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Maximum Entropy Regularisation Applied to Ultrasonic Image Reconstruction

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Abstract

Image reconstruction, in common with many other inverse problems, is often mathematically ill-posed in the sense that solutions are neither stable nor unique. Ultrasonic image reconstruction is particularly notorious in this regard, with narrow transducer bandwidths and limited - sometimes sparsely sampled apertures posing formidable difficulties for conventional signal processing. To overcome these difficulties, some form of regularisation is mandatory, whereby the ill-posed problem is restated as a closely related, well-posed problem, and then solved uniquely.

This thesis explores the application of maximum entropy (MaxEnt) regularisation to the problem of reconstructing complex-valued imagery from sparsely sampled coherent ultrasonic field data, with particular emphasis on three-dimensional problems in the non-destructive evaluation (NDE) of materials. MaxEnt has not previously been applied to this class of problem, and yet in comparison with many other approaches to image reconstruction, it emerges as the clear leader in terms of resolution and overall image quality. To account for this performance, it is argued that the default image model used with MaxEnt is particularly meaningful in cases of ultrasonic scattering by objects embedded in homogeneous media.

To establish physical and mathematical insights into the forward problem, linear equations describing scattering from both penetrable and impenetrable objects are first derived using the Born and physical optics approximations respectively. These equations are then expressed as a shift-invariant computational model that explicitly incorporates sparse sampling. To validate this model, time-domain scattering responses are computed and compared with analytical solutions for a simple canonical test case drawn from the field of NDE. The responses computed via the numerical model are shown to accurately reproduce the analytical responses

To solve inverse scattering problems via MaxEnt, the robust Cambridge algorithm is generalised to the complex domain and extended to handle broadband (multiple-frequency) data. Two versions of the augmented algorithm are then compared with a range of other algorithms, including several linearly regularised

algorithms and lastly, due to its acknowledged status as a competitor with MaxEnt in radio-astronomy, the non-linear CLEAN algorithm. These comparisons are made through simulated 3-D imaging experiments under conditions of both complete and sparse aperture sampling with low and high levels of additive Gaussian noise.

As required in any investigation of inverse problems, the experimental confirmation of algorithmic performance is emphasised, and two common imaging geometries relevant to NDE are selected for this purpose. In monostatic synthetic aperture imaging experiments involving side-drilled holes in an aluminium plate and test objects immersed in H_2O, MaxEnt image reconstruction is demonstrated to be robust against grating-lobe and side-lobe formation, in addition to temporal bandwidth restriction. This enables efficient reconstruction of 2-D and 3-D images from small numbers of discrete samples in the spatial and frequency domains.

The thesis concludes with a description of the design and testing of a novel polyvinylidene fluoride (PVDF) bistatic array transducer that offers advantages over conventional point-sampled arrays in terms of construction simplicity and signal-to-noise ratio. This ultra-sparse orthogonal array is the only one of its kind yet demonstrated, and was made possible by MaxEnt signal processing.

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List of Abbreviations

ADC	Analogue-to-Digital Converter
CLS	Constrained Least Squares
CPU	Central Processing Unit
DSP	Digital Signal Processing
DOA	Direction of arrival
DOF	Degree(s) of Freedom
EM	Expectation Maximisation
FFT	Fast Fourier Transform
FLOPS	Floating Point Operations per Second
GOF	Goodness of Fit
LS	Least-Squares
LMS	Least-Mean-Square
MAP	Maximum *a posteriori*
MNLS	Minimum-Norm Least-Squares
NDE	Non-Destructive Evaluation
NMR	Nuclear Magnetic Resonance
PAD	Positive Additive Distribution
PRF	Pulse Repetition Frequency
PSF	Point Spread Function
PVDF	Polyvinylidene-Fluoride
RF	Radio Frequency
ROI	Region of Interest
ROS	Region of Support
SAFT	Synthetic Aperture Focussing Technique
SNR	Signal-to-Noise Ratio
SVD	Singular Value Decomposition
TOFD	Time of Flight Diffraction

Glossary

A-scan	A broadband waveform representing the Amplitudes of waves reflected and diffracted from insonified objects.
Bistatic	A term describing any data acquisition geometry in which the transmitter and receiver experience relative motion.
B-scan	An unfocussed (usually two-dimensional) image of reflectivity formed by stacking multiple A-scans side by side and rendering the positive and negative amplitudes according to a grey scale.
Interrupt	A hardware signal used to temporarily redirect program execution to service the occurrence of some asynchronous external event.
Iso-surface	An extension of the concept of contour plotting to three dimensions, whereby data equalling or exceeding a set of threshold values are indicated by a corresponding set of geometric surfaces.
Monostatic	Any data acquisition geometry in which the transmitter and receiver remain stationary relative to each other.
Multiplexer	A device for selecting one or a combination of signals from a large number of input signals.
Trigger	A digital signal used to initiate a series of events.
Voxel	An extension of the concept of two-dimensional picture elements (pixels) to three-dimensional (volumetric) images.