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AN EPQ MODEL FOR NON-INSTANTANEOUS DETERIORATING ITEM IN WHICH HOLDING COST VARIES WITH TIME

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Abstract. *It is a production problem of non-instantaneous deteriorating item in which production and demand rate are constant. The holding cost varies with time. Completely deteriorated units are discarded. Partially deteriorated items are allowed to carry discount, no shortage is allowed. The optimal cycle time is derived and the result is applied to numerical problems*

Keywords. *EPQ, Non-instantaneous deterioration, Price discount, time dependent holding cost.*

1. Introduction

An economical production quantity (EPQ) model [3] is an inventory control model that determines the amount of product to be produced on a single facility so as to meet a deterministic demand over an infinite planning horizon.
In all inventory models a general assumption is that products generated have indefinitely long lives. In general, almost all items deteriorate over time. Often the rate of deterioration is low and

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there is little need to consider the deterioration in the determination of economic lot size. Nevertheless, there are many products in the real world that are subject to a significant rate of deterioration. Hence, the impact of product deterioration should not be neglected in the decision process of production lot size. Deterioration can be classified as age-dependent on-going deterioration and age-independent on-going deterioration. Blood, fish and strawberries are some examples of commodities facing age-dependent on-going deterioration. Volatile liquids such as alcohol and gasoline, radioactive chemicals, and grain products are examples of age-independent on-going deteriorating items. Legally these products do not have an expiry date; they can be stored indefinitely, though they suffer natural attrition while being held in inventory. In general, perishableness or deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero.

Mandal (1998), discusses the fuzzy inventory model of deteriorating items with stock- dependent demand using limited storage space. Maxwell (1964), considered the scheduling of economic lot sizes.

Misra (1975), first studied the EPQ model for deteriorating items with the varying and constant rate of deterioration. Choi and Hwang (1986), developed a model determining the production rate for deteriorating items to minimize the total cost function over a finite planning period. Raafat (1985), extended the model, given in Park (1983), to deal with a case in which the finished product is also subject to a constant rate of decay. Yang and Wee (2003), considered a multi-lot-size production-inventory system for deteriorating items with constant production and demand rates.

Aggarwal and Bahari-Hashani (1991), studied a model assuming that items are deteriorating at a constant rate, the demand rate is known and decreases negative exponentially, no shortages are allowed, and the production rate is known but can vary from one period to another over a finite planning period. Pakkala and Achary (1992), considered a production-inventory model of deteriorating items with two storage facilities and a constant demand rate, Shortages are allowed.Gary C.Lin, Dennis E. Kroll and C.J. Lin (2006), obtained common production cycle time for an ELSP with deteriorating items.

In this paper an EPQ model for single product subject to non-instantaneous deterioration under a production-inventory policy in which holding cost varies with time is considered. Each unit of the item is provided with price discount decayed units. In the next section, assumptions and notations that are employed for the development of the model are given. The optimal cycle time is derived in the Section 3. An illustrative numerical example and final concluding remarks are given in the subsequent sections.

2. Basic assumptions and notations

The following are the assumptions applied in the development of the model:

1. The demand rate for the product is known and finite.
2. Shortage is not allowed.
3. An infinite planning horizon is assumed.
4. Once a unit of the product is produced, it is available to meet the demand.

5. Once the production is terminated the product starts deterioration and the price discount is considered.
6. The deterioration follows an exponential distribution.
7. There is no replacement or repair for a deteriorated item.

The notations that are employed here:

- p : production rate per unit time.
- d : actual demand of the product per unit time
- A : Set up cost
- θ : a constant deterioration rate (unit/unit time).
- h= a+bt : inventory carrying cost per unit time, where a and b are positive constants.
- k : production cost per unit.
- r : price discount per unit cost
- T : optimal cycle time.
- T_1 : production period
- T_2 : time during which there is no production of the product i.e.,T_1= T- T_2.
- $I_1(t)$: inventory level for product during the production period, i.e., $0 \le t \le T_1$.
- $I_2(t)$: inventory level of the product during the period when there is no production i.e., $T_1 \le t \le T_2$.
- I(M) : maximum inventory level of the product.
- TVC(T) : total cost/unit time.

