



THE UNIVERSITY OF SYDNEY

Economics Working Paper Series

2011

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January 2011

Modelling asset correlations: A nonparametric approach*

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January 26, 2011

Abstract

This article proposes a time-varying nonparametric estimator and a time-varying semi-parametric estimator of the correlation matrix. We discuss representation, estimation based on kernel smoothing and inference. An extensive Monte Carlo simulation study is performed to compare the semiparametric and nonparametric models with the DCC specification. Our bivariate simulation results show that the semiparametric and nonparametric models are best in DGPs with gradual changes or structural breaks in correlations. However, in DGPs with rapid changes or constancy in correlations the DCC delivers the best outcome. Moreover, in multivariate simulations the semiparametric and nonparametric models fare the best in DGPs with substantial time-variability in correlations, while when allowing for little variability in the correlations the DCC is the dominant specification. The methodologies are illustrated by estimating the correlations for two interesting portfolios. The first portfolio consists of the equity sectors SPDRs and the S&P 500 composite, while the second one contains major currencies that are actively traded in the foreign exchange market. Portfolio evaluation results show that the nonparametric estimator generally dominates its competitors, with a statistically significant lower portfolio variance.

Keywords: *Semiparametric Conditional Correlation Model, Nonparametric Correlations, DCC, Local Linear Estimator, Portfolio Evaluation.*

JEL Classifications: C14; C58; G10.

*This research was initiated in May 2009, when the first author was visiting CREATES. The hospitality and stimulating research environment provided is gratefully acknowledged. Also, comments on an earlier version of the paper from seminar participants at the Universitat Rovira i Virgili, Aarhus University, The University of New South Wales, University of Sydney, La Trobe University, Macquarie University, University of Macedonia and Koç University as well as conference participants at the (EC)2-Conference 2009 and Australian Conference of Economists 2010 are greatly appreciated. We are also grateful to Mohamad Khaled, Asger Lunde, Oscar Martinez, Michael McAleer, Christos Ntantamis, Andreas Shrimpff and Valeri Voev for helpful suggestions and comments. I. Casas is also grateful for financial support to the Danish Social Sciences Research Council (grant no. FSE 275-08-0078). We bear sole responsibility, however, for the views expressed here.

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Introduction

The financial crisis beginning in September 2008 rattled the whole world and left a big global economical crisis which we are still feeling today. Given its recency, very little work has been undertaken on the crisis.¹ It is well known that in many crises the degree of co-movement between assets returns changes rapidly, partly as a result of generally increased uncertainty. Therefore, an accurate assessment of the correlation between assets during the crisis period is of particular interest. For instance, rapid changes in correlation patterns call for an immediate adjustment of portfolios. Furthermore, policy makers are also interested in these links because of their implications for systemic risk.

Methodologically, this article proposes a nonparametric and a semiparametric correlation approaches based on kernel smoothing techniques. These methods allow a great deal of flexibility on the correlation matrix whose functional form must only satisfy certain regularity conditions. In addition, the error term must be iid but not specific distribution is assumed. Therefore, these methods are desirable alternatives to parametric models where a misspecification of the functional form and distribution will result in inconsistent parameter estimators.

Hafner et al. (2006) and Long et al. (2010) present a semiparametric model that combines a parametric estimation of the volatility with a subsequent nonparametric estimation of the correlation of the returns.² The estimation consists of three steps. In the first step, the conditional variance of each asset is estimated separately using a parametric model, for example a GARCH(1,1). In the second step, the conditional covariance matrix estimator of the standardized returns is obtained with the nonparametric Nadaraya-Watson estimator. As noticed by Long et al. (2010) the key point behind their semiparametric model is that if the parametric estimation in the first step captures the main volatility features, the nonparametric estimation of the correlation in the second step will be easier, compared to the estimator of the whole covariance matrix. Finally, the resulting matrix is regularised to obtained a well-definted correlation estimator.

The semiparametric model that we propose here (denoted by SPCC) differs from those of Hafner et al. (2006) and Long et al. (2010) in Step 2. Both these studies assume that the correlations depend on one or several exogenous (or predetermined) variables. For example, the conditional correlation may depend on some sort of volatility proxy variable. This approach has two main drawbacks: 1) the choice of the exogenous variables is not always clear and 2) kernel smoothing methods are not feasible with many conditioning variables (e.g., ‘curse of dimension-

¹Exceptions are Fry et al. (2010) that use contagion tests to identify the transmission channels of the recent financial crisis, and Laurent et al. (2010) that compares several multivariate GARCH models in terms of forecasting during the 2007-2008 period crisis.

²Notice, however, the model of Long et al. (2010) is a semiparametric model for the conditional covariance matrix of raw returns where the nonparametric estimation serves as a correction for the parametric conditional covariance estimator.

ality', difficulties of interpretation). Thus, while the Hafner et al. (2006) and Long et al. (2010) estimators use information on the behaviour of the standardized returns around certain values of some exogenous variables, our method assumes that the correlation is a deterministic function of time and only information on the standardized returns in a neighbourhood of the interest point of time is used. Moreover, time variable, is rescaled by the total number of observations so the estimator depends on the sample size and therefore asymptotic results can be derived. The same rescaling device is commonly used in non-stationary processes (see Robinson, 1989; Drees and Stărică, 2002; Dahlaus and Rao, 2006, amongst others) to ensure the asymptotic behaviour of the estimator. Another important difference with the aforementioned studies is that they choose the Nadaraya-Watson (NW) as the kernel smoothing methodology, while we adopt the Local Linear (LL) estimator. The latter has a smaller bias and behaves better at the boundaries (Fan and Gijbels, 1992). The bandwidth is selected automatically through least squares cross-validation.

Long et al. (2010) mentions that if the volatility estimator in Step 1 is very far from reality, then the correlation estimator might be inconsistent. This problem may be resolved with our fully nonparametric approach (denoted by NPC). The NPC model consists of the same three steps as the SPCC with the difference that the estimation of the volatility in Step 1 is also done nonparametrically using the LL estimator. Therefore, the volatility is also assumed to be a smooth function of time, and therefore the resulting volatility and correlation estimators are consistent.

Note that a single step nonparametric model is possible using the returns series. The result is a nonparametric estimator of the covariance matrix which may be comparable with the parametric BEKK-MGARCH (Baba et al., 1991). In our experience, however, this estimator is worse than the three steps NPC because there is too much information to be estimated at once. In fact, correlations are less persistent than volatilities and therefore, it is better to divide the problem in two: first the volatility is estimated from the returns; and second, correlations are estimated from the standardised returns.

The parametric benchmark used in this paper is the Dynamic Conditional Correlation (DCC) model developed by Engle (2002). The DCC assumes that the correlation between assets evolves according to a simple GARCH-type structure. An attractive feature of this model is the so-called correlation targeting (substituting unconditional correlation by sample correlation), which reduces the number of estimated parameters. Other parametric methods available are the Smooth Transition Conditional Correlation (STCC) model proposed by Berben and Jansen (2005) and Silvennoinen and Teräsvirta (2005, 2009) and the Regime Switching Dynamic Correlation (RSDC) model of Pelletier (2006). The STCC method allows for the correlation of a Constant Conditional Correlation (Bollerslev, 1990) model to change smoothly over time, while in the Pelletier's model the correlation regimes depend on an unobserved Markov-Switching

process. Note, however, that correlation targeting is not possible with the last two models.

We follow a systematic procedure to compare the SPCC, NPC and DCC models by performing an extensive set of Monte Carlo simulations. In particular, we simulate bivariate processes and test the robustness of the results to a variety of misspecifications in volatility such as when the true variance is an asymmetric GARCH or when there are volatility spillovers. We find the following interesting points. In terms of mean absolute error (MAE) the semiparametric and nonparametric estimators are superior in DGPs with gradual changes or structural breaks in correlations. However, in DGP with rapid changes or constancy in correlations the parametric DCC model outperforms the NPC and SPCC models. This is observed throughout our simulations and is generally robust to misspecifications in the volatility processes. Finally, simulations based on calculating Value-at-Risk show that NPC and SPCC methods in general deliver more accurate results than the DCC model.

In addition, we further perform a multivariate simulation experiment to show the performance of the models in higher dimensions. To our knowledge, the current paper is the first attempt on a multivariate simulation of this kind. Our results show that the SPCC and NPC models are the best in DGPs with substantial time-variability in correlations, while when allowing for little variability in the correlations the DCC is the dominant specification.

With regard to the application, this paper estimates the conditional correlations of two interesting portfolios. The first portfolio, included in the Volatility Institute of the New York University, consists of the nine equity sectors SPDRs and the S&P 500 composite during 2004–2009. Although, there are considerable fluctuations in the correlation between the S&P 500 and the Materials, Utilities and Energy sectors, most correlations in this portfolio are high and nearly constant.

The second portfolio is a well diversified portfolio consisting of five major currencies plus two currencies from emerging economies that are actively traded in the foreign exchange market. Some of these currencies like the Swiss franc are considered as safe haven currencies as they offer investors the opportunity to protect wealth during adverse market conditions. A similar picture holds for the yen and the euro, although to a smaller extent. Other currencies like the Australian dollar and the Brazilian real have risen in recent years due to increased macroeconomic performance and higher interest rates. In our estimation, correlations among the major currencies shift to a higher level in the period 2002–2005 possibly reflecting the "global savings glut" phenomenon. On the other hand, the correlations of the Japanese yen dropped around 2006 and even became negative in the period afterwards. In general, the correlations of the currencies show more variability over time which implies a more frequent rebalancing of portfolios. We also perform a portfolio evaluation exercise to empirically compare the models by using standard portfolio weighting methods (equally weighted, minimum variance portfolios) as well as a carry trade weighting method for our application of currencies. Results show that the NPC

generally dominates the SPCC and DCC models, particularly in minimum variance weighted portfolios and to lesser extent for the carry trade portfolio. Also, the reduction in portfolio's variance obtained by the NPC is often statistically significant. All this information can be of great use in the fields of asset allocation and portfolio diversification.

The outline of the paper is as follows. Section 2 presents the SPCC, NPC and the DCC estimators. A detailed Monte Carlo experiment comparing their performance is shown in Section 3. Section 4 discusses the data and empirical results. Finally, in Section 5, we briefly summarize the main findings and give our conclusions.

1 Time-Varying Conditional Correlations

Let r_t denote an N -dimensional vector time series (zero-mean asset returns) with time-varying conditional covariance matrix

$$\text{Var}[r_t|\mathfrak{S}_{t-1}] = E[r_t r_t'|\mathfrak{S}_{t-1}] = H_t \quad (1)$$

where \mathfrak{S}_{t-1} is the information set at time t . The conditional covariance matrix can be decomposed as

$$H_t = D_t R_t D_t \quad (2)$$

where $D_t = \text{diag}(\sqrt{h_{1,t}}, \sqrt{h_{2,t}}, \dots, \sqrt{h_{N,t}})$ is a diagonal matrix with the square root of the conditional variances $h_{i,t}$ for each asset i at time t on the diagonal. The matrix R_t , with the (i,j) -th element denoted as $\rho_{ij,t}$, is the possibly time-varying correlation matrix with $\rho_{ii,t} = 1$, $i, j = 1, \dots, N$ and $t = 1, \dots, T$. The standardized residuals are denoted by $\epsilon_t = D_t^{-1} r_t$. They are independent and identically distributed with $E(\epsilon_i) = 0$ and $\text{Var}(\epsilon_i) = 1$.

The Constant Conditional Correlation (CCC) model assumes that R_t is constant over time, while the Semi-parametric Conditional Correlation (SPCC) and Dynamic Conditional Correlation (DCC) models allow distinct patterns of time-variation in R_t .

This paper compares the performance of the Nonparametric Correlation model (NPC) and the Semiparametric Conditional Correlation model (SPCC) with the Dynamic Conditional Correlation (DCC) model. The following three subsections describe in detail the three different methodologies.

1.1 Semiparametric Conditional Correlation Model

It is easy to show that $E(\epsilon_t \epsilon_t' | \mathfrak{S}_{t-1}) = R_t$ and therefore an estimator of the correlation can be obtained using nonparametric kernel regression methods. For instance, Hafner et al. (2006) and Long et al. (2010) use the classical Nadaraya-Watson estimator. Instead, we propose the Local

Linear (LL) estimator, which has smaller bias and behaves better at the boundaries (Fan and Gijbels (1996)). Another difference with the Hafner et al. (2006) and Long et al. (2010) is in the choice of the conditioning variable. The aforementioned studies assume that the correlations depend on a single exogenous (or predetermined) variable. Instead, we model the correlations as a deterministic function of time. Time may capture several economic factors expressing themselves mainly in the level of unconditional correlations. Thus, this paper proposes the Local Linear (LL) estimator in Fan and Gijbels (1992) with time as the dependent variable.

Notice that the difference between using time as the “dependent ”variable instead of a stochastic variable is that each point in time is visited only once and not previous information is used. In addition, increasing T does not increase the density of information, and therefore asymptotically speaking, it is the same to estimate the correlation using a large or a short T . Therefore it is necessary, as in Robinson (1989), to assume the existence of a smooth function $\rho_{ij}(t)$ on $(0, 1)$ such that:

$$\rho_{ij,t} = \rho_{ij}\left(\frac{t}{T}\right) \quad \text{for } t = 1, 2, \dots, T.$$

This condition ensures that the amount of local information around a point $\frac{t}{T} \in (0, 1)$ increases as T increases and therefore the bias and variance of an estimator of $\rho_{ij}(\frac{t}{T})$ will decrease. Thus, $\hat{\rho}_{ij,t} = \hat{\rho}_{ij}(\frac{t}{T})$.

