STATIC SHAPE CONTROL OF LAMINATED COMPOSITE PLATE SMART STRUCTURE USING PIEZOELECTRIC ACTUATORS ©

THE UNIVERSITY OF SYDNEY

Clinton Yat Kuan CHEE
BE (Mech), BSc (Math/Phys)

A thesis submitted in fulfilment
of the requirements for the degree of
Doctor of Philosophy

Department of Aeronautical Engineering
The University of Sydney

September 2000

ABSTRACT

The application of static shape control was investigated in this thesis particularly for a composite plate configuration using piezoelectric actuators. A new electro-mechanically coupled mathematical model was developed for the analysis and is based on a third order displacement field coupled with a layerwise electric potential concept. This formulation, TODL, is then implemented into a finite element program. The mathematical model represents an improvement over existing formulations used to model intelligent structures using piezoelectric materials as actuators and sensors. The reason is TODL does not only account for the electro-mechanical coupling within the adaptive material, it also accounts for the full structural coupling in the entire structure due to the piezoelectric material being attached to the host structure. The other significant improvement of TODL is that it is applicable to structures which are relatively thick whereas existing models are based on thin beam / plate theories. Consequently, transverse shearing effects are automatically accounted for in TODL and unlike first order shear deformation theories, shear correction factors are not required.

The second major section of this thesis uses the TODL formulation in static shape control. Shape control is defined here as the determination of shape control parameters, including actuation voltage and actuator orientation configuration, such that the structure that is activated using these parameters will conform as close as possible to the desired shape. Several shape control strategies and consequently algorithms were developed here. Initial investigations in shape control has revealed many interesting issues which have been used in later investigations to improve shape controllability and also led to the development of improved algorithms. For instance, the use of discrete actuator patches has led to greater shape controllability and the use of slopes and curvatures as additional control criteria have resulted in significant reduction in internal stresses. The significance of optimizing actuator orientation and its relation to piezoelectric anisotropy in improving shape controllability has also been presented. Thus the major facets of shape control has been brought together and the algorithms developed here represent a comprehensive strategy to perform static shape control.

ACKNOWLEDGMENTS

I would like to thank the many people who have been very supportive and generous in offering their time and assistance during my PhD program. First of all the greatest thanks to my supervisor Dr. Liyong Tong and associate supervisor Professor Grant Steven for their advice, support, numerous interesting ideas, wisdom, encouragement and patience without which the following work would not have been possible.

Also thanks to the staff members of the Department of Aeronautical Engineering, specifically, Dr. Osvaldo Querin for suggesting the use of algebraic integration and Dr. Douglas Auld for his technical support especially for things related to the computer network and resources.

In addition, thanks to all the staff of the Department of Aeronautical Engineering for their support throughout various stages of this PhD course, in particular Mr. Jehangir Madhani, Ms Yvonne Witting, Mr. Greg. Cumberland and Mr. Greg Riviere. And my colleagues in FEARC for the interesting discussions and especially Dr. Manuel Garcia, Dr. Ping Tan and Mr. Eugene Wu for sparking new ideas and techniques.

The Finite Element software PIEZFEP and SCONFEP and its associated files are mainly written in the Fortran90 language and numerical computations are performed on a Pentium III 500 MHz personal computer. The author would like to acknowledge and thank the support for the provisions of all the necessary research tools by the Department of Aeronautical Engineering - University of Sydney, the Australian Postgraduate Award & Supplemental Scholarship and the Australian Research Council under the large grant scheme (Grant No. A89905990).

PREFACE

This thesis is the accumulation of research work conducted in the field of smart structures technology. Since its inception, a little over ten years ago, it had attracted great interest among scientists and engineers in related fields and sparked the curiosity of those who are in less related fields. The potential applications are countless. The so called smart or intelligent structure can be integrated onto existing structures using existing materials, such as piezoelectric material which was discovered more than a century ago. Perhaps this is what makes it so exciting, the fact that previously passive structures can now be manipulated while in operation - hence its other name "active structures". Of course, the technological motivation in this field has also driven new innovations such as the creation of new types of piezoelectric materials with even higher actuation authority than previous ones which have been used for decades.

