A *pragmatic* logic for the expressive conception of norms and values and the Frege-Geach problem

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Abstract: In this paper I intend to overcome the incompatibility -basically expressed by the Frege-Geach problem- between the expressive conception of norms and values and the applicability of logic to them, as well as the widespread scepticism about a possible "logic of attitudes" (Hale). To this end, I present a logic for the *expressive* conception of norms and values providing a solution to the Frege-Geach problem, that is immune to the ambiguities affecting two previous attempts by Blackburn (1984 and [1988] 1993). In particular, I use and extend a *pragmatic language* L_p, that is an extension of the language of standard propositional logic L, which is obtained by adding two categories of logical-pragmatic signs to the vocabulary of L: the *signs of pragmatic mood* (├, ⊙ and $\mathcal{H}$, standing for 'assertion', 'obligation' and 'approval', respectively) and the *pragmatic connectives* (~, ∩, ∪, ⊃, ≡). The wffs of L are called radical formulas (rf) of L_p. By applying the signs of pragmatic mood to the rfs, we obtain elementary sentential formulas (sf) (assertive, normative, evaluative) of L_p, that can be connected by means of the pragmatic connectives, so obtaining complex sfs. Every rf of L_p has a *truth value* and every sf has a *justification value*, which is defined in terms of the intuitive notion of *proof* and which depends on the truth value of its radical subformulas. In this language, the notions of *pragmatic* validity, compatibility, satisfiability and inference are defined and some criteria of pragmatic validity are given. Therefore, in L_p, it is possible to carry out inferences between norms and values expressively understood and, thus, to adequately formalize Geach's problematic inferences.

1. The debate over the logic of norms: a brief survey

According to the famous Humean thesis – strongly supported by logical positivists as well as accepted by the majority of logicians and philosophers – normative and evaluative sentences are not truth-apt. If one accepts such plausible thesis, the question of the applicability of logic to normative and evaluative sentences becomes – as von Wright (1983) stressed – the fundamental philosophical problem of norms (and values).

The relevance of this problem becomes clearer if we consider that a *rational* theory of norms (and values) is possible only if one can establish logical relations of consistency, inconsistency, equivalence and inference between norms (or values). Since logical connectives and relations are canonically defined in terms of the notions of truth and falsity, logic is traditionally understood as solely applying to truth-apt sentences; but normative and evaluative sentences, having purely prescriptive and evaluative (and not descriptive) functions, are not truth-evaluable. They are adequately analysable in purely *expressive* terms using the Frege-Reichenbach pragmatic model of sentential analysis (Frege 1879, 1893, 1918; Reichenbach 1947; Alchourrón and Bulygin 1981), according to which any sentence can be analysed in terms of a *sign of pragmatic mood* (that does not describe, but only points to the pragmatic mood in which a proposition is used: as asserted, assumed, prescribed, approved of, etc.) and a *radical* or *radical formula* (that expresses a true or false proposition). In this sense, norms and values are the result of a *prescriptive or evaluative use of language*; more precisely, they are standard descriptive propositions *used in a prescriptive or evaluative pragmatic way*; therefore, as prescriptive (or evaluative) illocutionary acts, norms and values can be justified or unjustified (correct or incorrect,…) but never true or false.

In the Frege-Reichenbach model signs of pragmatic mood and radical formulas can be compounded according to the following rule, known as 'Frege's principle':

Signs of pragmatic mood (a) can't be iterated and (b) can't fall under the scope of a logical connective, but they can only be applied to a (simplex or complex) radical formula, considered as a whole.

Therefore, due to the clause (b) of Frege's principle, the Frege-Reichenbach sentential analysis restricts logical relations solely to radical formulas, so excluding the possibility of their application to sentential formulas and, consequently, the possibility of a logic of pragmatic moods and of norms and values expressively understood.

The crucial problem affecting the expressive conception of norms and values is classically expressed in two formulations: the Jørgensen's *Dilemma* (1937-38), that is more familiar in deontic logic and legal literature, and the Frege-Geach problem (Geach 1965), that is more familiar in the ethical literature.

Following Alchourrón and Bulygin (1989), the Jørgensen's *Dilemma* can be put as follows: either logical connectives and relations can only be defined in terms of truth and falsity and, thus, they cannot apply to norms and a logic of norms cannot exist; or logic can be applied to norms, but then it must be possible to define logical connectives and relations without any reference to the notions of truth and falsity.

Sticking to the traditional conception of logic, many prominent deontic logicians and legal philosophers (Jørgensen 1937-38; Kelsen 1960, 1979; Hart 1961, von Wright 1963, Ross 1941, 1968; Alchourrón and Bulygin 1971, 1981, 1989; Weinberger 1977 and Makinson 1999) couldn't see any other solution but opting for the first horn of the *dilemma*, concluding that logic cannot be directly applied to norms. However, some of them (see chiefly Kelsen 1960, von Wright 1963) have suggested an *indirect* solution to the problem, in the light of the fundamental distinction between genuine normative and evaluative sentences - expressing norms and evaluations that can be justified or unjustified - and norm-describing and value-describing sentences, expressing truth-evaluable norm-propositions or value-propositions that describe the existence of a norm or a value[1]. In particular, a norm- or value-proposition is true if and only if the norm or the value it describes is justified/exist. By drawing on such one-to-one correspondence between norms (or values) and norm- (or value-) propositions, the '*indirect*' solution consists in indirectly 'mirroring' the logical relations holding between norm-propositions onto the norms that they describe; that is, one obtains the indirect applicability of logic to norms *via* its direct applicability to the corresponding propositions describing them.

