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The Implications of Sequential Investment in the Property Rights Theory of the Firm

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Abstract

In the property rights theory of the firm, control over assets (ownership) affords bargaining power in the case of re-negotiation, providing incentives for parties to make relationship specific investments. The models predict that property rights will be allocated so as to maximise surplus generated from investment.

However, these models assume that investments are made simultaneously. In this thesis I extend the standard property-rights framework to allow for sequential investment; the model allows for two investment periods. If a party invests first (ex-ante), they sink their investment before any contracting is possible. The parties that invest second (ex-post) do so after some of the aspects of the project are tangible, so that they can contract on (at least some) of their investment costs.

As well as being empirically relevant, sequencing has several important theoretical implications. First, if a party gets to invest second, then – ceteris paribus – it has a greater incentive to invest. Second, the investment of parties that invest first are affected by a more than one influence. Anticipating higher ex-post investment, they can have a greater incentive to also increase their investments. However, higher ex-post investment leads to greater costs being borne by the ex-ante investors (via the cost sharing contracts); this reduces ex-ante incentives to invest. Overall either effect can dominate so that ex-ante investment can either increase or decrease as a result of sequential investment. Third, as noted, sequencing of investment provides the possibility to (partially) contract on ex-post investment and costs. This is an additional method of providing incentives to invest, beyond the allocation of property rights themselves. Consequently, ex-post investors can be protected (and be provided incentives to invest) via these contracts, whereas ex-ante investors – who can not contract on their investments at all – are more likely to require the protection of property rights (the allocation of asset ownership).

The addition of sequential investment alters some of the predictions of the standard models. For example, previously the literature found that if all assets are complements at the margin all agents should have access to all assets (Bel (2005)). However, when investment sequencing is possible, making a control structure more inclusive (increasing the number of agents who have access to assets) can reduce the incentives of the ex-ante investors, decreasing overall surplus; this is because increasing the property rights of ex-post investors increases the marginal costs borne by ex-ante investors, effectively reducing their claim on surplus, diminishing their incentives to invest. This result contradicts Bel (2005), and shows that even when all assets are complementary at the margin allocating access rights can be detrimental to incentives.

Furthermore, if assets are substitutes at the margin then transfer of assets from ex-ante investors to ex-post investors can increase ex-ante investment and surplus. This counter intuitive result can occur in the case when decreasing ex-post investment is necessary to provide an incentive to ex-ante investors to increase their investments.

(JEL classification: D23)

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Chapter 1

Introduction

When trade requires relationship-specific investments, incomplete contracting can lead to investment inefficiencies and under-investment in profitable projects. A clever re-assignment of property rights, such as access and veto, can change the bargaining power of the agents and alleviate the hold-up problem by protecting some of the returns to investment from opportunistic behaviour and expropriation at the time when re-negotiation occurs. According to this asset-based view, the boundaries of all firms are driven by the objective of minimising the ill-effects of the hold-up problem and to maximise the (overall) investment incentives of entrepreneurs, managers, employees and sub-contractors.

However, the traditional model is somewhat limited by the requirement that all relationship specific investments be made simultaneously before any contracts can be written. In reality, of course, investments might have to be made in a sequential process. Moreover, imposing simultaneous investment denies the real possibility that some agents invest first, when contracts are incomplete, helping make a project tangible so that the agents that invest second (finalising the project) do so in a more complete contracting environment. While allowing sequential investment adds realism, it also leads to interesting new economic predictions concerning the optimal allocation of property rights.

Broadly speaking, in the model parties either invest first (ex-ante) or second (ex-post). As in the standard model, if parties invest ex-ante they do so before contracting is possible. However, these initial investments allow the project to take shape, furthering the possibility of writing contracts concerning the investments of the ex-post parties. It is immediate that investing first or second has markedly different incentives. The first implication of the model is that the possibility of investment sequencing has a positive effect on ex-post incentives, because ex-post cost-sharing reduces the hold-up problem facing the parties who invest second. Solving backwards, this anticipated change in ex-post investment also affects ex-ante investors. A priori, ex-ante investments can increase or decrease as a result of sequencing, depending on whether the disincentive to invest from the greater marginal costs imposed on the ex-ante investors (due to ex-post cost sharing contracts) is outweighed by the greater incentive to invest that arises due to higher second period (ex-post) investment.

Altering the incentives to invest for both the first and second investors, can clearly have an impact on the optimal allocation of ownership (or property rights). In the first instance a natural consequence of investment sequencing is that ex-ante agents rely more heavily on property rights to protect their investment returns from hold-up than do ex-post agents who are protected because of cost sharing contracts.

Other predictions also arise, as an example when all assets are complementary at the margin then the simultaneous investment model of Bel (2005) finds that a more inclusive control structure always raises equilibrium investment – greater access rights increase a party's incentives to invest without diminishing any other party's incentives to invest. This is not always the case when investments are sequential because cost-sharing contracts and interdependence between first and second investors mean that if more control is given only to ex-post agents then it is possible that ex-ante incentives decrease. In some situations it may even be optimal to reduce the set of assets controlled by ex-post agents despite the fact that all assets are complementary. The reason is that, due to ex-post contracting, ex-ante agents pay for a share of the ex-post agents' investment costs. Taking away control decreases ex-post incentives which can decrease ex-ante marginal costs increasing ex-ante investment incentives.

Investment sequencing also has implications for the optimal allocation of property rights when assets are substitutes at the margin. The basic intuition is that in certain situations it may be optimal for ex-post agents to own more than one substitute asset (contrary to Bel's (2005) predictions) so as to decrease their incentives which, in turn, helps encourage the incentive to invest for the ex-ante investors.

The following example illustrates some of the alternate assumptions of the model.

Example A group of scientists and a large manufacturer of video game consoles are collaborating to develop and bring to market a new graphics processor unit that is to be included in the next generation of consoles. The two tasks (development of a new graphics processor by the scientists and the establishment of the production process by the manufacturer) could, under some circumstances be completed at the same time. This would be the assumption underlying the simultaneous investment (the standard) model. In this case, each party makes their relationship specific investment (which is sunk once made) prior to contracting being possible. Once both investments have been made, the project becomes tangible and the parties can re-negotiate and determine the distribution of surplus derived from trade. During re-negotiation the party's claim on surplus depends, in a large part, on their ownership of project-critical assets.

Of course, rather than investing in the two tasks at the same time, it could be the case that the scientific investment must be made first - this situation could arises when it is not possible to start establishing a manufacturing process before the exact nature of the graphics processor is known. Here we have sequential investment, as introduced to the model in this thesis. The investment process could proceed as follows. The scientists invest in developing the know-how and technologies required for the new graphics processor unit. None of these investments could have been adequately described in a contract ex-ante, but as the research proceeds the exact nature of the processor, its specifications and its manufacturing requirements become known and verifiable. It is the scientists' research that makes this possible. They are now in an environment in which contracting is (at least partly) possible, so that the manufacturer can write cost sharing arrangements prior to sinking his investments. The manufacture's investments complete the project and both parties are in a position to re-negotiate over the distribution of the gains from trade.

Making investments sequential changes the incentives to make relationship specific investments of both the scientists and the manufacturer. It is the aim of the following chapters to explore these changes and to illuminate their interesting consequences for the optimal allocation of property rights.

The thesis proceeds as follows: Chapter 2 reviews the literature and Chapter 3 introduces the model and analyses investment incentives when sequencing is introduced. Chapter 4 presents the main results and Chapter 5 concludes. The proofs are presented in Appendix A.

Chapter 2

Literature Review

Introduction

The neoclassical view of the firm has dominated mainstream economic thought until relatively recently. A firm is characterised by a given production technology and a profit-maximising entrepreneur who rents resources such as labour and capital and allocates them to their most efficient uses. The firm is thus a metaphorical *black box* because no attention is paid to its internal structure or organisation. It suffices to derive the firm's average cost curves in order to determine its cost minimising level of output and thus its optimal *plant size*.

In his famous essay, Coase (1937) argues that this view of the firm is very limited because optimal plant size does not explain optimal *firm size*; that is to say, the neoclassical theory is silent about who should own a firm, how many plants one firm should own or why there should be firm ownership in the first place. The neoclassical view does not explain why there are many firms in an economy instead of one all encompassing conglomerate and why there are many different ownership structures such as partnerships and large corporations.

Coase (1937) proposes that firms exists to minimise *transaction costs*. His fundamental insight is that the market and the firm are two inherently different governance structures and that transaction costs differ under the two structures. He argues that when market transaction costs are high then the firm as an alternative becomes efficient. However, increasing the number of transactions that are carried out inside the firm leads to decreasing efficiency of the entrepreneur (or owner-manager). According to Coase, a firm grows in size when the number of transactions it carries out increases. Thus, *efficient* firm size is reached when the cost of an additional transaction is less outside the firm than inside.

Property-rights theory

The there are two key problems with Coase's (1937) and other transaction cost based theories[1] that the property-rights view of the firm seeks to address.[2] According to Grossman & Hart (1986) the first deficiency is that in many situations transaction-cost theory fails to pin down exactly the boundaries of the firm because it does not explain exactly what kind of contract brings a transaction inside a firm (and under the control of the entrepreneur) and what kind of contract keeps it in the market. The problem is particularly acute when dealing with labour contracts. For example, transaction-cost theory does not differentiating between independent contractors and employees when the contractors agree to place their human capital under the direct supervision of the entrepreneur for some period of time. In response to this inherent difficulty, Grossman & Hart (1986) argue that it is more practical to define the firm in terms of the set of productive *assets* (or physical capital) that are owned together by one entrepreneur. This shift the view of the firm away from the number of transactions that are controlled by the entrepreneur to the number of assets that she owns and controls. From this perspective the difference between employees and independent contractors is that employees work with the assets provided by the entrepreneur whereas the independent contractors work with their own assets.[3] When the firm is

[1]See in particular Williamson (1975), Klein, Crawford & Alchian (1978) and Williamson (1985).

[2]See Grossman & Hart (1986), Hart & Moore (1990) and Hart (1995).

[3]The independent contractors are thus also entrepreneurs in his own right. Slight difficulty arises when the contractors use their own assets as well as those provided by the entrepreneur. However, such a situation could then be regarded as a joint venture or some kind of co-operation agreement.

defined in terms of asset ownership, it is possible to address the second key shortcoming of transaction cost based theories.

Modern transaction-cost theory (see for example Williamson (1985)) centres around investment inefficiencies that arise when trade requires relationship specific investments and contracting through the market is incomplete or altogether impossible. This situation has become known as the hold-up problem, the essence of which is that when contracts are incomplete and all parties anticipate re-negotiation after investments have been sunk then many agents will invest inefficiently if they expect not to recover the full marginal benefit of their investment return during re-negotiation (see Grout (1984)). Transaction-cost theorists argue that when the hold-up problem becomes too severe then bringing together all transacting parties within the same firm aligns their incentives so that the hold-up problem can be overcome. However, as pointed out by Grossman & Hart (1986), transaction cost theory fails to explain exactly *why* incentives change when a transaction is brought inside the firm. They argue that during re-negotiation the bargaining power of the parties (or agents) derives from two distinct sources. If the value of a transaction depends critically on one agent contributing her human capital then that agent can threaten to withhold her labour in order to extort a higher share of the surplus. This source of bargaining power can not be influenced through integration because a hostile supplier with critical human capital who negotiates a high supply price for his input will negotiate an equally high salary if he is to be hired by the entrepreneur.[4]

However, many transactions do not just depend on human capital but also require physical assets to generate surplus. In such a situation the owner of the necessary physical capital has bargaining power because she can threaten to withhold the assets and use them outside of the relationship instead. Naturally, the higher the value of the assets inside the relationship the greater is the bargaining power of the owner. Since most physical assets can be traded freely in the market, integration (of assets) does have a significant impact on

[4]In the absence of slavery human capital can never be sold in a market, it can at best be rented for certain periods of time and within narrowly defined contractual boundaries (job descriptions).

the nature of the hold-up problem.

The key findings of the property rights theory – Grossman & Hart (1986) and Hart & Moore (1990) – can be summarised as follows; as the contracting environment becomes less well defined, the *residual rights of control*[5] over critical assets become more important in determining the bargaining power of the agents. Under these conditions firm boundaries (or asset ownership) are driven to optimise the investment incentives of agents who make relationship specific investments by protecting them from opportunism when re-negotiation occurs.

Access, veto and asset relationships

Hart & Moore's (1990) model is frequently criticised for its broad and unrefined use of the term *ownership*. For example, Demsetz (1996) argues that residual control rights is an ambiguous concept and that assets have multiple attributes, each of which can have different owners. Similarly, Rajan & Zingales (1998) contend that a guarantee to freely access and use an asset can be just as effective and sometimes more effective than the allocation of ownership in overcoming the hold-up problem. Other authors pointed out that the original property rights model is at best a theory of the entrepreneurial firm but that it does not fit the picture of large modern-day corporations where ownership by shareholders is often separated from the day to day control of managers (Bolton & Scharfstein 1998).[6]

It follows that a more nuanced definition of property rights is needed to give a clearer understanding of real-world firm structures. Bel's (2005) model unbundles ownership into the right to *access* an asset and the right to *veto* access to an asset.[7] Using this refined notion of property rights, he is not only able to describe modern-day corporations with outside shareholders

[5]The owner of an asset controls all aspects of its use. He may chose to assign control rights over certain aspects of the asset's use to other parties but always retains residual control over all aspects of the asset's use which are not subject to any contracts.

[6]Bolton & Scharfstein (1998) also argue that the property-rights model ignores agency problems because owners are also managers of the firm.

