EUI WORKING PAPERS IN ECONOMICS

EUI Working Paper ECO No. 91/60

Inventory Holdings by a Monopolist Middleman

JOHN FINGLETON

European University Institute, Florence

Please note
As from January 1990 the EUI Working Paper Series is divided into six sub-series, each sub-series is numbered individually (e.g. EUI Working Paper **LAW** No. 90/1).

EUROPEAN UNIVERSITY INSTITUTE, FLORENCE

ECONOMICS DEPARTMENT

EUI Working Paper ECO No. 91/60

Inventory Holdings by a Monopolist Middleman

JOHN FINGLETON

BADIA FIESOLANA, SAN DOMENICO (FI)

Printed in Italy in December 1991
European University Institute
Badia Fiesolana
I–50016 San Domenico (FI)
Italy

Inventory Holdings by a Monopolist Middleman

John Fingleton[0]

Abstract

This paper uses results from optimal control theory to analyse the dynamic profit maximising inventory policy of a monopolist middleman in continuous time. The middleman has two choice variables - sales and purchases - and the first step is to change the problem to one in a single variable (net purchases, denoted x) enabling the use of a profit function $\pi(x,t)$. The optimal policy is a vector which maximises the integral of this profit function subject to a differential equation and some other constraints. The first section gives an overview of the recent, relevant literature on intermediation by middlemen. Section 2 gives necessary and sufficient optimality conditions for the relevant class of problem. Section 3 outlines the model and characterises and discusses the solution, an example of which is given in section 4. Section 5 discusses extensions and limitations. The concavity of the profit function is an important element in determining the type of solution which emerges, and this is analysed in the Appendix.

December, 1991.

Comments welcome to:

John Fingleton
Department of Economics
Trinity College
Dublin 2
Ireland

---

[0]Trinity College Dublin. This paper is based on Chapter 9 of my D.Phil. thesis, University of Oxford, 1991. I would like to thank Peter Hammond, Alan Kirman, and J.A. Mirrlees for many helpful suggestions.

1 Introduction

Middlemen received much attention from economic writers from the Middle Ages until the late eighteenth century (e.g. Cantillon (1755)). Of particular concern was the role of the middleman in the circular flow of income, in setting prices, breaking bulk, and holding inventories. Interest in these issues waned with the rising dominance of the Walrasian competitive paradigm and, in particular, with the increasing reliance on the fictive auctioneer. The resurgence of interest in intermediaries in recent years, both in finance and in economic theory, is more a consequence of the post-Walrasian analysis of the microstructure of markets than of a reappraisal of the works of the early writers.

Perhaps because of the neglect of intermediation, middleman is not uniquely defined in the specialist literature, and the word middleman has a pejorative connotation more generally. Contradictory definitions (often implicit) of both middleman and intermediary exist in the literature. For the purposes of this paper a middleman is defined as an agent who trades with both buyer and seller for profit rather than for personal use. Thus the middleman is differentiated from other agents who intermediate but who do not actually buy and sell the good (unless by consignment).

A striking feature of the recent, theoretical literature is that intermediated markets, because of their two-sided nature, may be characterised by inefficient outcomes. The intuition behind this general finding is that the interactions among the various participants in the market alters the nature of the game played by the middlemen or by the buyers and sellers. Stahl (1988) examines two-sided Bertrand competition and shows how the outcome of the interaction among the competitive middlemen when bidding for the sellers alters the subsequent game among the middlemen when selling to the buyers. Fingleton (1991) illustrates that the supply and demand in an intermediated market are interdependent if buyers and sellers have access to an active direct trade mechanism. Both these results suggest that it is not always appropriate to model an intermediated market as having a single price (the margin) for a single good (the service of intermediation).

This paper investigates and concentrates on one particular aspect of intermediation, namely inventory holding policy. A deterministic model is chosen for a number of reasons. First, the inventory policy of a middleman with deterministic and known demand and supply has not been properly investigated previously, and seems like a natural starting point. Second, there are inherent problems with uncertainty in this context. Zabel (1972) points out that the analysis of monopoly pricing with uncertain demand gives conflicting results according as the uncertainty in the demand function is additive or multiplicative (due to the variance properties). Precisely the same criticism applies when modeling a monopolist middleman facing uncertain demand (for instance, this point detracts considerably from the work of Irvine (1981)).