In short, the conclusion shared by most legal philosophers and deontic logicians is the following:

1. norms (expressing prescriptions) are not truth-apt; so there are no logical relations between norms, but only between propositions about norms;
2. standard deontic logic (SDL) –as an *alethic* language- is not a logic of norms, but of norm-describing sentences or *norm-propositions*).
3. the construction of such a logic cannot appeal to some already given logic of norms (see McNamara and Prakken, 1999: Introduction)

However, the tenability of this position has been brought into question by von Wright (1983) and Alchourrón and Bulygin (1989). In particular, von Wright remarks that "there can be some interest in conceiving a logic of descriptive sentences of norms, that be distinct from standard propositional logic, only if it is able to capture some logical characteristics that are peculiar to norms and that standard propositional logic is not able to capture. But, if there exist 'some logical characteristics that are specific to norms', then it seems natural to admit the existence of a logic of

[1] Norm-propositions are norm-describing sentences, stating that a norm to such and such effect has been issued (exists). Norms and their corresponding norm-propositions can be homophonic: one and the same deontic expression like "it is obligatory to pay taxes" may be used either *prescriptively*, to impose an obligation or *descriptively*, to state that a norm exists in a given normative system (see von Wright 1963, 1999; Bulygin 1982). The same holds for evaluative sentences, that are to be distinguished from the homophonic sentences describing values.

norms". In the light of this objection, a logic of norms is to be presupposed as "more fundamental than a logic of descriptive sentences of norms".

On the other hand, against the possibility of *directly* applying logic to normative (or evaluative) sentences (expressively understood) it rises the Frege-Geach problem (Geach 1965). Geach intends to show that, assuming the Frege-Reichenbach sentential model, the following – intuitively valid- instantiation of *Modus Ponens* is invalid:

1. lying is wrong
2. <u>if lying is wrong, then getting your little brother to lie is wrong</u>
3. getting your little brother to lie is wrong

This argument can be semi-formalized in expressive terms as follows:

1. wrong (α)
2. <u>wrong (α) → wrong (β)</u>
3. wrong (β)

where α and β are radical formulas (describing, respectively, the act of lying and the act of getting your little brother to lie) and "wrong" stands for the pragmatic mood of disapproval.

The reason why this inference (as any other inference in which normative and evaluative sentences in expressive sense occur as elementary formulas or as components of complex formulas) cannot be validly carried out is that sentences 1. and 3. are understood as having an evaluative illocutionary force, while the antecedent and the consequent of sentence 2. cannot be expressively understood without violating the clause (b) of Frege's principle – that Geach calls "the Frege's point" - forbidding a sign of pragmatic mood to fall under the scope of a truth-functional connective. However, if the antecedent and the consequent of 2. are regarded as descriptive sentences, then premise 2. is syntactically correct, but the *Modus Ponens* cannot be applied anymore, since the antecedent of 2. and premise 1. are not the same sentence anymore.

In order to solve this problem without abandoning the expressive conception, it is necessary to turn to the second horn of Jørgensen's *dilemma*, trying to define logical connectives and relations without making any reference to the notion of truth and falsity. This means to abandon the traditional conception of logic, extending it beyond the realm of truth.

The possibility of developing a logic of norms prescriptively understood has been generically suggested by von Wright (1957) and Weinberger (1977), who maintained that logic can be extended to norms, given that norms could have a new pair of values - such as justified/unjustified, valid/invalid - analogous to the pair of truth values. But Alchourrón and Bulygin (1989; see also Bulygin 1982) rightly remarked that merely suggesting this thesis is not enough; it is necessary to justify it and, to that end, it is not enough to show the analogy between the pair of truth values and the pair of validity values, nor to consider that we actually carry out normative inferences at an informal level. What is required is the real construction of a logic that is based on an alternative definition of the connectives and of the fundamental logical relations, that be alien to the notions of truth and falsity.

I have provided such a logic for assertions and norms (expressively understood) in Dalla Pozza 1991; 1995; 1997. I have constructed a formal pragmatic language $\mathbf{L_p}$, by *pragmatically* extending the standard language of classical propositional logic **L**, by adding to the logical vocabulary of **L** two categories of *logical-pragmatic signs*: *signs of pragmatic mood* (├ for assertion and ⓞ for obligation) and *pragmatic connectives* (~, ∩, ∪, ⊃, ≡). Using this extended vocabulary, two kinds of wffs of $\mathbf{L_p}$ are recursively defined: *radical formulas* (corresponding to the wffs in **L**) and *sentential formulas* –both elementary (assertive and normative), that are obtained by applying the signs of pragmatic mood to the radical formulas (atomic and molecular); and complex (including mixed formulas), that are obtained by combining sentential formulas through the pragmatic connectives.

The semantics of $\mathbf{L_p}$ is the same as for **L** and it provides the interpretation only of the radical formulas, assigning a truth value to them and interpreting propositional connectives as truth functions, in the standard way. On the contrary, the *pragmatic rules* of $\mathbf{L_p}$ provide a *pragmatic evaluation* of sentential formulas, assigning to each of them a *justification value* (justified/unjustified) and interpreting the pragmatic connectives as *partial justification functions*, having a logical behaviour of intuitionist kind.

On this language, in addition to the standard semantic notions of validity, consistency (satisfiability) and inconsistency (unsatisfiability) for radical formulas, the pragmatic notions of validity, consistency, inconsistency and inference for sentential formulas (assertive, normative and mixed) are also defined.

It is clear that in such language the limits of the Frege-Reichenbach model are overcome, given that the pragmatic connectives permit the construction of complex sentential formulas in which the signs of pragmatic mood can occur under the scope of pragmatic connective, thus permitting the emergence of logical relations between non truth-apt sentences.

I will now set out my pragmatic language $\mathbf{L_p}$ (1997), also extending it by adding the logical-pragmatic sign of approval $\mathcal{H}$ - corresponding to Blackburn's operator H! (Blackburn [1988] 1993), for evaluative sentential formulas of expressive kind.

2. The pragmatic language L_p

We define $\mathbf{L_p}$ by specifying its syntactic, semantic and pragmatic structure by means of the following definitions.