[7]Here Bel (2005) uses the properties of ownership – access, withdrawal, management exclusion and alienation – as defined in Schlager & Ostrom (1992) and groups them into access and veto.

and hybrid governance structures such as joint ventures, but he also gives conditions under which such institutional arrangements can be optimal. This is a radical departure from the original model by Hart & Moore (1990) where certain types of joint ownership are explicitly ruled out and outside veto is disallowed.[8]

By relaxing the assumption that all assets are complementary at the margin, Bel (2005) introduces another important innovation into the property-rights framework. The motivation here is that if the number of assets controlled by an agent increases then her human capital (for example, her management skills) is spread more thinly across the larger set of assets, which can reduce her marginal return to investment. This is essentially Coase's (1937) idea of diminishing returns to management applied to the property-rights paradigm.

The possibility to unbundle access and veto rights and to transfer them separately has far reaching consequences for the optimal allocation of property rights because the transfer of ownership alone (i.e., when access and veto are transferred together) is often too restrictive to solve complex incentive problems. For example, when all assets are complements Bel (2005) finds that it is optimal to give all agents access to all assets while veto powers should be given to none. Thus, there should be a kind of communal access to common resources, which ensures that the hold-up problem is minimised because nobody can threaten to withhold assets. Conversely, if all assets are substitutes at the margin then it is sometimes better for incentives to allocate more veto powers without allocating more access rights. For example consider a situation where an entrepreneur owns two substitute (at the margin) firms. The entrepreneur would have higher investment incentives if he could transfer ownership of the firm to someone else but it might be hard to find a buyer because all other entrepreneurs already own their own firms and do not wish to take control of a substitute asset. The solution is to give someone (outside) veto power over one of the firms without giving that person access. In this way the entrepreneur's return to investment increases because he no longer controls a substitute asset and can no longer bring it

[8] Propositions 3 and 4.

with him when he trades, but at the same time the incentives of the person exercising veto are unchanged.

The preceding discussion highlights that the transfer of ownership always involves a trade-off between the effects of access and veto on investment incentives. Thus, if access and veto can be transferred separately then the set of institutional arrangements to overcome incentives problems expands significantly.

Timing of investment

A related line of enquiry that draws on the insights developed in the incomplete-contracts literature and focusses on the timing of investments as an institutional arrangement to overcome the hold-up problem when other solutions – such as transfer of ownership – are unavailable (see Smirnov & Wait (2004*a*) and Smirnov & Wait (2004*b*)).

When the order of investment is given exogenously Smirnov & Wait (2004*b*) find that sequential investment improves the incentives of the second agent. The reason for this effect is that the second agent invests when contracting is possible and is thus not affected by the hold-up problem. However, the incentives of the first agent are reduced because the sequential investment regime delays the collection of ex-post returns. For the first agent's incentives to be adversely affected a discount factor has to be present in the model. If future payments are not discounted then the first agent's incentives are unaffected but the second agent's incentives still increase.[9]

The key insight of the literature on investment timing and hold-up is that investment incentives can change as a result of factors other than the re-allocation of property rights, for example by changing model parameters such as the time frame of investments. The literature also suggests that the impact of sequencing has an ambiguous effect on incentives, an insight which is explored further in this thesis.

[9] See Effects 2 and 3.

Sequential investment and property rights

The review of the property-rights literature shows that a proper assignment of access and veto rights can help to alleviate incentive problems when assets are complements or substitutes at the margin. At the same time, it seems clear that many economic situations require sequential investment. This means that some agents are more exposed to hold up than others and rely more heavily on property rights to protect their relationship-specific investments. Under certain conditions sequencing also introduces a Stackelberg type interaction between the investor groups which leads to additional affects on incentives.

In order to deepen our understanding of the property-rights framework and to better understand the evolution of firm boundaries, it is important to test the model with new real world modifications. The introduction of investment sequencing is a highly relevant addition to the model because many economic relationships (such as research and development) require staged investments. However, the findings of this thesis not only make an important contribution to the property rights literature but also help to confirm some of the findings of the literature on investment timing and hold-up.

Chapter 3

Model

This chapter draws on the ideas developed in the introduction and the literature review and formalises a theoretical model capable of analysing the the impact of investment sequencing on the optimal allocation of property rights when trade requires ex-ante investments and contracts are incomplete. Section 3.1 introduces the concepts of agents and assets and gives a timeline of the model. Section 3.2 formalises the concept of property rights and section 3.3 defines the value and cost functions and introduces some assumptions. First-best investment is analysed in section 3.4 and section 3.5 introduces the bargaining games. Section 3.6 introduces the maximisation problems of the different types of agents and compares investment incentives under sequential and simultaneous investment.

3.1 Agents, assets and timing of investments

This section introduces the basic structure and the timeline of the model. The economy is populated with a finite set of n agents. The grand coalition is denoted by N and can be divided into two mutually exclusive but collectively exhaustive subsets N_{ea} and N_{ep} ($N_{ea} \cap N_{ep} = \emptyset$ and $N_{ea} \cup N_{ep} = N$), such that all agents who invest ex-ante (see discussion below) are members of N_{ea} and all agents who invest ex-post are members of N_{ep}. There are I ex-ante agents and J ex-post agents. The set of productive assets $\underline{A}$ contains a finite number of m assets.

Simultaneous investment

Traditional two-period property rights-models – this section replicates the model used by Bel (2005) – feature simultaneous investment by all agents. By convention the first period $t = 1$ is referred to as *ex-ante* and the second period $t = 2$ is referred to as *ex-post*. Before investments are made the allocation of property rights is finalised. To ensure the most efficient allocation of access and veto rights it is assumed that the agents have no wealth restrictions and that they can use side payments to trade property rights. The ex-ante period – at $t = 1$ – represents the investment stage of the projects before trade can occur and by assumption all agents invest in this period (i.e., $N_{ea} = N$ and $N_{ep} = \emptyset$). At the end of the ex-ante period all relationship-specific investments have been made, so that all projects (or trade relationship) become tangible. Income (or surplus) is generated during the ex-post periods when production and trade occurs.

Ex-ante contracting is incomplete due to the intangible nature of the projects, which means that at $t = 1$, agents can not write contracts to govern their investment costs. Contracting becomes possible at the beginning of the ex-post period and it is then that bargaining occurs to divide up the surplus from production and trade. In a situation where ex-ante contracting is impossible, the agents derive their bargaining power from the set of assets that they control and from their inalienable human capital. The structure of Bel's (2005) model is summarised in Figure 3.1.

Sequential investment

There are two differences between Bel's (2005) simultaneous-investment model and the sequential-investment model presented here. (i) Perhaps not surprisingly, the set of agents who invest ex-post is no longer empty (i.e., $N_{ep} \neq \emptyset$) and (ii) ex-post agents invest after the projects become tangible, which means that they can contract on their investment costs. It is assumed that the sets N_{ep} and N_{ea} are determined outside of the model. Such a situation could arise from the nature of the projects; in some instances it might be necessary that one set of investments precedes another. The sequential investment model is summarised in Figure 3.2.

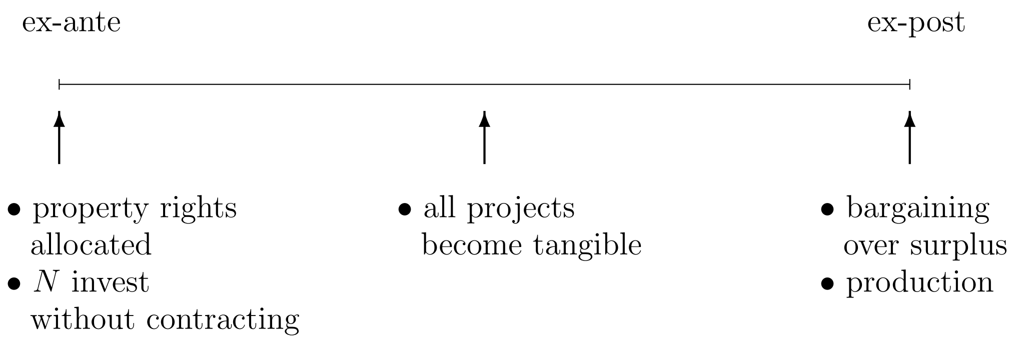

Figure 3.1: Simultaneous investment

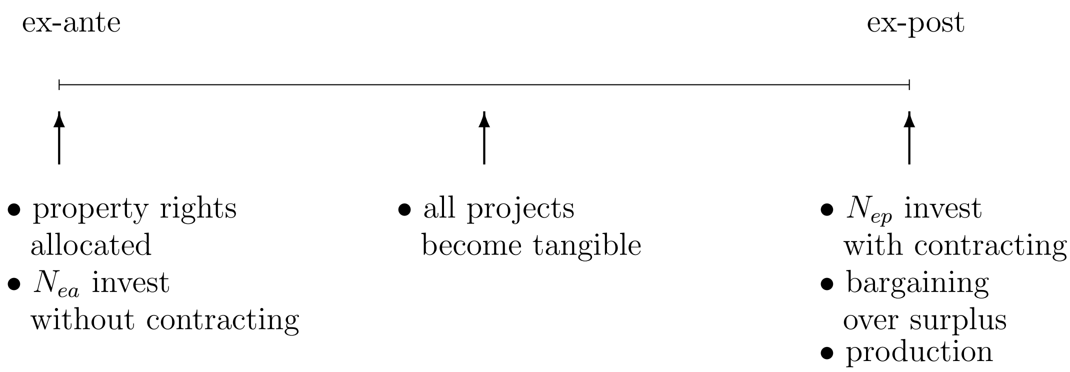

Figure 3.2: Sequential investment

3.2 Property rights

The model follows Bel (2005) in assuming that asset ownership can be unbundled into the right to access an asset and the right to veto access to an asset. This section formalises the definitions of access, veto and control structures and introduces the concept of single, multiple and joint property rights. The access structure of the economy is defined as follows:

Definition 1. Let the mapping υ from the set of subsets of N to the set of subsets of $\underline{A}$ be defined as the *access* structure of the economy. The mapping υ satisfies:

$$\upsilon(S') \subseteq \upsilon(S) \quad \forall\, S' \subseteq S \quad \text{and } \upsilon(N) = \underline{A} \tag{3.1}$$

Equation (3.1) says that if a sub-coalition accesses an asset then the full coalition must also access the asset and the grand coalition can access all assets. The structure of veto rights in the economy is defined in a similar way:

Definition 2. Let the mapping χ from the set of subsets of N to the set of subsets of $\underline{A}$ be defined as the *veto* structure of the economy. The mapping χ satisfies:

$$\chi(S') \subseteq \chi(S) \quad \forall\, S' \subseteq S \quad \text{and } \chi(N) = \underline{A} \tag{3.2}$$

Equation (3.2) says that if a sub-coalition vetoes an asset then the full coalition also vetoes the asset and the grand coalition always vetoes all assets. A coalition of agents S is said to control an asset a if and only if no coalition outside of S has veto over a. The control structure of the economy is important in determining the investment incentives of the agents because a coalition can only put an asset to productive use (and derive surplus from it) if it controls the asset. Formally:

Definition 3. A *control* structure is a mapping β from the set of subsets of N to the set of subsets of $\underline{A}$, such that $\beta(S) = \upsilon(S) \backslash \chi(N \backslash S)$. The control structure satisfies:

$$\beta(S') \subseteq \beta(S) \quad \forall\, S' \subseteq S \quad \text{and } \beta(N) = \underline{A} \tag{3.3}$$

Types of property rights

There exist different types of access, veto and control rights. For example, the access rights over an asset a_i or group of assets A can be allocated as follows:

Single access occurs when only a single agent can access an asset which implies that only coalitions that include the agent can access the asset ($\upsilon(i) = a_i, \upsilon(j) \neq a_i, \; \forall\, j \neq i \in N$).

Multiple access occurs when several agents can access an asset independent of each other and any coalition that includes at least one of the agents can access the asset ($\upsilon(i) = a_i$ for some $i \in N$).

Joint access occurs when several agents can access an asset only together and so only coalitions that include all of the agents can access the asset ($\upsilon(i) = \upsilon(j) = \emptyset, \upsilon(i,j) = a_i$ and $a_i \in \upsilon(S) \iff i, j \in S$).

Veto rights and control rights follow the same pattern.

3.3 Value and cost functions

Value function

The agents of a coalition use the human and physical capital they control to engage in economic production in order to generate income (or surplus). While the specific internal arrangements of a coalition can be complex, the model defines a value function that gives the maximum surplus achievable by a particular coalition, conditional upon the assumption that all human and physical assets are allocated to their most efficient uses. The value function is thus defined as follows:

Definition 4. Let $v(S, A \mid x)$ be the value function of a coalition $S \subseteq N$ in control of the subset of assets $A \subseteq \underline{A}$, where x is the vector of investments by all agents.[1]

The (maximum) value generated by a coalition S is sensitive to the relationships (i) between the agents in S, (ii) between the assets controlled by S and (iii) between the human-capital investments of the agents in S.[2] The model imposes a number of assumptions on these relationships:

[1]Note that $\beta(S) = A$.

[2]The same holds for the marginal return to investment $v^i(S, A \mid x)$.

Assumption 1 (Superadditivity of value function)**.** Agents and assets are always complementary;
i.e., $v(S, A \mid x) \geq v(S', A' \mid x) + v(S' \backslash S, A' \backslash A \mid x), \quad \forall\ S' \subseteq S$ and $A' \subseteq A$.