This paper assumes that the middleman faces deterministic, but not necessarily stationary, supply and demand functions in continuous time. The issue of interest is the middleman's intertemporal trade-off(s) over time regarding whether to buy now or later and whether to sell now or later, subject to constraints on forward selling, holding costs for inventories, and an end-date, after which stock is of no value. Notwithstanding the results of Stahl(1988) and Fingleton (1991), the problem can be considerably simplified by reducing it to one of a single variable; how much stock to accumulate each instant. To this end, it is assumed that the middleman is a monopolist[1] and that there is no direct trade, thereby avoiding the potential difficulties related to strategic interaction. Optimal control theory can then be applied to the problem. The solution specifies the quantities which the middleman, knowing his future opportunities, will trade at each instant. Under the assumption of regularly sloped supply and demand functions, these quantities may be converted into prices, with magnitudes of conversion which depend on elasticities in the normal way. The interested reader is referred to Fingleton (1991) Chapter 5, section 1 for a discussion of the optimal pricing policy of a monopolist middleman in a single period.

[1]Monopoly is a useful first step in that in enables a general characterisation of the market. Moreover, intermediated markets are often monopolistic in character (Fingleton 1991).

The middleman will only hold inventories if the intertemporal trade-off compensates for the holding cost of the inventories. When inventories are held, the middleman usually accumulates and depletes inventories in a gradual manner, and the level of inventory held is higher as the holding cost is lower, and the change in the supply or demand over time is greater. However, under certain circumstances the inventories will follow some chattering path whereby the middleman accumulates and depletes inventories extremely rapidly. In such a scenario, there is no optimal policy since any path may be overtaken by one with more rapid alteration between accumulation and depletion. The properties of the supply and demand functions under which this result arises are specified in the appendix, and shown to be unlikely but not impossible.

The structure of the paper is as follows. The next section cites results from optimal control theory which may be applied to the class of problem which is of interest. Section 3 outlines and analyses the problem, and the inventory policies are characterised. The fourth section fleshes out the insights obtained from the general structure by considering the case where the supply and demand are simple linear functions. Section 5 discusses the results generally, and suggests how they might be applied more widely and extended.

2 Conditions for Optimality

The general problem is to find a piece-wise continuous control variable $x(t)$ and an associated continuous and piece-wise differentiable state variable $I(t)$, defined on $[0,T]$ that will:

$$\text{maximise} \int_0^T f(x(t),I(t),t)dt \tag{1}$$

subject to

$$\dot{I} = g(x(t),I(t),t) \tag{2}$$

$$\phi(x(t),I(t),t) \geq 0 \tag{3}$$

$$I(0) = I_0 \quad (I_0 \text{ fixed point in } \mathbb{R}) \tag{4}$$

and also a terminal condition which takes one of the following three forms:

$$\begin{cases} I(T) = I_T & \text{(i)} \\ I(T) \geq I_T \quad (I_T \text{ fixed number}); & \text{(ii)} \\ I(T) \text{ free} & \text{(iii)} \end{cases} \tag{5}$$

Definition 1

A pair $(x(t),I(t))$ is <u>admissible</u> if it satisfies (2)-(5).

Definition 2

A pair $(\hat{x}(t),\hat{I}(t))$ is <u>optimal</u> if it is admissible and it maximises the integral in (1).

Assume that f and g are jointly continuous functions of (x,I,t) on the set $\{(x,I,t):\phi(x(t),I(t),t) \geq 0\}$ and that $\phi(t)$ is a continuous function. Next, introduce a function $\lambda(t)$ associated with the constraint $\phi(t)$, and a function $\mu(t)$ associated with $g(t)$, so as to form the generalised Hamiltonian L defined by

$$L(x,I,\lambda,\mu,t) = f(x,I,t) + \mu.g(x,I,t) + \lambda.\phi(x,I,t) \tag{6}$$

where $f(x,I,t) + \mu.g(x,I,t) = H(x,I,\mu,t)$ is the usual Hamiltonian.

Let $(\hat{x}(t),\hat{I}(t))$ be an admissible pair in the preceding problem. Suppose there exist functions $\mu(t)$ and $\lambda(t) \geq 0$, where $\mu(t)$ is continuous, and $\dot{\mu}(t)$, $\lambda(t)$ are piece-wise continuous. Then the necessary conditions for the existence of an optimum may be given formally as follows. The pair $(\hat{x}(t),\hat{I}(t))$ is optimal only if for all $t \in [0,T]$ where $\hat{x}(t)$ and $\lambda(t)$ are continuous, the following conditions are satisfied:

$$\dot{\mu} = -L_I(\hat{x}(t),\hat{I}(t),\mu(t),\lambda(t),t); \tag{7}$$

$$L_x(\hat{x}(t),\hat{I}(t),\mu(t),\lambda(t),t) = 0; \tag{8}$$

$$\lambda(t).\phi(\hat{x}(t),\hat{I}(t),t) = 0; \tag{9}$$

$$\begin{cases} \mu(T) \text{ free} & \text{i} \\ \mu(T) \geq 0 \ (= 0 \text{ if } I(T) > 0\,) & \text{ii} \\ \mu(T) = 0 & \text{iii} \end{cases} \tag{5'}$$

$\phi(x,I,t)$ is quasi-concave in (x,I), and differentiable in (x,I) at $(\hat{x}(t),\hat{I}(t))$.