The SPCC estimator that we propose for a value $\tau \in (0, 1)$ is defined as:

$$\hat{Q}_\tau^{\text{SPCC}} = \sum_{t=1}^T \hat{\epsilon}_t \hat{\epsilon}'_t K_b\left(\frac{t - T\tau}{T}\right) \frac{s_2 - s_1\left(\frac{t - T\tau}{T}\right)}{s_0 s_2 - s_1^2} \quad (3)$$

where $K_b(\cdot) = (1/b)K(\cdot/b)$, K is a symmetric kernel function heavily concentrated around the origin, b is the bandwidth parameter and τ is the focal point. In addition, $s_j = \sum_{t=1}^T \left(\frac{t - T\tau}{T}\right)^j K_b\left(\frac{t - T\tau}{T}\right)$ for $j = 0, 1, 2$. Equation (3) displays the nonparametric point estimator of the covariance of $\{\hat{\epsilon}_\tau\}$. Drees and Stărică (2002) and Dahlaus and Rao (2006) have used this rescaling device to estimate the time-varying volatility.

The estimators of the volatility $\hat{h}_{i,t}$, although consistent, may not converge fast enough to the true conditional standard deviation at time t for the finite sample. Then the diagonal of \hat{Q}_t^{SPCC} will not be close to the unity vector $\mathbf{1}$. Therefore, the quantity \hat{Q}_t^{SPCC} is typically rescaled using

$$R_t^{\text{SPCC}} = (Q_t^{\text{SPCC}*})^{-1} Q_t^{\text{SPCC}} (Q_t^{\text{SPCC}*})^{-1} \quad (4)$$

where $Q_t^{\text{SPCC}*}$ is a diagonal matrix composed of the square roots of the diagonal elements of Q_t^{SPCC} .

The bandwidth parameter b plays an essential role in nonparametric modelling. It is desirable to have a reliable data-driven and yet easily implemented bandwidth selection procedure.

Hafner et al. (2006) adopt a local bandwidth estimate which has been obtained *ad hoc* for the particular data set under study. On the other hand, Long et al. (2010) set $b = c\hat{\sigma}T^{-1/6}$ and follow a grid search over $c \in [0.5, 5]$ with $\hat{\sigma}$ being the empirical standard deviation of the conditioning variable. This choice aims at finding the bandwidth that ensures the optimal rate of convergence which minimises the asymptotic mean integrated square error of the estimator. However, although choosing the bandwidth amongst a grid of values is practical, it is not always correct. For example, infinite is the appropriate bandwidth for a constant or linear correlation and this value is not included in the grid. Therefore, we propose finding the global bandwidth through least squares cross-validation as it is defined in (5). In practice, we use a Newton-type minimization algorithm. As the Newton minimisers are susceptible to the starting point, we do several numerical minimizations with different starting points and choose the most appropriate bandwidth.

$$b^{\text{SPCC}} = \arg \min_b \sum_{t=1}^T [\text{vecl}(\hat{\epsilon}_t \hat{\epsilon}_t') - \text{vecl}(\hat{Q}_{-t}^{\text{SPCC}})]^2 \quad (5)$$

where $\hat{Q}_{-t}^{\text{SPCC}}$ is the nonparametric estimator obtained when pair $(t, \hat{\epsilon}_t)$ is left out. The *vecl* of a matrix takes the lower diagonal matrix, excluding the diagonal.

In summary, the SPCC estimator proposed here consists of three steps:

Step 1 Devolatilisation. In this step the data volatility of each return is estimated to obtain the standardised returns. Basically, \hat{D}_t is obtained by assuming a certain parametric model driving the volatility process, for example a GARCH(1,1). Therefore, $\hat{\epsilon}_t = \hat{D}_t^{-1} r_t$.

Step 2 Pseudo-correlation matrix. \hat{Q}_t^{SPCC} is obtained as in equation (3) for all $t = 1, \dots, T$.

Step 3 Matrix regularisation. To ensure that the estimators of the finite sample is between -1 and 1, $\hat{R}_t^{\text{SPCC}} = (\hat{Q}_t^{\text{SPCC}*})^{-1} \hat{Q}_t^{\text{SPCC}} (\hat{Q}_t^{\text{SPCC}*})^{-1}$.

1.2 Dynamic Conditional Correlation Model

Engle (2002) specifies the bivariate DCC model through the GARCH(1,1)-type process

$$Q_t^{\text{DCC}} = \Omega + \alpha \epsilon_{t-1} \epsilon_{t-1}' + \beta Q_{t-1}^{\text{DCC}} \quad (6)$$

where α is the news parameter and β is the decay parameter. A simple estimator for the intercept parameter matrix Ω is available through what is called correlation targeting. That is, using the estimator

$$\Omega = (1 - \alpha - \beta) \bar{Q} \quad (7)$$

where \bar{Q} is the sample unconditional correlation matrix between the standardized errors ϵ_t .

Substituting (7) into (6) gives the basic form for the mean-reverting DCC model given by

$$Q_t^{\text{DCC}} = \bar{Q} + \alpha(\epsilon_{t-1}\epsilon'_{t-1} - \bar{Q}) + \beta(Q_{t-1} - \bar{Q}) \quad (8)$$

It is easy to see how this model behaves. The correlations evolve over time in response to new information on the asset returns. When returns are moving in the same direction — either they are both moving up or they are both moving down— the correlations will rise above the average level and remain there for a while. Gradually this information will decay and correlations will fall back to the long-run average. Similarly, when assets move in opposite directions, the correlations will temporarily fall below the unconditional level. The two parameters (α, β) govern the speed of this adjustment. As before, we scale \hat{Q}_t^{DCC} to obtain a proper correlation matrix \hat{R}_t^{DCC} .

In a multivariate framework, the basic DCC specification may be too restrictive. In particular, note that the DCC model implies that all correlations pairs have the same dynamic pattern as implied by the parameters α and β . Cappiello et al. (2006) propose the Generalized DCC (G-DCC), which allows for correlation-specific news and decay parameters. The generalized DCC model is given by

$$Q_t^{\text{GDCC}} = (\bar{Q} + A'\bar{Q}A - B'\bar{Q}B) + A'\epsilon_{t-1}\epsilon'_{t-1}A + B'Q_{t-1}^{\text{GDCC}}B \quad (9)$$

where A and B are defined to be $N \times N$ parameter diagonal matrices. So, the basic DCC is obtained as a special case of the G-DCC if the matrices A and B are replaced by scalars. However, the number of parameters in the G-DCC increases rapidly with the dimension of the model. In particular, for our portfolio application (consisting of 10 assets in total—nine equity sectors SPDRs and the S&P 500) we propose the more parsimonious Semigeneralized DCC (SG-DCC) initially studied by Hafner and Franses (2009). This model allows for correlation-specific news parameter α and restricts only the decay parameter β to be the same across correlation pairs. It is expected that the news parameters varies across correlation pairs more than the decay parameter. The SG-DCC equation is given by

$$Q_t^{\text{SGDCC}} = (\bar{Q} + A'\bar{Q}A - \beta\bar{Q}) + A'\epsilon_{t-1}\epsilon'_{t-1}A + \beta Q_{t-1}^{\text{SGDCC}} \quad (10)$$

A sufficient condition for Q_t to be positive definite for all possible realizations is that the intercept, $\bar{Q} - A'\bar{Q}A - \beta\bar{Q}$, is positive semi-definite and the initial covariance matrix Q_0 is positive definite (Cappiello et al., 2006). As before, we rescale the quantity Q_t to obtain a proper correlation matrix.

In summary, the (SG)-DCC conditional correlation estimator \hat{R}_t^{SGDCC} is obtained in three steps:

Step 1 Devolatilisation. The conditional variances \hat{h}_t are obtained in this step in the same way

than for the SPCC model.

Step 2 The standardized returns $\hat{\epsilon}_t$ are then used to estimate the (SG)–DCC correlations by Gaussian Maximum Likelihood (ML) obtaining \hat{Q}_t^{SGDCC} .

Step 3 Matrix regularisation. The \hat{Q}_t^{SGDCC} is in general not a correlation matrix. Engle (2009) discusses how this is a technical problem that can be solved by rescaling as in equation (4).

Correlations $\rho_{ij,t}$ in the DCC models are assumed to depend on certain parameters α and β (A and β is the case of SG-DCC). These parameters do not change with time and therefore the DCC models may be restrictive to describe time-varying correlations.

1.3 Nonparametric Correlation Model

It is inevitable to wonder how the nonparametric estimator performs in comparison with the parametric and semiparametric estimators. As before, we estimate the model in three steps. First, as in Drees and Stărică (2002), the (unconditional) volatility of each individual asset is estimated using the LL:

$$\hat{h}_{i,\tau}^{\text{NPC}} = \sum_{t=1}^T r_{i,t}^2 K_b \left(\frac{t - T\tau}{T} \right) \frac{s_2 - s_1 \left(\frac{t - T\tau}{T} \right)}{s_0 s_2 - s_1^2}, i = 1, \dots, N \quad (11)$$

Here s_j are defined as in the SPCC model. Basically, one must assume that $h(t)$ is a smooth deterministic function defined on the interval $(0,1)$ such that $h_{j,t} = h \left(\frac{t}{T} \right)$. The positive aspect of this approach is that $\hat{h}_{i,\tau}$ is a consistent estimator of the volatility at point τ if τ is not a boundary point and the volatility function is continuous there (see Robinson, 1989). This is not always the case for the previous two models if the volatility is not well-modelled by the chosen parametric specification.

The second step consists on finding the pseudo-correlation:

$$\hat{Q}_\tau^{\text{NPC}} = \sum_{t=1}^T \hat{\epsilon}_t \hat{\epsilon}_t' K_b \left(\frac{t - T\tau}{T} \right) \frac{s_2 - s_1 \left(\frac{t - T\tau}{T} \right)}{s_0 s_2 - s_1^2}. \quad (12)$$

for $\tau \in (0,1)$. Finally, the appropriate matrix scaling the unconditional correlation matrix estimator is:

$$\hat{R}_t^{\text{NPC}} = (\hat{Q}_t^{\text{NPC}*})^{-1} \hat{Q}_t^{\text{NPC}} (\hat{Q}_t^{\text{NPC}*})^{-1} \quad (13)$$

The bandwidth is chosen by cross-validation as:

$$b^{\text{NPC}} = \arg \min_b \sum_{t=1}^T [\text{vecl}(\hat{\epsilon}_t \hat{\epsilon}_t') - \text{vecl}(\hat{Q}_{-t}^{\text{NPC}})]^2. \quad (14)$$

2 Monte Carlo Simulations

In this section, we compare the sample performance of the three models by examining certain characteristics of conditional correlations when the true correlation processes are observable. We simulate bivariate processes of length $T = 1000$ corresponding to four financial years. The parameter values for the DGPs are chosen from Engle (2002). In particular, we consider four different scenarios (DGPs) for the correlations:

Scenario 1: constant correlation, $\rho_t = 0.9$,

Scenario 2: correlation with weak seasonality, $\rho_t = 0.5 + 0.4 \cos(2\pi t/200)$,

Scenario 3: correlation with strong seasonality, $\rho_t = 0.5 + 0.4 \cos(2\pi t/20)$, and,

Scenario 4: correlation with a structural break, $\rho_t = 0.9 - 0.5(t > 500)$.

2.1 Experiment 1

Two series of returns are simulated with volatility following a GARCH(1,1) and the innovations are distributed as a bivariate normal with a vector zero as mean and a correlation matrix which is the interest.

A total $M = 200$ experiments were conducted for each scenario with the same model specification as in Engle (2002) which is transcribed below:

$$\begin{aligned} r_{1,t} &= \sqrt{h_{1,t}} \epsilon_{1,t}, & r_{2,t} &= \sqrt{h_{2,t}} \epsilon_{2,t} \\ h_{1,t} &= 0.01 + 0.05r_{1,t-1}^2 + 0.94h_{1,t-1} & h_{2,t} &= 0.5 + 0.2r_{2,t-1}^2 + 0.5h_{2,t-1} \end{aligned} \quad (15)$$

$$\begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_t \\ \rho_t & 0 \end{pmatrix} \right)$$

The DGP $h_{1,t}$ is chosen such that the unconditional variance is lower but the persistence is higher than in DGP $h_{2,t}$. Each estimate's performance is measured by the mean absolute error which for each simulation N is defined by:

$$\text{MAE}_M = \frac{1}{T} \sum_{t=1}^T |\rho_t - \hat{\rho}_t| \quad (16)$$

where $\hat{\rho}_t$ are the elements of \hat{R}_t .

Figure 1 shows the estimator from each model chosen as the typical sample whose MAE is equal to the median of all the MAE_M , while Figure 2 shows the boxplots of the MAE_M for the 200 simulations. The boxes represent the 25% and 75% quantiles (interquartile range, IQR) of the MAE_M . The line in the middle of the box represents the median of the MAE_M and therefore the error corresponding to the estimator in Figure 1. The end of the whiskers represent the 1.5IQR of the lower quartile and upper quartiles. As seen, the DCC is the best model for constant correlations (Figures 1-2 (a)). In this scenario, the SPCC also performs quite well choosing a very large bandwidth which results in a very smooth estimator. On the other hand, for Scenarios 2 and 4 the semiparametric and nonparametric estimators improve substantially on the DCC. Also, their performance is quite similar in those two scenarios. For instance, the NPC slightly outperforms the SPCC for periodic correlations with a yearly frequency (gradual changes) such as in Figures 1-2 (b), while the SPCC is more accurate for correlations with a regime switch (structural break) like in Scenario 4 (Figures 1-2 (d)). However, in Scenario 3 (periodic correlations with large frequencies—Figures 1-2 (c)) SPCC and NPC imply correlations that are too smooth and, therefore, are less accurate than the DCC model.

It must be note that the correlation function of Scenario 4 is not continuous at $t = 500$. Therefore the NPC and SPCC estimator of $\rho_{t-h}, \rho_{t-h+1}, \dots, \rho_{t+h}$ are not consistent. This could be improved using asymmetric kernels like in Qiu (2003).

2.2 VaR

Another performance measure that we use is the evaluation of models for calculating Value-at-Risk. In particular, for a bivariate portfolio with weighted vector $w'_t = (w_{1,t}, w_{2,t})$, the estimated Value-at-Risk under normality and for each simulation M is given by:

$$VaR_t^M(\alpha) = |\Phi_\alpha^{-1}| \sqrt{(w_t^M)' H_t^M w_t^M} \quad (17)$$

where $\Phi(\cdot)$ is the probability function of ϵ_t and α is the probability that the portfolio will fall in value.

