ESSAYS IN OPTIMAL AUCTION DESIGN

by
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Statement of Originality

This is to certify that to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or other purposes. I certify that the intellectual content of this thesis is the product of my own work and that all the assistance received in preparing this thesis and sources have been acknowledged.

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Abstract

Auctions are an ancient economic institution. Since Vickrey (1961), the development of auction theory has lead to an extremely detailed description of the often desirable characteristics of these simple selling procedures, in the process explaining their enduring popularity. Given the pervasiveness of auctions, the question of how a seller should engineer the rules of these mechanisms to maximize her own profits is a central issue in the organization of markets. The seminal paper of Myerson (1981) shows that when facing buyers with Independent Private Values (IPVs) a standard auction with a specifically selected reserve price (or prices) is *optimal*, that is, maximizes a seller's expected profits among all conceivable selling mechanisms. In this model, it is assumed that the buyers have perfect information as to the existence of gains from trade. We shall argue that the consequences of this assumption for the design of the optimal auction are not well understood, which motivates our analysis.

The three essays of this thesis relax the 'known seller valuation' assumption by examining the optimal auction program when the seller (and principal) holds private information representing her reservation value for the good. In the first essay we provide an original technique for comparing *ex ante* expected profits across mechanisms for a seller facing $N \geq 1$ potential buyers when all traders hold private information. Our technique addresses mechanisms that cannot be ranked point-by-point through their allocation rules using the *Revenue Equivalence Theorem.* We find conditions such that the seller's expected profits increase in the slope of each buyer's allocation probability function. This provides new intuition for the fact that a principal does not benefit from holding private information under risk neutrality. Monopoly pricing induces steep probability functions so the seller/principal benefits from announcing a fixed price, and implicitly her private information. An application is presented for the well known k double auction of the bilateral trade literature.

In the second and third essays of this thesis, we extend the above framework to allow for informational externalities. Specifically, we allow for the situation in which the seller's private information represents a common value component in buyers' valuations. Thus the seller's private information (say regarding the quality of the good) is of interest to bidders independently of any strategic effects. In recent work Cai, Riley, and Ye (2007) have demonstrated that a seller who holds private information about the quality of a good faces an extra consideration in designing an auction; the reserve price signals information to bidders. In a separating equilibrium signalling is costly in the sense that reserves are higher than would be optimal under complete information. We examine the returns to the seller in an English auction from using different types of secret reserve regimes. We find that immediate disclosure of a reserve is preferable to announcement after the auction in the form of a take-it-or-leave-it offer to the winning bidder. Sale occurs less often during the auction for a given reserve price *strategy* under secret reserve regimes, which increases the incentive for the seller to report more favourable information though the reserve price offer. Separating equilibria involving later announcement therefore generate even lower expected profits to the seller (signalling is more costly) than under immediate disclosure.

In the third essay we compare the benchmark signalling equilibrium of immediate disclosure to a screening regime which we call the *Right of Refusal*. In this extreme form of a secret reserve the seller never announces the reserve price, she simply accepts or rejects the auction price. We find that the *Right of Refusal* dominates immediate disclosure if the seller's valuation is a sufficient statistic for the private information of interest. Thus a seller with market-relevant private preference information can benefit from not exercising monopoly price setting power. The result also provides conditions under which a competitive screening equilibrium is more efficient than a signalling mechanism. Broadly speaking, screening is better when the common value aspect in the preferences of the informed and uninformed parties are 'aligned', and potential gains from trade to the uninformed party are significant. We believe this conclusion to be of particular interest to the design of privatization schemes.

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[1] A note to the reader on the presentation: Propositions, Theorems, Observations etc. are indexed to the subsection in which they appear for ease of reference; 3.2, 4.1 etc. The third number indicates the counter for the number of Propositions in the relevant subsection, eg 3.2.1, 3.2.2. Proofs are contained in the body of the thesis where they aid the interpretation of the results. Otherwise, they are relegated to the appendix.

Chapter 1

Introduction and Literature

The organization of markets is arguably the most fundamental economic problem. In deregulated markets, this question becomes considerably more focused; how should a seller design a trading mechanism in order to maximize her profits? Under complete information the solution is almost entirely described by the characteristics of supply, i.e. where the market lies on the spectrum from perfect competition to monopoly. Incomplete information changes the structure of this problem by restricting the extent of competition between sellers (since buyers face search costs), and providing potential surpluses to the buyers even under price discrimination (since their willingness-to-pay is private information). As a result of these considerations, auctions have become a powerful abstraction in the analysis of markets.

Auctions provide the necessary structure to represent the informational constraints of thin, high-search-cost or differentiated product markets . In most formalizations, buyers are subject to the supply decisions of a price-discriminating monopolist. The monopolist on the other hand is dealing with a fixed finite group of buyers with different preferences for the good. While she is aware of some limited properties of market demand such as the distribution of buyer valuations, the monopolist cannot observe their realizations. These markets can then be modeled as games of incomplete information with a fixed number of players in the sense of Harsanyi (1967). The application of the Bayesian equilibrium concept to this problem has lead to significant advancements not only in economic theory, but in the design of markets in practice, from electricity scheduling to the sale of natural resources, privatization and the organization of financial markets.

It is difficult to overstate the growth of interest in auction theory and the breadth of its application in the last 15 years. The global movement toward privatization of state-owned assets has seen a number of theorists involved in both the design of the sale procedure and its execution through bidding consultancies. In 1993-94 the format of the FCC's radio spectrum auction was significantly influenced by the advisory contributions of Preston McAfee, Paul Milgrom and Robert Wilson. Similarly in Britain, Ken Binmore and Paul Klemperer's input in the design of the 3G telephony auctions in 2000 helped raise US$34 billion in revenue.[1] Arguably the popularity of auctions as a means of allocating assets has been encouraged by the increasing body of theoretical research that casts doubt over the central tenet of the Chicago school; that efficiency is inevitable under ordinary market forces. When preference information is private, the logic of the Coase theorem is violated, and a secondary market will generally be inefficient.[2] Thus the initial allocation serves as a crucial factor in determining whether gains from trade will be exhausted.[3] Auctions help to alleviate these concerns by introducing competitive pricing which, as Al Gore noted in his address during the FCC auctions, is likely to put assets "*into the hands of those who value them the most.*"[4]

Of course, this logic is not without loss of generality. Any criticism than can be leveled at a secondary market on efficiency grounds can also be directed at the initial allocation, unless the existence of gains from trade between buyers and the seller is common knowledge, or is made irrelevant by a commitment to sell.[5] Maskin (2004) (pp. 1104) in reviewing Milgrom (2004) describes the problem as stemming from property rights. Ownership introduces a use value for the seller which is inevitably private information; "*there exists no nonconfiscatory mechanism...capable of attaining efficiency once the assets are in private hands*". The findings of Cramton, Gibbons,

[1]While this is a testament to the power of auctions, auction theory has a delicate structure. In Britain, as in the FCC auctions, the contributing theorists argued strongly that a complete understanding of all facets of the asset, the market and the players were vital to these successes, see Binmore and Klemperer (2002).

[2]The logic follows from the famous Myerson-Satterthwaite Theorem (see Myerson and Satterthwaite (1983)) which we discuss in Section 1.2.

[3]This argument and the importance of auction design that it connotes are elegantly put forth in Milgrom (2004).

[4]See again Milgrom (2004).

[5]In fact, even without such complications asymmetries between buyers can create inefficiencies in an auction that cannot be redressed by a secondary market. Krishna and Hafalir (2008) show that a first-price auction without a reserve price; a mechanism that is inefficient under asymmetry; remains inefficient even when a secondary market is added in which the bids from the auction are public information.

and Klemperer (1987) lay bare the constraint that property rights introduce. While a partnership of $N \geq 2$ can be dissolved efficiently (even when valuations are private) if no player owns too large an initial share, their Proposition 2 proves that no single-owner initial allocation can be efficiently dissolved.[6] If there is any possibility that gains from trade do not exist between the seller and buyers, and a seller does not eliminate this concern *ex ante*[7] an auction or any other mechanism will be inefficient. Further, the situation in which there is uncertainty as to the existence of gains from trade is arguably closer to the norm, given that a commitment to sell in every realization will only occur under extreme financial or political duress.[8] The extension to allow two-sided private information is therefore an important one. While the effect of this generalization on the efficiency objective has been adequately explained by the literature, this thesis will show that the players' priors regarding the existence of gains from trade are also influential for the optimality of the mechanism. We now review the foundations of auction theory and its relation to the design of optimal selling mechanisms as will be studied in this thesis.

1.1 Early Developments in Auction Theory

Auction theory has its genesis in the seminal work of Vickrey (1961), who analyzed equilibrium behaviour in first-price (Dutch) and second-price (English) auctions. Vickrey's crucial insight was that a bidder's problem is not only strategic but informational; she is competing against other bidders whose actions, as well as the preference information that informs them are unknown. His analysis revealed phenomena such as the dominance of truthful bidding in second-price auctions and revenue equivalence that were only to be fully understood 20 years later.

[6]As always, the type with the lowest interim utility is the type who does not expect to trade. However, partial ownership means that this type expects to retain her original share, and so has an 'average' valuation (in a sense that is dictated by the allocation rule). The IR constraint is less problematic here than in the single owner case because the lowest utility type experiences zero trade only on average. She earns profits if the types of others are higher or lower than expected, as opposed to the lowest type of a defined seller or buyer who never trades at all.

[7]Say by using a format in which the highest bidder wins and there is no reserve price.

[8]Privatization would be the most likely exception, although it is still hard to envisage a situation in which state assets would be sold at *any* price. Presumably the asset would have to have negative value to the state, in which case there would likely be few willing buyers.

As would be expected for a first investigation, Vickrey's work can be regarded as analytically positive, in the sense that he was concerned with explaining the properties of given auction forms. This scheme was broadly followed for some time, albeit using slightly more sophisticated methods. For example, Riley and Samuelson (1981) proved revenue equivalence in the common auction forms for general distribution functions, while Holt (1980) and Maskin and Riley (1984) demonstrated the failure of such equivalence under risk aversion. While certainly important, early advances in auction theory were somewhat piecemeal, where the characteristics of equilibria in the common auction forms were derived in increasingly general settings.[9] This was all to be changed by the path-breaking work of Myerson (1981). Myerson's use of the *Revelation Principle* and characterization of incentive compatibility facilitated general statements about utilities, expected payments and allocation probabilities in equilibrium across the space of all conceivable selling mechanisms. Remarkably, these results are contingent on just two conditions; statistical independence and risk neutrality. Myerson derived the most celebrated result in auction theory; the *Revenue Equivalence Theorem*, which states that any two mechanisms with the same allocation rule differ by at most a constant in terms of their expected revenues.[10] He then showed that under symmetry any of the common auction forms with a suitably selected reserve price implement the optimal auction, that is, maximize the seller's expected profits.[11] In the Myerson model, any *a priori* competitive effects between buyers and the seller are effectively assumed away in order to concentrate on the relationship between the ordering of bidders and the pricing rule.

The focus of auction theory today is still very much on the behaviour of bidders under different auction forms, valuation/information structures and

[9]The exception is Wilson (1967, 1969) who introduced the 'mineral rights' model, which proved to be a crucial development. In the general form of this model bidders hold private information that is a noisy signal of the object's true common value. This was to lay the foundations for the general interdependent values, affiliated signals model developed in the classic paper of Milgrom and Weber (1982a), which we discuss later.

[10]The importance of the *Revenue Equivalence Theorem* in the framework of auction theory notwithstanding, Milgrom (2004) (Chapter 3) emphasizes that the result is really a corollary of what he calls *Payoff Equivalence*, which can be demonstrated using the integral form of Hotelling's Lemma. Further, the intuition does not require private values. His Theorem 3.5 proves that *Payoff Equivalence* holds under interdependent valuations provided signals are independently distributed. A similar form of this result and its application to our analysis are discussed in Section 1.4.

[11]Riley and Zeckhauser (1983) give a similar result for a single buyer, but in a less general setting.

risk preferences, and relating these to the efficiency or revenue objective. The seller's role in this process is usually seen as passive, or at least non-competitive.[12] The seller facilitates the sale, and having voluntarily chosen to do so, it is presumed that, from the perspective of the participating buyers, gains from trade are known to exist. Because (or perhaps symptomatic) of this, most models assume, as did Myerson, that the seller's reservation value is public information. A striking feature of the optimal auction is that despite this restriction, and indeed the one-shot setting, the asset is withheld from the market with positive probability.[13] We argue in this chapter that the relationship between the optimal auction solution and this assumption has not been adequately explained by the literature, which motivates our contribution.

The majority of this thesis is dedicated to the relaxation of the 'known seller valuation' assumption. We investigate the influence of private seller information on optimal auction design in various theoretical and applied environments. The existing literature (see Williams (1987)[14]) suggests that allowing private seller information will not change the first-best solution in the IPV setting; the optimal auction of Myerson will remain optimal. The first part of our analysis will explain this phenomenon, and use these observations to present new results for auction design under two-sided private information in settings in which the first-best design cannot be implemented. Given this objective, it is instructive to first review the relevant results from bilateral trade. These models study the link between gains from trade and strategic considerations through a model in which two parties, buyer and seller, attempt an exchange when both hold private information as to their perception of an object's value. We now discuss this literature.

[12] For a thorough summary of the state of modern auction theory, see Krishna (2002). By contrast, Milgrom (2004) derives the central results by appealing to more general results from demand theory.

[13] Bulow and Roberts (1989) explain this phenomenon as a form of monopoly pricing, where the reserve price (a take-it-or-leave-it offer) introduces a marginal revenue effect. With asymmetric bidders, the mechanism is no longer anonymous, in the sense that strong players are penalized according to their identity (distribution function) in order to promote competition. The FCC (for this reason or for political motivations) offered discounts to small and minority-run businesses in their auction. Goeree and Offerman (2004) argue that the Amsterdam auction, a Dutch format for selling real estate that involves payments to the highest losing bidder, can be viewed as an attempt to implement the optimal mechanism under asymmetry. Lastly, Crémer and McLean (1985, 1988) prove that independence is also very important to Myerson's conclusion; any correlation between the buyers' information allows a monopolist to extract all expected surplus.

[14] We discuss this work in detail in the next section.

1.2 Bilateral Trade

Myerson's characterization of incentive compatibility and even more so the *Revenue Equivalence Theorem* has led to an explosion in the literature on Bayesian trading games. Perhaps the most important of these is his own contribution with Mark Satterthwaite (Myerson and Satterthwaite (1983)), which marks the first general analysis of selling procedures in which both sides of the market hold private information. Rather than restoring faith in the power of markets and rationality, the Myerson-Satterthwaite (M-S) theorem describes an impasse; any interim incentive compatible, individually rational mechanism that balances the budget (i.e. requires no external finance) is *ex post* inefficient with positive probability.[15] Much research since the M-S theorem is dedicated to describing the extent of this inefficiency, or finding relaxations of the incentive and budget constraints in order to restore efficiency.[16] For the most part however, it seems that some degree of inefficiency is an inevitable consequence of any plausible equilibrium of a trading game. Makowski and Mezzetti (1993) for example show that there exist interim incentive compatible, individually rational mechanisms that satisfy budget balance and *ex post* efficiency if there are at least two buyers, but the seller must commit to pay a 'probability bribe' even when the good is not sold.

Given that full efficiency is impossible, M-S present conditions under which a mechanism maximizes the *ex ante* gains from trade under interim incentive compatibility, individual rationality and *ex post* budget balance. Their class of 'α mechanisms' defines the *ex ante* efficient allocation rule as a condition on the weighted virtual valuations/costs of the buyer and seller. A solution in communication games is also included for the uniform $[0, 1]$ distribution, in which the incentive efficient allocation is implemented by the $k = \frac{1}{2}$ double auction.[17]

[15]This can also be proven indirectly by showing that the Vickrey-Clarke-Groves mechanism maximizes the seller's expected profits in interim incentive compatible individually rational mechanisms, then checking that this mechanism runs an expected deficit, see Krishna and Perry (1998). If we can dispense with the IR constraint full efficiency can be achieved, see D'Aspremont and Gérard-Varet (1979), Arrow (1979).

[16]The literature on efficiency is vast. Our focus on optimality rather than efficiency prohibits an extensive investigation of this work. Representative studies include Gresik and Satterthwaite (1989), Rustichini, Satterthwaite, and Williams (1994) and Satterthwaite and Williams (1989b, 2002), all of whom consider the influence of the number of traders on the size of the inefficiency.

[17]This was simultaneously shown for the restricted class of k-double auctions by Chatterjee and Samuelson (1983) by direct computation of equilibrium allocation probabilities.

Williams (1987) generalizes the *ex ante* efficiency concept in bilateral trade by examining convex combinations of buyer and seller utilities as opposed to an un-weighted sum as in M-S. The solution generates a similar set of solutions to M-S' α mechanisms, where α is further determined by the utility weightings.[18] This model incorporates as a special case the criterion of maximizing the seller's (*ex ante*) expected profits, and confirms that the specification of private seller information does not change the seller's optimal bilateral trading mechanism; she should make a take-it-or-leave-it-offer (appropriately selected) to the buyer.

It appears then that a relaxation of the optimal auction program to allow two-sided private information will not alter the structure of the solution, at least for Independent Private Values.[19] However, it is unknown whether private seller information is irrelevant only in the first-best solution, or whether this is true more generally. One cannot say whether the seller would value holding private information if the optimal auction were constrained in any way, or were subject to a more general valuation structure than IPV. Arguably, the source of this gap in understanding is the fact that it is unclear whether the irrelevance of a buyer's beliefs about the seller's information is intentional, or rather unavoidable from the seller's perspective. A simple example presented by Yilankaya (1999), which we shall now discuss, demonstrates the subtlety of the issue.

If the structure of the mechanism design problem were generically unchanged by the specification of private seller information (so the correspondence in designs is totally intentional), then the fact that the Myerson (1981) mechanism is optimal for every realization conditional on the seller's type being public information would imply that its *ex ante* optimality follows trivially. Yilankaya (1999) has shown that this is not so. When the seller's valuation is private information her interim expected profits in some realizations are higher in *ex ante* sub-optimal mechanisms than in the Myerson

[18]Building on these techniques, the existence and efficiency of the k-double auction from the *ex ante* and interim perspectives has been examined by Satterthwaite and Williams (1989a). Gresik (1991a, 1996, 1991b) address the representation of the *ex ante* efficient allocation rule as a plausible communication game. The first two of his papers discuss conditions under which one can transform an *ex ante* efficient allocation rule into a linear equilibrium of the k-double auction. The latter paper shows that the restriction to *ex post* individual rationality (as is common in practice; no sale means no transfer) does not constrain the efficiency objective since any level of trade induced under interim IR can be supported under *ex post* IR.

[19]Indeed, the first step of our analysis in Essay I (Proposition 2.2.1) is to show that this result can be extended to the case of $N > 1$ buyers.

(1981) mechanism.[20] However, any attempt by the seller to change the mechanism to her advantage would reveal her type, thus removing the anticipated benefits.[21] Therefore the averaging process that is undertaken in evaluating *ex ante* optimality is non-trivial, as is the link between the buyers' incentive constraints and their beliefs about the mechanism the seller would like to select. In the first paper of this thesis, we address directly the relationship between the incentives of buyers, their beliefs/allocation probability estimates and the design of the mechanism, and show why the seller chooses to 'undo' any buyer uncertainty associated with private seller information. The intuition is that, conditional on buyer beliefs being correct on average, greater certainty regarding the probability of allocation increases the convexity of buyer utilities, thus reducing information rents. We therefore provide an economic explanation for the correspondence between the optimal mechanisms of Myerson (1981) and Williams (1987). We then apply this intuition to derive several new results and interpretations for a well-known class of trading problems, the k-double auctions of bilateral trade. To further motivate this study, the following section discusses the relevant design literature and the limitations of its application to the problems studied in the first essay. We then follow with a similar analysis for the remaining essays. In the literature review for Essays II and III, we shall start with the general results for informed principal problems, then discuss the more auction-specific literature.

1.3 Essay I: Convex Utilities and Market Power in Trading Mechanisms

Literature: Mechanism Design and the Seller's Information (Private Values)

Maskin and Tirole (1990) (M-T) show that a principal who holds private information cannot benefit above the case in which her information is known to the agents under quasi-linear utilities. M-T use the abstraction of the

[20]The example he gives is that a seller with a valuation of 0 does better in the $k = \frac{1}{2}$ double auction than with her optimal take-it-or-leave-it offer ($k = 0$) when both traders' valuations are drawn from the uniform $[0, 1]$ distribution.

[21]The theory of inscrutable design of Myerson (1983) is applicable here. If the seller wants to change the mechanism conditional on her type, a 'grand' mechanism could always be conceived that is constant across her types that achieves this. The existence of mechanisms that are interim 'exogenously preferred' is therefore explained by the fact that profitable deviation to them would imply the existence of a 'grand' mechanism violating the *ex ante* optimality of the Myerson (1981) mechanism.

principal as a Walrasian trader who exchanges the slack on the agents' incentive constraints across realizations of her type. To explain their result, take the case in which the principal's type is always public information, conjecture a solution to be some profile of contracts (one for each type of principal) and compute the principal's expected payoff across all realizations. Now under quasi-linearity replacing the transfers from this profile with their mean does not change the principal's *ex ante* expected payoff. Also, in this equivalent mechanism the deterministic transfer will be 'differenced away' in the agents' IC constraints, since it is the same for any pair of types. Then in the design of any (principal-type) specific contract, the transfer only appears directly and in the agent's IR constraints, both of which are independent of the principal's type. Therefore the conditions that describe the optimal deterministic transfer are the same for all types of principal, as are the shadow prices on the incentive constraints. The different types of principal have no incentive to 'trade' these constraints across realizations, in the same sense that traders with the same endowments do not envy one another in Walrasian equilibrium.[22]

M-T prove that there is no benefit from allowing the agents' incentive constraints to hold on average over the principal's information rather than type-by-type, which appears to generalize to some extent the result of Williams (1987), that the specification of private seller information does not change the optimal selling mechanism. However, their argument requires the construction of a deterministic fixed transfer mechanism, which, as an abstraction, does not provide an economic explanation for the irrelevance of the agents' beliefs in equilibrium.[23] Further, given any other constraints that may apply to a trading problem, their technique can only be replicated if such constraints perturb the entire profile of contracts in one direction. For example, given two mechanisms that are not first-best, we can only conclude that one dominates the other without direct computation if transfers in one are higher for every realization of the principal's/seller's type.[24] The techniques introduced in our first paper will address these issues by provid-

[22]See Maskin and Tirole (1990) Section 4.

[23]M-T also require discrete types (as opposed to a continuous type space commonly assumed in trading games) and a sorting condition so that results relating the design problem to the efficiency of competitive equilibrium can be applied. This limitation of the applicability of their result to trading games is also noted by Yilankaya (1999).

[24]We also show that a similar problem is pervasive in auction theory with respect to the direct derivation of the optimal selling mechanism. Using *Revenue Equivalence* and Myerson's point-wise optimization method, one can only make a general statement about profits in two mechanisms if the allocation rule of one is 'closer' in every eventuality to the first-best solution than the other. We propose new techniques that avoid this problem.

ing an economic interpretation for the optimal belief structure of the agents as well as novel techniques for evaluating the performance of a mechanism.

1.4 Essays II, III: Auctions with an Informed Seller

Literature: Auction/Mechanism Design and the Seller's Information (Common Values)

In the first essay of the thesis, we study the scenario in which each buyer only cares about the seller's information to the extent that it determines the outcome of the mechanism. This is the case for which the mechanism design results of Williams (1987) and Maskin and Tirole (1990) apply. In the second and third essays we incorporate informational externalities, where the principal's private information is an explicit argument of the agents' utility functions. Gresik (1991c) and Maskin and Tirole (1992) have also addressed this problem. Gresik (1991c) proves that the M-S impossibility result generalizes to the case in which traders care directly about the valuation assessments of others. He also addresses the *ex ante* efficiency problem, where the solution is again characterized by an analogue of M-S' α-mechanisms, with virtual valuations adjusted for the interdependence between valuations.[25] Unfortunately, the strategic constraints on *ex post* efficiency are even more troublesome here than under private values, since traders have an extra incentive to misrepresent their information. First, fixing the beliefs of others, misrepresentation can achieve a trade on more favorable terms, as in the IPV model. Second and unique to the interdependent values set-up, it is also advantageous to distort other traders' perceptions of the value of the object through one's actions. We depart from the analysis of Gresik by addressing the profit objective and the implementation of auctions as communication games under such externalities in Essays II and III.

The externalities associated with interdependent valuations dictate that the imposition of private seller information is not only non-beneficial as in the IPV setting, but may even constrain the first-best solution. In the IPV case the seller can implement the optimal auction of Myerson (1981) whether or not her information is private. For example, in an English auction bidders have a dominant strategy to bid their value for any set of beliefs about the seller's information, or indeed the reserve price.[26] So, while the seller does not benefit from holding private information, she is certainly not worse off

[25]He does not, unlike Williams (1987), consider different weights on the utilities, so his work does not include a solution for the optimal mechanism.

[26]See Riley and Samuelson (1981).

for it. Alternatively, when the seller's assessment of the value of the object affects buyers' valuations, this generates some undesirable complementarities between the incentive constraints of buyers and the seller at the interim stage of the mechanism. Maskin and Tirole (1992) in extending their 1990 paper to incorporate such 'common value' effects discuss the adverse selection problem in some depth, "*the high (type principal) is likely to be hurt by the (agent's) incomplete information: either she will find herself "pooled" with her low (type) counterpart...or else she will have to undertake costly signaling activity...to distinguish herself.*"[27] For these reasons, the solution of the optimal auction problem under informational externalities does not follow from Myerson (1981). This distinction, of itself an interesting theoretical issue, provides the structure for our analysis of a long-standing question in auction theory, the pervasiveness of secret reserve prices.

In the second essay we examine the *ex ante* design problem of a seller designing an auction when she will hold private information as to the quality of the object. In this setting, any announced reserve price will be a signalling mechanism in the sense of Riley (1979). As is known from Akerlof (1970), a signalling equilibrium is inefficient since it results in a lower volume of trade than would occur if the informed party could credibly reveal her private information. In this essay we contrast the signalling equilibrium arising from public announcement of a reserve price before an auction with several secret reserve regimes. Here we consider secrecy as representing the ability to postpone the announcement (and possibly selection) of a reserve price until bidding has ended. In the event of non-clearance under secrecy, the reserve is announced by way of a take-it-or-leave-it offer to the highest bidder, a signalling outcome. If however the reserve is cleared, the object is sold at the auction price as determined by the strategic actions of the uninformed bidders, a screening outcome. We therefore test whether a principal can improve upon the costly signalling mechanisms discussed by M-T above by instead employing a mixed screening/signalling mechanism.[28]

[27] See Maskin and Tirole (1992), pp. 382.

[28] Different versions of the informed principal/auction design problem have been studied by Kremer and Skrzypacz (2004), Tisljar (2003), Watanabe and Yamato (2008) and Nagareda (2003). In Kremer and Skrzypacz (2004) and Tisljar (2003) the seller has private preference information that influences bidder valuations, and selects a game form conditional on this information. However, in Kremer and Skrzypacz (2004) signalling occurs through the auction form (eg English vs First Price), the reserve price plays no role because the seller does not value the good at all. Similarly in Tisljar (2003) the seller's information, which is private, is distinct from her valuation, which is common knowledge. Tisljar also assumes that the seller's signal gives her the complete distribution of bidder valuations, so she has no preference for different auction forms across states of the world,

It turns out that a famous result from auction theory attributable to Milgrom and Weber (1982a) (M-W) known as the *Linkage Principle* applies, but in an unexpected fashion. Their result states that the policy of announcing verifiable information does not affect the seller's expected profits under independence, which we also assume.[29] Our analysis differs from M-W since it examines the influence of non-verifiable information and the machinery by which information is revealed on the seller's expected profits. We show that the seller's profits from immediate announcement of a reserve price as in Cai, Riley, and Ye (2007) dominate those from later announcement. Prolonging bidders' uncertainty exacerbates the *winner's curse*, reducing reserve clearance rates at auction. By the *Linkage Principle*, there must be a compensating increase in the share of profits from post-auction negotiation, fixing a given reserve price strategy. Then equilibrium reserve prices will be higher under secrecy, since the seller's incentive to misrepresent her information is greater, making signalling even less efficient and reducing seller profits.