These are similar to the familiar (Pontryagin) necessary conditions, also seen in Takayama (1974) Theorem 8.C.1. Seierstad and Sydsaeter (1977) go further and give sufficient conditions for the existence of an optimum in this problem[2]. In effect, their theorem 6 states that the pair $(\hat{x}(t),\hat{I}(t))$ is optimal if both the necessary conditions hold and, in addition, the following condition is satisfied:

$H(x,I,\mu(t),t)$ is concave in (x,I) and differentiable w.r.t. (x,I) at $(\hat{x}(t),\hat{I}(t))$.

These give sufficient conditions for the existence of a solution for the extension of Pontryagin's Maximum Principle to deal with the case where there is an additional non-negativity constraint on the state variable. There is a stronger form of this theorem, due to Arrow, which does not require the Hamiltonian to be jointly concave in x and I; however, this theorem is more complicated and, as it is not necessary for the analysis in this paper, we refer the interested reader to Seierstad and Sydsaeter (1977).

3 Outline and Analysis of the Model

The structure of the model is as follows. The middleman buys and sells a single good. Quantity setting is assumed, and direct trade is excluded. Inverse supply and demand functions are denoted $p(s(t),t)$ and $P(b(t),t)$, where s and b are the number of sellers and buyers, respectively, who trade with the middleman, and p and P are the buying and selling prices, respectively. The model is in continuous time and the middleman operates for a fixed time of length T. The middleman may hold inventories $I(t)$ at cost h per unit held. Profit[3] at any

[2]Seierstad and Sydsaeter note that Takayama's result is not entirely correct. Takayama's second edition (1985) is open to the same criticism. The treatment here relies entirely on Seierstad and Sydsaeter.

[3]We distinguish between operating profit $\pi = P.b - p.s$ and overall profit $P.b - p.s - h.I$. The former might be thought of a short-run profit at any instant whereas the latter takes account of the middleman's longer term commitments.

instant t in [0,T] is $b.P(b,t) - s.p(s,t)$. Selling short is not permitted, so inventories are always non-negative, giving a constraint

$$I(t) \geq 0. \tag{10}$$

Define net purchases at any time t by $x(t) = s(t) - b(t)$. If $I(t)$ is differentiable, then $\dot{I} \leq x$ or, in the absence of dumping,

$$\dot{I}(t) = x(t). \tag{11}$$

Assumption 1 $P(b)$ and $p(s)$ are differentiable.

The overall profit of the middleman at any instant t is

$$b.P(b,t) - s.p(s,t) - h.I(t) \tag{12}$$

Let π be defined as a function of a single variable x as follows:

$$\pi(x,t) = \text{Max}\left\{ b.P(b,t) - s.p(s,t) \;\middle|\; s - b = x;\ s,\ b \geq 0 \right\} \tag{13}$$

The model may then be expressed as a problem as in (1) - (5) above where

$$f(x,I,t) = \pi(x,t) - h.I;$$

$$g(x,I,t) = x;$$

$$\phi(x,I,t) = I.$$

The problem is

$$\max \int_0^T [\ \pi(x(t),t) - h.I(t)]dt \quad \text{when} \quad \dot{I} = x(t) \quad \text{and} \quad I(t) \geq 0 \tag{14}$$

with boundary conditions

$$I(0) = I_0$$

$$I(T).\mu(T) = 0$$

Assumption 2 $\pi(x,t)$ is continuous in x.

This assumption is required to apply the theorem in the previous section, since f and g must be jointly continuous functions of (x,I,t) on the set $\{(x,I,t):I(t) \geq 0\}$. Both g and ϕ are continuous functions for the model that we are considering. The generalised Hamiltonian is for this model is

$$L(x,I,\mu,\lambda,t) = \pi(x,t) - h.I + \mu.x + \lambda.I \tag{6'}$$

where $\pi(x,t) - h.I + \mu.x = H(x,I,\mu(t),t)$ is the usual Hamiltonian.

The problem is now in a form which enables the application of the conditions for optimality from the previous section, giving a set of sufficient conditions for $(\hat{x}(t),\hat{I}(t))$ to be an optimal pair.

$$\dot{\mu} = h - \lambda(t) \tag{7'}$$

$$\pi_x(\hat{x}(t),t) + \mu(t) = 0; \tag{8'}$$

$$\lambda(t).\hat{I}(t) = 0; \tag{9'}$$

$$I(0) = I_0,\ I(T) = I_T \quad (I_0,I_T \text{ fixed numbers}) \tag{5'}$$

$\pi(x,t) - h.I + \mu.x$ is concave in (x,I) and differentiable w.r.t. (x,I) at $(\hat{x}(t),\hat{I}(t))$;

$I(t)$ is quasi-concave in (x,I) and differentiable in (x,I) at $(\hat{x}(t),\hat{I}(t))$;

Furthermore, it is required that $\mu(t)$ be continuous, $\dot{\mu}(t)$ and $\lambda(t)$ be piece-wise continuous, and that the above conditions hold for all $t \in [0,T]$ where $\hat{x}(t)$ and $\lambda(t)$ are continuous. The last of the above conditions is satisfied for our problem. The second last condition holds with the following assumption:

Assumption 3 $\pi(x,t)$ is concave in x and differentiable w.r.t. x at $(\hat{x}(t),\hat{I}(t))$.