We follow Engle (2002) and define a dichotomous variable called hit:

$$hit_t^M = \mathbf{1}((w_t^M)' r_t^M < VaR_t^M(\alpha)) - \alpha \quad (18)$$

where $\mathbf{1}(\cdot)$ is the indicator function and $\alpha=0.05$ is the level of significance. If the model is correct the hit variable should be unpredictable. To test this we perform the dynamic quantile test (Engle and Manganelli, 2004)— this is an F-test with null hypothesis “all coefficients as well as the intercept are zero” in a regression of the hit variable on its past (we use five lags) and lagged VaR. The number of rejections using a 5% critical value is a measure of model

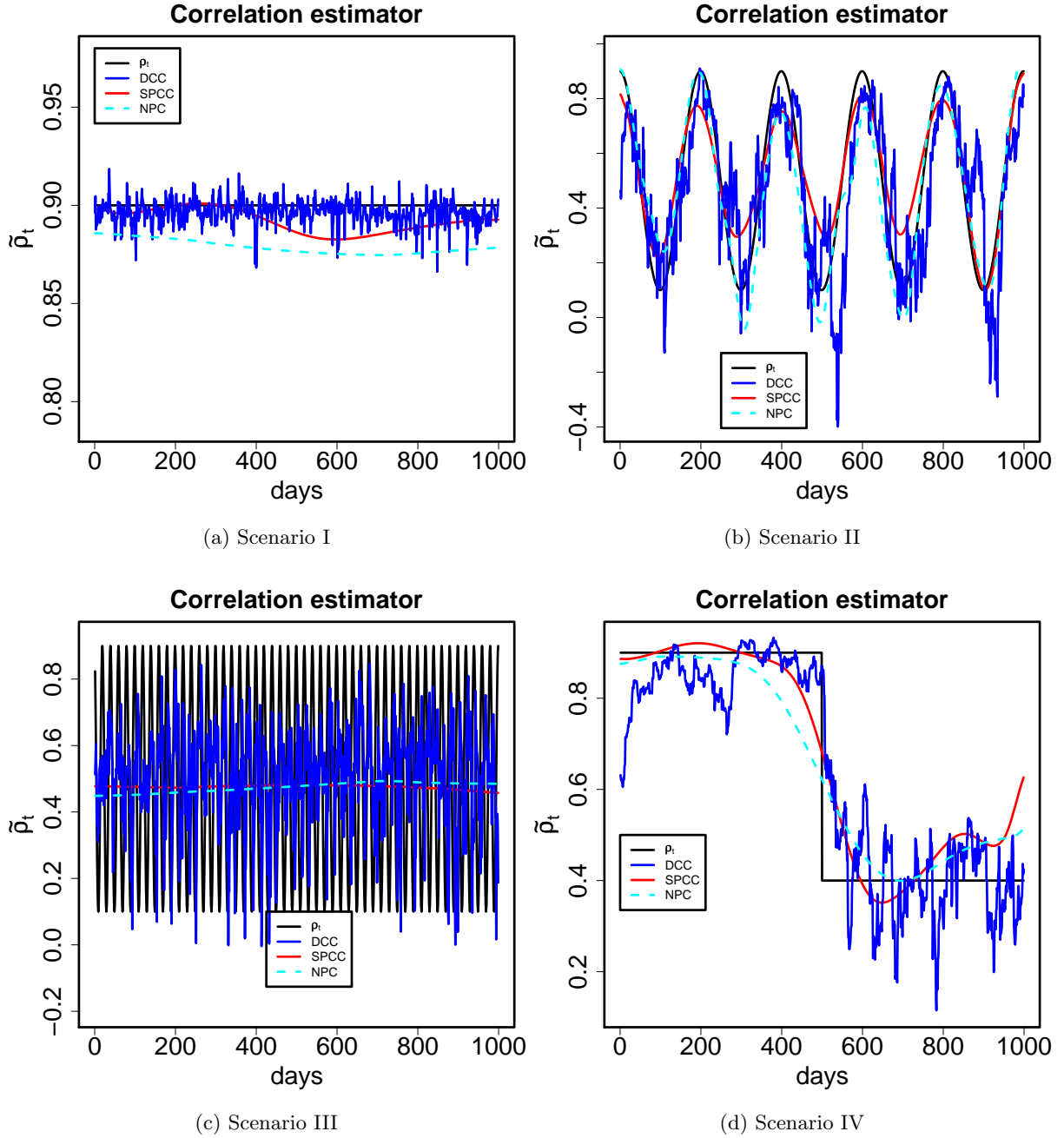
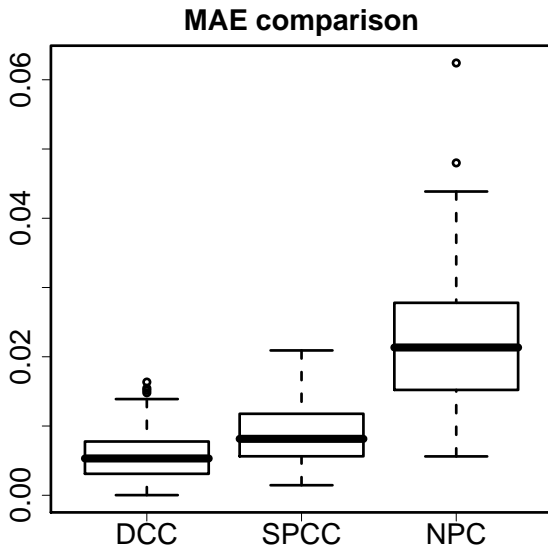


Figure 1: Correlation estimators for the different scenarios of Experiment 1.

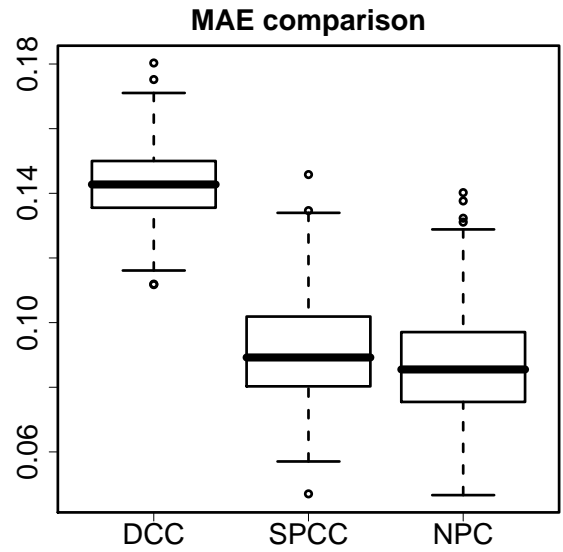
performance. We report results are for an equally weighted portfolio (EWP) and a minimum variance portfolio (MVP).

Table ?? shows the results for the dynamic quantile test using the data from Experiment 1 (with standard GARCH (1,1) processes³). As seen, for the equally weighted portfolio (EWP) and for Scenarios 1–3, the NPC delivers the most accurate results. That is, the number of

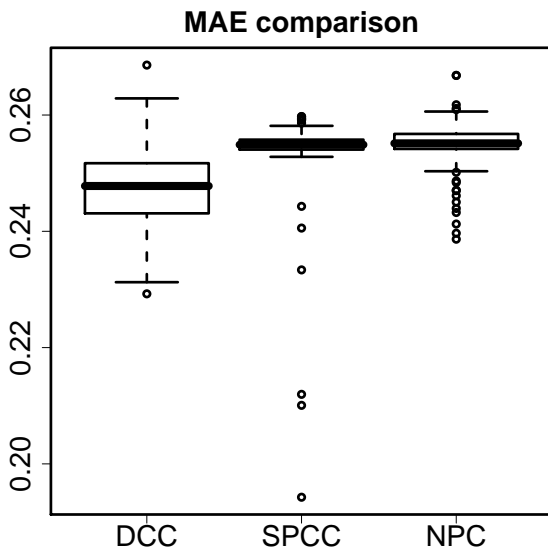
³Note that the data from experiments 2–3 deliver qualitatively similar results.



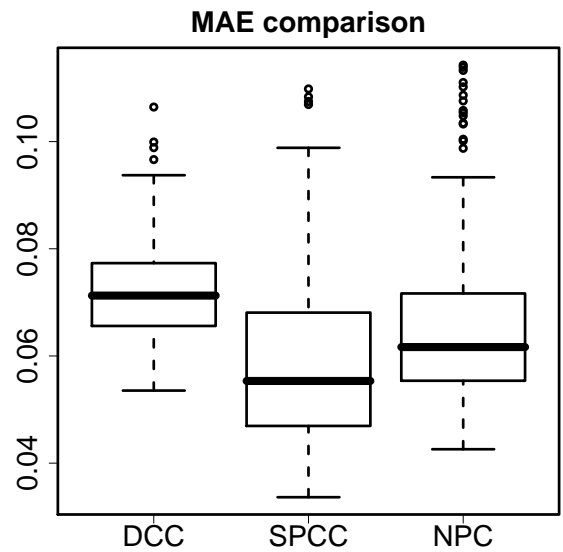
(a) Scenario 1



(b) Scenario 2



(c) Scenario 3



(d) Scenario 4

Figure 2: Distribution of MAE for the different scenarios of Experiment 1.

5% rejections is close to the 5% nominal level. On the other hand, for the minimum variance portfolio (MVP) the NPC generally over-rejects being the best only in Scenario 1. For this portfolio and for Scenarios 3–4 the SPCC is the most accurate with the DCC doing well in Scenario 2.

	Equal weights		
	DCC	SPCC	NPC
Scenario 1	0.0653	0.0553	0.0503
Scenario 2	0.0704	0.0201	0.0503
Scenario 3	0.0905	0.1005	0.0553
Scenario 4	0.0503	0.0352	0.0302
	Minimum variance weights		
	DCC	SPCC	NPC
Scenario 1	0.0653	0.0653	0.0603
Scenario 2	0.0603	0.0352	0.0101
Scenario 3	0.0704	0.0553	0.0402
Scenario 4	0.0754	0.0553	0.0302

Table 1: Hit results for Experiment 1 with $\alpha = 0.05$ and normal probability function.

2.3 Experiment 2

Typically, for stock returns negative shocks have a larger impact on volatility than positive shocks of the same magnitude (so-called leverage effect). In this experiment, we simulate DGPs from the following asymmetric GARCH(1,1) (Glosten et al., 1993) models:

$$h_{1,t} = 0.01 + 0.025r_{1,t-1}^2(1 - \mathbf{1}_{\{r_{1,t-1} < 0\}}) + 0.075r_{1,t-1}^2\mathbf{1}_{\{r_{1,t-1} < 0\}} + 0.94h_{1,t-1} \quad (19)$$

$$h_{2,t} = 0.5 + 0.1r_{2,t-1}^2(1 - \mathbf{1}_{\{r_{2,t-1} < 0\}}) + 0.3r_{2,t-1}^2\mathbf{1}_{\{r_{2,t-1} < 0\}} + 0.5h_{1,t-1}$$

We choose the parameter values such that the effect of a negative lagged return on current volatility is three times larger than the effect of a positive lagged return. In practice, we test the robustness of the previous results by estimating symmetric GARCH(1,1) processes. As for the correlation, we assume the same four scenarios described previously.

Figures 3–4 show that for Scenarios 1–3 the results are qualitative similar to the ones reported for symmetric GARCH(1,1). The DCC is the best model for constant (Scenario 1) and rapid changes in correlations (Scenario 3), while the NPC is the dominant specification for gradual changes in correlations (Scenario 2). In contrast, the results for Scenario 4 show that the SPCC becomes sensitive to misspecification in the conditional variance and performs the worst. In this scenario, the NPC outperforms its two competitors being in overall the best model in two out of four scenarios.

2.4 Experiment 3

The main drawback of the univariate (asymmetric) GARCH(1,1) processes simulated in Sections 2.1–2.3 is that they rule out potential feedback effects between the volatilities. In this

experiment, we simulate DGPs from the following bivariate unrestricted GARCH(1,1) system (Conrad and Karanasos, 2009):

$$\begin{aligned} h_{1,t} &= 0.1 + 0.03r_{1,t-1}^2 + 0.02r_{2,t-1}^2 + 0.3h_{1,t-1} + 0.1h_{2,t-1} \\ h_{2,t} &= 0.2 + 0.2r_{1,t-1}^2 + 0.05r_{2,t-1}^2 - 0.15h_{1,t-1} + 0.8h_{2,t-1} \end{aligned} \quad (20)$$

Notice that sign of the volatility feedback is different across the two equations. The second asset has a positive volatility effect on the first asset, while the first asset has a negative volatility effect on the second asset (for volatility spillovers see (Baele, 2005; Diebold and K., 2010). The parameter values for these DGPs are chosen from Conrad and Karanasos (2009).

Results in Figures 5 and 6 are very much in line with the results when the volatility is generated with a univariate symmetric GARCH(1,1) process. For instance, the NPC continues to outperform the DCC and only slightly the SPCC in Scenario 2 (gradual changes) while in Scenario 4 (structural break) the SPCC delivers the best outcome (again the performance of SPCC and NPC is very similar). The DCC, however, is the best specification in Scenario 1 (constant correlation) and Scenario 3 (rapid changes).

2.5 Experiment 4

In this part, we perform a multivariate simulation experiment to show the performance of the models in higher dimensions. Higher-dimensional models are of particular interest as portfolios are typically designed to include many assets. We decided to do an experiment with the returns of four assets that follow standard GARCH(1,1) processes. As before, the series have a length of $T = 1000$ and the number of simulations is $M = 200$.