3. Model development

At start t = 0, the inventory level is zero. The production and supply start simultaneously and the production stops at t =T_1 at which the maximum inventory I(M) is reached. The inventory built up at a rate is p-d in the interval $[0,T_1]$and there is no deterioration. After the time T_1, the produced units start deterioration and supply is continued at the discount rate. There is no fall in demand, when the inventory reduces to zero level and production run begins. The inventory level of the product at time t over period $[0, T]$ can be the represented by the differential equations:

$$\frac{dI_1(t)}{dt} = p - d \text{ , for } 0 \le t \le T_1 \qquad (1)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d \text{ for } 0 \le t \le T_2 \qquad (2)$$

The boundary conditions associated with these equations are: at $I_1(0)=0, I_2(T_2)=0$

$$I_1(t) = (p - d)t, \quad \text{for } 0 \le t \le T_1 \qquad (3)$$

$$I_2(t) = \frac{d}{\theta}\left[e^{\theta(T_2 - t)} - 1\right], \text{ for } 0 \le t \le T_2 \qquad (4)$$

Production cost: The production cost per unit time is given by

$$\text{PC} = pk\frac{T_1}{T} \quad (5)$$

Setup cost: The setup cost per unit time is given by

$$\text{SC} = \frac{A}{T} \quad (6)$$

Holding cost: The holding cost per unit time is given by

$$\text{HC} = \frac{1}{T}\left[\int_0^{T_1}(a+bt)I_1(t)dt + \int_0^{T_2}(a+bt)I_2(t)dt\right]$$

$$= \frac{1}{T}\left[\int_0^{T_1}(a+bt)(p-d)tdt + \int_0^{T_2}(a+bt)\frac{d}{\theta}\left(e^{\theta(T_2-t)}-1\right)dt\right] \quad (7)$$

Assuming $t\theta < 1$, an approximate value is got by neglecting those terms of degree greater than or equal to 2 in $t\theta$ in Tailors expansion of the exponential functions.

$$\text{HC} = a(p-d)\frac{T_1^2}{2T} + b(p-d)\frac{T_1^3}{3T} + \frac{adT_2^2}{2T} \quad (8)$$

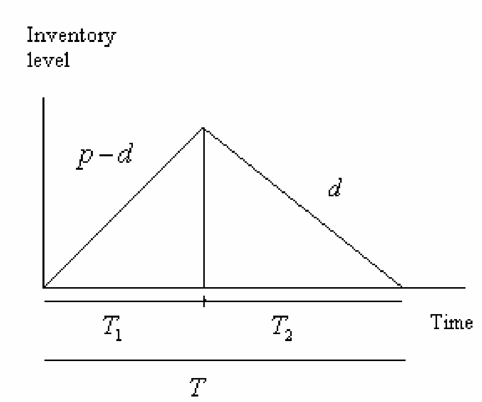

Figure.1 Representation of inventory model

Deterioration cost: The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand. Hence the deterioration cost per unit time is given as

$$\text{DC} = \frac{k}{T}\left[I_2(0) - \int_0^{T_2} d.dt \right] = \frac{kd\theta T_2^2}{2T} \tag{9}$$

Price discount: Price discount is offered as a fraction of production cost for the units in the period $[0, T_2]$

$$\text{PD} = \frac{kr}{T}\int_0^{T_2} d.dt = \frac{krdT_2}{T} \tag{10}$$

Therefore the average total cost per unit time is given by

$$\text{TVC (T) = PC+SC+HC+DC+PD}$$

$$= pk\left[\frac{T_1}{T}\right] + \frac{A}{T} + a(p-d)\frac{T_1^2}{2T} + b(p-d)\frac{T_1^3}{3T} + \frac{adT_2^2}{2T} + \frac{kd\theta T_2^2}{2T} + \frac{krdT_2}{T} \tag{11}$$

If (A.1) and (A.2) are used to express T_1 and T_2 in terms of T in equation (11) and if the third

and higher powers of θT terms are neglected for small values of θT then equation (11) becomes

$$\text{TVC(T)} = pk\left[\frac{d}{p}\right] + \frac{A}{T} + a(p-d)\frac{d^2T}{2p^2} + b(p-d)\frac{d^3T^2}{3p^3} + \frac{ad(p-d)^2T}{2p^2} + \frac{kd\theta(p-d)^2T}{2p^2} + krd\left(\frac{p-d}{p}\right) \tag{12}$$

Our objective is to minimize the total cost per unit time TVC (T). Differentiate TVC (T) with respect to T and set the result equal to zero. We get

$$\frac{dTVC(T)}{dT} = -\frac{A}{T^2} + a(p-d)\frac{d^2}{2p^2} + \frac{2d^3}{3p^3}Tb(p-d) + ad\frac{(p-d)^2}{2p^2} + \frac{k\theta d}{2}\left(\frac{p-d}{p}\right)^2 = 0 \tag{13}$$

$\frac{d^2TVC(T)}{dT^2} = \frac{2A}{T^3} + \frac{2(p-d)d^3}{3p^3} > 0$,i.e., the second derivative is found to be positive.

Example:

A = \$1000/set up, p = 100 units/unit time, d = 30 units/unit time, a =2,b = 0.1, k = \$40/ unit time, θ = 0.08, r = 0.02/ unit ,T= 4.7081unit time, TVC (T) = \$ 1640.2, HC=\$107.57

Total cost, Production cycle time, Production run time and Holding cost are computed for six different sets of deterioration rates and price discount. The results are compared. The following graphs show the total costs, the production cycle time and the production run time with reference to deterioration rate and price discount.