In the third essay we compare immediate announcement to the most extreme form of a secret reserve, one that is never announced. Under the *Right of Refusal*, the seller simply accepts or rejects the auction price decided by the bidders. This screening mechanism can dominate the monopoly pricing/signalling outcome of immediate disclosure, subject to some conditions on the information structure. The conclusion is that if the seller's preferences/valuation are a sufficient statistic for the market-relevant information,[30] then screening is more efficient and generates greater seller profits than signalling. In this essay we are therefore able to explain some well-known phenomena in auctions through a complementary analysis of auction

there are no adverse selection effects, and she can extract all surplus. In Watanabe and Yamato (2008) and Nagareda (2003) there are no informational externalities between buyers and the seller, and hence no adverse selection. Watanabe and Yamato (2008) model the seller's type information as summarizing moral hazard effects; whether or not she is a 'cheater'. There exist separating equilibria in which the cheating seller chooses a second-price auction and uses shill bids, while the honest seller uses a first-price auction. Nagareda (2003) allows the seller to choose her reserve price and whether or not to disclose it, but the lack of informational externalities means that Myerson (1981) applies, so we already know the first-best solution.

[29] Alternatively, we could phrase the result as a form of *Payoff Equivalence* as in Theorem 3.5 from Milgrom (2004). We use the *Linkage Principle* terminology due to its familiarity in the context of the seller's information policy. The implications of the Linkage Principle for ranking revenues across the major auction forms under interdependent values and affiliated signals is given in Milgrom and Weber (1982a).

[30] As opposed to the seller interpreting given information more favourably, say due to sentimentality (e.g. sale of a private residence).

theory and mechanism design under private seller information and interdependent valuations. The study also represents, to our knowledge, the first analysis of the relative performances of signalling and screening equilibria in an auction setting. This provides a framework for the principal's selection between the undesirable characteristics of pooling and signalling as identified by Maskin and Tirole (1992), and an explanation for the prevalence of secret reserves as used by auction houses.

Given that the mechanisms we consider in Essays II and III are auctions, the final body of literature we discuss here is that which is more specifically labeled as auction theory. The strands of this research that are relevant to our purposes address the role that asymmetric information plays in determining bidding behaviour.[31] Our work differs from the majority of these studies in that for the most part, these models have concentrated on effects between buyers, where each holds information that the other would like to know, or there is an 'insider' who has superior knowledge about the common value of the object. This tradition starts with Wilson (1967), and the so-called mineral-rights model. He considers a two bidder scenario in which the insider knows the common value of the object with certainty.[32] The other player receives only a noisy signal, so in equilibrium the *winner's curse* is severe for the uninformed party. Milgrom and Weber (1982b) prove more general results on this topic by supposing that the insider is characterized as having the only exclusive information. The informed bidder's profits are increasing in the amount of information she collects, and she always does better by acquiring this information overtly, because this makes public the increase in the *winner's curse* that is likely to be experienced if she is beaten. The seller can benefit from announcing her own private information only if this undoes some of the exclusivity of the insider's information. It is possible that the seller's information resolves the insider's residual uncertainty without providing the uninformed party with any useful information, which strengthens the insider to the detriment of revenues.[33]

[31] In Chapter 3 we give a succinct treatment of the theoretical and empirical work that relates to secret reserve prices.

[32] The usual analogy is that the asset is an oil or gas lease, and the insider has exclusive access to testing of the site or owns a neighbouring site.

[33] The example Milgrom and Weber (1982b) give for non-beneficial announcement is as follows. Say that the informed party has a signal X of the true value V, where $X = V + \epsilon$ and ϵ is an independent random error term. If the seller announces the value of ϵ, the insider completely knows her valuation, while the uninformed party learns nothing of value. Milgrom (2004) elaborates on these ideas, describing the *Linkage Principle* as generally having a *publicity effect* and a *weighting* effect that can limit or exacerbate the bidders' differential information. When bidders' access to the seller's information is

Goeree and Boone (2008) analyze a bidding situation of a similar flavour, but applied to privatization with the insider as an incumbent manager.[34] The presence of the insider is shown to be strongly detrimental to revenues, again due to the *winner's curse* that would be suffered by beating her. To remedy this problem, a two-stage mechanism is proposed involving initial non-binding bids as expressions of interest. Since the lowest bidder from the qualifying auction is excluded from competing, the first stage discriminates against the insider by revealing her information before the 'real' bids are submitted.

Hendricks, Porter, and Wilson (1994) present a masterly fusion of theory and econometrics in analyzing yet another variant of the mineral rights model. The insider's signal is assumed to be correlated with the unknown reserve price of the seller. The main result is that the distributions of the informed and uninformed bids are the same above the support of the reserve price, i.e. when the uninformed know their bids will be accepted and the common value exceeds some lower bound. At lower prices where the value of the insider's information is uncertain, the *winner's curse* takes over and informed bids stochastically dominate the uninformed.[35] The parts of our analysis that focus on asymmetric information share some aspects of the framework in Goeree and Boone (2008) in that we are concerned with "*situations where privatization involves substantial risk, i.e. when uncertainty about the assets common value is large relative to private-value cost differences.*"[36] Of course, while a reserve price can be viewed as a bid by an insider, in our model the seller benefits from using this bid to misrepresent her information when she loses, i.e. when she sells the good. The incumbent manager in Goeree and Boone (2008) on the other hand is a subsidiary or division of an organization that does not benefit from being outbid (although the 'parent' may). Similarly, while the effects of competition between an informed seller and bidders are borne out in Hendricks, Porter, and Wilson (1994), there is no insider to act as an unintentional surrogate for the seller in our model. Also, the authors only consider the most extreme form of a

symmetric, only the *publicity effect* exists and public announcement is always beneficial under affiliation.

[34]The authors augment Wilson's model with private value effects so that public knowledge of the common value component would not make the allocation or efficiency trivial.

[35]Engelbrecht-Wiggans, Milgrom, and Weber (1983) prove related results that are important to this conclusion. In a similar vein to Milgrom and Weber (1982b), the behaviour of informed and uninformed parties is compared. The distribution of bids from the unique insider is equal to the distribution of the maximum of the bids of the uninformed bidders.

[36]Goeree and Boone (2008) pp. 3.

secret reserve (akin to our *Right of Refusal*) due to their interest in applications to specific markets and data requirements. As the authors note, the presence of a secret/random reserve is exogenous and therefore unexplained in the Hendricks, Porter, and Wilson (1994) model; "*a random reservation price strategy is not unique to oil and gas lease auctions, but can also be found in the private sector. Therefore, the optimality and implications of different reserve price strategies for bidding behaviour may be of more general interest than suggested by our work here.*"[37] Thus our contribution in Essays II and III represents an expansion in both the theory of mechanism design by an informed principal, and in that of endogenous reserve price policies in auction theory.

[37]Hendricks, Porter, and Wilson (1994) pp. 1438.

Chapter 2

Convex Utilities and Market Power in Trading Mechanisms

2.1 Optimal Auctions and Private Information

One of the most significant achievements of mechanism design theory is the result that for symmetric potential buyers any of the common auction forms with a specifically selected reserve price maximize the seller's expected profits among all conceivable selling mechanisms.[1] This result, attributable to Myerson (1981) for the case in which the seller's reservation value is known can be simply extended to the case in which her preferences are private information. Williams (1987) for example has demonstrated that the Myerson mechanism for $N = 1$; a take it or leave it offer from seller to buyer; maximizes the seller's ex ante expected profits in a bilateral trade setting. A seller therefore gains nothing from holding private preference information if she can implement her first best design. Maskin and Tirole (1990) present a similar result in a very general setting for discrete types; a principal with private information should implement the complete information optimal contract under risk neutrality.[2]

On the other hand, there are other communication games (that are admitted by incentive compatible, individually rational direct mechanisms) in

[1] It is also required that the buyers are risk neutral and draw their types independently.

[2] By *complete information* we refer to the standard case in which the agents hold private information but the principal does not.

which a seller clearly prefers to have private information. For example, a seller in a bilateral trade market in which the buyer makes a take it or leave it offer will make zero profit if her private valuation is known; in any trade she receives the lowest possible price at which she would part with the good. If the seller's valuation is however unknown to the buyer, the former will earn some informational rents. An interesting question is why the optimal mechanism makes private information redundant for the seller, and at what point holding this information ceases to be useful. Clearly a more detailed explanation of how private information affects the seller's profits across the space of possible mechanisms is required in order to answer such questions.

A related issue of some interest to the practitioner is whether general properties exist that describe how restrictions on communication games (for example on the set of 'allowable' game forms) influence the seller's profits in the direct mechanisms through which market games are usually modelled. The standard technique used to address trading models is to apply the *Revenue Equivalence Theorem*, then check whether some given communication game implements the optimal allocation rule. This approach is simple, but is only instructive in comparing sub-optimal allocation rules (i.e. when constraints apply) when the tested allocation rules/probabilities can be ranked point by point, since the optimal allocation rule is derived by pointwise optimization. Given that applied restrictions on communication games need not map into allocation rules in such a fashion, a more flexible method of ranking mechanisms would be of some value.

We address these issues by attacking the mechanism selection problem from a new angle. We specifically investigate the role of the seller's private information by examining how she should select the allocation rule of a mechanism as reflected by the buyers' interim estimates of the probability of allocation. We demonstrate that the seller's profits are determined by two closely linked factors; (i) The set of buyer types that are totally excluded, i.e. have allocation probability and interim utility equal to zero;[3] (ii) The slope of the allocation probability function (or convexity of the utility function) for higher types. Further, our characterization of the interplay between (i) and (ii) allows us to make a general statement that links these factors to the seller's expected profits when a point by point ranking of allocation rules is not achievable, for example in comparing monopoly and monopsony pricing.

By the *Revenue Equivalence Theorem*, a buyer's information rents are completely determined by her allocation probability function. This seem-

[3]This is the focus of Myerson's (1981) analysis for the case in which the seller's reservation value is publicly known.

ingly introduces a tradeoff between allocating more often to a given buyer type, and increasing the rents that must be paid to higher types when sale occurs in equilibrium. Under our analogue of the well known regularity assumption and the informational requirements of Perfect Bayesian Equilibrium, we show that this tradeoff is trivial; any change in the profile of the allocation probability function that leads to a reduction information rents is worthwhile. One such profitable manipulation is to increase the slope of the probability function (and thus the convexity of the utility function). More monopolistic mechanisms induce steeper allocation probability functions since buyers have less control over the price, and therefore report their type more honestly. This means that any increase in a buyer's type will be translated with greater magnitude into her action/announcement in the communication game, so the probability function is steeper in her type. Our results therefore give a formalization of why market power (in the sense of price setting) is advantageous for the seller even under two-sided private information; it induces steeper probability functions reducing the set of 'lower' types a buyer could have masqueraded as and won in equilibrium. In this sense monopolistic mechanisms generate a greater level of *virtual competition* than their monopsonistic counterparts.

The generalization from the standard technique of ranking allocation rules to ranking their expectations (the allocation probabilities) is shown to be analogous to the relationship between first order and second order stochastic dominance. We apply our results and the stochastic dominance logic to explain some interesting properties of the k double auction for bilateral trade. This well known 1 parameter class of allocation rules cannot be ranked point by point, and hence, the global influence of shifts in the parameter k on the seller's profits is not well understood. On the other hand, our analysis demonstrates why the seller's expected profits are maximized by setting her price weighting $(1-k)=1$. An interesting corollary is that for $\hat{k} \in [0, \frac{1}{2})$, the buyer's interim probability of allocation in the $(1-\hat{k})$ double auction is second order stochastically dominated by that in the $\hat{k}$ mechanism. The seller therefore achieves the same ex ante volume of trade with lower buyer estimates of the interim probability of trade in the $\hat{k}$ mechanism. This increases the level of *virtual competition* in the seller's favour.

We proceed as follows. Section 2.2 introduces the model, characterizes incentive compatibility and individual rationality, and solves the mechanism design problem of selecting an optimal trading mechanism from the perspective of maximizing the seller's ex ante expected profits.[4] Section 2.3 outlines our new approach to ranking mechanisms. We then state and prove our main result regarding probabilities, utilities and profit rankings. Section 2.4 links our main result to the stochastic dominance criteria. The characteristics of monopoly with reference to the slope of the probability function are discussed, and the results are interpreted with respect to the role of private seller/principal information. Section 2.5 presents an application to the k double auction for bilateral trade. Section 2.6 concludes.

2.2 The Model

2.2.1 Preliminaries

Call the set of players $I = \{0, ..., N\}, \quad N \geq 1$, where Player 0 is the seller, and $I/\{0\}$ is the set of buyers. We shall use the term *auction* to refer to any situation in which $N \geq 2$. More generally we shall use the term *mechanism*. Player $j \in I$ receives a non-negative real valued signal labelled $X_0, X_1, X_2, ..., X_N$ respectively from the space $[\underline{\theta}_j, \overline{\theta}_j] = \chi_j \subset \Re^+$. This represents a player's private valuation of the good. We shall suppose that valuations/signals are statistically independent, and are distributed according to F_j with density f_j.[5] All distributions are assumed to have the familiar properties that ensure regularity, i.e. $x - \frac{1-F_j(x)}{f_j(x)}$ and $x + \frac{F_j(x)}{f_j(x)}$ are strictly increasing. We appeal to the *Revelation Principle*, momentarily restricting our attention to direct mechanisms in which players make (not necessarily truthful) announcements of their signals. A direct mechanism consists of an allocation rule and payment rule.

The allocation rule $Q(\hat{\mathbf{x}}) = (Q_1(\hat{\mathbf{x}}), ..., Q_N(\hat{\mathbf{x}}))$ maps a vector of players' announcements $\hat{\mathbf{x}} = (\hat{x}_0, \hat{x}_1, ..., \hat{x}_N)$ into an allocation of the good. The good is indivisible, so $\forall i \neq 0$ we must have $Q_i = 0$ or 1, and $Q_i = 1$ for at most one buyer.[6]

The payment rule $M(\hat{\mathbf{x}}) = (M_1(\hat{\mathbf{x}}), ..., M_N(\hat{\mathbf{x}}))$ similarly maps signal announcements of the players into individual transfers to the seller.

We define expected value functions for the allocation probability and expected transfer of Buyer i when she announces the signal $\hat{x}_i$ and all other

[4] This represents a generalization of Williams (1987) result for bilateral trade to the case of $N > 1$ potential buyers. To our knowledge this result is new.

[5] Henceforth realisations of a variable will be written in lower case, uppercase letters denote the random variable itself.

[6] From this point on we shall denote a generic buyer as i.

players announce truthfully as follows;

$$q_i(\hat{x}_i) = \int_{\chi_{-\mathbf{i}}} Q_i(\hat{x}_i, \mathbf{x_{-i}})\mathbf{f}(\mathbf{x_{-i}})d\mathbf{x_{-i}}$$

$$m_i(\hat{x}_i) = \int_{\chi_{-\mathbf{i}}} M_i(\hat{x}_i, \mathbf{x_{-i}})\mathbf{f}(\mathbf{x_{-i}})d\mathbf{x_{-i}}$$

where $\mathbf{x_{-j}} = (x_0, ..., x_{j-1}, x_{j+1}, ..., x_N)$, $\chi_{-\mathbf{j}} = \times_{h\neq j}\chi_h$ and $\mathbf{f}(\mathbf{z}) = \Pi_{j:x_j\in\mathbf{z}} f_j(x_j)$. All players are risk neutral, so the expected utility of Buyer i from announcing type $\hat{x}_i$ when her true type is x_i is

$$U_i(\hat{x}_i, x_i) = q_i(\hat{x}_i)x_i - m_i(\hat{x}_i)$$

The seller differs from the buyers since she initially holds the object and, we shall assume, receives all transfers. Our motivation for this assumption is that with $N \geq 2$ buyers, if we allow other kinds of transfers some of the fundamental results of the mechanism design/trading literature are known to break down. For example, Makowski and Mezzetti (1993) have shown that there exist interim incentive compatible, individually rational mechanisms that satisfy budget balance and ex post efficiency if there are at least two buyers and the seller can commit to pay a 'probability bribe' even when the good is not sold.[7] While these results are certainly of interest, in practice of course one does not observe transfers of this nature, so we rule them out. The seller's interim expected payoff is given by

$$U_0(\hat{x}_0, x_0) = m_0(\hat{x}_0) - q_0(\hat{x}_0)x_0$$

where

$$q_0(\hat{x}_0) = \sum_{i=1}^{N} \int_{\chi_{-\mathbf{0}}} Q_i(\hat{x}_0, \mathbf{x_{-0}})\mathbf{f}(\mathbf{x_{-0}})d\mathbf{x_{-0}}$$

$$m_0(\hat{x}_0) = \sum_{i=1}^{N} \int_{\chi_{-\mathbf{0}}} M_i(\hat{x}_0, \mathbf{x_{-0}})\mathbf{f}(\mathbf{x_{-0}})d\mathbf{x_{-0}}$$

The requirements of no external finance and Perfect Bayesian Equilibrium (PBE) impose the equilibrium budget balance condition for $\hat{\mathbf{x}} = \mathbf{x}$

$$E_{X_0}[m_0(x_0)] = \sum_{i=1}^{N} E_{X_i}[m_i(x_i)] \tag{2.1}$$

[7] The result that such mechanisms are impossible for bilateral trade was given in Myerson and Satterthwaite (1983). On a related note, Gresik (1991a) and Gresik (1996) show that in bilateral trade models, the imposition of ex post individual rationality on both traders does not constrain the standard model since a mechanism satisfying this condition can be constructed for any ex ante level of trade (efficiency) satisfying interim IR.

The next section sets up the seller's objective function plus the incentive compatibility and individual rationality constraints for the design problem.

2.2.2 Incentive Compatibility (IC) and Individual Rationality (IR)

Given the arbitrary allocation and payment rules described in Section 2.2.1, we now want to restrict ourselves to equilibria of the direct mechanism. That is, we would like to find rules such that it is a best response for each player to truthfully announce their signal, given that all others are doing likewise. This requirement is known as incentive compatibility, and is equivalent to the condition that

$$U_j(x_j) \equiv U_j(x_j, x_j) = \max_{\hat{x}_j \in \chi_j} \{U_j(\hat{x}_j, x_j)\} \quad \forall j$$

Myerson (1981) demonstrates that risk neutrality combined with IC has a powerful consequence known as the *Revenue Equivalence Theorem.* He shows that $U_i'(x_i) = q_i(x_i) \geq 0 \quad \forall i \neq 0$, so we can write[8]

$$U_i(x_i) = U_i(\underline{\theta}_i) + \int_{\underline{\theta}_i}^{x_i} q_i(t_i) dt_i \tag{2.2}$$

$$m_i(x_i) = -U_i(\underline{\theta}_i) + q_i(x_i)x_i - \int_{\underline{\theta}_i}^{x_i} q_i(t_i) dt_i \tag{2.3}$$

The requirement of Individual Rationality dictates that every type of buyer must receive an expected payoff at least as high as she would by not participating in the mechanism. Since U_i is non-decreasing, it is sufficient that this hold for the lowest type of buyer, so we must have $U_i(\underline{\theta}_i) \geq 0$. Similarly the seller's IC implies

$$U_0(x_0) = U_0(\overline{\theta}_0) + \int_{x_0}^{\overline{\theta}_0} q_0(t_0) dt_0 \tag{2.4}$$

and her IR condition is reduced to $U_0(\overline{\theta}_0) \geq 0$.[9] Given the characterization of the incentive constraints above, we now proceed to our analysis of the mechanism design problem. An instructive first step is to examine the ex ante design problem under two-sided private information from a conventional approach before introducing our new method.

[8] See Myerson (1981), Lemma 2.

[9] The application of the *Revenue Equivalence Theorem* logic to the seller's IC appeared first in Myerson and Satterthwaite (1983) Theorem 1, Equation (4).

2.2.3 Ex Ante Mechanism Design

Consider a seller who is ex ante designing a selling mechanism in order to maximize her own expected profits. This seller is choosing an optimal allocation and payment rule subject to the incentive compatibility and individual rationality of herself and the buyers at the interim stage. From the definitions of the expected utility terms, we can write the following;[10]

$$\begin{aligned}\sum_{i=1}^{N} E_{X_i}[U_i(x_i)] + E_{X_0}[U_0(x_0)] &= \sum_{i=1}^{N} E_{X_i}[q_i(x_i)x_i] - E_{X_0}[q_0(x_0)x_0] \\ &+ E_{X_0}[m_0(x_0)] - \sum_{i=1}^{N} E_{X_i}[m_i(x_i)] \\ &= \sum_{i=1}^{N} E_{X_i}[q_i(x_i)x_i] - E_{X_0}[q_0(x_0)x_0]\end{aligned}$$

The second line disappears due to the budget balance condition that follows from (2.1). Now substituting in the incentive compatibility conditions (2.2) and (2.4) for each U_i and U_0 we have

$$\begin{aligned}\sum_{i=1}^{N} E_{X_i}[q_i(x_i)x_i] - E_{X_0}[q_0(x_0)x_0] &= \sum_{i=1}^{N} U_i(\underline{\theta}_i) + U_0(\overline{\theta}_0) \\ &+ E_{X_0}\Big[\int_{x_0}^{\overline{\theta}_0} q_0(t_0)dt_0\Big] + \sum_{i=1}^{N} E_{X_i}\Big[\int_{\underline{\theta}_i}^{x_i} q_i(t_i)dt_i\Big]\end{aligned}$$

Call $\psi_i(x_i)$ the *virtual valuation* of Buyer i given by $\psi_i(x_i) = x_i - \frac{1-F_i(x_i)}{f_i(x_i)}$ and $\zeta_0(x_0)$ the *virtual cost* of the seller given by $\zeta_0(x_0) = x_0 + \frac{F_0(x_0)}{f_0(x_0)}$. Rearranging integrals and using the definition of q_j in the above expectations achieves

$$\sum_{i=1}^{N} \int_{\mathcal{X}} \big[\psi_i(x_i) - \zeta_0(x_0)\big] Q_i(\mathbf{x})\mathbf{f}(\mathbf{x})d\mathbf{x} = \sum_{i=1}^{N} U_i(\underline{\theta}_i) + U_0(\overline{\theta}_0) \tag{2.5}$$

The IR conditions are represented by the requirement that both sides of the above expression are non-negative.[11] The seller's ex ante expected profits

[10] In the derivations until Section 3 we shall be brief since the methods are well established from Myerson (1981), Myerson and Satterthwaite (1983) and Williams (1987).

[11] Given that (2.5) holds for some allocation rule we still need to verify IC by checking that $q_i' \geq 0$ and $q_0' \leq 0$.

are

$$E_{X_0}[U_0(x_0)] \quad = \quad \sum_{i=1}^{N} E_{X_i}[m_i(x_i)] - E_{X_0}[q_0(x_0)x_0]$$

Using (2.3), the definition of q_0 and rearranging the integrals in a similar fashion to (2.5), the profit equation becomes

$$E_{X_0}[U_0(x_0)] \quad = \quad -\sum_{i=1}^{N} U_i(\underline{\theta}_i) + \sum_{i=1}^{N} \int_{\mathcal{X}} \big[\psi_i(x_i) - x_0\big] Q_i(\mathbf{x})\mathbf{f}(\mathbf{x})d\mathbf{x} \tag{2.6}$$

Fixing an incentive compatible allocation rule, and thus both sides of (2.5), the seller's profits are decreasing in $\sum_{i=1}^{N} U_i(\underline{\theta}_i)$. Since we are still free to divide the sum $\sum_{i=1}^{N} U_i(\underline{\theta}_i) + U_0(\overline{\theta}_0)$ any way we see fit (subject to IC and IR), it is optimal to set $\sum_{i=1}^{N} U_i(\underline{\theta}_i) = 0$. Myerson and Satterthwaite (1983) show for bilateral trade that the payment rule of the mechanism can always be chosen such that $U_i(\underline{\theta}_i) + U_0(\overline{\theta}_0) = U_0(\overline{\theta}_0)$ for a generic IC, IR allocation rule.[12] For $N > 1$, the result immediately follows by re-weighting their constructed payment rule by N and ensuring that $U_i(\underline{\theta}_i) = 0 \quad \forall i$.[13] The Lagrangian for this problem can now be formed using the objective (2.6), the constraint that both sides of (2.5) are non-negative and without loss of generality, the substitution $\sum_{i=1}^{N} U_i(\underline{\theta}_i) = 0$;

$$\begin{aligned} \mathcal{L} \quad &= \quad \sum_{i=1}^{N} \int_{\mathcal{X}} \big[\psi_i(x_i) - x_0\big] Q_i(\mathbf{x})\mathbf{f}(\mathbf{x})d\mathbf{x} \\ &+ \quad \lambda \sum_{i=1}^{N} \int_{\mathcal{X}} \big[\psi_i(x_i) - \zeta_0(x_0)\big] Q_i(\mathbf{x})\mathbf{f}(\mathbf{x})d\mathbf{x} \end{aligned} \tag{2.7}$$

for $\lambda \geq 0$. We can now state the following result.[14]

Proposition 2.2.1. *The Myerson (1981) allocation rule maximizes the ex ante profits of a seller whose reservation value is private information if* $\underline{\theta}_0 \geq \underline{\theta}_i, \overline{\theta}_0 < \overline{\theta}_i \quad \forall i \neq 0$.

[12] See their Theorem 1.

[13] In the Appendix we present such payment rules.

[14] Williams (1987) considers the question of ex ante efficiency in a bilateral trade model, where the objective is to maximize a convex combination of buyer and seller expected payoffs subject to IR and IC constraints. Our result extends his Theorem 2(II); maximizing the seller's expected payoff (in his terminology, $\gamma = 1$) to the case of N bidders.

Proof. Any Q that maximizes (2.7) for some $\lambda \geq 0$ and satisfies IR and IC solves the seller's design problem. Rewrite the Lagrangian as

$$\begin{aligned}
\mathcal{L} &= \sum_{i=1}^{N} \int_{\chi} \left[(1+\lambda)\psi_i(x_i) - (1+\lambda)x_0 - \lambda \frac{F_0(x_0)}{f_0(x_0)}\right] Q_i(\mathbf{x})\mathbf{f}(\mathbf{x})d\mathbf{x} \\
&= (1+\lambda)\sum_{i=1}^{N} \int_{\chi} \left[\psi_i(x_i) - x_0 - \alpha \frac{F_0(x_0)}{f_0(x_0)}\right] Q_i(\mathbf{x})\mathbf{f}(\mathbf{x})d\mathbf{x} \qquad (2.8)
\end{aligned}$$

where we have defined $\alpha = \frac{\lambda}{1+\lambda} \in [0, 1)$.[15] Setting $\lambda, \alpha = 0$ the objective function collapses to that under the optimal auction programme when the seller's valuation is publicly known, as in Myerson (1981). We know that since ψ_i is increasing (i.e. the problem is regular) this programme is solved by using pointwise optimization to achieve the following allocation rule;

$$Q_i(\mathbf{x}) = \begin{cases} 1 & \text{if} \quad \psi_i(x_i) \geq \max_{j \neq i,0}\{\psi_j(x_j), x_0\} \quad i \neq 0 \\ 0 & \text{otherwise} \end{cases} \qquad (2.9)$$

Incentive compatibility is achieved for the buyers because the allocation rule is tied to each buyer's virtual valuation announcement, and we have assumed that this function is increasing. Note that $Q_i(\underline{\theta}_i, \mathbf{x}_{-i}) = 1$ only if $\underline{\theta}_i = \underline{\theta}_0$ and $x_0 = \underline{\theta}_0$, which is a zero probability event from the perspective of i, so $q_i(\underline{\theta}_i) = 0$. Given this, $U_i(\underline{\theta}_i) = 0$ requires $m_i(\underline{\theta}_i) = 0$, which can be achieved with the payment rule $M_i(\mathbf{x}) = Q_i(\mathbf{x})x_i - \int_{\underline{\theta}_i}^{x_i} Q_i(z_i, \mathbf{x}_{-i})dz_i$. Incentive compatibility for the seller follows since we know from Myerson (1981) that $\psi_i^{-1}(\hat{x}_0) = \psi_i^{-1}(x_0)$ is optimal for each seller type. The seller's IR constraint $U(\overline{\theta}_0) > 0$ is satisfied because $\overline{\theta}_i > \overline{\theta}_0$ and the highest type of seller trades with positive probability.[16] □

Given this technique for solving the design problem, the profit ranking with respect to any two IC, IR mechanisms A and B (or more generally, within any class of mechanisms) is reduced to a comparison of their respective allocation rules and inspection of Equation (2.9). This condition is only instructive if the allocation rules of A and B can be ranked point by point relative to the optimal allocation rule. That is, we require that $Q_i^A(x, x_0) \geq Q_i^B(x, x_0) \quad \forall (x, x_0), i$ and that A always allocates less often than the optimal mechanism (for A to dominate B) or B always allocates

[15] A very similar method to that implemented here is used in Myerson and Satterthwaite (1983) (Theorem 2) and Gresik (1991c) (Theorem 2) to maximize total expected gains from trade.