DEFINITION 1. *Syntax*.

(i) *Vocabulary*.

Descriptive signs: Propositional letters: $p_1, p_2, p_3, \ldots$

Logical-semantic signs: Semantic connectives: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$.

Logical-pragmatic signs:

-Signs of pragmatic mood: $\vdash$ (assertion), Θ (obligation in a prescriptive sense), $\mathcal{H}$ (approval)

-Pragmatic connectives: $\sim$ (negation), $\cap$ (conjunction), $\cup$ (disjunction),

$\supset$ (implication), $\equiv$ (equivalence).

(ii) *Formation Rules.*

- *Radical formulas* (rf) recursively defined through the following formation rules (**RFR**):

RFR1. (Atomic): Every propositional letter is a rf.

RFR2. (Molecular): (i) Let α be a rf; then $\neg\alpha$ is a rf.

(ii) Let α_1 and α_2 be rfs; then, $\alpha_1 \wedge \alpha_2$, $\alpha_1 \vee \alpha_2$, $\alpha_1 \rightarrow \alpha_2$, $\alpha_1 \leftrightarrow \alpha_2$ are rfs.

-*Sentential formulas* (sf) -assertive, normative, evaluative and mixed- recursively defined through the following formation rules (**SRF**):

SFR1. (Elementary): Let α be a rf; then $\vdash\alpha$, $\Theta\alpha$ e $\mathcal{H}\alpha$ are sfs.

SFR2. (Complex): (i) Let δ be a sf; then $\sim \delta$ is a sf.

(ii) Let δ_1 and δ_2 be sfs; then $\delta_1 \cap \delta_2$, $\delta_1 \cup \delta_2$, $\delta_1 \supset \delta_2$, $\delta_1 \equiv \delta_2$ are sfs.

Let us introduce by definition the signs of pragmatic mood $\mathcal{P}$ (permitted), $\mathcal{F}$ (forbidden), $\mathcal{T}$ (tolerated) e $\mathcal{B}$ (disapproved) as follows:

D1. $\mathcal{P}\alpha =_{def.} \sim \mathcal{O} \neg \alpha$

D2. $\mathcal{F}\alpha =_{def.} \mathcal{O} \neg \alpha$

D3. $\mathcal{T}\alpha =_{def.} \sim\mathcal{H} \neg \alpha$

D4. $\mathcal{B}\alpha =_{def.} \mathcal{H} \neg \alpha$

From these definitions, the following *pragmatic* equivalences hold:

EP1. $\sim \mathcal{P}\alpha \equiv \sim\sim \mathcal{O} \neg \alpha$

EP2. $\mathcal{P} \neg \alpha \equiv \sim \mathcal{O}\, \alpha$

EP3. $\sim\sim \mathcal{O}\, \alpha \equiv \sim \mathcal{P} \neg \alpha$

EP4. $\sim\mathcal{T}\alpha \equiv \sim\sim\mathcal{H} \neg \alpha$

EP5. $\mathcal{T} \neg \alpha \equiv \sim \mathcal{H}\alpha$

EP6. $\sim\sim\mathcal{H}\alpha \equiv \sim\mathcal{T} \neg \alpha$

Remarks about the syntax of L_p. It should be noticed that, because of the intuitionist behaviour of the pragmatic connectives, in particular of the pragmatic negation ~ (see the remarks about the pragmatic of $\mathbf{L_p}$ below) the pragmatic operators of the pairs $\mathcal{O}/\mathcal{P}$ and $\mathcal{H}/\mathcal{T}$ *are not dual*. This represents a crucial difference of the logical behaviour of the pragmatic operators, not only with respect to the standard deontic operators (that, as I recalled above, rather lend themselves to a *descriptive* interpretation), but also with respect to Blackburn's evaluative expressive operators H! and T!, that are mapped onto the deontic ones (Blackburn *op. cit.*).

Besides, we note that the subset of the elementary sentential formulas of $\mathbf{L_p}$ is defined by the rule SFR1 in strict conformity to the Frege-Reichenbach model of sentential analysis, according to which the signs of pragmatic mood $\vdash$, $\mathcal{O}$ and $\mathcal{H}$ cannot be iterated (since they can be applied only to radical formulas), nor they can occur under the scope of any classical connective, canonically interpreted as truth-functions. In this way, the signs of pragmatic mood are syntactically defined as operators that transform radical formulas into sentential formulas. This shows a fundamental syntactical difference between the signs of pragmatic mood and the signs of alethic modality; these latter, in our perspective, are to be syntactically understood as signs that transform radical formulas into modal radical formulas and that can thus be iterated, as well as they can fall under the scope of propositional connectives.

However, since normative and evaluative formulas containing iterated deontic end evaluative operators are indispensable for the formulation of metanorms and metavalues (v. Opfermann 1977; von Wright 1983), we'll see in the sequel how this result can be obtained in $\mathbf{L_p}$, by making use of normative and evaluative sfs of higher level, belonging to an appropriate modal extension of $\mathbf{L_p}$, including in the category of logical-semantic signs both deontic and evaluative operators of *descriptive* kind, that work as alethic modal operators.

By using the pragmatic connectives, the rule SFR2 defines the set of the complex sentential formulas, extending the Frege-Reichenbach model without violating 'Frege's point'. Such an extension is fundamental for the definition of logical relations between sentences that lack truth values. In particular, SFR2 permits the formulation of *mixed* sentential formulas, such as $\vdash\alpha \cup \mathcal{H}\beta$, that permit the correct formalization of couplings of norms, evaluative attitudes and beliefs, like Blackburn's "disjunctive commitment" that is inadequately formalized in his language as $\alpha \vee H!\beta$ (see Blackburn *op. cit.*). Such a formula is not a well formed formula in $\mathbf{L_p}$, since it attempts to combine a radical formula with a sentential formula by means of a truth-functional connective.

