Knowledge and Multisemiosis in Undergraduate Physics

Yaegan Doran

NOVEMBER 2010

This thesis has been submitted in partial fulfilment of the requirements for the degree of

Bachelor of Liberal Studies (Hons)

ACKNOWLEDGMENTS

While it's not possible to acknowledge all the people who have spurred me on, pointed me in the right direction or taken the time to discuss with me aspects of this thesis, there are a number of them who deserve particular thanks.

First and foremost, I would like to thank my two supervisors from different stages of the year, Shooshi Dreyfus and Jim Martin. The combination of their extraordinarily different styles encouraged me to both think and do, and to never be fully satisfied with what was found, pushing me increasingly deeper into the material. Without the time and effort both put in to ensuring I was on track, this thesis would have struggled to get off the ground.

I would also like to thanks Toni Borowsky who has been very generous throughout the year fixing many issues that have popped up. Also to the rest of the linguistics honours crew, particularly Beth, James and Matt, for helping me realise that, when it comes to writing a thesis, I am not alone in the regular shifts from excitement to trepidation to bamboozlement to satisfaction. The year would have been a much tougher journey had it not been for our weekly post-class lunch sessions.

Finally, my thanks to my mother, Sue Doran, for proof reading, for patiently listening to my regular spiels involving linguistics, sociology, physics and mathematics and above all for her unwavering support and encouragement

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Chapter 1 - Science and education

Science aims to understand the universe around us in a precise way. It aims to predict, describe and explain physical phenomena. As part of this mission, science construes the universe in very specific ways, relying on both experimental observation and theoretical modelling. Among the various disciplines of science, physics is one of the most fundamental. A first year university textbook explains that:

> 'Scientists of all disciplines make use of the ideas of physics, from chemists who study the structure of molecules to paleontologists who try to reconstruct how dinosaurs walked. The principles of physics play an essential role in the scientific quest to understand how human activities affect the atmosphere and oceans, and in the search for alternative sources of energy. Physics is also the foundation of all engineering and technology. No engineer could design any kind of practical device without first understanding the basic principles.' (Young and Freedman 2004: 1)

Despite the fundamental role physics plays in understanding the universe many people disengage with it at various levels of schooling. The overarching goal of the research presented in this thesis is to provide a step toward understanding why this is the case and, eventually, to develop educational systems to strengthen and broaden scientific literacy.

One possibility for this disengagement has to do with how knowledge in science is reconstrued and presented to students. Halliday and Martin (1993) argue that there is a 'language of science' that is markedly different to that of every day language. To use their example, when students are faced with a wording such as:

> 'One model said that when a substance dissolves, the attraction between its particles becomes weaker.' (Junior Secondary Science Project, 1968, pp 32 -33, in Halliday and Martin 1993: 2)

they have no trouble in recognising it as the language of a chemistry book.

Spoken and written language, however, is not the only means by which scientific knowledge is presented to students. The discourse of physics also includes many non-linguistic semiotic systems such as mathematics and images. Compounding any difficulties students may face with the language of science, both mathematics and images have the ability to alienate many people from the discourse of physics. The theoretical physicist, Stephen Hawking, in his best-selling book, *A Brief History of Time*, recounts that:

> 'Someone told me that each equation I included in this book would halve the sales' (Hawking 1988: vi)

It is clear then, that mathematics promotes some trepidation about reading physics among the general public. Despite this, however, the various methods for conveying knowledge to students may not be the only factor in the disengagement of students with physics.

Another possibility might simply have to do with the nature of the discipline of physics itself as compared to, say, history. Hawking (1988: 163) explains that the ultimate goal of physics is to find a complete, consistent, unified theory of everything in the universe. This goal has ramifications for how knowledge in physics is developed and related.

These two possibilities for disengagement frame the research presented in this thesis. Thus, this thesis aims to describe the nature of knowledge within the discipline of physics, as well as how this knowledge is conveyed to students.

1.1 Physics as a multisemiotic discipline

As mentioned, the knowledge within physics is presented multisemiotically. That is, physics relies on language, mathematical symbolism and images to construe its meanings. The fact that these three semiotic resources have been used consistently in physics since the time of Isaac Newton (around the seventeenth century) (O'Halloran 2005: 22), suggests that they are integral to the discipline – that knowledge within physics cannot be expressed fully without all three.

Within Systemic Functional Linguistics (SFL), research has focused on science since the late 1980s (See Halliday and Martin 1993, Martin and Veel 1998, Halliday 2004 for compilations of much of the early work on science). This early research was primarily concerned with the construal of scientific knowledge using language.

In the 1990s, SFL and Social Semiotics began to study the meanings made by semiotic resources extra to language, primarily images (O'Toole 1994, Kress and van Leeuwen 2006 (1st edition was 1996)). The study of multimodality was extended to mathematical symbolism by Jay Lemke (1998, 2003) and Kay O'Halloran (2003, 2005, 2007, 2008, 2010). As yet, however, there appears to have been no research to understand how the multisemiosis of written language, mathematical symbolism and images, construe knowledge within the physical sciences.

Basil Bernstein has developed a sociological theory of knowledge (Bernstein 1999), extended by various scholars in past decade (eg. Maton 2007, 2008, Forthcoming; Muller 2007; Maton and Muller 2006), that provides a framework for classifying and describing various academic disciplines. SFL has collaborated with this theory providing linguistic tools for description (Christie and Martin 2007, Christie and Maton 2011 in press, Martin 2006). Once again, however, this framework has not yet been comprehensively applied to physics (See Lindstrøm 2010, Draft, for non-linguistic research in this regard).

This thesis offers a first step into fully understanding how technical scientific knowledge is construed and conveyed. This includes the use of written language, mathematical symbolism and images. Secondly it presents the first data driven understanding of the nature of knowledge within physics.

1.2 Data and methodology

To understand how physics conveys its knowledge, excerpts from two undergraduate university physics textbooks, one first year and one third year, have been studied and compared. These two excerpts cover roughly the same topic: Schrödinger's equation in quantum mechanics.

The first text is an excerpt from a general undergraduate first year textbook titled University Physics (Young and Freedman 2004), that covers many sub-fields of physics (See Appendix 1.1). The excerpt is a section entitled *Wave Functions and the Schrödinger Equation*. This text is divided by sub-headings including: *Interpretation of the Wave Function*, *Stationary States*, *The Schrödinger Equation* and *Wave Packets*. This text will be referred to herein as the first year text.

The second text (See Appendix 1.2) in this study is an excerpt from a set of course notes (School of Physics, University of Sydney 2009) provided to third year quantum mechanics students in physics at Sydney University in lieu of a published textbook. The excerpt being studied is titled *Schrödinger's equation for a hydrogen-like atom*, containing sections sub-headed by *Solutions*, *Degeneracy*, *Expectation values* and *Radial probability density*. This text will be referred to as the third year text.

Each text is a prescribed text in their respective courses of physics at the University of Sydney. They were chosen as they covered a very similar area of physics, but used their written language, mathematical symbolism and images in distinctively different ways. The most obvious difference between the two is the comparatively high use of mathematical symbolism by the third year text, compared the first year.

The use of two texts focusing on the same topic allows for a detailed understanding of how each text conveys its knowledge to students. As the data used is a very small corpus focusing only on one sub-field within physics, however, conclusions drawn from this study are not necessarily representative of the entirety of the discipline of physics. Despite this, the study does present a first step into the research area, raising some important theoretical questions.

1.3 Structure and scope of thesis

In order to present the analysis of the two texts, this thesis is divided into five distinct chapters. Following this chapter, chapter two presents the theoretical foundations needed to understand the analysis, as well as the work on science and mathematics that has informed this thesis. This includes the Systemic Functional framework for language, mathematical symbolism and images, and the theory of knowledge developed by Bernstein et al.

Chapter three presents a genre survey, as well as the meanings made by written language in each text. Chapter four discusses the use of mathematical symbolism and images individually, as well as how the three semiotic resources work in conjunction with each other to produce cohesive meanings throughout the texts. Both chapter three and chapter four will apply their analyses of the texts to Bernstein's theory of knowledge, to understand the nature of the knowledge of physics.

The final chapter wraps up the study, linking the analyses presented in chapters three and four to understand how the texts convey their knowledge as well as what this says about the discipline of physics. Finally, this chapter presents the theoretical implications of the conclusions drawn in this thesis.

This study does not attempt to provide a comprehensive understanding of the nature of knowledge in physics, nor how this knowledge is construed within physics as a whole. It does, however, make an important contribution to the growing area of research into disciplinarity, multimodality and science education, uncovering a number of theoretical questions requiring further research.

Chapter 2 - Theoretical foundations

The study presented within this thesis utilises the framework of Systemic Functional Multimodal Discourse Analysis (SF-MDA) (O'Halloran 2005) in order to examine how written language, mathematical symbolism and images work together to convey physics knowledge to students within a series of university textbooks. SF-MDA has been developed as an extension to Systemic Functional Linguistic (SFL) theory which views language primarily as a resource for making meaning. This chapter will firstly present the theoretical framework of SFL, and how it has been used to analyse written language in relation to genre, discourse semantics and lexicogrammar. Following this, the SF-MDA frameworks for mathematical symbolism and images will be explained, including how they have been applied to similar work within science. Finally, a theory of knowledge will be presented, initially developed by Basil Bernstein (1999). This theory will be used to discuss the nature of the field of physics itself. Each theoretical framework described will provide the reader with the necessary tools to understand the analysis and conclusions of this thesis.

2.1 Systemic Functional Linguistics and language

Systemic Functional Linguistic theory considers language as a social semiotic, that is, as a resource for meaning making in social life. Based on the work of Michael Halliday (1978), Eggins (2004: 3) indicates four theoretical claims made by Systemic Functional linguists. These are that language use is functional, and that this function is to make meanings. These meanings are influenced by the social and cultural contexts in which they are exchanged, and finally that the process of using language is a semiotic process; a process of making meanings via a system of choices. The implication of these assumptions is that language is described by what it does rather than how it is structured.

2.1.1 Strata and rank

One of the cornerstones of Systemic Functional Linguistics is that language occurs simultaneously on multiple levels of abstraction, known as strata (Halliday and Mattiesson 2004: 24). The most concrete stratum is the expression plane which, in spoken language, is realised by phonology and in written language, graphology. The next level of abstraction is known as the content plane. This concerns the lexicogrammar and discourse semantic strata. Lexicogrammar, as the name suggests, concerns the lexis (content words) and the grammar of the language up to and including clause complexes. Discourse semantics concerns how meanings are built above the clause level across a whole text. Each stratum is organised compositionally, with smaller units realising larger units; this is called the rank scale. Table 2.1 (adapted from O'Halloran 2005:63) shows the various planes including their stratal realisations. The non-italic components of the Stratal Realisation column show the rank scale.

Plane	**Stratal Realisation**
Content	*Discourse Semantics* - Paragraph and Text
	Lexicogrammar - Clause Complex - Clause - Word group and phrase - Word
Expression	*Phonology* *Graphology/Typography*

Table 2.1

Strata for language

Thus, units within strata are organised by rank. Each stratum is connected to the other through a relationship of realisation. This means that SFL does not consider each stratum to be constructed out of the lower but rather the lower is a resource to realise the higher. This means, for example, that the meanings made by the lexicogrammar are provided for by the phonology/graphology instead of being built from it. From this perspective, social context fills the stratum above discourse semantics (Martin and Rose 2007:4). Thus, phonology/graphology realises lexicogrammar which realises discourse semantics which realises social context. Image 2.1 (appropriated from Halliday and Martin 1993: 25 & 32) provides a diagrammatic interpretation of this. It should be noted that social context is also treated as a stratified system, comprising the levels of register and genre (Martin 1997: 6). Genre will be presented in section 2.2.

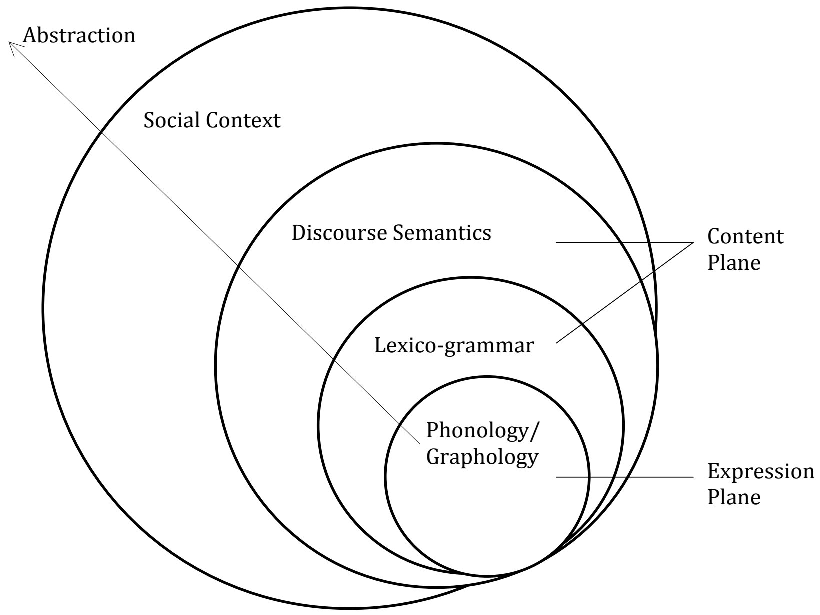

Image 2.1

Strata for language

2.1.2 Metafunctions

In order to analyse and understand the functionality of language, Halliday (Halliday and Matthiessen 2004: 29) has demonstrated that language has three broad meaning making functions that act simultaneously. These are known as the ideational, interpersonal and textual metafunctions. Each metafunction is construed by different grammatical resources.

The ideational metafunction is concerned with how we construe our experiences in language. This metafunction, in turn, is divided into two sub-types, experiential and logical which are sometimes referred to as metafunctions themselves (O'Halloran 2005: 64, Martin et al. 1997:100). The experiential metafunction is concerned with events and processes that take place within the clause, whereas the logical metafunction is orders the relations of one event to another between the clause.

The interpersonal metafunction deals with how language enacts personal and social relationships. Finally, the textual metafunction refers to how the meanings in a text are woven together into a coherent whole; it determines the way information is distributed throughout a text.

2.1.3 Paradigmatic choice and system networks

In order to use language functionally and to express meaning that is tailored to the individual need, SFL sees language as a system of choices. Meaning is generated through the choice of particular grammatical or lexical options above all available options. The extent of the choices at hand for a language user is known as the paradigm; the structural realisations of these choices are the syntagm. In order to model the paradigm of all possible choices, SFL uses diagrammatic system networks, read from left to right, where each feature is seen to be an entry condition to the next level of delicacy within the system. The complexity of the choices that can be made within the paradigm (such as simultaneous choice) can be shown using these system networks. The system networks show both the paradigmatic and syntagmatic, but are structured to foreground the paradigmatic, reflecting the overall theory. Image 2.2 shows a simplified example of the system network for major clauses within the system of Mood, including both the syntagmatic and paradigmatic choices.

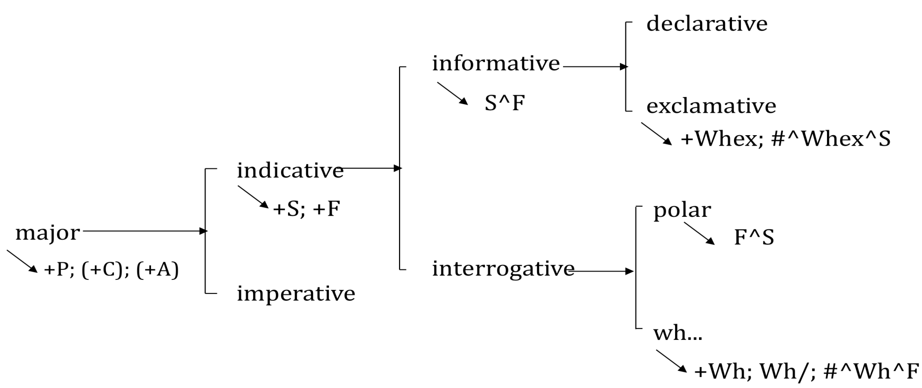

Image 2.2

System of MOOD (Martin 2009)

Within this system network, the horizontal arrows indicate the set of choices within a particular system. The square brackets indicate that the choice is oppositional. For example, if a clause is major, it is either an indicative or an imperative. ↘ indicates the realisation for a paradigmatic choice, linking the paradigm with the syntagm. ^ shows a sequence that isn't commutative, ie. a^b is not the same as b^a. + indicates that the realisation includes the feature following it, and finally, # indicates the beginning or end of a clause. Movement to the left and right within a system network, that is, moving to more general or specific choices, is known as movement in delicacy.

Now that the basic tenets of SFL have been introduced, we will turn our attention to the tools SFL has developed to analyse how meaning is construed through written texts, firstly via genre theory.

2.2 Genre

Genre refers to different types of texts that enact various social contexts (Martin and Rose 2007: 8). For many social contexts, meaning making patterns are relatively consistent, producing predictable structures within texts. Texts are classified into different genres according to their predictable meaning making patterns as well as their function in society. Within SFL, genres are seen as staged, goal-oriented social processes (Martin 1997: 13). 'Staged' refers to the characteristic of most genres to take more than single phase to unfold and 'goal-orientated' indicates they have a social purpose.

This section will introduce Christie's formulation of curriculum macrogenre, as well as outlining certain genres that are crucial to an understanding of the texts studied in this thesis. An analysis of genre allows for an overview of the social processes that texts produce, linking together the discursive and lexicogrammatical techniques used.

2.2.1 Curriculum macrogenre

Analysing the physics texts, it becomes clear that neither text simply conforms to a model of a single genre. Rather, the texts consist of a series of varying genres that relate to each other, whilst producing their own distinct meanings, each aimed at teaching a different piece of information to the student. Christie (1997) names this sequence of pedagogic genres, a curriculum macrogenre, and explains that:

> 'a curriculum macrogenre constitutes a sequence of curriculum genres in which new understandings and new forms of consciousness are taught and learned. A series of genres unfolds, each with its own elements of schematic structure, and the genres constitute important elements, in turn, of the macrogeneric structure, such that the genres stand in relation to each other.' (Christie 1997: 148)

This classification allows an understanding of how the texts are staged with different genres, to effectively teach students. (See Appendix 2 for the full analysis of the macrogenres). The most important genres to this study, that occur regularly within each text's macrogenre, are Causal Explanation and Descriptive Report.

2.2.2 Causal Explanation

Explanations are used to explain the processes involved in natural and social phenomena, how something works or why things are the way they are (Metropolitan East Disadvantaged Schools Program 1989: 16). There is considerable variation in different text types used to explain, and so, Explanations are divided into a series of sub-categories each with their own social purpose. This study, however, will focus on only one type of Explanation, that of a Causal Explanation.

Causal Explanations are concerned with explaining why things occur. To do this, they provide causal links between sequences of events. The generic structure of a Causal Explanation includes an optional Identification stage, identifying the issue at hand, followed by the Implication Sequence, to highlight the cause and effect sequence (Metropolitan East Disadvantaged Schools Program 1996: 69). Using a section from the third year text as an example, Causal Explanations can be modelled as in Table 2.2.

Identification	**2.2 Degeneracy** A state of a hydrogen-like atom is characterised by three *quantum numbers*: n, l, m_l.
Implication sequence	The energy of a state depends, however, upon the value of n only (equation 30). For a given value of n, the quantum number l can have the values $n - 1, n - 2, \ldots, 0$

	and for each l there are $2l + 1$ possible values of m_l: $l, l - 1, l - 2, \ldots, -(l - 1), -l$. The number of states with the same energy is therefore $$\sum_{l=0}^{n-1}(2l+1) = n^2 \qquad (38)$$ Thus energy levels are *degenerate* with a *degeneracy* of n^2: there are n^2 different quantum states, with different wave functions, which have the same energy E_n.

Table 2.2

Staging of Causal Explanation

2.2.3 Descriptive Report

Reports are factual texts aimed at describing the way things are (Metropolitan East Disadvantaged Schools Program 1989b: 2). They describe various features that enable a student to understand multiple facets of the thing under study. Again there are multiple types of Report, however here the focus is on the Descriptive Report only.

Descriptive Reports provide attributes (either physical or otherwise) about phenomena. Information on many different features can be described, providing a detailed picture of the phenomenon. The generic structure of a Report contains an Identifying stage, orientating the reader to the type of information in the Report. This is followed by a Descriptive stage. Table 2.4 provides a model of a brief Descriptive report from the third year text.

Identification	**2.4 Radial Probability Density**
Description	The probability that an electron is in the radial range $r \rightarrow r + dr$, P(r), is given by $$P(r)dr = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \;\; \psi^* \psi r^2 dr$$ P(r) depends on the values of n and l, but not on m_l, and is shown in Figure 3 for all l values corresponding to n = 1, 2, 3. Obviously $$\int_0^{\infty} P_{nl} dr = 1$$

Table 2.3

Staging of Descriptive Report

Genre realises the social context in which a text is placed. We now will turn our attention to how the meanings made by the texts themselves are realised through written language.