The existence of a solution requires that the maximum of the integral in (14) be finite. This will be so if, for example, revenue is bounded above. Such an assumption is reasonable, but stronger than is required. An alternative sufficient condition for boundedness is that the selling price falls to zero for some finite quantity sold.

Assumption 4 $\quad P(b) = 0$ for $b \geq \bar{b}$, where $\bar{b}$ is finite.

Finally, it is necessary that $\mu(t)$ be continuous, and that both $\dot{\mu}(t)$ and $\lambda(t)$ be piece-wise continuous. If so, then the necessary conditions above are satisfied by a solution to the following system

$$h - \lambda(t) = \dot{\mu}(t) \qquad (7')$$

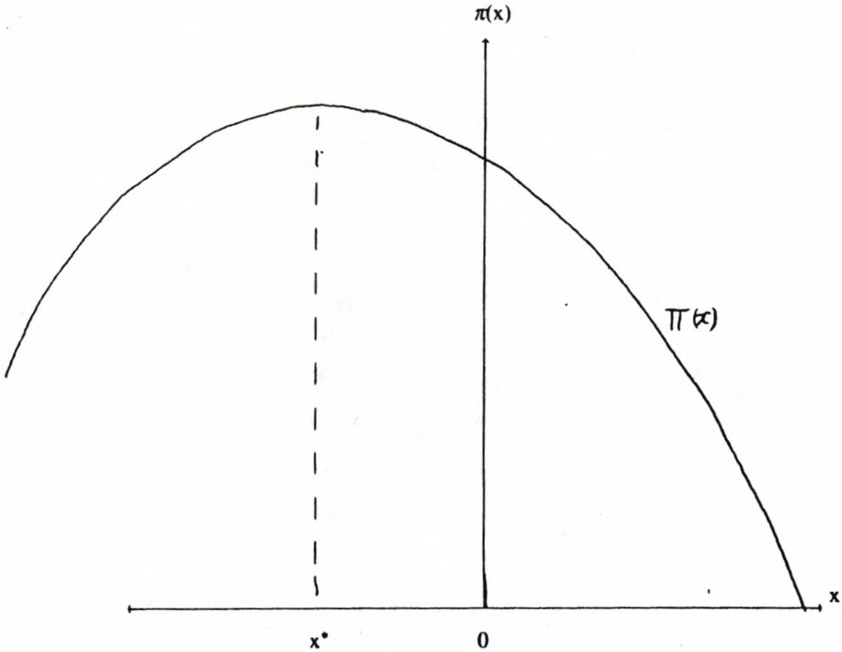
$$\pi_x(\hat{x}(t),t) + \mu(t) = 0 \qquad (8')$$

$$\lambda(t).\hat{I}(t) = 0 \qquad (9')$$

The next section considers an example with linear demand and supply functions which happens to be a case where the profit function is concave in x. A solution is obtained and discussed. For more general demand and supply functions it is necessary to construct a solution in stages, and to characterise the solution according to the path of inventories over the period. This characterisation reveals three alternative policies, which are to hold no inventories, to accumulate and deplete inventories gradually, and to accumulate and deplete inventories as rapidly as possible - which is more precisely defined below. The second of these is referred to as the "gradual inventory policy", and the third is referred to as the "bang-bang inventory policy".

Consider (7′) - (9′). Recall $\mu(t) \geq 0$ for all t which, from (8′), implies that the middleman only operates in the region of x for which $\pi(x,t)$ is non-increasing. A concave profit function is drawn in diagram 1 (for any arbitrary instant). The maximum occurs for a negative value of x (denoted x^*), since the profit is higher if the middleman sells more than he buys (provided, for example, that the inverse supply function is strictly increasing).

Diagram 1



Thus, the region of x of interest contains both positive and negative values of x so that inventories may be accumulated, left constant, or depleted.

Initial inventories, represented by I_0, will be either zero or a positive finite number. Proposition 1 demonstrates that final inventories will be zero if the buying price is strictly positive and there are no initial inventories.

Proposition 1

$I(T) = 0$ along all optimal paths if $p > 0$ and $I_0 = 0$.