A fundamental issue for any multivariate model is how to guarantee positive definiteness of the conditional covariance matrix. The solution we adopt here is choosing a seed matrix $\Sigma_t^{1/2}$ as in (21). The true correlation matrix is then generated as $R_t = (\Sigma_t^*)^{-1} \Sigma_t (\Sigma_t^*)^{-1}$, where Σ_t^* is a diagonal matrix composed of the square roots of the diagonal elements of Σ_t . This way we can ensure that we have a positive definite matrix to generate the innovations $\epsilon_t \sim N(\mathbf{0}, R_t)$ for $\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}, \epsilon_{4,t})'$.

$$\begin{aligned}
\Sigma_t^{1/2} = & \begin{pmatrix} 0.7 & 0.4 & 0 & -0.1 \\ 0.4 & 1 & -0.15 & 0 \\ 0 & -0.15 & 1.15 & -0.2 \\ -0.1 & 0 & -0.2 & 0.9 \end{pmatrix} \mathbf{1}_{\{1 \leq t < 300\}} \\
& + \begin{pmatrix} 0.7 & 0.4 & 0.15 & 0 \\ 0.4 & 1 & -0.15 & 0.1 \\ 0.15 & -0.15 & 1.15 & -0.1 \\ 0 & 0.1 & -0.1 & 0.9 \end{pmatrix} \mathbf{1}_{\{300 \leq t < 600\}} \\
& + \begin{pmatrix} 0.7 & 0.4 & 0.15 & 0 \\ 0.4 & 1 & -0.15 & 0.2 \\ 0.15 & -0.15 & 1.15 & 0.1 \\ 0 & 0.2 & 0.1 & 0.9 \end{pmatrix} \mathbf{1}_{\{600 \leq t \leq T\}}
\end{aligned} \tag{21}$$

Here, we allow for little time-variability in the correlations every 300 observations. In addition all pairwise correlations are discontinuous at $t = 300$ and 600 where the SPCC and the NPCC are not consistent.

As for the performance measure, we use an overall mean absolute error which for each simulation M is defined by:

$$\text{MAE}_M = \frac{1}{T} \sum_{t=1}^T |\text{vecl}(R_t) - \text{vecl}(\hat{R}_t)|. \tag{22}$$

Figure 7 plots the correlation pairs obtained from each model chosen as the typical sample whose MAE is equal to the median of all the MAE_M , while Figure 8 shows the boxplot of the MAE_M for the 200 simulations. As seen, the DCC model delivers the best results being slightly better than the SPCC. On the other hand, the NPC performs the worst. However, there is little variability in the correlations over time and in this sense it is not surprising that the DCC is the dominant specification. In fact, this result is consistent with the bivariate simulations where in all cases the DCC model is the best choice for the constant correlation scenario.

2.6 Experiment 5

In this experiment, we investigate the performance of the different multivariate (four assets) models when there is substantial time–variability in correlations. We use real data to estimate conditional correlations and then consider these correlations as the true DGP. In practice, the correlations are obtained from a DCC specification. Thus, this way we can ensure that we have a positive definite matrix to generate the innovations $\epsilon_t \sim N(\mathbf{0}, R_t)$ for $\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}, \epsilon_{4,t})'$.

As previously, for performance measure is the mean absolute error which for each simulation M is defined in (22).

Figure 9 plots the correlation pairs obtained from each model chosen as the typical sample whose MAE is equal to the median of all the MAE_M , while Figure 10 shows the boxplot of the MAE_M for the 200 simulations. As seen, the SPCC and NPC models outperform substantially the DCC specification. Thus, while in bivariate models there are occasions where the DCC can perform well, at a multivariate framework with substantial time–variability in correlations the semiparametric and nonparametric models clearly deliver the best results. Once again, the performances of the SPCC and the NPC are quite similar. Figure 10 also shows that for all three models there is very little dispersion in MAE values.

3 Empirical Results

Empirical examples of the above three correlation models are presented for two interesting portfolios. The first, referred to as SPDR, consists of the nine Select Sector SPDRs Exchange Traded Funds (ETF) that divide the S&P 500 index into sector index funds. The second portfolio consists of five major currencies plus two from emerging economies that are actively traded in the foreign exchange market. These portfolios have been studied by the Volatility Institute of the New York University.

Sector	Index
Materials Select Sector SPDR Fund	XLB
Energy Select Sector SPDR Fund	XLE
Financial Select Sector SPDR Fund	XLF
Industrial Select Sector SPDR Fund	XLI
Technology Select Sector SPDR Fund	XLK
Consumer Staples Select Sector SPDR Fund	XLP
Utilities Select Sector SPDR Fund	XLU
Health Care Select Sector SPDR Fund	XLV
Consumer Discretionary Select Sector SPDR Fund	XLY

Table 2: List of the nine sector indexes within the S&P 500.

3.1 Portfolio of Equity SPDRs and the S&P 500

We estimate the cross-correlation of a portfolio containing nine equity SPDRs and the S&P 500. The data of interest is daily from January 5, 2004 until December 21, 2009 (T=1504 observations). This period includes some years of market stability followed by the last financial crisis years. We expect a high correlation between the SPDRs and the S&P 500 which is built with the most representative companies of each sector. Table 2 summarises the sectors and their market index.

Figure 11 shows the correlation estimates obtained from the three correlation models. In practice, we estimate 45 correlation pairs but for presentation purposes, we only plot the correlation between the S&P 500 and the sectors. The volatility is assumed to be driven by a GARCH(1,1) process.

It can be seen that for most of the sample period the Financial, the Industrial, the Technology, the Consumer Staples, the Health Care and the Consumer Discretionary sectors show a stable as well as high correlation with the S&P 500. High correlation, however, means that there are little diversification opportunities in a portfolio including the S&P 500 and the sector indexes mentioned above. It is interesting to see though that the Utilities, Energy and, to a lesser extent, Materials sectors display a different behaviour. Their correlation with the S&P 500 experiences notable drops. For instance in the fourth quarter of 2006 and in the third quarter of 2008, correlations for these sectors decrease to values close to 0.3. The first period might be linked to the background of the financial crisis. In particular, one of the main causes of the crisis was the bursting of the housing bubble which peaked in approximately 2006. On the other hand, the second period corresponds to the time when the crisis hits its most critical stage. This distinct behaviour of the aforementioned correlations may be expected given that energy and utilities are considered noncyclical sectors.

Let us investigate the changes in the behaviour of the correlation of the Financial sector with the rest. This is of interest given that the current economic crisis was initiated by irregularities in the financial sector. The Financial index has a similar behaviour to the S&P 500. Figure 12 shows how the decay in the Financial index at the beginning of 2007 was followed closely by a decay in the S&P 500 only a few weeks later. The correlations of the Financial index with the other indexes are plotted in Figure 13. We observe that when the financial crisis hit its peak in September and October 2008⁴, we observe that there was a strong drop in the correlation with the Energy, Materials and Utilities sectors. This pattern is the same as the one for the S&P500 vs. the sectors. Indeed, the correlation between Financials and S&P 500 is quite stable and around 0.9 during the whole period. Also, it is interesting to notice that the correlation between

⁴During September–October 2008 several major financial institutions either failed, were acquired under duress, or were subject to government takeover. These included Lehman Brothers, Merrill Lynch, Fannie Mae, Freddie Mac, Washington Mutual, Wachovia and AIG.

the financial sector and health care decreases later on in the sample during the second and third quarters of 2009.

3.2 Portfolio of Currencies

We consider daily US dollar exchange rates of the Australian dollar (AUS), Swiss franc (CHF), euro (EUR), British pound (GBP), South African rand (RAND), Brazilian real (REALB), and Japanese yen (YEN) over the period from January 1, 1999 until May 7, 2010 (T=2856 observations).

This portfolio is a well diversified portfolio. In particular, the Swiss franc and to a smaller extent the Japanese yen are considered safe haven currencies. The National Bank of Switzerland used to back up grand part of the CHF value with gold and now investors are accustomed to invest in francs when uncertainty increases. The interest yield curve of the Japanese yen is very low and therefore it responds quickly to big drops in value. Therefore, these two currencies perform well during high risk financial times. On the other hand, the Australian dollar, the Brazilian real and the South African rand tend to drop during times of crisis. However, they recover the investors attention when their national banks set high interest rates. The euro-US exchange rate is the most active and liquid bilateral rate in the foreign exchange market. Although declining in importance, the pound sterling is still a key international currency and one of the most heavily traded one. On the other hand, the Brazilian real and the South African rand tend to drop during times of crisis. However, they recover investors' attention when their national banks set high interest rates.

The whole correlation matrix was estimated using the SG-DCC, the SPCC and the NPC for each time t . Figures 14–16 plot these correlation estimates two by two. As seen, correlations among the Australian dollar, Swiss franc, euro and British pound shift to a higher level in the period 2002–2005. This may be related to the “global savings glut” a situation where during the first half of the decade industrial countries received large amounts of excess savings created in other parts of the world (e.g., South–East Asia).⁵ Notice also that the correlation of the Japanese yen with the other currencies started to decrease around 2006 and became even negative in the period afterwards. This may reflect the severe financial problems of the Japanese economy. We further observe that the correlations of the South African rand experience pronounced shifts, presumably linked to the efforts of the South African Reserve Bank to keep inflation within the target range. On the other hand, the correlation of the Brazilian real against the other currencies steadily increases (with the exception of the yen). This may reflect the improved macroeconomic stability of the Brazilian economy during this decade. Finally, we remark that the chf-eur pair is the most stable correlation typically approaching one, except for drops during the crisis period

⁵ “Global savings glut” is a term coined by Ben Bernanke in his speech in 2005.

of 2008. Compared to the portfolio of the SPDRs, the correlations of the currencies show more variability over time implying also more frequent rebalancing of portfolios.

3.3 Evaluation of Models

We consider two types of criterion functions to evaluate the models in terms of portfolio's certain characteristics. The first one is the mean square error (MSE) loss function and second one is based on the portfolio's Value-at-Risk (VaR). Define the weighted in-sample portfolio's returns and variance as follows:

$$r_{p,t} = \omega_t' r_t \quad (23)$$

$$h_{p,t} = \omega_t' H_t \omega_t \quad (24)$$

where ω_t is a (possibly) time-varying weight vector and H_t is the in-sample covariance matrix. We consider the following portfolio weighting methods. First, the benchmark equally weighted portfolio (EWP) where the weights are constant and equal to $\omega_t = \omega = A^{-1}i$ and i is a $(A \times 1)$ vector of ones and A is the number of elements in the portfolio. Second, the minimum variance portfolio (MVP) where the weights are given by $\omega_t = H_t^{-1}i/(i'H_t^{-1}i)$. Note that the MVP weights are time-varying as they depend on the conditional covariance matrix. In practice, we allow the weight vector ω_t to change only after every 20 observations, so that the portfolios are rebalanced approximately every month.

The MSE loss functions for the EWP and MVP are defined respectively as:

$$MSE^{j,EWP} = T^{-1} \sum_{t=1}^T \left(\omega' \hat{H}_t^j \omega - \omega' r_t r_t' \omega \right)^2 \quad (25)$$

$$MSE^{j,MVP} = T^{-1} \sum_{t=1}^T \left(\omega_{t_{20}}' \hat{H}_t^j \omega_{t_{20}} - \omega_{t_{20}}' r_t r_t' \omega_{t_{20}} \right)^2 \quad (26)$$

where \hat{H}_t^j is the covariance matrix estimate obtained from model j and $r_t r_t'$ is the matrix of the cross-product of the returns. The second loss functions is defined in terms of VaR. More specifically, the VaR values of the EWP and MVP for model j at the confidence level α are given by:

$$VaR_t^{j,EWP}(\alpha) = \Phi_\alpha^{-1} \sqrt{\omega' \hat{H}_t^j \omega} \quad (27)$$

$$VaR_t^{j,MVP}(\alpha) = \Phi_\alpha^{-1} \sqrt{\omega_{t_{20}}' \hat{H}_t^j \omega_{t_{20}}} \quad (28)$$

where Φ_α is the standard normal probability function at tail probability $\alpha \in (0, 1)$. The cor-

responding VaR loss functions for model j from Koenker and Bassett (1978) is calculated as follows:

$$Q^{j,EWP}(\alpha) = T^{-1} \sum_{t=1}^T \left(\alpha - \mathbf{1}_{\{r_{p,t} < VaR_t^{j,EWP}(\alpha)\}} \right) \left(r_{p,t} - VaR_t^{j,EWP}(\alpha) \right) \quad (29)$$

$$Q^{j,MVP}(\alpha) = T^{-1} \sum_{t=1}^T \left(\alpha - \mathbf{1}_{\{r_{p,t} < VaR_t^{j,MVP}(\alpha)\}} \right) \left(r_{p,t} - VaR_t^{j,MVP}(\alpha) \right). \quad (30)$$

In practice, we use $\alpha = 5\%$.

For the portfolio of currencies we also consider a popular portfolio weighting method called 'currency carry trade'. The idea is to borrow from low interest rate currencies and invest in high interest rate currencies. The carry trade portfolio is at odds with the Uncovered Interest Parity (UIP) theory which states that exchange rate changes will eliminate any gain arising from the differential in interest rates across countries. However, there is overwhelming empirical evidence against the UIP (e.g. Burnside et al., 2007). Actually, the opposite is found to be true—high interest rate currencies tend to appreciate while low interest currencies tend to depreciate.

In our study, we adopt a carry trade portfolio similar to the one used by Christiansen et al. (2011). This is composed of a short position in the three currencies associated with the lowest interest rates and a long position in the four currencies with the highest interest rates. This portfolio is rebalanced every month though in practice we found that the weights are very stable. For instance, the carry trade portfolio is usually short in the CHF, EUR and YEN and long in the AUS, GBP, RAND and REALB. The portfolio is such that the weights add up to zero, which means that short positions have to be compensated by equivalent long positions. In particular, according to the ordering of currencies (AUS, CHF, EUR, GBP, RAND, REALB, YEN) the carry trade portfolio weight vector is given by, $w = (\frac{1}{4}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{3})'$.

