

ONE TRAIN OPERATOR AND TWO INFRASTRUCTURE MANAGERS: A SIMPLE MODEL TO EXPLORE THE ISSUE OF INFRASTRUCTURE CHARGING

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CONTEXT

- International link or « French concession »
- two infrastructure managers
- a certain degree of freedom to set infrastructure pricing.
- example here : Paris - Tour -Bordeaux. Tour-Bordeaux is operated in concession.
- Model during peak hours.

OPERATOR MODEL 1

- Linear demand :
- $q=q_0-b.p$, q = passengers demand, p = price.
- K = capacity of a train;
- C_1 = marginal cost of a train, constant.
- operator regulated to cover marginal cost.
- p_0 defined by $p_0.K=C_1$

OPERATOR MODEL 2

- $q_1=q_0-b.p_0$
- $\exists m$ such as :
- $m-1 \leq q_1/K < m$ (if $m=1$, no train)
- the demand is rationed to fill exactly $m-1$ trains, thus the price increases from p_0 to p , such as :
- $q_0-b.p=(m-1).K$

$$p = \frac{q_0 - (m-1).K}{b}$$

- Revenue :
- $(m-1)(p.k-C_1) > (m-1)(p_0.k-C_1) = 0$

OPERATOR MODEL 3

- The operator can choose to increase further the prices :
- $q=(m1-1)K$
- $p=(q0-(m1-1)K)/b$, there is a function* $p(m)$
- $Bn=$ « benefit » from operating n trains
 $= p.q-C= [(q0-n.K)/b] .n.K- n.C1$
- the reciproque function of $B(n)$ is $n(B)$

* a discrete function

OPERATOR MODEL 4/ EXISTENCE OF A MAXIMUM BENEFIT

- $Bn-1 - Bn > 0 \Leftrightarrow$
- $(q0-(n-1).K).(n-1).K/b-(n-1)C1 - (q0-n.K).n.K/b-nC1 > 0$
- \Leftrightarrow
- $n > ([q0.K-C1.b]/K^2+1)/2 \equiv n0$
- if $n > n0$, the benefit increases as the number of trains decreases.
- $B(n0) \equiv Bmax$, $b(p)$ is the « benefit » at price p .

TWO INFRASTRUCTURE MANAGERS, IM1 AND IM2

Each one is the owner of a part of the track

IM1 Example : Paris Bordeaux

IM2 has a « concession »

BOT for 65 years on Tour Bordeaux

IM2 In this model, the IMs will increase their infrastructure charges and thus diminish the number of trains to achieve their objectives IM2

We will consider only the traffic between two cities

A) If IM1 and IM2 are charging marginal cost, constant:

- same calculation as before with $C2=C1+cm1+cm2$
- in comparison to previous case; $q1$ is smaller; m is smaller; p is larger

B) IM2 pricing is marginal cost IM1 pricing is full cost (including capital cost)

- Let us denote $k1=K1.i$,
 $K1=$ capital spend to build infrastructure 1;
 $i=$ interest rate.
- Model with $C=C2=C1+cm1+cm2$.
- If $k1 < Bmax$, IM1 can recover its costs by iteration.
IM1 sets the price to $k1/(m-1)$, the number of train is $m1$
IM1 sets the price to $k1/m1$, the number of train is $m2$ and so forth as $k1 < Bmax - cm1 - cm2$, there is one me such as
 $B(me) > k1$ and $P(me) > k1/me$.

- IM2 is subsidizing IM1
- If $k_1 > B_{max}$, IM1 cannot fully recover its costs and IM1 needs subsidies.

C) Both IM want to recover their capital costs

- Not unlikely : IM1 wants to minimize the subsidies IM2 does not want to subsidize IM1.
- There are two cases :
 - 1) $k_1 + k_2 < B_{max}$
this case is similar to case B).
 - 2) $k_1 + k_2 > B_{max}$ 11

C) 2) $k_1 + k_2 > B_{max}$

- IM2 chooses $n(k_2)$, thus $p(n(k_2))$, IM2 don't know what infrastructure charging will be chosen by IM1.
- Similarly IM1 chooses $p(n(k_1))$.
- The outcome is a price superior to $p(n(B_{max}))$
- $(k_1 + k_2)/m > B_{max}/m \forall m$, number of trains running.
- It follows that they cannot reach a « benefit » of $k_1 + k_2$
- but the number of trains is reduced and the « benefit » is reduced with respect to B_{max} or anything between B_{max} and $b((k_1 + k_2)/m)$
- this situation is clearly sub optimal.

C) 2) $k_1 + k_2 > B_{max}$

How to exit this sub optimal situation ?

- This situation looks like a Bertrand curse : each one of the IM should benefit from a decrease in infrastructure charge but the one that decreases effectively its charge benefit less than the other.
- The solutions to exit this sub optimal situation could be :
 - long term contracts between the IM
 - regulation by a supervisory agency.¹³

CONCLUSIONS

- Possibilities of cross subsidies between IM
- Possibilities of sub optimal situation (the Bertrand curse)
- ==> necessity to co-ordinate and to regulate.