DEFINITION 2. *Semantics.*

We call *semantic interpretation of* $\mathbf{L_p}$ every pair ({T, F}, σ), where {T, F} is the set of truth values and σ is an assignment function, assigning to each radical formula of $\mathbf{L_p}$ a truth value, according to the usual truth rules of a classical (Tarskian) semantics.

The metalogical notions of validity (tautology), satisfiability, consistency and compatibility for the wffs of $\mathbf{L_p}$, are defined in the standard way of Tarskian semantics and will be omitted for the sake of simplicity.

DEFINITION 3. *Pragmatics*.

For every semantic interpretation σ, we call pragmatic interpretation of $\mathbf{L_p}$ associated to σ every pair ({J, U}, π_σ), where {J, U} is the set of justification values (justified/unjustified) and π_σ is a pragmatic evaluation function, assigning to every assertive, normative or evaluative sentential formula of $\mathbf{L_p}$ a justification value that depends on the assignments of truth values made by σ to its radical subformulas, in such a way that the following conditions or justification rules (**JR**) are satisfied.

JR1. (i) $\pi_\sigma(\vdash\alpha) = J$ iff a proof exists that α is true, i. e. that $\sigma(\alpha) = T$ (hence, $\pi_\sigma(\vdash\alpha) = U$ iff no proof exists that α is true).

(ii) Let **N** be a *normative* system. Then

$\pi_\sigma(\mathcal{O}\alpha) = J$ (relative to N) if and only if a proof exists that

a) the obligation satisfies the membership criteria of N (*existence condition*);
b) α describes a (kind of) act (*content condition*);
c) α is physically possible (*satisfiability condition*);
d) α is logically compatible with every radical formula β that occurs in a normative sentential formula of the form $\mathcal{O}\beta$ or $\mathcal{P}\beta$ belonging to N (*compatibility condition*).

Hence, $\pi_\sigma(\mathcal{O}\alpha) = U$ iff no proof exists that all the conditions a)-d) are satisfied.

(iii) Let **A** be an *axiological* system. Then

$\pi_\sigma(\mathcal{H}\alpha) = J$ (relative to A) if and only if a proof exists that

a) α is approved in A;

b) α is physically possible;

c) α is logically compatible with every radical formula β that occurs in an evaluative sentential formula of the form $\mathcal{H}\beta$ or $\mathcal{T}\beta$ belonging to A.

Hence, $\pi_\sigma(\mathcal{H}\alpha) = U$ iff no proof exists that all the conditions a)-d) are satisfied.

JR2. $\pi_\sigma(\sim\delta) = J$ iff a proof exists that δ is unjustified, i. e., that $\pi_\sigma(\delta) = U$.

JR3.

(i) $\pi_\sigma(\delta_1 \cap \delta_2) = J$ iff $\pi_\sigma(\delta_1) = J$ and $\pi_\sigma(\delta_2) = J$

(ii) $\pi_\sigma(\delta_1 \cup \delta_2) = J$ iff $\pi_\sigma(\delta_1) = J$ or $\pi_\sigma(\delta_2) = J$

(iii) $\pi_\sigma(\delta_1 \supset \delta_2) = J$ iff a proof exists that if $\pi_\sigma(\delta_1) = J$, then $\pi_\sigma(\delta_2) = J$

(iv) $\pi_\sigma(\delta_1 \equiv \delta_2) = J$ iff $\pi_\sigma(\delta_1 \supset \delta_2) = J$ and $\pi_\sigma(\delta_2 \supset \delta_1) = J$

Remarks about the pragmatics of L_p. Some elucidatory remarks seem necessary about the justification rules JR1-JR3 that recursively define the pragmatic concept of "*justified in L_p*", by interpreting the pragmatic connectives as *partial justification functions*.

1) First of all, it should be noticed that rule JR1 (ii)-(iii) defines the justification of elementary normative and evaluative formulas of $\mathbf{L_p}$ in a strictly parallel way. The reason of this large correspondence is that a moral system includes both a normative apparatus and an axiological apparatus that must be coherent if the whole system is to be consistent. This means that if in a moral system an action α is obligatory, then α is also approved; if α is forbidden, then α is disapproved and if α is permitted, then α is tolerated (see Kelsen 1979). Because of this, one requires that the logical structure of the normative apparatus of the system be mirrored into the logical structure of its axiological apparatus, on pain of inconsistency between normative and evaluative sphere of one and the same moral system.

Of course, values that are associated in this way to norms are to be regarded as fundamental and not as supererogatory; so that if $\mathcal{O}\alpha$ is justified in the system, then $\mathcal{T}\neg\alpha$ must be unjustified in the system. This correspondence between norms and values within one and the same system, however, does not mean or imply reducibility of values to prescriptions, as in Hare (1952): the evaluative function of evaluative sentences remains distinct from the prescriptive function of normative sentences.

2) It should be also noticed that rule JR1 (ii)-(iii) defines the justification of every elementary normative or evaluative sentential formula as relative to a normative or axiological system, as well as in terms of the existence of a proof that certain metalinguistic conditions (a)-(d) are satisfied. These conditions, however, fix some purely formal (logical) requirements on norms and values, without imposing any material or substantial condition on their content. In this way, such conditions are to be regarded as pure rationality conditions, that can be satisfied by competing moral systems. Moreover, as Dalla Pozza (1997) points out, the rule JR1 –by making the justification of a norm or a value depend "not generically upon the fact that conditions (a)-(d) obtain, but upon the *existence of a proof* that such conditions are satisfied…[it introduces] a *strong rationality criterion*", preventing the introduction of new norms (or values) into the system, *via* some procedures of hermeneutic interpretation that do not represent any correct proof procedure (see Kelsen, 1960).

3) The rules JR1-JR3 imply two important properties of the pragmatics of $\mathbf{L_p}$.