Table 1. Different sets of deterioration rate

Deterioration rate					
θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
.06	.08	.1	.125	.15	.175

Table 2.Different sets of Price discount

Price discount					
r_1	r_2	r_3	r_4	r_5	r_6
0.01	0.02	0.04	0.06	0.08	0.1

Figure.2 shows that when deterioration rate and discount increase production cycle time increases. Figure.3 shows that when deterioration rate and discount rate increase production runs time increases. Figure.4 shows that when cycle time increases the holding cost increases but the total cost per unit time decreases.

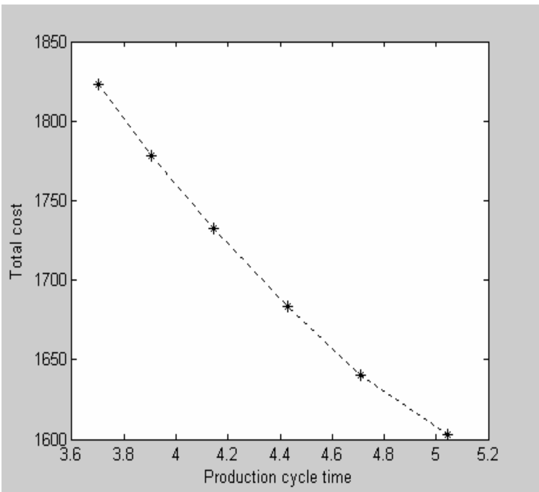

Figure 2.Comparision between Production cycle time and Total cost

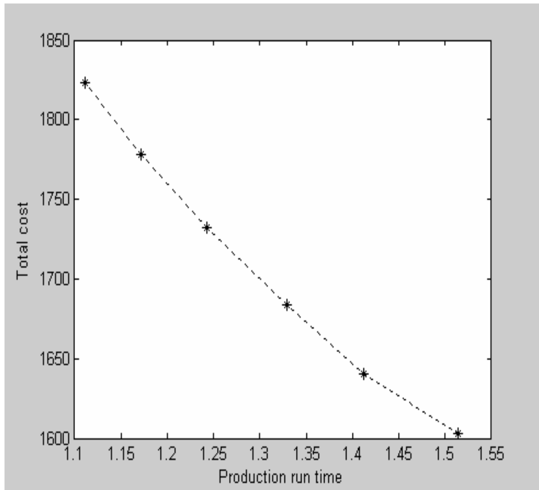

Figure 3.Comparision between Production run time and Total cost

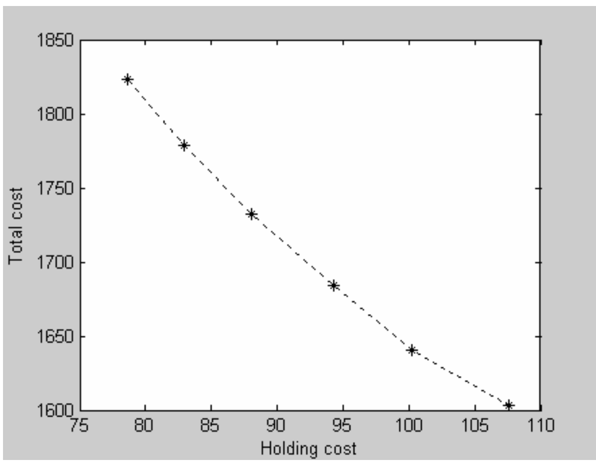

Figure 4.Comparision between Holding cost and Total cost

4. Conclusions

In this paper, an EPQ model for a single machine producing single item which under non-instantaneous deterioration and holding cost varies with time has been developed. Completely deteriorated items are discarded and partially deteriorated items are offered discount which maintains the demand which is more realistic. The production cycle time is derived and is found be a relatively simple expression. This study helps to reduce the total cost for non-instantaneous deterioration

APPENDIX A.1

To minimize the total cost per unit time TVC to express in terms of T in eq. (11) so that there is only one variable T in the equation. At the moment when production run is terminated during a cycle $\therefore I_1(T_1) = I_2(0)$

$$(p-d)\,T_1 = \frac{d}{\theta}\left(e^{\theta T_2} - 1\right)$$

Applying Tailor's expansion and approximation

$$(p-d)\,T_1 = d\left(T_2 + \frac{\theta T_2^{\,2}}{2}\right)$$

$$T_2 = \frac{(p-d)}{p}T \qquad \text{……….. (A.1)}$$

$$T = T_1 + T_2 = \frac{p}{d}T_1 \qquad \text{……….. (A.2)}$$

and $e^{\theta T_2} = 1 + \theta T_2 + \frac{(\theta T_2)^2}{2}$

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