AN EXAMINATION OF EFFICIENCY LEVEL VARIATIONS

FOR BUS SERVICES

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INTRODUCTION

In recent years significant progress has been made concerning measurement of efficiency in relation to productive activities, see e.g. Fried et al. (1993). In particular, non-parametric frontier methods such as Data Envelopment Analysis (put forward in Charnes et al. (1978)) and Free Disposal Hull (suggested by Deprins et al. (1984)) have been developed with applications across a wide range of sectors including transit services. This paper examines the efficiency variations of 157 of the 175 Norwegian subsidised bus companies using non-parametric frontier methods. A range of different efficiency measures within the non-parametric frontier tradition will be presented. In particular, radial and non-radial measures will be considered in order to determine the relevance of slacks. The efficiency measures will be decomposed into pure technical inefficiency, scale inefficiency and inefficiency due to the convexity assumptions included in Data Envelopment Analysis (DEA). As such this information will provide a very detailed picture of the differences in performance among the included bus services. Specific attention will be given to the efficient observations, in order to identify so-called super-efficient observations. In addition, to the calculation of efficiency measures emphasis will also be put on possible explanations of the obtained results. This work will be undertaken within a regression analysis framework, whereby the efficiency scores are related to a set of independent variables. Explanations are important in order to determine the scope for enhancing efficiency for specific observations. The key issue will concern the extent to which efficiency variations are caused by controllable factors. In some cases measured inefficiency may be caused by factors outside the control of the individual company, e.g. the topographic or demographic conditions.

These bus companies have previously been examined in Jørgensen et al. (1995), (1997). In Jørgensen et al. (1995) a translog cost function was estimated in order to examine the characteristics of bus operation costs in Norway. Jørgensen et al. (1997) examined the inefficiency of the Norwegian bus industry using a stochastic cost frontier model. As part of this paper the earlier efficiency results will be compared to the ones obtained using non-parametric frontier methods. This will establish the extent to which there is a positive association between the two sets of results.

The rest of the paper is structured as follows: Section 2 includes a brief overview of non-parametric efficiency measurement techniques emphasising the range of options available within this approach. In Section 3 the data used for the efficiency analysis are presented. The results of the efficiency analysis are presented in Section 4 including different types of efficiency measures and possible explanatory factors for the identified efficiency patterns. Section 5 concludes with final remarks including possible areas of further research.

METHODOLOGY

Data Envelopment Analysis (DEA) and Free Disposal Hull Analysis (FDH) examine the efficiency of similar production units using so-called dominance comparisons of the units' inputs and outputs. Each production unit is compared to the whole sample of production units in order to determine whether there exist other production units (or combinations of production units) using the same or less of the inputs to produce the same or more of the outputs. If this is the case, the production unit is declared inefficient. Otherwise, the production unit is efficient. In this way the efficiency concept is a relative one as it is only concerned with efficiency in relation to the sample and not some absolute efficiency standard.

Formally, assume there are n production units (indexed as k=1,...,n) using m inputs (indexed as j=1,...,m) to produce s outputs (indexed as i=1,...,s). The k'th production unit can now be described by the production vector (X_k,Y_k) where X_k $(X_k=(x_{k1},...,x_{kj},...,x_{km}))$ is the input vector and Y_k $(Y_k=(y_{k1},...,y_{ki},...,y_{ks}))$ is the output vector. Consider the dominance comparison for production unit k0 (where k0 belongs to the sample of n production units). DEA compares k0 to linear combinations of the n production units, i.e. $(\Sigma_k\lambda_kX_k, \Sigma_k\lambda_kY_k)$ with $\lambda_k\geq 0$ ($\lambda = (\lambda_1,\ldots,\lambda_n)$ is an intensity vector that forms convex combinations of observed input vectors and output vectors). Therefore, k0 is dominated in terms of inputs if $\Sigma_k\lambda_kx_{kj} \leq x_{k0j}$ holds for all inputs with strict inequality for at least one input and $\Sigma_k\lambda_ky_{ki} \geq y_{k0i}$ is satisfied for all outputs for at least one combination of production units. Similarly, if $\Sigma_k\lambda_kx_{kj} \leq x_{k0j}$ for all inputs and $\Sigma_k\lambda_ky_{ki} \geq y_{k0i}$ for all outputs with strict inequality for at least one output for at least one combination of production units, k0 is dominated in terms of outputs. Dominated production units are inefficient while undominated ones are efficient.

1.1 Production technology structure

If $\lambda_k\geq 0$ is the only restriction on λ then it is assumed that the underlying production technology satisfies constant returns to scale (CRS). The analysis with a variable returns to scale (VRS) technology can be undertaken by introducing the restriction that $\Sigma_k\lambda_k = 1$. Similarly, it is possible to construct non-increasing returns to scale (NIRS) and non-decreasing returns to scale (NDRS) technologies by changing the assumption that $\Sigma_k\lambda_k = 1$ to $\Sigma_k\lambda_k \leq 1$ (NIRS) or $\Sigma_k\lambda_k \geq 1$ (NDRS). Free Disposal Hull Analysis (FDH) restricts the dominance comparison for k0 to be with respect to other observed production units, i.e. FDH excludes linear combinations of production units from the analysis. Keeping the previous notation, FDH compares (X_{k0}, Y_{k0}) to ($\Sigma_k\lambda_k X_k$, $\Sigma_k\lambda_k Y_k$) where $\lambda_k \in \{0,1\}$ and $\Sigma_k\lambda_k$ = 1. The definition of dominance is as before, but the added restrictions on λ_k imply that it is less likely for a production unit to be dominated, i.e. inefficient.