2.3 SFL and written language

In terms of quantity, written language is the primary meaning making resource used in both texts. Language has been studied extensively within SFL providing a large number of analytical tools used to understand how meaning is construed. This section will outline a selection of these analytical tools used in discussions throughout the thesis. Within the discourse semantic stratum, two analytical tools will be presented: Periodicity and Taxonomic Relations (Martin and Rose 2007). Within the lexicogrammatical stratum, the system of TRANSITIVITY (Halliday and Matthiessen 2004) and the analysis of nominal groups will be given. In conjunction with these tools, the linguistic processes of grammatical metaphor and technicality (Halliday and Martin 1993) will be explained, in order to understand more fully how experiential meaning is construed within physics texts. As meaning is made on multiple strata, the use of these analytical tools will provide an understanding of how the various strata work simultaneously to produce the experiential meaning in physics.

2.3.1 Periodicity

Periodicity is concerned with information flow; the way in which meanings are structured for the reader to understand (This section is based on Martin and Rose 2007 chapter 6). Certain pieces of information are given prominence at multiple levels within texts. This prominent information can be classed into Theme or New.

At the lowest level of periodicity within the discourse, the clause, the peak of prominence is known as its Theme. Within written language, the Theme is essentially everything up to and including the Subject of the clause. Information prior to the subject is atypical and thus becomes more prominent. This information is known as the marked Theme. Marked Themes often include circumstantial elements, such as places or times, and usually signal new phases in a discourse. Also within a clause, a different kind of textual prominence occurs toward the end. This prominence is known as the New. The New has to do with the information that is to be expanded on as the text unfolds. The example below from the first year text shows the Theme in Bold, marked Theme underlined and the New in italics.

> **<u>In the photon interpretation of interference and diffraction</u>, the intensity at each point** is *proportional to the number of photons striking around that point.*

Theme and New work to package information at the clausal level. The various textual prominences can be reflected above the clause, however. Information is often predicted and distilled is long phases of the discourse. Paragraphs usually include a 'topic sentence' that introduces what will be said. This 'topic sentence' is a higher level Theme known as a hyperTheme. Once the information has been given within a phase of the discourse, it is often summarised by another sentence, known as the hyperNew. Where the hypertheme tells us what will be said, the hypernew tells us what has been said – it distils the new information into a small package. The example below from the third year text shows the hyperTheme in bold and the hyperNew in italics.

> **A state of a hydrogen-like atom is characterised by three quantum numbers, n, l and m_l . The energy of a state depends, however, upon the value of n only.**
> For a given value of n, the quantum number l can have the values n-1, n-2,…,0 and for each l there are 2l + 1 possible values of m_l: l, l – 1, l – 2, …, -(l-1), -l.
>
> The number of states with the same energy is therefore
>
> $$\sum_{i=0}^{n-1}(2l+1)=n^2$$
>
> *Thus energy levels are degenerate with a degeneracy of n^2: there are n^2 different quantum states, with different wave functions, which have the same energy E_n.*

In most texts, the patterning of information can extend to phases of discourse much larger than the paragraph. Any higher level Theme that predicts hyperThemes is known as macroTheme, and higher levels of New are macroNew. MacroThemes and macroNews order a text into a hierarchy that allocates varying degrees of prominence to information. Layering such as this can go on indefinitely depending on the complexity of the text. This layering is known as the hierarchy of periodicity and is vital to understanding the information being foregrounded by a text.

2.3.2 Taxonomic relations

Within the discourse semantic stratum, taxonomic relations are an aspect of the system of IDEATION that is concerned with how the discourse construes experience. It builds on the experiential meanings made through the experiential metafunction at the lexicogrammatical stratum. Martin and Rose (2007:73) explain that:

> '[IDEATION] focuses on sequences of activities, the people and things involved in them, and their associated places and qualities, and on how these elements are built up and related to each other as the text unfolds.'

Taxonomic relations refer to the way in which people and things (and also processes and qualities) are divided into categories and taxonomies with varying delicacy within a text. The divisions made are often not explicit, and need to be retrieved from the text via close study. There are two basic types of taxonomic relations: classification and composition.

Classification refers to taxonomies of class. That is, *x* is a type of *y* as opposed to *x* being a part of *y*. Individual participants within the taxonomy can have their relations described by the terms hyponomy, hyperonomy, co-hyponomy, superordination and subordination. The use of these terms will be illustrated by way of an example:

i. Electric charge is a hyperonym (also known as a superordinate) of a positive charge
ii. Positive charge is a hyponym (subordinate) of electric charge
iii. Positive charge and negative charge are co-hyponyms.

Compositional taxonomies are those which describe the part-whole relations between things. The relations of this type are described by meronomy and co-meronymy (or simply as part and whole). Another example will illustrate this:

i. The proton is a meronym of the atom. Conversely, the atom is a meronym of the proton. However this example could be described as the proton being a part of the atom, and the atom being a whole including the proton.
ii. The proton and neutron are co-meronyms.

2.3.3 TRANSITIVITY

The system of TRANSITIVITY concerns the experiential meanings made within a clause (Martin et al. 1997: 100). From this perspective, a clause consists of three components: a process unfolding through time, the participants involved in the process and the circumstances associated with the process (Halliday and Matthiessen 2004: 175). Processes and at least one participant are obligatory within the clause, however circumstantial elements are optional. As such, the process is the most central element in the configuration meaning that the participants within a clause will differ depending on the process type. Processes are typically realised by verbal groups, participants by nominal groups and circumstances by adverbial groups or prepositional phrases, as shown by Table 2.4, an example from the first year text.

We	know	the wave function	for a particular wave motion
Participant nominal group	Process verbal group	Participant nominal group	Circumstance prepositional phrase

Table 2.4

Example of clausal structure in TRANSITIVITY

This section will outline the two main process types used within the texts and their various participants. The processes are relational and material processes. A TRANSITIVITY analysis provides an understanding of how the lexicogrammatical stratum construes experiential meaning.

2.3.3.1 Relational processes

Relational processes are concerned with identifying and characterising (Halliday and Matthiessen 2004: 210). These processes are often, though not always, realised by the verb 'to be'. Two types of relational process used in the texts are identifying and attributive.

Relational Identifying processes, as the name suggests, aim to identify one participant in relation to another. The two participants within a relational identifying clause refer to the same thing, however, as the clause is not a tautology, there is a distinction. This distinction can be categorised in respect to the strata of language, as expression and content. From this categorisation, the respective grammatical labels for the participants are Token and Value. Either can be used to identify the other, with one of the features of identifying clauses being that they are reversible. Table 2.5 shows an example from the third year text.

a_0	is	the Bohr radius
Token	Process: Relational: Identifying	Value

Table 2.5

Relational Identifying process

As this clause can be reversed to form *the Bohr radius is* a_0, the clause is identifying.

Relational attributive clauses ascribe a class or attribute (known simply as Attribute) to what is labelled a Carrier. The Attribute is usually indefinite, and the clause cannot be reversed. Another example from the third year text (Table 2.6) is:

n	is	an integer
Carrier	Process: Relational: Attributive	Attribute

Table 2.6

Relational Attributive process

As *integer* is a class to which *n* is a member, the clause cannot be reversed. We cannot say *an integer is n* as not all integers are *n*.

2.3.3.2 Material processes

Material processes are processes of doing and happening (Halliday and Matthiessen 2004: 179). They construe actions that occur, in contrast to relational processes, which relate participants. Typically within a material clause, there are two possible participants, an Actor and a Goal. The Actor is the participant that does the process. The Goal is the participant that has the process directed to it. Table 2.7 from the first year text shows this:

We	use	a wider range of numbers
Actor	Process: Material	Goal

Table 2.7

Material process

2.3.3.3 Voice

Though not strictly part of the system of TRANSITIVITY, it is necessary here to explain the notion of Voice in regard to material clauses. Within English, a clause can be either active or passive (Thompson 2004: 92). Table 2.7 is an active material clause, as the Actor also functions as the subject (it is at the beginning of the clause). Within passive clauses the goal is moved to the clause to produce, as shown in Table 2.8.

A wider range of numbers	is used	by us
Goal	Process: Material	Goal

Table 2.8

Passive Material clause

In passive clauses, the Actor can be left off, producing, *a wider range of numbers was used*. This voice distinction will form an important part of our comparison of the two texts.

2.2.4 Nominal groups

Below the clause, combinations of words are built up on the basis of a particular logical relation (Halliday and Matthiessen 2004: 310). The combinations of words are known as groups and, as noted previously within TRANSITIVITY, different groups function to realise different experiential components. This section will describe in detail certain aspects of the structure of nominal groups as they are the most important for this study.

Nominal groups fulfil the participant role within a clause. To aid explanation Table 2.9 provides an example of a nominal group.

The	two	additional	sinusoidal	waves	with	slightly	different	frequencies
Deictic	Numerative	Post-Deictic	Classifier	Thing	Qualifier (prep phrase)			
					prep	nominal group		
						Epithet		Thing
						β	α	

Table 2.9

Nominal group

The central element of a nominal group is the Thing. It is the semantic core of the group (Halliday and Matthiessen 2004: 325), and answers the question, *what is it?* (ie., in the example above the answer must necessarily include *waves*, it is not, for example a *two*, or a *sinusoidal*).

The Classifier indicates a particular subclass of the Thing. The Epithet (shown in the embedded nominal group within the Qualifier), in contrast, indicates a quality of the Thing. The distinction between Classifier and Epithet is a very fine one. Epithets can take degrees of intensity or comparison, whereas the Classifier cannot. We can say the *very different frequencies*, however, we cannot say the *very sinusoidal waves*.

Occurring at the beginning of nominal groups is the Deictic. Deictics indicate whether or not a specific subset of the Thing is intended, and if so, which. In this case, the Deictic is realised by the determiner *the*. Following the Deictic, Numeratives indicate some numerical feature of the particular subset of the Thing. The final entity modifying the Thing is the Post-Deictic. Post-Deictics, like Deictics, also indicate a specific subset of the Thing, however, they do so by referring to its familiarity, its status or its similarity/dissimilarity (Halliday and Matthiessen 2004: 316).

Anything following the Thing in a nominal group is known as a Qualifier. Qualifiers, with only rare exceptions, are always rankshifted. This means that the Qualifier is realised by an embedded entity of a rank that is higher than or at least equivalent to that of a nominal group. The example in Table 2.9 contains a rankshifted prepositional phrase (*with slightly different frequencies*), which in turn contains a rankshifted nominal group. Rankshifting allows nominal groups and subsequently clauses to become very large and dense.

To this point, analytical tools at the discourse semantic and lexicogrammatical strata have been introduced. Now we turn to two linguistic resources of English that are crucial for meaning making in academic and scientific written language. These features are grammatical metaphor and technicality.

2.2.5 Grammatical metaphor

Grammatical metaphor is a language resource used regularly in abstract construals of experience. A grammatical metaphor is a lexical item that has undergone a change in its grammatical category. The change in category is used to expand the meaning potential, with different types of grammatical metaphor used in different discursive contexts. Martin and Rose (2007 pg 110) explain that grammatical metaphor is natural for readers with high levels of literacy. It only comes to our attention when it is used within unfamiliar discourse.

In science writing in particular, grammatical metaphor is used to a high degree. (Halliday 1989: 172) There are a number of types of grammatical metaphor, however only experiential metaphor will be explained, as it is by far the most highly relied upon within physics.

Experiential metaphor works to repackage experiential meaning and is often (but not always) realised by a process changing to a nominalised participant. This is shown in the example below, beginning with what is known as the congruent (non-metaphorical) realisation followed by a metaphorical realisation.

> The particle moved to the left.
>
> The motion of the particle was to the left.

In this example, the process *moved* was nominalised to become the participant *the motion*. By nominalising the process, the motion can now be described and qualified, becoming a fully-fledged participant of itself. The focus is then on an abstract entity of *motion* rather than on a concrete entity of *particle*. As physics deals with abstractions such as motion, nominalisations such as this are an important resource for construing meaning.

By repacking information through the use of grammatical metaphor, more information can be given more efficiently. Within non-familiar discourses, however, grammatical metaphor often has the effect of becoming very difficult to access due to its density and abstraction, possibly excluding the reader from this information. However, Grammatical metaphor can, at times, lead to the building of technical terms that are vital to an understanding of how science construes and explains the world around it. This is explained next.

2.2.6 Technicality and abstraction

When a term is introduced and named, encapsulating a particular aspect of the field of study, it has the opportunity to become part of the field itself. If the term has a field specific meaning that becomes part of the knowledge of the field, it is considered technicality (Wignell, Martin, Eggins 1993: 144). As it has become part of the field, technicality can thus be presumed within other texts, lessening the need for explanation and lightening the discourse. In order for something to become technicality, two process must be undertaken: distillation and transcendence of the text.

Distillation refers to technical language's ability to compact and change the nature of everyday words (Martin 1993c: 172). An example from the first year text shows this distillation (with the technicality in bold):

> 'Because the particle's probability distribution in such a state doesn't change with time, a state with a definite energy is called a **stationary state'**

The technical term compacts the meaning of all the previous entities into a single nominal group. Further, the technicality is not simply renaming other terms, it is encapsulating the meaning of all the other words and their relations, and as such it is changing the nature of the everyday words.

Simply because a term is distilled doesn't immediately mean it will become technicality, however. The term must also transcend the text, to become part of the knowledge of field itself. That is, it must come to represent the meanings it encapsulates throughout many texts within the field. If the term does not do this, and can only be recovered from the text it is introduced in, it is said to be instantial (Martin 2006: 13). Instantial namings are recoverable from the text, but not beyond, technicality is recoverable from the field itself. Technicality can be seen to transcend the text if it can be presumed without any introduction within other texts.

In terms of language's strata, technicality distils on the content plane, as the nature of the meaning is being modified along with its compaction (Halliday and Martin 1993: 30). In contrast, abbreviation simply condenses without changing any meaning. For example, using SFL instead of Systemic Functional Linguistics does not carry any variation in meaning. Abbreviation, therefore, condenses on the expression plane.

Within English, technical terms are almost exclusively nominal groups, such as *grammatical metaphor, wavefunction*. Martin (**1993a,b,c** :212) explains technical verbs are rare, and those that do exist are seldom used. As processes can be recategorised as participants via grammatical metaphor and subsequently become technical, they can then interact with and become defined by other technical terms. Thus, the discourse becomes increasingly abstract. That is, the discourse can move away considerably from what we can sense – see, touch, feel. This is entirely necessary for science as it provides the power to efficiently relate and describe quite complex phenomena. Between the abstraction, technicality and grammatical metaphor, however, the discourse can quickly become completely unintelligible and inaccessible to those not inducted into the field via education. For this reason it is important to understand how educational texts relate the common sense language of everyday life, to the uncommon sense language of academic fields.

In addition to technicality and abstraction within the lexicogrammar, physics also coopts the technicality of the discipline of mathematics – itself a complex and abstract discipline. Furthermore, mathematics and thus physics, makes use of non-linguistic meaning making resources such as symbolism and to a lesser extent, images. These semiotic resources are vital to understanding the field of physics. As such the next section will introduce the theoretical framework used for analysing how these resources complement written language in building our understanding of the universe.

2.4 Multimodal Discourse Analysis

As explained in section 2.1, language is seen as a social semiotic, a tool for producing functional meanings within everyday life. Within language, the interaction between different strata (section 2.1.1) work to produce very complex meanings. Language is not, however, the only semiotic tool available to us. Extra-linguistic semiotic systems, such as images and symbolism, regularly complement or replace language altogether, conveying meanings themselves. Moreover, much of the meaning made by these resources is not possible within written language. These meanings are frequently vital in the construction of many fields of knowledge. Within physics texts, images and mathematical symbolism are used on almost every page and are necessary for the teaching of the field. This section will outline the relevant tools from an SF-MDA framework for analysing the meaning made individually within mathematical symbolism and images (known as intrasemiosis (O'Halloran 2005:65)), how the complementarity of the three semiotic resources (including written language) work to produce meaning across the resources that is greater than the sum of its parts (known as intersemiosis) and the relevant multisemiotic work on mathematics and science that has informed this thesis.

The term *semiotic resource* is being used in line with O'Halloran (2010), which distinguishes between the various sensory modalities (visual, aural, haptic, etc.) and individual semiotic resources within each modality. Under this distinction, the semiotic resources of written language, mathematical symbolism and images are all within the visual modality.

2.4.1 Mathematical Symbolism

Mathematical symbolism has been studied within SF-MDA and Social Semiotics primarily by two researchers, Jay Lemke (Social Semiotics) and Kay O'Halloran (SF-MDA). Their research has found that, despite having originally evolved from written language (O'Halloran 2005:97), mathematical symbolism is constructed and organised very differently. The grammatical systems have developed from the symbolism's functions, and have allowed new meanings not possible within language to be produced and described. On the other hand, other meanings that can be made in written language have been contracted. As there are separate grammatical patterns and constructions, producing new meanings, the Systemic Functional framework for written language has been modified by O'Halloran (2005:98) in order to analyse mathematical symbolism. This framework will be outlined in the following section, followed by an explanation of the most important grammatical differences between language and mathematical symbolism. Finally a discussion of the variation in meanings that mathematical symbolism can produce will be given derived from the work of both O'Halloran and Lemke.

2.4.1.1 SF framework for mathematical symbolism

As mathematical symbolism has developed an entirely separate grammar and construction to that of written language, a separate stratal framework has been developed. An analysis of mathematical symbolism still uses the metafunctions described in section 2.1.2 as well as the theories on paradigmatic selection (section 2.1.3) and the principle that each level of strata realises it's higher level (section 2.1.1). The key difference is the strata itself, and its rank scale. This section presents the strata for mathematical symbolism developed by O'Halloran (2005). Eqn. 2.1 will be used to exemplify the different planes and ranks.

$$r = 4\pi\varepsilon_0 \frac{n^2\hbar^2}{mZe^2} = \frac{n^2}{Z}a_0$$

Eqn. 2.1

As with written language, there are two planes at work within mathematical symbolism. The content plane again is the highest. This is realised by the lower, known as the display plane, corresponding to the expression plane in written language. The term 'display plane' is used instead of 'expression plane' as a rank within the content plane has been named 'expression'.

The display plane contains the graphology and typography of the mathematics. Within mathematical symbolism, the graphology and typography (that is the size, font, style, colour etc.) is a functional element, considerably more so than in written language. For example, the two symbols *F* and ***F*** are distinct entities, with the bold signifying that the symbol is a vector (it has both a magnitude and direction), and a lack of bold signifying a scalar (purely a magnitude).

The content plane contains the grammar and discourse semantic strata. Within the grammar stratum, the lowest rank is of components. Components refer to the individual symbols within equations such as m, Z, ε_0.

Components realise the rank of expressions. Expressions are roughly comparable to word groups/phrase within written language (O'Halloran 2005: 108) and are made up of configurations between participants and operative processes (mathematical operations such as x, +, -, ÷ - to be discussed in more detail in the following section). Within Eqn. 2.1, one of the expressions is:

$$4\pi\varepsilon_0 \frac{n^2\hbar^2}{mZe^2}$$

Eqn 2.2

This example shows that the operative process of multiplication, x, is often ellipsed. Making ellipsis explicit in Eqn. 1 as well as indicating the embedded expressions by the use of [[..]] and the full clause with //…// shows the extent of rankshifting of expressions within mathematical symbolism:

$$//\, r = [\![4 \times \pi \times \varepsilon_0 \times [\![\frac{[\![n \times n \times \hbar \times \hbar]\!]}{[\![m \times Z \times e \times e]\!]}]\!]]\!] = [\![[\![\frac{[\![n \times n]\!]}{Z}]\!] \times a_0]\!] \,//$$

Eqn. 2.3

Above the rank of expression is that of clause and statement. The clause refers to two full expressions being related to by a single relational process such as =, <, ≥. A statement, on the other hand, is a sequence of horizontally alligned expressions related by one or more relational processes. Eqn. 2.1 is considered to be a statement as it contains multiple relational processes, however if only one relational process is used, it is considered both a clause and a statement. As such, this thesis will use the terms interchangeably. Both are denoted by //…//.

Within discourse semantic stratum are the inter-statemental relations. These, as the name suggests, takes into account multiple statements and how they produce meaning as a whole. Statements are usually aligned vertically down the page to form derivations and implication sequences, often incorporating both symbolism and written language in order to explain how the derivation works. The strata for mathematical symbolism is outlined in Table 2.10.