The first one is that such rules not always permit to determine the justification value of a complex sf, when all the justification values of its elementary components are known. For instance, $\pi_\sigma(\delta) = J$ implies $\pi_\sigma(\sim\delta) = U$, but $\pi_\sigma(\delta) = U$ does not necessarily imply that $\pi_\sigma(\neg\delta) = J$. Because of this the law of *excluded middle* doesn't hold in $\mathbf{L_p}$. The same thing happens when the pragmatic connectives $\supset$ and $\equiv$ occur in a sf. In this sense, the pragmatic connectives are said to express *partial* justification functions. It follows that no principle analogous to the truthfunctionality principle for classical connectives holds for the pragmatic connectives in $\mathbf{L_p}$: it is necessary to make reference to the concept of proof even when we evaluate the justification value of a complex sf, whose elementary components have well known justification values. In short, the pragmatics of $\mathbf{L_p}$ is not J-functional.

The second property is that pragmatic connectives, unlike semantical ones, are not interdefinable.

Because of these two properties, rules JR1-JR3 cause pragmatic connectives to have a logical behaviour of intuitionist kind. In technical terms, while semantic connectives of $\mathbf{L_p}$ constitute a classical Boolean algebra, pragmatic connectives of $\mathbf{L_p}$ constitute Heyting's intuitionist algebra. Therefore, as it is stressed in Dalla Pozza (1995, 1997), pragmatic connectives are not an *ad hoc* duplicate of standard semantic connectives and the pragmatic values 'justified' and 'unjustified' do not re-introduce any disguised truth values under a different name, in the attempt to make the applicability of logic to the expressive conception of norms [and values] plausible; we are dealing with really distinct concepts that satisfy logical laws of different kind and that give our pragmatically extended formal language a logical structure that is richer and more articulated than the one of the formal languages of standard logic, thus permitting to effectively extend logic beyond the field of truth-apt sentences.

3. Metalogical notions

Let us now introduce the fundamental metalogical notions of pragmatic validity, satisfiability, consistency and compatibility for the sentential formulas of $\mathbf{L_p}$, through the following definitions.

DEFINITION 4.

(i) A sf δ is *pragmatically valid* or *p-valid* (respectively, pragmatically invalid or p-invalid) if and only if for every assignment function σ and for every pragmatic evaluation function π_σ, $\pi_\sigma(\delta) = J$ (respectively, $\pi_\sigma(\delta) = U$).

(ii) A sf δ is *pragmatically satisfiable* if and only if there is at least one assignment function σ and a pragmatic evaluation function π_σ, such that $\pi_\sigma(\delta) = J$ (otherwise δ is unsatisfiable and, thus, p-invalid).

(iii) A sf δ is *pragmatically consistent* if and only if δ is pragmatically satisfiable.

(iv) Two sfs δ_1 and δ_2 are pragmatically *compatible* if and only if $\delta_1 \cap \delta_2$ is pragmatically satisfiable.

In connection to DEFINITION 4 (i), let us introduce some *decision procedures* (or criteria of pragmatic validity) for the set of all the p-valid sfs of $\mathbf{L_p}$.

Since, as we've seen above, the pragmatics of $\mathbf{L_p}$ is not J-functional, there is no direct general decision procedure for all the p-valid sfs of $\mathbf{L_p}$. However, by making use of rules JR1-JR3 and of DEFINITION 4 (i), it is possible to establish the following direct criteria of pragmatic validity (PV), that will facilitate the identification of some relevant subsets of p-valid sfs of $\mathbf{L_p}$.

PV1. Let α be a tautological rf (respectively, a contradiction); then $\vdash\alpha$, $\mathcal{O}\alpha$ e $\mathcal{H}\alpha$ are p-valid sfs (respectively, p-invalid).

PV2. Let δ be a sf; then $\sim\delta$ is p-invalid if δ is p-valid (hence, δ is p-invalid, if $\sim\delta$ is p-valid).

PV3. Let δ_1 and δ_2 be sfs; then

(i) $(\delta_1 \cap \delta_2)$ is p-valid iff (δ_1) and (δ_2) are p-valid.

(ii) $(\delta_1 \cup \delta_2)$ is p-valid iff (δ_1) is p-valid or (δ_2) is p-valid.

(iii) $(\delta_1 \supset \delta_2)$ is p-valid iff for every semantic assignment function σ and for every pragmatic evaluation function π_σ, then $\pi\sigma(\delta_2) = J$ every time that $\pi\sigma(\delta_1) = J$.

(iv) $(\delta_1 \equiv \delta_2)$ is p-valid iff $(\delta_1 \supset \delta_2)$ is p-valid and $(\delta_2 \supset \delta_1)$ is p-valid.

PV4. Let δ_1 and δ_2 be sfs and let $(\delta_1 \supset \delta_2)$ be p-valid; then, every time that δ_1 is p-valid, δ_2 is also p-valid; and every time that δ_2 is p-invalid, δ_1 is also p-invalid.

PV5. Let δ_1 and δ_2 be sfs and let $(\delta_1 \equiv \delta_2)$ be p-valid; then δ_1 is p-valid (respectively p-invalid) iff δ_2 is p-valid (respectively, p-invalid).

However, it is possible to provide an *indirect general criterion* for the pragmatic validity in $\mathbf{L_p}$. To that end, let us introduce a modal extension of $\mathbf{L_p}$, denoted as $\mathbf{L_p^M}$, that is obtained by adding to the logical-semantic signs of the vocabulary of $\mathbf{L_p}$, the alethic modal operators **Pr**, **O** and **H**, that are interpreted, respectively, as 'proved' (or 'provable'), 'obligatory' and 'approved' in a *descriptive* sense; and by adding to the formation rules for radical formulas the following rule:

RFR3: Let α be a rf; then **Pr**α, **O**α and **H**α are (modal) rfs of $\mathbf{L_p^M}$.

