

Image Thresholding Technique
Based On
Fuzzy Partition And Entropy Maximization

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Abstract

Thresholding is a commonly used technique in image segmentation because of its fast and easy application. For this reason threshold selection is an important issue. There are two general approaches to threshold selection. One approach is based on the histogram of the image while the other is based on the gray scale information located in the local small areas. The histogram of an image contains some statistical data of the grayscale or color ingredients. In this thesis, an adaptive logical thresholding method is proposed for the binarization of blueprint images first. The new method exploits the geometric features of blueprint images. This is implemented by utilizing a robust windows operation, which is based on the assumption that the objects have “C” shape in a small area. We make use of multiple window sizes in the windows operation. This not only reduces computation time but also separates effectively thin lines from wide lines. Our method can automatically determine the threshold of images. Experiments show that our method is effective for blueprint images and achieves good results over a wide range of images.

Second, the fuzzy set theory, along with probability partition and maximum entropy theory, is explored to compute the threshold based on the histogram of the image. Fuzzy set theory has been widely used in many fields where the ambiguous phenomena exist since it was proposed by Zadeh in 1965. And many thresholding methods

have also been developed by using this theory. The concept we are using here is called fuzzy partition. Fuzzy partition means that a histogram is parted into several groups by some fuzzy sets which represent the fuzzy membership of each group because our method is based on histogram of the image . Probability partition is associated with fuzzy partition. The probability distribution of each group is derived from the fuzzy partition. Entropy which originates from thermodynamic theory is introduced into communications theory as a commonly used criteria to measure the information transmitted through a channel. It is adopted by image processing as a measurement of the information contained in the processed images. Thus it is applied in our method as a criterion for selecting the optimal fuzzy sets which partition the histogram.

To find the threshold, the histogram of the image is partitioned by fuzzy sets which satisfy a certain entropy restriction. The search for the best possible fuzzy sets becomes an important issue. There is no efficient method for the searching procedure. Therefore, expansion to multiple level thresholding with fuzzy partition becomes extremely time consuming or even impossible.

In this thesis, the relationship between a probability partition (PP) and a fuzzy C-partition (FP) is studied. This relationship and the entropy approach are used to derive a thresholding technique to select the optimal fuzzy C-partition. The measure of the selection quality is the entropy function defined by the PP and FP. A necessary condition of the entropy function arriving at a maximum is derived. Based on this condition, an efficient search procedure for two-level thresholding is derived, which

makes the search so efficient that extension to multilevel thresholding becomes possible. A novel fuzzy membership function is proposed in three-level thresholding which produces a better result because a new relationship among the fuzzy membership functions is presented. This new relationship gives more flexibility in the search for the optimal fuzzy sets, although it also increases the complication in the search for the fuzzy sets in multi-level thresholding. This complication is solved by a new method called the "Onion-Peeling" method. Because the relationship between the fuzzy membership functions is so complicated it is impossible to obtain the membership functions all at once. The search procedure is decomposed into several layers of three-level partitions except for the last layer which may be a two-level one. So the big problem is simplified to three-level partitions such that we can obtain the two outmost membership functions without worrying too much about the complicated intersections among the membership functions.

The method is further revised for images with a dominant area of background or an object which affects the appearance of the histogram of the image. The histogram is the basis of our method as well as of many other methods. A "bad" shape of the histogram will result in a bad thresholded image. A quadtree scheme is adopted to decompose the image into homogeneous areas and heterogeneous areas. And a multi-resolution thresholding method based on quadtree and fuzzy partition is then devised to deal with these images.

Extension of fuzzy partition methods to color images is also examined. An adaptive thresholding method for color images based on fuzzy partition is proposed which can determine the number of thresholding levels automatically.

This thesis concludes that the "C" shape assumption and varying sizes of windows for windows operation contribute to a better segmentation of the blueprint images. The efficient search procedure for the optimal fuzzy sets in the fuzzy-2 partition of the histogram of the image accelerates the process so much that it enables the extension of it to multilevel thresholding. In three-level fuzzy partition the new relationship presentation among the three fuzzy membership functions makes more sense than the conventional assumption and, as a result, performs better. A novel method, the "Onion-Peeling method", is devised for dealing with the complexity at the intersection among the multiple membership functions in the multilevel fuzzy partition. It decomposes the multilevel partition into the fuzzy-3 partitions and the fuzzy-2 partitions by transposing the partition space in the histogram. Thus it is efficient in multilevel thresholding. A multi-resolution method which applies the quadtree scheme to distinguish the heterogeneous areas from the homogeneous areas is designed for the images with large homogeneous areas which usually distorts the histogram of the image. The new histogram based on only the heterogeneous area is adopted for partition and outperforms the old one. While validity checks filter out the fragmented points which are only a small portion of the whole image. Thus it gives good thresholded images for human face images.

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It has taken me three years to finish this thesis and during this time I became paralyzed after an accident. Running a program on my computer is not a major problem but typing up the results of my research with a stick attached to one hand became a big burden because I lost the good use of my hand. Also, the numerous health problems associated with the paralysis have always been in the way. This thesis could not have been completed without all the help and support I received from my supervisor Professor Hong Yan, my colleagues at the Image Processing Laboratory, and some of the staff at the School of Electrical and Information Engineering, the Engineering Faculty and the libraries at the University of Sydney.

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