This thesis represents a small contribution to the total body of on-going research work in this field. The new contribution in this thesis is twofold - the first is the development of a mathematical formulation that models the adaptive and the non-adaptive parts of the structure simultaneously and impartially, thereby incorporating not only the piezoelectric effects but as well as the physical presence of the adaptive material (e.g. stresses). In this sense, the adaptive material is truly recognized as being a constituent of the structure instead of a separate external component providing sensing or actuating capability. The second contribution is the development of shape control methods with the purpose of determining the necessary actuation in order to change the shape of the structure to conform to a desired shape. The mathematical formulation developed in the first part will be coded into a finite element software which will then be used for shape control calculations in the second part. The finite element software would be able to model laminated composite plate structures, in which any part of the structure can be modeled as being piezoelectric or non-piezoelectric materials.

Chapter 1 sets the scene for this research work. It introduces the field of smart structures, its engineering / technological significance and the variety of applications. Since this field is only limited by one's imagination, the introduction is not meant to be a comprehensive coverage of the research in this field. The various kinds of adaptive materials are described together with their advantages and disadvantages. The piezoelectric material, which is the preferred adaptive material in this research work, is described in greater detail in the following chapter. The rest of the chapter then surveys the advances of smart structures technology including various mathematical models that have been used. The chapter concludes by explaining the motivation and relevance behind this work and setting out the objectives of this research work.

In Chapter 2 the main adaptive material, piezoelectric materials, are explained in greater detail. It examines the origin of the linear theory of piezoelectricity by looking at the discovery of the direct and the converse piezoelectric effects and how the mechanical and the electrical phenomena was combined. The constitutive equations were also shown to be able to be developed quite naturally from a thermodynamic perspective and from this, some of the significance of the coupling properties emerged. The crystal structure received a passing mention, sufficient to explain the particular type of crystal structure of the piezoelectric material chosen to be used in this research. Issues such as non-linearity of the piezoelectric material were also presented. Following this, the practical properties of the piezoelectric material in relation to their use in smart structures is described as well as listing the main physical forms of piezoelectric materials that have currently been used.

Before going into the derivation of the mathematical model, the Chapter 3 first describes the existing methods and their shortfall for modeling intelligent composite plate structures. Then the high order displacement theory and the layerwise concept is explained. The mathematical formulation, known as TODL, is developed using HODT for mechanical and layerwise for the electrical components. The analytical formulation, derived from variational energy principles, fully integrates the entire smart structure because it models both piezoelectric and non-piezoelectric materials taking into account all mechanical, electrical and piezoelectric effects. Although the general formulation was done for a plate structure, a beam structure formulation was shown to be easily derived from it. The analytical formulation is complete for both the beam and the plate in the sense direct solutions are obtainable if desired. As an illustration, direct analytical solutions for two beam examples were presented.

The TODL plate analytical formulation developed in Chapter 3 is implemented into a Finite Element (FE) formulation in Chapter 4. Since FE is a numerical technique that has been established for over several decades, this thesis will assume that the reader is familiar with the fundamental concepts of FE. Thus Chapter 4 quickly launches into the FE formulation of TODL where the main goal is to show how the new features in TODL such as the 11 mechanical variables and the electric potential layerwise concept fit into the FE formulation. Although there are many new variables, their names and symbols have been kept consistent with traditional FE notation as far as possible. The TODL-FE is developed for an 8-node rectangular plate element since this element is quite adequate for the investigations in this thesis. The formulation can be readily modified to accommodate other types of elements. In addition, the FE implementation using a 2-node Hermitian beam element is also included.

The TODL-FE beam formulation is verified in Chapter 5 with numerical and experimental results found in the literature. The verification included conventional non-piezoelectric structures, structures with piezoelectric materials used as actuators, and structures with actuating / sensing capabilities. Following this is the presentation of new results of investigation of the structural effects using the beam TODL-FE. This include the effects on displacements, curvature and sensor voltages due to the rotation of the actuators, their selected positions and the electric field polarity. Finally it solved a cantilever beam example and compares with its exact solution derived in Chapter 3.