Besides, let us introduce the modal operators **P, F, T** and **B** (respectively interpreted as permitted, forbidden, tolerated and disapproved in a descriptive sense), through the following definitions corresponding to D1-D4:

D*1. $\mathbf{P}\alpha =_{def.} \neg \mathbf{O} \neg \alpha$

D*2. $\mathbf{F}\alpha =_{def.} \mathbf{O} \neg \alpha$

D*3. $\mathbf{T}\alpha =_{def.} \neg \mathbf{H} \neg \alpha$

D*4. $\mathbf{B}\alpha =_{def.} \mathbf{H} \neg \alpha$

We can interpret the modal radical formulas **Pr**α, **O**α, **P**α, **H**α, **F**α and **T**α as formulas that describe the illocutionary acts expressed by the corresponding assertive, normative and evaluative sfs ├α, $\mathcal{O}\alpha$, $\mathcal{P}\alpha$, $\mathcal{H}\alpha$, $\mathcal{F}\alpha$ e $\mathcal{T}\alpha$, respectively, and whose semantic interpretation is provided by a Kripkean interpretation (possible world semantics).

Therefore, in $\mathbf{L_p^M}$ we can establish one-to-one correspondences between sentential formulas and their corresponding modal radical formulas, through the following correlation schemes:

SCHEMA C1

├ α	**Pr** α
~├ α	**Pr** ¬ **Pr** α
├ α_1 ∩ ├ α_2	**Pr** $\alpha_1 \wedge$ **Pr** α_2
├ α_1 ∪ ├ α_2	**Pr** $\alpha_1 \vee$ **Pr** α_2
├ α_1 ⊃ ├ α_2	**Pr** (**Pr** $\alpha_1 \rightarrow$ **Pr** α_2)
├ α_1 ≡ ├ α_2	**Pr** (**Pr** $\alpha_1 \leftrightarrow$ **Pr** α_2)

Considering the correspondence introduced by SCHEMA C1, every assertive formula on the left is *justified* when the corresponding modal radical formula an the right is *true* (with respect to an appropriate kripkean interpretation) and *viceversa.*

Such correspondence is easily demonstrated by observing that the modal radical formulas on the right do nothing but displaying the justification conditions that are fixed by the pragmatic justification rules JR for the corresponding assertive formulas on the left.

SCEMA C1 permits to draw on as a validity *criterion* for the assertive formulas of $\mathbf{L_p^M}$ the general *criterion* of semantic validity for the radical formulas of a modal system KT4 (or S4) paired with a Kripke-style semantics.

SCHEMA C2

$\mathcal{O}\alpha$	$\mathbf{Pr}\ (\mathbf{O}\alpha)$
$\sim \mathcal{O}\alpha$	$\mathbf{Pr}\ \neg\ \mathbf{Pr}\ (\mathbf{O}\alpha)$
$\mathcal{O}\alpha_1 \cap \mathcal{O}\alpha_2$	$\mathbf{Pr}\ (\mathbf{O}\alpha_1) \wedge \mathbf{Pr}\ (\mathbf{O}\alpha_2)$
$\mathcal{O}\alpha_1 \cup \mathcal{O}\alpha_2$	$\mathbf{Pr}\ (\mathbf{O}\alpha_1) \vee \mathbf{Pr}\ (\mathbf{O}\alpha_2)$
$\mathcal{O}\alpha_1 \supset \mathcal{O}\alpha_2$	$\mathbf{Pr}\ (\mathbf{Pr}\ (\mathbf{O}\alpha_1) \rightarrow \mathbf{Pr}\ (\mathbf{O}\alpha_2))$
$\mathcal{O}\alpha_1 \equiv \mathcal{O}\alpha_2$	$\mathbf{Pr}\ (\mathbf{Pr}\ (\mathbf{O}\alpha_1) \leftrightarrow \mathbf{Pr}\ (\mathbf{O}\alpha_2))$

Analogously, a SCHEMA C3 for evaluative formulas can be obtained by simply replacing in SCHEMA C2 every occurrence of $\mathcal{O}$ in the left side with an occurrence of $\mathcal{H}$ and every occurrence of **O** in the right side with an occurrence of **H**.

Like SCHEMA C1, SCHEMAS C2 and C3 establish a correspondence between the justification of every normative and evaluative formula with the truth of the corresponding modal radical formulas. This correspondence is also easily demonstrated by observing that the modal radical formulas on the right do nothing but explicating the justification conditions that are fixed by the pragmatic justification rules JR for the corresponding normative and evaluative formulas on the left, provided that the justification of '$\mathcal{O}\alpha$' depends on the existence of a proof that the metalinguistic conditions introduced in the rules JR1 (ii) and (iii) are satisfied.

In this way it is possible to use again as a validity criterion for normative and evaluative formulas of $\mathbf{L_p^M}$ the general semantic criterion of validity for the radical formulas of a modal language paired with a more complex Kripke-style semantics for a multi-modal system in which KD is included in KT4 (for a model of this kind for my language see Bellin and Ranalter 2003).

We can also remark that it is possible to apply the rule SFR1 to the modal radical formulas of $\mathbf{L_p^M}$, so obtaining assertive, normative and evaluative formulas like, for example, $\vdash(\mathbf{O}\alpha)$, $\vdash(\neg\mathbf{P}\alpha)$, $\mathcal{O}(\mathbf{O}\alpha)$, $\mathcal{O}(\mathbf{P}\alpha)$, $\mathcal{P}(\mathbf{O}\neg\alpha)$, $\mathcal{P}(\mathbf{P}\alpha)$, $\mathcal{H}(\mathbf{H}\alpha)$, $\mathcal{H}(\mathbf{T}\alpha)$, $\mathcal{T}(\mathbf{H}\alpha)$, $\mathcal{T}(\mathbf{T}\alpha)$, $\mathcal{O}(\mathbf{H}\alpha)$, $\mathcal{P}(\mathbf{H}\alpha)$, $\mathcal{H}(\mathbf{O}\alpha)$, $\mathcal{H}(\mathbf{B}\alpha \rightarrow \mathbf{B}\beta)$, $\vdash(\mathbf{B}\alpha \cap \mathbf{B}\beta)$ ecc., that express sentential formulas of higher level; this makes it possible to express assertions about norm- and value-propositions, metanorms and metavalues (by iterating this procedures we can obtain in $\mathbf{L_p^M}$ some assertive, normative and evaluative formulas of further level, expressing meta-metanorms, meta-metavalues, meta-meta-metanorms, ecc.).