1.2 Efficiency measures

Thus, DEA and FDH can be used to classify a set of production units into two subsets: (a) efficient production units and (b) inefficient production units. Additional information about the inefficient production units' deviation from efficiency can also be derived using DEA or FDH through the calculation of efficiency measures for each production unit. The efficiency measure quantifies the distance from the observation to the best-practice technology; i.e. it projects an inefficient unit onto the frontier.

A range of different types of efficiency measures can be calculated within the DEA model, where two key distinctions can be drawn:

- Orientation of the efficiency measure: input orientation, output orientation, or base-orientation
- Radial or non-radial efficiency measures

1.3 Orientation

Input oriented efficiency measure compares the actual input level for a given production unit to the best practice input level (defined as the combination of production units that dominate k0 the most), holding the outputs constant, i.e. it quantifies the input reduction required for the production unit to become efficient. Similarly, an output oriented efficiency measure relates the actual output level of a production unit to the potential (best-practice) output level, holding the inputs constant, i.e. the efficiency measure quantifies the required output expansion to become efficient. Base-oriented quantifies necessary improvements for both inputs and outputs in order for a production unit to become efficient. The choice of orientation would depend on the extent to which inputs, outputs or both are controllable. In the context of the bus industry it appears that input oriented models are definitely valid. The applicability of output or base oriented models would depend on the outputs chosen, e.g. passenger kilometres vs. seat kilometres (the latter output may be controllable by the bus company; this is not the case with passenger kilometres).

Figure 1 illustrates the role of orientations in DEA in the single-input-single output case. In the case of Observation A (an inefficient observation) an input-oriented efficiency measure would concern reductions in the input level used at A along the horizontal arrow holding the output level constant (with efficiency being achieved at X). An output-oriented efficiency measure would involve expansions in output level at A along the vertical arrow holding the input level constant (with efficiency being achieved at Y). Notice that the X and Y may not be observed (efficient) production units, but could be formed through combinations of production units.

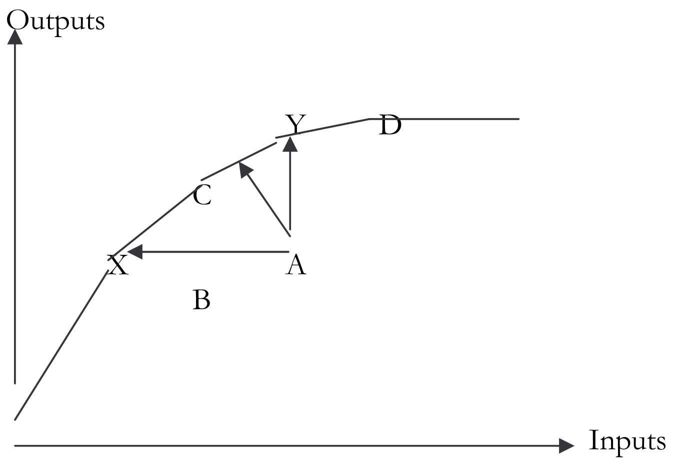

Figure 1: An Illustration of DEA Efficiency Analysis (Non-Increasing Returns to Scale).

1.4 Radial or non-radial efficiency measures

Radial efficiency measures (input, output or base orientation) determine the changes required for each observation in inputs and/or outputs to become efficient on the basis of equiproportionality, i.e. that all factors are changed by the same percentage.

For example, a radial input efficiency measure for k0 can be calculated as follows: For each dominating combination of production units, $(\Sigma_k \lambda_k X_k,\ \Sigma_k \lambda_k Y_k)$, compute the input ratios $(\Sigma_k \lambda_k x_{kj}) / x_{k0j}$. The smallest of these ratios $((\Sigma_k \lambda_k x_{kj}) / x_{k0j})^*$ which satisfies

$$\Sigma_k \lambda_k x_{kj} \leq ((\Sigma_k \lambda_k x_{kj}) / x_{k0j})^* \cdot x_{k0j}$$

for all inputs, is chosen as the input efficiency measure. The input efficiency measure will take values in the range from zero to one with inefficient production units having values below one. A necessary condition for a production unit to be input efficient is that the input efficiency measure is equal to one. A sufficient condition for input efficiency would require that

$$\Sigma_k \lambda_k x_{kj} = ((\Sigma_k \lambda_k x_{kj}) / x_{k0j})^* \cdot x_{k0j}$$

holds for all inputs. This problem is caused by the way the efficiency measure is calculated: it measures the proportionate reduction in the inputs necessary for a production unit to undertake in order to become efficient. However, after reducing all inputs proportionately further reductions for some inputs might be possible, i.e. slacks may exist. In a similar way a radial output or base-oriented efficiency measure can be derived for k0, but the details will not be included in this paper, see e.g. Fried et al. (1993).