The assumption that all assets are complementary is easily explained because it is hard to imagine a situation where the total income generated by a coalition decreases because it controls additional productive assets. The reason is that, if using these additional assets really decreased overall surplus then the firm could simply chose to ignore the new assets and produce only with those assets it had available originally. However, the superadditivity specification is not about assets alone. It also imposes gains from trade onto the economy. It says that adding more agents to a coalition is always weakly beneficial, which implies that the grand coalition will always produce the largest surplus.

Differentiating the value function with respect to human-capital investment gives the marginal return to investment of agent i in coalition S for a given vector of investments x.

$$\frac{\partial v(S, A \mid x)}{\partial x_i} \equiv v^i(S, A \mid x) \tag{3.4}$$

Several assumptions are imposed on the marginal-return function. Assumption 2, says that the addition of an agent to a coalition never decreases the marginal return to investment of coalition members. If the assumption holds, then the grand coalition necessarily maximises the return to investment of all agents in the economy for a given level of human capital investment. While seemingly a strong assumption it is not very restrictive because the focus of the thesis lies on asset relationships and investment timing:

Assumption 2. Agents are always complementary at the margin;
i.e., $v^i(S', A) \leq v^i(S, A) \leq v^i(N, \underline{A}), \quad \forall\ i \in N,\ \forall\ A \subseteq \underline{A},\ \forall\ S' \subseteq S \subseteq N$.

Assumption 3 specifies that there are diminishing returns to investment. This is a realistic but largely technical assumption that together with the functional form of the cost function guarantees a unique profit-maximising equilibrium. However, concavity also makes intuitive sense. For a given set of assets that an agent can work with and for a given set of co-workers, increasing investment in human capital will always increase productivity but because the other variables are fixed productivity will increase at a decreasing rate:

Assumption 3. The value function $v(S, \beta(S) \mid x)$ is strictly increasing in x_i and strictly convex;
i.e., $v^i(S, A \mid x) > 0$ and $v^{ii}(S, A \mid x) < 0$.

The following two assumptions specify the restrictions that are placed on the marginal relationships of human-capital investments. Assumption 4 is a simplifying assumption, used to emphasise that investments are not only relationship specific but also agent specific. In other words, the term *human capital* highlights the fact that the the investments are useless without the agent who made the investment.

Assumption 4. All investments are in human capital;
i.e., $v^i(S, A) = 0$ when $i \notin S$.

Assumption 5 is critical for many results of this thesis. It formalises the idea that the returns to human capital investments by different agents of a coalition are interdependent. The assumption suggests that agents who work with highly productive co-workers also tend to be more productive and therefore have greater incentives to invest in their own human capital. This assumption tends to compound the effect of productivity changes on investment incentives that directly affect only a few agents in the economy because these effects do not remain isolated. Some authors have placed restrictions on investment complementarity by assuming that $v^{ij}(S, A \mid x) = 0$ (see Hart & Moore (1988)). This restricts the effect of a change in investment on marginal return to the agent making the investment. This however, abstracts from the real-world dynamics of economic systems where shocks are rarely isolated to one coalition or subsection of the economy.

Assumption 5. Investments are strict strategic complements at the margin;
i.e., $v^{ij}(S, A \mid x) > 0, \ \forall \ j \neq i \in S$.

Cost function

In this economy the opportunity cost of making human capital investment $C_i(x_i)$ has the same unit of measurement as the value function. The model assumes that the cost function $C_i(x_i)$ is twice differentiable as well as strictly increasing and convex in x_i. Thus, the marginal cost of investment is increasing with the level of investment. This is summarised below.

Assumption 6. The cost function $C_i(x_i)$ is increasing in x_i and concave; i.e., $C_i'(x_i) \geq 0$ and $C_i''(x_i) \geq 0, \ \forall \ i \in N$.

3.4 First-best investment

It follows from superadditivity of the value function (Assumption 1) that the grand coalition N generates the largest surplus for any given level of human capital investment. The superadditivity assumption says that there are always gains from trade and that it is never optimal to deny some productive agents the possibility to trade with each other. The first-best level of investment is the welfare-maximising level of investment that would be chosen by a benevolent social planner. Similar to Bel (2005, 8-9) the planner's problem is:

$$\max_{x} \quad W(x) = v(N, \underline{A} \mid x) - \sum_{i \in N} C_i(x_i) \tag{3.5}$$

It follows that the welfare-maximising level of human capital investment (the vector x^*) solves the first-order condition (FOC):

$$v^i(N, \underline{A} \mid x^*) = C_i'(x_i^*) \quad \forall \ i \in N \tag{3.6}$$

The vector x^* maximises the surplus of the economy net of the sum of investment costs and a unique maximum is guaranteed because the value function of the grand coalition is concave (Assumption 3) and the cost functions of all agents are convex (Assumption 6).

The first-best investment goals are not necessarily obtainable as the objectives of the individual agents are not necessarily aligned with those of the social planner. Whereas the social planner maximises net social surplus the agents maximise their net private benefits.

3.5 Bargaining over surplus and costs

In the standard simultaneous investment model the agents bargain over the allocation of surplus in the ex-post period[3] and it is convention to use the Shapley value as the solution concept. One reason for why the Shapley value finds widespread application in the property rights literature is that it guarantees efficient ex-post bargaining. This restriction means that the possibility of hold-up is the only source of distortions to investment incentives in the model because a potential second cause of distortions arising from problems with the bargaining process are assumed away.[4] Let B_i be agent i's share of gross surplus. Then the following equality must hold:

$$\sum_{i \in N} B_i(\beta \mid x) = v(N, \underline{A} \mid x) \tag{3.7}$$

Equation (3.7) says that the surplus allocated to the agents must sum to total surplus generated by the grand coalition. Following the convention of Hart & Moore (1990) the Shapley value B_i is defined as follows:

Definition 5. Agent i's share of gross surplus B_i is given by the Shapley value.

$$B_i(\beta \mid x) = \sum_{S \mid i \in S} p(S)[v(S, \beta(S) \mid x) - v(S \backslash \{i\}, \beta(S \backslash \{i\}) \mid x)] \tag{3.8}$$

where $p(S) = \frac{(|S|-1)!(|N|-|S|)!}{(|N|)!}$ gives the probability that the particular coalition S occurs.

The possibility of sequential investment introduces a second type of bargaining game. As with the simultaneous case, the agents still bargain over

[3]Note that all projects are tangible ex-post.

[4]See (Grossman & Hart 1986) for reasons why the literature focusses on the hold-up problem rather than ex-post bargaining inefficiencies.

the division of surplus during the ex-post period but in addition they also bargain over how to share the investment costs of ex-post agents among themselves.[5]

Analogous to the simultaneous case, the representative agent's share of gross surplus B_i is given by the Shapley value but now let λ be the set of exogenously given sharing rules that determine how the costs of ex-post investments are shared among the agents of the economy. Let λ_{il} denote the proportion of ex-post agent i's investment cost paid by agent l and let the set of all sharing rules in the economy be defined as $\lambda = \{\lambda_{il} : \forall\ i \in N_{ep} \text{ and } \forall\ l \in N\}$. A sharing rules must be non-negative and less than or equal to one (i.e., $\lambda_{il} \in [0, 1]$) and also satisfy the condition $\sum_{l \in N} \lambda_{il} = 1$. Then the following equality holds:

$$\sum_{l \in N} B_l(\beta \mid x) - \sum_{i \in N_{ep}} \sum_{l \in N} \lambda_{il} C_i(x_i) = v(N, \underline{A} \mid x) - \sum_{i \in N_{ep}} Ci(x_i) \qquad (3.9)$$

Equation (3.9) gives the gross surplus net of all ex-post investment costs. The set of sharing rules is given exogenously and can be thought of as the closed-form solution to some cost-sharing bargaining game that is not explicitly included in the model.[6] The cost-sharing rule could be thought to arise from the relative bargaining strengths of the parties, relating for example to an extensive-form bargaining game such as Rubinstein (1982) or Binmore, Rubinstein & Wolinsky (1986).

Making the sharing rule exogenous could be troublesome if the results were sensitive to the assumption that the sharing rule does not depend on the allocation of property right. However, this is not the case. The results are also not sensitive to any particular sharing rule being used or any relationship between the sharing rule and the Shapley value. The results derive only from the fact that ex-post investment costs *can* be shared but it does not matter *how* they are shared.

Using the assumption that there are diminishing returns to investment,

[5]Recall that the agents in N_{ea} invest ex-ante and that the agents in N_{ep} invest ex-post and that the ex-post agents contract on their investment costs.

[6]In principle the solution concept of this game could also be the Shapley value.

that investments are complementary at the margin and that all investments are in human capital it is possible to characterise the behaviour of marginal return to investment. From Definition 5 and Assumption 4 it follows that agent i's marginal return to investment is given by:

$$\frac{\partial B_i(\beta \mid x)}{\partial x_i} = \sum_{S \mid i \in S} p(S) v^i(S, \beta(S) \mid x) \tag{3.10}$$

It is now convenient to define two matrices $\boldsymbol{W}_{ea}$ and $\boldsymbol{W}_{ep}$ that collect the second partial derivatives of the Shapley value for all ex-ante agents and ex-post agents:

$$\boldsymbol{W}_{ea} = \begin{bmatrix} \frac{\partial^2 B_1}{\partial x_{ea,1}^2} & \cdots & \frac{\partial^2 B_1}{\partial x_{ea,1} \partial x_{ea,I}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 B_I}{\partial x_{ea,I} \partial x_{ea,1}} & \cdots & \frac{\partial^2 B_I}{\partial x_{ea,I}^2} \end{bmatrix} \quad \boldsymbol{W}_{ep} = \begin{bmatrix} \frac{\partial^2 B_1}{\partial x_{ep,1}^2} & \cdots & \frac{\partial^2 B_1}{\partial x_{ep,1} \partial x_{ep,I}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 B_I}{\partial x_{ep,I} \partial x_{ep,1}} & \cdots & \frac{\partial^2 B_I}{\partial x_{ep,I}^2} \end{bmatrix}$$

The elements on the main diagonal of $\boldsymbol{W}_{ea}$ and $\boldsymbol{W}_{ep}$ are all negative because there are diminishing returns to investment (Assumption 3 and Definition 5 together imply that $\frac{\partial^2 B_l}{\partial x_l^2} < 0 \ \forall \ l \in N$) and all elements off the main diagonal are all positive because investments are assumed to be strict strategic complements (Assumption 5 and Definition 5 together imply that $\frac{\partial^2 B_l}{\partial x_l \partial x_k} > 0 \ \forall \ l \neq k \in N$). In addition, the following assumption is made:

Assumption 7. The matrices $\boldsymbol{W}_{ea}$ and $\boldsymbol{W}_{ep}$ are negative definite.

The matrices $\boldsymbol{W}_{ea}$ and $\boldsymbol{W}_{ep}$ contain the second partial derivatives of the Shapley value for either ex-ante or ex-post agents. Assuming that they are negative definite can be justified intuitively. It means that a change in an agent's own investment x_i has a relatively greater absolute impact on his marginal return than combined changes to other agents' investments $x_{l,l \neq i}$. In other words, Assumption 7 says that an agent's marginal return is more sensitive to changes in his own investment that to changes in other agents' investments.

3.6 Maximisation problem

Simultaneous investment

The agents in the economy are self interested and seek to maximise their ex-post surplus. Under *simultaneous* investment the first-order conditions of the agents' maximisation problems are solved by the equilibrium investment vector x^e. The first-order conditions are given below:

$$\frac{\partial B_i(\beta \mid x^e)}{\partial x_i} = \frac{\partial C_i(x_i^e)}{\partial x_i} \quad \forall\, i \in N \tag{3.11}$$

Condition (3.11) states that in equilibrium, profit-maximising agents set marginal return to investment equal to marginal cost of investment. It is the standard maximisation condition of the property-rights literature and all results derive from it. Note that $\frac{\partial B_i(\beta|x^e)}{\partial x_i} \equiv \sum_{S|i\in S} p(S)v^i(S, \beta(S) \mid x^e)$ due to Definition 5 and Assumption 4. Reallocation of property rights alters the control structure β and therefore changes marginal return, which will affect each party's level of equilibrium investment. The following sections however, are not concerned with the allocation of property rights but rather take these as given and discusses the consequences for incentives when sequential investment is introduced into the economy.

Ex-post investments

The introduction of investment *sequencing* changes the maximisation problem of the ex-post agents because ex-post contracting on investments is possible, reducing their exposure to the hold-up problem by spreading the costs of their investments. Let ex-ante and ex-post investments be such that $x_{ea} \cap x_{ep} = \emptyset$ and $x_{ea} \cup x_{ep} = x$, then (3.12) gives the ex-post agents' maximisation problem:

$$\max_{x_i} \quad B_i(\beta \mid x_{ea}, x_{ep}) - \sum_{k\neq i\in N_{ep}} \lambda_{ki} C_k(x_{ep,k}) - \lambda_{ii} C_i(x_{ep,i}) \tag{3.12}$$

where the elements of x_{ea} are treated as exogenous. The first term from the left gives agent i's share of gross surplus and the second term gives i's share of other ex-post agents' investment costs. The term on the right gives i's own investment cost and because contracting is allowed, if $\lambda_{ii} < 1$ then i does not pay the full cost of investment. The vector x^e_{ep} of ex-post equilibrium investments then solves the first-order conditions given below:

$$\frac{\partial B_i(\beta \mid x_{ea}, x^e_{ep})}{\partial x_{ep,i}} = \lambda_{ii} \frac{\partial C_i(x^e_{ep,i})}{\partial x_{ep,i}} \quad \forall\ i \in N_{ep} \tag{3.13}$$

where $\frac{\partial B_i(\beta|x_{ea}, x^e_{ep})}{\partial x_{ep,i}} \equiv \sum_{S|i \in S} p(S) v^i(S, \beta(S) \mid x_{ea}, x^e_{ep})$ (using Definition 5 and Assumption 4). From condition (3.13) it follows that ex-post equilibrium investment is implied by the exogenous variables, namely the control structure of the economy β, the set of sharing rules λ and to the level of ex-ante investment x_{ea}. This relationship is formalised in Definition 6:

Definition 6. Let the vector of equilibrium investments be governed by the implicit function $x^e_{ep} = R(\beta, \lambda | x_{ea})$ such that $x^e_{ep,i} = R_i(\beta, \lambda | x_{ea})\ \forall\ i \in N_{ep}$.