By means of such modal extension of $\mathbf{L_p}$ it becomes possible to make it syntactically precise the fundamental distinction between normative (or evaluative) sentences and descriptive sentences of norms and values; particularly, normative and evaluative formulas with signs of pragmatic mood correspond to the former, while descriptive formulas of norms and values with operators "acting in semantic capacity" (alethic modal) correspond to the latter.

In $\mathbf{L_p}^{\mathbf{M}}$ it also becomes possible to demonstrate the close correspondence between norms or values and assertions about norms or values. In fact, by making use of SCHEMAS C1, C2 and C3, one demonstrates that the following equivalences are p-valid in $\mathbf{L_p}^{\mathbf{M}}$:

$\mathcal{O}\alpha \equiv \vdash (\mathbf{O}\alpha)$

$\mathcal{H}\alpha \equiv \vdash (\mathbf{H}\alpha)$

that establish a pragmatic equivalence between normative and evaluative sentences and assertions about norm- and value-propositions, thus proving that a normative or evaluative sentence expressively understood is justified if and only if the assertion of the radical formula describing that norm or that value is justified. In this way, the correspondence between the logic of norms and values and the logic of sentences describing norms or values is set up again.

4. Applications

By making a direct use of the justification rules JR1-JR3 and of the pragmatic validity criteria PV1-PV5 (or an indirect use of SCHEMA C1-C3), we can identify some p-valid sfs of $\mathbf{L_p}$. Using the sign Φ as metalinguistic sign that stands for any *primitive* sign of pragmatic mood of $\mathbf{L_p}$, let us see some general schemes of p-valid sfs. From each of such schemes a p-valid sentential formula –respectively, assertive, normative or evaluative- can be obtained by uniformly replacing the sign Φ with $\vdash$, $\mathcal{O}$ or $\mathcal{H}$. For the proof of these formulas, see Dalla Pozza (1997).

1) $(\Phi\neg\alpha) \supset (\sim\Phi\alpha)$
2) $(\Phi\alpha_1 \cap \Phi\alpha_2) \equiv \Phi(\alpha_1 \wedge \alpha_2)$
3) $(\Phi\alpha_1 \cup \Phi\alpha_2) \supset \Phi(\alpha_1 \vee \alpha_2)$
4) $\Phi(\alpha_1 \rightarrow \alpha_2) \supset (\Phi\alpha_1 \supset \Phi\alpha_2)$
5) $\Phi(\alpha_1 \leftrightarrow \alpha_2) \supset (\Phi\alpha_1 \equiv \Phi\alpha_2)$

The pragmatic validity of the schemes 1)-5) is important, since it fixes some fundamental relations between classical and pragmatic connectives.

The following schemes are also p-valid:

6) $\Phi(\alpha \leftrightarrow \neg\neg\alpha)$
7) $\Phi\alpha \equiv \Phi\neg\neg\alpha$
8) $\Phi\alpha \supset \sim\Phi\neg\alpha$
9) $\Phi\alpha \supset \sim\sim\Phi\alpha$
10) $\sim\sim\sim\Phi\alpha \equiv \sim\Phi\alpha$

The pragmatic validity of the schemes 6)-10) permits to demonstrate some interesting properties of the connectives $\neg$ and $\sim$.

Let $\top$ be any tautological radical formula and let $\bot$ be any contradictory radical formula, then, every sf $\Phi\top$ is p-valid and every sf $\Phi\bot$ is p-invalid. From schemes 1), 7), 8), 9) and 10) we can obtain the p-validity of $\sim\Phi\neg\top$, $\sim\sim\Phi\top$ e $\sim\sim\sim\Phi\neg\top$, and the p-invalidity of $\sim\Phi\neg\bot$, $\sim\sim\Phi\bot$ e $\sim\sim\sim\Phi\bot$.

It is important to notice that the inverses of 1), 3), 4), 5), 8) and 9) are not p-valid.

The following schemes – representing, respectively, the strong and the weak version of the law of excluded middle - are not p-valid either:

11*) $\Phi\alpha \cup \Phi\neg\alpha$

12*) $\Phi\alpha \cup \sim \Phi\alpha$

The non-validity of 11*) and 12*) and of the inverses of 1), 3), 4), 5), 8) e 9) is an obvious consequence of the intuitionist logical behaviour of the pragmatic connectives.