[16] This also ensures that complimentary slackness is achieved at $\alpha = 0$.

more often than the optimal mechanism (for B to dominate A). Alternatively, our method can provide a profit ranking, and importantly, intuition for this result where the above methods fail.

2.3 Ranking Mechanisms: Calculus of Variations Method

2.3.1 The Seller's Expected Profits

Start as before by writing the seller's ex ante expected profits in the expected revenue minus expected cost form

$$E_{X_0}[U_0(x_0)] = \sum_{i=1}^{N} E_{X_i}[m_i(x_i)] - E_{X_0}[q_0(x_0)x_0] \tag{2.10}$$

From the *Revenue Equivalence Theorem*, we have an expression for $m_i(x_i)$ that depends up to an additive constant on the allocation rule $Q_i(\mathbf{x})$ through $q_i(x_i)$. As in Section 2.2.3, we can without loss of generality set $\sum_{i=1}^{N} U_i(\underline{\theta}_i) = 0$ to achieve

$$\begin{aligned} E_{X_0}[U_0(x_0)] &= \sum_{i=1}^{N} \int_{\underline{\theta}_i}^{\overline{\theta}_i} \Big\{ q_i(x_i)x_i - \int_{\underline{\theta}_i}^{x_i} q_i(t_i)dt_i \Big\} f_i(x_i)dx_i - \int_{\underline{\theta}_0}^{\overline{\theta}_0} q_0(x_0)x_0 f_0(x_0)dx_0 \\ &= \int_{\chi} \Big[\sum_{i=1}^{N} \Big\{ q_i(x_i)x_i - \int_{\underline{\theta}_i}^{x_i} q_i(t_i)dt_i \Big\} - q_0(x_0)x_0 \Big] \mathbf{f}(\mathbf{x})d\mathbf{x} \end{aligned} \tag{2.11}$$

The second line follows because we can take expectations for each m_i over all other x_j's since q_i depends only on x_i, then factor out the joint pdf $\mathbf{f}(\mathbf{x})$ due to independence. In Section 2.2.3 we evaluated these terms using the definitions of the q_j's and rearranged the integrals to yield an expression identical to that in Myerson (1981). This expression was maximized pointwise with respect to Q. Here we instead phrase the profit maximization problem in terms of selecting the functions that determine each buyer's (and the seller's) interim estimate of the probability she receives or parts with the object, $\mathbf{q} = (q_0, ..., q_N)$. Note that we are only free to choose these functions such that the probability estimates are correct on average as must be the case in equilibrium. These restrictions are defined as follows, where S_i is Buyer i's ex ante share of total trade;

$$J_i(\mathbf{q}) \equiv \int_{\underline{\theta}_i}^{\overline{\theta}_i} q_i(x_i)f_i(x_i)dx_i - S_i \int_{\underline{\theta}_0}^{\overline{\theta}_0} q_0(x_0)f_0(x_0)dx_0 = 0 \quad \forall i \neq 0 \tag{2.12}$$

We can now outline our method for ranking the profitability of different mechanisms for the seller given the selection of $\mathbf{q}$. The following section presents a brief review of the mechanics of the Calculus of Variations before applying these techniques to the design problem.

2.3.2 Calculus of Variations

The simplest problem in the multidimensional calculus of variations involves choosing a vector valued function $\mathbf{y}$ of $\mathbf{a}$ and its gradient $\nabla\mathbf{y}$, where $\mathbf{a} = (a_1, ..., a_l)$, $\mathbf{y} = (y_1(a_1), ..., y_l(a_l))$. These functions are chosen in order to find an extremal for an integral over a subspace of the domain of $\mathbf{a}$.[17] Take an integral of the form

$$\pi = \int_\Omega \Phi(\mathbf{y}, \nabla\mathbf{y}; \mathbf{a}) d\mathbf{a} \tag{2.13}$$

where Φ is a scalar function, Ω is a bounded domain, and the boundary $d\Omega$ is regular. The seller's profit maximization problem has a similar form, since she is choosing $l = N + 1$ functions (the q_j's), each of a single variable (x_j), and we can define $\Omega = \chi$ which, as a finite Cartesian product of compact subsets of $\Re^+$ is closed and has a regular boundary. A necessary condition for the chosen functions $\mathbf{y}$ and $\nabla\mathbf{y}$ to achieve an extreme value for π is that no nearby trajectories do better. An alternative trajectory is represented by a perturbation $\delta\mathbf{y}$, and we are interested in the change in π conditional on this perturbation, or $\delta\pi = \pi(\mathbf{y} + \delta\mathbf{y}) - \pi(\mathbf{y})$. For infinitesimal $\delta\mathbf{y}$ and $\delta\nabla\mathbf{y}$, we can linearize $\Phi(\mathbf{y} + \delta\mathbf{y})$ to write

$$\begin{aligned}\Phi(\mathbf{y} + \delta\mathbf{y}, \nabla(\mathbf{y} + \delta\mathbf{y}); \mathbf{a}) &= \Phi(\mathbf{y}, \nabla\mathbf{y}; \mathbf{a}) + \Phi_{\mathbf{y}}\delta\mathbf{y} + \Phi_{\nabla\mathbf{y}}\delta\nabla\mathbf{y} \\ &+ o(|\delta\mathbf{y}|, |\nabla\delta\mathbf{y}|)\end{aligned}$$

Here $\Phi_{\mathbf{y}}$,$\Phi_{\nabla\mathbf{y}}$ denote the partials of the integrand Φ with respect to the vectors $\mathbf{y}, \nabla\mathbf{y}$ respectively ignoring their dependence on $\mathbf{a}$. So we have

$$\delta\pi = \int_\Omega \Big[\Phi_{\mathbf{y}}\delta\mathbf{y} + \Phi_{\nabla\mathbf{y}}\delta\nabla\mathbf{y}\Big] d\mathbf{a} + o(|\delta\mathbf{y}|, |\nabla\delta\mathbf{y}|) \tag{2.14}$$

[17] All techniques in this subsection are standard in the calculus of variations and can be found in for example Kamien and Schwartz (1981). A similar principle to that applied here has also been used for a one dimensional problem in the context of auction theory by Hafilir and Krishna (2008). They prove that with asymmetric bidders and resale, the expected revenue from a First Price auction is exceeds that from a Second Price auction. This difference is also shown to be increasing in the degree of bidder asymmetry.

The linearity of the operators δ and ∇ means that their order can be interchanged. Making this substitution and using the multivariable version of integration by parts the second term in the integral from (2.14) above can be written as

$$\int_{\Omega} \Phi_{\nabla \mathbf{y}} \delta \nabla \mathbf{y} d\mathbf{a} = -\int_{\Omega} \delta \mathbf{y} \nabla \Phi_{\nabla \mathbf{y}} d\mathbf{a} + \int_{d\Omega} \delta \mathbf{y} \Phi_{\nabla \mathbf{y}} d\theta \tag{2.15}$$

where θ is the vector of coordinates on the boundary $d\Omega$. Therefore

$$\begin{aligned} \delta\pi &= \int_{\Omega} (\Phi_{\mathbf{y}} - \nabla \Phi_{\nabla \mathbf{y}}) \delta \mathbf{y} d\mathbf{a} + \int_{d\Omega} \Phi_{\nabla \mathbf{y}} \delta \mathbf{y} d\theta d\mathbf{a} \\ &+ o(|\delta \mathbf{y}|, |\nabla \delta \mathbf{y}|) \end{aligned} \tag{2.16}$$

For infinitesimal $\delta \mathbf{y}$, $o(|\delta \mathbf{y}|, |\nabla \delta \mathbf{y}|)$ disappears, and the effect of the variation on π is given by two terms; the first describes the change in the objective fixing the values of $\mathbf{y}$ at the boundary coordinates, the remainder is the effect of changing the boundary values of $\mathbf{y}$ themselves. Initially, we shall consider perturbations of $\mathbf{y}$ such that the boundary coordinates and the value of the alternative function $\mathbf{y} + \delta \mathbf{y}$ at these coordinates are fixed.[18] In this case, we say the boundary conditions are *posed*, which means that $\delta \mathbf{y} = 0$ on $d\Omega$ and $\int_{d\Omega} \Phi_{\nabla \mathbf{y}} \delta \mathbf{y} d\theta d\mathbf{a} = 0$. Then for $\mathbf{y}$ to achieve an extreme value for π, $\mathbf{y}$ and $\nabla \mathbf{y}$ must satisfy

$$\delta\pi = \int_{\Omega} \Big\{ \Phi_{\mathbf{y}} - \nabla \Phi_{\nabla \mathbf{y}} \Big\} \delta \mathbf{y} d\mathbf{a} = 0 \tag{2.17}$$

The Fundamental Lemma of the Calculus of Variations states that since the choice of $\delta \mathbf{y}$ is arbitrary, the only way for this condition to be generically satisfied is for $\mathbf{y}$ and $\nabla \mathbf{y}$ to be such that

$$\Phi_{\mathbf{y}} - \nabla \Phi_{\nabla \mathbf{y}} = 0 \quad \forall \mathbf{a} \tag{2.18}$$

This system describes l conditions referred to as the Euler-Lagrange equations, which are solved for each y along with the boundary conditions. The final issue we must address before subjecting the profit maximization problem to this method is the incorporation of constraints. Recall that we are not free to choose the allocation probability functions in $\mathbf{q}$ as we please; this selection is subject to the equilibrium requirements of incentive compatibility, individual rationality and the implementation of Bayes' Rule in PBE,

[18] Shortly we fully specify the objective function, and verify that boundary constraints are trivially satified for the selection of q_0. In Section 2.4.4 we consider variations in the other selected functions at the boundaries.

as summarized by $J_i(\mathbf{q}) = 0 \quad \forall i \neq 0$ in (2.12). Thankfully, this concern is easily dealt with, since when the selected functions are subject to integral constraints in the calculus of variations, the variation is simply applied to the augmented objective function given the familiar Lagrangian multiplier method, in our case;[19]

$$\begin{aligned}\pi &= \int_{\chi} \Big[\sum_{i=1}^{N} \Big\{ q_i(x_i)x_i - \int_{\underline{\theta}_i}^{x_i} q_i(t_i)dt_i \Big\} - q_0(x_0)x_0 \Big] \mathbf{f}(\mathbf{x})d\mathbf{x} + \sum_{i=1}^{N} \lambda_i J_i(\mathbf{q}) \\ &= \int_{\chi} \Big[\sum_{i=1}^{N} \Big\{ q_i(x_i)(x_i + \lambda_i) - \int_{\underline{\theta}_i}^{x_i} q_i(t_i)dt_i \Big\} - q_0(x_0)(x_0 + \sum_{i=1}^{N} \lambda_i S_i) \Big] \mathbf{f}(\mathbf{x})d\mathbf{x}\end{aligned} \tag{2.19}$$

The multiplier λ_i applies to the constraint that relates Buyer i's ex ante share of trade to the total volume of trade, as in (2.12). It should now be evident from examination of (2.19) that we do not need to consider any boundary conditions for the selection of q_0. The variation in the boundary term $\int_{d\Omega} \Phi_{\nabla y} \delta y d\theta da$ is equivalent to the partial of Φ with respect to the derivative of the function under consideration, evaluated at the boundary values of a. For the selection of q_0, Φ is independent of q_0', so the condition $[\Phi_{q_0'}]_{\underline{\theta}_0}^{\bar{\theta}_0} = 0$ is always satisfied. With the objective specified, we now prove a general result that will allow us to rank expected profits to the seller across mechanisms.

2.3.3 The Main Result

To apply the calculus of variations method, we first set $\Omega = \chi$ and $\mathbf{a} = \mathbf{x}$. We are primarily interested in the role of private seller information, and thus the aspect of the allocation rule that describes trade between the seller and each buyer. We shall therefore assume symmetry in the distribution functions of $i \neq 0$, so that $f_i = f$ and $F_i = F$.[20]

Lemma 2.3.1. *Under symmetry, for any vector of functions* $\mathbf{q_{-0}}$ *setting* $\lambda_i = -x_0$ *for* $i \neq 0$ *is optimal.*

Proof. The weighting $f_i = f$ is the same on every term in $\{.\}$ from Equation (2.19), and since these have a symmetric form, it is optimal to set $S_i =$

[19] Again we use independence to factor out the joint pdf.

[20] We envisage our result being applied to problems in which constraints on the allocation rule are imposed symmetrically across $i \neq 0$ for $N > 1$ so that the above restriction is harmless.

$\frac{1}{N}$, $\lambda_i = \lambda \quad \forall i \neq 0$. Each buyer receives the same ex ante share of trade and has her expected payments weighted equally in the objective. Then $\sum_{i=1}^{N} \lambda_i S_i = \lambda$, and from (2.19) the Euler-Lagrange equation for the function q_0 is

$$\begin{aligned} \Phi_{q_0} - \frac{d}{dx_0}\Phi_{q_0'} = -(x_0 + \sum_{i=1}^{N} \lambda_i S_i)\mathbf{f}(\mathbf{x}) &= 0 \\ \lambda &= -x_0 \end{aligned} \tag{2.20}$$

By inspection of (2.19), concavity of the objective is satisfied because Φ_{q_0} is positive for $\lambda < -x_0$ and negative for $\lambda > -x_0$. □

Φ is independent of q_0', so the second term in the Euler-Lagrange equation disappears. This identifies λ, which describes the optimal weighting of q_0 relative to each q_i in the objective function, subject to satisfaction of the constraints. Having established a link between q_0 and each q_i under the constraint, we can now move on to the more instructive part of our analysis, identifying a method for ranking profits across mechanisms in terms of the q_i's. The *Revenue Equivalence Theorem* dictates that the integrand of expected revenue is a linear function of the information rent term $\int_{\underline{\theta}_i}^{x_i} q_i(t_i)dt_i = U_i$ and its derivative $q_i(x_i)$. Given our symmetry assumption, for $i \neq 0$ let $q_i = q$, $U_i = U$. Then we can define for $i \neq 0$ the representative functions $y = U = \int_{\underline{\theta}}^{x} q(t)dt$, $y' = U' = q(x)$ to be inserted into the N remaining Euler-Lagrange conditions. These facts, combined with our previous work give us the following result.

Proposition 2.3.1. *Fix $U(\overline{\theta}) = \overline{U}$, $U(\underline{\theta}) = 0$, and say that $-f(x) - \frac{d}{dx}[f(x)(x - x_0)] < 0$ for $x \in [a, b] \subseteq [\underline{\theta}, \overline{\theta}]$. Then any perturbation of the interim expected utility $U(x) = \int_{\underline{\theta}}^{x} q(t)dt$ toward 0 for $x \in [a, b]$ such that $\delta U = 0$ at all other values increases the seller's expected profits.*

Proof. Given that $U(\underline{\theta}) = 0$ and $U(\overline{\theta}) = \overline{U}$ the values of U are fixed at the boundaries, so for each $i \neq 0 \quad \delta\pi = \int_{\underline{\theta}}^{\overline{\theta}} \{\Phi_U - \frac{d}{dx}\Phi_{U'}\}\delta U dx \quad \forall x$. Now compute the N identical expressions from the left hand side of (2.18) using (2.19) so that for we have $\Phi_U = -f(x)$, $\frac{d}{dx}\Phi_{U'} = \frac{d}{dx}[(x + \lambda)f(x)]$. With the substitution $\lambda = -x_0$, this yields

$$-f(x) - \frac{d}{dx}[(x - x_0)f(x)] < 0 \quad x \in [a, b] \tag{2.21}$$

Under this condition any reduction in $\int_{\underline{\theta}}^{x} q(t)dt$, i.e. $\delta U < 0$ for $x \in [a, b]$ fixing $\delta U = 0$ elsewhere sets $\delta\pi > 0$, and so must increase the seller's expected profits. □

The constraint condition $\lambda = -x_0$ reflects the fact that the cost of increasing the probability of allocation in any solution, i.e. fixing every q_i for $i \neq 0$ is x_0; the seller would forego her cost/valuation by allocating the good. Given this constraint, the calculus of variations problem describes the tradeoff between reducing a buyer's information rent $U = \int_{\underline{\theta}}^{a} q(t)dt$ at say $x = a$, given that this must reduce the seller's efficient payoff $q(x)(x - x_0)$ from buyer types below a. The condition (2.21) in Proposition 2.3.1 ensures that this tradeoff is worthwhile; the probability of allocating to any type given the possible surplus $(x - x_0)$ should be sacrificed in order to reduce the information rents on those types above. Although Proposition 2.3.1 states that any reduction in U over some domain is beneficial, we shall be primarily interested in situations in which U is reduced $\forall x \in (\underline{\theta}, \overline{\theta})$. Therefore, for the remainder of the analysis we shall assume that (2.21) holds $\forall x$.

A natural question is how reductions in interim utility can be achieved. Clearly if the problem is not constrained in any way the solution is trivial; one should set $q(x) = 0$ and thus $U(x) = 0 \quad \forall x \in (\underline{\theta}, \overline{\theta})$. There are three reasons why this cannot occur in equilibrium. First, if we fix some allocation rule such that the object is allocated with positive probability, then the constraint $J_i(\mathbf{q}) = 0 \quad \forall i \neq 0$ ensures that q will be positive over some domain. The seller cannot deceive buyers into believing they cannot win when this is a possibility. Second, the seller's interim IC will indeed ensure that the good will be allocated with positive probability, since she will seek positive profits through trade. Third, we must ensure that $U(\overline{\theta})$ is fixed at $\overline{U}$ so reductions in U for types below $\overline{\theta}$ cannot be so dramatic as to prevent this. Rather, we shall be looking at more subtle reductions in $U(x)$ that relate to the shape of expected utility, and the economics of how market power and the pricing rule determines this shape, which is the subject of Section 2.4.2. Before presenting this analysis, we first discuss the intuition and implications of Proposition 2.3.1 in further detail.

2.4 Probabilities and Convexity

2.4.1 Ranking Mechanisms Using Stochastic Dominance

How would one compare the condition in (2.21) to the requirement of increasing virtual valuations (regularity) as used in the standard proof of Proposition 2.2.1? Regularity can be written as

$$2f(x) + f'(x)\frac{1 - F(x)}{f(x)} > 0 \tag{2.22}$$

Equation (2.21) is equivalent to

$$2f(x) + f'(x)(x - x_0) > 0 \tag{2.23}$$

If $f'(x) \geq 0$ at every point, then both conditions are satisfied. If $f'(x) < 0 \quad \forall x$, it is easily verified that regularity is stronger, i.e. (2.22) implies (2.23) when $\psi(x) \geq x_0$, and weaker when $\psi(x) \leq x_0$. Then in some sense our condition (2.23) is an 'average' around the pivot point $\psi^{-1}(x_0)$ from the pointwise optimization. This is intuitive given that we are concerned with the selection of q, itself an average over the subject of the pointwise optimization, Q. Our quest for a method that is more flexible than pointwise ranking has therefore been accommodated by a smoothed version of the regularity condition. The interpretation of Proposition 2.3.1 as an average and a generalization of the pointwise method is further strengthened when we consider the characteristics of the probability function.

The probability function q is the expectation over the types of all other players such that a buyer wins the object in equilibrium. Maintaining our assumption of symmetry in distributions and the allocation rule for $i \neq 0$, and considering only the probability of transfer between the seller and some Buyer i, we can characterize q as a mechanism dependent probability function. Let us present this now formally.

Observation 2.4.1. In any incentive compatible direct mechanism A that is symmetric among $i \neq 0$, we can write q^A as a joint probability function characterized by the threshold type $\underline{x}^A(x_0)$;

$$\underline{x}^A(x_0) = \begin{cases} \inf\{x : Q_i^A(X_i = x, X_0 = x_0, X_j < x, \forall j \neq i, 0) = 1\} & \text{if this exists,} \\ \overline{\theta} + c, \quad c > 0 & \text{otherwise} \end{cases}$$

The probability function $q^A(t)$ can be written as $Pr[\underline{x}^A(X_0) \leq t]F(t)^{N-1}$, where $\underline{x}^A(X_0)$ is a mechanism dependent random variable. Two mechanisms can then be compared in terms of the distributions of the threshold values they induce.

The threshold value $\underline{x}^A(x_0)$ is the lowest type of Buyer i that can win the object given seller type x_0, assuming that i is the highest buyer type.[21] When winning the good is impossible in equilibrium conditional on x_0, we set $\underline{x}^A(x_0) > \overline{\theta}$ so that the buyer assigns zero probability of winning to all such values of x_0.[22] The random variables $(\mathbf{X}_{-(i,0)}, \underline{x}^A(X_0))$ have their own

[21] Of course, $\underline{x}^A(x_0)$ is independent of the identity of the buyer.

[22] The larger the set of seller types for whom $\underline{x}^A(x_0) > \overline{\theta}$ the lower $q^A(t)$ for every t.

joint cdf F^A which inherits the usual properties of being non-decreasing on $[\underline{\theta}, \overline{\theta}]^{N-1} \times [\underline{x}^A(\underline{\theta}_0), \underline{x}^A(\overline{\theta}_0)]$ and taking the values of 0 and 1 at the boundaries. However, we are interested in the function $q^A(t) = Pr[\underline{x}^A(X_0) \leq t]F(t)^{N-1}$ which applies F^A to the argument $t \in [\underline{\theta}, \overline{\theta}]$. Since $[\underline{x}^A(\underline{\theta}_0), \underline{x}^A(\overline{\theta}_0)]$ and $[\underline{\theta}, \overline{\theta}]$ need not perfectly overlap, in general $Pr[\underline{x}^A(X_0) \leq \overline{\theta}] \leq 1$, and there is no guarantee that in comparing two mechanisms, $Pr[\underline{x}^A(X_0) \leq \overline{\theta}] = Pr[\underline{x}^B(X_0) \leq \overline{\theta}] = 1$. Recognizing that q^A is characterized by F^A over the restricted domain $[\underline{\theta}, \overline{\theta}]$, we can recast the result of Proposition 2.3.1 in terms of stochastic dominance criteria.

A distribution H dominates another distribution G over the domain $[\underline{\theta}, \overline{\theta}]$ according to *Restricted First Order Stochastic Dominance* (FOSD) if and only if $H(t) \leq G(t) \quad \forall t \in [\underline{\theta}, \overline{\theta}]$ with at least one strict inequality. H dominates G in terms of *Restricted Second Order Stochastic Dominance* (SOSD) if and only if $\int_{\underline{\theta}}^{x} H(t)dt \leq \int_{\underline{\theta}}^{x} G(t)dt \quad \forall x$, and with at least one strict inequality.

Proposition 2.4.1. *Say that* $U^A(\underline{\theta}) = U^B(\underline{\theta}) = 0$, $U^A(\overline{\theta}) = U^B(\overline{\theta}) = \overline{U}$ *and (*2.21*) holds* $\forall x$. *Then Mechanism A generates greater seller profits than Mechanism B if* F^A *dominates* F^B *in terms of SOSD.*

Proof. The proof follows by substituting $q^A(t) = Pr[\underline{x}^A(X_0) \leq t]F(t)^{N-1}$ and $q^B(t) = Pr[\underline{x}^B(X_0) \leq t]F(t)^{N-1}$ as per Observation 2.4.1 into the definition of SOSD then applying Proposition 2.3.1, which is allowable because the boundary conditions and (2.21) are satisfied. We call this case the weak form of SOSD because the condition holds with equality at $x = \overline{\theta}$.[23] □

The stochastic dominance perspective is useful in several senses. First, it shows that our calculus of variations method of proof does indeed generalize the standard point by point method since, given Observation 2.4.1, the latter is equivalent to establishing First Order Stochastic Dominance, i.e. pointwise ranking of distributions/probabilities, which implies but is stronger than SOSD.[24]

[23] Note that the following is true for any distribution over any subset $[\underline{\theta}, \overline{\theta}]$ of its domain; $\int_{\underline{\theta}}^{\overline{\theta}} F(t)dt = \overline{\theta}F(\overline{\theta}) - \underline{\theta}F(\underline{\theta}) - \int_{\underline{\theta}}^{\overline{\theta}} tf(t)dt$. In the standard unrestricted implementation of stochastic dominance, two distributions are compared over their supports, which are assumed identical. Then $F^A(\overline{\theta}) = 1$, $F^A(\underline{\theta}) = 0$ and similarly for B, so the weak case of SOSD will imply that the distributions have the same mean. Under our restricted definition of SOSD this not guaranteed, since $F^B(\overline{\theta})$ need not equal $F^A(\overline{\theta})$. It follows that our definition of SOSD is not in general equivalent to the concept of a mean preserving spread.

[24] While FOSD implies SOSD, Proposition 2.3.1 does not as yet imply that FOSD of

Further, this generalization provides us with an obvious approach for achieving new profit ranking results; search for conditions where SOSD holds and FOSD does not. As we shall see, this leads us to examine some interesting classes of mechanisms, in particular explaining why the seller prefers more monopolistic to monopsonistic pricing. SOSD intuitively relates to the variance of a distribution, and it is simple to see that the probability function $q(x)$, when positive, is spread over more values of x under monopsony than monopoly. The reason is that monopsonistic mechanisms smooth a buyer's uncertainty regarding whether or not she will win the object over more realizations of her type, since no particular type would choose to lock-in a zero probability of trade. On the other hand, monopolistic mechanisms totally exclude some low buyer types, thus pushing the mass of the q's toward the set of high types. The relationship between monopoly pricing, the shape of the utility and probability functions and the seller's expected profits is now discussed.

2.4.2 Probabilities/Convexity and Monopoly Pricing

We are interested in making some statements about expected profits in mechanisms in which the allocation probabilities cannot be ranked point by point. That is, we are searching for conditions where SOSD will succeed where FOSD fails. It is straightforward to demonstrate that the criterion that links SOSD to the allocation rule in such cases is convexity in utilities, or equivalently, the slope of the allocation probability function. The proposition that follows is an exposition of this fact. The remainder of the section is dedicated to an examination of how more monopolistic pricing can be used to facilitate convexity.

Proposition 2.4.2. *Say that $U^A(\overline{\theta}) = U^B(\overline{\theta}) = \overline{U}$, $U^A(\underline{\theta}) = U^B(\underline{\theta}) = 0$ and q^A and q^B intersect at $x = x'$, where $U^A(x'), U^B(x') > 0$.*[25] *Then if $U''^A(x) > U''^B(x)$ wherever both are positive, Mechanism A generates greater ex ante expected profits to the seller than Mechanism B.*

Proof. The convexity condition; $U''^A(x) > U''^B(x)$ wherever both are positive is equivalent to the statement $q'^A(x) > q'^B(x)$ wherever positive. Since

itself can be used to achieve a profit ranking. The reason is that FOSD implies the strong form of SOSD, i.e. that the boundary conditions are violated. In Section 2.4.4 we provide conditions such that the strong form of SOSD is sufficient for a profit ranking through a treatment of the effect of variations at the boundaries.