Proof:

Suppose $(\hat{x}(t), \hat{I}(t))$ is an optimal path satisfying $\int_0^T \hat{x}(t)dt = \hat{I}(T) > 0$. Consider any other admissible pair $(\tilde{x}(t), \tilde{I}(t))$ which satisfies $\tilde{x}(t) \leq \hat{x}(t)\ \ \forall\ t \in [0,T]$ and

$\int_0^T \tilde{x}(t)dt < \int_0^T \hat{x}(t)dt.$

The total profit from policy $(\tilde{x}(t), \tilde{I}(t))$ is strictly greater than that from policy $(\hat{x}(t), \hat{I}(t))$ since $\pi(\tilde{x}) \geq \pi(\hat{x})\ \ \forall\ t \in [0,T]$ and $\pi(\tilde{x}) > \pi(\hat{x})$ for some part of the interval. Thus $(\hat{x}(t), \hat{I}(t))$ is not optimal. ||

In this case, there is no optimal path with positive inventories at time T because the middleman could have done better by not having bought the inventories. The result holds when there are modest initial inventories. If there are substantial initial inventories, then the middleman might have to dump them at the end in order to avoid selling beyond the profit maximisation level.

Consider an optimal policy.

Differentiate (8′) $$\pi_{xx}.\dot{x} + \pi_{xt} + \dot{\mu} = 0$$

and combine (7′) $$\dot{x}(t) = -\left(\frac{1}{\pi_{xx}}\right)(\pi_{xt} + h - \lambda(t)) \qquad (15)$$

This gives the time derivative of the change in inventories. The conditions above are sufficient for optimality for concave $\pi(x)$. Hence, for strictly convex $\pi(x)$, the sign of $\dot{x}(t)$ is given by the sign of $\pi_{xt} + h - \lambda(t)$ along any optimal path. If $x(t)$ is continuous, then gradual accumulation of inventories followed by gradual depletion is characterised by $\dot{x}(t) < 0$. From equation 9′

$$I(t) > 0 \implies \lambda(t) = 0$$

$$\lambda(t) > 0 \implies I(t) = 0$$

Therefore, if inventories are held, the sign of $\dot{x}(t)$ is the same as the sign of $\pi_{xt} + h$.

Diagram 2a: Path of x(t)

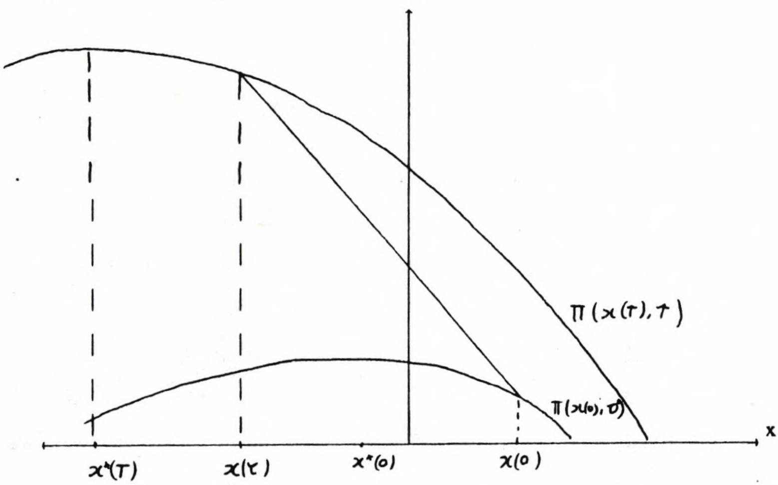
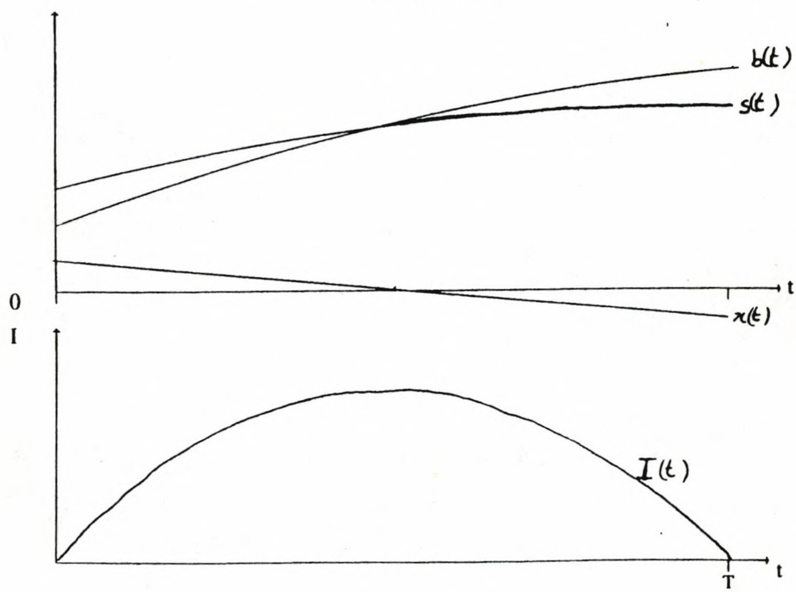