The problem of slacks associated with radial efficiency measures can be addressed through so-called non-radial efficiency measures. A non-radial efficiency measure can be calculated in different ways, but the most common is the Färe-Lovell measure, see Färe & Lovell (1978). In the following, we will concentrate on the Färe-Lovell measure. The key element in the Färe-Lovell measure is the calculation of specific efficiency measures for each input and/or output. These specific efficiency measures should be determined such that the average required improvement across the inputs and/or outputs is maximised. In the case of the Färe-Lovell measure it is important to notice that a value equal to one is a necessary and sufficient conditions for efficiency as it would imply that each of the input and/or output specific efficiency measures are equal to one.

1.5 Examples of the mathematical programming problems for DEA/FDH efficiency measures

The calculation of efficiency measures can for both DEA and FDH be formulated as mathematical programming problems, see e.g. Fried et al. (1993) for an overview. For example, *the radial input efficiency measure* with CRS can be calculated through the LP problem

$$[1] \quad \text{MIN}_{\lambda_k} \ \theta_{k0}$$

$$\text{s.t.}$$

$$\begin{gathered}\Sigma_k \ \lambda_k x_{kj} \leq \theta_{k0} x_{k0j} \\ \Sigma_k \ \lambda_k y_{ki} \geq y_{k0i} \\ \lambda_k \geq 0\end{gathered}$$

where θ_{k0} is the efficiency measure. This measure takes values between 0 and 1.

Similarly, the *radial FDH output efficiency measure* can be determined in the Integer Programming problem

$$[2] \quad \text{MIN}_{\lambda_k} \ \theta_{k0}$$

$$\text{s.t.}$$

$$\begin{gathered}\Sigma_k \ \lambda_k x_{kj} \leq x_{k0j} \\ \Sigma_k \ \lambda_k y_{ki} \geq y_{k0i}/\theta_{k0} \\ \lambda_k \geq 0 \\ \Sigma_k \lambda_k = 1 \\ \lambda_k \in \{0,1\}\end{gathered}$$

Super-efficiency

The measure of super-efficiency was put forward by Andersen and Petersen (1993) as a way to distinguish between the efficient observations. In particular, the super-efficiency measure examines the maximal radial change in inputs and/or outputs for an observation to remain efficient, i.e. how much can the inputs be increased (or the outputs decreased) while not become inefficient. The larger the value of the super-efficiency measure the higher an observation is ranked among the efficient units. Super-efficiency measures can be calculated for both inefficient and efficient observations. In the case of inefficient observations the values of the efficiency measure do not change, while efficient observations may obtain higher values. Values of super-efficiency are therefore not restricted to 1 (for the efficient observations), but can in principle take any value greater than or equal 1.

Super-efficiency measures are calculated on the basis of removing the production unit from the best-practice reference technology. This explains why the inefficient observations do not change value by calculating super-efficiency measures, as the inefficient observations are not influencing the best-practice technology.

1.6 Strengths and weaknesses

A number of advantages of DEA and FDH analysis can be identified. One of the main advantages is that no functional form regarding the relation between inputs and outputs is necessary in order to compute the efficiency measures. Secondly, the techniques allow for multiple inputs and multiple outputs without the use of weighting factors. In this way a more valid model of production activities is provided. This implies that DEA/FDH can be applied in situations where inputs and/or outputs are measured in physical units creating the possibility for efficiency analysis for sectors without well-defined input prices and/or output prices. Furthermore, since DEA and FDH are based on a best-practice frontier, each observation is compared to an efficient unit or a combination of efficient units thereby providing guidance for the inefficient units concerning which areas of their activities to improve and by how much. In this sense the efficient units can act as peers for the inefficient ones. Overall, the best-practice units will be those, which not only are efficient but also, are included at least once as peer unit for an inefficient observations. Finally, the DEA/FDH techniques are consistent with the production theoretic concept of efficiency as this is based on the maximum output for given input levels.

However, DEA and FDH have also disadvantages where some of these are specific to these methods and others are pertinent to other performance measurement techniques as well. Firstly, it is assumed that it is possible to define and measure a set of inputs and outputs for each production unit and that these appropriately characterise the production activities. Related to the input-output specification is the issue of similarity. It is important that the production units included are similar in the sense that they can be described by identical input and output categories. Otherwise, observations can be declared as efficient due to a special output/input profile, which would imply meaningless results from the analysis. This problem is parallel to the problems of outliers. Production units with an extreme production structure (e.g. specialisation into a single output) may be declared as efficient simply because of the special production structure. Possible outlier influence is increased since DEA is an extreme point technique, implying the risk that even measurement error can have significant influence. The problems of non-similarity and outlier influence can imply that it is not possible to achieve a complete ranking of the production units because relative many will be characterised as efficient (the development of super-efficiency measures can address this problem, see above). In general, there is a trade-off between a realistic description of the production profile and a complete ranking. If the efficiency analysis is based on a few number of variables then it is likely that a complete ranking can be obtained but restricting the number of variables to describe the production might not give a realistic impression of the production activities. On the other hand, inclusion of many variables will provide a more reliable description of the production activities, but this increases the possibility for specialisation and therefore makes a complete ranking less likely. This problem has been addressed in two recent studies. In Olesen & Petersen (1993) a test is developed that determines the optimal number of variables to include in a DEA analysis. Kittelsen (1992) suggests a procedure that could establish a statistical optimal data specification.