The first point of interest is to characterise the behaviour of equilibrium ex-post investment when either ex-ante investment changes or when the set of sharing rules changes. Lemma 1 shows how ex-post investment responds to changes in ex-ante investment:

Lemma 1. *For a given control structure and set of sharing rules, if any ex-ante agent increases investment then – ceteris paribus – all ex-post agents increase equilibrium investment. I.e,* $\frac{\partial x_{ep,i}}{\partial x_{ea,j}}|_{x_{ep}=x^e_{ep}} > 0\ \forall\ i \in N_{ep}$ *and* $\forall\ j \in N_{ea}$ *or equivalently* $\frac{\partial R_i(\beta,\lambda|x_{ea})}{\partial x_{ea,j}} > 0\ \forall\ i \in N_{ep}$ *and* $\forall\ j \in N_{ea}$.

Proof: See Appendix A

The intuition of Lemma 1 is straight forward and depends on the assumption that all investments are strictly complementary. It says that if an ex-ante agent increases her level of investment then the marginal productivity of all ex-post agents also increases because the ex-ante agent will be a more productive co-worker. Ceteris paribus; the increase in marginal productivity of the ex-post agents raises their investment incentives and leads to higher ex-post equilibrium investment.

Consider next the response of ex-post investment to changes in the set of sharing rules:

Lemma 2. *For a given control structure and vector of ex-ante investments, if λ_{ii} decreases then – ceteris paribus – all ex-post agents decrease equilibrium investment; i.e.,* $\frac{\partial x_{ep,i}}{\partial \lambda_{ll}}|_{x_{ep}=x^e_{ep}} \leq 0 \ \forall \ i,l \in N_{ep}$ *or equivalently* $\frac{\partial R_i(\beta,\lambda|x_{ea})}{\partial \lambda_{ll}} \leq 0 \ \forall \ i,l \in N_{ep}$.

Proof: See Appendix A

The intuition behind Lemma 2 is that the investment incentives of ex-post agents depend – among other things – on their marginal costs. Agent i's marginal costs depend on his sharing rule λ_{ii}, where a value closer to one means that the agent has higher marginal costs. Thus, a small increase in his sharing rule decreases his investment incentives and – ceteris paribus – causes him to invest less in equilibrium. However, Lemma 2 says more; because investments are complementary the lower equilibrium investment by i also lowers the marginal productivity (and return) of all other ex-post agents because he is a less productive co-worker. Thus, decreasing equilibrium investment of all other ex-post agents. Of course, this effect is reversed if the sharing rule of the ex-post agent decreases rather than increases.

Ex-ante investments

Consider next the maximisation problem of the ex-ante agents. In contrast to the ex-post group, the ex-ante agents can not contract on their investment costs and are thus more exposed to the hold-up problem. Also, the fact that ex-ante agents sink their investments before ex-post agents make their investment decisions gives rise to a Stackelberg type strategic interaction if investments are assumed to be strategic complements.

Recall that ex-post equilibrium investment is implied by the structural parameters of the economy (such as the control structure and the set of sharing rules) and the level of ex-ante investment. Ex-ante investors make use of this fact by incorporating the (implicit) ex-post reaction functions $R = \{R_i : \forall\, i \in N_{ep}\}$[7] into their maximisation problems. Thus, internalising the impact on ex-post equilibrium investment due to changes in ex-ante investment. A representative ex-ante agent $j \in N_{ea}$ solves:

$$\max_{x_j} \quad B_j(\beta \mid x_{ea}, x^e_{ep}) - \sum_{i \in N_{ep}} \lambda_{ij} C_i(x^e_{ep,i}) - C_j(x_{ea,j}) \tag{3.14a}$$

Replacing x^e_{ep} with R:

$$\max_{x_j} \quad B_j(\beta \mid x_{ea}, R) - \sum_{i \in N_{ep}} \lambda_{ij} C_i(R_i) - C_j(x_{ea,j}) \tag{3.14b}$$

Similar to the ex-post maximisation problem, the first term on the left gives j's share of gross surplus. Notice that the ex-post equilibrium investments are endogenous, which is signified by the optimal response function R which depends on x_{ea} (see Definition 6). The term on the far right gives the cost of investing $x_{ea,j}$ in human capital. It is missing a sharing rule because ex-ante agents can not contract on their investment costs. The summation in the middle specifies how much of the ex-post investment costs are paid by the ex-ante agent j (a consequence of the fact that ex-post agents can contract on their investments).

Remark 1. Ex-post contracting means that ex-ante agents pay for a share of ex-post investment costs (i.e., the term $\sum_{i \in N_{ep}} \lambda_{ij} C_i(x^e_{ep,i})$ appears in the ex-ante maximisation problem (3.14a)) and therefore it is possible that the set of sharing rules is such that net surplus is negative for some ex-ante agents. If that is the case then those agents do not invest. While such an outcome is possible, this thesis assumes that $B_j(\beta \mid x_{ea}, x^e_{ep}) - \sum_{i \in N_{ep}} \lambda_{ij} C_i(x^e_{ep,i}) - C_j(x_{ea,j}) \geq 0 \;\forall\, j \in N_{ea}$, thus restricting subsequent analysis on the marginal impacts of investment sequencing.

Subject to Remark 1, the vector of ex-ante equilibrium investments x^e_{ea}

[7] See Definition 6 for details.

solves the first-order conditions:

$$\frac{\partial B_j(\beta \mid x^e_{ea}, R)}{\partial x_{ea,j}} + \sum_{i \in N_{ep}} \left[\frac{\partial B_j(\beta \mid x^e_{ea}, R)}{\partial R_i} - \lambda_{ij} \frac{\partial C_i(R_i)}{\partial R_i} \right] \frac{\partial R_i}{\partial x_{ea,j}} = \frac{\partial C_j(x^e_{ea,j})}{\partial x_{ea,j}} \tag{3.15}$$

$\forall\ j \in N_{ea}$, where $\frac{\partial B_j(\beta | x^e_{ea}, R)}{\partial x_{ea,j}} \equiv \sum_{S|j \in S} p(S) v^j(S, \beta(S) \mid x^e_{ea}, R)$ (using Definition 5 and Assumption 4) is simply the marginal return to investment and $\frac{\partial C_j(x^e_{ea,j})}{\partial x_{ea,j}}$ is the standard marginal cost term. Comparing condition (3.11) to (3.15) reveals that ex-ante investment incentives change significantly due to the appearance of $\sum_{i \in N_{ep}} \left[\frac{\partial B_j(\beta | x^e_{ea}, R)}{\partial R_i} - \lambda_{ij} \frac{\partial C_i(R_i)}{\partial R_i} \right] \frac{\partial R_i}{\partial x_{ea,j}}$. This new term arises under sequential investment because ex-ante agents *internalise* the effect that their investment choices have on ex-post equilibrium investment. It is therefore convenient to name this term the *internalisation effect* and to introduce the following notation in order to simplify the discussion:

Definition 7. Let the *internalisation effect* be denoted by:

$$S_j \equiv S_j(\beta, \lambda | x_{ea}) \equiv \sum_{i \in N_{ep}} \left[\frac{\partial B_j(\beta \mid x^e_{ea}, R)}{\partial R_i} - \lambda_{ij} \frac{\partial C_i(R_i)}{\partial R_i} \right] \frac{\partial R_i}{\partial x_{ea,j}} \quad \forall\ j \in N_{ea}$$

From Definition 7 it is immediately clear that the internalisation effect impacts the ex-ante maximisation problem if and only if ex-post equilibrium investment is sensitive to changes in ex-ante investment (i.e., iff $\frac{\partial R_i}{\partial x_{ea,j}} \neq 0\ \forall\ i \in N_{ep}$ and $\forall\ j \in N_{ea}$). Given the assumptions on diminishing returns to investment, investment complementarity and human capital investment Lemma 1 showed that $\frac{\partial R_i}{\partial x_{ea,j}} > 0\ \forall\ i \in N_{ep}$ and $\forall\ j \in N_{ea}$ and therefore the internalisation effect impacts on ex-ante investment incentives.

The internalisation effect can be broken up into two parts. The term $\sum_{i \in N_{ep}} \frac{\partial B_j(\beta | x^e_{ea}, R)}{\partial R_i} \frac{\partial R_i}{\partial x_{ea,j}}$ denotes the impact of the internalisation effect on ex-ante marginal return to investment. It captures the fact that an increase in ex-ante investment raises ex-post equilibrium investment which has two opposing effect. On the one hand, higher ex-post investment makes all ex-post agents more productive increasing gross surplus, which in turn raises ex-ante marginal return to investment. But on the other hand, more productive ex-post agents demand a greater share of gross surplus, which reduces ex-ante marginal return. Thus, it is ambiguous whether the internalisation effect increases or decreases marginal return.[8]

The term $\sum_{i\in N_{ep}} \lambda_{ij} \frac{\partial C_i(R_i)}{\partial R_i} \frac{\partial R_i}{\partial x_{ea,j}}$ denotes the impact of the internalisation effect on ex-ante marginal costs. It captures the fact that an increase in ex-ante investment not only raises ex-post equilibrium investment but also ex-post investment costs. As a consequence of ex-post contracting ex-ante investors share some of the ex-post investment costs, which means that an increase in ex-ante investment leads to an increase in ex-ante costs. It follows that the internalisation effect always increases ex-ante marginal costs.

The thesis assumes that the impact of internalisation on ex-ante investment incentives is independent of the level of ex-ante investment. In other words, the change in ex-ante marginal return and costs that occurs because ex-ante agents internalise the effect of their investment choice on ex-post equilibrium investment stays constant at any level of ex-ante investment. While this is a reasonable assumption to make it also makes the analysis more tractable when changes in the control structure are examined in Chapter 4. However, relaxation of Assumption 8 could be a fruitful avenue of future research.

Assumption 8. The internalisation effect is independent of the level of ex-ante investment; i.e., $\frac{\partial S_j}{\partial x_{ea,l}} = 0 \; \forall \; j, l \in N_{ea}$.

The preceding discussion makes clear that it is not possible to determine the sign of the of the internalisation effect without imposing further assumptions on investment relationships. This makes it difficult to compare ex-ante (and ex-post) equilibrium investment under the simultaneous and sequential regimes. However, it is possible to characterise the response of ex-ante equilibrium investment to a change in the internalisation effect. Lemma 3 does exactly that:

[8]To see this more clearly; use Definition 5 to get $\sum_{i\in N_{ep}} \frac{\partial B_j(\beta|x^e_{ea},R)}{\partial R_i} \frac{\partial R_i}{\partial x_{ea,j}} = \sum_{i\in N_{ep}} \left[\sum_{S|j\in S} \left[v^i(S,\beta(S)|x_{ea},R) - v^i(S\backslash\{j\},\beta(S\backslash\{j\})|x_{ea},R)\right]\right] \frac{\partial R_i}{\partial x_{ea,j}}$. By Assumption 6 $v^i(S,\beta(S)|x_{ea},R) > 0$ and $v^i(S\backslash\{j\},\beta(S\backslash\{j\})|x_{ea},R) > 0$ which means that the sign of the internalisation effect on marginal return is ambiguous.

Lemma 3. *For a given control structure; if the internalisation effect increases for any ex-ante agent then – ceteris paribus – all ex-ante agents increase their equilibrium investments; i.e.,* $\frac{\partial x_{ea,j}}{\partial S_l}|_{x_{ea}=x_{ea}^e} > 0 \; \forall \; j, l \in N_{ea}$

Proof: See Appendix A

Lemma 3 says that if the equilibrium value of the internalisation effect of any ex-ante agent changes such that marginal return increases relative to marginal costs, then all ex-ante agents have higher investment incentives. The reason is that the incentive to invest increases for the first agent, which causes her to invest more. Her higher level of investment makes her a more productive co-worker which, in turn, increases the marginal return and equilibrium investment of all other ex-ante agents.

So far it was shown that investment incentive of ex-post agents rise if ex-ante agents invest more and if ex-post agents pay for a smaller share of their investment costs. With regards to ex-ante agents it was shown that if the internalisation effect increases then ex-ante investment incentives rise. These insights are now combined to show that ex-ante investment incentives can rise as a consequence of investment sequencing and that the response of ex-post incentives depends partially on the response of ex-ante incentives:

Proposition 1. *For a given control structure; (i) if sequential investment is introduced then the response of ex-ante equilibrium investment is ambiguous; (ii) if at least one ex-ante agent increases equilibrium investment an no ex-ante agents decrease equilibrium investment then ex-post equilibrium investment always increases; however, (iii) if some ex-ante agents decrease equilibrium investment then the response of ex-post equilibrium investment to the introduction of sequential investment is ambiguous.*

Proof: See Appendix A

Proposition 1 summarise the impact on investment incentives and equilibrium investment when sequential investment is introduced and ex-post contracting on investment costs is allowed. While the proposition does not give conditions for the *behaviour* of ex-ante investment, it does highlight the fact that ex-ante equilibrium investment *can* increase when a sequential regime is introduced.

