# A COMPARATIVE STUDY OF AMERICAN OPTION VALUATION AND COMPUTATION

Karl Rodolfo

A thesis submitted in fulfilment
of the requirements for the degree of
Doctor of Philosophy

School of Mathematics and Statistics
The University of Sydney

November 17, 2007

# Abstract

For many practitioners and market participants, the valuation of financial derivatives is considered of very high importance as its uses range from a risk management tool, to a speculative investment strategy or capital enhancement. A developing market requires efficient but accurate methods for valuing financial derivatives such as American options.

A closed form analytical solution for American options has been very difficult to obtain due to the different boundary conditions imposed on the valuation problem. Following the method of solving the American option as a free boundary problem in the spirit of the "no-arbitrage" pricing framework of Black-Scholes, the option price and hedging parameters can be represented as an integral equation consisting of the European option value and an early exercise value dependent upon the optimal free boundary.

Such methods exist in the literature and along with risk-neutral pricing methods have been implemented in practice. Yet existing methods are accurate but inefficient, or accuracy has been compensated for computational speed.

A new numerical approach to the valuation of American options by cubic splines is proposed which is proven to be accurate and efficient when compared to existing option pricing methods. Further comparison is made to the behaviour of the American option's early exercise boundary with other pricing models.

# Acknowledgements

This thesis will not have been possible without the guidance of my supervisor, Dr Peter Buchen. With Peter's tireless efforts and astuteness to all things related to my PhD, I always knew that I will never be left in the dark. Equally important he is always a good friend to have a chat about golf and cooks a fantastic barbecue. I am truly thankful Peter.

Secondly many thanks to the lecturers and support staff at School of Mathematics and Statistics who throughout the years I spent there, provided me with the resources, knowledge and encouragement to excel.

To my professional colleagues and directors at Opes Prime Stockbroking, I thank you for your support and allowing me to concentrate time towards my studies.

Lastly but by no means least my beautiful wife Tracey who has supported me tremendously in her own unique and colourful way. Thank you my love. To my parents Bing and Tess, my siblings Katrina and Albin and to all my friends, thank you for your endless support.

Karl Rodolfo
School of Mathematics and Statistics
The University of Sydney
September 2007

# Contents

# List of Figures

# List of Tables