Plane	Stratal Realisation
Content	*Discourse Semantics* - Inter-Statemental relations
	Grammar - Statements - Clause (//…//) - Expressions ([[…]]) - Components
Display	*Graphology and Typology*

Table 2.10

Strata for mathematical symbolism

O'Halloran explains (2005: 97) that despite the functionality of mathematical symbolism it nonetheless requires surrounding linguistic co-text to contextualise the symbolic descriptions. The implications of this is that the discourse semantic stratum of mathematical symbolism is necessarily multisemiotic.

Now that the SF framework for mathematical symbolism has been introduced, the important grammatical features of mathematical symbolism for this thesis will be explained. This facilitates an understanding of how symbolism can produce varying meanings from written language.

2.4.1.2 Grammatical features of mathematical symbolism

Mathematics developed as a semiotic resource which aimed to unambiguously encode experiential and logical meaning with maximum economy and condensation (O'Halloran 2005: 97). By encoding this meaning, mathematics worked to describe and predict the world around it in a very specific way. This required a new paradigmatic set of choices, leading to a new grammar. This section will explain the two most important aspects of this grammar in comparison to written language. These are, firstly, the use of operative processes and rankshift, and secondly, the use of the spatial positioning in relation to the display strata.

As mentioned previously, operative processes correspond to arithmetic and algebraic operations such as addition, multiplication etc. These processes function in very different ways to other processes, in that they can be modified and rearranged, whilst still maintaining the process-participant configurations. That is, clauses and expressions can be rearranged, changing the types of operative processes and the rankshifted configurations of the participants (O'Halloran 2005: 104). By rearranging statements, new results are achieved, however, they can be changed back without any information lost. For example:

$$F = \frac{mv^2}{r}$$ **Eqn. 2.3**

can be rearranged as:

$$v = \sqrt{\frac{Fr}{m}}$$ **Eqn. 2.4**

and subsequently arranged back into the orginal equation, without losing any information. This allows the dependency and implication relations to be mapped much more accurately than in written language, as each variable can be predicted from knowledge of every other.

The second grammatical feature of mathematical symbolism involves the display plane and the layout of equations. Mathematic operations are not read from left to right as in written language, rather they are determined by rules known as the order of operations. The order of operations is brackets, powers, division/multiplication and finally addition/subtraction. This means that the spatial layout of mathematical equations is very important as it classes each participant into its requisite expression for the order of operations.

The grammatical features outlined have been developed in order to provide the descriptive and predictive functions of mathematics. These functions are fulfilled by the new possibilities in meanings allowed by the grammatical structure of mathematics. These new meanings are explained in the following section.

2.4.1.3 Mathematical symbolism and meaning

The development of mathematical symbolism varied the types of meaning that could be expressed. This involved a contraction in some meanings and an expansion in others. This section will discuss these expansions and contractions in terms of, firstly, the metafunctions introduced in section 2.1.2, and secondly, the social semiotic perspective given by Jay Lemke (1998, 2003).

2.4.1.3.1 Metafunctions and mathematical symbolism

O'Halloran (2005:97) explains that:

> 'Mathematical symbolism developed as a semiotic resource with a grammar through which meaning is *unambiguously* encoded in ways which involve *maximal economy* and *condensation.*' (Original italics)

In order to unambiguously and economically encode the symbols with meaning, interpersonal and some experiential meanings are contracted if they are peripheral or unimportant to the goal of describing and predicting. Interpersonal meaning is severely contracted. Mathematical symbolism does not make use of choices from the system of APPRAISAL for graduations of evaluation and attitude (eg. that is nice, I am sad, I want that). Furthermore choices of TENSE, MODALITY (probability and usuality) and MOOD ADJUNCTS (eg. readiness, obligation, time, typicality, obviousness, intensity, degree) are typically excluded or are given precise measures eg. for probability, p <0.5). This contraction works to make the discourse appear non-evaluative, value-free, unambiguous and entirely factual (O'Halloran 2005: 116).

Certain aspects of experiential meaning are also contracted within symbolism. Material processes (action verbs, eg. to run, hit), mental processes(eg. to like, hear), behavioural processes (eg. to breathe, dream) and verbal process (to say, flatter) are absent. This leaves the symbolism to focus purely on the meanings made by the relational and operative processes. The grammatical possibilities allowed for by these processes combined with the textual layout between statements allows very precise and efficient logical implication sequences to be expressed. These implication sequences provide mathematics with the ability to describe and predict in a very powerful way.

2.4.1.3.2 Typological and topological meaning

Jay Lemke's social semiotic work on mathematical symbolism offers another insight into how mathematics constructs the world differently to written language. Lemke (1998, 2003) makes a distinction between two different types of meaning that various semiotic systems can make use of. The first relates to meaning by kind; that is, discrete categorical distinctions, known as typological meaning. The second relates to meaning by degree, or continuous variation – known as topological meaning.

Lemke argues that written language is particularly well developed in describing typological meanings, but is relatively poor in resources for topological meaning (Lemke 1998: 87). Visual images, on the other hand, can more readily represent topological meaning. As an example, a Cartesian graph is shown in Image 2.3:

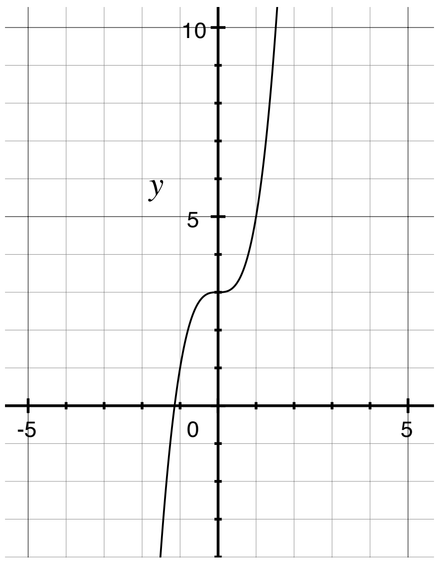

Image 2.3

Written language has no way to describe this line exactly. Individual features can be described for any point, but the overall line cannot be described. That is, written language can describe the image typologically by classifying and naming features, but cannot describe topologically, by describing precisely the continuous variation. Mathematical symbolism, on the other hand, can describe this continuous variation, as $y = 2x^3+3$. In doing so, individual symbols representing distinct typological meanings are used, culminating in a statement that describes topological meaning. Lemke argues this to be a major power of mathematical symbolism – a bridge between the typological meanings of written language and the topological meanings of images (Lemke 2003). The significance is that distinct categories can have complex co-varying relationships described precisely via symbolism.

Due to its grammatical framework, mathematical symbolism can produce meaning that is extra to that of written language. This is very important to the understanding of why physics uses mathematics. The next section will focus on the analytical framework for images, to determine how they present their meanings.

2.4.2 Images

Visual images play a large role in the understanding of physics. As such, this section will introduce the Systemic Functional framework used to analyse images in this thesis. Each text being studied regularly refers to images in order to aid students' understanding. Due to visual images' large role within wider society, it has been relatively widely studied when compared to mathematical symbolism and other non-written language semiotic resources (O'Halloran 2003, 2005, 2007, 2008, Kress and van Leeuwen 2006, O'Toole 1994, Lemke 1998).

2.4.2.1 SF framework for visual images

This thesis uses the SF framework for images presented in O'Halloran (2005: 133), who in turn has modified O'Toole's (1994) framework. O'Halloran's framework again makes use of the metafunctional approach outlined in section 2.1.2 and the theories on paradigmatic selection in section 2.1.3. This section will outline the strata used to analyse images within the physics texts, using Image 2.4 as an example (sourced from a segment of the third year text outside that studied in this thesis).

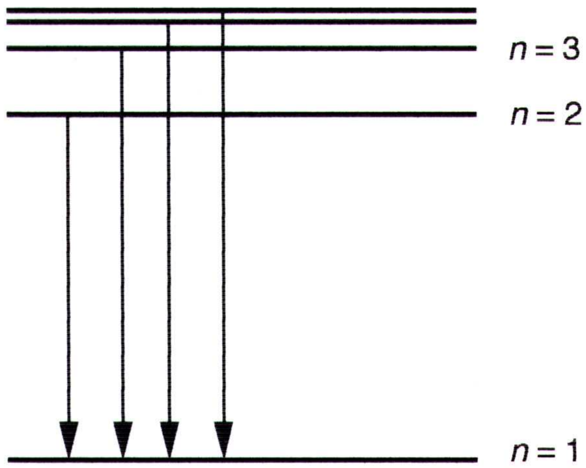

Image 2.4
Energy level transitions

As with mathematical symbolism, images contain two planes, the content plane and the display plane. The content plane contains the discourse semantic and grammar strata, the display plane contains an image's graphics.

The discourse semantic stratum contains the rank of work (the entire image) and inter-visual relations (between multiple works).

Within the grammar stratum, the highest rank is the Episode and concerns the configurations of process-participant relations and circumstance within the work. Episodes include the lower rank of Figure, which are the individual participants within an image. Each line within Image 2.4 is an individual Figure, acting as a participant relating to the other participants via to its spatial positioning. The lowest rank within the grammar stratum is the Part – the features which make up the figure. Within Image 2.4 the arrow heads and black lines are the two Parts making up certain Figures.

In a similar way to mathematics, images within physics contract any peripheral or non-essential information. Image 2.4 is a good example of this, where the entire work consists purely of interactions between figures. There is no contextualising circumstance. For this reason, the Episode within Image 2.4 is also the entire work (discounting the symbolism used on the side). Table 2.10 provides an overview of the strata for images.

Plane	**Stratal Realisation**
Content	*Discourse Semantics* - Inter-visual Relations - Work
	Grammar - Episode - Figure - Parts
Display	*Graphics*

Table 2.11

Strata for visual images

O'Halloran explains that mathematical images are typically multi-semiotic in that they often include both linguistic and symbolic figures in the form of titles, labels and captions (O'Halloran 2005: 134). Both Image 2.3 and 2.4 show this. The interaction between semiotic systems produces extra to that made by each individual semiotic resource. As such, the following section will outline the analytical framework studying the meanings made across semiotic systems.

2.4.3 Intersemiosis and complementarity

To this point, the analytical frameworks have been introduced for studying each semiotic resource individually (for studying *intrasemiosis*). The texts being studied, however, are multisemiotic in nature. The semiotic resources are not individual elements, which simply co-occur on the page with no bearing on the meaning of each other. Rather, each resource works to complement the others to produce a meaning from the text that is greater than the sum of its parts (Royce 1998: 27). Lemke (1998: 92) argues that within multisemiotic genres:

> 'meanings made with each functional resource in each semiotic modality can modulate meanings of each kind in each other semiotic modality, thus *multiplying* the set of possible meanings that can be made' (original emphasis)

The meanings made across semiotic systems is known as *intersemiosis* (or, as Royce refers to it, intersemiotic complementarity). This section will outline some of the mechanisms for intersemiosis presented in O'Halloran (2005).

2.4.3.1 Mechanisms for intersemiosis

Multisemiotic texts necessarily have discernable units of each semiotic resource (eg. graphs, tables, diagrams, stretches of linguistic text and symbolic equations). O'Halloran (2005: 169) refers these units as Items. Transitions between Items of different semiotic systems occur at two levels: Macrotransitions occur at the rank of discourse, where Items of primarily one semiotic system give way to Items of another. Microtransitions occur at the rank of grammar where elements of one semiotic system are contained in another (for example the symbolism used in Image 2.4).

O'Halloran (2005: 169) describes six mechanisms within texts which work toward intersemiosis, involving both micro and macrotransitions:

- *Semiotic Cohesion*: System choices function to make the text coherent across different semiotic resources

- *Semiotic Mixing*: Items consist of system choices from different semiotic resources and involve microtransitions (such as symbols within images as in Image 2.4).

- *Semiotic Adoption*: Similar to semiotic mixing except that a resource from one semiotic system has transcended all systems and has become a choice in another semiotic system in its own right.

- *Juxtaposition*: Items and components within Items are compositionally arranged to facilitate intersemiosis

- *Semiotic Transition*: Macrotransitions indicating discursive moves.

Although each mechanism is not necessarily used within every text (and further research is needed to make this list exhaustive), when used, they ensure that each Item is working with other Items to produce meanings greater than the sum of the parts.

2.5 Knowledge structure

Throughout this chapter, the tools that have been used to analyse how the texts produce meaning have been introduced. We now turn our attention to the nature of the field of knowledge of physics itself and how the discourse represents this. Bernstein (1999) distinguishes between two different types of discourse that convey differing types of knowledge, named horizontal and vertical discourse. This section will explain this distinction, as well as the concept of 'knowledge structure' and the factors that build these knowledge structures. Following this, an overview of some of the tools used to analyse texts in terms of their knowledge structure, developed by Karl Maton's Legitimation Code Theory (Maton 2008, forthcoming) and SFL, will be presented. This will be concluded with an overview of the very small amount of work applying to physics and other sciences that has occurred within this framework.

2.5.1 Horizontal and vertical discourse

The most common form of discourse in everyday society is that of horizontal discourse. This is usually typified as common-sense knowledge and is potentially accessible by all. In Bernstein's words, horizontal discourse:

> 'is likely to be oral, local, context dependent and specific, tacit and multi-layered, and contradictory across but not within contexts…. The crucial feature is that it is segmentally organized.' (Bernstein 1999: 159)

By segmentally organised, Bernstein is referring to how different types of common-sense knowledge apply to varying activities, and are usually not linked in any way, often being contradictory. As an example, the knowledge of how to cook a pie has little, if anything, to do with how to cross the street.

Vertical discourse, on the other hand:

> 'takes the form of a coherent, explicit, and systematically principled structure' (Bernstein 1999: 159)

This type of discourse concerns that which is taught formally within schooling and will be the type of discourse studied in this thesis. Within vertical discourse different fields contain different knowledge structures.

2.5.1.1 Hierarchical and horizontal Knowledge Structures

Vertical discourses are explicitly taught within an education system, however the varying fields of study: from physics to history, medicine to visual art, mathematics to literary criticism, have vastly different ways of building the knowledge needed to fully access the field. The internal relationship of the various aspects of knowledge within a field is known as the field's knowledge structure. Bernstein distinguishes between two types of knowledge structures within vertical discourse; hierarchical and horizontal.

Bernstein describes hierarchical knowledge structures as attempting to create:

> 'very general propositions and theories, which integrate knowledge at lower levels, and in this way shows underlying uniformities across an expanding range of apparently different phenomena. Hierarchical knowledge structures appear, by their users, to be

> motivated towards greater and greater integrating propositions, operating at more and more abstract levels.' Bernstein (1999: 162)

This type of knowledge structure is said to describe the sciences, which work to produce a coherent description of their specific field. This coherent description involves subsumption and integration of knowledge; that is, any higher order or more general descriptions necessarily subsumes lower knowledge and the lower orders are consistent with each other. Hierarchical knowledge structures aim to provide general propositions that describe a wide range of phenomena. Bernstein (1999: 162) uses the image of a triangle to represent hierarchical knowledge structures, where the pointy top represents the very general propositions and theories that subsume the wider range of more specific theories lower down.

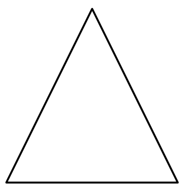

Image 2.5
Hierarchical knowledge structure

Horizontal knowledge structures, on the other hand, do not necessarily always work toward a coherent picture of the world across the entire field.

> 'Horizontal knowledge structures consist of a series of specialized languages with specialized modes of interrogation and criteria for the construction and circulation of texts.' Bernstein (1999: 162)

The specialised language within horizontal knowledge structures are often completely at odds with each, and do not work to produce general propositions that integrate each language. This type of knowledge structure describes the humanities, where, to use Bernstein's example, sociology includes functionalism, post-structuralism, post-modernism, Marxism etc and within each broad category or language, there are specific theories (Bernstein 1999: 162). Horizontal knowledge structures build their own field via an accumulation of languages, as opposed to integration in hierarchical knowledge structures.

To this point, physics has been considered to be the canonical hierarchical knowledge structure (Bernstein 1999, Martin 2006, Maton 2008), however there has been very little research done into whether this is the case, and if this is true, how this knowledge structure is built (See Lindstrøm 2010, forthcoming, for the only non-linguistic research into this). The following sections will introduce the concepts of verticality and grammaticality, semantic density and semantic gravity, which will be used to describe the knowledge structure of physics.

2.5.1.2 Verticality

The classification of knowledge structures has not, so far, provided tools with which to analyse different fields. As such, Muller (2007, 2006) has introduced two variables, verticality and grammaticality, used to describe different fields' knowledge structures.

Verticality concerns how theory develops through integration and subsumption, through ever more integrative and general propositions (Muller 2007). The higher the level of subsumption and integration, the higher the verticality. High verticality suggests a hierarchical knowledge structure. The two factors indicating verticality are firstly, the ability of the discipline to subsume knowledge into increasingly general propositions, and secondly, the degree of integration between different aspects of the discipline (Muller 2007).

Muller (2007: 78) describes the level of subsumption between different disciplines as their:

> ' - capacity to subsume statements into logical types (syntactic/semantic axis);
>
> - therefore their relative expressibility in terms of general and particular statements (general/particular axis);

- and therefore their relative expressibility in terms of propositional content and stylistic content (content/form axis).'

In essence what he means by this is that a discipline with high subsumption has the ability to order knowledge into taxonomies and that the various levels of delicacy within these taxonomies can be expressed in terms of their generality or specificity - that is, the range of situations they describe.

A field's ability to subsume is related to its ability to integrate, however integration is considerably less well defined. For this thesis, the difference between subsumption and integration will be as follows: Where subsumption involves providing ever more general propositions that incorporate all below them, integration involves linking aspects of the field not necessarily in subsumptive relationships so that they are consistent and do not contradict each other. To again use the triangle metaphor, subsumption and integration can be represented as:

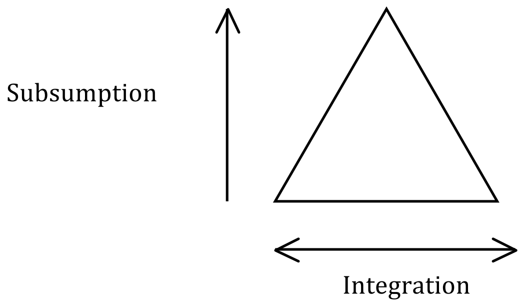

Image 2.6
Subsumption and integration

The sciences are considered to be hierarchical as they have a high verticality – that is, high levels of subsumption and integration. Horizontal knowledge structures, on the other hand, have a low verticality as the field is not subsumptive nor integrative. Horizontal knowledge structures do not look to integrate all languages or schools of thought within the discipline.

2.5.1.3 Grammaticality

The second variable, Grammaticality, has to do with how the theory deals with the world around it.

> 'the stronger the… grammaticality of a language, the more stably it is able to generate empirical correlates and the more unambiguous because more restricted the field of referents; the weaker it is, the weaker its capacity to stably identify empirical correlates and the more ambiguous because much broader is the field of referents.' (Muller 2007:71)

Thus, grammaticality describes a discipline's capacity to unambiguously describe the outside world. Physics is said to have very strong grammaticality (Muller 2007, Maton 2008). To illustrate this, at a very basic level, the concept of velocity has very definite referents of displacement per unit time, where displacement is the vector quantity of distance. Moreover, this concept can be applied precisely to the external world. Within music criticism, on the other hand, an artist may be labeled as being 'raw' without raw having any definite meaning (ie. It could mean good/bad, unproduced, gravelly vocals with an Australian accent, unspoiled by stardom). Its meaning is greatly dependent on its context. Thus, music criticism has weak grammaticality.

In summary, physics is said to have both high verticality and grammaticality. That is, it has an integrated and subsumptive field, and an ability to describe unambiguously the outside universe. The following sections describe factors within the discourse that allow an analysis of both verticality and grammaticality, and thus, the knowledge structure of a field.

2.5.1.4 Semantic density and semantic gravity

Karl Maton (2008, forthcoming), as part of his Legitimation Code Theory, introduces two concepts for analysing how the discourse encodes grammaticality and verticality. These are semantic density and semantic gravity. As Maton explains:

> '*Semantic density* (SD) refers to the degree to which meaning is condensed within symbols (a term, concept, phrase, expression, gesture, etc.). Where semantic density is stronger (SD+), the symbol has more meaning condensed with it; where semantic density is weaker (SD-), the symbol condenses less meaning.' (Maton 2008: 8)

Semantic gravity is the degree of abstraction from concrete particulars within specific contexts. Or, as Maton puts it, semantic gravity (SG):

> 'refers to the degree to which meaning is dependent on its context. Semantic gravity may be relatively stronger (+) or weaker (-). Where semantic gravity is stronger (SG+), meaning is more closely related to its context; when weaker, meaning is less dependent on its context.' (Maton 2008: 7)

Maton (2008: 20-21) argues that high verticality relates to the ability for weaker semantic gravity and stronger semantic density, as the subject matter becomes more abstracted whilst also condensing more meaning into individual terms. The basis for strong grammaticality, however, is the ability for stronger semantic gravity and weaker semantic density, with more specific propositions being more closely related to the concrete world.