At this stage it is interesting to compare the results from this chapter to the results from the literature on investment timing and hold-up. Consider for example Smirnov & Wait (2004*b*) who analyse a buyer-seller scenario where the seller faces the prospect of hold-up because trade requires significant relationship specific investments. They find that when the seller is allowed to invest second (ex-post) and contract on her investment costs then she is no longer held up and has a greater incentive to invest. This result is analogous to Lemma 2, which stated that if an ex-post agent can share a larger proportion of his investment costs then – ceteris paribus – his investment incentives rise. Smirnov & Wait (2004*b*) also find that the incentives of the ex-ante agent decrease as a result of sequencing. But, this is a consequence of their discount factor, which when set to one (no discounting) makes ex-ante investment incentives neutral to the introduction of sequential investment. The reason is that their Assumption 2 sets the *internalisation effect* to zero which removes the interdependence of ex-ante and ex-post investment from their model.

Chapter 4

Main Results and Discussion

The aim of the following section is to develop a deeper understanding of the optimal allocation of property rights when trade requires sequential investments and ex-ante contracting is incomplete.

A key insight of Bel (2005) is that the response of investment incentives to a change in the allocation of property rights is sensitive to the *asset relationships* in the economy. For example, if two assets are always complementary at the margin, the marginal return to investment of all agents in a coalition increases if the coalition controls both assets together instead of only one. The intuition is that certain assets impose positive externalities on each other when they are controlled together, which raises the return generated by an additional unit of human capital investment. Section 4.1 analyses the behaviour of incentives when all assets are complementary at the margin.

However, if two assets are substitutes at the margin then the marginal return to investment of all agents in a coalition decreases if it controls the assets together. Bel (2005) argues that substitution at the margin occurs frequently because if two assets are controlled together then the human capital of the agent(s) is spread more thin, which can decrease production efficiency and lower the return generated by an additional unit of human capital investment. Section 4.2 considers the case where all assets are substitutes at the margin.

4.1 All assets are complementary at the margin

In the following discussion all assets are assumed to be complementary at the margin. This is clearly a strong assumption but it serves the purpose of isolating the behaviour of investment incentives when complementary assets (or firms) are integrated or separated. This section tests Bel's (2005) optimality conditions derived under simultaneous investment and suggests modifications where necessary. Assumption 9 formalises the concept of asset complementarity:

Assumption 9. All assets are complementary at the margin; i.e., $v^i(S, A|x) \geq v^i(S, A'|x), \ \forall \ i \in S \subseteq N$ and $A' \subseteq A$.

4.1.1 Simultaneous investment

In order to contrast the behaviour of incentives under the sequential investment regime to the simultaneous case it is necessary to first give an overview of Bel's (2005) results. When all assets are complementary at the margin he finds that property rights are assigned optimally if each agent *individually* accesses all the assets while only the agents of the grand coalition *jointly* veto all assets. Formally: $\upsilon(i) = \underline{A}, \chi(i) = \emptyset, \ \forall \ i \in N$ and $\chi(N) = \underline{A}$. Allocating access and veto rights in this way means that the control structure is highly inclusive because every agent controls every assets. This observation leads to the following definition:

Definition 8. Control structure β is said to be more *inclusive* than control structure β' if and only if $\beta'(S) \subseteq \beta(S), \ \forall \ S \subset N$ and $\beta'(S) \subset \beta(S)$, for at least one $S \subset N$. Conversely, control structure β' is said to be more *exclusive* than control structure β if and only if $\beta'(S) \subseteq \beta(S), \ \forall \ S \subset N$ and $\beta'(S) \subset \beta(S)$, for at least one $S \subset N$.[1]

[1]This definition reflects the fact that the grand coalition always controls all assets and can not be made more inclusive or exclusive.

Definition 8 can be interpreted as follows; if either the set of assets controlled by some coalitions increase or if the number of coalitions who control some set of assets increases – ceteris paribus – the control structure is said to be more inclusive. Conversely, if the set of assets controlled by some coalitions decreases or if the number of coalitions who control some set of assets decreases – ceteris paribus – the control structure is said to be more exclusive.

Bel's (2005) result can be characterised in terms of Definition 8. Thus, under simultaneous investment the most inclusive control structure is optimal if all assets are complementary at the margin. To better understand the intuition behind this result, recall the example developed in the introduction. The set of assets could be thought of as the physical assets of the development project, such as processor prototypes and detailed construction plans. Then, in order to minimise the chance of hold-up it would be optimal to give both the scientists and the console manufacturer joint control (creating a joint venture) over the physical assets[2] of the project because doing so prevents either party from threatening to withhold critical assets.

It is interesting to note that the two property rights *access* and *veto* have conflicting effects on the control structure. Allocating access rights is said to be inclusive because increasing the number of assets accessed by a coalition potentially increases the number assets it controls without affecting the control rights of other coalitions. The allocation of veto rights on the other hand is said to be exclusive because increasing the number of assets vetoed by a coalition has no effect on the number of assets it controls but it potentially reduces the number of assets controlled by other coalitions.

4.1.2 Sequential investment

The introduction of sequential investment changes the incentives of both ex-ante and ex-post agents. It was shown in Lemmas 1 and 2 that although the incentives of ex-post agents benefit from ex-post contracting on investment costs, their incentives are nevertheless sensitive to the level investment of ex-

[2]Recall that human capital is inalienable.

ante agents when investments are complementary. The incentives of ex-ante investors on the other hand are changed because of the internalisation effect. As shown by Lemma 3 this effect can increase or decrease their incentives.

Investments are neutral at the margin

The internalisation effect on ex-ante incentive is only significant if Assumption 5 holds because ex-post investment must be sensitive to changes in ex-ante investment if ex-ante agents are to internalise the impact of their investment choices.

However, investment incentives do not change only as a result of the internalisation effect. In fact, the introduction of investment sequencing brings with it ex-post contracting, which changes ex-post incentives. By replacing Assumption 5, with Assumption 10 it is possible to isolate the impact of ex-post contracting on investment incentives and to determine whether the optimal allocation of property rights changes as a result of ex-post contracting only.

Assumption 10. Investments are neither complements nor substitutes; i.e., $v^{ij}(S, A \mid x) = 0, \ \forall \ j \neq i \in N$.

When Assumption 10 replaces Assumption 5 then Lemma 1 no longer holds.[3] It follows that the internalisation effect drops out of the ex-ante first-order conditions:

$$\frac{\partial B_j(\beta \mid x^e_{ea,j}, x_{-j})}{\partial x_{ea,j}} = \frac{\partial C_j(x^e_{ea,j})}{\partial x_{ea,j}} \quad \forall \ j \in N_{ea} \tag{4.1}$$

where the vector x_{-j} contains the exogenous investments of the other agents. Furthermore, because Assumption 10 holds, the marginal return of an agent j is independent of all other investments by agents $l \neq j$, which means that the maximisation problem of each ex-ante agent can be analysed individually.

[3] To verify, apply Assumption 10 rather than 5 to equation (A.5) in the proof of Lemma 1 to get $\frac{\partial x_{ep,i}}{\partial x_{ea,j}} = 0$.

The same reasoning applies to the ex-post maximisation problem:

$$\frac{\partial B_i(\beta \mid x_{ep,i}, x_{-i})}{\partial x_{ep,i}} = \lambda_{ii} \frac{\partial C_i(x^e_{ep,i})}{\partial x_{ep,i}} \quad \forall\, i \in N_{ep} \tag{4.2}$$

where the vector x_{-i} contains the exogenous investments of the other agents and the sharing rule λ_{ii} appears because of ex-post contracting.

The following proposition summarises the impact of a more inclusive control structure on investment incentives when Assumption 10 holds and all assets are complementary at the margin:

Proposition 2. *If all assets are complementary but human capital investments are neutral at the margin then the optimal control structure is the most inclusive control structure.*

Proof: See Appendix A

Proposition 2 is an important result because it shows that ex-post contracting alone does not change the optimal allocation of property rights. In other words, Bel's (2005) result still holds when investments are independent despite the introduction of sequential investment. However, Assumption 10 is restrictive because it removes all positive externalities of human capital investment from the model.

Investments are complementary at the margin

This section reintroduces investment complementarity; that is, Assumption 5 replaces Assumption 10. As a result, Lemma 1 holds and the internalisation effect on ex-ante incentives reappears.

Once again it is possible to solve the investment game backwards. For this purpose, consider Lemma 4. It characterises the change of ex-post investment incentives when the control structure becomes more inclusive:

Lemma 4. *If all assets are complementary at the margin and the control structure becomes more inclusive then – ceteris paribus – ex-post investment incentives and ex-post equilibrium investments increase; i.e.,* $R(\beta, \lambda | x_{ea}) \geq$

$R(\beta', \lambda | x_{ea})$*, if* β *is more inclusive than* β'*.*

Proof: See Appendix A

Holding ex-ante investment constant, Lemma 4 says that when all assets are complementary at the margin and the control structure becomes more inclusive such that at least one coalition containing ex-post agents controls a larger set of assets then all ex-post agents have higher marginal return to investment and invest more in equilibrium. The intuition is that all ex-post agents who are part of coalitions that control larger sets of assets have a higher return to investment and increase equilibrium investment, which also increases the marginal return of all other ex-post agents because higher investment has positive externalities for other agents (i.e., investments are complementary).[4] The converse it is also true; if the control structure becomes more exclusive then ex-post incentives fall.

In contrast to the ex-post group, ex-ante investors do not contract on investment costs but internalise their effect on ex-post incentives when making their investment choices. Using Definition 7, the ex-ante first-order conditions are given by:

$$\frac{\partial B_j(\beta \mid x_{ea}, R)}{\partial x_{ea,j}} + S_j(\beta, \lambda | x_{ea}) - \frac{\partial C_j(x_{ea,j})}{\partial x_{ea,j}}|_{x_{ea}=x_{ea}^e} = 0 \qquad \forall\ j \in N_{ea} \quad (4.3)$$

The key insight of conditions (4.3) is that ex-ante agents increase their investment until marginal return equals marginal costs, while taking into account the impact of their investment choices on the investment incentives of ex-post investors because the response of ex-post investment affects ex-ante surplus and costs. A more detailed discussion of the intuition underlying conditions (4.3) is given in Section 3.6.

It was shown previously that ex-post investment incentives increase when the control structure becomes more inclusive (Lemma 4). Making use of this insight, it is possible to show that, contrary to Bel's (2005) result (derived

[4]Lemma 4 extends the results of Lemmas 1 and 2, which show that ex-post incentives increase when either ex-ante investment rises or ex-post agents pay a smaller share of their investment costs.

under simultaneous investment), ex-ante equilibrium investment can in fact decrease when the control structure becomes more inclusive. Proposition 3 formalises this result:

Proposition 3. *If all assets are complementary at the margin and the control structure becomes more inclusive then ex-ante investment incentives and equilibrium investment can decrease; i.e., it is possible that* $x^e_{ea,j}(\beta) \leq x^e_{ea,j}(\beta')$ *for some or all* $i \in N_{ea}$*, if* β *is more inclusive than* β*.*

Proof: See Appendix A

The intuition underlying Proposition 3 is the following; it was shown in Lemma 4 that a more inclusive control structure increases ex-post investment incentives, meaning that for a given level of ex-ante investment all ex-post agents invest more in equilibrium. Increased investment of course means that investment costs are higher and since the cost function is convex this implies that ex-post marginal costs are also higher for any given level of ex-ante investment. Further, ex-ante agents internalise the response of ex-post equilibrium investment to changes in ex-ante investment, which means that when an ex-ante agents considers whether to invest more, they take into account that an increase in ex-ante investment not only raises ex-post investment but also ex-post investment costs of which they pay a share. This gives the following situation; under a more inclusive control structure ex-post marginal costs are higher, which means that ex-post costs increase faster for a given increase in ex-ante investment relative to a more exclusive control structure. In other words, the marginal costs of ex-ante agents have increased. Finally, if the increase in ex-ante marginal costs described above, is greater than the increase in ex-ante marginal return (brought about by the more inclusive control structure) then ex-ante investment incentives and equilibrium investment can decrease when the control structure becomes more inclusive.

The example developed in the introduction is useful to further illuminate the intuition of Proposition 3. Recall that a group of scientists and a large video games console manufacturer are collaborating on a joint research project. Suppose now that property rights are re-allocated so that some critical assets that were previously under the control of the scientists are now controlled jointly (i.e., a more inclusive control structure is implemented). This clearly increases the incentives of the manufacturer because the scientists can no longer threaten to withhold theses assets. However, this change has potentially made investment more costly for the scientists because they realise that every extra dollar they invest into the project will increase the value of the manufacturer's investment by more than previously. It also increases their share of the manufacturer's investment costs by more than previously. If this negative effect dominates the positive effect on their marginal return then a more inclusive control structure can be bad for the investment incentives of the scientists and for welfare.

4.1.3 Protecting ex-ante incentives

A control structure can be made more inclusive by re-allocating property rights so that some coalitions control larger sets of assets while taking care that no coalitions control smaller sets of assets. It is clear that this can be achieved in different ways. For example, it is possible to target certain subsets of agents with a more inclusive (or exclusive) control structure while not affecting others.