Menkhoff et al. (2010) argue that carry trade portfolios are negatively related to foreign exchange (FX) volatility. In particular, they find that high interest rate currencies are negatively related to innovations in FX volatility while low interest rate currencies provide a hedge. In this light, we also used a flexible carry trade portfolio where during periods of high FX volatility the weight vector switches sign. In particular, short positions in the three currencies with the lowest interest rates become long positions in the new portfolio. Similarly, long positions in the four currencies with the highest interest rates become short positions in the new portfolio. More specifically, our time-varying (carry trade) weight vector is given by:

$$w_t = [1 - 2I(FX_{vol_{t-1}}^{innovations})]w \quad (31)$$

for

$$I(FX_{\text{vol}_{t-1}}^{\text{innovations}}) = \begin{cases} 1 & FX_{\text{vol}_{t-1}}^{\text{innovations}} \geq 0 \\ 0 & FX_{\text{vol}_{t-1}}^{\text{innovations}} < 0 \end{cases}$$

As seen, the weight vector changes over time depending on FX volatility. For instance, during tranquil times FX volatility is low (thus negative innovations) and the weight vector is w . As before, this weight vector consists of short positions in the three currencies with the lowest interest rates and long positions in the four currencies with the highest interest rates. On the other hand, during times of financial distress FX volatility is high (thus positive innovations), investors hedge and the weight vector now becomes $-w$ – long positions in the three currencies with the lowest interest rates (“safe haven” currencies) and short positions in the four currencies with the highest interest rates (“risky” currencies).

As for the measure of FX volatility innovations, we follow Menkhoff et al. (2010) and use a simple average of absolute exchange rates (log–returns) of the five major currencies in our portfolio (AUS, CHF, EUR, GBP, YEN). We then calculate FX volatility innovations (denoted by $FX_{\text{vol}_{t-1}}^{\text{innovations}}$) by estimating a simple AR(1) for the FX volatility level and take the residuals as a proxy for innovations.⁶

Table 3 shows the criterion functions of the estimated conditional correlation models. We also check the robustness of the results to the inclusion of the crisis period (with its macroeconomic context) in the sample. In particular, we split the overall sample into two sub-samples, one before and another one after 2007. Overall, the results show that the NPC dominates the SPCC and SG–DCC models. For instance, for the portfolio of currencies the MSE values of the NPC indicate substantial improvement particularly for the MVP and to lesser extent for two carry trade portfolios. However, these improvements are less marked in the case of the VaR loss values. On the other hand. For the portfolio of the SPDRs, however, the values of the criterion functions as well as the differences across models are small. Notice also that although the NPC generally outperforms the other two models, the SPCC model appears now to be the most accurate specification in quite a few cases (e.g., in the two sub–samples).

Defining $Var_{\text{NPC}} = \sum_t h_{p,t}^{\text{NPC}}$ as the sum of the NPC portfolio variances during the whole period. If the ratio of two different model variances is close to 1 then the two models have statistically equal variance. Otherwise, they are different. We next perform a comparison of covariance matrix estimators by employing the methodology proposed by Engle and Colacito (2006). Suppose that we have two alternative estimators of the covariance matrix, one produced by the SPCC model and one produced by the SG–DCC model. In each period, a set of minimum variance portfolio weights and therefore portfolio returns (eg. $r_{p,t}^{\text{SPCC}}$ and $r_{p,t}^{\text{SG–DCC}}$) is constructed

⁶Menkhoff et al. (2010) also experimented with different weight vectors. For example, they weighted the volatility contribution of different currencies by their share in international currency reserves but found qualitatively similar results.

based on each model. Let the difference between the squared portfolios returns be denoted by

$$u_t = (r_{p,t}^{\text{SG-DCC}})^2 - (r_{p,t}^{\text{SPCC}})^2.$$

The null hypothesis of interest is that the portfolio variances are equal. This can be tested by running an OLS regression and testing whether the mean of u_t is zero.

Table 4 reports the portfolio variance ratios for the three models as well as the p-values of the Engle–Colacito test for MVP. We evaluate the models two by two. Results are summarized as follows. Regarding the SPDR application and over the full sample the smallest portfolio variance is obtained by the SG–DCC model. Moreover, according to the Engle–Colacito test the improvement of the SG–DCC over the NPC model is statistically significant at 7%. On the other hand, compared to the SPCC model there is no statistically significant reduction in the portfolio’s variance obtained by the SG–DCC (p-value=0.144). Looking at the two sub-samples, however, the most accurate model is now the SPCC with the SG–DCC continuing to outperform the NPC. These differences are also supported by the Engle–Colacito test particularly during the crisis period (Subsample 2). The poor performance of the NPC was not unexpected as there is little variability in the correlations of the SPDRs. This result is in line with the simulation exercise where in all cases with constant correlation scenario the NPC was not performing well. As for the portfolio of currencies, the picture now is very different. Both NPC and SPCC outperform the SG–DCC model and the reduction in variance is highly significantly (p-values are 0) for the full sample and subsample 1. During the crisis period (subsample 2), the SPCC is the best model though.

4 Conclusion

In this paper, we compare three promising methodologies of time-varying asset correlations. The popular parametric DCC model, a semiparametric model and a fully nonparametric approach. In terms of Monte Carlo simulations and bivariate processes the semiparametric and nonparametric models perform well when correlations experience gradual changes or a structural break. On the other hand, the DCC model is the best in DGPs with rapid changes or constancy in correlations.

Moreover, in a multivariate framework the semiparametric and nonparametric models are the best in DGPs with substantial time-variability in correlations, while when allowing for little variability in the correlations the DCC is the dominant specification. With regard to the application we consider two asset portfolios during the recent financial crisis. The first portfolio consists of equity sectors and the S&P 500, while the second one contains major currencies that are actively traded in the foreign exchange market. We carry out a portfolio evaluation exercise and show that the nonparametric model generally dominates its competitors, particularly in

minimum variance weighted portfolios and to lesser extent for the carry trade portfolio. However, our application considers portfolios based only on in-sample results. As a future research, we feel this can be improved by considering out-of-sample forecasts of the conditional correlation/covariance matrix. Also, it would be interesting to look at other applications of the above models including bonds, international stock markets and commodities prices.

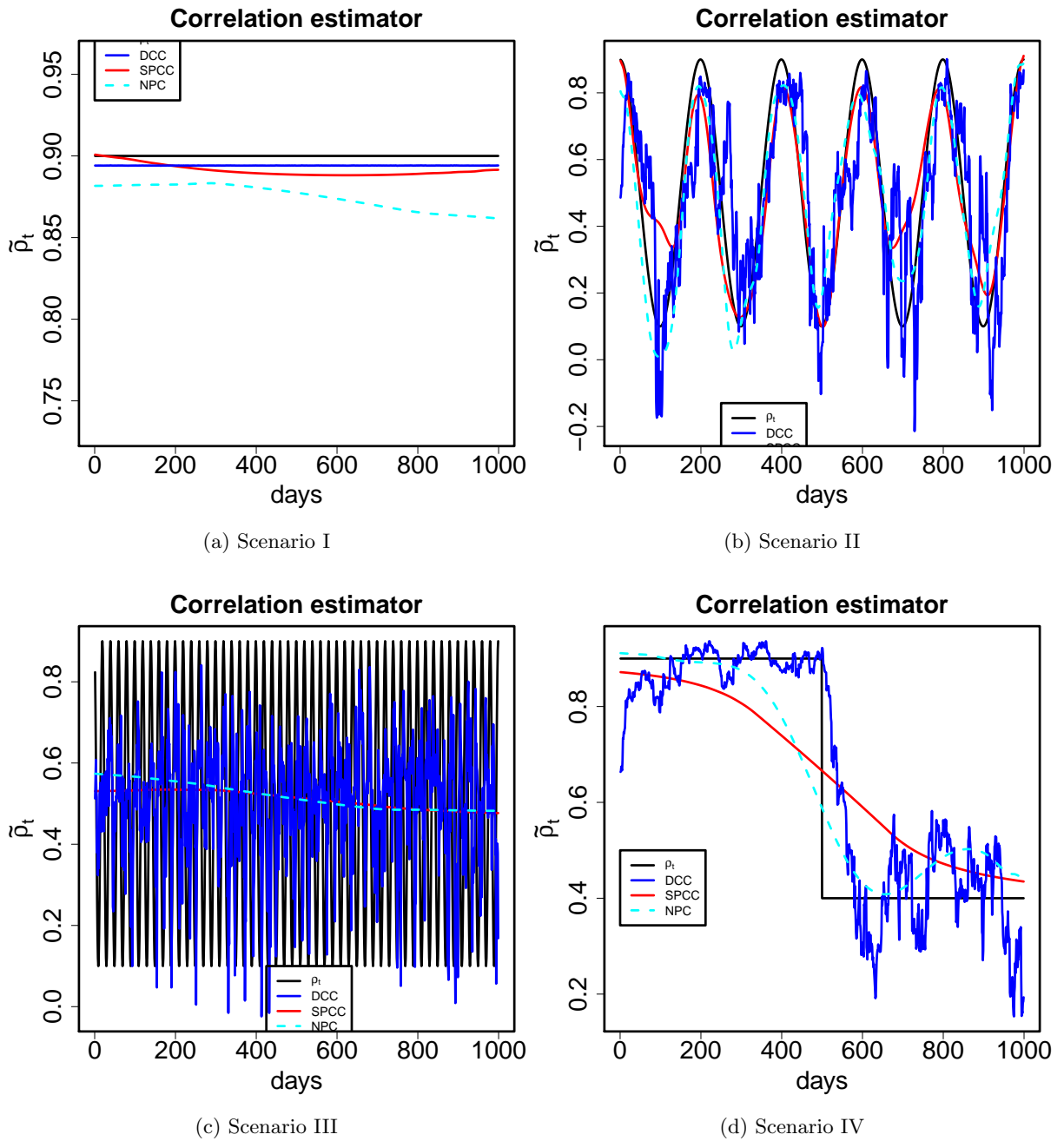
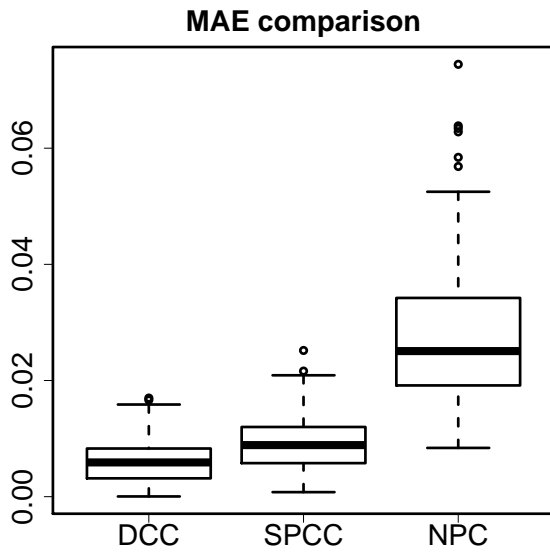
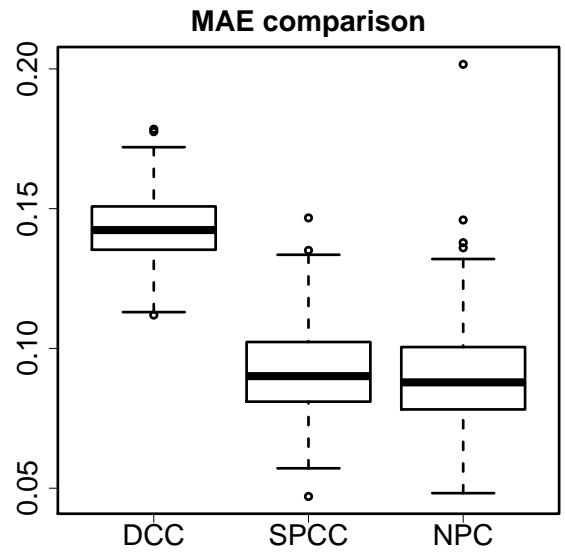


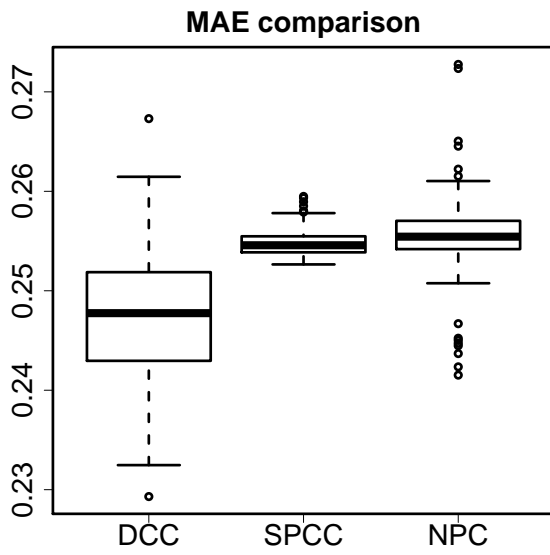
Figure 3: Correlation estimators for the different scenarios of Experiment 2.



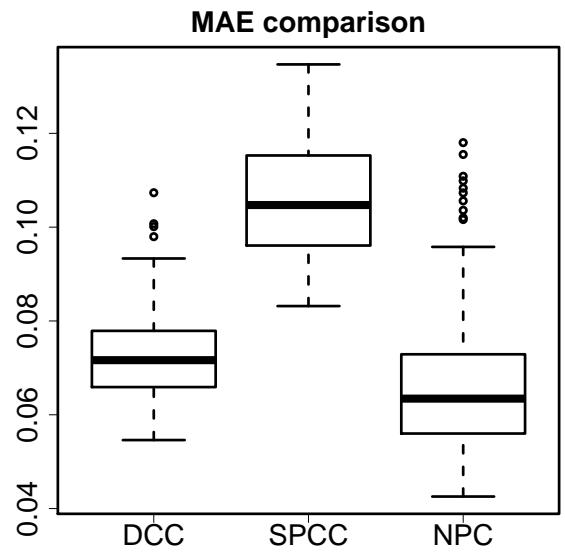
(a) Scenario 1



(b) Scenario 2



(c) Scenario 3



(d) Scenario 4

Figure 4: Distribution of MAE for the different scenarios of Experiment 2.