The next section will briefly outline the developments SFL has produced that inform the framework of knowledge structures. This primarily revolves around the use of grammatical metaphor and technicality.

2.5.2 Technicality, grammatical metaphor and knowledge structure

Martin (2006) offers an explanation of certain linguistic resources that construe various knowledge structures. Firstly, he argues that grammatical metaphor is the key linguistic resource used to construe vertical discourse (Martin 2006: 18). This allows information to be repackaged into nominalised form, allowing abstract descriptions and relations to be built.

Secondly, Martin (2006:20-21) argues high verticality has a strong correlation with technicality. Grammatical metaphor is distilled into technicality, allowing for taxonomies of greater abstraction to be presented within a text, providing a lightening of the discourse and allowing for greater condensations of meaning. The abstraction developed by the use of technicality allows knowledge to be easily subsumed, making it easier for the discourse to develop ever more general and integrative theories.

2.6 Conclusion

This chapter has explained the key theoretical frameworks that are used in this thesis. Analytical tools from Systemic Functional Linguistics across a range of strata will be used to analyse the written language component of the texts, as well as their genres. Kay O'Halloran's Systemic Functional Multimodal Discourse Analysis will be primarily used to analyse how meaning is made within and across other semiotic systems such as mathematical symbolism and images. Finally, sociology of knowledge (Bernstein 1999) will be used to understand how these meanings are built into knowledge structures.

This theoretical framework, although diverse, allows for a powerful analysis of how visual texts within physics convey their field to students. A study such as this is important to eventually understand why some students simply give up on science early in their schooling career.

The following chapter will analyse the genre and written language of each text, showing how the field is built for students across multiple strata. Further it will use this analysis to assess the verticality within physics.

Chapter 3 - Building knowledge in undergraduate physics through genre and written language

The following two chapters are focused on how the first and third year texts work to convey physics knowledge and build the field for students. In particular, these two chapters will study the role that each semiotic resource - written language, mathematical symbolism and images – plays. The two texts demonstrate similarities in how these meaning making resources are used, but also show noticeable differences, producing quite dissimilar texts. As an overarching framework, the linguistic analysis presented will inform a discussion of verticality, grammaticality and knowledge structure of physics.

This chapter focuses on how ideational meaning is made through genre and written language and the implications this has for physics' knowledge structure. At the discourse semantic stratum the field of physics will be explained through an analysis of its taxonomic relations. Following this, a genre analysis provides an overall understanding of how the texts present the field through the various genres and stages that make up their macrogenres. At the clausal stratum, nominal group analysis shows the high density of each text through their use of technicality. The technicality and subsequent abstraction allows for a discussion of subsumption leading to verticality apparent within the texts. Finally, the results of a TRANSITIVITY analysis will provide an understanding of the process and participant choices each text utilises to construe the physical world.

A multistratal approach to analysis such as this is vital to understanding how knowledge of physics is conveyed, as meaning is made throughout all strata, as well as across all three semiotic resources. Mathematical symbolism and images will be explained in the next chapter. From simply looking at the written language within both texts however, it becomes apparent that each text caters to different audiences. The first year text allows for a gradual easing into the field through careful induction. The third year text, on the other hand, puts forward its experiential meaning in precise, but inaccessible, terms reflecting an understanding that its student audience possesses considerable experience in negotiating the language of the field.

To begin the analysis a map of the field is presented via its taxonomic relations, providing a general understanding of the knowledge the texts are conveying.

3.1 The fields of the texts

The fields of the texts are mapped through the development of a taxonomy of all participants, using Martin (1992) distinction between classification and composition. The taxonomies of both texts can be seen in Appendix 3. Entities were divided loosely into 'concrete' and 'abstract'. Concrete entities are those that exist within the physical world (such as atoms), including their properties (eg. charge). Abstract entities are those that do not; that is, they exist at a level of abstraction far removed from that which is easily grasped in the physical realm (such as partial differential equations). The ensuing discussion of the field begins with the third year text.

The most striking observation to come out of the third year taxonomy (Appendix 3.2) is the number of entities under the abstract classification compared to those within the concrete. Distinct abstract entities occur at a rate of around five to every one concrete entity. These participants are primarily mathematical, dealing with aspects or types of equations. Examples of this are various types of *function*, the classing of different equations as *partial differential equations* or *ordinary differential equations* and the system of *spherical coordinates*.

Thus, the third year text is concerned primarily with discussing the mathematics, rather than the physical systems involved with the field. This is not surprising for a section of text entitled *Schrödinger's equation for a hydrogen-like atom* which includes the mathematical entity, *equation* as the Thing.

The prevalence of abstract participants means a high level of technical knowledge is needed. In order to convey this knowledge efficiently, the text must necessarily use a high degree of technicality. This is because these concepts are not used within common sense, everyday language. This immediately makes the text impenetrable for those who are not inducted into the field of physics and mathematics to the degree necessary for engagement.

This high degree of technicality is extended by those few entities that are classed as concrete, as even these are technical and not visible with the naked eye. Participants such as a *hydrogen-like atom* and properties such as *nuclear charge of +Ze* are not seen in every day situations, and as such must be grasped via definition, placing them outside the realm of common-sense language.

In a similar way to the third year text, the first year text also includes a very high level of abstract mathematical participants. In contrast however, it includes a much higher ratio of concrete participants. This is significant, allowing for a more easily accessible text. From the taxonomy of the 1st year text (Appendix 3.1), it can be seen that the set of concrete participants and their properties include many concepts that are outside what is described by Schrödinger's equation and quantum mechanics, such as *pressure* and *electric field*. These entities, which are presumed to be known by the student, are used as examples helping to introduce the new concepts of wave function and eventually Schrödinger's equation. This observation leads to the next discussion of the macrogenre and staging within the texts. This analysis shows that each text employs distinct methods to achieve their pedagogic goals.

3.2 Macrogenre and staging

Both the first and third year texts consist of various genres providing different pieces of information. Furthermore, each genre consists of multiple stages dividing the information even more. The conglomeration of the various genres and stages into a text that provides a coherent meaning forms the text's macrogenre. This section will discuss the results of a genre analysis of each text, beginning with the first year text. This allows for an overview of how the text conveys its field to the student. From this analysis, initial evidence for verticality - both subsumption and integration - has been found, and will be presented in section 3.2.4.

3.2.1 Macrogenre and staging of the first year text

The first year text is a macrogenre consisting of a series of staged genres, each conveying different meanings (See Appendix 3.1 for full genre analysis). The genres relate to each other, producing a macrogeneric structure that conveys the knowledge of the field. Being a first year undergrad resource, the text works to introduce the main concepts of *wave function* and *Schrödinger's equation* gradually, referring back to previous knowledge from other fields of physics.

The opening genre of the text is a variation on a Descriptive Report (Metropolitan East Disadvantaged Schools Program 1989). It begins with a stage functioning as the Identification providing a brief bridge to the previous section that also functions as the macroTheme for the entire text:

> '**39.5 Wave Functions and the Schrödinger Equation**
>
> We have now seen persuasive evidence that on an atomic or subatomic scale, a particle such as an electron cannot be described simply as a point. Instead we use a *wave function* to describe the state of a particle. Let's describe more specifically the kinematic language we must use to replace the classical scheme of describing a particle by its coordinates and velocity components.' (Young and Freedman pg 1503)

This Identification indicates that the social purpose of this genre is to describe language that is used within the text. The Description stage is unusual, however. Before the description of the language occurs, it is likened to the language used in other sub-fields of physics. The hypertheme for the descriptive stage is thus:

> 'Our new scheme for describing the state of a particle has a lot in common with the language of classical wave motion' (ibid)

The following two paragraphs explain three different aspects of wave physics (transverse waves on a string, sound waves and electromagnetic waves) in reference to their respective wave functions (named as $y(x, t)$ for transverse waves and $p(x, t)$ for sound waves). It is during this phase that we see many of the more concrete participants and their properties that have little to do with quantum mechanics introduced.

A comparison such as this suggests that the genre may be a Comparative Report (Metropolitan East Disadvantaged Schools Program 1996: 42), however, this is not the case. Within this phase, various descriptions of the wave function occur; they just happen to occur within the context of other, known, physical systems. This shows that the overall purpose of the genre is not to compare the language to other systems, but rather to describe the key elements of the language (ie the wave function). This is confirmed by the final phase within the genre that includes:

> 'Thus it is natural to use a wave function as the central element of our new language. The symbol customarily used for this wave function is Ψ or ψ. In general, Ψ is a function of all space coordinates and time, whereas ψ is a function of the space coordinates only – *not* of time.' (ibid.)

The phase similar to a comparison, albeit not a feature of a canonical Descriptive Report, is simply a tool used to ease the student into the description of the wave function. Within the comparison, the wave function is described in relation to various known systems within physics. By opening the text with this genre, this fundamental concept to quantum mechanics is described in a manner that is relatively easily grasped by the student.

Two Descriptive Reports and a Causal Explanation follow, focusing on introducing new terminology and explaining via mathematical symbolism. The second of these two stages, sub-headed *The Schrödinger Equation* introduces a particular type of the Schrödinger equation in its mathematical form.

The fourth genre is a very important one. It contains the vast majority of the mathematics for this text, employing a mathematical example to demonstrate that the Schrödinger equation is in fact true. It is an example of a genre not previously described. The term Derivation will be coined to represent this genre.

The Derivation provides a series of implication sequences in which the written language is primarily concerned with manipulating symbolic mathematics. The genre concludes with a macroNew in the form of a mathematical result that is interpreted by written language. Without going into detail about the structure of the Derivation, what this genre within the first year text shows to a student is that mathematics can be used for reasoning and testing the real world. An equation is presented, and then shown to be true using a mathematical description of a real world scenario. Despite the genre focusing on the mathematical symbolism, however, considerable prompting from the written language is required as the students are not yet fully inducted into field of physics and as such cannot independently grasp technical meanings purely from the symbolism. A more detailed description of the genre of Derivation, including its generic staging structure, will be given in the next section in relation to the third year text.

Three more Descriptive Reports are presented to a student before the text is concluded with another Causal Explanation. The genre given provides an explanation of the link between two mathematical functions given within an image. It opens with an Identification stage of:

> 'There is an important relation between the two functions $\psi(x)$ and $A(k)$.' (Young and Freedman: 1509)

The following Implication Sequence uses the relationship between the two functions to eventually show that two fundamental - but to this point segmented - principles within quantum mechanics are consistent with one another:

> 'What we are seeing is the uncertainty principle in action. A narrow range of k means a narrow range of $p_x = \hbar k$ and thus a small Δp_x; the result is a relatively large Δx. A broad range of k corresponds to a large Δp_x and the resulting Δx is smaller. Thus we see that the uncertainty principle $\Delta x \Delta p_x \geq \hbar$ is an inevitable consequence of the de Broglie relation and the properties of integrals such as Eq. (39.25) [not shown].' (ibid.)

The Explanation provides an integrative link between two fundamental principles of physics. Moreover it utilises written language, mathematical symbolism and images (not shown). As such, it not only explicitly teaches the field, but also implicitly conveys that the form of the field of physics is multisemiotic.

This analysis has shown that the first year text is indeed a macrogenre consisting of multiple staged genres. Each genre conveys separate but interrelated pieces of knowledge in order to produce a coherent text that eases the student into the fundamentals of quantum mechanics. Further, the analysis has shown that the text is multisemiotic utilising the meaning making resources of written language, mathematical symbolism and images. The third year text is also multisemiotic, and consists of the same set of genres as the first year.

3.2.2 Macrogenre and staging of the third year text

The third year text again is a macrogenre consisting of a series of smaller, though not less distinct, genres (See Appendix 3.2). This text presents its knowledge, however, in a considerably different fashion to the first year text. The most noticeable difference is the dramatic decrease in the ratio of written language words to mathematical statements. Both texts include a similar number of statements however the third year text contains about a sixth of the number of words. This immediately suggests that much of the introductory and peripheral information given in the first year text is lacking in the third year.

This is best exemplified by the opening clause complex, which functions as the macroTheme for the entire text:

> 'Using spherical coordinates, Schrödinger's equation for a hydrogen-like atom with a nuclear charge of $+Ze$ is
>
> $$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi$$
>
> where $\psi = \psi(r, \theta, \phi)$, and
>
> $$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$
>
> $$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

Unlike the first year text, there is no introduction nor explanation of any of the technical terms including many of the mathematical symbols, nor is there any mention of the equation's significance or relevance to other fields of physics. It is entirely presumed that the student will be able to understand this equation, and why it is being studied.

The analysis of the field via its taxonomic relations showed that the text is primarily concerned with abstract mathematical participants. An analysis of the genres within the text confirms this. The first two genres, arguably the most important, are dedicated firstly to solving the equation given in the macroTheme, and secondly to discussing the resulting solutions.

The text opens with another example of a Derivation. This is a very common genre within physics, which has not yet been described by genre theory. In the case of the third year text the genre opens with an Identification stage (given as the macroTheme above) that presents the equation. This is followed by a Goal, which is to solve Schrödinger's equation identified at the beginning of the text. This goal is given implicitly within the hypertheme:

> 'It is solved by a technique known as *separation of variables*' (School of Physics, University of Sydney pg 9)

This is followed by a sequence of steps used to achieve this goal. This structure resembles a Procedure (Metropolitan East Disadvantaged Schools Program 1989: 12). There are, however, two key differences between a Derivation and a Procedure.

Firstly, unlike a Procedure, the steps are implication sequences that include declarative, not imperative clauses (see image 2.2 - the system network for MOOD). That is, the text is not telling the student how to solve the equation, it is actually solving it. This is in contrast to a Procedure that simply states the process that needs to be taken to achieve the Goal.

Secondly, the genre concludes with a stage that presents a new result not necessarily previously known. A Procedure allows for the Goal to be achieved external to the text. Moreover, the end result of the Procedure (ideally) is specifically what the Goal states. For example, if the goal was to make a pie, the end result is that a pie is made outside the text.

A Derivation, on the other hand, presents the result of the steps within the text. Furthermore, the end result is something that was not necessarily known at the outset of the Derivation. By undertaking the Derivation, something new is found. Within the third year text this stage, known as the Result, is:

> 'The resulting equations are:
>
> $$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2\mu}{\hbar^2}\left[E-V(r)\right]R=l(l+1)\frac{R}{r^2}$$
>
> $$-\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\frac{m_l^{\;2}}{\sin^2\theta}\Theta=l(l+1)\Theta$$
>
> $$\frac{d^2\Phi}{d\phi}=-m_l^{\;2}\Phi$$
>
> Thus the partial differential equation with three independent variables [the Schrödinger equation given above] has been converted to three ordinary differential equations – one for each independent variable. In the process two constants, m_l and l, have been introduced.'(School of Physics, University of Sydney pg 10)

As the text illustrates, the series of mathematical statements, including the introduced constants, were not known at the outset of the Derivation. The Derivation has achieved the Goal of solving the equation but in doing so has produced new knowledge.

Both the third and first year texts show that Derivations are primarily constituted by mathematical symbolism, with written language being used to explicitly state the steps needed to manipulate and solve the equations. In other contexts (such as in student exam solutions) it is not necessary to involve written language in the steps of a Derivation. Rather, the meaning making is entirely dependent on mathematical symbolism and the sequence taken from the textual layout.

The social purpose of a Derivation is to find new mathematical results and to show how these results can be found from previously known mathematical equations. Derivations of this form are very common within physics and are regularly used as assessable tasks in an educational setting.

In summary, the stages of a Derivation are as follows: Identification ^ Goal ^ Steps (1-n) ^ Results. The Identification stage names and introduces the equation used in the Derivation. The Goal outlines the purpose of the Derivation or the process to be undertaken. The Steps manipulate mathematical symbolism until the Result is finally given. Table 3.1 provides the staged framework for the derivation genre, as applied to this text.

Derivation	
Social Purpose: To find new mathematical results and to show how they can be achieved from known results.	
Stage	**Text**
Identification	'Using spherical coordinates, Schrödinger's equation for a hydrogen-like atom with a nuclear charge of +*Ze* is $-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi$ where $\psi = \psi(r, \theta, \phi)$, and $V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r}$ $-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi$
Goal	It is solved by the technique known as separation of variables.
Steps, 1-n	- $\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$ is substituted into Equation 17 - the resulting equation is then divided through by $R(r)\Theta(\theta)\Phi(\phi)$ - The result can be rearranged to give $\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = \text{a function of } r \text{ and } \theta$ - This equation will be true for all r, θ, ϕ only if both sides are equal to a constant, written (by

	convention as $-m_l^2$.) - Equation 25 follows from the left hand side. The right hand side can be rearranged into the form: a function of *r* = a function of *θ* -which will be true for all r and *θ* only if both functions are equal to a constant, written (by convention) as $l(l+1)$.
Result	'The resulting equations are: $V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r}$ $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$ $\frac{d^2\Phi}{d\phi} = -m_l^2\Phi$ Thus the partial differential equation with three independent variables has been converted to three ordinary differential equations – one for each independent variable. In the process two constants, m_l and *l*, have been introduced.

Table 3.1

Derivation genre

After the Derivation, the rest of the text contains a series of small Descriptive Reports and Causal Explanations, each focusing on a different mathematical feature of the Schrödinger equation or the Results of the Derivation. These stages are brief however they provide the student with various mathematical tools used to analyse the physical world that they have not been taught previously.

One example of this series of genres is the final genre within the text, sub-headed *2.4 Radial probability density*. The sub-heading is the Identification stage, leaving the rest of the text to provide the Description. This is as follows:

> The probability that an electron is in the radial range $r \to r + dr$, *P*(r), is given by
>
> $$P(r)dr = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \ \psi^* \psi r^2 dr \tag{40}$$
>
> *P*(r) is depends on the values of *n* and *l*, but not m_l, and is shown in Figure 3 for all *l* values corresponding to *n* = 1, 2, 3. Obviously
>
> $$\int_0^{\infty} P_{nl} dr = 1 \tag{41}$$

This genre provides both mathematical and non-mathematical descriptions of various features of the radial probability density of a system described by the Schrödinger equation. This is typical of most of the text after the Derivation, providing descriptions of terminology and tools used to describe physical systems.

3.2.3 Summing up macrogenre

The genre analysis shows that both the first and third year texts are multisemiotic macrogenres incorporating a conglomeration of different genres. The first and third year texts use the same three genres to present knowledge to their student – Descriptive Report, Causal Explanation and Derivation. The difference between the two, however, is what these genres are presenting. The third year text opens with a mathematical equation with no explanation. The rest of the text is then geared toward describing and explaining features of this equation. The first year text, on the other hand, uses its stages to gradually introduce terminology, eventually leading to the mathematical equation. It is only after the terminology is introduced that features and implications of the equation are presented to the student. The differences between the two texts reflect their readership. Students learning from the first year text are less likely to have been inducted into the field, needing considerably more grounding in the terminology. Furthermore, they are not likely to be as experienced in dealing with mathematics as their third year counterparts. The third year texts' readership, on the other hand, has a considerable amount more experience in the field. This text, therefore, does not need to introduce the underlying concepts to the same extent.

The following section will use this analysis to inform a preliminary discussion of the verticality of physics apparent within the texts.

3.2.4 Verticality shown by genre analysis

Analysis of the macrogenres of each text assists an initial understanding of the subsumption and integration of physics. Subsumption is indicated by the repetition of the same general topic area across multiple years. Integration, on the other hand, is shown by the first years' repeated attempts to link principles and terminology to other sub-fields within physics.

3.2.4.1 Subsumption between texts

The repetition of the same topic at greater complexity across multiple years is evidence for subsumption. Lindstrøm (Draft: 16) explains that within physics, topics are returned to year after year, where each year the curriculum deepens knowledge previously taught. By repeating topics across years other topics can also be taught in between, building both the knowledge and the skill base needed to understand the more complex aspects of each sub-field. Lindstrøm (ibid.) explains that within quantum mechanics, the most fundamental concepts (eg. wave function) are far removed from everyday reality (they are abstract - see section 2.3.2), as they operate on length scales so small that we cannot possibly visualise them. This means that the base of the quantum mechanics hierarchy must be formally taught before the generalised statements can be understood.

Muller (2007: 81) argues that the repetition of topics is caused by the fact that the reconstituted logic of a discipline ordered into levels of complexity does not necessarily provide an optimal learning path. This limits the possibility of a linear representation of content within a curriculum where a sub-field within a discipline can be introduced and taught in its entirety in isolation from all other sub-fields, and from therein presumed. The reason for this, he argues, is that within the disciplines, *imperfect subsumption* applies. That is:

> 'the same semantic topics (the same particulars) play different roles in different generals' (Muller 2007: 81)

He points out that this imperfect subsumption is reflected in the curriculum by the repetition of the same topic across multiple years (ibid.). This is precisely what occurs in the two texts in this study. The same topic (Schrödinger's equation) is repeated in first and third year in varying forms. The equations given, however, are not in a perfectly subsumptive relationship. The third year equation is not simply a generalised form of the first year. They both describe particular systems at varying levels of complexity. By teaching the equations in different years, different particulars are being learnt. The more complex particulars, however, still subsume a large amount of the knowledge introduced in the less complex.