Proposition 3 showed that a more inclusive control structure can lower investment incentives of ex-ante agents. However, so far conditions are missing that define when ex-ante incentives are likely to rise or fall when the control structure changes. The question arises naturally, whether targeting either ex-ante or ex-post agents with a more inclusive control structure has different consequences for ex-ante investment incentives. Proposition 4 gives an interesting result:

Proposition 4. *A more inclusive control structure targeted only at ex-post agents is more likely to decrease ex-ante incentives than a more inclusive control structure targeted at both investor groups.*

Proof: See Appendix A

This is an interesting result because it contradicts Bel (2005) who finds no such effect when investments are simultaneous. The intuition underlying Proposition 4 is straight forward. When a more inclusive control structure is targeted only at ex-post agents then there is no direct change of ex-ante marginal return because, by assumption, all coalitions containing ex-ante investors control the same set of assets. However, ex-ante marginal return may still increase, since ex-ante investors anticipate the improved incentives of the ex-post parties (some of whom are their colleagues), who invest more for a given level of ex-ante investment. The internalisation effect also increases ex-ante marginal costs, because ex-ante investors anticipate an increase in ex-post costs marginal costs of which they pay a share, this in turn reduces incentives. Thus, when control becomes more inclusive without directly affecting ex-ante parties and any increase in ex-ante marginal return occurs because of the anticipated increase in ex-post investment then the increase in marginal costs is relatively more important and has a stronger negative impact on investment incentives, which makes it more likely that ex-ante incentives fall.

The proposition also has empirical implications. In situations where the ex-post investor (such as the console manufacturer) has fast increasing (highly convex) marginal investment costs, it is expected that the set of assets controlled by the ex-post investor is relatively small, which is done to protect the ex-ante investor from the fast increasing ex-post marginal investment costs. However, there is a trade-off involved. Protection of the ex-ante investor leaves the ex-post investor highly exposed to hold-up, which reduces his incentives (something that it is meant to do) but if the investment of the ex-post investor is critical to the success of the project then this may not be optimal. Thus, in order to determine whether more protection of the ex-ante investor is warranted, it is crucial to determine which agent's investment is more important to the project.

4.2 All assets are substitutes at the margin

This section discusses the impact on the optimal allocation of property rights when sequential investment is introduced into an economy where all assets are substitutes at the margin in absence of some agents. This restriction on asset relationships is formalised below:

Assumption 11. All assets are substitutes at the margin in absence of some agents in N; i.e., $v^i(S, A|x) \leq v^i(S, A'|x)\ \forall\ i \in S \subset N$ and $A'(\neq \emptyset) \subseteq A$ $(v^i(S, A) \geq v^i(S, \emptyset)\ \forall\ i \in S \subseteq N)$.

Assumption 11 says that increasing the set of assets controlled by any coalition S that is smaller than the grand coalition decreases the marginal return of all agents in S.[5]

The focus of this section lies on one of Bel's (2005) seemingly trivial results. He shows that when Assumption 11 holds and if the optimal control structure is given by β^* then any agent either controls no assets individually or exactly one asset individually but never more than one. Formally, $|\beta^*(i)| = 1\ \forall\ i \in N_1$ and $|\beta^*(i)| = 0\ \forall\ i \in N_0$.[6] The intuition for this result works by contradiction. Suppose that $\hat{\beta}$ is the optimal control structure, it is similar to β^* in all respects except that it has $|\hat{\beta}(i)| = 2$ for one agent $i \in N$ (i.e., one of the agents controls two assets rather than only one). Because assets are substitutes at the margin it is possible to increase this agent's marginal return to investment and investment incentives by giving veto rights to one of his assets to an outside party. This not only increases the agent's marginal return to investment when he produces alone ($v^i(i, \{a, b\}) \leq v^i(i, a)$ and $v^i(i, \{a, b\}) \leq v^i(i, b)$) but it also increases his marginal return in all coalitions that he is a member of because he no longer contributes the substitute asset. Therefore, $\hat{\beta}$ can not be optimal.[7]

The optimality conditions change when sequential investment is introduced, because contrary to Bel (2005), it may be optimal for some ex-post

[5] The grand coalition N always controls all assets $\beta(N) = \underline{A}$ and therefore no assets can be added to the set controlled by the grand coalition.

[6] Where $N_1 \cap N_0 = \emptyset$ and $N_1 \cup N_0 = N$.

[7] This paragraph is not meant to replicate Bel's (2005) proof. See his Proposition 3 for more details.

agents to control more than one asset:

Proposition 5. *When all assets are substitutes at the margin, ex-ante and ex-post equilibrium investment can increase if some ex-post agent individually controls two assets rather than one.*

Proof: See Appendix A

The intuition for this result is that if control of a second substitute asset is given to an ex-post agent then ex-post investment incentives fall, which means that for a given level of ex-ante investment ex-post equilibrium investment is lower. By familiar argument, this has two implications for ex-ante investors. On the one hand, lower ex-post incentives decrease ex-ante marginal return as a consequence of the internalisation effect. But on the other hand, the internalisation effect also decreases ex-ante marginal costs because ex-post marginal costs are lower for a given level of ex-ante investment. Thus, if the decrease in marginal costs is greater than the decrease in marginal return, then ex-ante investment incentives can increase. In other words, sometimes it can be optimal to decrease ex-post incentives by making control more inclusive for ex-post agents in order to foster ex-ante investment.

Using the example from the introduction it is possible to develop this result more clearly. Start by assuming that both the group of scientists and the console manufacturer control a set of complementary assets that they require to work on the project.[8] Suppose that one of the scientists' assets is a large testing laboratory and that the manufacturer also controls a similar facility. If joint control of the scientists' laboratory is given to both parties then assume that the laboratory acts as a substitute to the set of assets already controlled by the manufacturer, which has the consequence of reducing the manufacturer's marginal return to investment. The incentives of the scientists can increase as a result because the manufacturer invests less for a given level of the scientists' investment, which, in addition, means that scientists have lower marginal costs (recall that the manufacturer can write a cost sharing contract). Thus, in situations where the incentives of

[8] These assets could be thought of as research labs, supercomputers or training facilities.

the scientists are very sensitive to the manufacturers marginal costs, it might be optimal to transfer control of a substitute asset to the manufacturer in order to decrease its incentives.

Proposition 5 provides an interesting contrast to Proposition 4; when all assets are complementary it may be optimal to take control away from ex-post agents in order to protect ex-ante investment but when assets are substitutes it may be optimal to give ex-post agents more control in order to protect ex-ante investments. Both findings contradict the results derived by Bel (2005) under simultaneous investment, demonstrating that the introduction of sequential investment is an important extension of the property rights framework.

Chapter 5

Concluding Remarks

The property-rights theory of the firm developed by Hart & Moore (1990), and its many refinements such as Bel (2005)), has made an invaluable contribution to the understanding of organisational forms and firm boundaries when relationship specific investments are subject to the hold-up problem. The theory finds widespread application in many economic contexts, such as the design of financial structures and take-over analysis (Hart 1995).

This thesis extends the scope of the property-rights framework even further by examining its predictions when sequencing of investments is allowed. Sequential investment is particularly important during research and development projects where a group of firms (or scientists) must collaborate in order to develop new technologies. It is often not possible for all parties to work on a project simultaneously because the nature of research is that progress builds on the findings of others. Consequently it of great importance to protect the incentives of agents who are early contributors to a project when the contracting environment is less complete.

The introduction of sequencing has a number of implications for incentives. When investment is sequential ex-ante investors anticipate the impact of their investment choices on ex-post investment which in turn influences ex-ante marginal return (by changing total surplus and by changing the surplus-share of ex-post agents) and marginal costs (by changing ex-post investment costs). This is the internalisation effect, which is only present when investments of ex-ante and ex-post parties are interdependent.

Perhaps the most important result is that when all assets are complementary at the margin, investment sequencing can mean that a common access (or ownership) structure is not optimal to protect ex-ante incentives. This can occur for example when the ex-post party to a project has fast increasing marginal costs, making it beneficial for the ex-ante investors to re-allocate property rights such that ex-post investment for a given level of ex-ante investment decreases. The implication is that it is not always necessary to look for asset substitutability in order to explain less inclusive ownership structures.

Another original finding suggests that when all assets are substitutes it may strengthen incentives of ex-ante parties to a project when ownership of substitute assets is transferred to ex-post parties in order to decrease their investment incentives, in turn lowering their marginal costs. This has further implications for optimal ownership, suggesting that under certain conditions assets substitutability can be the driver for more inclusive ownership structures.

The internalisation effect is not well behaved because no assumptions have been imposed on the relative changes of total surplus and the surplus-share of ex-post agents. However, in future applications it may be fruitful to restrict the behaviour of this effect so that the response of ex-ante investment to changes in the control structure can be clarified.

The current analyses focusses only on the two extreme cases where all assets are either complements or substitutes at the margin but neglects the general case where some assets are substitutes and some are complements. Bel (2005) finds that the general case provides a rich set of optimality conditions for many different organisational forms such as joint ventures and outside shareholding. It is therefore likely that the interaction of sequential investment and general asset relationships would provide interesting auxiliary results.

The literature on investment timing and hold-up derives several interesting results when discounting of future returns matters (Smirnov & Wait 2004*a*). It remains to be seen whether the current model could benefit from the introduction of discounting but it is clear that any such modification would have to weigh up its benefits and costs. On the one hand the presence of discounting would increase the number of interactions in the model, which may uncover previously hidden implications for property rights. But at the same time, the model would lose some of its clarity and it may become impossible to separate some of the effects.

As an empirical application of the framework developed here it would be interesting to analyse the the ownership structures of real world collaborative research and development projects in order to determine whether the timing of major investments influences the ownership structure of the projects as predicted by this thesis.[1]

[1] It is common practice for large infrastructure projects to combine the research expertise of several industrial companies. One such example is the German ICE high speed train project where the Canadian train manufacturer Bombardier is collaborating with the German industrial conglomerate Siemens.

Appendix A

Proofs

Proof of Lemma 1

The Nash equilibrium ex-post investments x^e_{ep} are characterised by (3.13)

$$\frac{\partial B_i(\beta \mid x_{ea}, x_{ep})}{\partial x_{ep,i}} - \lambda_{ii}\frac{\partial C_i(x_{ep,i})}{\partial x_{ep,i}}|_{x_{ep}=x^e_{ep}} = 0 \qquad i \in N_{ep}.$$

Totally differentiating the above gives

$$\boldsymbol{J}_{ep}\begin{bmatrix} dx_{ep,1} \\ \vdots \\ dx_{ep,I} \end{bmatrix} = \begin{bmatrix} -\sum_{j\in N_{ea}} \frac{\partial^2 B_1}{\partial x_{ep,1}\partial x_{ea,j}} dx_{ea,j} \\ \vdots \\ -\sum_{j\in N_{ea}} \frac{\partial^2 B_I}{\partial x_{ep,I}\partial x_{ea,j}} dx_{ea,j} \end{bmatrix} + \begin{bmatrix} \frac{\partial C_1}{\partial x_{ep,1}} d\lambda_{11} \\ \vdots \\ \frac{\partial C_I}{\partial x_{ep,I}} d\lambda_{II} \end{bmatrix} \tag{A.1}$$

where $\boldsymbol{J}_{ep}$ is the Jacobian matrix

$$\boldsymbol{J}_{ep} = \boldsymbol{W}_{ep} + \begin{bmatrix} -\frac{\partial^2 C_1}{\partial x^2_{ep,1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{\partial^2 C_I}{\partial x^2_{ep,I}} \end{bmatrix}. \tag{A.2}$$

By Assumption 7 $\boldsymbol{W}_{ep}$ is a negative definite matrix. It follows that $\boldsymbol{J}_{ep}$ is also a negative definite. Thus, $\boldsymbol{J}_{ep}$ is invertible. Pre-multiplying both sides of (A.1) by the inverse Jacobian $\boldsymbol{J}_{ep}^{-1}$ gives

$$\begin{bmatrix} dx_{ep,1} \\ \vdots \\ dx_{ep,I} \end{bmatrix} = \boldsymbol{J}_{ep}^{-1} \begin{bmatrix} -\sum_{j \in N_{ea}} \frac{\partial^2 B_1}{\partial x_{ep,1} \partial x_{ea,j}} dx_{ea,j} + \frac{\partial C_1}{\partial x_{ep,1}} d\lambda_{11} \\ \vdots \\ -\sum_{j \in N_{ea}} \frac{\partial^2 B_I}{\partial x_{ep,I} \partial x_{ea,j}} dx_{ea,j} + \frac{\partial C_I}{\partial x_{ep,I}} d\lambda_{II} \end{bmatrix}. \tag{A.3}$$

Set $dx_{ea,k} = 0 \ \forall \ k \neq j \in N_{ea}$ and $d\lambda_{ii} = 0 \ \forall \ i \in N_{ep}$. Dividing both sides by $dx_{ea,j}$ gives[1]

$$\begin{bmatrix} \frac{\partial x_{ep,1}}{\partial x_{ea,j}} \\ \vdots \\ \frac{\partial x_{ep,I}}{\partial x_{ea,j}} \end{bmatrix} = \boldsymbol{J}_{ep}^{-1} \begin{bmatrix} \frac{\partial^2 B_1}{\partial x_{ep,1} \partial x_{ea,j}} dx_{ea,j} \\ \vdots \\ \frac{\partial^2 B_I}{\partial x_{ep,I} \partial x_{ea,j}} dx_{ea,j} \end{bmatrix} \tag{A.4}$$

Then, the response of ex-post agent i's equilibrium investment to a small change in investment of ex-ante agent $j \in N_{ea}$ is given by

$$\frac{\partial x_{ep,i}}{\partial x_{ea,j}} = \sum_{k \in N_{ep}} \frac{C_{ki}}{|\boldsymbol{J}_{ep}|} \left(-\frac{\partial^2 B_k}{\partial x_{ep,k} \partial x_{ea,j}} \right) \qquad \forall \ i \in N_{ep} \text{ and } \forall \ j \in N_{ea} \tag{A.5}$$

where $|\boldsymbol{J}_{ep}|$ is the determinant and C_{ki} is the cofactor of element c_{ki} in $\boldsymbol{J}_{ep}$. Recall that $\boldsymbol{J}_{ep}$ is negative definite, which means that its inverse $\boldsymbol{J}_{ep}^{-1}$ is non-positive. It follows that $\frac{C_{ki}}{|\boldsymbol{J}_{ep}|} < 0 \ \forall \ k, i \in N_{ep}$. Also, by Definition 5 and Assumption 5 $\frac{\partial^2 B_k}{\partial x_{ep,k} \partial x_{ea,j}} > 0 \ \forall \ k \in N_{ep}$ and $j \in N_{ea}$. The right hand side of (A.5) must therefore be positive (i.e., $\frac{\partial x_{ep,i}}{\partial x_{ea,j}} > 0$). Hence, an increase in investment by any ex-ante agent increases ex-post equilibrium investment of all ex-post agents. *Q.E.D.*

[1] Here use is made of the fact that $\frac{dx_{ep,i}}{dx_{ea,j}}$ must be interpreted as the partial derivative $\frac{\partial x_{ep,i}}{\partial x_{ea,j}}$ because $dx_{ea,k} = 0 \ \forall \ k \neq j \in N_{ea}$.