Chapter 6 is the counterpart of Chapter 5 for the plate TODL-FE formulation. The verification and the investigations of the composite plate model are more extensive since the development of the TODL-FE for the composite plate structure is one of the main focus of this thesis. Verification of non-piezoelectric structures have shown the capability of TODL for analyzing moderately thick structures where results correlate well with 3D elasticity exact solutions. Verification with structures containing piezoelectric materials are also in good agreement. Exceptions to this arise most notably in structures with actuator patch configurations where the difference between this TODL formulation and others based on the Induced Strain concept, is reflected in the numerical results. New investigations for structural actuation included a parametric study on actuator locations, electric field directions, actuator orientations, anisotropy of piezoelectric actuators and how they affect shape characteristics such as bending and twist. The understanding of these factors is significant in the development of shape control algorithms in subsequent chapters. The variety of verification and new examples tested successfully testify to the robustness of TODL-FE.

Chapter 7 onwards represent the second major part of this thesis which is the application of shape control using piezoelectric actuators. This chapter uses displacement as the only criterion to measure the conformity between the actual and the desired shapes. This thesis defines shape control as the determination of actuation parameters such as electric voltages or structural parameters such as actuator orientation angle which are required to manipulate the actual structure to conform to a desired shape. The SC algorithms in Chapter 7 would calculate these parameters given the desired shape. A novel SC algorithm, BVD, was developed here and compared with some generic optimization algorithms. The limited amount of examples available in the literature has been verified successfully here. In addition, new investigative studies reveal various issues such as the fact that not all desired shapes can be generated exactly and the degree of shape controllability depends on the number of independent actuator patches. The chapter concludes with a manual process of actuator location optimization by using a "Displacement Patch Insensitivity" index.

Chapter 8 extends the concept of displacement based shape control (DBSC) to using higher order attributes of shape such as slopes (SDSC) and curvatures (CDSC). The limitation of DBSC were identified, viz. bumpiness of the resultant structure and the severe localized stresses due to having discrete patch actuators. Another novel algorithm called PBVD was developed for SDSC and CDSC each; mainly because generic multi-criteria optimization algorithms are not suited to this application due to conflicting effects of displacement, slope and curvature measures. The PBVD method, designed specifically for shape control, is more robust and applicable because it allows the user to specify the degree of conformity between the actual and desired shapes. Results show that PBVD is able to smooth out the bumpiness that occur in DBSC. More importantly it shows significant reduction in stresses thus implying that in SC, applying the voltage configuration calculated by PBVD does not generate as much stress as other methods of SC.

The optimization of the angle / orientation of piezoelectric actuators in SC, or AOSC is considered in Chapter 9. Several brief findings from chapter 6 in regards to the effects of actuator orientation and material anisotropy on the shape of the structure, have been found to be useful in understanding the concepts of actuator orientation optimization in SC. Another iterative-heuristic algorithm called, BOD was developed for this particular type of shape control in which the criteria is the displacement measure while the output is the actuator orientation configuration. The advantage of performing AOSC, as shown by the results of using BOD, is a clear improvement on the shape controllability for some structural configurations.

Finally the thesis concludes with the major findings discovered in this research work and highlights the mathematical model and SC algorithms developed here.

NOTATION

Symbols

General Variables

ε	mechanical strain
Λ	actuation strain
σ	mechanical stress
ϕ	general electric potential vector
$\phi(x,y)$	interface functions
χ	electric permittivity
c	mechanical stiffness
d	piezoelectric strain constant
D	electric displacement
e	piezoelectric stress constant
E	electric field
h	total thickness/height of composite
K	kinetic energy
L	Lagrangian
L_{id}, L_{iu}	layerwise functions
P	potential energy
Rc	mechanical anisotropy
Rd	piezoelectric anisotropy
T	temperature
u	general displacement vector
(U,V,W)	total displacement components
V	voltage
W	work

Finite Element Variables

B	matrices relating variables to FE nodal variables

D	definition matrices of some variable with derivatives
F, Q	mechanical and electrical load vectors
J	Jacobian matrix
$Kuu, Ku\phi, K\phi u, K\phi\phi$	electro-mechanical stiffness matrices incorporating piezoelectric effects
N^{q8}	8-node serendipity element shape functions
Q	2D transformation matrix - rotation about z-axis
R	transformation between engineering and infinitesimal strain tensor
T, T_{3D}	3D transformation matrix - rotation about z-axis
(ξ,η)	local planar coordinate of element
(x,y)	global planar coordinate system

