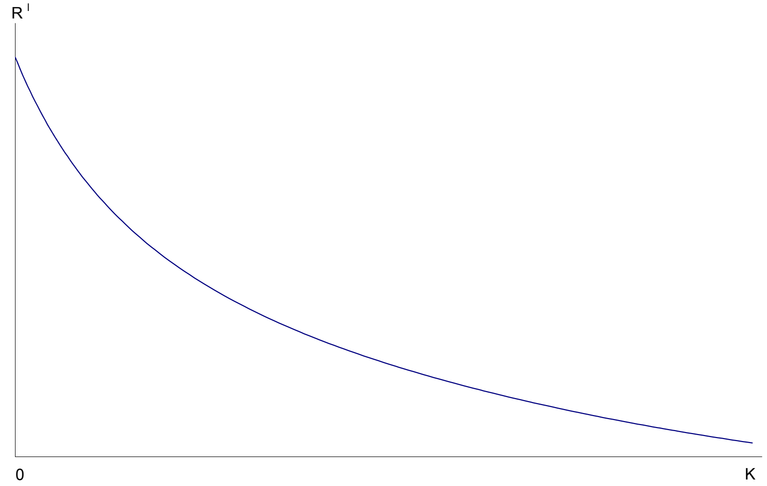

Figure 1b: Borrowing

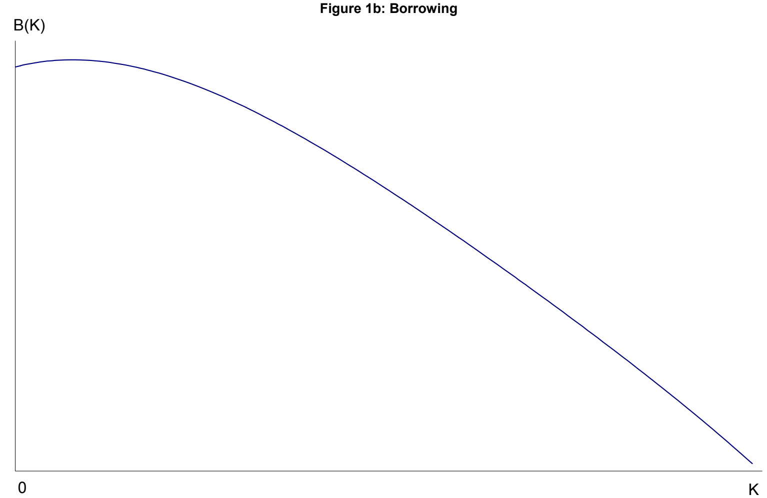

Figure 2: Steady States

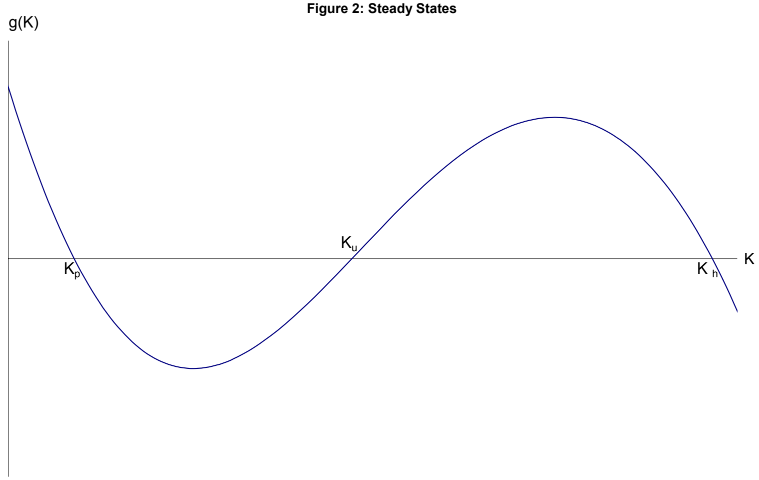

Figure 3a: IRF for capital due to a 1% productivity shock

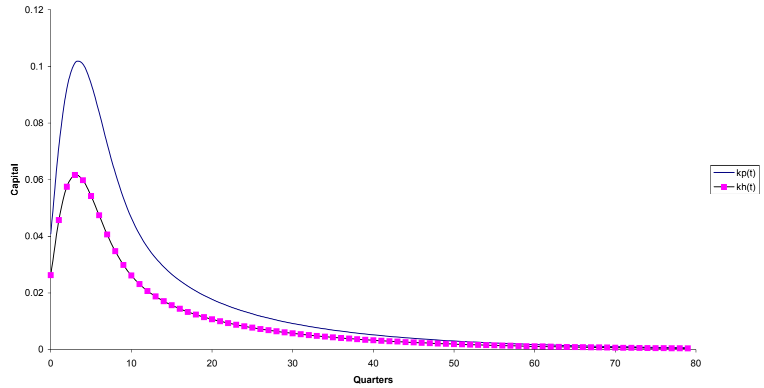

Figure 3b: IRF for output due to a 1% productivity shock

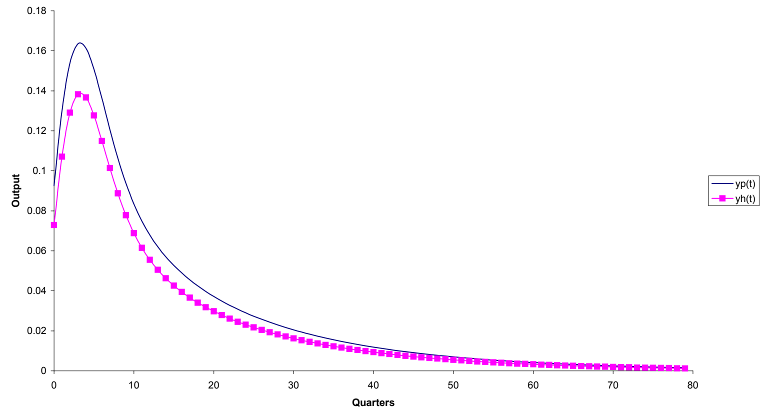

Figure 4a: IRF for capital due to a interest rate shock

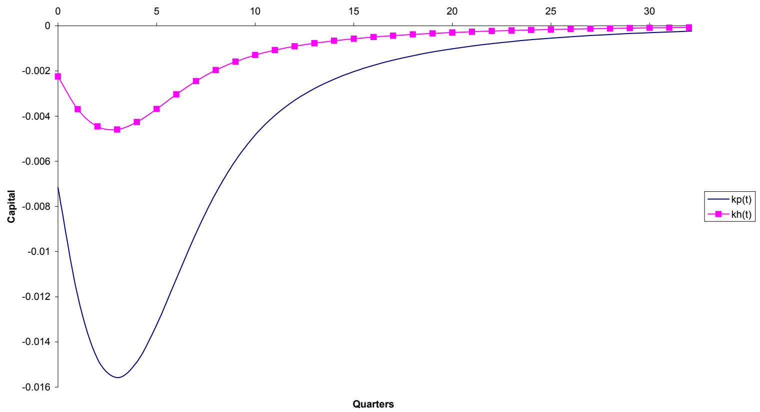

Figure 4b: IRF for output due to a interest rate shock