Diagram 2b



When initial inventories are zero, this gives a policy of no inventories for $\pi_{xt} \geq - h$. The middleman would make no more profit by accumulating inventories initially and selling running them down later. In fact, the middleman would prefer to sell forward and hold negative inventories in the beginning, but this is not allowed. The constraint $I(t) \geq 0$ is binding and the solution is characterised by

$$I(t) = 0, \;\; x(t) = 0 \;\text{ for all t.}$$

Otherwise, $\pi_{xt} < - h$ and the middleman first accumulates inventories at a decreasing rate, and then depletes them at an increasing rate. The exact path is determined by the terminal conditions. $\pi(x(t),t)$ is moving over time in such a way that its slope at $x = 0$ is decreasing (becoming steeper), as shown in diagram 2a (based on the linear example from the following section). Diagram 2b depicts the optimal path of the quantities chosen, and of inventories. In this case, inventories are accumulated in earlier periods, sacrificing a relatively small profit, and sold later, giving a policy of gradual accumulation followed by gradual depletion. Note that it is not required that the levels of profit obtained in later periods be higher, only that the slope become steeper (negative) at $x = 0$.

Next suppose that the middleman holds positive initial inventories. If $\pi_{xt} \geq - h$, then the optimal policy is to deplete inventories at a decreasing rate. The holding costs cause the middleman to deplete faster in the earlier stages than in the later ones. The rate of change of depletion is determined by the consideration of the marginal profit and the marginal holding cost. To see this, consider $\pi_{xt} = 0$. The rate of change of inventory depletion is given by $\dot{x}(t) = - \frac{h}{\pi_{xx}} > 0$. Thus higher holding costs are likely to lead to more rapid depletion of inventories. If π_{xx} is larger, then the marginal profit from a change in x is decreasing faster, so a smaller change in x is necessary to achieve the correct balance between marginal profit and holding cost, and hence the change in x is slower. The length of the time horizon has no effect on the change in net purchases, but may affect $x(0)$ - the starting point. If the inventory holding costs are sufficiently high, or the initial inventories are sufficiently small, then inventories may be exhausted at some time $\tau < T$. The function $x(t)$ might be discontinuous at τ with $x(t) = 0$ for all $t \in (\tau,T]$. However, this seems unlikely given concavity - see the example in the following section - as the trade-off between holding inventories and operating profit increasingly favours the former as x increases. This suggests that a policy with discontinuous $x(t)$ would not be optimal, as it would be dominated by one with a higher $x(0)$ and slower depletion.

Diagram 3a illustrates one possible situation where $x(0)$ is strictly negative. The distance between $x(0)$ and $x(\tau)$ is directly related to the magnitude of the holding cost, and the average (per period) profit, as represented by some convex combination of $\pi(x(0),0)$ and $\pi(x(\tau),\tau)$, is higher as the holding cost is higher and as the initial inventories are higher. If initial inventories are sufficiently high, the middleman would operate near x^* for the entire period and dump stock at the end. However, this is an extreme case and not particularly likely. Diagram 3b shows the optimal path of quantities, net purchases, and inventories for the case where inventories are exhausted before the end of the period.

If there are initial inventories and $\pi_{xt} < - h$, the middleman will deplete inventories gradually over the period, in a manner similar to that described in the discussion of diagram 2 above. Initial inventories alter the path by reducing the levels of both x_0 and x_1 in that diagram. That is because the middleman does not need to accumulate as many inventories. The overall profit will also be higher - a consequence of his having more initial inventories.

In summary, optimal inventory holdings over time are determined by three principle factors: the level of initial inventories; the level of holding costs; and the time derivative of the marginal profit. Higher initial inventories and a lower level of the holding cost act in same direction, in that inventories are held for a longer time and that overall profit (profit plus holding costs) is higher. The holding cost (partially) determines the rate of depletion of the stock of inventory, and the size of the stock of inventory affects the initial point $x(0)$. The middleman will never accumulate inventories unless there is a strong negative time effect on marginal profit. Only in this case will the higher future profit at the margin make it worthwhile to accumulate inventories. The linear example in the next section shows more precisely the manner in which the above factors determine the optimal inventory path.