Shape Control Variables

$w, S_x, S_y, K_{xx}, K_{yy}, K_{xy}$	Transverse displacement, slopes and curvatures respectively.
$C^w, C^{Sx}, C^{Sy}, C^{Kxx}, C^{Kyy}, C^{Kxy}$	Influence coefficient matrix for displacement, slopes and curvatures respectively, in response to electric voltage.
c_{ij}	Coefficient based on the displacement measure of the previous iteration.
γ	Gamma factor - intermediate cost function in BVD.
Δw	Least squares difference of transverse displacement.
$\Delta Kxx, \Delta Kyy, \Delta Kxy$	Area integrated squared difference of curvatures.
$\Delta Sx, \Delta Sy$	Area integrated squared difference of slopes.
Lnm	Lshape normalized by 4*Nn
LSd	Displacement tolerance of PBVD
Lshape	Self-normalized displacement measure.
LSw, LSSx, LSSy, LSKxx, LSKyy, LSKxy	Normalized least squares difference measure between actual and desired shape of the structure.
N_K	Total number of curvature points.
N_n	Total number of nodes of the FE model of a structure

N_p	Total number of active patches.
N_S	Total number of slope points.
p	Slope - displacement tolerance factor.
s	Slope tolerance factor.
SR	Selection Rate of patches in BVD.

Symbols which have only appeared briefly in this thesis are not included here. Also due to the large number of quantities, it is unavoidable to have some symbols representing more than one quantity. In such cases, it is important to be aware of the context in which the symbols appear. The list of symbols here is divided into several sections to assist in identifying the context of the symbols.

Acronyms - Glossary

AOSC	*Actuator Orientation Shape Control* - shape control that optimizes the orientation angle of the piezoelectric actuators.
BOD	*Buildup Orientation Distribution* - shape control algorithm developed in this research to optimize actuator orientation configuration using displacement measures.
BVD	*Buildup Voltage Distribution* - shape control algorithm developed in this research to optimize voltage configuration using displacement measures.
CDSC	*Curvature-Displacement based Shape Control* - shape control that uses curvatures and transverse displacement to match the desired shape.
CLPT	*Classical Laminated Plate Theory* - well known model for composite laminates.
CPI	*Curvature Patch Insensitivity index* - a measure of insensitivity of actuator patches to affect curvature, given input voltage.
DAP	*Directionally Attached Piezoelectric* - technique of attaching actuators to preferentially enhance certain effect such as twist.
DBSC	*Displacement based Shape Control* - shape control that uses transverse displacement to match the desired shape.
d.o.f.	*Degree of Freedom*
DPI	*Displacement Patch Insensitivity index* - a measure of insensitivity of actuator patches to affect displacement, given input voltage.
DQM	*Dual Quadratic Minimization* - technique of finding the minimum point between

	two intersecting, upright parabola.
EPA	*Extensional Piezoelectric Anisotropy* - ratio of d_{31}:d_{32}
ERF	*Electro-Rheological Fluid* - a fluid with coupled electrical and rheological properties.
FE/FEA/FEM	*Finite Element / ....Analysis / .... Method* - computational analysis method.
FOSDT	*First Order Shear Deformation Theory*
GA	*Genetic Algorithm* - an optimization routine based on the principles of biological evolution.
HODT	*Higher Order Displacement Theory* - displacement field to model deformations with order greater than 1.
IC	*Influence Coefficients* - coefficients relating variables with a linear relationship such as voltages and displacements.
IDE	*Inter-Digitated Electrodes* - a layout of electrodes pioneered by the MIT group.
LCW	*Lo, Christensen, Wu* (displacement theory) - third order displacement field used by the authors mentioned.
LDU	*LDU Decomposition* - a mathematical technique to decompose a matrix.
LLS	*Linear Least Squares* - a technique that minimizes the sum of least square difference between functions which are linear.
LPCE	*Linear Piezoelectric Constitutive Equations*
LS	*Least Squares*
McLLS	*Multi Criteria Linear Least Squares* - the LLS method applied for multi-criteria cases.
MCSC	*Multi Criteria Shape Control* - shape control application based on using more than one criteria.
MIMO	*Multi Input Multi Output* - control strategy having more than one input and output.
NPCE	*Non-linear Piezoelectric Constitutive Equations*
PBVD	*Perturbation Buildup Voltage Distribution method* - One of the heuristic methods developed in this thesis, based on BVD, for shape control applications.
PFC	*Piezoelectric Fiber Composites* - composite in which its fibres are piezoelectric material.
PIEZFEP	*Piezoelectric Finite Element Program* - developed in this thesis using the TODL-FE formulation.