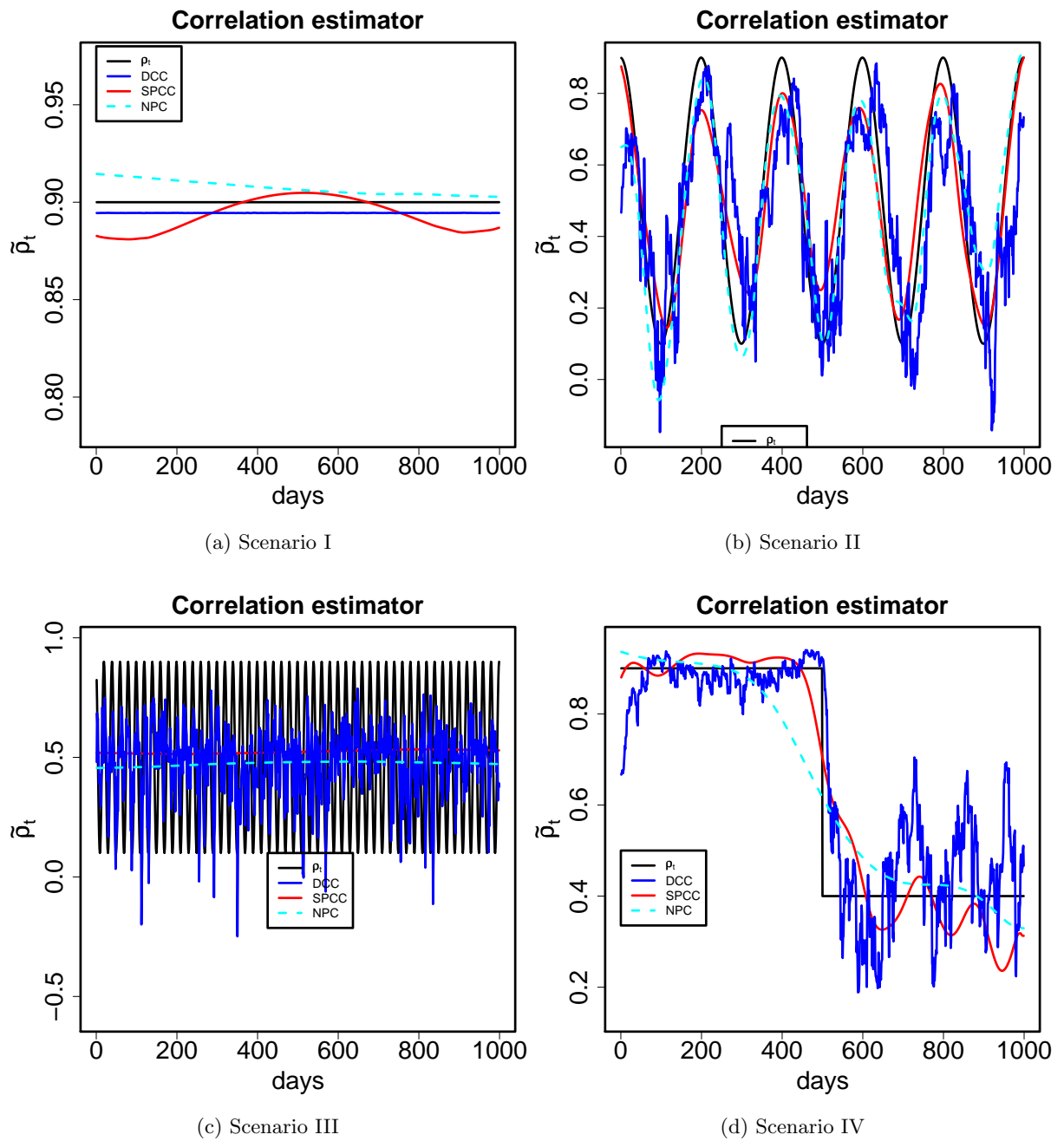
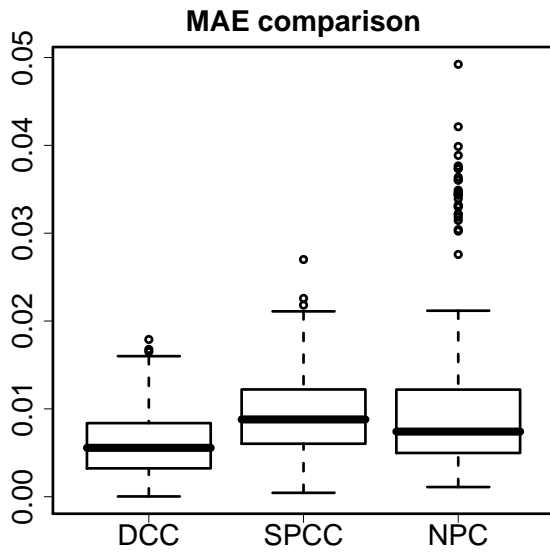
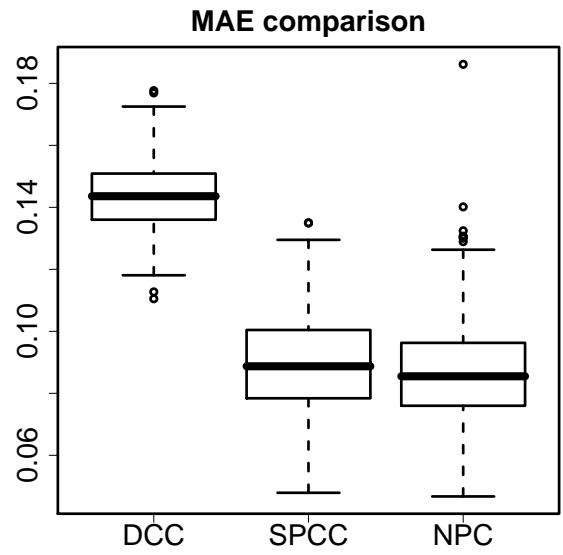


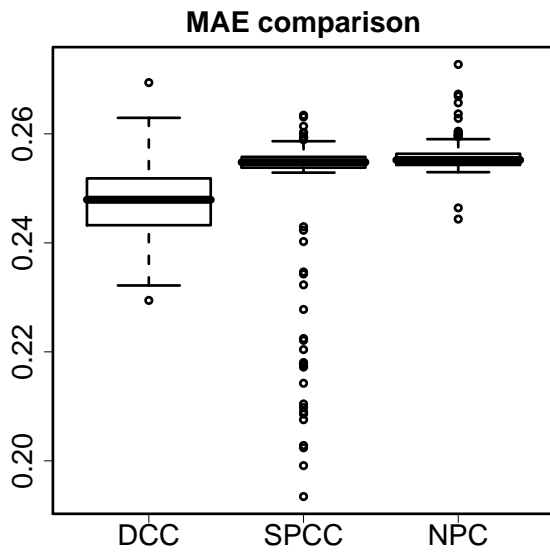
Figure 5: Correlation estimators for the different scenarios of Experiment 3.



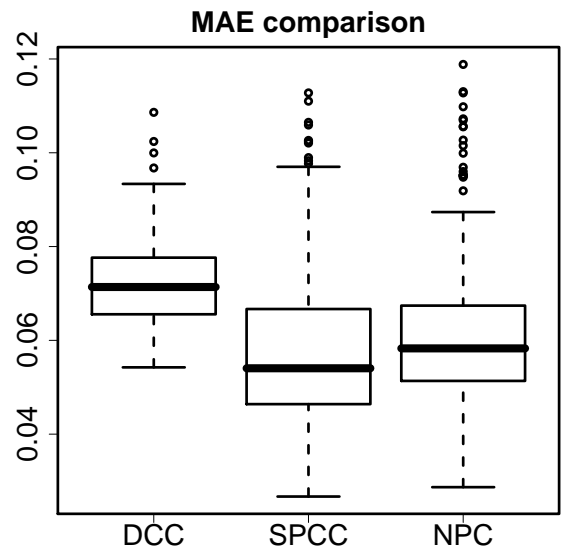
(a) Scenario 1



(b) Scenario 2



(c) Scenario 3



(d) Scenario 4

Figure 6: Distribution of MAE for the different scenarios of Experiment 3.

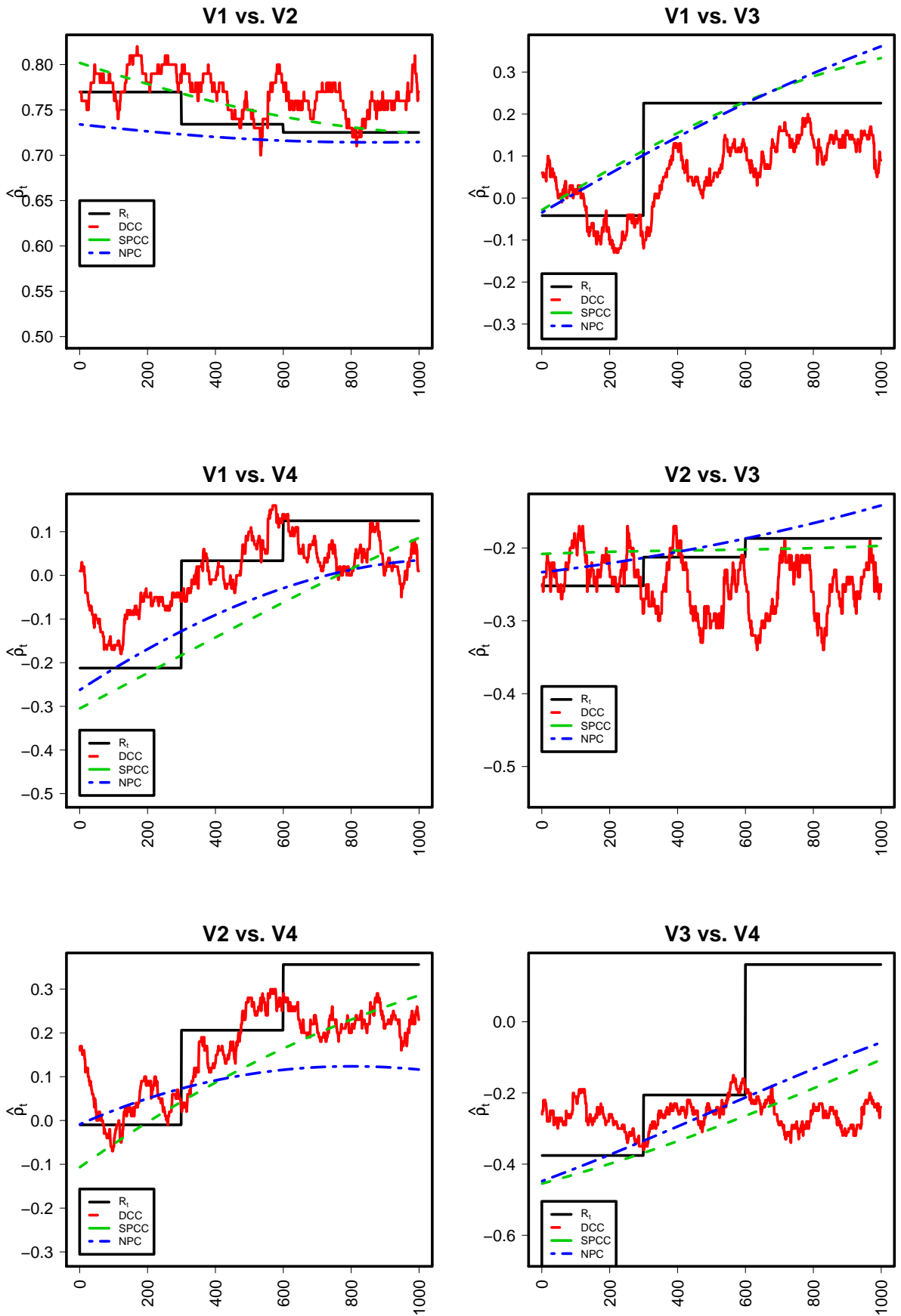


Figure 7: Correlation estimators for Experiment 4.

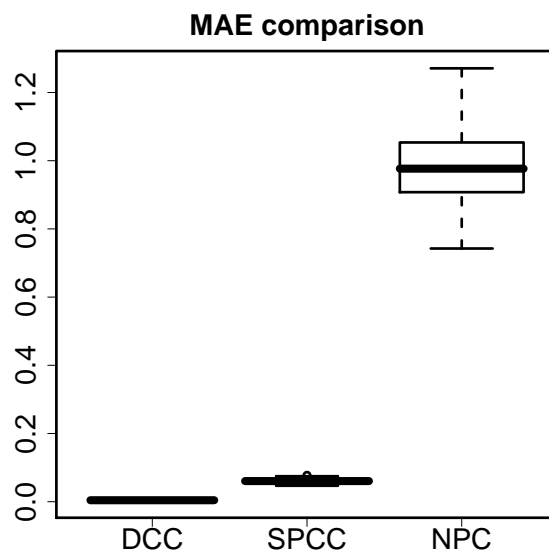


Figure 8: Distribution of MAE for Experiment 4.