The genre analysis of both texts has shown that the first year text introduces and describes fundamental concepts within quantum mechanics, such as the wave function, primarily using Descriptive Reports. The third year text, on the other hand, assumes this knowledge. It uses its Descriptive Reports, Derivation and Explanation to teach more complex aspects of the topic that subsumes the wave function. Thus, the first evidence from the texts of subsumption, albeit imperfect, comes from the repetition of the topic.

The genre analysis of the first year text provides evidence for the second aspect of verticality, integration.

3.2.4.2 Integration within the first year text

The first year text provides two distinct pieces of evidence for the integration of various principles and sub-fields within physics. The first occurs within the opening Descriptive Report where the wave function is described in relation to other known sub-fields. The second is the final Casual Explanation that integrates two fundamental principles of quantum mechanics. As has been mentioned, the definition of integration is not yet fully formed, nor have sufficient data driven linguistic factors been found to comprehensively determine the level of integration of a discipline. The evidence put forward for integration throughout this thesis, therefore, must be read in this equivocal light.

The initial Descriptive Report likens the terminology of quantum mechanics to that of other fields of physics. This provides the student with the knowledge that the tools used within quantum mechanics are not so different (at least, at this early stage) from those they already know. Although the terminology has little, if anything, to do with the theories cohering and being integrated in the knowledge structure, linking the language of the theories allows the student to understand that the descriptive tools cohere, and that there are consistent approaches to many systems within physics. This integrates the student's idea of the knowledge structure. Once integrated, the student can draw parallels between the systems and synthesise conclusions.

The final Explanation, on the other hand, shows that two principles crucial to quantum mechanics (the Heisenberg uncertainty principle, and de Broglie's relations) are consistent within the discipline itself. This can be seen from an excerpt from the Explanation:

> 'What we are seeing is the uncertainty principle in action. A narrow range of k means a narrow range of $p_x = \hbar k$ and thus a small Δp_x; the result is a relatively large Δx. A broad range of k corresponds to a large Δp_x, and the resulting Δx is smaller. Thus we see that the uncertainty principle $\Delta x \Delta p_x \gtrsim \hbar$ is an inevitable consequence of the de Broglie relation and the properties of integrals such as Eq. (39.25)[Within the text, but not shown in this thesis].

Before this explanation, the de Broglie relations and the Hiesenberg uncertainty principle were segmented, without an apparent link and discussed in isolation from each other. The text has provided an explicit integration of the two concepts, welding the respective segments of knowledge together thereby building a more cohesive knowledge structure.

Analysis of the macrogenres has shown that the texts utilise various genres to convey their meanings to students. The meanings that are being made by the two texts, however, vary. The first year text provides more fundamental knowledge lower down the hierarchy, introducing it gradually. The third year text, on the other hand, assumes a greater level of knowledge presenting much of its information without the same degree of introduction. This thesis will now turn to the meanings made by the semiotic resource of written language, at the lexicogrammatical stratum.

3.3 Written language in the physics texts

Despite the use of mathematical symbolism and images, both texts rely heavily on written language as the main method of explanation. There are two reasons for this. Firstly, students have considerably more experience reading written language and grasping new information from causal explanations than they do from mathematical symbolism and images, and as such need written language to steer the discourse. There is a decrease in the reliance on written language from first to third year, which signals the students' increasing competency in grasping meanings made from mathematical symbolism and images. The second reason for the use of written language is because mathematical symbolism and images must necessarily be encoded with meaning. If the meanings of the symbols and images cannot be presumed from a student's previous experience (as is the case especially in the first year text), then written language is the primary resource to encode it with meaning. This point, however, will be discussed in more detail in the following chapter.

The remainder of this chapter will study how written language, at the lexicogrammatical stratum works to build the field within each text. Section 3.3.1 provides uses a nominal group analysis to describe the technicality, abstraction and the resulting density within each text. A discussion will follow of the subsumption due to written language. Section 3.3.2 will provide the results of a TRANSITIVITY analysis of each text. This analysis will assist an understanding of how the processes, participants and circumstances realise each text's experiential meaning.

3.3.1 Density, technicality and abstraction

Halliday (1993a, 1989) illustrates that scientific discourse can be very dense. That is, many words with considerably abstract meanings are often packed into very large clauses. This section will show the lexical density (the number of lexical words within a clause) that occur within the texts. Further, the semantic density within technical terms used in each text will be compared, linking this result to the level of subsumption within the texts. These two factors show the high level meaning that is packed into the discourse within science, making it virtually inaccessible to the uninducted reader.

3.3.1.1 Lexical density

Lexical density is a measure of the density of information within a text, according to how tightly the lexical items (content words) have been packed into the grammatical structure. It can be measured as the number of lexical words per clause (Halliday 1989: 168). Nominal groups contain most of this density, becoming very large and complex. The analysis of nominal groups within each text shows that the density within science described by Halliday does indeed occur in both texts. The clauses introducing Schrödinger's equation in each text will form the comparison.

The first year text introduces its Schrödinger equation with:

The simplest form of the Schrödinger equation	is	for [[a particle of mass m that moves in one dimension only, so that the spatial wave function is a function only of x]]
nominal group	process	prepositional phrase

Table 3.2

First year introduction of Schrödinger equation

This clause contains sixteen lexical items. This is very large, allowing a huge amount of information to be packed into one clause (Halliday (1989: 76) notes that in informal spoken language, the lexical density is usually two words). Table 3.2 and its nominal group analysis (Appendix 5.1 pg:180-181) shows that the vast majority of the lexical words are packed into the second nominal group embedded within the final prepositional phrase. Just as Halliday argued, the nominal groups contain most of the lexical density. Although this is an extreme case, it is not uncommon for the first year text to have a density of over ten lexical words.

This density also occurs within the third year text. The introductory clause for Schrödinger's equation is:

Schrödinger's equation for a hydrogen-like atom with a nuclear charge of +Ze	is	$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi$
nominal group	process	//statement//

Table 3.3

Third year introduction of the Schrödinger equation

Disregarding the mathematical statement (but including the mathematical components within the initial nominal group), the clause contains ten lexical items. These are entirely packed within one very dense nominal group. Further, both the first and third year examples show that mathematical symbolism in the form of components or statements can be included in a written language clause. This is known as semiotic mixing (O'Halloran 2005: 169). The inclusion of mathematical symbolism can significantly increase the density of a clause.

While a simple quantitative measure of the number of words within a clause is useful, another more qualitative measure of density reveals a dramatic difference between the first and third year text.

3.3.1.2 Semantic density in technicality

A simple measure of the lexical words per clause does not tell us how conceptually dense each individual word is. A more qualitative measure of semantic density (Maton 2008) allows for an understanding of the degree of abstraction and condensation of knowledge in individual words.

A much higher degree of technicality occurs within the third year example above, compared to the first year. This means that it takes considerably more training in physics for a student to understand precisely what *a hydrogen-like atom with a nuclear charge of +Ze* means, than it does to understand any of the terms in the first year example. The consistent use of higher-abstraction technical terms by the third year text can also be seen in the following examples of nominal groups from very similar passages within each text:

First year text:

> 'the probability of finding the particle around that point.'

Third year text:

> 'the probability that an electron is in the radial range $r \to r + dr$, $P(r)$'

These examples show that the third year text uses a higher level of technicality and with it, abstraction, ultimately finishing with more precise terminology. Where the first year text offers a vague location of *around that point*, the third year text provides a precise location of *in the radial range* $r \to r + dr$. In order to achieve this precision, the third year text necessarily requires the use of a number of technical terms (eg. *radial, range*), including those from mathematical symbolism (*r*, *dr*). Each of these terms is far removed from everyday common-sense knowledge, unlike those in the location within the first year's introduction. These technical terms require explicit teaching, making them both more precise, but also, less accessible. The use of higher order technicality increases the text's semantic density.

The two strategies used by the texts again reflect their audience. The first year text is aimed at those who are not already inducted into the field. It makes a trade-off between accessibility and precision. The third year text's audience are more competent readers, meaning precision can be presumed to be understood without explanation.

This analysis shows that both texts have very high lexical density however the third year text uses technical words that are considerably more abstract than the first year. This adds another layer of density in terms of the overall meanings made. The final point relating to density comes about from the incorporation of mathematical symbolic statements into the clausal structure of written language. The use of technicality leading to increased density is a factor in building verticality as discussed in the following section.

3.2.1.3 Written language technicality producing verticality

Martin (2006: 21) argues that the degree of verticality a knowledge structure exhibits correlates strongly with its technicality. The use of technicality allows very large condensations of meaning to be related efficiently. By distilling something to a technical term, the technicality is subsuming knowledge of the original statement. A taxonomy is being set up, where the delicacy is determined by the level of subsumption. To put it into Maton's (2008) terms, by technicalising a statement, the semantic density of the term is strengthening; there is a higher degree of meaning condensation.

Following this understanding of technicality it is no surprise that the third year text displays a much higher level of written language technicality than the first year. As the student moves through the learning levels of physics, technical terms subsume relations of common sense and other technical terms, creating increasing semantic density. By the time they reach third year, the semantic density of technical terms is very high. The lexical density remains the same, meaning texts increasingly produce larger packages of meaning. As we have seen, however, with technicality, abstraction follows. The technicality and abstraction mean that the terms become less dependent on their context. This can be seen in the example given previously, regarding the probability of finding a particle in an electron. The third year text uses technicality to provide a precise location *in the radial range* $r \rightarrow r + dr$. The first year text, on the other hand provides the less specific, more context dependent location *around that point*. Again in Maton's terms, this means that the semantic gravity weakens.

As mentioned in section 2.4.1.4, Maton (2008: 20-21) argues that a discipline with high verticality has the capacity for stronger semantic density and weaker semantic gravity. Based on this view, the increase in technicality between the texts that greatly strengthens semantic density and weakens semantic gravity, provides further evidence of the high verticality within physics.

The discussion on density has focused on the density of meanings that are made primarily within participant nominal groups. The following section will discuss the results of a TRANSITIVITY analysis, shaping an understanding of the types of experiential meanings made through the configuration of processes and participants.

3.3.2 TRANSITIVITY of written language

By studying processes, participants and circumstances, the experiential meaning that is being construed by each text is laid bare. A transitivity analysis reveals both similarities and difference between the first and third year texts. A quantitative survey of the instances of written language process types within each text is given in Table 3.4.

Process Type	**Relational**	**Material**	**Mental**	**Verbal**	**Existential**
First Year	30%	49%	4%	12%	5%
Third Year	50%	35%	3%	8%	3%

Table 3.4

Processes used in texts

This snapshot shows that the two main process types used within the texts are Relation and Material. As such, these will be the focus of the next two sections.

3.3.2.1 Relational processes in the texts

Halliday (1998: 193) and O'Halloran (2005: 75) point out that relational processes are favoured within science, allowing series of descriptive relations to be established between participants. This is true for both texts, with relational processes accounting for a third and half of all processes within the first and third year texts respectively.

Clauses with relational processes often include mathematical symbolism (either an individual component or a full statement) as one of the participants, becoming a method by which the symbolism is encoded with meaning. Relational attributive clauses are used to class certain types of symbolism, to provide boundaries as to what the symbolism can do or to provide a description. For example:

$F_{l\|ml\|}(\cos\theta)$	are	polynomials in $\cos\theta$
Carrier	Process: Relational: Attributive	Attribute

Table 3.5

Relational Attributive process 1

solutions	are	finite	only for integer values of l and values of m_l
Carrier	Process: Relational: Attribute	Attribute	Circumstance

Table 3.6

Relational Attributive process 2

the value Ψ^2	at each point	is	independent of time
Carrier	Circumstance	Process: Relational: Attributive	Attribute

Table 3.7

Relational Attributive process 3

Relational Identifying clauses are used for technical definitions (Wignell, Martin and Eggins 1993: 149) however, in both texts, they are outnumbered by relational attributive clauses. This is despite the high degree of technicality that needs defining. The relational identifying clauses that are used, however, are almost exclusively geared toward defining equations, symbols or graphs. Table 3.8 shows an example of this:

$\|\Psi(x, y, z, t)\|dV$	is	the probability of finding the particle within a volume dV around the point (x, y, z) at time t
Token	Process: Relational: Identifying	Value

Table 3.8

Relational Identifying process

The issue of the lack of relational identifying processes is solved by a study of the mathematical symbolism. Within every mathematical statement in the texts, the equals sign = is used. According to O'Halloran (2005: 107), = is a relational identifying process. The use of =, thus, shifts the technical defining somewhat into the realm of mathematical symbolism. In the third year text, if mathematical symbolism is taken into account within the transitivity analysis (discounting operative processes), then relational identifying processes are the most common processes used. This reaffirms Halliday's assertion that relational processes are the favoured type within science.

3.3.2.2 Material processes

The other process type used most frequently within the texts is the material process. The experiential meanings made by the material processes in each text are similar, however the texts vary in the choice of Voice.

The third year text shows a very clear pattern of material processes consistently used in passive clauses, usually without an agent. Where an agent does occur, the agent is not a human participant. Rather, it is usually an abstract mathematical participant – often symbolism. For example:

The resulting equation	is then divided through by	$R(\mathrm{r})\Theta(\theta)\Phi(\phi)$
Goal	Process: Material	Actor

Table 3.9

Third year passive clause

In the third year text, the primary function of material processes is to manipulate mathematical equations. Table 3.9 shows an example of this. Because of this, the Derivation genre uses a high proportion of material processes to realise the mathematical Steps. These material processes often can be translated into mathematical symbolism as operative processes. For example *is divided by* equates to ÷, / or –. Although it is not seen in either text, this allows Derivation to be written exclusively in mathematical symbolism in other contexts.

The first year text uses material processes in a slightly different way to the third year. Whilst still regularly used for manipulating equations:

we	substitute	$\psi(x)$	into the left-hand side of the Schrödinger equation
Actor	Process: Material	Goal	Circumstance

Table 3.10

First year material process

material processes are not used exclusively in the passive voice. Rather, the information is presented in active voice with a human participant, *we*, functioning as the Actor. *We* is used very regularly throughout the text and provides interpersonal rather than experiential meaning. The use of *we* reflects a more personal approach to the construction of knowledge, by drawing the (first year) reader into a shared knowledge space. The third year text presumes the reader already shares this space, thus no longer needing to provide the extra interpersonal meaning.

3.3.3 Summing up written language

The analysis of written language indicates a shift away from the relative accessibility and inclusiveness of the first year text, to the less accessible efficiency of the third year text. Both texts are very lexically dense and concerned with describing abstract mathematics, however the third year text compounds this by dramatically increasing the semantic density of its technicality. This increased semantic density allows for decreased semantic gravity, producing more precise descriptions. Moreover, the increasing use of technicality between the texts suggests very strongly that subsumption and thus verticality occurs within the discipline.

The transitivity analysis has shown that the discourse of both texts is concerned with describing, defining, classing and manipulating mathematical symbolism. The first year text takes an inclusive approach, using active voice with the human participant, *we*. The third year text, on the other hand, uses agentless passive voice to manipulate mathematical statements. Further, it shifts many of its relational identifying processes into mathematical symbolism.

3.3 Conclusion

This chapter has shown that the texts under study are concerned primarily with very abstract mathematical concepts. The methods used by each text to convey these concepts reflect their readership. The first year text eases its novice audience into the field, using its macrogenre to introduce the fundamental concepts of quantum mechanics in relatively simple language. The third year text, on the other hand, assumes a much higher degree of knowledge, using its macrogenre to present much more subsumptive information. In doing this, it uses very technical language, providing much denser meanings to its comparatively experienced students.

The genre analysis and written language has also provided evidence for the assertion of physics' high verticality. The repetition of topics across years and the increasing level of technicality show subsumption within the discipline. The first year text shows integration by linking the terminology of multiple sub-fields and providing a consistent link between two fundamental principles within quantum mechanics.

Written language, however, is not the only meaning making resource used within physics. The next chapter will focus on the extra-linguistic semiotic resources of mathematical symbolism and images. These resources produce meanings in their own right that complement those made in written language.

Chapter 4 – Building knowledge in undergraduate physics through mathematical symbolism and images

The study of written language has shown that both texts are concerned mainly with abstract mathematical entities. In order to convey the meanings associated with these entities, the text uses very dense written language. This chapter will now examine the intrasemiotic meanings produced by mathematical symbolism and images, as well as the intersemiosis of the three semiotic resources. Section 4.1 will focus on the meanings encoded into mathematical symbolism, leading to an explanation of hypertechnicality. The use of mathematics within physics will also be discussed, concluding with the large role mathematics plays in building the knowledge structure of physics. Section 4.2 describes the use of images within each text and the interpersonal and experiential meanings they construe. Finally, section 4.3 discusses the intersemiotic mechanisms that produce the complementarity between the three resources, multiplying the intrasemiotic meanings of each. A full multisemiotic study is vital to understanding the nature of physics' knowledge structure, and how this knowledge is conveyed to students.

4.1 Mathematical Symbolism

Mathematics is an integral part of physics because it allows the universe to be precisely described and predicted (see Parodi (2010) for a quantitative survey of mathematics in physics). Within SFL and social semiotics, mathematical symbolism has been studied by two theorists, Jay Lemke (1998, 2003) and Kay O'Halloran (2003, 2005, 2007, 2010). Lemke has explained the types of meaning mathematical symbolism makes that is extra to written language and images. O'Halloran has described the grammatical systems in mathematical symbolism that achieves these meanings. Neither, however, has studied how mathematical symbolism is used within other fields, such as physics. The framework developed by O'Halloran is useful due to its ability to lay bare the meaning making resources within mathematics, and as such will be applied to the mathematical symbolism within the physics texts in this study in order to understand how physics uses mathematics.

This chapter will show that mathematical symbolism can contain technical meanings that are used to precisely describe the real world. Section 4.1.1 will provide the methods used in the physics texts of encoding technical meaning into mathematical symbolism. Section 4.1.2 will show that mathematical symbolism can transcend the text, functioning, in a similar way to written language, as technicality. The variation in use of mathematics between the texts will be described in section 4.1.3, and finally, mathematics role in building the knowledge structure of physics will be explained in section 4.1.4. The analysis presented illustrates that mathematics is a fully functioning meaning making resource that is integral to how physics understands the world and conveys its knowledge.

4.1.1 Mathematical symbolism and technical meaning

As demonstrated in chapter three, the fields represented in both texts are very abstract and technical. Mathematics contributes to building this field however symbolism does not have any inherent physical meaning. In order to convey technical information mathematical symbolism must be encoded with meaning by other semiotic resources. The texts show that technical meaning can be encoded into mathematics at two levels: individual components (symbols) and entire statements (equations). There are three main methods of encoding meaning into mathematical symbolism shown within the texts. These methods involve relational processes, apposition or images. Once physical meaning has been encoded into the symbolism, complex relations between very abstract entities can be described precisely. The section will begin by outlining the techniques used to encode meaning into individual components.

4.1.1.1 Relational processes encoding meaning into components

The most common method for encoding meaning into mathematical symbolism is via relational identifying processes (implied in O'Halloran 2005: 178). This is in line with the assertion that identifying processes are used for definition (Wignell, Martin and Eggins 1993: 149). Table 4.1 (third year) and 4.2 (first year) show examples of this.

a_0	is	the Bohr radius
Token	Process: Relational: Identifying	Value

Table 4.1

Relational processes encoding technical meaning in third year text

y	represents	the displacement from equilibrium, at time *t*, of a point on the string at a distance *x* from the origin
Token	Process: Relational: Identifying	Value

Table 4.2

Relational processes encoding technical meaning in first year text

These examples show that both semantically and lexically dense nominal groups function as the Value that is represented by a symbolic component as the Token. The first example from the third year text has a relatively small nominal group as the Value but includes more semantically dense technicality. Conversely, the second example taken from the first year text includes a much larger nominal group with considerably less semantically dense technicality. A nominal group analysis of the first year example (Appendix 5.1 pg: 173) shows that the Thing in the Value – displacement - is qualified by a series of prepositional phrases. This is common with physics texts, as it is vitally important to understand the precise situations under which the component applies. The third year example does not include a qualifier as it has been subsumed by the previously taught technicality, *Bohr radius*. It is presumed that students know that the Bohr radius only applies to the radius of an electron's orbit around a hydrogen nucleus. The use of relational processes, therefore, can transfer precise technical meanings from written language to mathematical symbolism.