Explaining efficiency

An important issue of the efficiency analysis is not only to determine the efficiency levels but also to be able to explain the variation with reference to characteristics of the production units. One possible approach is to interpret the efficiency measures as a dependent variable that is determined by a set of production unit characteristics, see e.g. Fried et al (1993a). Let $\theta = (\theta_1,\ldots,\theta_n)$ denote the vector of efficiency scores for the n observations and Z be a n×L matrix of L production unit characteristics. Thus a general regression model can be formulated as:

$$[3] \quad \theta_k = f(z_k;\beta) + e_k, \qquad k = 1,\ldots,n$$

where β are the parameters to be estimated, z_k is the vector of characteristics for the k'th unit and e_k is a disturbance term for the k'th unit. In order to estimate the vector of parameters β, assumptions about the functional form of $f(z_k,\beta)$ have to be made. This specification could be non-linear and thus require non-linear estimation techniques. However, since no apriori knowledge about the relationship between θ and z_k are available the tradition of assuming a linear relationship is adopted, i.e. the model

$$[4] \quad \theta = Z\beta + e,$$

This model can be estimated by Ordinary Least Squares (OLS), although it should be noted that the restrictions on the efficiency scores $0 < \theta \leq 1$ (or $0 < \theta$ in the case of super efficiency models) imply biased and inconsistent estimates of β unless a transformation of θ is undertaken. In Lovell et al. (1990) it is demonstrated that $\ln(\theta)$ will provide consistent and unbiased estimates of β provided θ is only restricted to take values above 0 (super-efficiency models). Otherwise the transformation $\ln((1-\theta)/\theta)$ is required.

DATA

The data used for the efficiency analysis is based on information for 157 of the 175 Norwegian subsidised bus companies. These data have been provided from official reports from the bus companies to the county councils for the 1991 calendar year. The complete database covers all 175 bus companies but 18 companies had to be discarded due to extreme observations and missing data for key variables to be used as inputs. Four companies appeared to have reported inaccurate data. Three other companies were considered to operate in incomparable conditions with reference to the other companies in the database (one of these is the main bus operator in Oslo, the other one is a small company with very low costs because some routes are served by hired taxi caps). Data for 11 companies could not be used in the analysis due to missing information on costs. Each Norwegian county is represented by at least one bus company and most counties have a number of entries in the database (the only exception is Finnmark County, the county furthest to the North with only a single bus company). The company size in the data set varies considerably; if number of vehicle kilometres is used as an indicator of size then the smallest company achieves approx. 11500 vehicle kilometres, the largest company provides 8.9 mill vehicle kilometres, while the average bus company provides 1.6 mill vehicle kilometres.

For each bus company the following data are available:

Continuous variables

- Vehicle kilometres
- Passengers
- Passenger kilometres
- Fuel costs
- Driver costs
- Total costs (incl. Capital costs)
- Fleet size
- Seats
- Standing places
- Bus size (sum of seating capacity and standing places)

- Seat kilometres
- Number of passengers boarding the buses of the company per vehicle km (derived from information on passengers and vehicle kilometres)

Dummy variables

- Bus company is engaged or not in sea transport
- Bus company is operates in a coastal area or not
- Bus company is publicly owned and faces a subsidy policy based on cost norm or not
- Bus company is privately owned and has the ability to negotiate with the county council over the size of the subsidy or not
- Bus company is privately owned and faces a subsidy policy based on cost norm or not

In addition, there is information about the county in which the bus company operates.

Below, descriptive statistics is given for the continuous variables including average, standard deviation, median, maximum and minimum, see Table 1. In Table 2 qualitative information about the sample is provided on the basis of the dummy variables.

	Vehicle kilometre	**Passengers**	**Passenger kilometres**	**Fuel Costs (Nkr)**	**Driver Costs (Nkr)**	**Total Costs (Nkr)**
Mean	1602695	1321076	17162420	1542060	9215587	22867571
Median	920000	385900	8793600	826797	4079758	11215782
Standard deviation	1818244	2499785	26366662	1896458	12827809	30370638
Maximum	8863117	16584953	208364607	9775000	72129317	176000000
Minimum	11500	4545	60840	2907	64000	123329
	Fleet Size	**Seat capacity**	**Standing places**	**Bus Size**	**Seat kilometres**	**Number of passengers boarding**

						per vehicle kilometres
Mean	43	1836	545	49	91318473	0.64
Median	28	1118	165	50	46332000	0.53
Standard deviation	43	1906	947	13	116970172	0.42
Maximum	204	9000	6729	86	620418190	2.66
Minimum	1	7	0	7	161000	0.05

Table 1: Descriptive Statistics for the Norwegian Bus Company Sample

Table 1 shows the variation in the scale of bus operation for the included companies; the smallest companies (according to fleet size) have only one bus while the largest one has over 200 buses (this company is operating in Hordaland County where Norway's 2nd largest city is placed (Bergen).

	Percentage of sample
Bus companies with sea transport	10.0
Bus companies without sea transport	90.0
Bus companies operating in a coastal area	47.0
Bus companies not operating in a coastal area	53.0
Publicly owned bus companies with subsidy allocation based on cost norm	9.0
Publicly owned bus companies with subsidy allocation based on negotiation	14.0
Privately owned bus companies with subsidy allocation based on negotiation	33.0
Privately owned bus companies with subsidy allocation based on cost norm	44.0

Table 2: Characterisation of Bus Sample

The table shows that a majority of bus companies do not operate sea transport. Table 2 confirms that the majority of subsidised bus companies are privately owned, 77 per cent in 1991. A slight majority (53 per cent) of bus companies received subsidy from the county council based on cost norms in 1991.