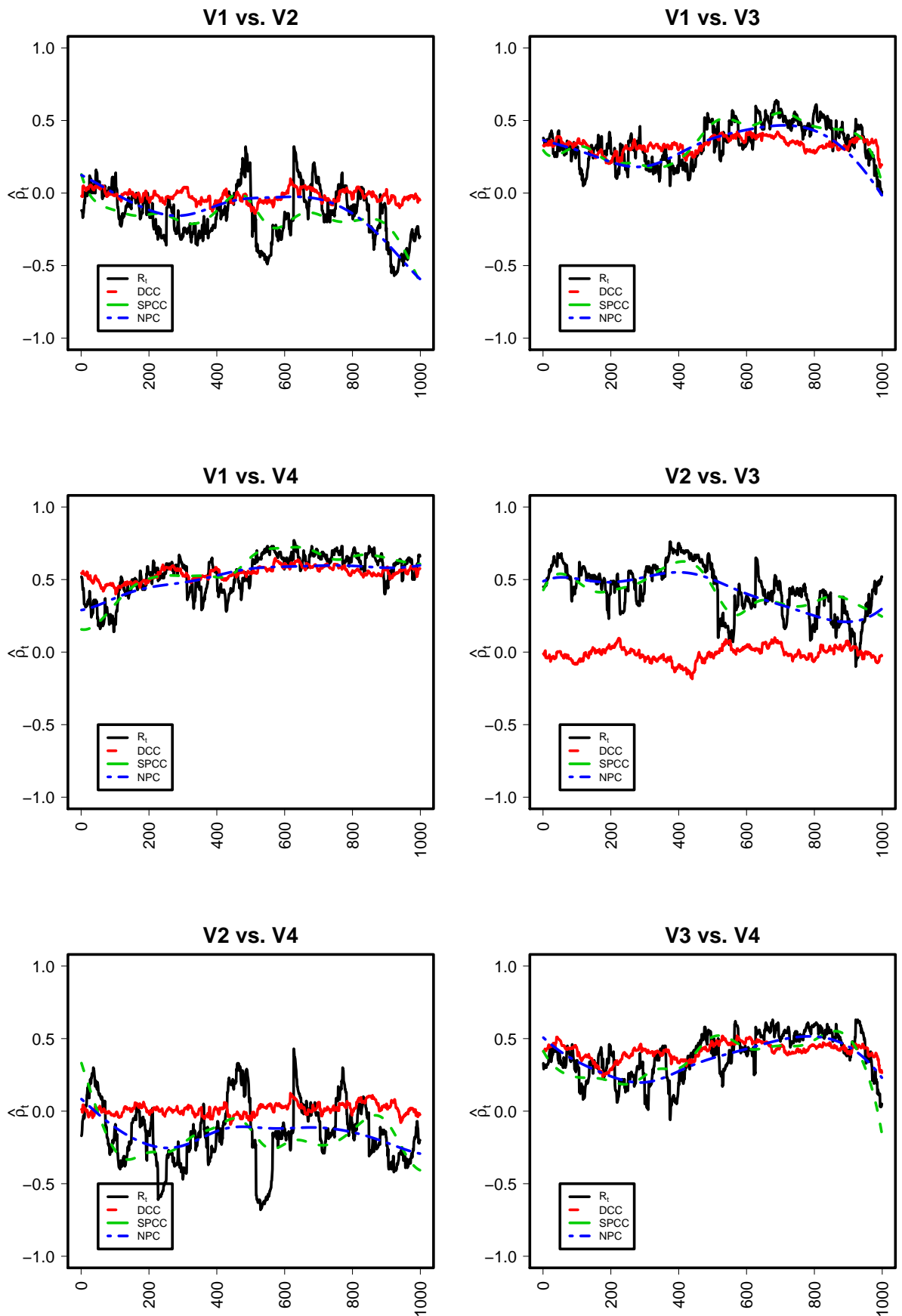


Figure 9: Correlation estimators for Experiment 5.

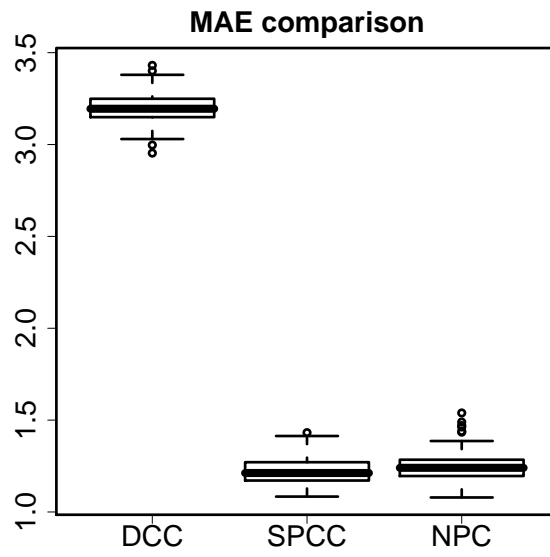


Figure 10: Distribution of MAE for Experiment 5.

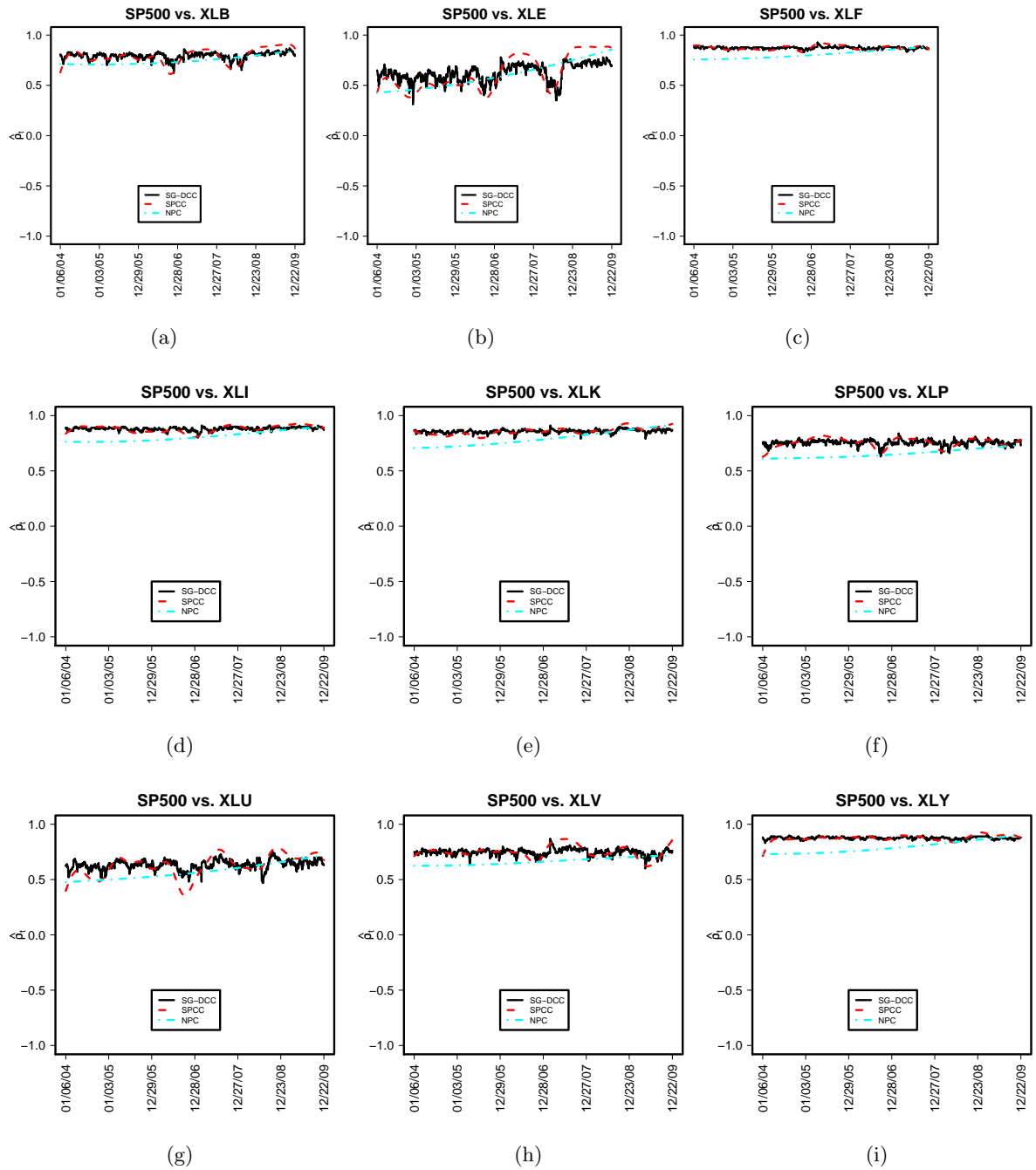


Figure 11: Correlations of S&P500 with the rest of the sectors when the volatility process is estimated with a GARCH(1,1) process.

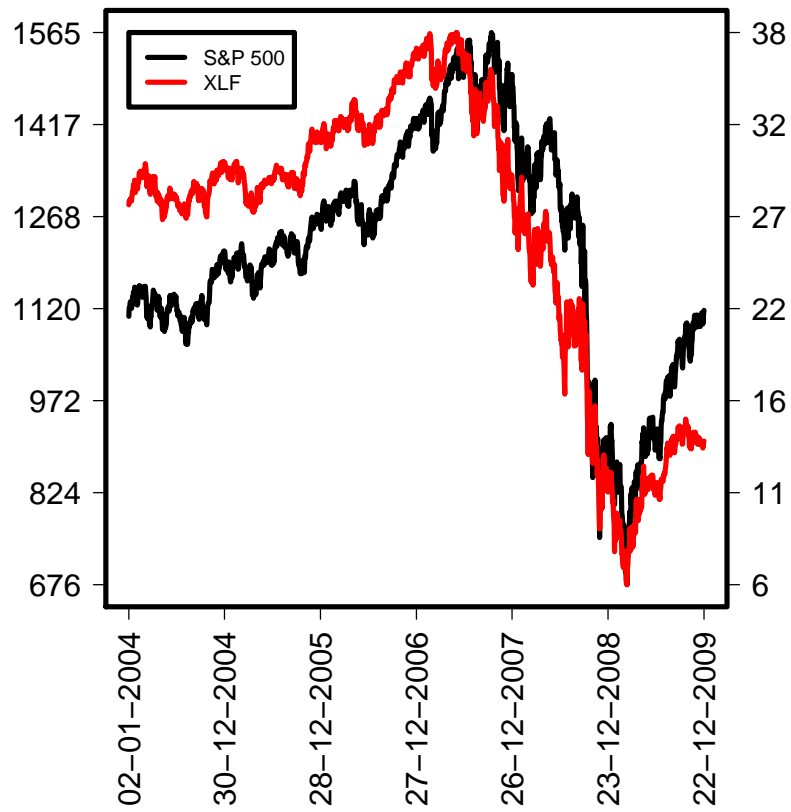


Figure 12: Comparison of the S&P 500 and the Financial sector index. The left axis displays the values of the S&P 500 and the right axis displays the values of the XLF.

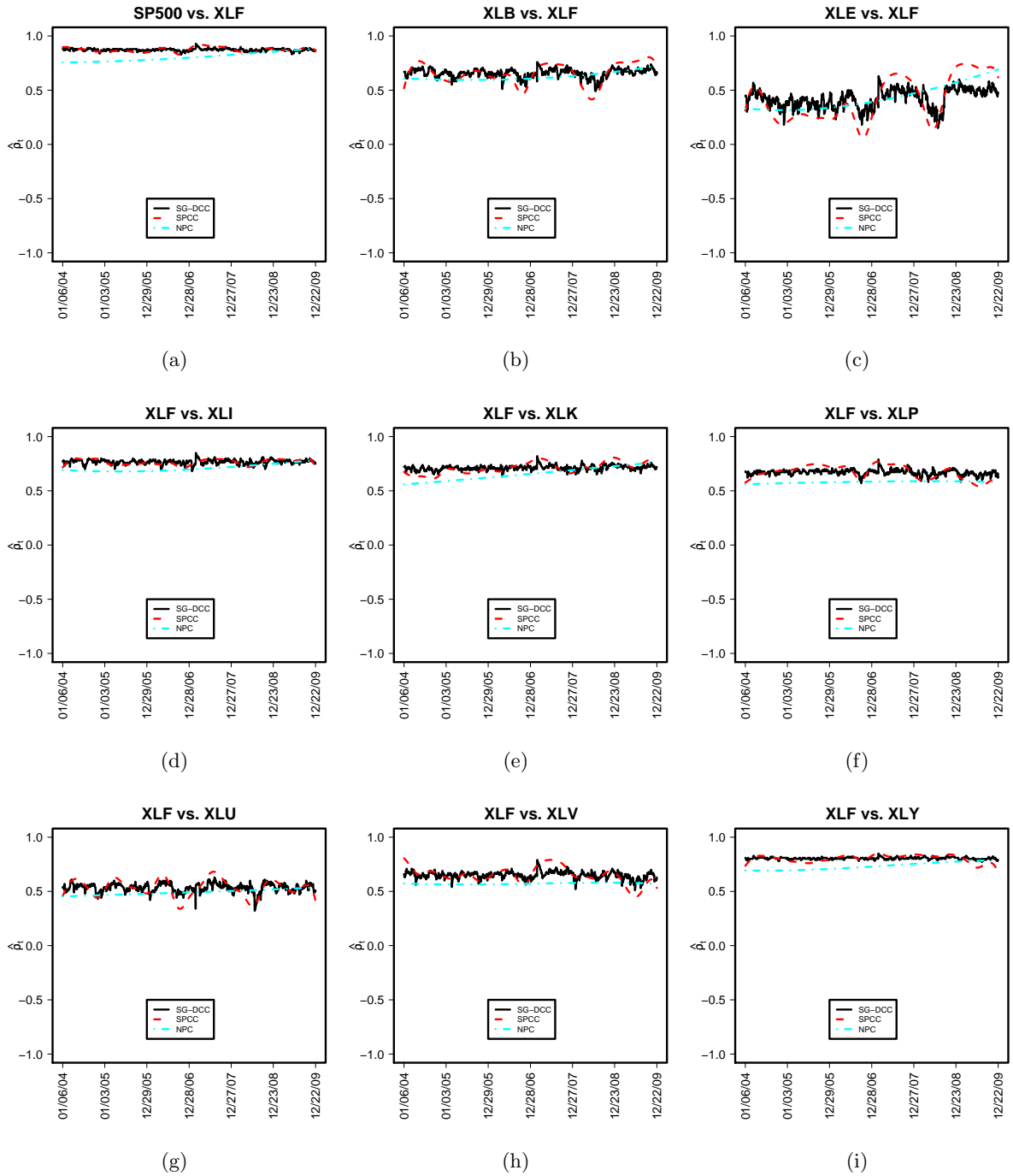


Figure 13: Correlations of XLF with the rest of the sectors when the volatility is estimated with a GARCH(1,1) process.