[25]The alternative is that q^A and q^B don't intersect (or equivalently, they do but trivially at $q(x') = 0$), in which case the allocation rules can be ranked point by point.

q^A and q^B cross where both are positive at x', we have $q^A(x) < q^B(x)$ for $x < x'$, $q^A(x) > q^B(x)$ for $x > x'$ and $\underline{x}^A(\underline{\theta}_0) > \underline{x}^B(\underline{\theta}_0)$. These facts imply that $U^A(x) < U^B(x)$ for $x \leq x'$. For $x \geq x'$, take the points $U^A(x') < U^B(x')$ and $U^A(\overline{\theta}) = U^B(\overline{\theta})$. Then $U^A(x)$ and $U^B(x)$ cannot intersect between x' and $\overline{\theta}$, since this would imply that U^B is steeper than U^A over some subset of $[x', \overline{\theta}]$, which we have shown is impossible. Then $U^A(x) \leq U^B(x) \quad \forall x$, with strict inequality for $x < \overline{\theta}$, so by Proposition 2.3.1 Mechanism A dominates Mechanism B. □

Convexity of the utility function describes the slope of the allocation probability function q_i from the perspective of a generic buyer. Observation 2.4.1 shows that, fixing symmetry among $i \neq 0$, in any two Mechanisms A and B we have $q^A(x) = Pr[\underline{x}^A(X_0) \leq x]F(x)^{N-1}$ and $q^B(x) = Pr[\underline{x}^B(X_0) \leq x]F(x)^{N-1}$. The purpose of this observation was purely expository; to identify q as a probability function over a mechanism dependent random variable so that stochastic dominance intuition could be invoked. An equivalent but more natural way to express q in the context of equilibrium in the mechanism would be for it to reflect a buyer's direct expectation over the unknown types of the other players, applied to the threshold values required for Buyer i to win in equilibrium. In this vein, we can alternatively write $q^A(x) = F_0(\overline{x}_0^A(x))F(x)^{N-1}$ where

$$\overline{x}_0^A(x) = \begin{cases} \sup\{x_0 : Q_i^A(X_i = x, X_0 = x_0, X_j < x, \forall j \neq i, 0) = 1\} & \text{if this exists,} \\ \underline{\theta}_0 & \text{otherwise} \end{cases}$$

and similarly for B.[26] The threshold value $\overline{x}_0$ defines the highest type of seller that allows Buyer i to win the object in equilibrium, conditional on her being the highest type among $j \neq 0$. If winning is impossible conditional on the type realization x, then $\overline{x}_0(x) = \underline{\theta}_0$ and the cdf over X_0 ensures that the buyer's interim probability estimate is $q(x) = 0$. Recalling Proposition 2.4.2, we are interested in differences in the convexity of utilities in A and B, where positive, as described by differences in the slopes of q^A and q^B, again where positive. Under the conditions of Proposition 2.4.2, the seller should then choose the mechanism with the higher $q'(x)$.

Observation 2.4.2. The seller's threshold type function $\overline{x}_0(x)$ is steeper in her preferred mechanism at $x = x'$.

Proof. To see this, use $q(x) = F_0(\overline{x}_0(x))F(x)^{N-1}$ and we have

$$q'(x) = f_0(\overline{x}_0(x))\frac{d\overline{x}_0(x)}{dx}F(x)^{N-1} + F_0(\overline{x}_0(x))\frac{d}{dx}F(x)^{N-1} \tag{2.24}$$

[26] Again, $\overline{x}_0^A(x)$ depends only on the buyer's type realization x, not on her identity.

Given that q^A and q^B intersect at $x = x'$, the convexity condition becomes $q'^A(x) > q'^B(x)$ if and only if $\frac{d\overline{x}_0^A(x)}{dx} > \frac{d\overline{x}_0^B(x)}{dx}$ at $x = x'$. □

Of course, the threshold type cannot be separately chosen at each point, it is an increasing and continuous function reflecting the allocation rule of some equilibrium. Arbitrarily choosing $\overline{x}_0^A(x)$ to be steeper over some domain may alter the probability functions q^A and q^B such that they no longer intersect, rendering Proposition 2.4.2 inapplicable. In fact, we demonstrate that for a broad class of allocation rules the seller can indeed chose a steeper function for $\overline{x}_0(x)$ while ensuring that intersection in probability functions occurs. Moreover, for a general class of mechanisms this can be achieved by a convergence toward monopoly pricing. We argue that the use of monopoly pricing to achieve steeper threshold type functions, or equivalently greater convexity in utilities, provides intuition for the result that the specification of private seller information does not change the optimal selling mechanism.

2.4.3 Ex Ante Efficiency and the Convexity Condition

Williams (1987) defines for bilateral trade ($i = \{1\}$) a group of allocation rules parameterized by $t, s \in [0, 1]$ as follows

$$Q_1^{(t,s)}(x, x_0) = \begin{cases} 1 & \text{if} \quad \psi_s(x) \equiv x - s\frac{1-F(x)}{f(x)} \geq x_0 + t\frac{F_0(x_0)}{f_0(x_0)} \equiv \zeta_t(x_0) \\ 0 & \text{otherwise} \end{cases} \tag{2.25}$$

Specific cases of the (t, s) allocation rule are shown by Williams (1987) to maximize the ex ante total expected gains from trade (efficiency), where t and s are dictated by the weights on the buyer's and seller's utilities respectively.[27] The following proposition examines the relationship between the slope of $\overline{x}_0(x)$ and its level.

Proposition 2.4.3. *Among the set of ex ante efficient allocation rules, there is no tradeoff between the level and slope of the allocation probability function.*

Proof. First define $\overline{x}_0^{(t,s)}(x) = \zeta_t^{-1}(\psi_s(x))$, which is decreasing in t and s for $x \geq \underline{x}^{(t,s)}(\underline{\theta}_0)$. On the other hand, it is easily verified using the regularity conditions that $\frac{d\overline{x}_0^{(t,s)}(x)}{dx}$ is decreasing in t and increasing in s, again for

[27] Under our symmetry assumption the analogue for $N > 1$ is identical once we index Player 1 as the highest type of buyer. It is easily verified that $(t, s) = (0, 1)$ describes the allocation rule that maximizes the seller's profits from Proposition 2.2.1.

$x \geq \underline{x}^{(t,s)}(\underline{\theta}_0)$. Then by a continuity argument, the seller can always decrease t and increase s to increase the slope of $\overline{x}_0(x)$ at a given intersection point x' without affecting $\overline{x}_0^{(t,s)}(x')$. The key to this is that from (2.25) $\underline{x}^{(t,s)}(\underline{\theta}_0)$ is independent of t. The seller can make her threshold type function steeper while achieving a given level for $\overline{x}_0(x')$ by making $\overline{x}_0(x)$ 'start' later, i.e. by increasing $\underline{x}^{(t,s)}(\underline{\theta}_0)$. □

The seller's threshold type function is so steep under monopoly because as a price-taker, the buyer truthfully announces her valuation, so any increase in her type will be mapped directly into an increase in her allocation probability. Having made this observation, a simple computation shows that $s = 1$ maximizes the slope of $\overline{x}_0^{(t,s)}$ for any t, minimizing the slope of $\underline{x}^{(t,s)}(x_0)$ and thus the set of seller types who never trade.

Maskin and Tirole (1990) have shown that a principal does not benefit from holding private information under risk neutrality. That is, she chooses the same profile of contracts conditional on her type and reaps the same ex ante profits. Our exposition of the link between monopoly price setting and the seller's preference for convex utilities/steep allocation probabilities provides a new interpretation for this fact.[28] The lowest type of seller ignores the effects of holding private information because buyers do; they always view themselves as competing against her, since they must announce a type above $\underline{x}(\underline{\theta}_0)$ to have any chance of winning. Conditional on this, to allow the allocation rule to reflect in any way the buyer's beliefs (in the above class $t \neq 0$) would force some types of seller to lock in a zero probability of trade, since the allocation probability must be decreasing in x_0 and the set of ex ante excluded types is so high. Given that monopoly pricing is an equilibrium regardless of whether the seller's type is private information, the seller has benefited from undoing the effects of holding private information because monopoly maximizes the slope of the threshold type function $\overline{x}_0$, and thus convexity. Under the conditions of Proposition 2.4.2, greater convexity leads to greater seller profits.

Our calculus of variations method ranks mechanisms through a condition on the buyers' interim utilities, so it is of interest to know how their average utilities, or ex ante expected surplus is affected. Reducing $U(x)$ for (almost) every x must also reduce its expectation, so ranking mechanisms through SOSD achieves a lower expected surplus for the buyers. The

[28] In fact, while the intuition is similar to that in Maskin and Tirole (1990), Bayesian trading games are not a special case of their analysis since they assume discrete types.

following identity can be used to demonstrate this intuition;

$$\begin{aligned} U(\overline{\theta}) &\equiv U(\overline{\theta})F(\overline{\theta}) - U(\underline{\theta})F(\underline{\theta}) \\ \int_{\underline{\theta}}^{\overline{\theta}} q(x)dx &= \int_{\underline{\theta}}^{\overline{\theta}} U(x)f(x)dx + \int_{\underline{\theta}}^{\overline{\theta}} U'(x)F(x)dx \\ \int_{\underline{\theta}}^{\overline{\theta}} q(x)[1-F(x)]dx &= E_X[U(x)] \end{aligned} \tag{2.26}$$

Increasing the convexity of utilities means that although the q's will be higher 'at the top',[29] they are weighted by $[1 - F(x)]$, which is lower as we approach $\overline{\theta}$. The content of Proposition 2.4.2 is that regardless of the differences in $q(\overline{\theta})$ across mechanisms, stacking the probability function to be higher 'at the top' reduces the left hand side of the above expression and thus the buyer's expected surplus. While it may be intuitive that the seller wishes to capture the buyers' surplus, the real value of Proposition 2.4.2 is that we have argued that such a redistribution is always advantageous (given the distribution assumption of Proposition 2.3.1), regardless of any effects on the level of total surplus. That is, we are not relying on a division of surplus argument. The intuition of our results can be further extended so that they do not require fixing the boundary values of U. The next section develops this result. In Section 2.5 we apply our results to a well-known class of mechanisms, the k double auctions of bilateral trade.

2.4.4 Free Boundary Conditions and $U(\overline{\theta})$

We assumed in Section 2.3.2 that the variation, that is, the types of functions for U we were comparing differed in the interior of their domains, but not on the boundaries. In this section we relax the fixed boundary assumption at $x = \overline{\theta}$ and provide conditions such that the intuition of Proposition 2.3.1 carries through. We ignore the boundary condition for $x = \underline{\theta}$ since we are only concerned with mechanisms in which $U_i(\underline{\theta}) = 0 \quad \forall i \neq 0$ as per the arguments of Section 2.2.3.

Corollary 2.4.1. (Proposition 2.3.1) Taking the variation including the boundary term from Eq. (2.16), in order for reductions in U to remain

[29]Recall we are presuming that the q's intersect.

advantageous the N variations on U_i for $i \neq 0$ must satisfy $\forall \delta U < 0$

$$\begin{aligned} \int_{\underline{\theta}}^{\overline{\theta}} [\Phi_U - \frac{d}{dx}\Phi_{U'}]\delta U dx + \Phi_{U'}|_{x=\overline{\theta}}\delta U(\overline{\theta}) &> 0 \\ -\int_{\underline{\theta}}^{\overline{\theta}} [f(x) + \frac{d}{dx}[f(x)(x-x_0)]]\delta U dx + f(\overline{\theta})[\overline{\theta} - x_0]\delta U(\overline{\theta}) &> 0 \end{aligned} \quad (2.27)$$

If we maintain (2.21) $\forall x$ the first term is positive for $\delta U < 0$. However, the addition of the variation on the boundary term introduces a countervailing force; if U is reduced for every x then $\delta U(\overline{\theta}) < 0$ and the second term in (2.27) will be negative for some values of x_0.[30] The net effect of the variation on the seller's profits will be dictated by the relative magnitudes of δU and $\delta U(\overline{\theta})$. A specific example of this condition is computed and verified in the application we consider in the next section. When (2.27) holds $\forall \delta U < 0$, we have the following;

Observation 2.4.3. If the allocation rules of Mechanisms A and B can be ranked point by point, so can their respective probability functions q^A and q^B. Then if (without loss of generality) $q^A(x) \leq q^B(x) \quad \forall x$ with strict inequality for some subset of $[\underline{\theta}, \overline{\theta}]$, F^A dominates F^B in terms of FOSD, and therefore also in terms of SOSD. From Corollary 2.4.1, Mechanism A dominates Mechanism B if (2.27) holds.[31]

Again, it is only appropriate to state this observation after proving Corollary 2.4.1, because FOSD implies the strict form of SOSD; $\int_{\underline{\theta}}^{\overline{\theta}} F^A(t)dt < \int_{\underline{\theta}}^{\overline{\theta}} F^B(t)dt$. This means that $U^A(\overline{\theta}) < U^B(\overline{\theta})$ so the boundary conditions are not fixed and Proposition 2.3.1 cannot be applied.

2.5 Applications: k-Double Auctions in Bilateral Trade

We now apply our results to analyze some known, but curious results from a particularly interesting case of bilateral trade, the linear equilibria of

[30] The only exception is where x_0 is always greater than $\overline{\theta}$, in which case gains from trade do not exist.

[31] Of course, (2.27) can hold even when FOSD does not. For example, take the conditions of Proposition 2.4.1, in which q^A and q^B cross, but say that $U^A(x) < U^B(x) \quad \forall x$ so that $U^A(\overline{\theta}) \neq U^B(\overline{\theta})$. This is equivalent to the strict form of SOSD. We consider such a case in the next section.

the sealed bid k double auction as modelled by Chatterjee and Samuelson (1983). In this format, a buyer and seller submit sealed tenders B and S respectively, with trade occurring when $B \geq S$, at a price determined by the parameter $k \in [0,1]$ according to $P = kB + (1-k)S$. Chatterjee and Samuelson (1983) characterize equilibrium strategies, allocation rules and probabilities and expected (ex ante) profits for each party. For buyer and seller valuations distributed uniformly on $[0,1]$, they derive the result that the seller's expected profits are maximized at $k = 0$.[32] Therefore the seller wishes to maximize her market (price setting) power, as captured by the price weighting $(1-k)$. On the other hand, the ex ante volume of trade varies non-monotonically over k according to $v(k) = \frac{1}{8}(-k^2+k+2)$. It is not well understood whether the seller's preference for the lowest possible k is purely an artefact of the simple linear structure of the example, or whether it reflects some deeper economic reasoning. Arguably the source of this gap in understanding is the fact that mechanisms in this class are completely described by the parameter k, and the sign of the relationship between k and the equilibrium allocation rule (over and above its ex ante expectation $v(k)$) varies over the domain of signals. In other words, the allocation probabilities of mechanisms in this class cannot be ranked point by point. To see this, examine the bid and offer functions below;

$$\begin{aligned} S(x_0) &= \frac{x_0}{2-k} + \frac{1-k}{2} \\ B(x) &= \frac{x}{1+k} + \frac{k(1-k)}{2(1+k)} \end{aligned}$$

Given the bid and offer functions in the k double auctions, a buyer of type x wins the object in equilibrium iff $x_0 \leq \overline{x}_0^k(x)$;

$$\overline{x}_0^k(x) = \begin{cases} \frac{2-k}{1+k}\left[x - \frac{1-k}{2}\right] & \text{if} \quad x \geq \frac{1-k}{2} \\ 0 & \text{otherwise} \end{cases} \tag{2.28}$$

From these definitions we have the following result.

Proposition 2.5.1. *No two mechanisms in the class of k double auctions can be ranked point by point, or, equivalently, by FOSD.*

Proof. To demonstrate that the point by point method is inapplicable in this scenario, note that given some k, the buyer's allocation probability estimate for types at or above $\frac{1-k}{2}$ is $q^k(x) = F_0(\overline{x}_0^k(x)) = \overline{x}_0^k(x)$ under the uniform

[32] See Chatterjee and Samuelson (1983) Example 1 pp. 842.

$[0,1]$ distribution. The value $\underline{x}(0) = \frac{1-k}{2}$ is decreasing in k, so for $k' > k$ there exist buyer types such that $q^k(x) < q^{k'}(x)$. Alternatively, $q^k(1) = \frac{2-k}{2}$ which is decreasing in k, so $q^k(1) > q^{k'}(1)$. It follows that no two allocation rules in the class of k double auctions can be ranked point by point. □

On the other hand, our technique can be applied. The following proposition compares allocation rules in pairs of mechanisms subject to fixed boundary conditions, i.e. where $U^k(0) = U^{k'}(0) = 0$ and $U^k(1) = U^{k'}(1)$.

Proposition 2.5.2. *For $\hat{k} \in [0, \frac{1}{2})$, the buyer's interim probability of allocation in the $\hat{k}$ double auction dominates that in the $(1-\hat{k})$ double auction by weak SOSD. By Propositions 2.3.1, 2.4.1 the $\hat{k}$ double auction generates greater profits to the seller than the $(1-\hat{k})$ double auction.*

Proof. Our previous discussion has already shown that the allocation probabilities of $\hat{k}$ and $1-\hat{k}$ intersect. Now compute $q'^k(x)$ when $x \geq \frac{1-k}{2}$ to give $\frac{2-k}{1+k}$. Since this is decreasing in k, so is the convexity of utility. To check the remaining conditions for Propositions 2.3.1 and 2.4.1 note that the condition in (2.21) is satisfied because under any uniform distribution $f(x)$ we have $f'(x) = 0$, so the condition collapses to $-2 < 0 \quad \forall x$. By inspection of (2.28), $q(0) = 0$ and thus $U(0) = 0$ for any k. We can now compare any two mechanisms in the class of k double auctions according to our convexity criteria provided they have the same $U(1)$. Under the uniform $[0,1]$ distribution we can say that $U^k(1) = \int_0^1 q^k(t)dt = \int_0^1 q^k(t)f(t)dt = v(k)$. So any two mechanisms with the same volume of trade have the same $U(1)$. Chatterjee and Samuelson (1983) show that in the k double auction the volume of trade $v(k)$ is symmetric in k around $k = \frac{1}{2}$, which proves the result.[33] □

Having fixed $U(1)$ and decreased $U(x)$ at every other x, we must have reduced the buyer's expected surplus. A useful property of the k double auction under uniform beliefs is that both the volume of trade and the total surplus are pinned down once we have $U(1)$.[34] The seller therefore achieves a better division of the fixed surplus. The more convex and more monopolistic

[33]Myerson and Satterthwaite (1983) provide more general conditions for a mechanism to be ex ante efficient, which includes a demonstration of Chatterjee and Samuelson's probability of trade result, that the $k = \frac{1}{2}$-double auction maximizes the expected gains from trade for the uniform [0,1] distribution.

[34]In the proof above we have used the fact that $U(1) = v(k) = v(1-k)$. Total surplus is pinned down because v and total surplus are proportional. Recall from our discussion in Section 2.4.2 that in general our method does not require fixing the total surplus, although this will certainly achieve the result.

mechanism $\hat{k}$ reduces the utilities of all but highest type of buyer, reducing rents and instilling a greater level of *virtual competition* while simultaneously fixing the total volume of trade. Our methods demonstrate that the seller prefers $\hat{k}$ to $(1-\hat{k})$. Can we say more? To derive the result that $k = 0$ is optimal, we first use the fact that from Proposition 2.5.2 we need only consider $k \in [0, \frac{1}{2}]$. Now observe the effect of reducing k from $k = \frac{1}{2}$, given that the volume of trade $v(k)$ and hence $U^k(1) = v(k)$ are increasing in k. Using the criterion of strict SOSD, it is straightforward to show that increasing convexity (i.e. reducing utilities) remains advantageous despite the fact that it reduces the volume of trade, and thus total surplus.

Proposition 2.5.3. *For $k \in (0, \frac{1}{2}]$, decreasing k increases the seller's profits. Given Proposition* 2.5.2*, the seller's profits are maximized by setting* $k = 0$.

From (2.27) and our distributional assumptions the variation must satisfy

$$\begin{aligned} 2\int_{\underline{\theta}}^{\overline{\theta}} \frac{dU^k(x)}{dk}dx &> (1-x_0)\frac{dU^k(1)}{dk} \\ 2\frac{d}{dk}\int_{\underline{\theta}}^{\overline{\theta}} U(x)f(x)dx &> (1-x_0)v'(k) \\ \frac{d}{dk}E_X[U(x)] &> \frac{(1-x_0)}{2}\frac{(1-2k)}{8} \end{aligned} \tag{2.29}$$

Proof. The second line follows from $U^k(1) = v(k)$ and $f(x) = 1 \quad \forall x$ under the uniform distribution. Chatterjee and Samuelson (1983) show that $E_X[U(x)] = (1+k)^2(2-k)$, so the inequality holds $\forall k, x_0$ by a tedious calculation. Reducing k therefore increases the seller's expected profits for $k \in (0, \frac{1}{2}]$, which combined with Proposition 2.5.2 ensures that $k = 0$ is optimal $\forall k$. □

2.6 Monopoly Pricing as the Optimal Mechanism

The correspondence between the optimal mechanisms of Myerson (1981) and Williams (1987) can be broadly described as follows; the seller prefers to earn monopoly rents than to surrender pricing power and earn information rents. Despite the significant body of literature devoted to the classification of incentive compatibility under different information structures, this observation, and more generally, the effect of market power on mechanism design under two-sided private information has not to this point been adequately explained. Arguably this is due to the fact that differences in pricing rules typically do not map monotonically into differences in allocation rules, making employment of the *Revenue Equivalence Theorem* problematic. Our analysis represents a first step in addressing these problems. We have introduced new techniques to deal with the comparison of mechanisms in which allocation rules cannot be interim ranked in every eventuality. These tools have proven effective for the analysis of monopoly pricing, because market power is described by the shape of the allocation rule, rather than its values. Monopoly pricing is a powerful device because it reduces the set of buyer types who win with positive probability. For buyers who do win, there are fewer lower types one could have masqueraded as and won in equilibrium, and thus less surplus required to induce incentive compatibility. This increase in *virtual competition* reduces the information rents of the buyers in the seller's favour.

In Essays II and III, the interdependence between the valuations on each side of the market hinders the principal's ability to implement monopoly pricing, such that the conclusions derived here for the IPV setting cannot be extended. We therefore view the 'adversarial' nature of the non-cooperative bargaining solution found here to be relevant only for parties whose preferences are absolutely determined.

Chapter 3

Auctions with an Informed Seller: Signalling

3.1 Disclosed vs Secret Reserve Prices

As we discussed in Section 1, in auction theory the seller's valuation and reserve price are typically treated as publicly known. Within the standard framework of Independent Private Values (IPV) models, these impositions do not restrict the predictions of the theory in any significant way. For example, bidding one's value remains a weakly dominant strategy in an English or Second Price Sealed Bid auction for any set of bidder beliefs regarding the seller's preferences or the reserve price.[1] Further, as was shown in our first essay, the seller cannot use her private information to extract any more rents than she could if her valuation were known; the optimal auction of Myerson (1981) remains optimal when the seller's valuation is private information. The standard paradigm of IPV models therefore instructs a seller as to the reserve she should select, but gives her no reason to favour disclosing or withholding it.

This is surprising given the pervasiveness of secret reserve prices. In many market situations bidders do not know the reserve when they are bidding, and may even be unsure as to whether one exists. Ashenfelter (1989) comments on this phenomenon in auctions for wine and art; '*...every item is hammered down and treated as though it were sold. Only after the auction does the auctioneer reveal whether and at what price the item may have actually been sold. In short, the auctioneers do not reveal the reserve price*

[1] Provided bidders incur no participation costs.

and make it as difficult as they can for bidders to infer it.' This behaviour is also common in auctions of real estate in New South Wales. The NSW Office of Fair Trading website reports that *'the seller will nominate a reserve price which is usually not told to the interested buyers.'*[2] One possible explanation for the discrepancy between the IPV theory and auction design in practice is the specification of private values. In particular the standard models ignore the fact that the seller's pricing decisions, and announcements of those decisions can influence buyers' opinions regarding the quality of a good. In many situations the seller, as a market expert or as custodian of the object holds private information as to its quality.[3] It is then reasonable that a buyer's estimate of the value of the good would by influenced by the seller's assessment. To the extent that the seller's valuation, and thus the reserve price she will set are strictly increasing in her signal, an announced reserve price represents an indirect announcement of this information. It is of interest to know whether the seller benefits from making such announcements in this context, or should rather use a secret reserve price. Cai, Riley, and Ye (2007) have characterized the separating equilibrium that describes reserve price signalling through disclosure before bidding commences. In this essay we compare the expected profits to the seller from immediate announcement of the reserve as in their model to those under two different secret reserve regimes;

(EN) *Envelope*: The seller, upon observing her signal and before bidding commences, selects a reserve price, but it is kept secret, sealed in an envelope, to be revealed once bidding has ended. In the event of non-clearance at auction, the reserve is announced as a take-it-or-leave-it offer to the winner.

(WS) *Wait and See*: The reserve is selected and revealed once bidding ends. In the event of non-clearance at auction, the reserve represents a take-it-or-leave-it offer to the winner.

A celebrated result in auction theory seems applicable here;[4]

[2]See NSW Office of Fair Trading; Reserve Prices, Website: http://www.fairtrading.nsw.gov.au/realestaterenting/buyingselling/buyingatauction.html.

[3]This is often referred to as the 'lemons' problem, and was first addressed by Akerlof (1970).

[4]The general argument and several implications for revenue rankings in the various auction forms were originally presented in Milgrom and Weber (1982a). Their Theorem 7 derives the result for the announcement of verifiable information in a Second Price Sealed Bid (SPSB) auction.

Theorem 3.1.1. *Linkage Principle* (Milgrom and Weber (1982a), informal statement[5]): If signals are independent, any two auctions generate the same expected profits to the seller in symmetric equilibria. The *direct* announcement of *verifiable* information has no effect on the seller's expected profits.

A casual application of this result suggests that the seller should have no preference between announcing the reserve price (and thus, indirectly her private information) and keeping it secret, provided bidders interpret announcements correctly so that the ordering of bidders and thus symmetry is preserved.[6] We demonstrate that this intuition is incorrect. The allocation rule of an auction specifies not only an ordering of players (i.e. selects a 'winner'), but also defines the rules of trade between buyer and seller, that is, the reserve price. The problems of reserve price selection and information disclosure cannot be separated (and thus the Linkage Principle cannot be applied) because both affect the incentive constraints of the seller. The set of allocation rules that can be implemented in equilibrium is changed by the timing of information announcements.

A significant part of our analysis examines whether different actions (immediate or later announcement) implementing the same strategy (reserve price) are mappings from incentive compatible allocation rules. In the standard theory, one can remain agnostic about the role of the reserve in price determination, since for a given reserve price any of the common auction forms will induce the same equilibrium allocation rule, and hence the same expected revenues.[7] It does not matter whether the reserve is a direct monopoly price setting device as in an English or SPSB auction, or an indirect type-selecting device, as in a First Price Sealed Bid (FPSB) auction. In our model, these considerations matter. For example, the conventional set-up of the English auction involves the reserve price acting as a take-it-or-leave-it offer to the winner in the event of non-clearance at auction.[8]

[5] The above is an informally stated special case of the *Linkage Principle* result. Our focus in the current section is on the implications of the theorem for the information disclosure problem in the particular environment we shall study. We therefore withhold a more rigorous statement of the result and direct the reader to Milgrom and Weber (1982a), Theorem 7.

[6] Here we address the case in which there is no impartial third party who can be *costlessly* employed to verify quality.

[7] See Myerson (1981) and Riley and Samuelson (1981).