RESULTS

Input-output specification

A basic model for the productive activities undertaken by the bus companies was used for the calculation of the different efficiency measures. This model included four inputs and one output:

Inputs

- Fuel costs
- Driver costs
- Other costs
- Bus fleet size

Outputs

- Seat kilometres

The other costs component is calculated by subtracting fuel and driver costs from total costs. Other costs includes both operating costs and capital costs.

All efficiency measures have been calculated using the Efficiency Measurement System (EMS) software developed by Holger Scheel at University of Dortmund, Germany. This software is for Windows 9x/NT where data can be analysed through either Excel or textfiles.

DEA-C

Efficiency measures with a constant returns to scale technology have been calculated in input, output and base-oriented versions. In the following we will concentrate on the efficiency results with reference to input-oriented measures as the constant returns to scale technology assumption implies that input and output oriented efficiency measures obtain the same value. The same does not hold though for non-oriented efficiency measures, the required improvement will as a general property be smaller for non-oriented measures than for either input or output oriented efficiency measures.

In the case of the input-oriented efficiency, the average value is 0.68 (counting all efficient units with a value equal to one). This average is the outcome of significant variation in the efficiency scores obtained for the different bus companies ranging from 0.19 (the minimum) to 1.00 (the maximum) with an overall standard deviation of 0.18.

Out of the 157 observations 7 have obtained an efficiency score equal to one, where it should be noticed that no slacks exist for these observations, i.e. they can be characterised as efficient in accordance with the definition in economic theory. In Table 3 the results of a further analysis of the efficient observations are shown in terms of super efficiency scores and the number of times each of these observations are identified as benchmarks for inefficient observations.

	Super efficiency	Benchmark frequency
DMU10	1.07	95
DMU14	1.02	34
DMU16	1.90	82
DMU54	1.07	23
DMU128	1.02	24
DMU152	1.01	3
DMU164	1.38	128

Table 3: Super efficiency and Benchmark Frequency

These results indicate a positive correlation between super-efficiency and benchmark frequency although the correlation is not perfect (the correlation coefficient is 0.54). Three of the seven efficient units are placed in the same county, Østfold (with a relative high population density, 64). This county is located in the Southeast of Norway, next to the county with Norway's capital, Oslo. On average bus companies in Østfold have significant higher efficiency scores compares to the sample average. The remaining 4 bus companies are placed in different counties with no clear-cut trend with respect to the role of population density. This issue will be considered further as part of the explanation of the efficiency variation within a regression analysis approach (see below).

Concerning the inefficient observations the results suggest that input slacks are present after the equiproportionate reduction, i.e. inputs can be further reduced. Table 4 shows the average results in terms of percentages with radial slack to best-practice and non-radial slack to best-practice (total slack is the sum of the two percentages). The table shows that non-radial slacks are present for all four input variables, in particular with respect to driver costs and number of buses (where reductions of 23.5% and 20.1% respectively are possible).

	Total slacks (%)	Radial slacks (%)	Non-radial slacks (%)
Fuel costs	66.2	63.1	3.2
Driver costs	106.2	82.6	23.5
Other costs	74.2	65.6	8.6
Buses	110.4	90.3	20.2

Table 4: Slack Analysis

DEA-V

Efficiency scores calculated within a variable returns to scale technology will be greater than or equal the ones obtained within a constant returns to scale because the scale of operation for each observation is assumed given. Inefficiency under variable returns to scale cannot be the result of operating on a too high or too low scale. The results for the Norwegian bus companies confirm this property: average input efficiency is equal to (0.735), while output oriented efficiency is slightly lower (0.726). Results for average base-oriented efficiency indicate a required improvement in inputs and outputs of 16.7% in order for the inefficient observations to move to best practice. The variable returns to scale technology assumption also implies that more observations have the possibility to be declared efficient, indeed our results demonstrate that in input terms 21 observations have an efficiency score equal to one, while 20 observations have an efficiency score equal to one in terms of outputs. However, one of the observations with an efficiency score equal to one in input terms has non-radial slacks and is therefore not efficient. This conclusion is confirmed from the output efficiency score for this observation, as it is lower than one. As such this observation serves as an illustration of the need for careful examination of the results obtained in order to formulate appropriate conclusions.

In Table 5 further information about the efficient units is provided concerning benchmark frequency and super-efficiency.

	Super efficiency	Benchmark frequency
DMU5	1.065	20
DMU8	1.241	1
DMU10	1.087	78
DMU14	1.033	28
DMU15	1.018	3
DMU16	1.909	58
DMU21	1.038	12
DMU38	1.065	17
DMU54	1.197	29
DMU55	4.362	68
DMU61	1.004	0
DMU93	1.006	13

DMU97	Big (Note)	1
DMU128	1.142	19
DMU132	1.021	1
DMU146	1.238	24
DMU151	1.061	4
DMU152	1.050	2
DMU162	2.501	7
DMU164	1.517	106

Note: The efficiency score = big appears within the super-efficiency model when a unit remains efficient under arbitrary large increased inputs (input oriented) or decreased outputs (output oriented), respectively.