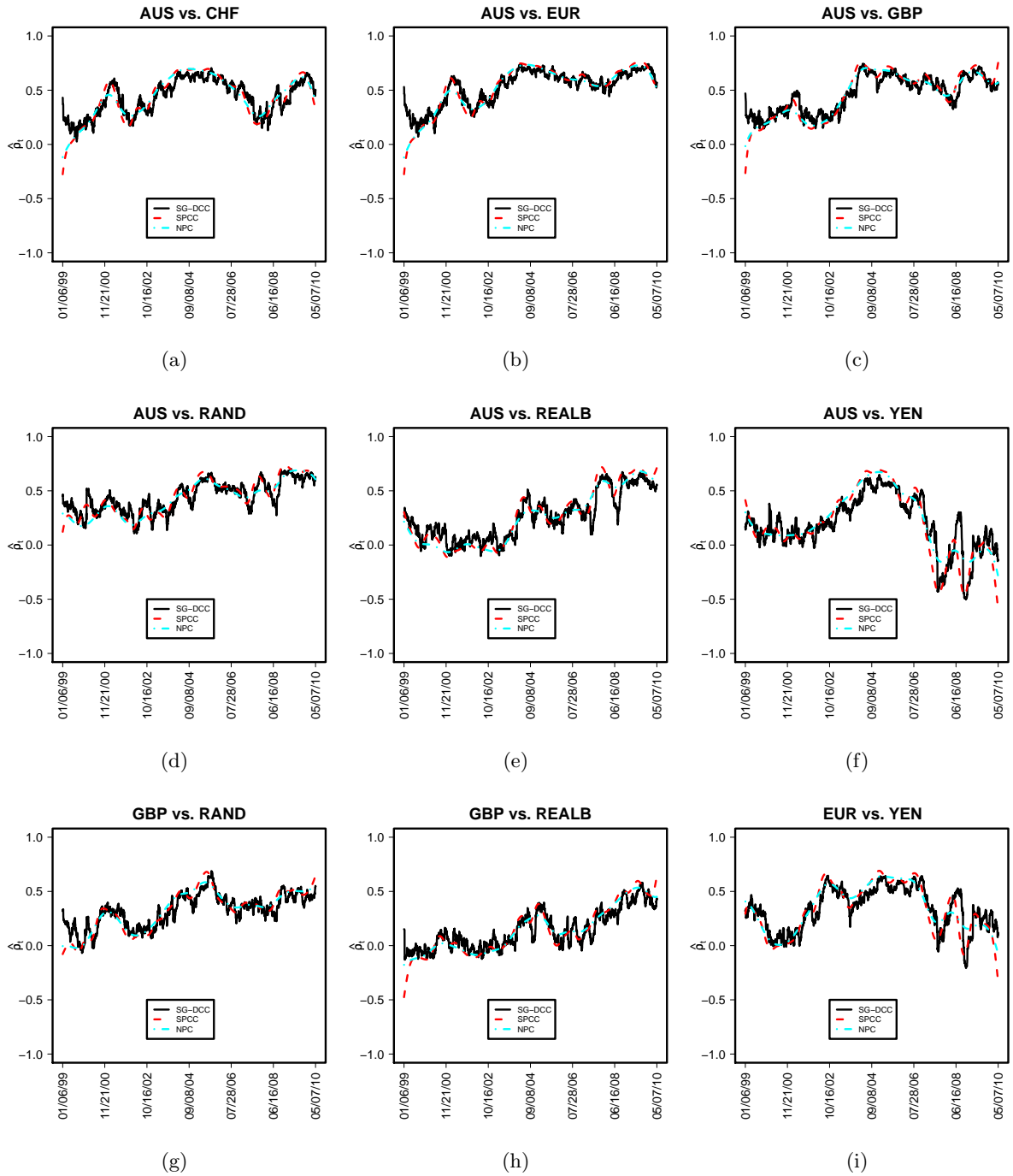


Figure 14: Correlations of AUS and GBP with the rest of currencies when the volatility is estimated with a GARCH(1,1) process.

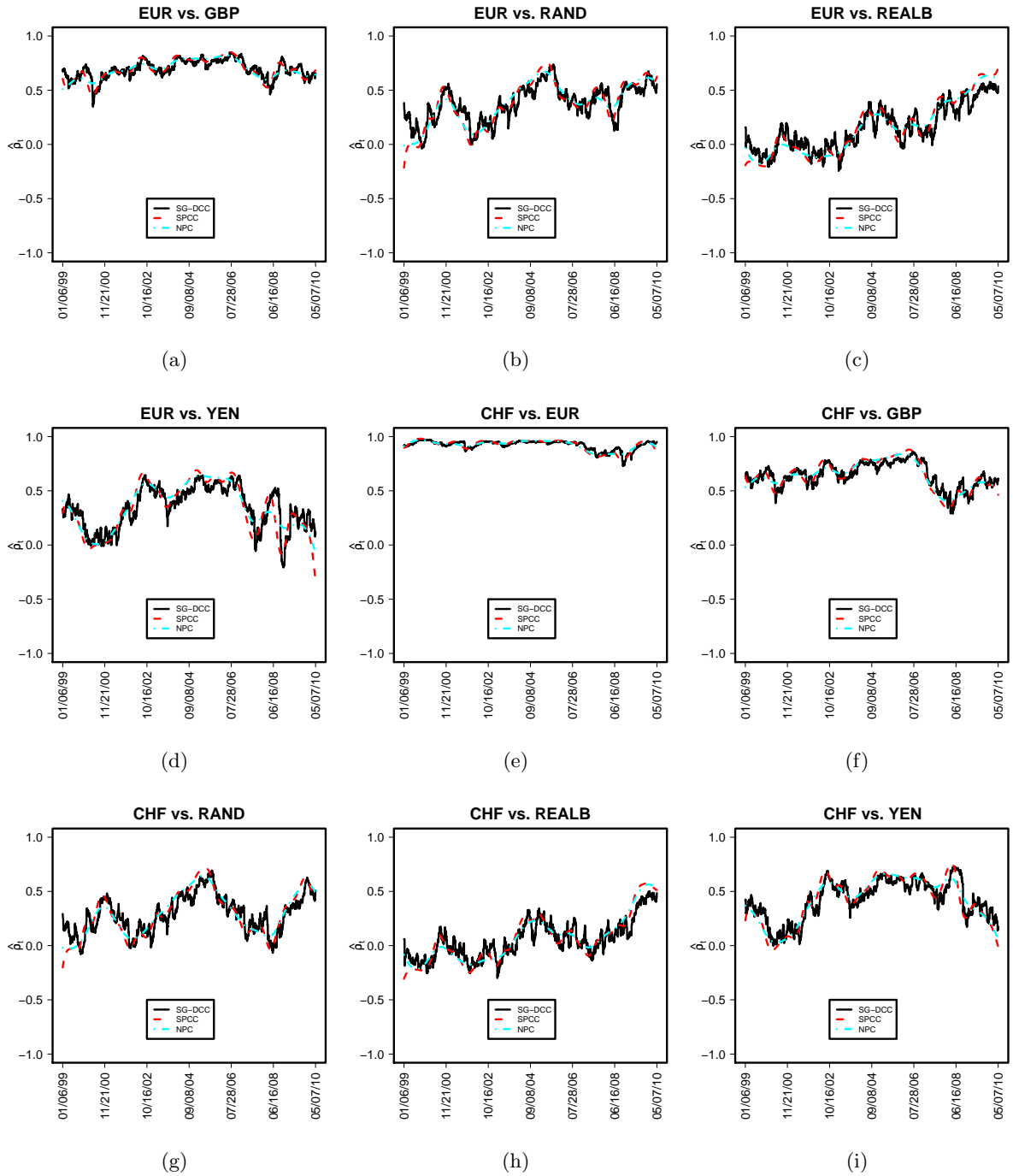
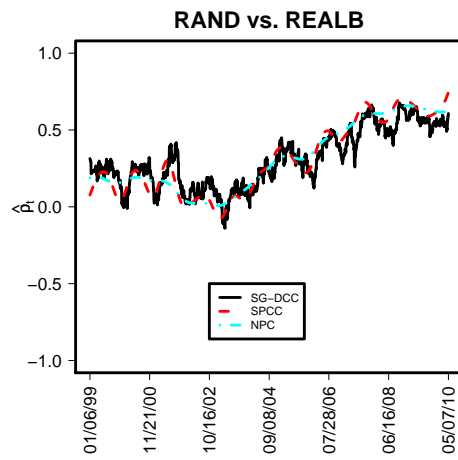
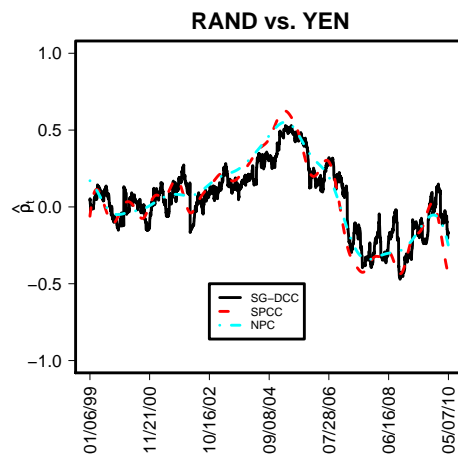


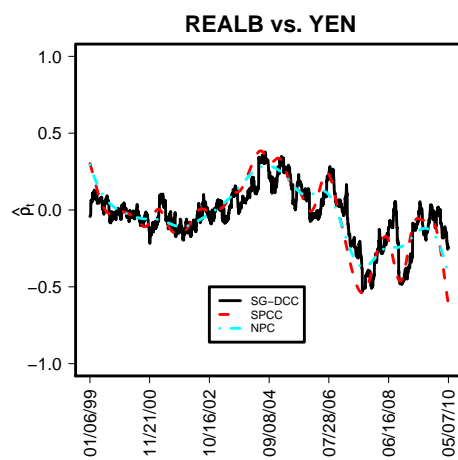
Figure 15: Correlations of EUR and CHF with the rest of currencies when the volatility is estimated with a GARCH(1,1) process.



(a)



(b)



(c)

Figure 16: Correlations of RAND, REALB and YEN with the rest of currencies when the volatility is estimated with a GARCH(1,1) process.

Panel A: MSE loss

	SPDR			Currencies		
	SG-DCC	SPCC	NPC	SG-DCC	SPCC	NPC
Full Sample	Jan 04 – Dec 09			Jan 99 – May 10		
Equal weight	3.46e-07	3.44e-07	2.87e-07	0.4300	0.4276	0.3856
Minimum variance	9.13e-08	1.09e-07	2.71e-08	0.1975	0.1724	0.0826
Hedging/Carry trade	3.49e-08	3e-08	2.1e-08	3.0935	3.0675	2.4594
Carry trade given FX volatility				3.0935	3.0675	2.4594
Subsample 1	Jan 04 – Dec 06			Jan 99 – Dec 06		
Equal weight	5.03e-09	4.94e-09	4.22e-09	0.1384	0.1360	0.1244
Minimum variance	6.15e-09	6.56e-09	4.93e-09	0.0899	0.0705	0.0375
Hedging/Carry trade	6.47e-09	6.45e-09	5.29e-09	0.4075	0.4	0.3393
Carry trade given FX volatility				0.4075	0.4	0.3393
Subsample 2	Jan 07 – Dec 09			Jan 07 – May 10		
Equal weight	9.39e-07	9.36e-07	5.72e-07	1.4855	1.4739	1.006
Minimum variance	6.29e-07	6.70e-07	5.57e-08	1.384	1.0217	0.3342
Hedging/Carry trade	6.37e-08	6.33e-08	3.67e-08	12.625	12.707	7.494
Carry trade given FX volatility				12.625	12.707	7.494

Panel B: VaR loss at 5%

	SPDR			Currencies		
	SG-DCC	SPCC	NPC	SG-DCC	SPCC	NPC
Full Sample	Jan 04 – Dec 09			Jan 99 – May 10		
Equal weight	0.0173	0.0173	0.0170	0.8005	0.7993	0.7789
Minimum variance weight	0.0106	0.0106	0.0118	0.6286	0.6159	0.5530
Hedging/Carry trade	0.0150	0.0143	0.0135	1.0106	1.0035	0.9802
Carry trade given FX volatility				1.0042	0.9972	0.9701
Subsample 1	Jan 04 – Dec 06			Jan 99 – Dec 06		
Equal weight	0.0111	0.0107	0.0100	0.7414	0.7351	0.7070
Minimum variance	0.0083	0.0081	0.0091	0.5973	0.5846	0.5162
Hedging/Carry trade	0.0117	0.0122	0.0116	0.8842	0.8602	0.8216
Carry trade given FX volatility				0.8702	0.8466	0.8051
Subsample 2	Jan 07 – Dec 09			Jan 07 – May 10		
Equal weight	0.0139	0.0141	0.0242	0.7976	0.7984	0.9498
Minimum variance	0.0102	0.0098	0.0146	0.5761	0.5505	0.6408
Hedging/Carry trade	0.0118	0.0110	0.0155	1.1163	1.1486	1.3569
Carry trade given FX volatility				1.1012	1.1322	1.3414

Table 3: Evaluation of empirical results of both portfolios.

	Var_{SPCC}/Var_{SG-DCC}	Var_{NPC}/Var_{SG-DCC}	Var_{NPC}/Var_{SPCC}
SPDR minimum variance portfolio			
Full sample	1.0192 (0.1440)	1.3515 (0.0704)	1.326 (0.0483)
Subsample 1	0.9457 (0.0132)	1.4025 (0.0558)	1.4830 (0.0026)
Subsample 2	0.8341 (0.0019)	3.9867 (0.0003)	4.7794 (1.98e-05)
Currencies minimum variance portfolio			
Full sample	0.9699 (6.92e-12)	0.8377 (1.18e-14)	0.8637 (1.26e-09)
Subsample 1	0.9665 (7.62e-10)	0.8038 (1.84e-09)	0.8317 (1.40e-06)
Subsample 2	0.9297 (1.30e-07)	1.9050 (6.90e-10)	2.0491 (3.08e-09)

Table 4: Portfolio variance ratios. P-values of the Engle and Colacito (2006) test are displayed in brackets.

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