A Parallel Navier–Stokes Solver for Natural Convection and Free Surface Flow

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Doctor of Philosophy

by

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Abstract

A parallel numerical method has been implemented for solving the Navier–Stokes equations on Cartesian and non-orthogonal meshes. To ensure the accuracy of the code first, second and third order differencing schemes, with and without flux-limiters, have been implemented and tested. The most computationally expensive task in the code is the solution of linear equations, and a number of linear solvers have been tested to determine the most efficient. Krylov space, incomplete factorisation, and other iterative and direct solvers from the literature have been implemented, and have been compared with a novel black-box multigrid linear solver that has been developed both as a solver and as a preconditioner for the Krylov space methods. To further reduce execution time the code was parallelised, after a series of experiments comparing the suitability of different parallelisation techniques and computer architectures for the Navier–Stokes solver.

The code has been applied to the solution of two classes of problem. Two natural convection flows were studied, with an initial study of two dimensional Rayleigh–Bénard convection being followed by a study of a transient three dimensional flow, in both cases the results being compared with experiment.

The second class of problems modelled were free surface flows. A two dimensional free surface driven cavity, and a two dimensional flume flow were modelled, the latter being compared with analytic theory. Finally a three dimensional ship flow was modelled, with the flow about a Wigley hull being simulated for a range of Reynolds and Froude numbers.

Declaration

I hereby declare that the work presented in this thesis is solely my own work and that to the best of my knowledge the work is original except where otherwise indicated by reference to other authors. No part of this work has been submitted for any other degree or diploma.

Stuart Norris

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Contents

List of Symbols

Cr	Courant number	$U\,\Delta t/\ell$
c_p	constant pressure specific heat	
d	diffusion flux	
Fo	Fourier number	$\nu t/\ell^2$
Fr	Froude number	$U/\sqrt{g\ell}$
g	acceleration due to gravity	
h	specific enthalpy	
k	thermal conductivity	
m	mass flux	
m_α	mass fraction of species α	
ℓ	length	
N	number of equations	
n	number of processors	
Nu	Nusselt number	$q\,\ell/k\,\Delta T$
O	order	
$\mathcal{P}$	multigrid prolongation operator	
p	pressure	
Pe	Péclet number	$U\ell/\Gamma$
Pr	Prandtl number	ν/α
$\mathcal{R}$	multigrid restriction operator	
r	residual	
Ra	Rayleigh number	$g\beta\Delta T\ell^3/\nu\alpha$
Re	Reynolds number	$U\ell/\nu$
T	temperature	
$\mathbf{u}$	velocity	
u	x axis component of velocity	
v	y axis component of velocity	
w	z axis component of velocity	
α	thermal diffusivity	$k/\rho c_p$
β	coefficient of volumetric expansion for temperature	
ϵ	error	
Γ	diffusivity	
λ	wavelength	
μ	dynamic viscosity	
ν	kinematic viscosity	μ/ρ
Ω	volume	
ψ	streamfunction	
ρ	density	
τ	non-dimensional time	$\nu t/\ell^2$
θ	dimensionless temperature	$T/\Delta T$

Commonly Used Acronyms

ADI	Alternating Direction Implicit (linear solver)
BiCG	Bi-Conjugate Gradient (linear solver)
BiCGSTAB	Bi-Conjugate Gradient Stabilised (linear solver)
CFD	Computational Fluid Dynamics
CG	Conjugate Gradient (linear solver)
CGS	Conjugate Gradient Squared (linear solver)
CMSSL	Connection Machine System Scientific Library (numerics library)
FLOPS	Floating point Operations Per Second
FOU	First Order Upwind (discretisation scheme)
GMRES	General Minimalised Residual (linear solver)
HPF	High Performance Fortran
IC	Incomplete Cholesky factorisation (linear solver)
ICCG	Incomplete Cholesky–Conjugate Gradient (linear solver)
ILU	Incomplete Lower–Upper factorisation (linear solver)
K & R	Kernighan and Ritchie (the original dialect of C)
LDL	Lower Diagonal Lower^T factorisation (linear solver)
LU	Lower Upper factorisation (linear solver)
MAC	Marker And Cell
MG	Multi-Grid (linear solver)
MPI	Message Passing Interface (parallelisation library)
MSI	Modified Strongly Implicit procedure (linear solver)
MSOU	Monotonic Second Order Upwind (flux-limited discretisation scheme)
PDE	Partial Differential Equation
PVM	Parallel Virtual Machine (parallelisation library)
QMR	Quasi-Minimalised Residual (linear solver)
QUICK	Quadratic Upwind Interpolation for Convective Kinematics (discretisation scheme)
RBSOR	Red Black Successive Over Relaxation (linear solver)
SAXPY	A Scalar $a \times x + y$ operation
SIMPLE	Semi-Implicit Method for Pressure-Linked Equations
SD	Steepest Descent (linear solver)
SIP	Strongly Implicit Procedure, Stone's method (linear solver)
SOR	Successive Over Relaxation (linear solver)
SSOR	Symmetric Successive Over Relaxation (linear solver)
SOU	Second Order Upwind (discretisation scheme)
ULTRA	Universal Limiter for Tight Resolution and Accuracy (flux-limiter)
VOF	Volume Of Fluid