Table 5: Super Efficiency and Benchmark Frequency

In this case there is no correlation between super-efficiency and benchmark frequency. The reason for the possibility of lack of association between these two measures is that a high super-efficiency score can be obtained through specialisation whereas a high benchmark frequency cannot.

Scale-efficiency

DEA can be used to provide information about scale efficiency for each observation in terms of inputs and outputs respectively. The ratio of the DEA-C efficiency score to the DEA-V input oriented efficiency score (output oriented efficiency score) determines the input (output) oriented scale efficiency measure. This scale efficiency measure can take values in the interval]0,1], where 1 will imply scale efficiency. A value of the scale efficiency measure equal to one reflects that the DEA-C and DEA-V scores are identical, i.e. the efficiency score of a given observation is not influenced by moving from a constant returns to scale technology to a variable returns to scale technology. The results for the Norwegian bus company sample indicate high levels of scale efficiency in both input and output terms, 0.93 and 0.94 respectively. In this case the majority of the detected inefficiency under constant returns scale is not caused by bus companies operating on a too high or too low scale.

A DEA analysis can also establish the direction of scale inefficiency, i.e. too high scale (decreasing returns to scale, DRS) or too low scale (increasing returns to scale, IRS). If an observation operates according to constant returns to scale, it is declared scale efficient. In the case of the Norwegian bus companies the results suggest that a majority of the 157 companies operate under IRS (91). 59 companies produce under DRS, while 7 observations produces according to constant returns to scale. Therefore, a majority of the bus companies should increase the scale of operation in order to achieve the optimal scale.

Table 6 demonstrates the high degree of variation concerning the scale of operation for the scale efficient observations. The average fleet size among these (7) observations is equal to 31.6 (approx. 10 buses lower than the overall sample average). This average is the result of fleet size variation from 4 to 96.

	Vehicle kilometres	Fleet size	Seat kilometres
DMU10	2790000	53	181350000
DMU14	353900	8	21587900
DMU16	230000	15	11270000
DMU54	4900500	96	357736500
DMU128	94000	4	4888000
DMU152	752400	13	48906000
DMU164	1321317	32	80600337
Average	1491731	31.57	100905534
Min	94000	4.00	4888000
Max	4900500	96.00	357736500
Standard deviation	1766406	33.03	128513873

Table 6: Scale Efficiency Variations

FDH

FDH efficiency scores have been calculated for the 157 bus companies in terms of inputs and outputs. The use of FDH implies that the efficiency scores will be greater than or equal compared to the scores obtained with DEA-V and increases the probability for observations with efficiency score equal to one. Overall, the average output efficiency score is equal to 0.941 while the average input efficiency score is equal to 0.939. A larger number of observations obtain an efficiency score equal to one, 102 in terms of inputs and 98 in terms of outputs. The four additional observations with input efficiency score equal to one compared to the number with output efficiency measures are not efficient in the sense that non-radial slacks are present for these observations with respect to three out of four inputs. The only input without slacks for these observations is number of buses. Furthermore, some of the observations with an efficiency score equal to one are not dominating any other observations in the sample. In this sense such observations can be said to be efficient by default. In Table 7 the average values of the efficiency measures for DEA-C, DEA-V and FDH are shown providing the possibility to decompose overall efficiency into the sub-components of pure technical efficiency, scale efficiency and convexity efficiency.

	Output efficiency	Input efficiency
DEA-C	0.680	0.680
DEA-V	0.726	0.735
FDH	0.941	0.939
Pure technical efficiency	0.941	0.939
Convexity efficiency	0.772	0.783
Scale efficiency	0.939	0.930
DEA-C	0.680	0.680

Table 7: Decomposition of Efficiency

Convexity efficiency is determined as the ratio of DEA-V and FDH efficiency scores (in input and output terms). If efficiency scores calculated with DEA-V and FDH are identical it would imply that the convexity efficiency score is equal to one. Otherwise, the convexity efficiency score will take values between zero and one. In this way the convexity efficiency score can be used to assess the impact of assuming convexity on the efficiency results obtained. Table 7 shows that convexity does have a significant influence on the level of efficiency.

Färe-Lovell measures

The non-radial Färe-Lovell efficiency measure contains detailed information concerning the performance for each of the included inputs and/ or outputs. For example, the input-oriented Färe-Lovell measure will not only provide an overall efficiency score but also determine input specific efficiency scores (the Färe-Lovell measure is then calculated as the average of these individual efficiency scores).

An illustration of the results from the Färe-Lovell input oriented efficiency measure is given in Table 8. The results shown are calculated with an assumption about constant returns to scale, although changing to another technology assumption would not influence the interpretation of the concepts. Table 8 includes information about two of the observations in the Norwegian bus company sample, DMU1 and DMU54. DMU1 is inefficient with a Färe-Lovell efficiency score equal to 0.723. Furthermore, it can be seen that the required improvements differ between the four inputs. In the case of DMU1 inefficiency is higher with respect to number of buses compared to other input elements such as fuel costs. The table also demonstrates that in order for an observation to get a Färe-Lovell efficiency score equal to one, it is necessary that each of the individual input efficiency scores are equal to one. This is the case for DMU54. On average the inputs with the highest degree of efficiency are other costs and fuel costs, whereas buses and driver costs should be improved to a larger extent.

