

An Investigation Of Mathematical Models For Animal Group Movement, Using Classical And Statistical Approaches

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Abstract

Collective actions of large animal groups result in elaborate behaviour, whose nature can be breathtaking in their complexity. Social organisation is the key to the origin of this behaviour and the mechanisms by which this organisation occurs are of particular interest. In this thesis, these mechanisms of social interactions and their consequences for group-level behaviour are explored. Social interactions amongst individuals are based on simple rules of attraction, alignment and orientation amongst neighbouring individuals.

As part of this study, we will be interested in data that takes the form of a set of directions in space. In Chapter 2, we discuss relevant statistical measures and theory which will allow us to analyse directional data. These statistical tools will be employed on the results of the simulations of the mathematical models formulated in the course of the thesis.

The first mathematical model for collective group behaviour is a Lagrangian self-organising model, which is formulated in Chapter 3. This model is based on basic social interactions between group members. Resulting collective behaviours and other related issues are examined during this chapter.

Once we have an understanding of the model in Chapter 3, we use this model in Chapter 4 to investigate the idea of guidance of large groups by a select number of individuals. These individuals are privy to information regarding the location of a specific goal. This is used to explore a mechanism proposed for honeybee (*Apis mellifera*) swarm

migrations. The spherical theory introduced in Chapter 2 will prove to be particularly useful in analysing the results of the modelling.

In Chapter 5, we introduce a second mathematical model for aggregative behaviour. The model uses ideas from electromagnetic forces and particle physics, reinterpreting them in the context of social forces. While attraction and repulsion terms have been included in similar models in past literature, we introduce an orientation force to our model and show the requirement of a dissipative force to prevent individuals from escaping from the confines of the group.

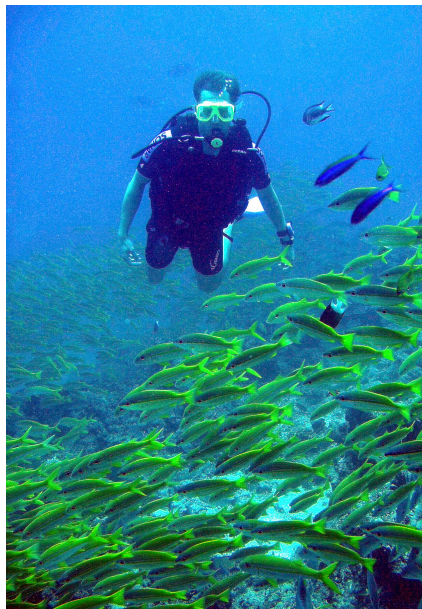
Declaration of originality

I declare this thesis to be wholly my own work, unless stated otherwise. No part of this thesis has been used in the fulfilment of any other degree.

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