



Numerical Study on Some Rheological Problems of Fibre Suspensions

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Declaration

I declare that this thesis is based on my own research work in past years and it does not incorporate any material previously submitted for a degree or diploma in any university. To my knowledge and belief it does not contain any material previously published or written by another person where due reference is not made in the text.

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Abstract

This thesis deals with numerical investigations on some rheological problems of fibre suspensions: the fibre level simulation of non-dilute fibre suspensions in shear flow; the numerical simulation of complex fibre suspension flows and simulating the particle motion in viscoelastic flows. These are challenging problems in rheology.

Two numerical approaches were developed for simulating non-dilute fibre suspensions from the fibre level. The first is based on a model that accounts for full hydrodynamic interactions between fibres, which are approximately calculated as a superposition of the long-range and short-range hydrodynamic interactions. The long-range one is approximated by using slender body theory and includes infinite particle interactions. The short-range one is approximated in terms of the normal lubrication forces between close neighbouring fibres. The second is based on a model that accounts only for short-range interactions, which comprise the lubrication forces and normal contact and friction forces. These two methods were applied to simulate the microstructure evolution and rheological properties of non-dilute fibre suspensions.

The Brownian configuration method was combined with the highly stable finite element method to simulate the complex flow of fibre suspensions. The method is stable and robust, and can provide both micro and macro information. It does not require any closure approximations in calculating the fibre stress tensor and is more efficient and variance reduction, compared to CONNFFESSITT, for example. The flow of fibre suspensions past a sphere in a tube and the shear induced fibre migration were successfully simulated using this method

The completed double layer boundary element method was extended to viscoelastic flow cases. A point-wise solver was developed to solve the constitutive equation point by point and the fixed least square method was employed to interpolate and differentiate data locally. The method avoids volume meshing and only requires the boundary mesh on particle surfaces and data points in the flow domain. A sphere settling in the Oldroyd-B fluid and a prolate spheroid rotating in shear flow of the Oldroyd-B fluid were simulated. Based on the simulated orbit of a prolate spheroid in

shear flow, a constitutive model for the weakly viscoelastic fibre suspensions was proposed and its predictions were compared with some available experimental results.

All simulated results are in general agreement with experimental and other numerical results reported in literature. This indicates that these numerical methods are useful tools in rheological research.

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