	θ_1 (fuel costs)	θ_2 (driver costs)	θ_3 (other costs)	θ_4 (buses)	θ_{FL} (Note)
DMU1	0.795	0.711	0.700	0684	0.723
DMU54	1.000	1.000	1.000	1.000	1.000
Average	0.610	0.498	0.712	0.597	0.604

Note: θ_{FL} is calculated as the average of θ_1-θ_4

Table 8: The Non-Radial Färe-Lovell Input Oriented Efficiency Measure

Efficiency explanation model

The available information provided the possibility to examine the extent to which the efficiency scores can be explained using a number of factors that may be of importance in shaping performance of bus companies. In particular, the following factors were considered as possible explanatory variables (involving a combination of continuous and dummy variables:

- Bus company is publicly owned and faces a subsidy policy based on cost norm or not (H1)
- Bus company is privately owned and has the ability to negotiate with the county council over the size of the subsidy or not (H2)
- Bus company is privately owned and faces a subsidy policy based on cost norm or not (H3)
- Bus company is engaged or not in sea transport (D1)
- Bus company is operates in a coastal area or not (D2)
- Average bus size (Z1)
- Number of passengers boarding the buses of the company per vehicle-km (Z2)
- Population density (DENSE)

Regressing the logarithm to the DEA-C efficiency measure (with super-efficiency) on these variables gives a rather high R^2 (0.86) although only four variables are significant at a 5 per cent level (the full model). Therefore, it was decided to exclude these variables in another model (the reduced model). In Table 9 the estimated values for the coefficients in the two models are shown together with the t-statistics.

	Full Model		Reduced Model	
	Coefficient	t-values	Coefficient	t-value
Intercept	-1.712	-31.494	-1.687	-38.383
H1	0.022	0.516		
H2	-0.027	-0.825		
H3	0.030	0.942		
D1	0.062	1.839		
D2	0.064	2.850	0.054	2.428
Z1	0.012	12.309	0.012	12.721
Z2	0.331	14.977	0.330	14.798
DENSE	-0.000	-3.027	-0.000	-3.289

Table 9: Regression Results

The reduced model can also explain a high proportion of the variation in the dependent variable, ln(θ), as reflected by $R^2 = 0.85$. Parameter estimates in the reduced model are not significantly different from the ones obtained in the full model. It should be noticed that among the variables with apparent insignificant contribution to the explanation in efficiency variation are the policy variables (h1, h2, h3) relating to subsidy form and ownership dimensions. The findings suggest that higher efficiency is associated with operation in inland area rather than coastal area (D2), bus size (Z1), and number of passengers boarding per vehicle kilometre (Z2).

Comparison between non-parametric and parametric results

A comparison between the non-parametric efficiency results presented in this paper and the parametric ones included in Jørgensen et al. (1997) is not straightforward, although the same data are used. In particular, the efficiency results are based on different models. The parametric efficiency results are derived taking into account exogenous factors, whereas the approach in this paper calculates the efficiency results and then explains the efficiency variation according to exogenous factors. Therefore, it should be expected that the results are different with the parametric efficiency being at a higher level than non-parametric efficiency because of the approach towards the exogenous factors. Indeed, in Jørgensen et al. (1997) average inefficiency is between 7.2 per cent and 13.7 per cent, whereas DEA results indicate a higher level of inefficiency. Average base-oriented DEA-C improvement has been determined to be 20.5 per cent, while average base-oriented

DEA-V improvement is equal to 16.7 per cent. Overall, the Pearson Correlation Coefficient for the non-parametric and parametric results has been estimated to be between 0.28 and 0.42.

CONCLUSIONS

This paper has presented the results of an analysis of efficiency patterns for Norwegian bus companies using the non-parametric techniques DEA and FDH. Overall, the paper has demonstrated that it is feasible to use these techniques to examine the productive performance of bus companies. In particular, the application has shown that DEA and FDH can provide useful information regarding the efficiency patterns. This information relates both to the industry as well as to the individual companies. In the Norwegian bus industry a relative high inefficiency level was detected. Obviously, the efficiency results depend on the technology assumption used. However, the difference between DEA-C and DEA-V was relatively small indicating a high level of scale efficiency. In contrast, the change from a DEA to a FDH model resulted in significant changes in efficiency level demonstrating the importance of the convexity assumption. In the paper it was also shown the significance of slacks in the inputs and/or outputs emphasising the need for careful analysis of observations with efficiency scores equal to one. The scope for providing valid explanations of the efficiency patterns was examined, where the research revealed that a relative simple model with four variables could explain around 85 per cent of the variation in efficiency.

Future research could consider the extent to which it is possible to develop alternative output measures in order to allow for consideration to the quality of the bus service provision in the measurement of efficiency. Furthermore, at a more theoretic level it could of importance to examine the scope for converging non-parametric approaches towards parametric approaches and vice versa. Indeed, it could be of importance to develop non-parametric efficiency measurement techniques with a stronger statistical basis. Similarly, possible improvements in the parametric approach could accommodate for more flexible functional forms concerning the linkage between inputs and outputs.

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REFERENCES

ANDERSEN, P. & Petersen, N.C. (1993) A procedure for ranking efficient units in data envelopment analysis. *Management Science*, **39** (10), pp. 1261-1264.