Diagram 3a

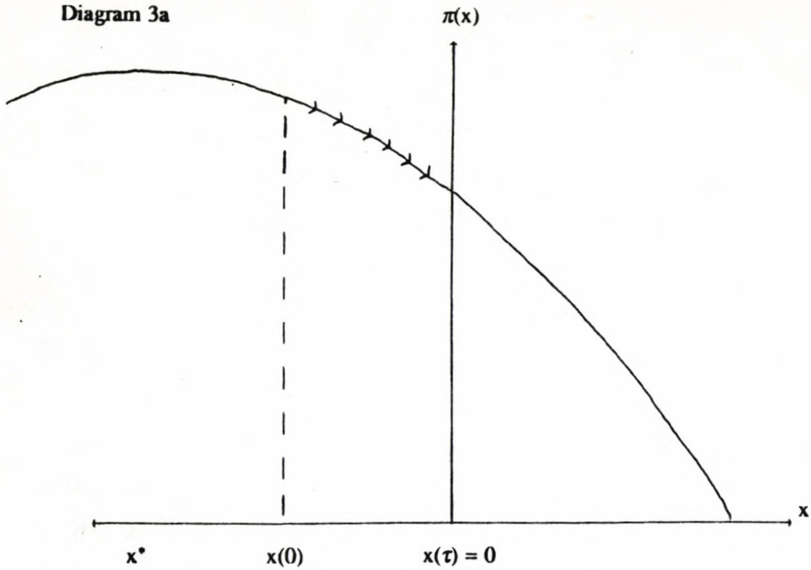
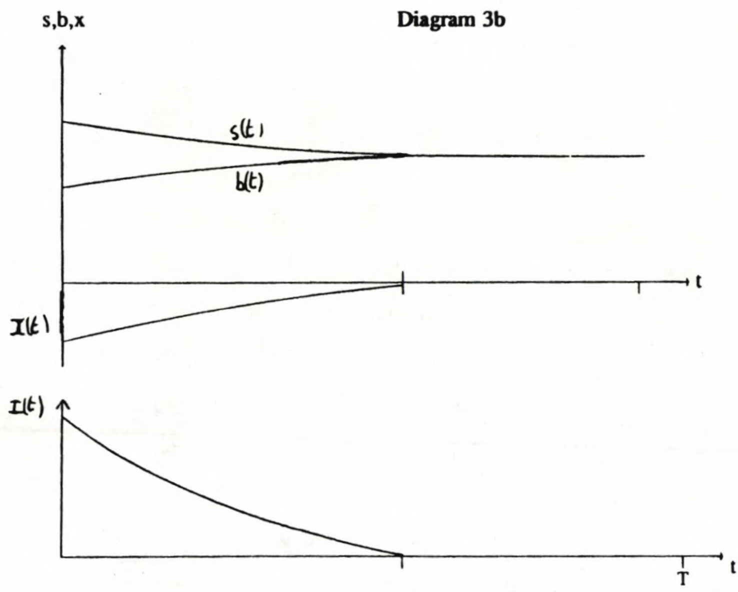


Diagram 3b



Next, consider the solution for $\pi(x)$ non-concave in x. When $\pi(x)$ is convex at $x = 0$, it is possible to concavify it by choosing $y, z \in \mathbb{R}$, and $\alpha \in [0,1]$ such that

$$z \leq 0 \leq y;$$

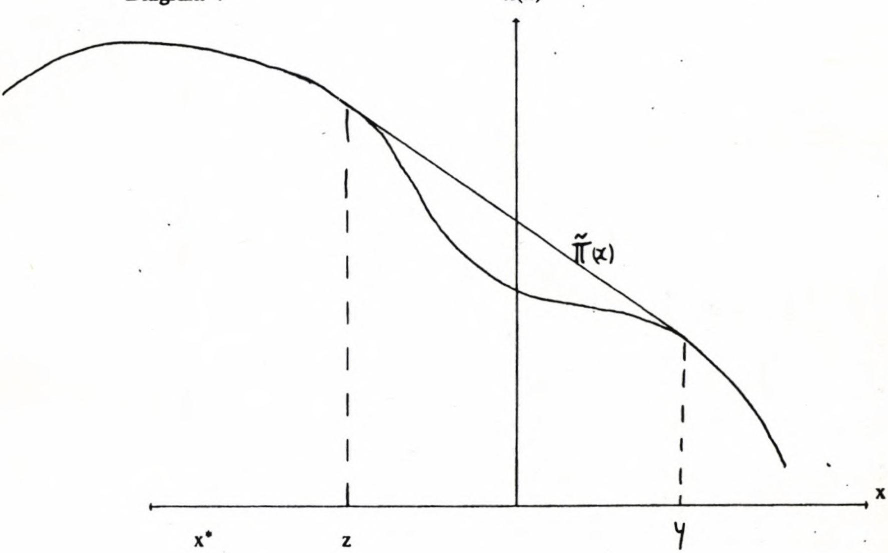
$$\alpha y + (1-\alpha)z \geq 0;$$

Suppose that in any interval $(t,t+\varepsilon)$, $x = y$ for proportion α and $x = z$ for the remaining $1 - \alpha$. Let $\xi = \alpha y + (1-\alpha)z$.