Proof of Lemma 2

The proof is similar to the proof of Lemma 1. Starting from (A.1) set $dx_{ea,j} = 0 \;\forall\; j \in N_{ea}$ and pre-multiply both sides with $\boldsymbol{J}_{ep}^{-1}$ to get

$$\begin{bmatrix} dx_{ep,1} \\ \vdots \\ dx_{ep,I} \end{bmatrix} = \boldsymbol{J}_{ep}^{-1} \begin{bmatrix} \frac{\partial C_1}{\partial x_{ep,1}} d\lambda_{11} \\ \vdots \\ \frac{\partial C_I}{\partial x_{ep,I}} d\lambda_{II} \end{bmatrix}. \tag{A.6}$$

Then, for agent $i \in N_{ep}$ the response of equilibrium investment to small changes in the set of sharing rules is given by

$$dx_{ep,i} = \sum_{k \in N_{ep}} \frac{C_{ki}}{|\boldsymbol{J}_{ep}|} \frac{\partial C_k}{\partial x_{ep,k}} d\lambda_{kk} \qquad \forall\; i \in N_{ep}. \tag{A.7}$$

Now, setting $d\lambda_{kk} = 0 \;\forall\; k \neq l \in N_{ep}$ and dividing both sides by $d\lambda_{ll}$ gives

$$\frac{\partial x_{ep,i}}{\partial \lambda_{ll}} = \frac{C_{li}}{|\boldsymbol{J}_{ep}|} \frac{\partial C_l}{\partial x_{ep,l}} \qquad \forall\; i, l \in N_{ep}. \tag{A.8}$$

Analogous to the proof of Lemma 1, it must be that $\frac{C_{li}}{|\boldsymbol{J}_{ep}|} < 0 \;\forall\; l, i \in N_{ep}$. Also by Assumption 6 it must be that $\frac{\partial C_l}{\partial x_{ep,l}} \geq 0 \;\forall\; l \in N_{ep}$. It follows that the right hand side of (A.8) is non-positive (i.e., $\frac{\partial x_{ep,i}}{\partial \lambda_{ll}} \leq 0$). Hence, an increase in the share of investment costs payable by any ex-post agent decreases equilibrium investment of all ex-post agents. *Q.E.D.*

Proof of Lemma 3

The Nash equilibrium ex-ante investments x_{ea}^e are characterised by (3.15)

$$\frac{\partial B_j(\beta \mid x_{ea}, R)}{\partial x_{ea,j}} + S_j - \frac{\partial C_j(x_{ea,j})}{\partial x_{ea,j}} |_{x_{ea} = x_{ea}^e} = 0 \qquad \forall\; j \in N_{ea}.$$

Totally differentiating the above and rearranging gives

$$\left[\frac{\partial^2 B_1}{\partial x_{ea,1}^2} - \frac{\partial^2 C_1}{\partial x_{ea,1}^2}\right] dx_{ea,1} + \sum_{k \neq 1 \in N_{ea}} \frac{\partial^2 B_1}{\partial x_{ea,1} \partial x_{ea,k}} dx_{ea,k} = -dS_1$$

$$\cdots$$

$$\left[\frac{\partial^2 B_J}{\partial x_{ea,J}^2} - \frac{\partial^2 C_J}{\partial x_{ea,J}^2}\right] dx_{ea,J} + \sum_{k \neq J \in N_{ea}} \frac{\partial^2 B_J}{\partial x_{ea,J} \partial x_{ea,k}} dx_{ea,k} = -dS_J$$

or in matrix notation

$$\boldsymbol{J}_{ea} \begin{bmatrix} dx_{ea,1} \\ \vdots \\ dx_{ea,J} \end{bmatrix} = \begin{bmatrix} -dS_1 \\ \vdots \\ -dS_J \end{bmatrix} \tag{A.9}$$

where $\boldsymbol{J_{ea}}$ is the Jacobian matrix

$$\boldsymbol{J}_{ea} = \boldsymbol{W}_{ea} + \begin{bmatrix} -\frac{\partial^2 C_1}{\partial x_{ea,1}^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{\partial^2 C_I}{\partial x_{ea,I}^2} \end{bmatrix}. \tag{A.10}$$

By Assumption 7 $\boldsymbol{W}_{ea}$ is a negative definite matrix, which implies that $\boldsymbol{J}_{ea}$ is also negative definite and invertible. Then, pre-multiplying both sides of (A.9) by the inverse Jacobian $\boldsymbol{J}_{ea}^{-1}$ gives

$$\begin{bmatrix} dx_{ea,1} \\ \vdots \\ dx_{ea,J} \end{bmatrix} = \boldsymbol{J}_{ea}^{-1} \begin{bmatrix} -dS_1 \\ \vdots \\ -dS_J \end{bmatrix}. \tag{A.11}$$

Then, for agent $j \in N_{ea}$ the response of equilibrium investment to small changes in the internalisation effect of all ex-ante agents is given by

$$dx_{ea,j} = \sum_{k \in N_{ep}} \frac{C_{kj}}{|\boldsymbol{J}_{ea}|}(-dS_k) \qquad \forall\, j \in N_{ea} \tag{A.12}$$

where $|\boldsymbol{J}_{ea}|$ is the determinant and C_{kj} is the cofactor of element c_{kj} in $\boldsymbol{J}_{ea}$. Now, setting $dS_{ea,k} = 0 \; \forall \; k \neq l \in N_{ea}$ and dividing both sides by $dS_{ea,l}$ gives

$$\frac{\partial x_{ea,j}}{\partial S_l} = -\left(\frac{C_{lj}}{|\boldsymbol{J}_{ea}|}\right) \qquad \forall \; j, l \in N_{ea}. \tag{A.13}$$

Recall that $\boldsymbol{J}_{ea}$ is negative definite, which implies that its inverse $\boldsymbol{J}_{ea}^{-1}$ is non-positive. Thus, $\frac{C_{ki}}{|\boldsymbol{J}_{ep}|} < 0 \; \forall \; k, i \in N_{ep}$. It follows that the right hand side of (A.13) must be positive (i.e., $\frac{\partial x_{ea,j}}{\partial S_l} > 0$). Hence, an increase in the internalisation effect of any ex-ante agent increases equilibrium investment of all ex-ante agents. *Q.E.D.*

Proof of Proposition 1

(i) For a given control structure β; when investments are simultaneous then the internalisation effect is zero by assumption $S_j = 0 \; \forall \; j \in N_{ea}$ but when investments are sequential it can be negative, zero or positive. Lemma 3 showed that when the internalisation effect increases then ex-ante equilibrium investments increase. Therefore, if the internalisation effect is sufficiently large (positive) in equilibrium then ex-ante equilibrium investment can be higher under sequential investment than under simultaneous investment. The converse is also true, if the internalisation effect is negative and sufficiently large it is possible that ex-ante equilibrium investment is lower under investment sequencing than under simultaneous investment.
(ii) If at least one ex-ante agent increases equilibrium investment and no ex-ante agents decrease equilibrium investment then by Lemma 1 investment incentives of all ex-post agents increase because investments are complementary. Also, the introduction of sequencing makes ex-post contracting possible which by Lemma 2 increases ex-post incentives. These two effects work in the same direction and hence ex-post equilibrium investment always increases when investment sequencing is introduced and some ex-ante agents increases equilibrium investment.
However, (iii) if some ex-ante agents decrease equilibrium investment, then by Lemma 1 the some ex-post agents may reduce equilibrium investment.

But the introduction of sequencing also makes ex-post contracting possible which by Lemma 2 increases ex-post incentives. These two effects work in opposite directions and hence the impact of investment sequencing on ex-post equilibrium investment is ambiguous when ex-ante investment falls. *Q.E.D.*

Proof of Proposition 2

Define function

$$G(\beta, \lambda|x) = \sum_S p(S)v(S, \beta(S)|x) - \sum_{j \in N_{ea}} C_j(x_{ea,j}) - \sum_{i \in N_{ep}} \sum_{l \in N} \lambda_{il} C_i(x_{ep,i})$$

and let β be a more inclusive control structure than β'.
For a representative ex-ante agent $j \in N_{ea}$ define the function

$$f(\alpha, x_{ea,j}) = \alpha G(\beta, \lambda|x_{ea,j}, x_{-j}) + (1-\alpha)G(\beta', \lambda|x_{ea,j}, x_{-j})$$

for $\alpha \in [0,1]$, where x_{-j}[2] is exogenous and let $x_{ea,j}(\alpha)$ solve

$$\frac{\partial f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j}} = 0. \tag{A.14}$$

Totally differentiating (A.14) and taking α as the exogenous variable, gives

$$\frac{dx_{ea,j}}{d\alpha} = -\frac{\frac{\partial^2 f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j}\partial \alpha}}{\frac{\partial^2 f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j}^2}} \tag{A.15}$$

where $\frac{\partial^2 f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j}^2} = \alpha \left[\frac{\partial^2 B_j(\beta)}{\partial x_{ea,j}^2} - \frac{\partial^2 C_j}{\partial x_{ea,j}^2}\right] + (1-\alpha)\left[\frac{\partial^2 B_j(\beta')}{\partial x_{ea,j}^2} - \frac{\partial^2 C_j}{\partial x_{ea,j}^2}\right] < 0$ by Assumptions 3 & 6, while $\frac{\partial^2 f(\alpha, x_{ea,j}(\alpha))}{\partial x_{ea,j}\partial \alpha} = \left[\frac{\partial B_j(\beta)}{\partial x_{ea,j}} - \frac{\partial C_j}{\partial x_{ea,j}}\right] - \left[\frac{\partial B_j(\beta')}{\partial x_{ea,j}} - \frac{\partial C_j}{\partial x_{ea,j}}\right] \geq 0$ follows from Assumption 9. Hence, $\frac{dx_{ea,j}}{d\alpha} \geq 0 \ \forall j \in N_{ea}$ and $x_{ea,j}(1) \geq x_{ea,j}(0)$ or $x^e_{ea,j}(\beta) \geq x^e_{ea,j}(\beta') \quad \forall j \in N_{ea}$.
For ex-post agents, the proof that $x^e_{ep,i}(\beta) \geq x^e_{ep,i}(\beta') \quad \forall i \in N_{ep}$ is analogous and will be omitted.
Thus, a more inclusive control structure always increases ex-ante and ex-post

[2] $x_{-j} = \{x_{ep}, x_{ea,1}, \ldots, x_{ea,j-1}, x_{ea,j+1}, \ldots, x_{ea,J}\}$ is the set of all other investments.

investment incentives and increases equilibrium investment. Therefore, the most inclusive control structure gives the highest investment incentives and maximises (second best) equilibrium investment and is optimal. *Q.E.D.*

Proof of Lemma 4

Define function

$$G(\beta,\lambda|x_{ea},x_{ep}) = \sum_S p(S)v(S,\beta(S)|x_{ea},x_{ep}) - \sum_{j\in N_{ea}} C_j(x_{ea,j}) - \sum_{i\in N_{ep}}\sum_{l\in N}\lambda_{il}C_i(x_{ep,i})$$

where x_{ea} is observed and taken as given by the ex-post agents.[3] Let β be a more inclusive control structure than β' and define function

$$f(\alpha,x_{ep}) = \alpha G(\beta,\lambda|x_{ea},x_{ep}) + (1-\alpha)G(\beta',\lambda|x_{ea},x_{ep})$$

for $\alpha\in[0,1]$ and let $x_{ep}(\alpha)$ solve

$$\frac{\partial f(\alpha,x_{ep}(\alpha))}{\partial x_{ep,i}} = 0 \qquad \forall\, i\in N_{ep}. \tag{A.16}$$

Totally differentiating (A.16) and taking α as the exogenous variable gives

$$\boldsymbol{H}_{f(\alpha,x_{ep})}\begin{pmatrix} dx_{ep,1}\\ \cdots \\ dx_{ep,I}\end{pmatrix} = -\begin{pmatrix} \frac{\partial G(\beta,\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,1}} - \frac{\partial G(\beta',\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,1}} \\ \cdots \\ \frac{\partial G(\beta,\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,I}} - \frac{\partial G(\beta',\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,I}}\end{pmatrix} d\alpha \tag{A.17}$$

where $\boldsymbol{H}_{f(\alpha,x_{ep})}$ is the Hessian matrix of $f(\alpha,x_{ep})$. By Assumptions 3 and 6 the elements on the main diagonal are negative and by Assumption 5 the off diagonal elements are all positive. It follows that $\boldsymbol{H}_{f(\alpha,x_{ep})}$ is negative definite and $\boldsymbol{H}^{-1}_{f(\alpha,x_{ep})}$ is non-positive. Pre-multiplying both sides by $\boldsymbol{H}^{-1}_{f(\alpha,x_{ep})}$ and