[8] At least, this is the treatment the reserve usually receives in auction theory. It is this property that leads the seller to act like monopolist, setting a price above her private valuation/marginal cost, see Bulow and Roberts (1989). The reserve price offer is also

Cai, Riley, and Ye (2007) also address reserve prices of this form. In this essay we remain faithful to this structure by considering secret reserves as representing the ability to *delay* the announcement of a reserve until the offer stage, as in the EN and WS regimes. In our third essay we address the most extreme form of a secret reserve, one that never determines the price since it is never announced.

Whenever the seller announces her reserve price, she wants to claim favourable information, i.e. high quality. Conditioning on a given set of bidder beliefs about the reserve price rule, the seller can always announce a higher reserve to falsely represent higher quality. On the other hand, the incentive to do this is tempered by a reduction in the volume of sales. The higher the true quality, and thus seller's reservation value of the good, the less she loses from increasing the reserve and not selling it. Under fairly permissive conditions, it is possible to find prices where these incentives are balanced, so that signalling can occur and information can be completely revealed in equilibrium. However Cai, Riley, and Ye (2007) show that under immediate announcement of the reserve a lower level of trade (and profits) are realized than if the seller's information were publicly known.[9] We derive the separating equilibria of the EN and WS regimes, and demonstrate that later announcement, to the seller's detriment, shifts the undesirable characteristics of signalling from the bidding stage to the negotiation that follows the auction.

For a given reserve price *strategy*; (1) Bids clear the reserve less often at auction under secret reserve regimes than under immediate disclosure. In the separating equilibrium under immediate disclosure, bidders learn the seller's information before the auction and so bid their valuations. Bids are less likely to clear any given reserve price under secrecy, since at the margin–i.e. for signal values such that a given bidder's *valuation* would just reach the reserve–her bid under secrecy instead incorporates 'bad news', conditioning on all values of S that are low enough to be consistent with her bid clearing the reserve. However; (2) After accounting for transactions that

vital for the English auction to represent an implementation of the optimal auction of Myerson (1981).

[9]This is true for all except the lowest type of seller. They also demonstrate that the reserve price is increasing in the number of bidders even with independent signals. This is in contrast to the optimal reserve price under complete information and independence which is not a function of N. Jullien and Mariotti (2006) prove the existence of adverse selection in a very similar model for less general valuation functions than Cai, Riley, and Ye (2007). They show that the announced reserve price/monopoly mechanism can be less *ex ante* efficient than a second-price auction facilitated through an uninformed monopoly broker who buys from the seller and sells to the bidders after soliciting a fee.

occur though the reserve price offer the allocation rule is preserved (i.e. the *Linkage Principle* can be applied) and these mechanisms generate the same expected profits. Expected profits to the seller are the same for a *given reserve price strategy* under these regimes, because any differences in the bidders' estimates of S are resolved in expectation through the announcement of the reserve that occurs in the negotiation game. Thus (2) can be viewed as an application of the *Linkage Principle*; For a given allocation rule information effects do not influence the seller's expected profits under independence. It is the combination of (1) and (2) that allows us to perform a comparative static on reserve price selection in the separating equilibria of these regimes. (1) and (2) together imply that a greater share of profits are accrued through sale at the reserve price for a given reserve strategy under EN and WS than for immediate disclosure. This shifts more importance to the reserve price announcement under EN and WS and thus requires that signalling is even more costly to be credible in equilibrium. The prevalence of secret reserves cannot be explained in our framework as representing the ability to *delay* announcement. If a reserve will be announced eventually, then the sooner the better.

In related work, Vincent (1995) and Horstmann and LaCasse (1997) have also addressed the prevalence of secret reserves through a departure from the IPV framework. Vincent demonstrates in a note that an announced reserve price restricts bidder participation. A secret reserve therefore aggregates more bidder information increasing *Linkage Principle* effects to the seller's advantage. His model relies on affiliation style arguments and endogenous entry which we do not consider here. Horstmann and LaCasse (1997) consider secret reserves as a means of facilitating non-commitment to a given reserve. This allows a seller who has perfect information as to the object's common value to signal greater value for a future resale. Their model differs from ours since their common values specification implies that there can be no separating equilibrium in announced reserve prices that generates positive trade (see their Section III(A)). In contrast we analyze the comparative performance of secret reserves when full revelation *is* possible. Further, in our third essay we show that secrecy can generate greater profits from a given set of bidders than disclosure in a one-shot game.[10]

[10]It should be noted that departures from the standard utility framework have also been shown by Brisset and Naegelen (2006) and Rosenkranz and Schmitz (2007) to be possible explanations for a seller's preference for secret reserve prices. The former identify risk-averse bidders as preferring secret reserves, the latter use reference-based utility. An announced reserve triggers a reduction in utility related to the premium of the price paid over the reserve. This induces a lower probability of clearance for a given reserve price,

Despite the prevalence of secret reserves, their is little empirical evidence regarding their optimality. For the most part, this is due to the lack of a strong theoretical model against which one can test observed behaviour. Elyakime, Laffont, Loisel, and Vuong (1994) and Eklöf and Lunander (2003) for example find that secret reserves hurt the seller, however their results are unsurprising given that they compare actual bidding behaviour with simulations generated from an original hypothesis of Independent Private Values. As such they do not tell us anything that a simple understanding of the IPV model does not. Since we allow for interdependencies, their conclusions have no bearing here. In a more promising study, Katkar and Reiley (2006) conduct a field experiment by selling matched pairs of Pokémon cards using the same reserve price, half secret, half disclosed. They find that secret reserves deter 'serious' bidders who can instead compete under the disclosed reserve, knowing that they are 'in the running'. Given that the cards were purchased for the experiment at an average price of \$7.19,[11] the value of the good is unlikely to generate the common value effects we examine here. Arguably preferences for Pokémon cards are better described by the IPV model. These findings aside, we take the *Wilson Doctrine*[12] seriously, arguing that practitioners usually know what they are doing, and that it is the job of the theorist to explain why they are correct. One implication of our results (Essay III) is that we would expect secret reserves to be more popular when those setting them are experts, which seems sensible given the practice of auction houses.

We proceed with some definitions and a description of the rules and timing of each regime. Sections 3.3 and 3.4 characterize equilibrium behaviour in the negotiation stage, and equilibrium bidding under the secret reserve regimes. Section 3.5 compares the probability of allocation and expected profits across the three regimes for a fixed reserve price strategy. Section 3.6 reviews Cai et. al.'s treatment of the separating equilibrium under immediate disclosure, and provides sufficient conditions for a profit ranking with the EN and WS regimes. We then derive equilibria in these regimes in Section 3.7 and prove that immediate disclosure dominates later announcement of a reserve price. Section 3.8 recaps the results and discusses the possibility of improving upon the secret reserve mechanisms, which is the subject of the third essay in Chapter 4.

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reducing the effectiveness of announced reserves.

[11]See Katkar and Reiley (2006) pp. 7.

[12]See Krishna (2002) pp. 75.

3.2 The Model

3.2.1 Preliminaries

The set of players is given by $I = \{0, 1, ..., N\}$, $N \geq 2$ where Player 0 is the seller, and $I \setminus \{0\}$ is the set of bidders. Throughout the paper, the set of bidders is fixed and common knowledge. Each bidder receives a real-valued signal distributed on $[\underline{\theta}, \overline{\theta}]$ according to F with density f.[13] We require that the hazard rates on the X_i's are increasing. The seller's signal S is distributed according to F_S with density f_S on $[\underline{\theta}_S, \overline{\theta}_S]$. All signals are statistically independent.

The valuation of each bidder i is symmetric, non-negative and continuous, given by $V_i = v(X_i, S)$. The function v is assumed to be strictly increasing in bidder i's own signal (X_i) as well as that of the seller (S). Thus, each bidder cares about the seller's signal, in addition to her own, but not about the signals of the other bidders. We define the expected valuation of a bidder with signal x when $S \leq s$ as

$$w(x, s) \equiv E_S[v(x, S)|S \leq s]$$

The seller's valuation is $v_0(S)$, which is also strictly increasing. We shall also assume that v is separable in the private and common value components, and that $\frac{d^2 v(x,s)}{dx^2} \leq 0$.[14]

3.2.2 The Game

A single indivisible object is to be sold by way of an English auction, to be modelled in the familiar 'clock' format.[15] We compare the *ex ante* expected profits to the seller in such an auction given her choice among several reserve price regimes. Events unfold in four stages.

Stage 1: Reserve Regime Selection

The seller decides, before knowing the realization of her signal, which of the following three reserve price regimes she will employ.

[13] Again realisations of a variable will be written in lower case, uppercase letters denote the random variable itself.

[14] The preceding conditions on signals and valuation functions represent a special case of Cai, Riley, and Ye (2007). They assume the X's are affiliated (but independent of S), and they allow valuations to depend upon the signals of other bidders.

[15] This set-up is standard in the literature: see Milgrom and Weber (1982). Cai, Riley, and Ye (2007) model a Second Price Sealed Bid auction. In our set-up valuations do not depend on the signals of other bidders, so under immediate disclosure these auctions are equivalent.

(FD) *Full Disclosure*: The seller, upon observing her signal and before bidding commences, selects and announces a reserve price.

(EN) *Envelope*: The seller, upon observing her signal and before bidding commences, selects a reserve price, but it is kept secret, sealed in an envelope, to be revealed once bidding has ended. Thus, the seller is committed to this secret reserve.

(WS) *Wait and See*: The seller selects and reveals her reserve once bidding ends.

The seller publicly announces the regime she chooses. The regimes EN and WS are alike in the sense that they are *secret reserve regimes*: the particular value of the reserve is unknown to bidders during the bidding process. However, once bidding has ended, all of the regimes become similar because the reserve is public information at that point.

Stage 2: Signals Observed

Once the seller has selected among the three reserve price regimes, all players observe their signals. The seller then selects a reserve price under regimes FD and EN, announcing it under FD and privately committing to it without announcing it under EN.

Stage 3: Bidding

The bidders bid in an English auction. The price at the auction ascends with the bidders indicating their continued participation at each level. A bidder who is willing to buy the object at any price p is said to be *active* at p. Bidders exit publicly, voluntarily and irrevocably. As soon as only one bidder remains, bidding ends and this individual is said to have won the auction. The price at which this takes place is the *auction price*. We sometimes refer to this stage as the *bidding game*.

Stage 4: Negotiation and Allocation

Under EN, the reserve is now revealed. Under WS, the reserve is selected and announced. We say the reserve is *cleared* if the auction price is at least as high as the (by now public) reserve.[16] In that event, the highest bidder at the auction wins the object at the auction price. If the reserve is not cleared, we enter a simple *negotiation game*. The reserve price acts as a take-it-or-leave-it offer by the seller to the highest bidder. If the highest bidder accepts the offer, she buys the object at the reserve price.

[16] Note this treatment of FD is without loss in generality; bidders incur no participation cost so bidding then withdrawing below the reserve is equivalent to not participating.

3.2.3 Equilibrium

A seller's *reserve price strategy* is an increasing function $R : [\underline{\theta}_S, \overline{\theta}_S] \to \Re_+$, so a typical reserve price r can be written as $r = R(s)$. Given a reserve price strategy R, a *secret reserve bidding strategy* is a symmetric increasing function $\beta_R : [\underline{\theta}, \overline{\theta}] \to \Re_+$ in the secret reserve regimes. Under FD, the reserve price announcement that occurs before bidding represents an announcement of the seller's private information $\hat{s} \in [\underline{\theta}_S, \overline{\theta}_S]$, which bidders will use in addition to their own information in forming their bids. In this regime, a *disclosed reserve bidding strategy* is a symmetric increasing function $\hat{\beta}_R : [\underline{\theta}, \overline{\theta}] \times [\underline{\theta}_S, \overline{\theta}_S] \to \Re_+$. A *bidder negotiation strategy* is an increasing function $P_R : [\underline{\theta}, \overline{\theta}] \times [\underline{\theta}_S, \overline{\theta}_S] \to \Re_+$ that represents the highest price she will accept as a take-it-or-leave-it offer of the reserve price given the reserve price strategy R, the associated announcement $\hat{s} \in [\underline{\theta}_S, \overline{\theta}_S]$ and her signal $x \in [\underline{\theta}, \overline{\theta}]$.

A *signalling equilibrium* is defined as a triple (R, β_R^a, P_R^a) for $a \in \{FD, EN, WS\}$ that are best responses in the following sense. For the seller, the strategy R is a best response given her signal s and the symmetric behaviour of the set of bidders under β_R^a in the bidding game and under P_R^a in the negotiation game. R is therefore a best response in the selection of a minimum type of bidder who will clear the reserve under β_R^a, and accept the reserve price offer under P_R^a. In these three regimes, R is also a best response in terms of the seller's type announcement $\hat{s} = R^{-1}(r)$ and the influence on bidders' behaviour through P_R^a in each regime, and also through β_R^{FD} in FD. β_R^{EN} and β_R^{WS} are best responses for a bidder given her signal, and the bidding behaviour of others under the reserve price strategy R and the allocation rule of the regime. The same is true for FD, except that β_R^{FD} is a best response given one's signal and that of the seller under R, where $\hat{s} = s$, so $\beta_R^{FD} = \hat{\beta}_{R|\hat{s}=s}$. P_R^a is similarly a best response for a bidder given x and $\hat{s} = s$, which is known given the reserve price strategy R once the reserve offer r is announced. Next, we consider equilibrium behaviour in the final stage of the mechanism, the negotiation game.

3.3 Negotiation and Allocation

Recall that the object goes to the highest bidder at the auction price provided this is no lower than the (by now public) reserve. Should the auction price be too low, we enter a negotiation stage where the reserve price acts as a take-it-or-leave-it offer to the highest bidder. It is clear that the optimal action of the highest bidder at this stage is to accept the offer if and only if her valuation conditional on learning the reserve and thus s is at least as high as the offer, so we have $P_R^a(x,s) = v(x,s)$ for $a = \{FD, EN, WS\}$. For a reserve price strategy R and a seller's signal s, we define the signal value $s_R(x)$ by

$$s_R(x) := \sup\{s : v(x,s) \geq R(s)\} \tag{3.1}$$

Thus, $s_R(x)$ represents the highest seller signal such that a valuation meets or exceeds the reserve. Whenever $s_R(x) \in [\underline{\theta}_S, \overline{\theta}_S]$ exists and is strictly increasing, in equilibrium a bidder with signal x will accept any take-it-or-leave-it offer up to $R(s_R(x))$. The key to this is that R is strictly increasing, so that announcement of a particular reserve allows bidders to 'learn' s by inverting R. To address the properties of $s_R(x)$, consider the closely related signal value $m_R(s)$ defined as

$$m_R(s) := \inf\{x : v(x,s) \geq R(s)\} \tag{3.2}$$

For signalling to occur in equilibrium the probability of allocation (selling the good) must be decreasing in the reserve price. Otherwise, the seller can raise the reserve price and costlessly signal better information.[17] This requirement is equivalent to the statement that $m_R(s) \in [\underline{\theta}, \overline{\theta}]$ exists and is strictly increasing in s. $s_R(x)$ and $m_R(s)$ are inverses, so in a signalling equilibrium $s_R(x)$ exists for $x \geq m_R(\underline{\theta}_S)$. We shall suppose further that $m_R(\overline{\theta}_S) = \overline{\theta}$, so $s_R(x)$ is strictly increasing for $x \geq m_R(\underline{\theta}_S)$. As we shall see, this statement is without loss of generality for our ranking of the FD, EN and WS regimes. The reason is that taking the lowest type seller for whom trade occurs with zero probability, say s', we have $m_R(s') = \overline{\theta}$, and the good will never be allocated for $S > s'$ under FD. Our analysis can therefore be viewed as addressing the set that remains after these high types have withdrawn from the market. One of our main findings is that the excluded set of seller types can only be larger under the EN and WS regimes, so consideration of the set that leaves the market under FD only serves to consolidate our profit ranking result; that FD dominates EN and WS. In what follows, we therefore assume for simplicity (unless otherwise stated) that $m_R(s)$ exists and is strictly increasing for all s, and $s_R(x)$ exists and is strictly increasing $\forall x \geq m_R(\underline{\theta}_S)$.

[17] We show more rigourously that this condition will hold when we derive separating equilibria in reserve prices later.

3.4 Bidding under Secret Reserve Regimes

3.4.1 Participation

The bidding games in the secret reserve regimes look very similar; bidders know their own signal and are competing against each other in the presence of an unknown reserve. A bidding function in an English auction under a secret reserve regime is a symmetric function β_R that specifies for each bidder signal X given the reserve price rule R a price $\beta_R(X)$ at which she will withdraw from the auction. Our task is to find particular functions that are best responses for a bidder given the symmetric play of other bidders, the reserve price strategy R and (where appropriate) the best response function of the bidder in the negotiation stage that will follow.

Given an increasing reserve price strategy R, a natural starting point is to ask whether a bidder should participate in the auction or not. Equivalently, we could ask whether her bid clears the reserve with positive probability.[18] It is important to note that in secret reserve regimes the definition of a 'serious' bid is weaker than when the reserve is public information. Since the particular value of the reserve is unknown to a bidder, she views her bid as 'serious' as long as it exceeds the lowest possible reserve price. Now the reserve represents the lowest price at which a bidder can acquire the object, so a necessary condition for participation is that for some s, $v(x, s) \geq R(s)$. If this is not the case, then winning the object always results in an *ex post* negative payoff to the bidder, and she will not want to participate in the auction.[19] By the definition of $s_R(x)$ in (3.1) and our condition that $m_R(s)$ exists, is increasing and $m_R(\overline{\theta}_S) = \overline{\theta}$, for every type of bidder $x \geq m_R(\underline{\theta}_S)$ there exist prices which ensure an *ex post* positive profit, so a necessary condition for participation is achieved. In the derivation of equilibrium bidding strategies in each of the secret reserve regimes that will follow, we verify that this condition is also sufficient. We therefore have the following;

Observation 3.4.1. In secret reserve regimes, a bidder submits a 'serious' bid, i.e. one she believes will clear the reserve with positive probability, as long as $s_R(x)$ is defined for that bidder type, or $x \geq m_R(\underline{\theta}_S)$.

From this point on, we consider the equilibrium behaviour of participating bidders. We shall now present our analysis of bidding behaviour in the secret reserve regimes.

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[18]See Footnote 16.

[19]In this scenario we assume that the bidder would submit a bid below $R(\underline{\theta}_S)$.

3.4.2 Bidding under the EN and WS Regimes

Given an increasing reserve price strategy R and an increasing symmetric bidding function β_R, we can express the expected profit of a bidder with signal x facing an unknown reserve from announcing type z when all others announce truthfully as[20]

$$
\begin{aligned}
E[\Pi(z,x)] = & \int_{\underline{\theta}}^{z} \int_{\underline{\theta}_S}^{\widetilde{s}_R(y)} [v(x,s) - \beta_R(y)] f_S(s) f_Y(y) ds dy \\
& + \int_{\underline{\theta}}^{z} \int_{\widetilde{s}_R(y)}^{\max\{s_R(x),\widetilde{s}_R(y)\}} [v(x,s) - R(s)] f_S(s) f_Y(y) ds dy
\end{aligned} \tag{3.3}
$$

where Y and f_Y represent the highest of $N-1$ draws from F and the density of this variable respectively. Thus $\beta_R(y)$ is the highest bid of one's competitors. The signal value $\widetilde{s}_R(x)$ is defined as follows[21]

$$\widetilde{s}_R(x) := R^{-1}(\min\{\beta_R(x), R(\overline{\theta}_S)\}) \tag{3.4}$$

We shall suppose for now that $R^{-1}(\beta_R(x)) \geq \underline{\theta}_S$ for $x \geq m_R(\underline{\theta}_S)$ so $\widetilde{s}_R(x)$ is always defined for participating bidders. This will be verified in the derivation of the equilibrium bidding function.[22] The first component of Equation (3.3) represents the expected profits from announcing type z and winning the auction at a price above the unknown reserve. This involves taking expected values over the events in which one wins ($Y \leq z$), and the best competitor's bid clears the reserve, so $\beta_R(y) \geq R(s)$ or equivalently $S \leq \widetilde{s}_R(y)$ so the price paid upon winning will be the auction price. When $\beta_R(y) \leq R(\overline{\theta}_S)$ we have $\widetilde{s}_R(x) \leq \overline{\theta}_S$ and the bidder incorporates the 'bad news' about the seller's information that is consistent with the reserve being cleared in order to avoid the winner's curse. Otherwise, the first term involves the unconditional expectation over S. The second term corresponds to the expected profits from receiving a take-it-or-leave-it offer of the reserve price, which occurs when one wins the auction, but at a price (highest competitor's bid) below the reserve. The boundary terms reflect the fact that this event only yields surplus to the bidder when the reserve *may* not be cleared, so $\widetilde{s}_R(y) < \overline{\theta}_S$, and she would accept the reserve price offer, which from (3.1) is equivalent to $S \leq s_R(x)$. If the values of Y and X are such that the winner

[20]Here we appeal to the *Revelation Principle.*

[21]Since this value depends on the equilibrium bidding function, it will differ across the secret reserve regimes. For notational simplicity we do not index this dependence, the relevance to a particular regime will be made clear in each case.

[22]See proof of Proposition 3.4.1.

would not accept the reserve price offer when the reserve is not cleared, the boundary terms set this surplus equal to zero. Maximizing with respect to z and solving the first order condition yields the symmetric equilibrium of the bidding game, recorded in the proposition below.

Proposition 3.4.1. *Given the reserve price strategy R and associated value $s_R(x)$, the lowest participating bidder, type $m_R(\underline{\theta}_S)$ bids $\beta_R^{EN,WS}(m_R(\underline{\theta}_S)) = R(\underline{\theta}_S)$ in the EN and WS regimes. For types above, the symmetric equilibrium bidding function is given by*

$$\beta_R^{EN,WS}(x) := \begin{cases} b = w(x, R^{-1}(b)) + \frac{\int_{R^{-1}(b)}^{s_R(x)} [v(x,s)-R(s)] f_S(s) ds}{F(R^{-1}(b))} & \text{if} \quad R(\overline{\theta}_S) \geq E_S[v(x,S)] \\ E_S[v(x,S)] & \text{if} \quad R(\overline{\theta}_S) \leq E_S[v(x,S)] \end{cases} \tag{3.5}$$

Proof. See appendix, Section 5.2. □

High type bidders who always clear the reserve bid their unconditional expected value since they imply no bad news from clearing the reserve. For the remaining types, the first term of the bidding function is the expected value of the object conditional on clearing the secret reserve with $\beta_R(x) = \beta_R(y)$. The bidder adjusts her valuation of the good since beating the reserve implies an upper bound on the seller's signal S. The second component represents the option value of learning the reserve, and hence one's valuation, from an offer in the negotiation stage. Having observed an offer $R(s)$ and learned her valuation $v(x, s)$, the winner can do no better than to accept the price iff $v(x, s) \geq R(s)$ or, from the perspective of a bidder who does not yet know what this offer will be, $S \leq s_R(x)$. The second term therefore incorporates the expected value of the surplus from the negotiation game. Note that the upper boundary term in the option component of the bid has been replaced with $s_R(x)$, which implies that $s_R(x) > \widetilde{s}_R(x)$, so that the option value of receiving the reserve price offer is positive for $x : E_S[v(x, S)] \leq R(\overline{\theta}_S)$. The intuition is that bidders respond to the fact that they can avoid the winner's curse by symmetrically underbidding; rather than overcommit in the auction, bidders leave 'room' for trades to occur in the negotiation stage when they will know the true value of S. This is a necessary condition for equilibrium bidding in the EN and WS regimes that has very important consequences for the ranking of expected profits relative to FD which will be developed in Section 3.5. We now further explore the intuition of the underbidding phenomenon in terms of the role of private seller information.

In the secret reserve regimes, the equilibrium bid is independent of S, so it will typically differ from the true valuation $v(x, s)$, leading to different bidding behaviour than if the value of S were known. In particular, players will bid too little for high values of S and too much for low values. It is important to note that the bidding behaviour outlined above is not a strategic response to a secret reserve *per se*, but to the beliefs that are induced about the value of the object when one wins under a secret reserve. To demonstrate the influence of private seller information on the auction outcome in terms of a pure information effect, we can fix a particular mechanism, i.e. a regime and reserve price strategy R, and compare bidding behaviour in this regime across two exogenously given scenarios; when S is public information, and when it is unknown to bidders. The example that follows in Section 3.4.3 is dedicated to such an exploration. Section 3.5 then compares the probability of reserve clearance for a fixed regime and reserve price strategy when S is unknown to that when it is public information in more general terms. It turns out that even though bids can be higher in either situation depending on the realisation of S, for a given reserve price strategy R, the reserve is cleared more often when S is public information *in every realisation.* The example that now follows is a demonstration of this property. We then show that this interesting but somewhat artificial result can be simply translated to a ranking of reserve clearance rates across regimes; for fixed R the reserve is cleared more often under FD than in the secret reserve regimes due to pure information effects.

3.4.3 Example: Reserve Clearance

A single indivisible object is offered for sale by a seller to N bidders by way of an English auction under the EN regime. Let these players receive signals S, $X_1, \ldots, X_N$ that are all distributed identically, independently and uniformly on $[0, 1]$. The bidders have symmetric valuations of the form

$$V_i = X_i + S$$

Say that the seller's reserve price strategy is

$$R(S) = \frac{3}{2}S$$

In order to examine the effect of private seller information on bidders' behaviour, we now want to investigate how a representative bidder would behave in this auction if the value of S were common knowledge. In this scenario, a bidder with signal x has a 'complete' valuation, and in an English auction she can do no better than to bid this value:[23]

$$\hat{b} = v(x, s) = x + s$$

The lowest type of bidder that can clear the reserve when the seller's type is s is then given by $x = \frac{s}{2}$. Does this type of bidder clear the reserve when S is unknown? This can be simply tested by asking whether type $x = \frac{s}{2}$ is active when the price reaches $b = R(2x)$. Substituting $R(s_R(x))$ into the bidding function, the option component disappears because the option is worth nothing if one wins at $R(s)$. For type x to be active at $R(s_R(x))$, we must have the condition $b \leq w(x, R^{-1}(b))$. At any particular price b, a bidder with signal x can evaluate the expected value conditional on winning the object at b. This is given by

$$w(x, R^{-1}(b)) = x + E[S|S \leq \frac{2}{3}b]$$

Given our distributional assumptions, type x is only active for unknown S if

$$\begin{aligned} b &\leq x + \frac{2}{3} \cdot \frac{b}{2} \\ b &\leq \frac{3}{2}x \end{aligned}$$

But of course $b = R(s_R(x)) = \frac{3}{2}2x = 3x$, so the above inequality is not satisfied. Since we demonstrated that type x is inactive at $R(s_R(x))$ when S is unknown for arbitrary x, the probability of reserve clearance at auction is always higher when S is public information. Note that this phenomenon does not follow from bids being universally larger when S is known. In fact, bids can be higher in either situation depending on the value of S. For example, taking $s = 0$ bids are higher for unknown S since $\beta_R^{EN}(x) \geq \frac{3}{2}x \geq x = \hat{b}$.