On the contrary, the following schemes representing some versions of the non contradiction principle for the sfs of $\mathbf{L_p}$ are p-valid:

13) $\sim (\Phi\alpha \cap \Phi\neg\alpha)$

14) $\sim (\Phi\alpha \cap \sim\Phi\alpha)$

The following schemes that represent the pragmatic analogues of the classical logical laws expressing the interdefinability of *standard* connectives are not p-valid:

15*) $(\Phi\alpha \cap \Phi\alpha) \equiv \sim (\sim\Phi\alpha \cup \sim\Phi\alpha)$

16*) $(\Phi\alpha \cup \Phi\alpha) \equiv \sim (\sim\Phi\alpha \cap \sim\Phi\alpha)$

17*) $(\Phi\alpha \supset \Phi\alpha) \equiv (\sim\Phi\alpha \cup \Phi\alpha)$

On the contrary, the following schemes showing that the previous laws hold at a pragmatic level in a weakened form are p-valid:

18) $(\Phi\alpha \cap \Phi\alpha) \supset \sim (\sim\Phi\alpha \cup \sim\Phi\alpha)$

19) $(\Phi\alpha \cup \Phi\alpha) \supset \sim (\sim\Phi\alpha \cap \sim\Phi\alpha)$

20) $(\sim\Phi\alpha \cup \Phi\alpha) \supset (\Phi\alpha \supset \Phi\alpha)$

Besides, the following schemes representing well known logical laws also holding for sfs of $\mathbf{L_p}$, and that correspond to as much important *rules of inference for sfs*, are p-valid:

21) $(\delta_1 \cap \delta_2) \supset \delta_1$ (*Reduction*)

22) $\delta_1 \supset (\delta_1 \cup \delta_2)$ (*Addition*)

23) $(\delta_1 \cap (\delta_1 \supset \delta_2)) \supset \delta_2$ (*Modus Ponens*)

24) $(\delta_1 \supset (\delta_2 \supset \delta_3) \equiv (\delta_1 \cap \delta_2) \supset \delta_3$ (*Import./Export.*)
where δ_1, δ_2 and δ_3 stand for elementary or complex sfs.

In addition to the previous p-valid schemes that concern all the sfs of $\mathbf{L_p}$ (assertive, normative, evaluative), the following schemes that more specifically concern normative and evaluative sfs are p-valid:

25) $\mathcal{O}\alpha \supset \mathcal{P}\alpha$ $\quad\quad$ $\mathcal{H}\alpha \supset \mathcal{T}\alpha$

26) $\mathcal{O}\alpha \supset \sim\mathcal{P}\neg\alpha$ $\quad\quad$ $\mathcal{H}\alpha \supset \sim\mathcal{T}\neg\alpha$

27) $\sim\mathcal{O}\alpha \equiv \mathcal{P}\neg\alpha$ $\quad\quad$ $\sim\mathcal{H}\alpha \equiv \mathcal{T}\neg\alpha$

28) $\sim \sim\mathcal{O}\alpha \supset \sim\mathcal{P}\alpha$ $\quad\quad$ $\sim \sim\mathcal{H}\alpha \supset \sim\mathcal{T}\alpha$

Lastly, the following mixed equivalences

29) $\vdash\alpha \equiv \mathcal{O}\alpha$

30) $\vdash\alpha \equiv \mathcal{H}\alpha$

31) $\Theta\alpha \equiv \mathcal{H}\alpha$

are p-valid every time that α is a *tautology* or a *contradiction*.

It is important to remark that every p-valid sf of the form $\delta_1 \supset \delta_2$ logically corresponds to a form (or rule) of inference between sfs of $\mathbf{L_p}$.

Therefore, in $\mathbf{L_p}$, one can specify both classical logical relations between radical formulas –the only ones that are admitted by Hare's *principle of 'dictive' indifference*- and logical relations of intuitionist kind between sentential formulas (assertive, normative, evaluative and mixed), *that enable to extend logic to normative and evaluative language expressively understood.*

5. The pragmatic language L_p and the Frege-Geach problem

We can say we have built in $\mathbf{L_p}$ a logic of intuitionist kind for the expressive conception of norms and values, that does not present any of the limits we can see in the previous attempts by Hare and Blackburn.

In particular, we are able to formalize the moral inference proposed by Geach as follows:

(I)

1. $\mathcal{B}(\alpha)$
2. $\underline{\mathcal{B}(\alpha) \supset \mathcal{B}(\beta)}$
3. $\mathcal{B}(\beta)$

Using DEFINITION D4 in section 2, (**I**) can be re-formulated in terms of:

(I*)

1. $\mathcal{H}(\neg\alpha)$
2. $\underline{\mathcal{H}(\neg\alpha) \supset \mathcal{H}(\neg\beta)}$
3. $\mathcal{H}(\neg\beta)$

Inference (**I**) and (**I***) are instances of the application of *Modus Ponens* for sentential formulas of $\mathbf{L_p}$ $\dfrac{\delta_1,\ \delta_1 \supset \delta_2}{\delta_2}$, corresponding to the p-valid scheme 23) in section 4.

It should also be noticed that, on the ground of the equivalence $\mathcal{H}\alpha \equiv \vdash(\mathbf{H}\alpha)$ in section 3, if (**I***) holds, then also the following inference between the corresponding assertions about values holds:

(I)**

1. $\vdash(\mathbf{H}\neg\alpha)$
2. $\underline{\vdash(\mathbf{H}\neg\alpha) \supset \vdash(\mathbf{H}\neg\beta)}$
3. $\vdash(\mathbf{H}\neg\beta)$

that reconstructs in $\mathbf{L_p^M}$ the correspondence between the logic of evaluative sentences and the logic of sentences descriptive of values and whose correctness can be demonstrated in a Kripkean semantics, by making use of SCHEMA C1.

Obviously, the same inferences can be obtained between norms and descriptive sentences of norms, by replacing in (**I***) every occurrence of $\mathcal{H}$ with $\mathcal{O}$ and in (**I****) every occurrence of **H** with **O**.

Having provided a logic of intuitionist kind that is adequate to the expressive conception of norms and values and that is able to give a positive solution to the *Frege-Geach Problem*, we can say we are able to give a positive answer to the question asked in the title of Hale's famous article "Can there be a logic of attitudes?" (1993), so eliminating any reason for scepticism about it.

We can, thus, maintain that the main obstacle to a full endorsement of Non-Cognitivism has been removed. Should Non-Cognitivism still be considered untenable, this cannot be due to the *logical reasons* on which the main objection levelled against it was grounded; not even to the fear that Non-Cognitivism could exclude ethics from the sphere of rationality.

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