4.1.1.2 Apposition encoding meaning into components

Another very common method of encoding individual components with technical meaning is through apposition (Halliday and Matthiessen 2004: 489). Apposition occurs where a nominal group complex consists of two nominal groups that are in a relationship of paratactic elaboration. In the text, to encode symbolism, one nominal group contains written language, and the other a mathematical component. Table 4.3 illustrates this.

The probability that an electron is in the radial range $r \rightarrow r + dr$,	$P(r)$	is given by	$P(r)dr = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \ \psi^* \psi r^2 dr$
Goal nominal group complex		Pro: Material	Actor
1	=2		

Table 4.3

Apposition encoding technical meaning

This example illustrates one of the benefits of using apposition for encoding meaning into symbolism. Via apposition, mathematical components can be introduced using words, then immediately used in a mathematical statement and related. This very quickly increases the semantic density of the symbol.

4.1.1.3 Images encoding meaning into components

Outside written language, it is not uncommon for images to be used to encode technical meaning into components. This can be seen in the following example:

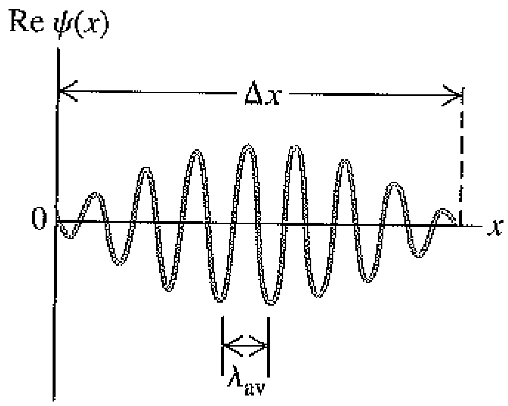

Image 4.1

Images encoding technical meaning

This image, taken from the first year text, is a visual representation of a wave packet. The entire length indicated by arrows surrounding Δx is the wave packet, containing multiple individual waves. The arrows surrounding λ_{av} indicates the length of the individual waves. By indicating the lengths with arrows and placing the symbolic components where they do, the images encode meaning into the components. That is, from therein within the text, Δx is the length of the wave packet, and λ_{av} is the length of waves, and can be used within the discourse. Images produce meaning aside from that which is encoded into symbolism, however this will be discussed in section 4.2. For now, attention will turn to the second rank at which meaning can be encoded into symbolism; statements.

4.1.1.4 Encoding meaning into statements

The previous sections have shown that within the physics texts meaning is often encoded into individual components which function as the equivalent of participants in language. These individual components combine to make statements similarly to the way participants and processes combine to make clauses in written language. Unlike in written language however, in mathematical symbolism not only do components have technical meaning encoded into them, but also statements. The meanings encoded into statements are extra to those made by the configuration of components. This is best shown by the introduction of first statement in the third year text:

Schrödinger's equation	for a hydrogen-like atom with a nuclear charge of +*Ze*	is	$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi$
Value	Circumstance: Matter	Pro: Rel: Identfying	Token

Table 4.4

Encoding technical meaning into statements

The mathematical statement has been rankshifted down to a nominal group functioning as the Token within the written language clause. Using the relational identifying process *is* the statement is equated to the first nominal group, *Schrödinger's equation*, functioning as the Value. The circumstance provides specific boundaries as to the systems in which this mathematical statement applies. Using this construction, the technical meaning associated with the nominal group, *Schrödinger's equation*, as well as the circumstance, is encoded into the full mathematical statement. No individual component contains the meaning of Schrödinger's equation. Moreover, the meaning made by the statement cannot be fully gathered by the conglomeration of the meanings of the individual components. This means that any meaning subsumed by the technicality *Schrödinger's equation* will now be included in the statement. Thus the semantic density of statements can become very large.

Encoding statements with meanings such as this allows the mathematics to represent large segments of fields - often entire theories. This has implications for the knowledge structure of physics and will be discussed in section 4.1.3. The next section will show that mathematical symbolism can transcend the text, functioning as technicality.

4.1.2 Mathematical symbolism and hypertechnicality

In written language, when something is named it has the possibility of functioning as technicality (Martin 2006: 13). In order for it to become technicality, it must transcend the text to become part of the knowledge of the field. If the name does not become part of the field, with its meaning only recoverable from the text in which it is introduced, it is known as being instantial (ibid.)

Martin explains that the two processes needed for technicality are distillation (condensation and a change in nature) (Martin 1993c: 172), and transcendence of the text. Evidence for this is if the term can be presumed within another text.

Mathematical symbolism regularly transcends both texts at the rank of both component and statement. To begin with components, the third year text uses the symbol ψ, among others, without any introduction. ψ is presumed within this text, as the student is expected to understand it from previous dealings with physics. It can be modified using a qualifier, such as $\psi(r, \theta, \phi)$, that presents the variable on which it depends, but nevertheless, the meaning of ψ is presumed in the third year text. This is in contrast to the first year text that introduces the symbol with:

> 'it is natural to use a wave function as the central element of our new language. The symbol customarily used for this wave function is Ψ or ψ. In general Ψ is a function of all space coordinates and time, whereas ψ is a function of the space coordinates only – *not* of time.'

The third year text uses ψ in the way described in the first year text. It should be noted, however, that as there is a limited set of symbols that can be used (mainly from the Roman and Greek alphabets), many symbols represent multiple concepts within different fields. This means that the symbols, despite being technical, may need to be introduced within pedagogic texts to clarify in which sense the symbol is being used. Eventually, however, the students can tell from the field which sense of the symbol is being used.

Similarly to components, full statements can also transcend the text becoming technical. Thus, mathematics has the ability to technicalise relations among components as well as the components themselves. In order to see this clearly, it is necessary to look outside the two texts, to a formula sheet given to students during their third year quantum mechanics exam (School of Physics, University of Sydney 2009b: 2), given in Image 4.2.

FORMULAE:

$$L = mvr = n\hbar$$

$$r = 4\pi\epsilon_0 \frac{n^2\hbar^2}{mZe^2} = \frac{n^2 a_0}{Z}$$

$$\mu = \frac{mM}{m+M}$$

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

$$\overline{\mathcal{P}} = \int \psi^*_{nlm_l} \mathcal{P} \psi_{nlm_l} d\tau$$

$$\hat{L}_y = i\hbar\left(-\cos\phi\frac{\partial}{\partial\theta} + \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right)$$

$$\hat{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial^2\phi}\right]$$

$$\boldsymbol{\mu}_s = -\frac{2\mu_b}{\hbar}\mathbf{S}$$

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$$

$$P = \frac{4\pi^3\nu^4}{3\epsilon_0 c^3}p^2$$

$$\left|\int \psi_1^* e\mathbf{r}\psi_2 d\tau\right| \equiv p_{12}$$

$$\lambda = \frac{h}{p} \sim \frac{h}{mv_{th}} \sim \left(\frac{h^2}{3mkT}\right)^{1/2}$$

$$g = 1 + \frac{j'(j'+1) + s'(s'+1) - l'(l'+1)}{2j'(j'+1)}$$

$$I = \mu R_0^2$$

$$E_v = (v + \frac{1}{2})h\nu_0$$

$$v = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{n\hbar}$$

$$E = -\frac{mZ^2e^4}{(4\pi\epsilon_0)^2 2\hbar^2}\frac{1}{n^2} = -\frac{Z^2 13.6\text{eV}}{n^2}$$

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi$$

$$-\frac{\hbar^2}{2\mu}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\hat{L}_x = i\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right)$$

$$\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$$

$$\mu_l = -\frac{\mu_b}{\hbar}\mathbf{L}$$

$$\Delta E = -\boldsymbol{\mu}.\mathbf{B}$$

$$\mathbf{B} = -\frac{1}{c^2}\mathbf{v} \times \mathbf{E}$$

$$E_{nj} = -\frac{\mu e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2}\left[1 + \frac{\alpha^2}{n}\left(\frac{1}{j+1/2} - \frac{3}{4n}\right)\right]$$

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{r})e^{-iEt/\hbar}$$

$$\boldsymbol{\mu}_I = g\frac{\mu_N}{\hbar}\mathbf{I}$$

$$\Delta E = \mu_b B g m_j'$$

$$\Delta E = \mu_b B(m_l' + 2m_s')$$

$$E_r = \frac{L^2}{2I} = \frac{\hbar^2}{2I}r(r+1)$$

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$

Image 4.2

Quantum mechanics formula sheet

Schrödinger's equation is given in the third year text is third from the top in the right hand column. This formula sheet presumes knowledge of the meanings of each statement. They are not introduced. Although it is not expected that the students can remember precisely the composition of each statement, it is expected that they understand each statement's meaning, and more specifically, the circumstances to which each apply and how they are used. As the statements have transcended the pedagogic text and become part of the field they can be used without question or explanation.

O'Halloran implies the transcendence of mathematical statements by saying that mathematics relies on

> 'previously established results which are formalized as definitions, axioms, theorems, laws and so forth' (O'Halloran 2005: 119).

If a student was to undertake a mathematical problem or derivation independently within physics, it is entirely reasonable (or rather, expected) that they use previously established statements without explaining where they come from, assuming they understand when they are applicable. There are, however, instantial statements that do not necessarily have this privilege.

This section so far has shown that both components and statements can transcend the text, functioning as technicality. There is, however, a subtle yet distinct variation between this and technicality in written language. As mentioned in section 2.2.5, the distillation of technicality occurs within language's content plane, both condensing and changing the nature of the meanings of common sense words. Abbreviation, on the other hand, simply condenses on the expression plane, without any variation in meaning.

By encoding meaning from written language into mathematical symbolism, a concept is being shifted into a different semiotic system. Thus its nature is being changed as it now has the ability to be used within mathematics. The concept can now have an algebraic and numerical value. The written language term used to signify this concept, however, is not necessarily condensed when moved into symbolism, as mathematical statements regularly become long and unwieldy (as Image 4.2 illustrates). But, by shifting it into symbolism it becomes possible to use the concept fully within the discourse of mathematics.

So, in brief, mathematical statements can be encoded with meaning. By doing this, however, the written language is not necessarily being condensed, rather it is having its nature changed by shifting into a different semiotic resource. The symbolism can then transcend the text to become part of the presumed knowledge of the field. As this is a subtle shift from technicality, this process I will term hypertechnicality[1].

From a stratal perspective, technicality condenses on the content plane within language, whereas hypertechnicality shifts across semiotic resources. This is presented diagrammatically in Image 4.3.

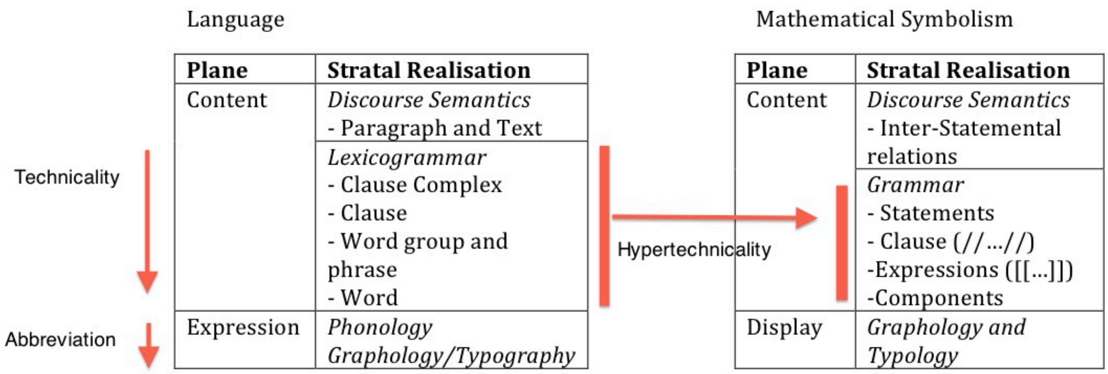

Image 4.3

Technicality, abbreviation and hypertechnicality

This definition encompasses both components and statements. Further research is needed to determine whether hypertechnicality occurs at different ranks within mathematical symbolism. Section 4.2.1 will argue, however, that images also have the power to become hypertechnical.

In a similar way to written language technicality, hypertechnicality allows mathematics to subsume knowledge, to produce more semantically dense texts, allowing descriptions of increasingly abstract and technical knowledge. Section 4.1.4 will discuss the role of mathematics in building physics' knowledge structure, through both its verticality and grammaticality.

[1] The term hypertechnicality was proposed by Jim Martin in conversation about this work.

So far, this section has shown how mathematical symbolism can produce technical meanings. It has not, however, described specifically how each text uses its mathematics. This is the purpose of the next section.

4.1.3 Usage of mathematics within the texts

Both the first and the third year texts use mathematics in slightly different ways to supplement explanations and descriptions provided by written language. The first year text uses mathematics within a Causal Explanation as evidence to confirm assertions made in written language. The third year text uses mathematics within its Derivation to extend the field and within Descriptive Reports to describe features.

Beginning with the first year text, an assertion is made that:

> 'if the particle is in a state of definite energy, such as an atomic electron in an atom in a definite energy level, the value of $|\Psi|^2$ at each point is *independent* of time.'

This assertion leads to the Identification stage of a Causal Explanation:

> 'Why is $|\Psi|^2$ independent of time if the particle is in a state of definite energy?'

The text does not attempt to answer this via explaining any physical phenomena in written language. Rather it immediately turns to mathematics by saying:

> 'To answer this question, we first note the following result from quantum mechanics: for a particle in a state of definite energy E, the time-dependent wave function $\Psi(x, y, z, t,)$ can be written as a product of a time-*independent* function $\psi(x, y, z)$ and a simple function of time:
>
> $$\Psi(x,y,z,t) = \psi(x,y,z)e^{-iEt/\hbar}$$
>
> (time-dependent wave function for a stationary state)'

This is followed by an implication sequence including mathematical statements with written language providing commentary to interpret the conclusions that can be drawn from the mathematics. This culminates in:

> 'Hence

$$|\Psi(x,y,z,t)|^2 = \Psi^*(x,y,z,t)\Psi(x,y,z,t)$$

$$= \psi^*(x,y,z)\psi(x,y,z)e^{+iEt/\hbar}e^{-iEt/\hbar}$$

$$= \psi^*(x,y,z)\psi(x,y,z)e^0 = |\psi(x,y,z)|^2 \quad (39.17)$$

> Since $|\psi(x, y, z)|^2$ does not depend on time, Eq. (39.17) shows that the same must be true for the probability distribution function $|\Psi(x, y, z, t)|^2$. This justifies the term *stationary state* for a state of definite energy.'

In this example, the mathematics is being used as evidence to justify the assertion that a state of definite energy is independent of time. A similar thing occurs in the first year Derivation. Within physics, mathematical justifications such as this are positioned to be indisputable (for a discussion of this within mathematics see Veel (1999), O'Halloran (2005:67)). This means that the mathematics holds a privileged place within physics, as the canonical method of theorising, with all theory being based on mathematics. Indeed the term 'theoretical physics' is synonymous with mathematical physics and refers to the sub-field that almost exclusively employs mathematical models to describe the universe. Thus, whilst the first year text offers more explanation to the reader by way of written language, it very quickly turns to and includes mathematics in its explanations.

The third year text uses it Derivation to extend the field, introducing new participants. The Identification stage presents a mathematical statement (Schrödinger's equation), with Goal to solve it. During the Steps of the Derivation, two new participants are introduced: m_l and l. The hypernew in the Results stage concludes by saying:

> 'Thus the partial differential equation with three independent variables has been converted to three ordinary differential equations – one for each variable. In the process, two constants, m_l and l, have been introduced.'

The two constants (l being the azimuthal quantum number describing the subshell of an electron and m_l being the magnetic quantum number describing the orbital within the subshell – another example of technical meaning within individual symbols) are very important for a student's understanding, not only of this section but of the entire course in Quantum Mechanics. These were not known, however, before the mathematical Derivation had occurred. It was only via the manipulation of mathematical symbolism that these participants were introduced. The field has been extended as a direct result of the Derivation involving mathematical symbolism.

The third year text uses mathematics in each of its Descriptive Reports to describe entities' features. This can be seen from the following example taken from the Description stage of a section titled *Radial probability density*:

> 'The probability that an electron is in the radial range $r \rightarrow r + dr$, P(r), is given by
>
> $$P(r)dr = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \ \psi^* \psi r^2 dr$$
>
> P(r) is depends on the values of n and l, but not m_l, and is shown in Figure 3 for all l values corresponding to n = 1, 2, 3. Obviously
>
> $$\int_0^{\infty} P_{nl} dr = 1$$

The final sentence in this example is of particular note. Other than the marked interpersonal Theme *obviously*, the entire clause is made up of a mathematical statement. This mathematical statement describes the feature of normalisation for the radial probability. The written language term normalisation, however, is not mentioned in reference to the radial probability. The feature is entirely described by the mathematics.

Further research is needed to fully understand the use of mathematics within physics but this section has shown that mathematics can function to explain, describe and extend the field when applied to physics. The following section will use the analysis provided so far in this chapter to discuss the role mathematics has in building physics' knowledge structure.

4.1.4 Mathematics' role in the knowledge structure of physics

A quantitative survey of various non-linguistic Items within a large corpus of texts from different academic disciplines was presented in Parodi (2010). This survey found a significantly higher use of non-linguistic semiotic resources within science, compared to the humanities. In particular, mathematical formulae was found to be used in physics to a much greater extent than all other disciplines studied (he did not include mathematics within the study). Image 4.4 summarises the results of this study:

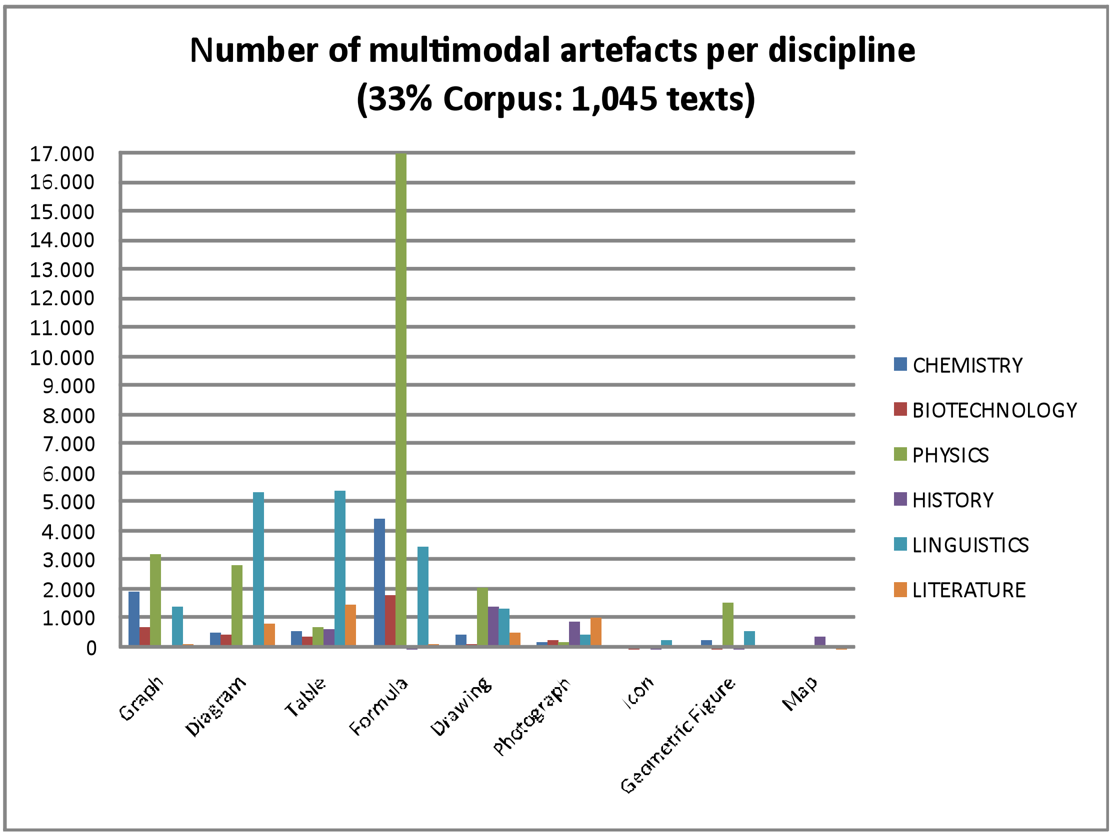

Image 4.4

Multisemiotic Items per discipline

Dreyfus (forthcoming) takes up these results and questions whether, if physics is indeed the most hierarchical knowledge structure, this survey indicates a causal link between multisemiosis and verticality. This question is examined herein and extended to include grammaticality in relation to mathematical symbolism. An application of the analysis of mathematics so far in this thesis, provides an understanding that mathematics plays a significant role in the subsumption, integration and grammaticality present within the discipline of physics.