Rather, the reduced probability of reserve clearance is a general property of secret reserve regimes that can be explained as a simple consequence of the winner's curse. For any realization of the reserve price $R(s)$, type x will bid her valuation $v(x, s)$ if she knows the value of s. For unknown S, bidders in the EN and WS regimes always leave 'room' for trades to occur at the reserve price in the negotiation stage. Recalling the bidding function from (3.5) the equilibrium bid contains an expected value component and

[23] Again, the fact that the secret reserve may be inferred by the bidders through their knowledge of s plays no role here. As long as they incur no participation cost, their dominant strategy is unchanged for any set of beliefs about the reserve price.

an 'option' component. If it were possible for one's bid to meet the secret reserve under EN or WS where the true valuation, or equivalently, the bid for known S would not, then for that type of bidder x we have $R^{-1}(b) \geq s_R(x)$. By swapping the boundaries of the integral in the option term, the bidding function can then be expressed as

$$\beta_R^{EN}(x), \beta_R^{WS}(x) := b = w(x, R^{-1}(b)) + \frac{\int_{s_R(x)}^{R^{-1}(b)} [R(s) - v(x,s)] f(s) ds}{F(R^{-1}(b))} \quad (3.6)$$

Note that the option component is again positive by the definition of $s_R(x)$. However, this means that at the point where the bid exactly meets the secret reserve, the bidder would have paid for an option that has zero value; she would never accept the reserve price offer (once announced), because $S > s_R(x)$. In equilibrium bidders do not systematically bid too high, they don't pay a premium for a right they would never exercise. The option component of the bid is only worth paying for if the secret reserve would not be cleared, but, once announced, the reserve price offer would be accepted. A necessary condition for this is that for unknown S the bid must be below the highest price one would accept from a reserve price offer; one's valuation. Since it is a dominant strategy to bid one's valuation when S is known, the reserve must be cleared more often in the EN and WS auctions when S is known. The bidder underestimates the value of the object at the margin where $S = s$ but all $S \leq s$ are consistent with her bid clearing the reserve. The reserve is therefore cleared more often when S is public information in all secret reserve regimes. The next section is dedicated to a formal statement of this proposition, and links this result to reserve clearance in the FD regime.

3.5 Reserve Clearance and Seller Profits for Fixed R

3.5.1 The Probability of Reserve Clearance for Fixed R

Recall that reserve clearance in an English auction occurs whenever the second highest bid meets the reserve. Then in a given regime (and fixing the reserve price strategy R) the effect of private seller information on the probability of reserve clearance can be described by ranking the lowest bidder signal value that facilitates reserve clearance when S is public information, and when it is unknown. When S is known to bidders, they have complete valuations. In an English auction, bidders have a weakly dominant strategy to bid this valuation *under any reserve regime.* The relevant threshold signal for reserve clearance in this case is $m_R(s)$, the lowest bidder type whose valuation exceeds the reserve given the seller's signal s, as introduced in Section 3.3. When the seller's information is private, bidding behaviour and therefore the set of reserve clearing bidders depends on the equilibrium bidding function in the regime. We therefore define given regime $a = \{FD, EN, WS\}$ and fixed R the value $\widetilde{m}_R^a(s)$, where

$$\widetilde{m}_R^a(s) := \begin{cases} \inf\{x : \beta_R^a(x) \geq R(s)\} & \text{if} \quad S \leq R^{-1}(\beta_R^a(\overline{\theta})) \\ \overline{\theta} & \text{if} \quad S \geq R^{-1}(\beta_R^a(\overline{\theta})) \end{cases}$$

This represents the lowest bidder signal value that results in reserve clearance given the bidding strategy β_R^a when $S = s$, if reserve clearance is possible at all. Otherwise the seller sets probability zero to such an event ($\widetilde{m}_R^a(s) = \overline{\theta}$ achieves this). Given this definition, it is clear that the relative probabilities of reserve clearance in a regime for known and unknown S will be described by the ranking of $\widetilde{m}_R^a(s)$ and $m_R(s)$. Moreover, it is simple to illustrate that this intuition also applies across regimes. Under FD, S is known to bidders at the time they place their bids. In an English auction, bidders will therefore bid their valuations, leading to outcomes *in the bidding game* that are equivalent to those when S is specified as common knowledge. We therefore have $\widetilde{m}_R^{FD}(s) = m_R(s)$. We now prove the following result.

Proposition 3.5.1. *For any given strictly increasing R and for every value of s;*
(i) *The reserve is cleared at least as often in the secret reserve regimes when S is public information than when it is unknown, or $m_R(s) \leq \widetilde{m}_R^a(s)$.*
(ii) *The reserve is cleared at least as often in FD as in the secret reserve regimes.*

Proof. For (i), we have proven in Lemma 5.2.1 (see the appendix Section 5.2) that $s_R(x) \geq \widetilde{s}_R(x)$ for all participating bidders, with strict inequality for $x > m_R(\underline{\theta}_S)$. Just as s_R and m_R are inverses, so too are $\widetilde{s}_R$ and $\widetilde{m}_R$. We therefore have $\widetilde{m}_R(x) \geq m_R(x)$ for participating bidders, with strict inequality for $x > m_R(\underline{\theta}_S)$. For $x < m_R(\underline{\theta}_S)$, the reserve is never cleared in either regime. (ii) follows from (i), given that $\widetilde{m}_R^{FD}(s) = m_R(s)$. □

So, for a fixed reserve price strategy, differences between the regimes can be summarized in terms of a pure information effect; FD facilitates an announcement of S, which means that the reserve is cleared more often under

FD than in the secret reserve regimes. From this point on (unless otherwise specified), we shall return to the structure which we initially specified, that where S is private information for the seller and can only be made public through her actions. The next subsection investigates the repercussions of (ii) for a profit ranking, again for a fixed reserve price strategy R. Despite the difference in reserve clearance rates as described in (ii), after accounting for sales in the negotiation stage the signalling regimes generate the same expected profits to the seller.

This result may be viewed as a prediction of the *Linkage Principle*; the disclosure of information has no effect on expected profits in symmetric equilibria under statistical independence. This fact, combined with (ii) will prove extremely useful to the ranking of expected profits in equilibrium in these regimes. The results allow us to perform a comparative static on the reserve prices that will be selected in a signalling equilibrium as the share of profits from the negotiation game increases. This share is greater under the secret reserve regimes, which further increases equilibrium reserve prices relative to FD, so that immediate disclosure is preferable to later announcement.

3.5.2 Ranking Expected Profits for Fixed R

In this section we compute expected profits to the seller in the EN and WS regimes, and compare this to profits under FD for a fixed reserve price strategy R. Proposition 3.5.1 shows that $m_R(s) \leq \widetilde{m}^a_R(s)$ for $a = \{EN, WS\}$. However, the lower probability of reserve clearance also means a greater probability of receiving a reserve price offer in the EN and WS regimes. We now prove that (fixing a reserve price strategy) this trade-off is perfectly balanced by the expectations in the secret reserve bidding function, which yields an equivalence between profits under FD, EN and WS.

Proposition 3.5.2. *For any strictly increasing R, the EN and WS regimes yield the same expected profits to the seller as under FD.*

Proof. The *ex ante* expected profit in EN and WS from using the increasing secret reserve price strategy R and associated m_R given the bidding function

β_R^a and associated $\widetilde{m}_R^a$ for $a = \{EN, WS\}$ is given by[24]

$$\begin{aligned} E[\Pi(\text{EN,WS})] &= \int_{\underline{\theta}_S}^{\overline{\theta}_S} \int_{\widetilde{m}}^{\overline{\theta}} [\beta(x) - v_0(s)] f_2(x) f_S(s) dx ds \\ &+ \int_{\underline{\theta}_S}^{\overline{\theta}_S} [F_2(\widetilde{m}) - F_1(m)][R(s) - v_0(s)] f_S(s) ds \end{aligned}$$

F_1, F_2, f_1 and f_2 represent the distribution functions of the highest and second highest of the X_i's and their densities respectively. These terms represent the expected profits to the seller from sale at the auction price; when the second highest bidder's signal is above $\widetilde{m}$, and from sale at the reserve price; when the auction price is too low but the highest bidder's signal is above m so she will accept the reserve price offer.[25] Changing the order of integration in the first term and substituting in the equilibrium bidding function from (3.5) obtains[26]

$$\begin{aligned} E[\Pi(\text{EN,WS})] &= \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}_S}^{\widetilde{s}(x)} [w(x, \widetilde{s}(x)) - v_0(s)] f_S(s) f_2(x) ds dx \\ &+ \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}_S}^{\widetilde{s}(x)} \frac{\int_{\widetilde{s}(x)}^{s(x)} [v(x,s) - R(s)] f_S(s) ds}{F_S(\widetilde{s}(x))} f(s) f_2(x) ds dx \\ &+ \int_{\underline{\theta}_S}^{\overline{\theta}_S} [F_2(\widetilde{m}) - F_1(m)][R(s) - v_0(s)] f_S(s) ds \end{aligned}$$

Recalling that $w(x, \widetilde{s}(x))$ is the expected value conditional on $S \leq \widetilde{s}(x)$, integrating the first two terms over S then changing the order of integration

[24] In this section we suppress the dependence of $m_R(s)$ and $\widetilde{m}_R^{EN}(s)$ on s, the subscript R and superscript EN where appropriate, and similarly for WS. Note that under the definition of $\widetilde{m}$, if for some s; $\widetilde{m}(s) = \overline{\theta}$ the first term in the profit function disappears.

[25] Here we are using the fact that $F_1(.) < F_2(.)$, and $\widetilde{m} \geq m$.

[26] From the arguments of Section 3.3 we consider reserve price strategies and seller types such that $s(x) \leq \overline{\theta}_S$ so $\widetilde{s}(x) < \overline{\theta}_S$ and we use the bidding function for $x : R(\overline{\theta}_S) \geq E_S[v(x,S)]$.

the profit function becomes

$$\begin{aligned}
E[\Pi(\text{EN,WS})] &= \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}_S}^{s(x)} [v(x,s) - v_0(s)] f_S(s) f_2(x) ds \\
&- \int_{\underline{\theta}}^{\overline{\theta}} \int_{\widetilde{s}(x)}^{s(x)} [R(s) - v_0(s)] f_S(s) f_2(x) ds dx \\
&+ \int_{\underline{\theta}_S}^{\overline{\theta}_S} [F_2(\widetilde{m}) - F_1(m)][R(s) - v_0(s)] f_S(s) ds \\
&= \int_{\underline{\theta}_S}^{\overline{\theta}_S} \int_{m}^{\overline{\theta}} [v(x,s) - v_0(s)] f_2(x) f_S(s) dx ds \\
&+ \int_{\underline{\theta}_S}^{\overline{\theta}_S} [F_2(\widetilde{m}) - F_2(\widetilde{m}) + F_2(m) - F_1(m)][R(s) - v_0(s)] f_S(s) ds \\
&= E[\Pi(\text{FD})] \qquad (3.7)
\end{aligned}$$

The last line follows from noting that under FD players bid their values. The seller therefore receives the second highest valuation when it exceeds the reserve ($X_2 \geq m$), and sells the object at the reserve price when $X_1 \geq m > X_2$. □

In the EN and WS regimes the reserve is cleared less often than under FD, however secrecy does not hurt the seller because she can still sell in the negotiation stage when the reserve is not cleared at auction. Bidders then incorporate the value of receiving this offer in their bids when the reserve is secret, and their estimation of the value of receiving this offer is correct on average. We could view this result as a corollary of Milgrom and Weber's (1982) *Linkage Principle*, in its weak form for independent signals. They use this principle to demonstrate that the direct announcement of verifiable information has no impact on expected revenues. Proposition 3.5.1(i) proves that for any s, allocation in the bidding game is only affected by secrecy since it reduces the probability of reserve clearance. The post auction offer of the reserve price resolves any such differences, preserving the allocation rule.[27] In our result, the fixed reserve price strategy therefore plays the dual roles of fixing the allocation rule and ensuring that the information is verifiable so that the *Linkage Principle* applies. Alternatively, in equilibrium the seller's information cannot be made public 'for free' unless the value

[27] Proposition 3.5.1 also means that we don't have to worry about a bidder paying too much under secrecy, clearing the reserve when she would not purchase under complete information.

of S can be costlessly verified. While she may prefer for her type to be known in terms of a pure information effect, the intuition of our example and Proposition 3.5.2 can only be applied to exogenously given mechanisms. Thus we have demonstrated that a secret reserve has no *direct* information effects on expected profits. The seller must also however take into account the fact that if S is to be made public, it will be via some mechanism that is likely to distort equilibrium in other ways. The remainder of the analysis in this essay compares possible methods (reserve price regimes) of disseminating this information credibly, and the associated effects on reserve price selection and hence profits.

Once we consider equilibrium reserve price selection, the equivalence fails; Full Disclosure is preferable to EN and WS. Moreover, our analysis will demonstrate that in equilibrium Full Disclosure is preferable despite the fact that signals are independent, *as a direct consequence* of the Linkage Principle/Proposition 3.5.2 and Proposition 3.5.1. While expected profits are identical for fixed R, the share that arises from sale at the reserve price is higher under secrecy for any s. Increasing the share increases the importance of the reserve price announcement, and thus the incentive for the seller to falsely claim better information. Later announcement of a reserve therefore results in a higher equilibrium reserve price which reduces expected profits. To explicitly compare these equilibria, we now review the treatment of equilibrium under the benchmark case of Full Disclosure as per Cai, Riley, and Ye (2007), and develop conditions for a profit ranking. In Section 3.7, we analyze reserve price setting behaviour in the EN and WS regimes, then apply our condition to prove that FD is preferable.

3.6 Full Disclosure (FD)

Recalling the definition of $m_R(s)$ from (3.2), the task is to specify a reserve price strategy that results in a separating equilibrium under FD. Cai et. al. (2007) express this problem as a maximization in terms of the seller's announced type $\hat{s}$, and the threshold type m subject to incentive compatibility, so that $m(\hat{s}) = m(s)$. It is known from Riley (1979) that if the lowest type of informed agent selects an action that would be optimal if her type were known, then this separating equilibrium is unique. In our analysis of the signalling regimes FD and EN we restrict our attention to such equilibria. The interim expected profit function for this problem under

FD can be written as follows;

$$U(s,\hat{s},m)=\int_m^{\bar{\theta}}[v(x,\hat{s})-v_0(s)]f_2(x)dx+[F_2(m)-F_1(m)][v(m,\hat{s})-v_0(s)]$$

Here F_1 and F_2 again represent the distributions of the first and second order statistics of bidder types respectively, and $v(m,\hat{s})=R(\hat{s})$ by definition. The first component describes the interim expected profit from sale at the auction price. The other component represents profits from the post auction take-it-or-leave-it offer of the reserve price to the winning bidder. Taking the first order conditions for m and $\hat{s}$ obtains[28]

$$\begin{aligned}\frac{dU}{dm}&=f_1(m)\Big[v_0(s)-v(m,\hat{s})+\frac{dv}{dx}|_{x=m}\Big(\frac{F_2(m)-F_1(m)}{f_1(m)}\Big)\Big]\\ \frac{dU}{d\hat{s}}&=\frac{dv}{d\hat{s}}[1-F_1(m)]\end{aligned}\tag{3.8}$$

Define m^* to be the be the solution to $\frac{dU}{dm}=0$ given that $\hat{s}=s$. Then m^* identifies the optimal reserve price under complete information.[29]

Theorem 3.6.1. **(Cai et. al.)** For separable v, there is a unique separating equilibrium under Full Disclosure that is described by the differential equation

$$\frac{dm_{FD}}{ds}=\frac{\frac{dv}{ds}[1-F_1(m_{FD})]}{f_1(m_{FD})\Big[v(m_{FD},s)-v_0(s)-\frac{dv}{dx}|_{x=m_{FD}}\Big(\frac{F_2(m_{FD})-F_1(m_{FD})}{f_1(m_{FD})}\Big)\Big]}$$

with initial condition $m_{FD}(\underline{\theta}_S)=m^*_{FD}(\underline{\theta}_S)$, if

$$\frac{d}{dx}\Big[v(x,s)-\frac{dv(x,s)}{dx}\Big(\frac{F_2(x)-F_1(x)}{f_1(x)}\Big)\Big]>0\tag{3.9}$$

for all s.

Proof. See Cai et. al. Theorem 1 (2007).[30] □

[28] Here we make use of our assumption that the valuation function $v(x,s)$ is separable in the private and common value components. Cai et. al. do not require this assumption.

[29] This solution is an analogue of the standard optimal auction reserve, the price that sets virtual valuations equal to the seller's valuation, see Myerson (1981). We use the phrase 'complete information' to describe the standard case in which the agents (bidders) have private information but the seller's preferences are commonly known. Similar terminology is also used in Maskin and Tirole (1990, 1992).

[30] Cai, Riley, and Ye (2007) show that when signals are independent (3.9) is satisfied if $\frac{d^2v(x,s)}{dx^2}<0$ for all s and the distribution of x is regular. Equation (3.9) is therefore also satisfied for our model.

In this equilibrium, the lowest type of seller chooses the reserve that is optimal under complete information. It is straightforward to demonstrate that every other type of seller selects a higher reserve price, and therefore a higher threshold signal m than she would if her type were known to bidders. A separating equilibrium solves[31]

$$\frac{d}{d\hat{s}}[U(s,\hat{s},m(\hat{s}))] = \Big(\frac{dU}{d\hat{s}}\Big)|_{\hat{s}=s} + \Big(\frac{dU}{dm}\Big)|_{\hat{s}=s}\Big(\frac{dm}{ds}\Big)|_{\hat{s}=s} = 0$$

Now $\frac{dU}{d\hat{s}}|_{\hat{s}=s} > 0$ for all s, and $\frac{dm}{ds} > 0$ at equilibrium. Therefore at the equilibrium solution $\frac{dU}{dm}|_{\hat{s}=s} < 0$ so under FD seller types above $\underline{\theta}_S$ select a higher m than under complete information. Equation (3.9) implies that $\frac{dU}{dm}|_{\hat{s}=s}$ is decreasing in m from (3.8) so it also follows that increasing m further from m_{FD} for each S reduces the interim expected profit function $U(\hat{s},s,m(\hat{s}))|_{\hat{s}=s}$. We can immediately prove the following proposition.

Proposition 3.6.1. *In the signalling equilibria of the English auction, if the secret reserve price is higher than would be announced under Full Disclosure for every S, then Full Disclosure is preferable to later announcement.*[32]

Proof. Recall from Proposition 3.5.2 that expected profits in EN and WS are equivalent to that under FD for the same reserve price strategy. Profits in these mechanisms therefore differ only in the sense that the two auctions may select a different reserve price strategy, and thus a different m for each s. We have demonstrated that the interim profit function $U(\hat{s},s,m(\hat{s}))_{|\hat{s}=s}$ is decreasing in m for $m \geq m_{FD}$. Any secret reserve regime that selects a higher m than under FD for each s therefore reduces the integrand of $E[\Pi(\text{EN,WS})]$ in (3.7) at every s, and results in lower expected profits. □

To demonstrate that equilibrium under Full Disclosure (FD) is preferable to any involving announcement after the auction, it now suffices to verify that the separating equilibria in the Envelope (EN) and Wait and See (WS) regimes involve higher reserve prices than under FD. Section 3.7 derives equilibrium reserve prices in these regimes and verifies this condition.

[31] Again, see Cai et. al. Equation (8).

[32] Recall that in equilibrium $m_R(s)$ is strictly increasing, and we are presuming that this always exists. For other cases the proof follows directly by restricting the comparison to seller types where $m_R(s)$ is defined, and noting from Proposition 3.5.1 that FD can do no worse than EN and WS for the remaining set of seller types.

3.7 Equilibrium in the EN and WS Regimes

3.7.1 Envelope Regime (EN)

In this section we consider the reserve price setting behaviour of a seller who selects and commits to a reserve price before bidding commences, but seals it in an envelope (perhaps to be handed to a committed third party, e.g. an auctioneer) to be announced after the auction has ended. The interim expected profit to the seller can be written as follows;

$$U_{EN}(s,\hat{s},m) = \int_{\widetilde{m}(m)}^{\overline{\theta}} [\beta^{EN}(x) - v_0(s)] f_2(x) dx + [F_2(\widetilde{m}(m)) - F_1(m)][v(m,\hat{s}) - v_0(s)]$$

Here $\beta^{EN}(x)$ is the symmetric equilibrium that describes bidding behaviour conditional on the equilibrium reserve price strategy that will be selected. The probability that a bid clears the reserve is summarized by $\widetilde{m}$. We shall write this (with a slight abuse of notation) as $\widetilde{m}(m)$, since $\widetilde{m}$ is a function of the reserve price that is summarized by m for each s. Again following the methodology of Riley (1979), we have the following result.

Theorem 3.7.1. There exists a unique separating equilibrium under the EN regime given by

$$\frac{dm_{EN}}{ds} = \frac{\frac{dv}{ds}[F_2(\widetilde{m}(m_{EN})) - F_1(m_{EN})]}{f_1(m_{EN})\Big[v(m_{EN},s) - v_0(s) - \frac{dv}{dx}|_{x=m_{EN}}\Big(\frac{F_2(\widetilde{m}(m_{EN})) - F_1(m_{EN})}{f_1(m_{EN})}\Big)\Big]}$$

with initial condition $m_{EN}(\underline{\theta}_S) = m^*_{EN}(\underline{\theta}_S)$ equal to the minimum bidder type that would be selected under complete information provided

$$\frac{d}{dx}\Big[v(x,s) - \frac{dv(x,s)}{dx}\Big(\frac{F_2(\widetilde{m}(x)) - F_1(x)}{f_1(x)}\Big)\Big] > 0 \qquad (3.10)$$

Proof. See appendix, Section 5.4. □

We now show that $m_{FD}(s) < m_{EN}(s)$, which, recalling Proposition 3.6.1 is sufficient for Full Disclosure to dominate the Envelope regime.

Proposition 3.7.1. *The separating equilibrium under Full Disclosure generates greater expected seller profits than that under the EN regime.*

Proof. Given (3.10), $m^*_{EN}(\underline{\theta}_S) = m^*_{FD}(\underline{\theta}_S)$ since both set $\frac{dU}{dm} = 0$ at $\hat{s} = s = \underline{\theta}_S$ in their respective objective functions, and in the envelope auction, we know that[33]

$$\frac{dU}{dm}|_{\hat{s}=s} = f_1(m)\Big[v_0(s) - v(m,s) + \frac{dv}{dx}|_{x=m}\Big(\frac{F_2(\widetilde{m}(m)) - F_1(m)}{f_1(m)}\Big)\Big]$$

This function only differs from that under FD where $F_2(m)$ is replaced by $F_2(\widetilde{m}(m))$. We know that $\widetilde{m} = m$ for any R at $s = \underline{\theta}_S$, so $m^*_{EN}(\underline{\theta}_S) = m^*_{FD}(\underline{\theta}_S)$. For all higher type sellers, $\frac{dm_{EN}}{ds} > \frac{dm_{FD}}{ds}$ because we have increased the numerator and decreased the denominator by replacing $F_2(m)$ with $F_2(\widetilde{m}(m))$, and $\widetilde{m} < m$ for $S > \underline{\theta}_S$. So $m_{EN} \geq m_{FD}$ with strict inequality for $S > \underline{\theta}_S$, and we can apply the logic of Proposition 3.6.1, Full Disclosure generates greater expected profits than the Envelope auction. □

We briefly withhold a discussion in order to demonstrate a very similar result for the Wait and See (WS) auction.

3.7.2 Wait and See Regime (WS)

In the Wait and See auction the seller selects and announces a reserve price at the auction's end. The seller therefore takes the observed bids of all but the winner as given, and selects a reserve price with the sole intention of negotiating with the high bidder.[34] The interim expected profit is then[35]

$$U(s,\hat{s},m) = \frac{1 - F_1(m)}{1 - F_1(\beta^{-1}(p))}[v(m,\hat{s}) - v_0(s)]$$

given that the auction ended at price p. Equilibrium is characterized below.

Theorem 3.7.2. There exists a unique separating equilibrium under the WS regime given by

$$\frac{dm_{WS}}{ds} = \frac{\frac{dv}{ds}[1 - F_1(m_{WS})]}{f_1(m_{WS})\Big[v(m_{WS},s) - v_0(s) - \frac{dv}{dx}|_{m_{WS}}\Big(\frac{1-F_1(m_{WS})}{f_1(m_{WS})}\Big)\Big]}$$

with initial condition $m_{WS}(\underline{\theta}_S) = m^*_{WS}(\underline{\theta}_S)$ sets $\frac{dU}{dm} = 0$ at $\hat{s} = s = \underline{\theta}_S$, provided

$$\frac{d}{dx}\Big[v(x,s) - \frac{dv(x,s)}{dx}\Big(\frac{1 - F_1(x)}{f_1(x)}\Big)\Big] > 0 \qquad (3.11)$$

[33] We again refer the reader to the appendix.

[34] Of course if the selected reserve price is below the second highest bid she will sell at the auction price.

[35] The conditional probability in the denominator terms here plays no role as the seller gains no useful information about the high type from observing all others when signals are independent, see the proof of Theorem 3.7.2 in the appendix.

Proof. See appendix, Section 5.5. □

Following our previous method, we now prove that FD also dominates the WS regime.

Proposition 3.7.2. *The separating equilibrium under Full Disclosure generates greater expected seller profits than that under the Wait and See (WS) regime.*

Proof. Given (3.11) holds, in the WS auction we know that[36]

$$\frac{dU}{dm}|_{\hat{s}=s} = \frac{f_1(m)}{1 - F_1(\beta^{-1}(p))}\Big[v_0(s) - v(m, \hat{s}) + \frac{dv}{dx}|_{x=m}\Big(\frac{1 - F_1(m)}{f_1(m)}\Big)\Big]$$

The only difference between this condition and that under Full Disclosure is that $F_2(m)$ has been replaced by 1 and the whole expression has been divided by $[1 - F_1(\beta^{-1}(p))] \leq 1$. It follows that $m^*_{WS}(\underline{\theta}) > m^*_{FD}(\underline{\theta})$. Also, replacing $F_2(m)$ with 1 in the $\frac{dm}{ds}$ equation increases the numerator and decreases the denominator. Thus $\frac{dm_{WS}}{ds} > \frac{dm_{FD}}{ds}$, so $m_{WS} > m_{FD}$ for all S, and by Proposition 3.6.1 Full Disclosure is preferable to the Wait and See regime. □

3.8 Secrecy As Delayed Announcement

In the EN and WS regimes bidders treat the reserve price as secret during the auction. For any given reserve price strategy, $\widetilde{m} > m$ for all s, so players are less likely to clear the reserve than if the same reserve were announced. However, Proposition 3.5.2 demonstrates that for the same reserve price strategy, this does not affect expected profits, because sale will occur more often after the auction to compensate. This means that we can fix a probability of allocation $[1 - F_1(m)]$ and analyze the effects of an increase in the share of profits from the negotiation stage. In the EN and WS regimes, this shift toward the bargaining component of the mechanism increases the stakes of selecting the reserve for the seller, and thus increases her incentive to misrepresent her information. It therefore requires a higher reserve price to credibly signal the same information. We are moving to an even more costly signalling mechanism for the seller, since the reserve is higher than she would like ($\frac{dU}{dm} < 0$) conditioning on $\hat{s} = s$. This decreases expected profits relative to Full Disclosure.

[36]Again refer to the appendix.

The prevalence of secret reserve auctions in practice cannot be explained as a profit-increasing manipulation of signalling equilibria by the seller. Contrary to intuition, the forces of competition cannot be used to diminish the importance, and thus inefficiency of the signalling mechanism. Rather, information as reflected through prices is more adequately transmitted earlier, to be incorporated into the competitive aspects of the mechanism when the stakes are lower. In our final essay, we test whether the failure of secrecy in our model can be overturned, and thus a satisfactory explanation for the pervasiveness of secret reserves uncovered, by restricting the rights and information of bidders in the negotiation stage.

Chapter 4

Auctions with an Informed Seller: Screening

4.1 Signalling vs Screening

4.1.1 The Value of Winning: Information and Rights

In the FD, EN, and WS regimes, and in the theory of English auctions more generally, the reserve acts as a take-it-or-leave-it offer to the high bidder in the event of non-clearance at auction. The fact that winning the auction assigns one exclusive rights to post-auction negotiation and any associated extra information is neither a crucial nor commonly noted feature of this set up. This phenomenon is common however in real world auctions, and is protected by law in real estate auctions in NSW, *'the highest bidder is the purchaser, subject to any reserve price'*.[1] It is therefore of interest to note that, as shown in our second essay, this right is an important feature of equilibrium under the EN and WS regimes. Recall that the relative failure of these regimes stems from postponement of the signal announcement; in equilibrium bidders symmetrically underbid in order to avoid the winner's curse and purchase the object when they will know their true valuation. In some sense, the ability of the winner to learn new information and act on it in the negotiation stage creates these undesirable outcomes. This motivates our final essay, in which we consider secret reserve regimes with variations on the rules of the negotiation game in order to avoid the underbidding phenomenon. As we shall see, underbidding is not caused by the exclusivity

[1] Again, see http://www.fairtrading.nsw.gov.au/realestaterenting/buyingselling/buyingatauction.html.

of rights to negotiation and information that are gained by the winner, but by the information itself.