[3] The proofs of Lemma 4 and Proposition 3 are similar to the proof of Proposition 1 in Hart & Moore (1990).

dividing by $d\alpha$ gives

$$\begin{pmatrix} \frac{\partial x_{ep,1}}{\partial \alpha} \\ \cdots \\ \frac{\partial x_{ep,I}}{\partial \alpha} \end{pmatrix} = -\boldsymbol{H}^{-1}_{f(\alpha,x_{ep})} \begin{pmatrix} \frac{\partial G(\beta,\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,1}} - \frac{\partial G(\beta',\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,1}} \\ \cdots \\ \frac{\partial G(\beta,\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,I}} - \frac{\partial G(\beta',\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,I}} \end{pmatrix}. \quad \text{(A.18)}$$

where $\frac{\partial G(\beta,\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,i}} - \frac{\partial G(\beta',\lambda|x_{ea},x_{ep}(\alpha))}{\partial x_{ep,i}} \geq 0 \ \forall \ i \in N_{ep}$ follows from Assumption 9. Hence, $\frac{\partial x_{ep,1}}{\partial \alpha} \geq 0 \ \forall \ i \in N_{ep}$, which implies $x_{ep}(1) \geq x_{ep}(0)$ or $R(\beta,\lambda|x_{ea}) \geq R(\beta',\lambda|x_{ea})$. *Q.E.D.*

Proof of Proposition 3

Define function

$$G(\beta,\lambda|x_{ea},R) = \sum_S p(S)v(S,\beta(S)|x_{ea},R) - \sum_{j\in N_{ea}} C_j(x_{ea,j}) - \sum_{i\in N_{ep}} \sum_{l\in N} \lambda_{il} C_i(R_i)$$

where $R = R(\beta,\lambda|x_{ea})$ is the implicit optimal response function of the ex-post agents. Let β be a more inclusive control structure than β' and define function

$$f(\alpha,x_{ea}) = \alpha G(\beta,\lambda|x_{ea},R) + (1-\alpha)G(\beta',\lambda|x_{ea},R)$$

for $\alpha \in [0,1]$ and let $x_{ea}(\alpha)$ solve

$$\frac{\partial f(\alpha,x_{ea}(\alpha))}{\partial x_{ea,j}} = 0 \qquad \forall \ j \in N_{ea}. \quad \text{(A.19)}$$

Totally differentiating (A.19) and taking α as the exogenous variable gives

$$\boldsymbol{H}_{f(\alpha,x_{ea})} \begin{pmatrix} dx_{ea,1} \\ \cdots \\ dx_{ea,J} \end{pmatrix} = - \begin{pmatrix} \frac{\partial G(\beta,\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,1}} - \frac{\partial G(\beta',\lambda|x_{ea}(\alpha),R))}{\partial x_{ea,1}} \\ \cdots \\ \frac{\partial G(\beta,\lambda|x_{ea}(\alpha),R))}{\partial x_{ea,J}} - \frac{\partial G(\beta',\lambda|x_{ea}(\alpha),R))}{\partial x_{ea,J}} \end{pmatrix} d\alpha \quad \text{(A.20)}$$

where $\boldsymbol{H}_{f(\alpha,x_{ea})}$ is the Hessian matrix of $f(\alpha, x_{ea})$. By Assumptions 3, 6 and 8 the elements on the main diagonal are negative and by Assumptions 5 and 8 the off diagonal elements are all positive. It follows that $\boldsymbol{H}_{f(\alpha,x_{ea})}$ is negative definite and $\boldsymbol{H}^{-1}_{f(\alpha,x_{ea})}$ is non-positive. Pre-multiplying both sides by $\boldsymbol{H}^{-1}_{f(\alpha,x_{ea})}$ and dividing by $d\alpha$ gives

$$\begin{pmatrix} \frac{\partial x_{ea,1}}{\partial \alpha} \\ \cdots \\ \frac{\partial x_{ea,I}}{\partial \alpha} \end{pmatrix} = -\boldsymbol{H}^{-1}_{f(\alpha,x_{ea})} \begin{pmatrix} \frac{\partial G(\beta,\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,1}} - \frac{\partial G(\beta',\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,1}} \\ \cdots \\ \frac{\partial G(\beta,\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,J}} - \frac{\partial G(\beta',\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,J}} \end{pmatrix}. \tag{A.21}$$

Decomposition of $\frac{\partial G(\beta,\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,j}} - \frac{\partial^2 G(\beta',\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,j}}$ $\forall\ j \in N_{ea}$ reveals

$$\left[\frac{\partial B_j(\beta \mid x_{ea}(\alpha), R(\beta))}{\partial x_{ea,j}} - \frac{\partial B_j(\beta' \mid x_{ea}(\alpha), R(\beta'))}{\partial x_{ea,j}}\right] \\ + \left[\frac{\partial C_j(x_{ea,j}(\alpha))}{\partial x_{ea,j}} - \frac{\partial C_j(x_{ea,j}(\alpha))}{\partial x_{ea,j}}\right] + [S_j(\beta,\lambda|x_{ea}(\alpha)) - S_j(\beta',\lambda|x_{ea}(\alpha))]$$

$\forall\ i \in N_{ea}$. Clearly, $\frac{\partial C_j(x_{ea,j}(\alpha))}{\partial x_{ea,j}} - \frac{\partial C_j(x_{ea,j}(\alpha))}{\partial x_{ea,j}} = 0$ and by Assumption 9 and Lemma 4 $\frac{\partial B_j(\beta|x_{ea}(\alpha),R(\beta))}{\partial x_{ea,j}} - \frac{\partial B_j(\beta'|x_{ea}(\alpha),R(\beta'))}{\partial x_{ea,j}} \geq 0\ \forall\ i \in N_{ea}$. However, no restriction has been placed on the internalisation effect. It follows that if $S_j(\beta,\lambda|x_{ea}(\alpha)) - S_j(\beta',\lambda|x_{ea}(\alpha)) < 0$ for some $i \in N_{ea}$ then it is possible that $\frac{\partial x_{ea,i}}{\partial \alpha} \leq 0$ for some or all $i \in N_{ea}$. Thus, $x_{ea,j}(1) \leq x_{ea,j}(0)$ or $x^e_{ea,j}(\beta) \leq x^e_{ea,j}(\beta')$ for some or all $i \in N_{ea}$ is a possibility when all assets are complementary at the margin. *Q.E.D.*

Proof of Proposition 4

Suppose that control structure β is more inclusive than control structure β' but that only ex-post agents are targeted, which means that $\beta(S) = \beta'(S)\ \forall\ S \mid S \cap N_{ea} \neq \emptyset$ (the control structure of all coalitions containing ex-ante agents remains unchanged).

Proceed as in the proof of Proposition 3 and derive (A.21). The situation is very similar to Proposition 3 and $S_j(\beta,\lambda|x_{ea}(\alpha)) - S_j(\beta',\lambda|x_{ea}(\alpha)) < 0\ \forall\ i \in N_{ea}$ is a possibility. However, while still positive, $\frac{\partial B_j(\beta|x_{ea}(\alpha),R(\beta))}{\partial x_{ea,j}} -$

$\frac{\partial B_j(\beta'|x_{ea}(\alpha),R(\beta'))}{\partial x_{ea,j}} \geq 0$ is smaller $\forall\ i \in N_{ea}$ compared to a more inclusive control structure that also targets ex-ante agents, because here control rights of ex-ante investors are explicitly assumed to be unchanged (no coalitions containing ex-ant investors control more assets).

The reason why the term is still positive, is that ex-ante investors anticipate the increase in ex-post investment that is associated with a more inclusive control structure (Lemma 4), which means they will be working with more productive co-workers.

Thus, if $S_j(\beta, \lambda|x_{ea}(\alpha)) - S_j(\beta', \lambda|x_{ea}(\alpha)) < 0\ \forall\ i \in N_{ea}$ (for example, because marginal costs increase strongly) then the negative impact on ex-ante incentives is greater than under a more inclusive control structure that targets both investor groups, because the increase in marginal return is smaller.

Q.E.D.

Proof of Proposition 5

Suppose that control structures β' is such that $|\beta'(l)| = 1\ \forall\ l \in N_1 (N_1 \neq \emptyset)$ and $|\beta'(l)| = 0\ \forall\ l \in N_0$ and that control structure β is similar in all respects except that $|\beta(i)| = 2$ for exactly one agent $i \in N_1 \cap N_{ea} (N_1 \cap N_{ea} \neq \emptyset)$. Then β is more inclusive than β'.

It can be shown that $R(\beta, \lambda|x_{ea}) \leq R(\beta', \lambda|x_{ea})$ when Assumption 11 holds. Call this Fact 1. The proof is analogous to Lemma 4 and will be omitted.

Define $G(\beta, \lambda|x_{ea}, R)$ and $f(\alpha, x_{ea})$ as in the proof of Proposition 3 and proceed identically to derive (A.21).

Decomposition of $\frac{\partial G(\beta,\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,j}} - \frac{\partial^2 G(\beta',\lambda|x_{ea}(\alpha),R)}{\partial x_{ea,j}}\ \forall\ j \in N_{ea}$ reveals

$$\left[\frac{\partial B_j(\beta \mid x_{ea}(\alpha), R(\beta))}{\partial x_{ea,j}} - \frac{\partial B_j(\beta' \mid x_{ea}(\alpha), R(\beta'))}{\partial x_{ea,j}}\right] + [S_j(\beta, \lambda|x_{ea}(\alpha)) - S_j(\beta', \lambda|x_{ea}(\alpha))]$$

$\forall\ i \in N_{ea}$.[4] By Assumption 9 and Fact 1 $\frac{\partial B_j(\beta|x_{ea}(\alpha),R(\beta))}{\partial x_{ea,j}} - \frac{\partial B_j(\beta'|x_{ea}(\alpha),R(\beta'))}{\partial x_{ea,j}} \leq 0\ \forall\ i \in N_{ea}$.

[4]Note that $\left[\frac{\partial C_j(x_{ea,j}(\alpha))}{\partial x_{ea,j}} - \frac{\partial C_j(x_{ea,j}(\alpha))}{\partial x_{ea,j}}\right] = 0$ and drops out.

The response of the internalisation effect S_j to the change in property rights is not restricted, and therefore, due to Fact 1 ($R(\beta) \leq R(\beta')$) it is possible that

$$\sum_{i \in N_{ep}} \left[\frac{\partial B_j(\beta \mid x_{ea}(\alpha), R(\beta))}{\partial R_i} \frac{\partial R_i(\beta)}{\partial x_{ea,j}} - \frac{\partial B_j(\beta' \mid x_{ea}(\alpha), R(\beta'))}{\partial R_i} \frac{\partial R_i(\beta')}{\partial x_{ea,j}} \right] <$$
$$\sum_{i \in N_{ep}} \left[\lambda_{ij} \frac{\partial C_i(R_i(\beta'))}{\partial R_i} \frac{\partial R_i(\beta')}{\partial x_{ea,j}} - \lambda_{ij} \frac{\partial C_i(R_i(\beta))}{\partial R_i} \frac{\partial R_i(\beta)}{\partial x_{ea,j}} \right]$$

$\forall\, j \in N_{ea}$, which implies that $S_j(\beta, \lambda | x_{ea}(\alpha)) - S_j(\beta', \lambda | x_{ea}(\alpha)) > 0\ \forall\, i \in N_{ea}$ is feasible. In other words, it is possible that the decrease of the internalisation effect on ex-ante marginal costs (right hand side of the inequality) is greater than the decrease of the internalisation effect on ex-ante marginal return (left hand side of the inequality).
It follows that $S_j(\beta) - S_j(\beta') > \frac{\partial B_j(\beta' | x_{ea}(\alpha), R(\beta'))}{\partial x_{ea,j}} - \frac{\partial B_j(\beta | x_{ea}(\alpha), R(\beta))}{\partial x_{ea,j}}\ \forall\, j \in N_{ea}$ is also possible. Hence, $\frac{\partial x_{ea,j}}{\partial \alpha} \geq 0\ \forall\, j \in N_{ea}$ is feasible, which implies that $x_{ea,j}(1) \geq x_{ea,j}(0)$ or $x^e_{ea,j}(\beta) \geq x^e_{ea,j}(\beta')\ \forall\, j \in N_{ea}$ is a possibility when all assets (in the absence of some agents) are substitutes at the margin.
Recall Lemma 1, which states that $\frac{\partial R_i(\beta, \lambda | x_{ep,i})}{\partial x_{ea,j}} > 0\ \forall\, i \in N_{ep}$ and $\forall\, j \in N_{ea}$. Thus, it is possible that the decrease in ex-post incentives (due to Fact 1) can be offset by the increase in ex-ante equilibrium investment which increases ex-post incentives (Lemma 1). Hence, $x^e_{ep,j}(\beta) \geq x^e_{ea,j}(\beta')\ \forall\, i \in N_{ep}$ is a possibility.
It follows that giving an ex-post agent control over two substitute assets can increase not only ex-ante equilibrium investment ($x^e_{ea,j}(\beta) \geq x^e_{ea,j}(\beta')\ \forall\, j \in N_{ea}$) but can also, counter-intuitively, increase ex-post equilibrium investment ($x^e_{ep,j}(\beta) \geq x^e_{ea,j}(\beta')\ \forall\, i \in N_{ep}$) under the right conditions. *Q.E.D.*

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