4.1.4.1 Mathematics and subsumption in physics

In line with analysis of symbolism so far in this thesis, subsumption occurs within mathematics at both the component and statement rank. Components have the ability to contain increasing levels of semantic density. Statements, on the other hand, can represent various levels in a cline of generality with more general statements entirely subsuming more particular. Together, the components and statements provide great subsumptive power to physics.

Individual components can have very complex relationships packed into them. This greatly increases their semantic density, as the symbols now subsume the relationships between other components. This is shown in the opening clause complex of the third year text:

> 'Using spherical coordinates, Schrödinger's equation for a hydrogen-like atom with a nuclear charge of +*Ze* is
>
> $$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V(r)\psi = E\psi$$
>
> where $\psi = \psi(r, \theta, \phi)$, and
>
> $$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$
>
> $$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

This explicitly shows that the two components ψ, $V(r)$ as well as the expression ∇^2 represent much larger sets of relations. Furthermore, each component within the statements that are being subsumed are technical, containing their own semantic density and representing relations between other technical participants (eg. E), precise numerical constants (eg. μ, e, π, ε_0) or real world measures (Z, r, θ, ϕ). The semantic density of ψ, $V(r)$ and ∇^2 has increased dramatically. If components could not subsume complex relations such as in this example, mathematical statements would become increasingly long, unwieldy and unappealing for use. Subsumption by components allows for the mathematical discourse to be presented relatively efficiently, albeit with a very high level of semantic density.

Subsumption at the statement level occurs in a considerably different fashion. Hypertechnical statements can describe various systems at various levels of generality. Less general hypertechnical statements (particular statements) often are derivable from more general statements. These more general statements describe in broad terms a very large range of systems, however they often cannot be used to describe precisely an instance of a particular system. In order to describe these particular systems, more specific particular statements are derived from general statements. This creates a hierarchy of statements ordered in regards to their generality. As mentioned in section 2.4.1.2, this is an aspect of subsumption described by Muller (2007: 78). This hierarchy is reflected in the texts.

As mentioned before, the Schrödinger equations given in the first and third year texts describe particular systems, but neither subsumes the other. Further into the third year text (but out of the segment that has been studied in detail in previous chapters), a more general form of Schrödinger's equation is given:

$$\hat{H}_0\psi = E\psi$$

Eqn 4.1

This general equation tells us very little if anything about a specific physical system. The semantic density of Eqn 4.1 is much stronger, however its semantic gravity is much weaker. From this statement, however, the more particular equations in both the first and third year texts can be derived. The general statement subsumes the more particular statement, and in doing so, comes to represent the peak of the knowledge structure of Schrödinger's equation. These ever more general statements become hypertechnical, representing large segments of the field. In fact, it is very common for equations to represent the peak of a sub-field within physics, from which all else can be derived (eg. Maxwell's equations in electromagnetism, Newton's second law in classical mechanics). The third year text also provides statements lower down the hierarchy, in an entire page of examples shown in Image 4.5.

Quantum Numbers n	l	m_l	Eigenfunctions
1	0	0	$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$
2	0	0	$\psi_{200} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(2-\frac{Zr}{a_0}\right) e^{-Zr/2a_0}$
2	1	0	$\psi_{210} = \frac{1}{4\sqrt{2\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0} e^{-Zr/2a_0}\cos\theta$
2	1	± 1	$\psi_{21\pm1} = \frac{1}{8\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Zr}{a_0} e^{-Zr/2a_0}\sin\theta\, e^{\pm i\varphi}$
3	0	0	$\psi_{300} = \frac{1}{81\sqrt{3\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(27-18\frac{Zr}{a_0}+2\frac{Z^2r^2}{a_0^2}\right) e^{-Zr/3a_0}$
3	1	0	$\psi_{310} = \frac{\sqrt{2}}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6-\frac{Zr}{a_0}\right)\frac{Zr}{a_0} e^{-Zr/3a_0}\cos\theta$
3	1	± 1	$\psi_{31\pm1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(6-\frac{Zr}{a_0}\right)\frac{Zr}{a_0} e^{-Zr/3a_0}\sin\theta\, e^{\pm i\varphi}$
3	2	0	$\psi_{320} = \frac{1}{81\sqrt{6\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}(3\cos^2\theta-1)$
3	2	± 1	$\psi_{32\pm1} = \frac{1}{81\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}\sin\theta\cos\theta\, e^{\pm i\varphi}$
3	2	± 2	$\psi_{32\pm2} = \frac{1}{162\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\frac{Z^2r^2}{a_0^2} e^{-Zr/3a_0}\sin^2\theta\, e^{\pm 2i\varphi}$

Image 4.5

Eigenfunctions in third year text

These statements describe the individual wave functions of specific systems and can be derived from the third year Schrödinger equation. Thus they are subsumed by that equation, which in turn is subsumed by the more general Eqn 4.1. Subsumption such as this allows mathematics to contain a cline of semantic gravity, moving from general to specific descriptions, allowing a large range of phenomena to be described consistently.

Mathematics, however, does not simply subsume, but also integrates aspects of physics not in subsumptive relationships.

4.1.4.2 Mathematics and Integration in Physics

When technical terms are introduced they are given meanings and arranged into an implicit taxonomy. The taxonomies set up within physics become very deep, meaning that the relationships between different terms not in subsumptive relationships can be very difficult to explain. Moreover, even if their relationship can be qualitatively described, written language struggles to show how the measures of these terms (their numerical quantity) vary precisely and consistently in relation to others. Mathematical symbolism, on the other hand, is well versed at precisely relating these participants.

Lindstrøm (Draft : 18) suggests that written language more easily establishes concepts, whereas mathematics is the preferred language for communicating relationships between these concepts. The analysis of the texts in the previous chapters agrees with this. Describing the relationships between different entities from different aspects of theory, allows the discipline to be integrated. Within physics, mathematical symbolism does not have external technical meaning when developed entirely in isolation. It can produce new participants, however these have no meaning outside the internal grammar of mathematics without written language (or sometimes images) encoding technical meaning into them. Moreover, activity sequences set up by mathematical symbolisms, even with technical meanings involved, are not necessarily understood by students as readily as activity sequences in written language. As such, written language is the preferred semiotic system for establishing concepts; for introducing the various levels of delicacy within taxonomies.

Once these concepts have been introduced and understood, however, mathematics can more easily describe the precise relations between participants. A relatively simple example from the third year text will be used to illustrate this:

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$

Eqn 4.2

This algebraic equation relates all seven participant components precisely. The relations between them are seen to be very complex when translated into written language:

> Potential energy as a function of the radius equals the negative of the atomic number multiplied by the exponential function squared, all divided by four multiplied by pi multiplied by the permittivity of free space multiplied by the radius.

Aside from removing syntactic ambiguities in this sentence (eg. *the negative of the atomic number multiplied by the exponential function squared* can represent $(-Ze)^2$, $-(Ze)^2$, $-(Ze^2)$ or $(-Ze)^2$) through spatial positioning and rule of order operations (O'Halloran 2005: 124), mathematical symbolism allows the subject to be changed. By changing the subject, each component can be expressed as a relationship of all other components, whilst still conserving the configuration of operative processes (ibid: 128). For example, the statement can be rearranged to form:

$$r = \frac{-Ze^2}{4\pi\varepsilon_0 V(r)}$$

Eqn 4.3

or

$$e = \sqrt{\frac{-4\pi\varepsilon_0 r V(r)}{Z}}$$

Eqn 4.4

The rearranging of these equations can be reversed to find the original. This is practically impossible within written language to this extent. This point is the key to the discussion of integration. Where written language has greater power to introduce individual participants, mathematics has greater power to relate them. Mathematics can precisely describe the relationship between any number of participants. Moreover, given the requisite mathematical skills, the mathematics can be rearranged so that each participant within a statement can be moved to the subject of the equation, allowing its precise relationship to be described.

This means that mathematics allows various aspects of physical theories to be related to each other, so that they cohere. If the mathematical formulations within two theories are not consistent with each other, the theories cannot be integrated perfectly. Thus, the main method of integration within physics is via mathematical symbolism. By regularly producing consistent mathematical theories, physics proves itself to be an integrated field.

This and the previous section have shown that mathematics provides both subsumption and integration for physics. Thus, this provides evidence to support Dreyfus' (forthcoming) suggestion of a causal link between multisemiosis and verticality. The study so far is by no means exhaustive, however it is clear that mathematics does indeed help build physics' verticality. The final factor in a discipline's knowledge structure is its grammaticality. This is where we now turn.

4.1.4.3 Mathematics and the Grammaticality of Physics

To revisit section 2.4.1.3, Muller (2007: 71) explains that grammaticality has to do with how theory deals with the world. That is, how theoretical statements deal with their empirical referents.

> 'The stronger the (external) grammaticality of a language, the more stably it is able to generate empirical correlates and the more unambiguous because more restricted the field of referents; the weaker it is, the weaker its capacity to stably identify empirical correlates and the more ambiguous because much broader is the field of referents... In other words, grammaticality determines the capacity of a theory or language to progress through worldly corroboration. (Muller 2007: 71)'

In essence, a strong grammaticality has two factors, unambiguous boundaries on terminology and the ability to relate them to the real world. From this definition physics is said to have a strong grammaticality (Maton: 2008). As with verticality, however, evidence for this has so far been thin on the ground.

This section describes how physics accesses the strong grammar of mathematics and provides it with technical meaning, to ensure strong grammaticality. This grammaticality links theory to the real world.

As the field of physics is realised multisemiotically, its knowledge structure incorporates various knowledge structures of mathematics to suit its needs. O'Halloran explains that:

> '*science progresses through accessing the strong grammar of mathematics which is free to proliferate to provide alternative approaches and descriptive tools*. These approaches are integrated with the hierarchical knowledge structure of science to serve particular agendas relating to the description of material reality' (O'Halloran 2007: 213 – original emphasis)

O'Halloran (2007: 209) argues that mathematics possesses a very strong grammar as its symbolic referents contain definite boundaries and are not open to interpretation (unlike, say, terminology in sociology). Pure mathematics as a discipline, however, does not have the necessary real world referents that is the second factor of grammaticality.

This contradiction opens an important question for physics: if mathematics can not describe the real world on its own, how does it manage to do so with great power when placed into the discipline of physics?

O'Halloran (2007: 211) attempts to answer this question by explaining that:

> 'the grammar looks inwards (internally) in the case of mathematics and the grammar looks outwards (externally) in the case of science.'

In other words, the referents in mathematics are within the language of the discipline and have no external meaning, whereas in science, the referents refer to things in the real world. The change comes about because in physics, mathematical symbolism is encoded with real world technical meanings that have been developed by written language. As has been detailed, this technical meaning is encoded into either individual symbolic components or into statements, leading to the possibility of hypertechnicality. By combining the precise referents of mathematics with the real world meanings of physics, the mathematics becomes very effective in describing empirical relations in the external world. Thus, the strong grammaticality of physics comes from its written language representation of the world, and the precise relations set up by mathematical symbolism.

The discussion of mathematics in relation to physics' grammaticality and verticality has shown that symbolism is an integral part of the knowledge structure of mathematics, allowing theory to move from general to specific whilst relating precisely and coherently multiple aspects of the field and describing the real world.

4.1.5 Summary of mathematics in the physics texts

The discussion of mathematics within the physics texts has shown that mathematical symbolism is a functional element within the discourse of physics that can be used to explain, describe and extend the field. Technical meaning is encoded into symbolism at the ranks of component and statement using relational processes, apposition and images. Once the symbolism has been encoding with meaning, they have the possibility of becoming hypertechnical, by transcending the text to become part of the knowledge of the field itself.

Components can subsume large and complex relations between many abstract and technical participants, condensing the discourse. Hypertechnical statements, on the other hand, produce a cline of semantic gravity that allows mathematics to move from general to particular statements. General statements subsume more particular statements, allowing them to describe a wide range of phenomena. Further to this, mathematical statements have the ability to integrate various aspects of a discipline, by precisely relating multiple participants. Thus, to a large extent, mathematics builds verticality within physics. Further to this, the ability for mathematics to have technical meaning encoded within it, allows physics to describe precisely the real world. This means that mathematics also has a hand in the grammaticality of physics.

To this point, the intrasemiosis of written language and mathematical symbolism has been described in relation to the physics texts. From here, the focus moves to the final semiotic resource used in the texts; images, followed by the three resources' intersemiotic complementarity.

4.2 Images in the physics texts

Visual images are an important resource for students within physics, and despite not being utilised to the same extent as written language or mathematical symbolism, they can be of great assistance in understanding challenging concepts. Both O'Halloran (2005: 129) and Lemke (1998: 107) argue that images within science and mathematics can provide a visually intuitive understanding of the more precise technical meaning afforded by the mathematics. That is, images often present a less precise, but more easily understandable representation of technical meaning. Both texts use images to this effect, however they differ in how it is they provide this understanding. This section will compare how each text uses images to convey meaning.

Section 4.2.1 illustrates that the first year text uses images to present both interpersonal and experiential meaning. It is argued that the graphs used to present experiential meaning, become hypertechnical, becoming part of the assumed knowledge of the field. The third year text, on the other hand, does not uses hypertechnical images. It does, however, again use graphs to present experiential meaning that is not mentioned explicitly within the written language. Despite the differences, both texts use images to allow the students to gain a deeper understanding of the field being presented.

4.2.1 Use of images in the first year text

The first year text uses two different types of images: photos and graphs, each type conveying different meanings. The three photos in the margin of the text, two showing portraits of prominent physicists, and one a photo of a dust mite taken from a scanning electron microscope, provide interpersonal meanings.

The Image 4.6 shows the portrait provided of Erwin Schrödinger.

Image 4.6
Erwin Schrödinger

These photos are not placed to provide any extra experiential or logical meaning. That is, they are not used at all to build the field of quantum mechanics for the student. Rather, they have a purely interpersonal role, humanising the subject area, just as the use of *we* does. In the case of the dust mite, the photo works to provide an example for the student of what the field is used for, shunting the abstract back into the concrete world.

The Cartesian graphs used further into the first year text illustrate the experiential meaning associated with the wave functions and wave packets introduced in the written language. Images 4.7 and 4.8 reproduce two of these graphs.

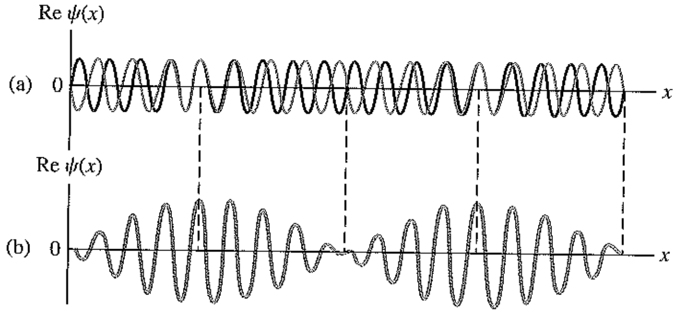

Image 4.7

Graph of superposition of wave functions

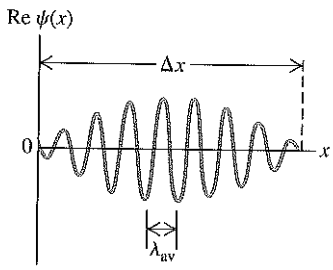

Image 4.8

Wave packet

In order to understand the benefit of these graphs, the notion of conceptual metaphor must be introduced, described in relation to physics in Brookes and Etkina (2007). Within the text (and indeed within the field of physics itself), electrons are described both as a particle and, implicitly, as a wave (by having a wave function). In reality, an electron is neither a particle nor a wave in the classical sense. Rather under certain conditions it displays properties similar to a particle, and under other conditions displays properties similar to a wave. This apparently contradictory set of descriptions is known as 'wave-particle duality'. As electrons are so small, we cannot possibly see them with the naked eye (their size is less than the wavelength of visible spectrum). In order to discuss them, physics has developed the conceptual metaphor of electrons being particles and waves in particular situations. This metaphor allows an understanding of the electron to be grasped and used in a relatively simple way. Students learn the limitations of each metaphor, and when each is applicable.

Image 4.7 and 4.8 are used within the first year text to convey to students the conceptual metaphor of the electron as a wave and a particle. Without going into technical description, the written language explains that in relation to image 4.8:

> 'we have something that begins to look like both a particle and a wave. It is a particle in the sense that it is localised in space; if we look from a distance, it may look like a point. But it also has a periodic structure that is characteristic of a wave.'

That is, the image has been encoded with technical meaning, and is used to represent both the wave particle duality of matter.

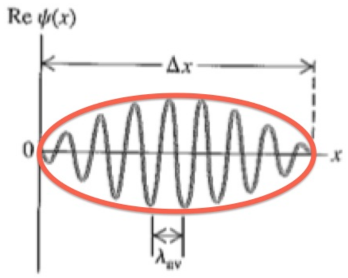

Image 4.9

Hypertechnical figure – wave packet

The entirety of the periodic line, indicated by the oval in Image 4.9 (which is at the rank of Figure; see section 2.4.2.1), is named as a *wave packet*. In written language, the term *wave packet* is an instance of technicality. Similarly, it can be argued that the periodic line in Image 4.9 as a visual representation of the wave packet, is an instance of hypertechnicality. Although further research is needed to confirm this, the image is regularly used within physics to represent the wave particle duality of matter without introduction. Moreover, at a pedagogic level, it is used to reason with and explain physical phenomena. Indeed it is immediately used within the same text to aid the Causal Explanation linking the Heisenberg uncertainty principle and de Broglie's relations (see section 3.2.1). By becoming hypertechnical, the image subsumes any knowledge used to encode meaning into it, and can be used elsewhere without introduction.

The third year text does not use hypertechnical images. It does, however, use its images to present experiential meaning not given within the written language.

4.2.2 The use of images in the third year text

The third year text presents a series of Cartesian graphs, from which students are expected to draw physical conclusions that are not explicitly stated in written language. Image 4.10 shows these graphs, with the caption providing the technical meaning necessary to understand them:

‘Radial probability functions for hydrogen-like atoms. The quantity P_{nl}(r) is plotted as a function of $r/(a_0/Z)$. The solid triangles indicate the expectation (average) value of *r*; the dashed vertical lines indicate the radius values given by Bohr’s theory.’

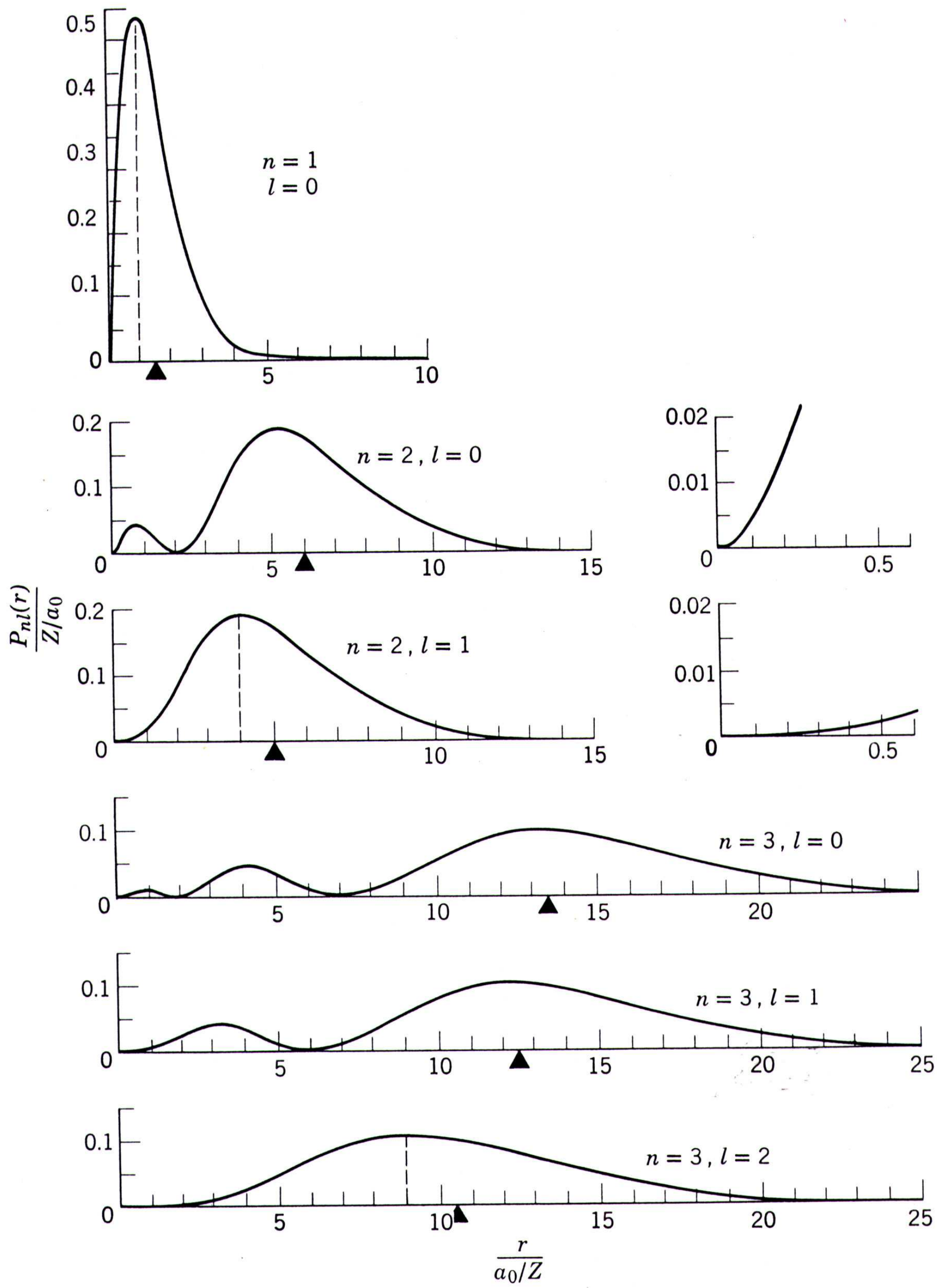

Image 4.10

Third year Cartesian graphs

The graphs show plots of specific probability functions described in general by the mathematical statement:

$$P(r)dr = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \ \psi * \psi r^2 dr$$

Eqn 4.5

Neither the written language nor the mathematics, however, explicitly describes the physical implications of the graphs. It is assumed the student has enough experience in dealing with images of this kind to draw conclusions themselves.