The secret reserve regimes we analyze in this essay are characterized by the fact that given non-clearance, winning the auction involves either no rights to further information or negotiation, or no such *exclusive* rights. These regimes are described as follows.

(RR) *Right of Refusal*: The seller never announces a reserve, she simply chooses whether or not to sell the object at the auction price.

(PP) *Posted Price*: The reserve is selected before the auction, but is initially kept secret, acting as a posted price in the event of non-clearance. In this situation the object is won by lottery among the bidders who are willing to pay the posted price.

After the auction in the RR regime, the seller announces whether or not the good has been sold, but this is not the same as announcing the reserve price. Consequently, the role the reserve price plays in EN and WS differs from that in RR. The reserve is always unknown to bidders in RR: the auction price acts as a take-it-or-leave-it offer to the seller. Winning the auction at the price p therefore imparts no information that has not already been taken into account by the bidder. If the seller refuses to sell, then any implications that may be drawn regarding her information from this action are irrelevant. If the good is sold, the winning bidder has learnt nothing that she had not already inferred, and deemed acceptable, from being active at p. Under the Posted Price regime, in the event of non-clearance the winner does learn new information upon observing the price, but cannot rely upon being able to purchase the object whenever she likes after the auction. The fact that the negotiation game is either trivial, or seemingly unrelated to the auction that precedes it in these regimes would suggest that similar incentives are at play in their respective bidding games. The next section demonstrates that this is not the case. Removing exclusivity cannot resolve the reserve clearance problem. Rather, the existence of new information itself in the negotiation stage is the source of the inefficiency. This identifies signalling as the root cause of inefficiency, and leaves the screening regime RR as the most likely candidate to overturn the result that disclosure dominates secrecy.

4.1.2 Bidding Under the PP and RR Regimes

In what follows, we retain the model, assumptions and notation from Chapter 3, and simply expand the choice of reserve regimes. The intuition in examining the PP regime is that underbidding may be avoided while retaining the monopoly pricing aspect of the reserve price offer. This logic seems promising, since having reached the negotiation stage, a bidder's profits from the posted price are independent of one's actions in the auction. It of course ignores the fact that bidders anticipate the effect of their bidding behavior on the probability of reaching the lottery. To see how this will influence equilibrium bids, we again write the expected payoff from announcing type z given true type x under the candidate equilibrium β_R;

$$\begin{aligned} E[\pi^{PP}(z,x)] &= \int_{\underline{\theta}}^{z}\int_{\underline{\theta}_S}^{\widetilde{s}_R(y)}[v(x,s)-\beta_R(y)]f_S(s)f_Y(y)dsdy \\ &+ E[\pi^{PP}(z,x)|\text{win lottery}] \end{aligned}$$

As before, the first term captures profits from winning at a price above the secret reserve. The problem is that the profits from winning the lottery are in fact affected by one's behaviour in the auction, since the probability that the lottery occurs is influenced by one's bid. In any symmetric equilibrium a bidder assigns positive probability to being the second highest bidder (type). A bidder who comes second but clears the reserve receives zero payoff. However, a slight reduction in her bid, fixing the behaviour of others, would increase the probability of the reserve not being cleared, and give the bidder a chance to win the object through the posted price lottery. Since the PP regime is a secret reserve auction with signalling in the negotiation stage, we can immediately transplant the intuition from Proposition 3.6.1 to achieve the following.

Observation 4.1.1. The reserve price selected under PP will be higher for (almost every) s than in the FD regime. By again applying Proposition 3.6.1, FD dominates the PP regime. Therefore it is better to make a take-it-or-leave-it offer to the winner in the event of non-clearance than to use a posted price.[2]

This result demonstrates that the assignment of winner's rights in the secret reserve regimes that involve reserve price offers, for example in real estate markets, achieves little and may even be detrimental to profits.[3] We

[2]To verify that the seller's profit function is indeed identical to that in the EN and WS regimes for a given reserve price strategy, note that expected profits to the seller from the lottery can be written as $[F_2(\widetilde{m})-F_1(m)][v(m,s)-v_0(s)]$. The probability of at least one bidder being willing to pay the posted price is $[1-F_1(m)]$.

[3]We have not considered here a ranking of PP against the EN and WS regimes.

now return to the RR regime and compare equilibrium expected profits to those under Full Disclosure.

The only further characterization of an equilibrium that we require here is that in the RR regime, a *seller negotiation strategy* is represented by her reserve price strategy, this is the lowest auction price she will accept. An *equilibrium* in RR is defined as a double (R, β_R^{RR}). R is a best response for the seller given her signal. β_R^{RR} is a best response given a bidder's signal, the symmetric behaviour of other bidders, and the reserve price strategy R, which dictates the set of auction prices the seller will accept. It is immediate that the optimal action of the seller is to accept if and only if the auction price is at least as high as her valuation $v_0(s)$. Thus, in equilibrium under RR, $R(s) = v_0(s)$ will become the *de facto* reserve. Since we initially wish to compare bidding behaviour across regimes for a fixed strategy R, we momentarily withhold from imposing this equilibrium condition, and simply say that under RR the reserve price $R(s)$ represents the lowest price the seller will accept. For a general reserve price strategy R, we assume that withdrawal under RR is irrevocable, since then we do not have to worry about whether a solution of the equilibrium bidding function is unique.[4]

In the RR regime a bidder with signal x who announces type z facing the reserve price strategy R when all other bidders play β_R has an expected payoff given by

$$E[\pi^{RR}(z, x)] = \int_{\underline{\theta}}^{z} \int_{\underline{\theta}_S}^{\widetilde{s}_R(y)} [v(x, s) - \beta_R(y)] f_S(s) f_Y(y) ds dy \tag{4.1}$$

A bidder with signal x is allocated the object when she wins the auction $(Y \leq z)$ at a price above the secret reserve $R(S) \leq \beta(y)$. Again, the boundary term $\widetilde{s}_R(y)$ is present because if one wins at a price above the highest possible reserve, no 'bad news' is inferred about the value of S.[5] In equilibrium setting $z = x$ is a best response, and we see from (4.1) that z only appears in determining the boundary of the integral where $Y = z$. It is well known that this leads to truthful 'break-even' bidding; a bidder would receive zero expected surplus in the event of winning in a tie with another bidder. We now present this formally.

[4] Irrevocability is only a constraint when bidders learn new information during the auction that makes it worthwhile to re-enter. In the appendix (proof of Proposition 4.1.1) we provide a condition on the seller's valuation such that the assumption of irrevocability is not required in equilibrium, i.e. when $R(s) = v_0(s)$.

[5] Again, this integral is always positive since we are considering 'serious' bidders, so $\widetilde{s}_R(y) = R^{-1}(\min\{\beta_R(y), R(\overline{\theta}_S)\}) > \underline{\theta}_S$.

Proposition 4.1.1. *Given the reserve price strategy R in the Right of Refusal (RR) regime, the symmetric equilibrium bidding function is given by*

$$\beta_R^{RR}(x) := \begin{cases} b = w(x, R^{-1}(b)) & \text{if} \quad R(\overline{\theta}_S) \geq E_S[v(x,S)] \\ E_S[v(x,S)] & \text{if} \quad R(\overline{\theta}_S) \leq E_S[v(x,S)] \end{cases} \tag{4.2}$$

Proof. See the appendix, Section 5.6. □

Bids are equal to the expected value conditional on clearing the secret reserve. This is similar to the derivation of equilibrium in the second price sealed bid auction with interdependent values in Milgrom and Weber (1982a). In our case, the signals are independently distributed and the interdependence of valuations is limited: a bidder's valuation depends on the seller's signal but not on the signals of other bidders. This introduces asymmetry between the bidder and seller strategies and valuation functions, so that the winner's curse information cannot be summarized solely by monotonicity in types. As in Chapter 3 we allow the possibility that some high types of bidder ($x : R(\overline{\theta}_S) \leq E_S[v(x,S)]$) always clear the reserve. These types bid their unconditional expected value because no bad news is implied from winning. In the Right of Refusal regime the negotiation stage is trivial in the sense that no new information is revealed; the allocation rule is determined completely by the set of bids. The only way to win the object is through sale at the auction price, which occurs whenever the reserve is cleared during the bidding game. If we ignore the issue of equilibrium reserve price selection, we can apply a Corollary of Proposition 3.5.1;

Proposition 4.1.2. *For a fixed reserve price strategy R, allocation occurs more often in the FD regime than under RR, and FD dominates RR.*

Proof. The result follows from our proof of Proposition 3.5.1 (Lemma 5.2.1) and noting that in the RR regime, there is no distinction between reserve clearance and allocation, because the reserve represents the lowest price the seller will accept. Since negotiation under FD represents another opportunity to sell the good, allocation must occur more often under FD than in the RR regime. □

The importance of the probability of reserve clearance to this argument can be seen using a slight variation of Example 3.4.3. Fixing distributions, valuation functions and the reserve price strategy from that example, we can compare the seller's expected profits *from the bidding game* in the FD and RR regimes; from Proposition 3.5.1 (ii), if these profits are greater under

FD, total profits from the mechanism are as well. For the RR regime, the equilibrium bid is $b = w(x, R^{-1}(b))$, where

$$w(x, R^{-1}(b)) = x + E[S|S \leq \frac{2}{3}b] \tag{4.3}$$

So $\beta_R^{RR}(x) = \frac{3}{2}x$.[6] A bidder clears the reserve in this case whenever $\frac{3}{2}s \leq \frac{3}{2}x$ or $s \leq x$. The auction price is the point at which the second last bidder withdraws (the second highest bid) and calling $f_2(x)$ the density of the second highest bidder type, we can express the seller's expected profit under RR as

$$\begin{aligned}\Pi^{RR} &= \int_0^1 \int_0^x (\frac{3}{2}x - \frac{3}{2}s)f(s)f_2(x)dsdx \\ &= \frac{3}{2}\int_0^1 (x^2 - \frac{x^2}{2})f_2(x)dx \\ &= \frac{3}{4}\int_0^1 x^2 f_2(x)dx\end{aligned}$$

For FD, bidders bid their values; $\hat{b} = x + s$. The reserve is therefore cleared whenever $x \geq \frac{s}{2}$. We use these facts to separate the expected profits from the bidding game as follows;

$$\Pi^{FD}(\text{Bidding Game}) = \Pi^{FD}(\frac{s}{2} \leq x \leq s, \text{Bidding Game}) + \Pi^{FD}(x \geq s, \text{Bidding Game}) \tag{4.4}$$

Concentrating on the second term in (4.4), we have

$$\begin{aligned}\Pi^{FD}(x \geq s, \text{Bidding Game}) &= \int_0^1 \int_0^x (x + s - \frac{3}{2}s)f_2(x)f(s)dsdx \\ &= \int_0^1 [x^2 - \frac{1}{4}x^2]f_2(x)dx \\ &= \frac{3}{4}\int_0^1 x^2 f_2(x)dx \\ &= \Pi^{RR}\end{aligned}$$

[6] An almost identical calculation is given in the original example. Note that by construction of this example all but the highest type of bidder experience 'bad news', since meeting the reserve implies $S = x$, which is a positive probability event for all but the highest type of bidder. Recall from the equilibrium bidding function (4.2) that this is by no means a general feature.

So (4.4) becomes

$$\Pi^{FD}(\text{Bidding Game}) = \Pi^{FD}(\frac{s}{2} \leq x \leq s, \text{Bidding Game}) + \Pi^{RR}$$

Note that bids can be higher in either regime depending on the value of S. Again using the example $s = 0$ bids are higher under RR since $\frac{3}{2}x \geq x$, while for $s = 1$ valuations exceed expected valuations. Regardless, we have found that expected profits from the bidding game, and, therefore, in the regime itself are higher under FD because the reserve is cleared more often. Bids incorporate a conditional expectation over S when it is unknown. If we integrate over all outcomes in which this expectation is correct, i.e. when the secret reserve is cleared ($x \geq s$), we achieve the expected value of the second highest valuation in that range. The same expected profits would be achieved for signals in this region if bidders knew S, since then bids are equivalent to valuations. So the profit ranking follows from the fact that the good is allocated more often under FD, as a result of the avoidance of the winner's curse under RR.

Once we consider equilibrium reserve price selection, this result need not hold. It is clear that the strongest form of a secret reserve, one that is never announced as in the RR regime can avoid the undesirable characteristics of signalling equilibria. Further, this screening regime most closely resembles those run by auction houses as described by Ashenfelter (1989), recall; '*Only after the auction does the auctioneer reveal whether and at what price the item may have actually been sold. In short, the auctioneers do not reveal the reserve price and make it as difficult as they can for bidders to infer it*'. The seller's equilibrium reserve price will be much lower than under signalling; she has a dominant strategy to accept any price above her reservation value. She therefore faces a tradeoff between perhaps allocating more often, thus avoiding the inefficiencies of signalling equilibria, but losing monopoly price setting power. In the next section we show that if uncertainty about reserve prices (in equilibrium, the seller's reservation value) predominantly reflects uncertainty about the 'bidder relevant' component of the seller's information, then the Right of Refusal dominates Full Disclosure. We can therefore explain the prevalence of secret reserves as used by auction houses as a response to the seller's inability to cheaply and/or credibly reveal her private information. Further, this conclusion is arguably of some interest to the more general issue of asset pricing under asymmetric information that was foreshadowed by Akerlof (1970), and Maskin and Tirole (1992)'s discussion regarding the relative difficulties of implementation via signalling and screening by an informed principal under 'common' values.[7]

4.2 The Right of Refusal

We have shown that a seller who will eventually announce a reserve price should do so earlier to signal the same information in a less costly manner. The possibility remains however that Full Disclosure may be dominated by a policy of full secrecy. That is, it may be optimal to never reveal the reserve price and instead operate under the Right of Refusal (RR). There is reason to be optimistic about such a policy, since if the seller never reveals her information, bidders are never concerned about the seller's incentives and costly increases in the reserve price above the complete information case may be avoided. The cost of such a policy is that the seller must sacrifice potential profits by refusing sale when there is a possibility that the winner would be willing to pay a higher price if she knew the value of S. To investigate this, recall the equilibrium bidding function under RR from (4.2) and call $\widetilde{m}^{RR}(s)$ the threshold value for reserve clearance in the equilibrium of the RR regime. Assume that $\widetilde{m}^{RR}(s) < \overline{\theta}$, so $\forall s \quad \exists x : w(x,s) \geq v_0(s)$. The interim expected profits to the seller in this regime are

$$\widetilde{U}(s, \widetilde{m}^{RR}(s)) \quad = \quad \int_{\widetilde{m}^{RR}(s)}^{\overline{\theta}} [\beta^{RR}(x) - v_0(s)] f_2(x) dx$$

Recall that in equilibrium under FD we had

$$\begin{aligned} U(\hat{s}, s, m^{FD}(\hat{s}))_{|\hat{s}=s} = & \int_{m^{FD}(s)}^{\overline{\theta}} [v(x,s) - v_0(s)] f_2(x) dx \\ & + [F_2(m^{FD}(s)) - F_1(m^{FD}(s))][v(m^{FD}(s), s) - v_0(s)] \end{aligned}$$

In the equilibrium of FD we considered, the seller's interim expected profit at her lowest signal value is equivalent to that which she would earn under complete information. Cai, Riley, and Ye (2007) show that this is described by[8]

$$m^{FD}(\underline{\theta}_S) = \begin{cases} \underline{\theta} & \text{if} \quad J(\underline{\theta}, \underline{\theta}_S) \geq v_0(\underline{\theta}_S) \\ J_s^{-1}(v_0(\underline{\theta}_S)) & \text{if} \quad J(\underline{\theta}, \underline{\theta}_S) \leq v_0(\underline{\theta}_S) \leq J(\overline{\theta}, \underline{\theta}_S) \\ \overline{\theta} & \text{if} \quad J(\overline{\theta}, \underline{\theta}_S) \leq v_0(\underline{\theta}_S) \end{cases}$$

[7] Recall our treatment of Maskin and Tirole's work in Chapter 1.

[8] See their Equation (7).

where

$$J(x,s) = v(x,s) - \frac{dv}{dx}\Big(\frac{F_2(x) - F_1(x)}{f_1(x)}\Big) \tag{4.5}$$

$J_s^{-1}(.)$ is the inverse of J for fixed $S = s$. $J(x,s)$ is the generalization of the *virtual valuation* concept from Myerson (1981), and recall from (3.9) that we have assumed $J(x,s)$ is strictly increasing, which corresponds to Myerson's *regular* case.[9] To achieve a profit ranking for FD and RR, it is helpful to know how interim profits change with S in these regimes. For the RR regime, we have

$$\begin{aligned} \frac{d}{ds}[\widetilde{U}(s,\widetilde{m}^{RR}(s))] &= -[\beta(m^{RR}(s)) - v_0(s)]f_2(\widetilde{m}^{RR}(s))\frac{d\widetilde{m}^{RR}(s)}{ds} \\ &- v_0'(s)[1 - F_2(\widetilde{m}^{RR}(s))] \\ &= -[1 - F_2(\widetilde{m}^{RR}(s))]v_0'(s) < 0 \end{aligned} \tag{4.6}$$

In the first line, the expression in the square brackets becomes 0 because either $\beta(m^{RR}(s)) = v_0(s)$ (if $\exists x : \beta^{RR}(x) = R(s)$) or $\frac{dm^{RR}(s)}{ds} = 0$ (if $\beta^{RR}(\underline{\theta}) > v_0(s)$). The seller receives lower expected profits as her signal increases because her reservation value increases, but bids do not as they are independent of S. Her highest interim profits are therefore achieved at her lowest signal value, where she benefits from the externality that occurs due to the existence of higher possible types of herself.

Performing the same operation for interim profit in the FD regime achieves

$$\begin{aligned} \frac{d}{ds}[U(\hat{s},s,m^{FD}(\hat{s}))|_{\hat{s}=s}] &= \frac{dU}{d\hat{s}}|_{\hat{s}=s} + \Big(\frac{dU}{dm^{FD}(\hat{s})}\Big)|_{\hat{s}=s}\Big(\frac{dm^{FD}(\hat{s})}{d\hat{s}}\Big)|_{\hat{s}=s} + \Big(\frac{dU}{ds}\Big) \\ &= -v_0'(s)[1 - F_2(m^{FD}(s)) + F_2(m^{FD}(s)) - F_1(m^{FD}(s))] \\ &- [1 - F_1(m^{FD}(s))]v_0'(s) \\ &= -[1 - F_1(m^{FD}(s))]v_0'(s) < 0 \end{aligned} \tag{4.7}$$

By a standard envelope theorem argument, infinitesimal changes in the seller's type only influence her utility through their direct effect on her objective function in equilibrium. At $s = \underline{\theta}$, the seller allocates as she would under complete information, and her interim profits fall as S increases, despite the fact that valuations, and thus bids increase. It is as if she keeps selling the good in the same circumstances, and forfeits a greater cost, the increase given by $v_0'(s)$. All revenue benefits accrued from higher bidder

[9] To achieve his pure private values model, set $v(x,s) = x$ in $J(x,s)$, and note that $\frac{F_2(x)-F_1(x)}{f_1(x)} = \frac{1-F(x)}{f(x)}$ under independence.

valuations must be 'paid for' with suboptimal monopoly pricing in order for the information to be transmitted credibly. Using (4.7) and (4.6) with the Fundamental Theorem of Calculus we can find new expressions for *ex ante* expected profits. For the RR regime;

$$\widetilde{U}(s,\widetilde{m}^{RR}(s)) = \widetilde{U}(\underline{\theta}_S,\widetilde{m}^{RR}(\underline{\theta}_S)) - \int_{\underline{\theta}_S}^{s}[1-F_2(\widetilde{m}^{RR}(t)))]v_0'(t)dt$$

Taking expectations over S and rearranging the integrals achieves the following;

$$\begin{aligned} E_S[\widetilde{U}(s,\widetilde{m}^{RR}(s))] &= \widetilde{U}(\underline{\theta}_S,\widetilde{m}^{RR}(\underline{\theta}_S)) \\ &- \int_{\underline{\theta}_S}^{\overline{\theta}_S}[1-F_S(s)][1-F_2(\widetilde{m}^{RR}(s))]v_0'(s)ds \end{aligned} \tag{4.8}$$

Similarly for the FD regime;

$$\begin{aligned} E_S[U(s,\hat{s},m^{FD}(\hat{s}))_{|\hat{s}=s}] &= U(\underline{\theta}_S,\hat{s},m^{FD}(\hat{s}))_{|\hat{s}=\underline{\theta}_S} \\ &- \int_{\underline{\theta}_S}^{\overline{\theta}_S}[1-F_S(s)][1-F_1(m^{FD}(s))]v_0'(s)ds \end{aligned} \tag{4.9}$$

Using these expressions, the following theorem presents sufficient conditions for the RR regime to dominate Full Disclosure.

Proposition 4.2.1. *If $\widetilde{m}^{RR}(s) < \overline{\theta} \quad \forall s$ and at $s = \underline{\theta}_S$ expected profits under RR exceed that under FD by at least $E_S[v_0(s)] - v_0(\underline{\theta}_S)$ then RR ex ante dominates FD.*

Proof. If $\widetilde{m}^{RR}(s) < \overline{\theta} \quad \forall s$ then *ex ante* expected profits in RR and FD can be computed using (4.8) and (4.9). RR auction dominates FD iff

$$\widetilde{U}(\underline{\theta}_S,\widetilde{m}^{RR}(\underline{\theta}_S)) - U(\underline{\theta}_S,\hat{s},m^{FD}(\hat{s}))_{|\hat{s}=\underline{\theta}_S}$$
$$> \int_{\underline{\theta}_S}^{\overline{\theta}_S}\Big\{[1-F_S(s)][F_1(m^{FD}(s)) - F_2(\widetilde{m}^{RR}(s))]v_0'(s)\Big\}ds$$

Since $[F_1(m^{FD}(s)) - F_2(\widetilde{m}^{RR}(s))] < 1 \quad \forall s$, and the other terms in the integrand are non-negative, the following is sufficient;

$$\begin{aligned} \widetilde{U}(\underline{\theta}_S,\widetilde{m}^{RR}(\underline{\theta}_S)) - U(\underline{\theta}_S,\hat{s},m^{FD}(\hat{s}))_{|\hat{s}=\underline{\theta}_S} &\geq \int_{\underline{\theta}_S}^{\overline{\theta}_S}[1-F_S(s)]v_0'(s)ds \\ &= E_S[v_0(s)] - v_0(\underline{\theta}_S) \end{aligned} \tag{4.10}$$

□

This condition is particularly convenient since we do not need to compute anything that depends upon the complicated signalling equilibrium under FD. The inequality can be directly computed from the valuation and distribution functions of the bidders and the seller, and (similarly) the seller's optimal reserve when her type is known $s = \underline{\theta}_S$. Intuitively, the condition compares the costs and benefits of screening relative to signalling. The LHS reflects the benefit of selling at average information prices despite holding poor information, relative to monopoly profits. The right hand side reflects the fact that when S is high and the good is sold under RR the seller is paid the same as she would be under FD, she misses out on the increase in the fair compensation $E_S[v_0(s)]$. We now revisit our earlier example to glean further insight as to the conditions under which (4.10) holds and RR dominates FD.

4.2.1 Linear Valuations: RR Dominates FD

Say that there are two bidders, with bidder valuations and signal distributions fixed as per the example in Section 3.4.3, so $x \sim U[0,1]$ and $v = x + s$. Let the seller's valuation be $\gamma s : \gamma > 0$, and say that S is distributed uniformly on $[0, \omega]$. We are therefore allowing flexibility in the type of uncertainty that bidders face when S is unknown; that regarding the reserve price, or seller's valuation, and the information of interest that determines the reserve price. The threshold value under FD at $s = 0$ can be computed by solving for $m^{FD}(0)$ from (4.5), which gives $m(0) = \frac{1}{2}$. Given the bidders' valuation structure, at $s = 0$ the model under FD conveniently collapses to the private values case. The expected profits under FD are likewise the same, equal to $\frac{5}{12}$.[10] Now we compute expected profits under RR. Using the bidding function from (4.2) we have[11]

$$\beta^{RR}(x) := \begin{cases} \frac{\gamma}{\gamma - \frac{1}{2}} x & \text{if} \quad x \leq \omega(\gamma - \frac{1}{2}) \\ x + \frac{\omega}{2} & \text{if} \quad x \geq \omega(\gamma - \frac{1}{2}) \end{cases}$$

At $s = 0$, the seller's valuation is zero and the good is always allocated. If $\gamma \leq \frac{1}{2}$, then $\omega(\gamma - \frac{1}{2}) \leq 0$ and all types of bidder will bid their unconditional expected value. Cases in which all types bid their unconditional expected value may appear to be of little practical interest, since this implies that

[10]Note this is true for any γ, and the reserve and bids are independent of the seller's distribution, meaning this result is also independent of ω.

[11]It is simple to verify in this instance that the condition $\widetilde{m}^{RR}(s) < \overline{\theta} \quad \forall s$ is satisfied.

the secret reserve under RR will always be cleared, a restriction that presumably does not hold in most applications. Nevertheless, it can be verified that for $\gamma \leq \frac{1}{2}, \omega \geq \frac{1}{3}$ RR dominates FD, which shows that a seller with private information should not attempt any monopoly pricing if gains from trade are 'significant' despite the fact that this would be desirable under complete information (since $m^*(s) \geq \frac{1}{2}$).[12] In this case the RR reserve is equivalent to not having a reserve at all, so this feature corresponds to a well known result in signalling games; banning the signalling activity can generate Pareto improvements.[13] A more interesting case is where RR dominates FD and the reserve is not always cleared. To analyze such cases, consider $\omega(\gamma - \frac{1}{2}) \in (0, 1)$. Some bidders (types below $\omega(\gamma - \frac{1}{2})$) know they will not clear the reserve in some realizations and so bid their conditional expectations. Expected profits to the seller at $s = 0$ under RR are

$$\begin{aligned}
\widetilde{U}(0, \widetilde{m}^{RR}(0)) &= \int_0^{\omega(\gamma-\frac{1}{2})} \frac{\gamma}{\gamma - \frac{1}{2}} x f_2(x)dx + \int_{\omega(\gamma-\frac{1}{2})}^1 (x + \frac{\omega}{2}) f_2(x)dx \\
&= \int_0^1 x f_2(x)dx + \frac{\frac{1}{2}}{\gamma - \frac{1}{2}} \int_0^{\omega(\gamma-\frac{1}{2})} x f_2(x)dx + \frac{\omega}{2}[1 - F_2(\omega(\gamma - \frac{1}{2}))] \\
&= \frac{1}{3} + \frac{\omega}{2} + \frac{1}{(\gamma - \frac{1}{2})}[\frac{x^2}{2} - \frac{x^3}{3}]_0^{\omega(\gamma-\frac{1}{2})} - \frac{\omega}{2}[2\omega(\gamma - \frac{1}{2}) - \omega^2(\gamma - \frac{1}{2})^2] \\
&= \frac{1}{3} + \frac{\omega}{2} + \frac{\omega^3(\gamma - \frac{1}{2})^2}{6} - \frac{\omega^2(\gamma - \frac{1}{2})}{2}
\end{aligned}$$

Again using (4.10) RR dominates FD if

$$2\omega^3(\gamma - \frac{1}{2})^2 - 6\omega^2(\gamma - \frac{1}{2}) - 6\omega(\gamma - \frac{1}{2}) + 3\omega - 1 \geq 0 \qquad (4.11)$$

So taking (for example) the parameters $\gamma = \frac{5}{8}$ and $\omega = 2$, the above holds.[14] Holding ω constant and differentiating the left hand side of (4.11), we find that this expression is strictly decreasing in $(\gamma - \frac{1}{2})$ whenever $\omega(\gamma - \frac{1}{2}) \in [0, 1]$. Thus the likelihood of RR dominating FD (in terms of our condition) is decreasing in the seller's signal weighting above $\frac{1}{2}$. This reflects the fact that reducing γ increases gains from trade, lessening the 'bad news' implied from clearing the secret reserve, since $E_S[S \leq \frac{b}{\gamma}]$ is decreasing in γ. Decreasing γ

[12] An example would be the state designing their selling procedure for future privatizations. Arguably it is known that there are will be significant gains from trade in all realizations and the government will be committed to selling the asset.