BANKER, R.D., CHARNES, A. & COOPER, W.W. (1984) Models for estimation of technical and scale inefficiencies in Data Envelopment Analysis. *Management Science*, **30** (9), pp. 1078-1092.

BANKER, R.D. & MOREY, R.C. (1986a) Efficiency analysis for exogenously fixed inputs and outputs. *Operations Research*, **34** (4), pp. 513-521.

BANKER, R.D. & MOREY, R.C. (1986b) The use of categorical variables in Data Envelopment Analysis. *Management Science*, **32** (12), pp. 1613-1627.

BANKER, R.D. & THRALL, R.M. (1992) Estimation of returns to scale using Data Envelopment Analysis. *European Journal of Operational Research*, **62** (1), pp. 74-84.

CARLQUIST, E. & JOHANSEN, K.W. (1999) Local public transport systems – financial and organisational frameworks in Norway and abroad. *TØI Report 451/1999*.

CHARNES, A., COOPER, W.W. & RHODES, C. (1978) Measuring the efficiency of decision making units. *European Journal of Operational Research,* **2** (6), pp. 429-444.

DEPRINS, D., SIMAR, L. & TULKENS, H. (1984) Measuring labour efficiency in post offices. In Marchand, M., Pestieau, P., & TULKENS, H. (eds.) *The Performance of Public Enterprises.* Amsterdam, North-Holland.

FILLIPINI, M., MAGGI, R. & PRIONI, P. (1992) Inefficiency in a regulated industry: the case of Swiss regional bus companies. *Annals of Public and Cooperative Economics*, **63** (3), pp. 437-455.

FRIED, H.O., LOVELL, C.A.K. & SCHMIDT, S. S. (1993) *The Measurement of Productive Efficiency: Techniques and Applications*. Oxford University Press, Oxford.

FRIED, H.O., LOVELL, C.A.K. & VANDEN EECKAUT, Ph. (1993a) Performance of US credit unions. *Journal of Banking and Finance*, **17**, pp. 251-265.

FÄRE, R. & LOVELL, C.A.K. (1978) Measuring the technical efficiency of production. *Journal of Economic Theory*, **19** (1), pp. 150-162.

FÄRE, R., LOVELL, C.A.K. & ZIESCHANG, K. D. (1983) Measuring the technical efficiency of multiple output production technologies. In EICHHORN, W., HERN, K., NEUMANN, K. & R. SHEPHARD, R. (eds.) *Quantitative Studies on Production and Prices*. Physica-Verlag: Wien, Vienna.

FØRSUND, F. & HJALMARSSON, L. (1979) Generalised Farrell Measures of efficiency: an application to milk processing in Swedish dairy plants. *Economic Journal,* **89** (354), pp. 294-315.

GATHON, H.-J. (1989) Indicators of partial productivity and technical efficiency in the European urban transit sector. *Annals of Public and Co-operative Economics*, **60** (1), pp. 43-59.

GROSSKOPF, S. (1986) The role of the reference technology in measuring productive efficiency. *Economic Journal,* **96** (382), pp. 499-513.

JØRGENSEN, F., PEDERSEN, P.A. & SOLVOLL, G. (1995) The costs of bus operations in Norway. *Journal of Transport Economics and Policy*, **24** (3), pp.253-262.

JØRGENSEN, F., PEDERSEN, P.A. & VOLDEN, R. (1997) Estimating the inefficiency in the Norwegian bus industry from stochastic cost frontier models. *Transportation* **24**, pp.421-433.

KITTELSEN, S. (1992) Stepwise DEA: Choosing variables for measuring technical efficiency in the norwegian electricity distribution. *Paper presented at the Nordic Workshop on Productivity and Growth in Gothenburg 25-27 November 1992*.

KERSTENS, K. & VANDEN EECKAUT, P. (1995) The economic cost of subsidy-induced technical inefficiency: a methodological postscript. *International Journal of Transport Economics*, **22** (2), pp. 225-229.

NORHEIM, B. & CARLQUIST, E. (1999) Markedseffektiv kollektivtransport? En analyse av utviklingen I Oslo, Bergen, Trondheim, Kristiansand og Tromø (title in Norwegian). *TØI Report 428/1999*.

OBENG, K. (1994) The economic cost of subsidy-induced technical inefficiency. *International Journal of Transport Economics*, **21** (1), pp. 3-20.

OBENG, K. (1995) The economic cost of subsidy-induced technical inefficiency: a reply. *International Journal of Transport Economics*, **22** (2), pp. 231-236.

OBENG, K. & AZAM, (1995) The intended relationship between federal transit operating subsidy and cost. *Public Finance Quarterly*, **23** (1), pp. 72-94.

OLESEN, O. & PETERSEN, N.C. (1993): Indicators of ill-conditioned data sets and model misspecification in Data Envelopment Analysis: aggregation as a potential remedy. *Publications from Department of Management, Odense University, no. 3/1993*.

OUM, T.H. & YU, C. (1994): Economic Efficiency of Railways and Implications for Public Policy: A Comparative Study of the OECD Countries Railways. *Journal of Transport Economics and Policy,* **28** (2), pp. 121-138.

TULKENS, H. (1993): On FDH Efficiency Analysis: some methodological issues and applications to retail banking courts and urban transit. *Journal of Productivity Analysis,* **4** (½), pp.183-210.