The vertical alignment of the graphs in order of their quantum numbers (indicated by the *n* and *l* values in each graph) means a trend is shown of the plotted line becoming flatter and wider.

From the technical meanings encoded into the graph by the caption and the mathematical symbolism, the student can draw conclusions about the physical phenomena the graphs describe. This conclusion is that, as the quantum numbers increase for an electron, the range in which it is likely to be found increases. The written language, however, does not explicitly state this; the images construe this experiential meaning.

Analysis of the images within the two texts, shows in both cases, that images are used in physics to convey technical experiential meaning that is extra to that produced by mathematical symbolism or written language. Moreover, these images have the potential to be hypertechnical, becoming part of the assumed knowledge of the field.

The three sections dealing with written language, mathematical symbolism and images, produce meanings intrasemiotically. Each section, however, has also shown that each semiotic resource does not function in isolation. There is a regular interplay between written language, mathematical symbolism and images. Each meaning making resource builds on the meanings made by the others, complementing them to produce meaning within the text that is greater than the sum of its parts. This complementarity and the intersemiotic mechanisms used to build it will be formalised in the next, and final section of this chapter.

4.3 Intersemiosis and complementarity of meaning making resources

In order to discuss intersemiotic complementarity, the terminology presented in section 2.3.3 will be revisited. Royce (1998: 26) argues that within multisemiotic texts,

> 'visual and verbal modes semantically complement each other to produce a single textual phenomenon.'

He names this idea intersemiotic complementarity and explains, as does Lemke (1998), that the product of the complementarity is 'synergistic' or 'multiplicative'. That is, the resulting meanings are greater than the sum of the individual meanings of each individual semiotic resource.

This multiplicative complementarity is represented diagrammatically in Image 4.11.

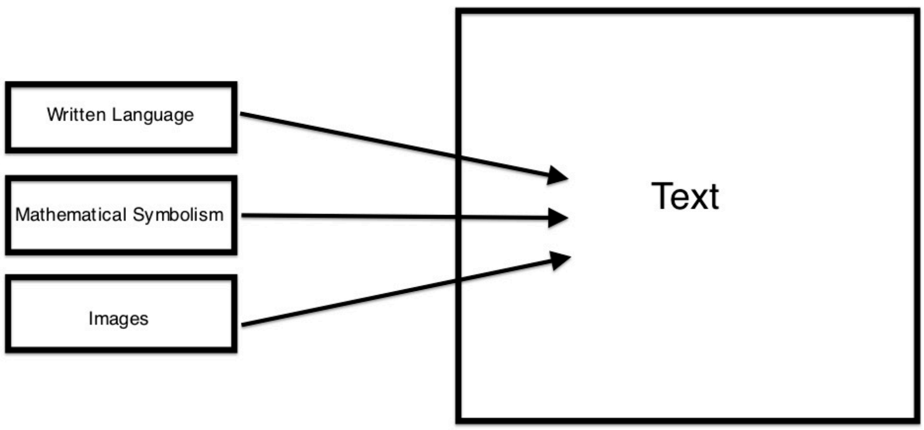

Image 4.11
Multiplicative intersemiotic complementarity

O'Halloran (2005: 159) explains that within mathematical discourse:

> 'Language, symbolism and visual images function together in mathematical discourse to create a semantic circuit which permits semantic expansions beyond that conceivable through the individual contributions.'

This is true of the texts in this study, each allowing the various semiotic Items to cohere, producing complementarity. Section 4.3.1 will discuss the main intersemiotic mechanisms (O'Halloran 2005: 169) used within the texts to demonstrate how the texts build this complementarity. The following and final section of this chapter, section 4.3.2, will illustrate the integration that results from this complementarity.

4.3.1 Intersemiotic mechanisms producing complementarity

The most prominent intersemiotic mechanism used within both texts is semiotic mixing (O'Halloran 2005: 169), producing microtransitions throughout the text. This involves Items of primarily one semiotic system incorporating other entities of other semiotic systems as functional elements. This has been shown many times already, especially in the discussion of technical meaning being encoded into mathematical symbolism. This can be seen, for example, in Table 4.1.

The probability that an electron is in the radial range $r \rightarrow r + dr$, $P(r)$,	is given by	$P(r)dr = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi\ \psi * \psi r^2 dr$
Goal	Process: Material	Actor

Table 4.1

Semiotic mixing within the clause

This written language clause incorporates selections from the system of mathematical symbolism, giving them the function of participants. Microtransitions happen at the lexicogrammatical rank, such as this, occur regularly within both texts, and allow the symbolism to gain meaning (See section 4.1.1. Image 4.11 shows that semiotic mixing also occurs in images.

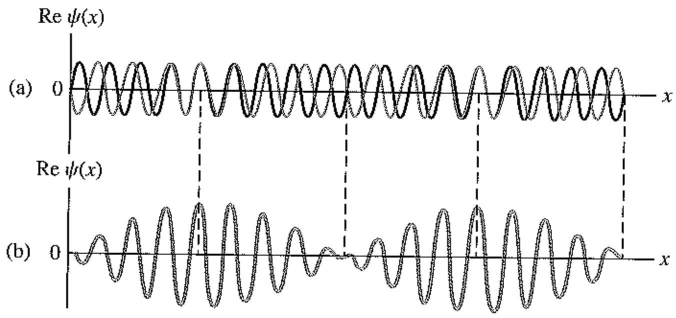

Image 4.12
Semiotic mixing in images

Image 4.11, previously shown, illustrates that mathematical symbolism is used as a functional element within images, providing meaning for the circumstance of the axes. Without the labelling of the axes the image would have no meaning. Semiotic mixing allows the text to cohere, providing an important factor in the 'semiotic circuit', where each resource modifies the meanings of the others to create the entire text.

As images cannot be incorporated into written language, the texts provide coherence by referencing the images within the written language. This is done via a system of numbered images for example:

> 'Figure 39.14a is a graph of the real parts of the individual wave functions for the case $A_1 = -A_2$; Fig. 39.14b graphs the real part of the combined wave function $\psi(x)$ given by Eq. (39.24).' [2]

This referencing works to provide a cohesive link between the resources.

By providing intersemiotic cohesion in the form of microtransitions such as semiotic mixing as well as referencing that signals macrotransitions between Items, the texts lay the platform for the multiplicative complementarity discussed by Royce and Lemke. This has been touched upon already in the description of the use of images within the third year text.

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[2] The referencing system used in the original texts is different to that of this thesis.

The series of graphs in the third year text, shown previously in Image 4.10, uses semiotic mixing in the form of captioning – both the symbolic caption on each graph and the written language caption - and symbolically labelled axes. The individual graphs, without the technical and contextualising symbols and written language, provide almost no information to the students about the physical system they are studying.

Similarly, if the mathematical symbolism or the written language in Image 4.10 were to be seen in isolation, the meanings would be considerably less, or possibly non-existent. It is only thanks to the layout of all the plots, the contexualising information given by the written language and symbolism, and the technical knowledge developed by the written language and mathematical symbolism elsewhere in the text, that the student can grasp that these images show a consistent trend relating the position of an electron and its energy state. To put it another way, the individual semiotic resources do not provide this meaning, rather it is the text as a whole incorporating Items of these resources that does. This is what is meant by a multisemiotic text being 'synergistic' and 'multiplicative' – that the result is greater than the sum of its parts.

The final section will discuss how the intersemiotic complementarity effects the integration and thus the verticality of physics.

4.3.2 Intersemiotic complementarity and verticality

Intersemiotic complementarity provides the final piece of evidence for the integration present in the knowledge structure of physics. If each semiotic resource is taken to be a distinct descriptive language of its own, the study of complementarity has shown that these languages are integrated to produce meanings above what each individually can produce. This has the result that the discipline is, by its nature, realised multisemiotically.

If the explanatory powers of one semiotic resource break down under certain conditions, another fills the void. This can be seen by way of a solution to an example given in Martin (2006: 23). Martin argues that within certain fields (such as SFL), complementary hierarchies are needed to explain the full range of phenomena. That is, the entire field cannot be derived from one overarching principle. Within SFL, the two complementary hierarchies are the realisation hierarchy (for strata) and the classification hierarchy (system networks).

Martin applies this to physics by saying:

> 'In physics, the well known complementarity of light as a wave and light as a particle illustrates the modeling issue here – at times, for certain phenomena, integration under a single generalizing proposition is not possible.' (Martin 2006: 23)

The apparent wave-particle conflict within physics is known as the principle of complementarity[3] (Young and Freeman: 1478), and determines the written language explanations of differing systems. As mentioned previously, at the quantum level the matter dealt with is neither a wave nor a particle, it is something we can't visualise; it simply displays properties of each under varying circumstances. For this reason, the conceptual metaphor of wave-particle duality is used leading to two seemingly irreconcilable hierarchies for description.

While I am making no judgment about Martin's theory of complementary hierarchies, in the case of wave-particle duality within physics, it does not need to apply. The hierarchies are integrated by the multisemiotic nature of physics. Mathematical symbolism again holds the key. Where integration and the subsumption into one general proposition breaks down within written language, the mathematical theory of Quantum Electrodynamics (QED) describes and predicts both the wave and particle nature of each interpretation (Young and Freedman: 1479). This mathematical theory (of which, I admit, my undergraduate training in physics does not allow me to fully understand) describes the universe in a way that can be applied to the whole range of systems that would otherwise be divided into wave and particle phenomena. As the first year text explains:

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[3] In this case, complementarity has a technical meaning within physics separate to that used in this in linguistics.

> 'the wave function $\Psi(x, y, z, t)$ for a particle contains all the information that can be known about the particle.'

This wave function under certain conditions describes phenomena similar to that which occurs with particles and under other conditions that which occur for waves. Unfortunately, it takes a considerable amount of training to be able to access the information the wave function holds. Moreover once this information has been teased out, the results cannot necessarily be described fully by written language. It remains only accessible via the semiotic system of mathematical symbolism. As mathematics isn't as easily understood as written language or images, however, any explanation of the implications of the mathematics reverts back to the wave-particle duality.

Thus intersemiotic complementarity provides integration between different aspects of the theory by allowing written language explanations to be subsumed by mathematical symbolism. Where one semiotic system falls down another takes up the slack within the hierarchy. Mathematics has the potential to describe that which cannot be visualised; written language and images, on the other hand, allows for more readily understandable explanatory meanings to be made. Moreover, written language can name anything mathematics develops, thereby potentially encapsulating an entire field in one nominal group for those who are sufficiently trained.

4.4 Conclusion

The previous two chapters have attempted to describe how the two excerpts from university physics textbooks work to construe their field. The chapters have drawn on Halliday's and Martin's descriptions of written language in science as well as O'Halloran's descriptions of mathematical symbolism and images, and Royce's, Lemke's and O'Halloran's theories of intersemiotic complementarity. These have been combined to build an understanding of how the texts utilise the three semiotic resources of written language, mathematical symbolism and images to construe the field.

This analysis has shown that the field within each text is very dense and abstract, dealing with conceptualisations far removed from every day experience. These conceptualisations are encoded into technical terms within written language allowing them to be efficiently related to each other. It has been found, however, that technicality is not exclusive to written language. Both mathematical symbolism and images can not only contain technical knowledge, but also become hypertechnical; part of the assumed knowledge of the field. This allows both participants and their relations to be assumed within the discipline, facilitating the building of increasingly dense meanings to be made.

Reflecting its audience, the first year text eases its students into the knowledge of the field. The level of technicality is considerably lower than that of the third year text. Furthermore, the macrogenre within the text aims to link knowledge that is being learnt to that which is already known. In doing this, it employs interpersonal tactics such as the use of *we* and contextualising images, to make the text appear more inclusive. The first year text does, however, use mathematical symbolism to explain.

The third year text, on the other hand, is less concerned with making the text accessible. It presumes the readers are already inducted into the field, meaning that it can focus on providing the highly technical experiential meaning. Mathematical symbolism is used to describe and extend the field, and images are used purely to convey experiential meaning without prompting from written language.

Both the first and third year text allow for an understanding, albeit partial, of the nature of the discipline of physics itself. Verticality is built through a number of means. The repetition of topics allows knowledge to be subsumed, and deepened, using increasingly dense technicality. The intersemiotic complementarity compounds physics' integration allowing different semiotic resources to fill various levels within the knowledge structure. Mathematical symbolism both integrates and subsumes knowledge using hypertechnicality, increasing its semantic density. Mathematics also provides physics' strong grammaticality, as technical meanings can be encoded into symbolism, allowing for precise descriptions of the real world. These two factors strongly suggest that physics is indeed a hierarchical knowledge structure.

The texts in this study, along with many others of all forms, provide a pathway for students to be inducted fully into the field of physics, so that they, eventually, can use their knowledge and skills to extend our knowledge of the universe.

Chapter 5 – Knowledge in Physics

This thesis has studied the way two texts construe physics knowledge, and present it to students. Using Systemic Functional Multimodal Discourse Analysis, the semiotic resources of written language, mathematical symbolism and images were shown to produce meanings in their own right, as well as in conjunction with each other, intersemiotically. Further to this, a discussion of the nature of knowledge within physics itself has revealed that the discipline is integrated, and can be applied to the real world.

The findings of this thesis are significant on a number of fronts. They constitute a contribution to the growing body of work using SF-MDA to analyse how meaning is construed through multisemiotic texts, as well as the body of work applying sociology of knowledge to describing various disciplines. In particular, this thesis has provided a first thorough description of the meaning making resources used within physics, as well as provide evidence for the assertion that physics is the canonical hierarchical knowledge structure.

5.1 Theoretical implications

Four important issues have been raised from the analysis in this thesis that require further research and theorising. The first is in regard to hypertechnicality. This thesis has illustrated that hypertechnicality occurs at both the rank of component and statement, allowing relationship between participants to be technicalised. Furthermore, it argues that images can also become hypertechnical at the rank of figure. Further research is needed, however, in order to understand the full extent of hypertechnicality in both images and symbolism, as well as the possibility of hypertechnicality in other semiotic resources (such as non-mathematical symbolism and film) and modalities (such as aural and haptic).

The second avenue for research concerns the Derivation genre in both the first and third year text. The classification of this genre shows that science contains genres not realised purely by language. I suspect that there are multiple sub-genres within the genre of Derivation. An understanding of these genres will greatly contribute to the understanding of the challenges students face when studying science.

The third research question is to what extent do students engage with technical meaning made at both the statement and component level. When not inducted into the field, do students simply see statements as a conglomeration of components, or can the engage with statements as meaning making resources in their own right? And what effect does this have on their learning?

The final issue brought forward by this thesis relates to the framework developed to describe knowledge structures. The analysis in this thesis illustrates that currently there is a lack of linguistic factors that have been discovered to describe knowledge structures. Moreover, the factors that have been developed are currently under defined, which has the effect of offering vague interpretations of texts.

These four issues alone present enormous opportunities for further avenues of research, and is only compounded by the limitations of this thesis.

5.2 Limitations

The largest single limitation of this thesis concerns its scope. The very small corpus containing two texts focusing on the same area within physics limits the extent to which the findings can be regarded as representative across all fields of physics. A brief survey of other physics texts shows that within the visual modality, physics is not simply realised by mathematical symbolism, written language and images. Non-mathematical symbolism is also used across many sub-fields of physics, producing its own meanings. Moreover, students do not simply gain their knowledge from textbooks. They attend lectures, watch films and undertake experiments.

Despite these limitations, however, the findings of this thesis do represent an important step toward a full understanding of both the knowledge structure of physics, and how this knowledge structure is conveyed to students.

5.3 Construing knowledge in physics

It is clear that the pedagogic physics texts are multisemiotic macrogenres, employing a number of resources in order to construe and convey knowledge to students. Each text contains a series of staged genres that have a particular structure and purpose. There are three main genres included in each text. Descriptive Reports are used to describe features and implications of concepts. Causal Explanations provide understandings of how physical principles work, while Derivations are used to test theories and extend the field. The texts combine these genres to create a coherent macrogenre in order to effecitvely build the knowledge for its students.

Written language is used in both texts to establish unfamiliar concepts. Relational processes are used to define and classify participants, creating expansive and abstract taxonomies whilst condensing technical meaning. Material processes, on the other hand, are used to manipulate mathematical statements, particularly within Derivations.

Both texts are lexically dense, containing very large nominal groups. This allows a large amount of information to be conveyed efficiently. The semantic density, on the other hand, varies between the texts. The third year text relies on a greater level of technicality to provide precision in description, however his has the effect of greatly reducing the accessibility for those not inducted into the field. The first year text, on the other hand, uses less technicality, producing a less precise, but more accessible text. This each text reflects its audience in construing technical meanings.

Once the technical meanings have been established, they are often encoded into mathematical symbolism at the ranks of component and statement. The symbolism can then become hypertechnical allowing precise relationships to become part of the knowledge of the field and used without introduction in other texts. Mathematical symbolism is used in each type of genre within the text. It describes features, it aids explanation, and is the only necessary resource within a Derivation.

Images, though used to a lesser extent than written language and mathematical symbolism, produce both experiential and interpersonal meanings on their own. The first year text uses photos to humanise the subject matter, creating a more inclusive text. Cartesian graphs, on the other hand, are used by both texts to aid explanation and show relationships. This is done by providing more easily understandable visualisations of phenomena than mathematical symbolism and written language. Although not shown within the studied texts, images can also become hypertechnical, representing specific knowledge.

The three semiotic resources construe meanings intrasemiotically. They do not, however, function in isolation. Knowledge is also built intersemiotically, allowing each resource to multiply the meanings of the others.

Thus, the pedagogic purposes of the physics texts are realised through their multisemiosis. Each resource has certain advantages over all others to convey meanings and can become technical, construing knowledge throughout the entire discipline.

5.4 Knowledge Structure of Physics

The analysis presented in this thesis strongly suggests that physics is indeed a hierarchical knowledge structure with both high verticality and strong grammaticality. The repetition of topics across multiple years allows for knowledge to be subsumed providing a more comprehensive understanding of physical processes. This subsumption is realised in written language by the increasing use of technicality.

The key to both verticality and grammaticality, however, appears to come from the use of mathematics. Hypertechnical symbolism achieves two subsumptive purposes. Firstly, it allows great condensation and increasing semantic density within both components and statements to be used without introduction throughout the field. Secondly, it can order statements into a cline of generality, where the more general subsumes the more particular, and describes a broader range of phenomena. Hypertechnical statements also produce integration by relating concepts from various areas of physics, producing consistent theories. The intersemiotic complementarity of the three semiotic resources is such that, where one resource wanes in its descriptive power, another can fill the void, ensuring that descriptions of phenomena are integrated and are consistent throughout the discipline.

The interaction between mathematics and written language also develops physics' strong grammaticality. Mathematics can precisely relate many participants, however they do not have any inherent external meaning. Written language, on the other hand, contains meanings that describe the real world but struggles to produce relationships to the same complexity of mathematics. By encoding technical meaning from written language into mathematics, then, physics can precisely describe very complex relationships between phenomena in the outside world.

Although this study is by no means exhaustive, it does show that mathematics is crucial to the nature of physics. It allows physics to become a subsumptive and integrative hierarchical knowledge structure that can be related precisely to the real world.

This thesis has added to the growing body of work that attempts to understand how science construes the universe around it. Although there is still much to be done, it is hoped that the culmination of research in this area can eventually lead to educational tools that can broaden and strengthen scientific literacy within our society.

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