PVDF	*Polyvinylidene Fluoride* - a particular type of piezoelectric polymeric material.
PZT	*Lead Zirconate Titanate* - a particular type of piezoelectric ceramic material.
RHOD	*Reddy's HOD* - Reddy's (1984) version of the Higher Order Displacement Theory.
S/A	*Sensors and Actuators*
SA	*Simulated Annealing* - a minimization technique for multidimensional and multi-modal objective functions.
SC	*Shape Control* - the application of manipulating a structure to induce conformity to a desired shape.
SCF	*Shear Correction Factor* - a correction factor used with FOSDT.
SCONFEP	*Shape Control Finite Element Program* - implements the shape control algorithms developed in this thesis and uses PIEZFEP.
SDSC	*Slope-Displacement based Shape Control* - shape control that uses slopes and transverse displacement to match the desired shape.
SMA	*Shape Memory Alloy* - adaptive material which changes shape in response to temperatures.
SPA	Shear Piezoelectric Anisotropy - ratio of $d_{15}:d_{24}$
SPEC	*Successive Peak Error Correction* - Heuristic optimization algorithm for shape control developed by Subramanian & Mohan(1996).
SPI	*Slope Patch Insensitivity index* - a measure of insensitivity of actuator patches to affect slope, given input voltage.
SR	*Selection Rate* - parameter of BVD and BOD.
SSC	*Static Shape Control* - shape control application in a static manner.
TODL	*Third Order Displacement Layerwise* - The analytical formulation developed in this thesis that incorporates a third order displacement field for mechanical field and layerwise concept for electrical field.
TODL-FE	*Third Order Displacement Layerwise Finite Element* -The finite element program written by the author based on the TODL formulation.
UCS	*Unified Condensed Storage* - a storage method developed in this research as part of the implementation of TODL-FE.

LIST OF PUBLICATIONS

Chee C., L. Tong & G. Steven. 1998. “A Review on the Modelling of Piezoelectric Sensors and Actuators Incorporated in Intelligent Structures”, *Journal of Intelligent Material Systems and Structures*, 9(1):3-19.

Chee C., L. Tong & G. Steven. 1999a. “A Mixed Model for Composite Plates With Piezoelectric Actuators and Sensors”, *ACAM99, The Second Australasian Congress On Applied Mechanics* 10-12 February 1999, Canberra, Australia.

Chee C., L. Tong & G. Steven. 1999b. "A Mixed Model for Composite Beams with Piezoelectric Actuators and Sensors", *Smart Materials and Structures,* 8:417-432.

Chee C., L. Tong & G. Steven. 2000a. "Static Shape Control of Composite Plates Using a Slope-Displacement Based Algorithm", *41st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference,* Atlanta, Georgia, U.S.A., April 3rd - 7th, 2000.

Chee, C., L. Tong & G. Steven. Jun 2000b. “A Mixed Model for Adaptive Composite Plates with Piezoelectric for Anisotropic Actuation”, *Computers and Structures,* 77(3):253-268.

Chee, C., L. Tong & G. Steven. 2000c. "A Buildup Voltage Distribution (BVD) algorithm for shape control of smart plate structures", *Computational Mechanics*, 26(2):115-128.

Chee, C., L. Tong & G. Steven. 2000d. "Shape Functions Generation using Macsyma", *Communications in Numerical Methods in Engineering,* 16(10):705-719.

C.Chee, L.Tong & G.P.Steven, "Static Shape Control of Composite Plates Using a Slope-Displacement Based Algorithm", *AIAA Journal,* (under review).

C.Chee, L.Tong & G.P.Steven, "Static Shape Control of Composite Plates Using a Curvature-Displacement Based Algorithm", *International Journal of Solids and Structures,* (under review).

C.Chee, L.Tong & G.P.Steven, "Piezoelectric Actuator Orientation Optimization for Static Shape Control of Composite Plates", *International Journal of Numerical Methods for Engineering*, (under review).

TABLE OF CONTENTS