[13] See Mas-Colell, Whinston, and Green (1995) Section 13C.

[14] Given these parameters the condition $\omega(\gamma - \frac{1}{2}) = \frac{1}{4} \in [0, 1]$ is also satisfied.

also increases the set of bidders who bid their unconditional expected value, pushing us toward the result for $\gamma < \frac{1}{2}$ above. Alternatively, fixing the spread of information that is relevant to bidders (given by ω), increasing γ means that clearing a secret reserve price conveys more bad information under RR, and signalling becomes easier to implement under FD since the increase in the seller's reservation value is greater for each realization of her signal. Finally, note that the degree of bidders' uncertainty regarding a secret reserve is increasing in both γ and ω, since higher values of either of these parameters increase the 'spread' of possible reserves the bidder could be facing. Despite the simple linear structure of this example, we can nevertheless uncover an interesting link between the source of bidders' uncertainty and the dominance of RR. To analyze this, first re-express (4.11) as

$$\omega(2c^2 - 6c + 3) - 6c - 1 \quad \geq \quad 0 \tag{4.12}$$

where $c = \omega(\gamma - \frac{1}{2})$. We are fixing a bidder's level of total uncertainty c which dictates whether or not she views herself as competing against the seller, i.e. whether or not her bid may fail to meet the reserve. Now say that $(2c^2 - 6c + 3) \geq 0$, which must be the case for RR to dominate FD under our condition.[15] Then the LHS of (4.12) is increasing in ω. Since we must reduce γ if we are to increase ω keeping c fixed, the dominance of RR must increase in the *seller's information share of uncertainty.* The seller's decision regarding a secret reserve is therefore more subtle than the range of possible secret reserve prices that bidders may face. Fixing the spread of private seller information, stronger seller preferences (higher γ) makes disclosure more likely to be selected, since the seller's reservation value increases making reserve price announcements more credible. If however a given spread of reserve prices reflects significant underlying uncertainty about the seller's information itself, then RR becomes preferable. Thus if the seller's valuation is sufficiently relevant to the market, RR dominates FD.

4.3 Conclusion

We have investigated the properties of secret reserve regimes in a setting in which the seller holds private information about the quality of the object. Somewhat surprisingly, we have found that later announcement of a reserve

[15] This is equivalent to $c \leq \frac{1}{2}(3 - \sqrt{3}) \approx 0.63$

exacerbates the seller's incentive problem, so that Full Disclosure is preferable to later announcement of a reserve. (Almost) every type of seller will therefore select a higher reserve price under secrecy, so FD is preferable to EN and WS. The undesirable characteristics of the signalling equilibria can be avoided if the seller uses the Right of Refusal, provided the variability of reserve prices stems primarily from the variability of the underlying information that is relevant to bidders. We believe this conclusion to be particularly important for the design of institutions in which the seller's role is purely that of a trader, in which case all information contained in her reservation value is relevant to the market. The most obvious example of such a third party seller is an auction house, in which the value of withholding supply reflects the discounted value of revenue from future sales. This interpretation can be easily accommodated in our model by presuming that the game (including regime selection) is repeated in the event of non-clearance with a new set of bidders.[16] Our model therefore provides an explanation for the observations of Ashenfelter (1989) regarding the prevalence of secret reserves in these institutions. However, the conclusion that secrecy is best may also hold more broadly in markets for financial assets (e.g. privatization, mergers and acquisitions) in which asymmetric information is a powerful force and private perceptions of value are universally relevant.

[16]Note the contrast with Horstmann and LaCasse (1997) which requires the same set of bidders in each round so that the information gleaned from the initial auction can influence revenues in future auctions.

Chapter 5

Appendices

5.1 Construction of Payment Functions

Given a generic allocation rule, for each $i \neq 0$ construct the following payment rule;

$$\begin{aligned} M_i(\mathbf{x}) &= \underline{\theta}_i q_i(\underline{\theta}_i) + \int_{\underline{\theta}_i}^{x_i} t_i q_i'(t_i) dt_i \\ &+ \frac{1}{N}\int_{\underline{\theta}_0}^{x_0} t_0 q_0'(t_0) dt_0 - \frac{1}{N}\int_{\underline{\theta}_0}^{\overline{\theta}_0} t_0(1-F_0(t_0)) q_0'(t_0) dt_0 \end{aligned}$$

Taking expectations, the terms in the second line are differenced away to achieve the following expression for $m_i(x_i)$

$$m_i(x_i) = \underline{\theta}_i q_i(\underline{\theta}_i) + \int_{\underline{\theta}_i}^{x_i} t_i q_i'(t_i) dt_i$$

By construction, $U_i(\underline{\theta}_i) = \underline{\theta}_i q_i(\underline{\theta}_i) - \underline{\theta}_i q_i(\underline{\theta}_i) = 0$. Incentive compatibility for each buyer follows the demonstration in Myerson (1981) (Theorem 1). For the seller, the expected receipts are

$$\begin{aligned} m_0(x_0) - m_0(z_0) &= \sum_{i=1}^{N} \frac{1}{N}\int_{\underline{\theta}_0}^{x_0} t_0 q_0'(t_0) dt_0 - \sum_{i=1}^{N}\frac{1}{N}\int_{\underline{\theta}_0}^{\overline{\theta}_0} t_0(1-F_0(t_0)) q_0'(t_0) dt_0 \\ &= \int_{z_0}^{x_0} t_0 q_0'(t_0) dt_0 \end{aligned}$$

In the first line, the terms in $M_i(\mathbf{x})$ that are independent of x_0 are differenced away. The second line follows because the terms that remain are independent of i. From this point, the proof of IC for the seller is analogous to that for the buyers.

5.2 Proof of Proposition 3.4.1.

Proof. Differentiating (3.3) with respect to z yields[1]

$$\frac{dE[\Pi(z,x)]}{dz} = \int_{\underline{\theta}_S}^{\widetilde{s}(z)} [v(x,s)-\beta(z)]f_S(s)f_Y(z)ds + \int_{\widetilde{s}(z)}^{\max\{s(x),\widetilde{s}(z)\}} [v(x,s)-R(s)]f_S(s)f_Y(z)ds \tag{5.1}$$

by Liebniz' rule. Setting $z = x$, it is simple to verify that $\beta(x) = v(m_R(\underline{\theta}_S), \underline{\theta}_S) = R(\underline{\theta}_S)$ sets the first order condition equal to 0 for $x = m_R(\underline{\theta}_S)$. Thus $\beta_R(m_R(\underline{\theta}_S)) = R(\underline{\theta}_S)$, and the lowest participating bidder under secrecy is type $m_R(\underline{\theta}_S)$, as it is under disclosure. We can also conclude that $s_R(x)$ is defined for all participating bidders, because it is the inverse of m_R and is defined for $x \geq m_R(\underline{\theta}_S)$. This proves the first part of the proposition. To derive the equilibrium bidding function, replace $\beta(z)$ with b and z with x in (5.1), then solve

$$\begin{aligned} F_S(R^{-1}(\min\{b, R(\overline{\theta}_S)\}))b &= \int_{\underline{\theta}_S}^{R^{-1}(\min\{b,R(\overline{\theta}_S)\})} v(x,s)f_S(s)ds \\ &+ \int_{R^{-1}(\min\{b,R(\overline{\theta}_S)\})}^{\max\{s(x),R^{-1}(\min\{b,R(\overline{\theta}_S)\})\}} [v(x,s)-R(s)]f_S(s)ds \end{aligned} \tag{5.2}$$

in b. Is there a solution to this equation for $b \geq R(\overline{\theta}_S)$? Substituting $b \geq R(\overline{\theta}_S)$ into (5.2) yields $b = E_S[v(x,S)]$. It follows that bidder types for whom the unconditional expected value exceed the highest possible reserve price should bid this value. This is consistent with the implementation of Bayes' Rule (and avoidance of the winner's curse), since if bidding in such a fashion always causes one to clear the reserve, no 'bad news' should be inferred about the value of S. We now address the remaining case where $x \geq m_R(\underline{\theta}_S)$ and $E_S[v(x,S)] < R(\overline{\theta}_S)$. To do this, we need the following Lemma.

[1] We suppress the subscript R where possible in the algebra here for notational convenience.

Lemma 5.2.1. *For any strictly increasing R and m_R, $s_R(x) \geq \widetilde{s}_R(x)$ for $x \geq m_R(\underline{\theta}_S)$, with strict equality for $x > m_R(\underline{\theta}_S)$. Equivalently, $\forall s \quad \widetilde{m}_R(s) \geq m_R(s)$ with strict equality for $s > \underline{\theta}_S$.*

Proof. Say that the proposition is false, so that for some x there is a solution to (5.2) in b for $R(\underline{\theta}_S) \leq b \leq R(\overline{\theta}_S)$, and $\widetilde{s}_R(x) > s_R(x)$. Then $\beta(x) = w(x, \widetilde{s}_R(x))$, because the 'option' component of the bid becomes zero. The option will never be exercised since $\beta(x) < R(s)$ implies $v(x,s) < R(s)$. Now given s we can write the bid at $x = \widetilde{m}(s)$ as $\beta(\widetilde{m}(s)) = w(\widetilde{m}(s), \widetilde{s}(\widetilde{m}(s)))$. But

$$
\begin{aligned}
R(s) = \beta(\widetilde{m}(s)) &= w(\widetilde{m}(s), \widetilde{s}(\widetilde{m}(s))) \\
&= w(\widetilde{m}(s), s) \\
&\leq v(\widetilde{m}(s), s) \qquad (5.3)
\end{aligned}
$$

So type $\widetilde{m}(s)$ is still active when S is known. $m(s)$ is strictly increasing by assumption, and $\widetilde{m}(s)$ is strictly increasing because $\beta_R(x)$ is independent of s and $R' > 0$. It follows that $m(s) \leq \widetilde{m}(s)$, with the equality coming from the event in (5.3) where $s = \underline{\theta}_S$. Similarly, $s(x) \geq \widetilde{s}(x)$ for $x \geq m(\underline{\theta}_S)$. □

Given this result, we can substitute $s_R(x) \geq R^{-1}(b)$ for $b \leq R(\overline{\theta}_S)$ into (5.2). This yields

$$
\begin{aligned}
F_S(R^{-1}(b))b &= \int_{\underline{\theta}_S}^{R^{-1}(b)} v(x,s) f_S(s) ds + \int_{R^{-1}(b)}^{s(x)} [v(x,s) - R(s)] f_S(s) ds \\
&\equiv \mathcal{F}(b) \qquad (5.4)
\end{aligned}
$$

We now prove that there is a unique solution to this equation for $x : E_S[v(x,s)] \leq R(\overline{\theta}_S)$, $x \geq m_R(\underline{\theta}_S)$, which will complete the proof. Differentiating the right hand side of this expression with respect to b, we have

$$
\begin{aligned}
\mathcal{F}'(b) &= v(x, R^{-1}(b)) f_S(R^{-1}(b)) \frac{dR^{-1}(b)}{db} - [v(x, R^{-1}(b)) - b] f_S(R^{-1}(b)) \frac{dR^{-1}(b)}{db} \\
&= f_S(R^{-1}(b)) \frac{dR^{-1}(b)}{db} b > 0
\end{aligned}
$$

because f_S, R' and $b > 0$. Alternatively differentiating $F_S(R^{-1}(b))b$ gives

$$
\begin{aligned}
\frac{d}{db}[F_S(R^{-1}(b))b] &= f_S(R^{-1}(b)) \frac{dR^{-1}(b)}{db} b + F_S(R^{-1}(b)) \\
&= \mathcal{F}'(b) + F_S(R^{-1}(b))
\end{aligned}
$$

So the LHS term of (5.4) starts below $\mathcal{F}(b)$, but increases in b at a greater rate than the RHS. We have already shown that the lowest bid from a participating bidder is $R(\underline{\theta}_S)$. It follows that for $x > m_R(\underline{\theta}_S)$, at $b = R(\underline{\theta}_S)$ the LHS of (5.4) is zero, while the RHS is positive. Then we only need find some $b \in [R(\underline{\theta}_S), R(s(x))]$, such that $F(R^{-1}(b))b \geq \mathcal{F}(b)$ to ensure the existence and uniqueness of a solution to (5.4). Taking $b = R(s(x)) = v(x, s(x))$, the LHS term is $F_S(s(x))v(x, s(x))$. Evaluating $\mathcal{F}(b)$ at the same point gives

$$\mathcal{F}(v(x, s(x))) = \int_{\underline{\theta}_S}^{s(x)} v(x, s) f_S(s) ds < F_S(s(x)) v(x, s(x))$$

Then at $b = v(x, s(x))$ we have $F_S(R^{-1}(b))b > \mathcal{F}(b)$, so continuously increasing b from $R(\underline{\theta}_S)$ achieves a solution to (5.2) by the Intermediate Value Theorem. This completes the proof. □

5.3 Bidding Function: Discussion

Note that the bidding function for $E_S[v(x, s)] \leq R(\overline{\theta}_S)$ can also be expressed as

$$\beta(x) = w(x, R^{-1}(\beta(x))) + \frac{\int_{R^{-1}(\beta(x))}^{s(x)} [v(x, s) - R(s)] f_S(s) ds}{F_S(s(x)) - F(R^{-1}(\beta(x)))} \times \frac{F_S(s(x)) - F_S(R^{-1}(\beta(x)))}{F_S(R^{-1}(\beta(x)))}$$

the second term is the expected surplus from receiving a reserve price offer weighted by the probability of this event relative to winning at the auction price. Again, we have proven in Lemma 5.2.1 that such an offer yields positive expected surplus for all types of bidder who bid below their unconditional expected value. On the other hand, the expected value in the first term $w(x, R^{-1}(\beta(x)))$ effectively has a weight of 1. These weights sum to

$$\frac{F_S(s(x))}{F_S(R^{-1}(\beta(x)))} > 1$$

The fact that the equilibrium bid is not (indeed it exceeds) a convex combination of expected values seems to suggest players are overbidding. In fact, this is not so; bidders can afford to do this because when they win and the reserve is not cleared, their bid becomes irrelevant and they have made no commitment.[2] They can decide later whether or not to buy at a price the seller will decide.

[2] Recall that in this event their bid remains unobserved.

A general feature of the English auction (and also Second Price Sealed Bid auction) is that players bid such that the expected value from 'just' winning (i.e. when there is a tie with another bidder) equals their expected payment in this outcome.[3] Since bidders are price takers, all they can do is make sure that they participate in all trades that yield non-negative expected profit, so the marginal trade (when the object is won in a tie with another bidder) results in an expected net payoff of zero. Equation (5.5) also exhibits this 'break even' characteristic; expected payments at the marginal trade equal the expected gross payoff given a tie. The only distinction is in the definition of the marginal trade. The expected value of winning from a tie is equivalent to the Right Hand Side of (5.5) and includes both winning at the auction price and from a reserve price offer. However, the Left Hand Side differs slightly, because the marginal trade is defined by winning *at the auction price only*, since this is the only instance where one's bid in the auction determines whether trade occurs. If the reserve is not cleared trade is determined by the bidder's acceptance or rejection of the reserve price offer, which is independent of her bid in the auction. A bidder therefore sets the expected payment from the marginal trade (i.e. when the second highest bid is equal to her own and above the reserve) equal to the gross expected payoffs from winning in a tie, which can occur at the auction price or some offer above.

5.4 Proof of Theorem 3.7.1.

Proof. Take the first order conditions of $U_{EN}(s, \hat{s}, m(\hat{s}))$ with respect to $\hat{s}$ and m;

$$
\begin{aligned}
\frac{dU}{d\hat{s}} &= [F_2(\widetilde{m}(m)) - F_1(m)]\frac{dv}{d\hat{s}} \\
\frac{dU}{dm} &= f_2(\widetilde{m}(m))\frac{d\widetilde{m}(m)}{dm}[v(m,\hat{s}) - \beta(\widetilde{m}(m))] \\
&- f_1(m)[v(m,\hat{s}) - v_0(s)] + [F_2(\widetilde{m}(m)) - F_1(m)]\frac{dv}{dx}|_{x=m}
\end{aligned}
$$

Following Cai, Riley, and Ye (2007) (Theorem 1), we can verify that the single crossing condition holds, namely that $\frac{d}{ds}\left(\frac{dU/dm}{dU/d\hat{s}}\right) > 0$, because $\frac{dU}{d\hat{s}}$ is not a function of s and $\frac{dU}{dm}$ is increasing in s. Recall that a separating

[3] See for example Krishna (2002) Section 8.2; Second Price Auctions with Interdependent Values.

equilibrium constitutes a minimum bidder type $m(s)$ such that $\frac{dm}{ds} > 0$, and $\hat{s} = s$ maximizes $U_{EN}(s, \hat{s}, m(\hat{s}))$. Setting $\hat{s} = s$ in the First Order Conditions yields

$$\begin{aligned}
\frac{dU}{d\hat{s}} &= [F_2(\widetilde{m}(m)) - F_1(m)]\frac{dv}{d\hat{s}}|_{\hat{s}=s} \\
\frac{dU}{dm} &= - f_1(m)[v(m,s) - v_0(s)] + [F_2(\widetilde{m}(m)) - F_1(m)]\frac{dv}{dx}|_{x=m} \\
&= f_1(m)\Big[v_0(s) - v(m,s) + \frac{dv}{dx}|_{x=m}\Big(\frac{F_2(\widetilde{m}(m)) - F_1(m)}{f_1(m)}\Big)\Big]
\end{aligned}$$

The first term in the original expression for $\frac{dU}{dm}$ disappears because either $\beta(\widetilde{m}(m)) = v(m,s) = R(s)$ or $\beta(\widetilde{m}(m)) < R(s) = v(m,s)$ so $\frac{d\widetilde{m}(m)}{dm} = 0$. Thus by Theorem 1 of Cai, Riley, and Ye (2007), (3.10) is sufficient for the existence of a unique separating equilibrium given by

$$\begin{aligned}
\frac{dm_{EN}}{ds} &= -\frac{\frac{dU_{EN}}{d\hat{s}}|_{\hat{s}=s}}{\frac{dU_{EN}}{dm_{EN}}|_{\hat{s}=s}} \\
&= \frac{\frac{dv}{ds}[F_2(\widetilde{m}(m_{EN})) - F_1(m_{EN})]}{f_1(m_{EN})\Big[v(m_{EN},s) - v_0(s) - \frac{dv}{dx}|_{x=m_{EN}}\Big(\frac{F_2(\widetilde{m}(m_{EN})) - F_1(m_{EN})}{f_1(m_{EN})}\Big)\Big]}
\end{aligned}$$

with initial condition $m_{EN}(\underline{\theta}_S) = m^*_{EN}(\underline{\theta}_S)$. □

5.5 Proof of Theorem 3.7.2.

Proof. We have the following first order conditions for $U_{WS}(\hat{s}, s, m(\hat{s}))$;

$$\begin{aligned}
\frac{dU}{d\hat{s}} &= \frac{1 - F_1(m)}{1 - F_1(\beta^{-1}(p))}\Big(\frac{dv}{d\hat{s}}\Big) \\
\frac{dU}{dm} &= \frac{1}{1 - F_1(\beta^{-1}(p))}\Big[- f_1(m)[v(m,\hat{s}) - v_0(s)] + [1 - F_1(m)]\frac{dv}{dx}|_{x=m}\Big]
\end{aligned}$$

Again, $\frac{dU}{d\hat{s}}$ is independent of s and $\frac{dU}{dm}$ is increasing in m, so the single crossing condition holds, and at $\hat{s} = s$ we have

$$\begin{aligned}
\frac{dU}{d\hat{s}} &= \frac{1 - F_1(m)}{1 - F_1(\beta^{-1}(p))}\Big(\frac{dv}{d\hat{s}}_{|\hat{s}=s}\Big) \\
\frac{dU}{dm} &= \frac{f_1(m)}{1 - F_1(\beta^{-1}(p))}\Big[v_0(s) - v(m,\hat{s}) + \frac{dv}{dx}_{|x=m}\Big(\frac{1 - F_1(m)}{f_1(m)}\Big)\Big]
\end{aligned}$$

Thus by Theorem 1 of Cai et. al., (3.11) is sufficient for the existence of a unique separating equilibrium given by

$$\begin{aligned}\frac{dm_{WS}}{ds} &= -\frac{\frac{dU_{WS}}{d\hat{s}}|_{\hat{s}=s}}{\frac{dU_{WS}}{dm_{WS}}|_{\hat{s}=s}} \\ &= \frac{\frac{dv}{ds}[1-F_1(m_{WS})]}{f_1(m_{WS})\Big[v(m_{WS},s)-v_0(s)-\frac{dv}{dx}|_{x=m_{WS}}\Big(\frac{1-F_1(m_{WS})}{f_1(m_{WS})}\Big)\Big]}\end{aligned}$$

with initial condition $m_{WS}(\underline{\theta}_S) = m^*_{WS}(\underline{\theta}_S)$. □

5.6 Proof of Proposition 4.1.1.

Proof. Differentiating (4.1) with respect to z yields the first order condition

$$\Big[\int_{\underline{\theta}_S}^{\widetilde{s}(z)} v(x,s)f(s)ds - \beta(z)F_S(\widetilde{s}(z))\Big]f_Y(z) = 0$$

Setting $z = x$ and rearranging for the symmetric equilibrium β^{RR}, we have

$$\beta_R^{RR}(x) := w(x,\widetilde{s}(x)) = \begin{cases} b = w(x,R^{-1}(b)) & \text{if} \quad R(\overline{\theta}_S) \geq E_S[v(x,S)] \\ E_S[v(x,S)] & \text{if} \quad R(\overline{\theta}_S) \leq E_S[v(x,S)] \end{cases}$$

If $E_S[v(x,s)] \geq R(\overline{\theta}_S)$, we have $\beta_R^{RR}(x) = E_S[v(x,s)]$. Mimicking our arguments from Proposition 3.4.1, at $x = m_R(\underline{\theta}_S)$ we have $\beta(m_R(\underline{\theta}_S)) = v(m_R(\underline{\theta}_S)) = R(\underline{\theta}_S)$, and the lowest type of participating bidder is the same under RR and for known S for a given reserve price strategy. Further, we have shown that a solution to (5.5) must exist in our proof of Lemma 5.2.1.

We now prove that if $R(s)$ (and in equilibrium $v_0(s)$) is not too concave, in a sense that will be specified, then any solution to (5.5) below $R(\overline{\theta}_S)$ is unique. Under this condition, our assumption of irrevocable exit is not required in equilibrium. Suppose the solution to (5.5) is not unique, so that that there are multiple solutions to $b = w(x, R^{-1}(b))$, and call the first p^*. Clearly these other solutions must occur in $(p^*, R(s_R(x)))$, since any solution at or above $R(s_R(x))$ would involve $w(x,R^{-1}(b)) \geq v(x,R^{-1}(b))$, which is impossible for $R^{-1}(b) > \underline{\theta}_S$. Further, note that one solution alone cannot exist in this range, at least two must. The reason is that the expected value function w must drop back below b before $b = R(s_R(x))$, so that $w(x,s_R(x)) < v(x,s_R(x))$. Call the first two of these (possibly many) extra solutions p^1 and p^2, with $p^* < p^1 < p^2 < R(s_R(x))$. Since p^* and p^1

both lie on the line $b = w(x, R^{-1}(b))$, the slope between them is one. This immediately implies the existence of at least one point $p' \in [p^*, p^1]$ such that

$$\frac{d}{db} w(x, R^{-1}(b))_{|b=p'} = 1$$

by the Mean Value Theorem. By a similar argument another point $p'' \in [p^2, R(s_R(x))]$ must exist such that

$$\frac{d}{db} w(x, R^{-1}(b))_{b=p''} = 1$$

We can write the derivative $\frac{d}{db} w(x, R^{-1}(b))$ explicitly as

$$\frac{d}{db} w(x, R^{-1}(b)) = \frac{1}{R'(s)_{|s=R^{-1}(b)}} \frac{f_S(R^{-1}(b))}{F_S(R^{-1}(b))} [v(x, R^{-1}(b)) - w(x, R^{-1}(b))]$$

Since p^1 and p^2 are the smallest solutions above p^*, the definitions of p' and p'' imply that

$$\begin{aligned} p' >& \quad w(x, R^{-1}(p')) \\ p'' <& \quad w(x, R^{-1}(p'')) \end{aligned}$$

or

$$\begin{aligned} v(x, R^{-1}(p')) - w(x, R^{-1}(p')) >& \quad v(x, R^{-1}(p')) - p' \\ v(x, R^{-1}(p'')) - w(x, R^{-1}(p'')) <& \quad v(x, R^{-1}(p'')) - p'' \end{aligned}$$

From the definition of $\frac{d}{db} w(x, R^{-1}(b))$, and substituting in its value of 1 at p' and p'', we have

$$\begin{aligned} v(x, R^{-1}(p')) - p' <& \quad R'(s)_{|s=R^{-1}(p')} \frac{F_S(R^{-1}(p'))}{f_S(R^{-1}(p'))} \\ v(x, R^{-1}(p'')) - p'' >& \quad R'(s)_{|s=R^{-1}(p'')} \frac{F_S(R^{-1}(p''))}{f_S(R^{-1}(p''))} \end{aligned}$$

We know that $[v(x, R^{-1}(b)) - b]$ falls with b, because $\frac{dv}{ds} < R'(s)$ is necessary for $m_R(s)$ to be increasing. So the above implies

$$R'(s)|_{s=R^{-1}(p')} \frac{F_S(R^{-1}(p'))}{f_S(R^{-1}(p'))} > R'(s)|_{s=R^{-1}(p'')} \frac{F_S(R^{-1}(p''))}{f_S(R^{-1}(p''))} \tag{5.5}$$

Calling $\frac{F_S}{f_S} = \sigma$, the function $R'(s)_{|s=R^{-1}(b)} \sigma(R^{-1}(b))$ is non-decreasing in b provided

$$R'' \geq -\frac{\sigma'}{\sigma} R' \quad \forall s \tag{5.6}$$

By regularity, $-\frac{\sigma'}{\sigma}R' < 0$, so (5.6) is equivalent to the statement that the reserve price strategy R is not too concave. Under (5.6), the fact that $p'' > p'$ by definition gives us a contradiction in (5.5). Thus any solution to (5.5) is unique. In the equilibrium of the RR regime, we can replace $R(s)$ with $v_0(s)$, and (5.6) reduces to a condition on the seller's valuation and distribution functions. In general, we will only consider the first solution to (5.5), and rely on the assumption of irrevocable exit so that other solutions are irrelevant.

So for $R(\overline{\theta}_S) \geq w(x, \overline{\theta}_S) = E_S[v(x,S)]$ we have a solution to $b = w(x, R^{-1}(b))$ by the Intermediate Value Theorem.If on the other hand the unconditional expected value exceeds the highest possible seller value, then we have $b = E_S[v(x,S)]$. Bidding the unconditional expected value ensures that a bidder who wins the auction concludes nothing about the value of S, her bid does not determine the price she pays and she makes positive expected profit from winning. She can therefore do no better than to bid this expected value. So we have

$$\beta_R^{RR}(x) := \begin{cases} b = w(x, R^{-1}(b)) & \text{if} \quad R(\overline{\theta}_S) \geq E_S[v(x,S)] \\ E_S[v(x,S)] & \text{if} \quad R(\overline{\theta}_S) \leq E_S[v(x,S)] \end{cases}$$

□

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