Define $\tilde{\pi}(\xi,t)$ $$\tilde{\pi}(\xi,t) = \mathrm{Max}\{\alpha\pi(y) + (1-\alpha)\pi(z) \mid \xi \geq 0, y \geq 0\} \qquad (16)$$

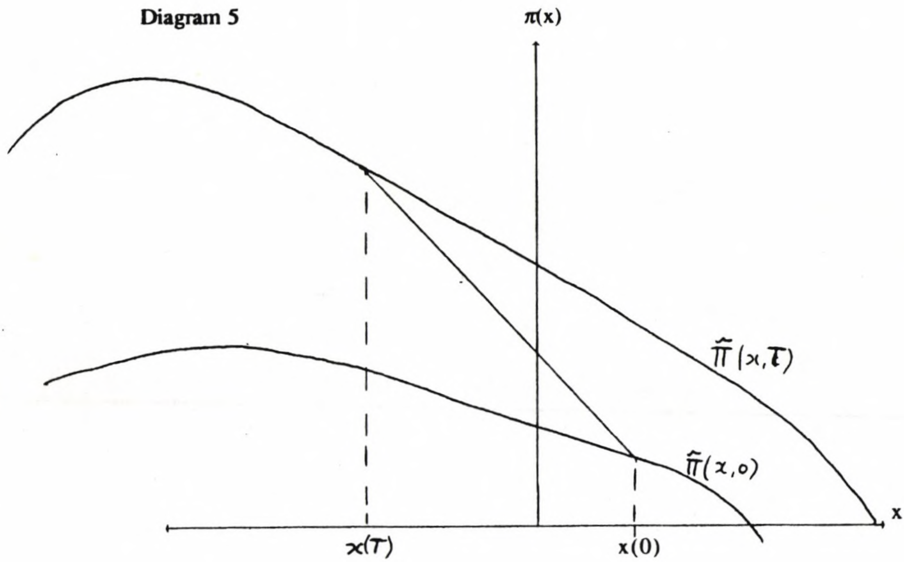
$\tilde{\pi}(\xi,t)$ is a concave function (see diagram 4) and, consequently the first order conditions from section 2 may be applied to $\tilde{\pi}(\xi,t)$ and a solution for ξ may be found in a manner similar to that for x above. To see that y is well-defined and bounded above, note that z is bounded below by x^* and that, for x sufficiently large, $b > \underline{b}$ and profit is non-positive.

Diagram 4



Consider first the case where there are no initial inventories and $\tilde{\pi}_{xt} > - h$. This suggests that the optimal policy is $\xi = 0$. However, this permits a bang-bang inventory policy whereby inventories are accumulated for an instant (at point y) and depleted in the next instant (at point z). The inventory policy is characterised by inventories alternating back and forth between y and z with resting times at the two points such that $\xi = 0$. The limit of the overall profit, as this alternation becomes faster, is the supremum for the problem, but is never attained as there is an infinitesimal holding cost. There is no well-defined function for $\dot{x}(t)$, and there is no optimal solution, as every path is dominated by some other path with faster oscillation. Such a phenomenon is known in the literature as chattering, and Romer (1986) discusses this phenomenon. For h = 0 there is a continuum of solutions in which the supremum is attained, all of which will be typified by x resting at z or y and in proportion to their distances from the origin.

Diagram 5



If there are no initial inventories and $\tilde{\pi}_{xt} \leq -h$, the inventory policy is a gradual one as before. This is illustrated in diagram 5, where point A is intended to represent the supremum of the profit from a chattering path, namely $\alpha\pi(y) + (1-\alpha)\pi(z)$. The second profit schedule is drawn for a later instant $(t > 0)$ and so that the slope of $\tilde{\pi}(\xi,t)$ at $\xi = 0$ is becoming steeper. If $\tilde{\pi}_{xt} = -h$, the middleman earns the supremum of the chattering profit by pursuing a gradual accumulation-depletion policy and this is then the optimal policy.

If there are initial inventories, then there will either be chattering or gradual depletion (possibly followed by chattering). The chattering in this case will be slightly different than that described above, in so far as the initial inventories enable the middleman either to rest for a greater proportion at the negative level or to deplete the initial inventories before the chattering commences. (In either case it seems that α is non-increasing in the level of initial inventories.)

The non-existence of an optimal inventory policy presents a problem which may be resolved in a number of ways, some of which are mentioned in section 5. Moreover, the analysis in the appendix suggests that convex $\pi(x)$ is relatively rare. The example of linear demand and supply which is considered in the next section gives $\pi(x)$ concave so that chattering is not seen. Examples may be conceived which exhibit convexity; however, these usually involve higher order polynomials (of at least order four for $\pi(x)$) or other non-linear functional forms, and are not investigated further.

4 An Example with Linear Supply and Demand

The example uses linear supply and demand functions. The possibility that the profit function varies over time is incorporated in the demand side by assuming that there is a trend component, r, in the intercept of the demand curve. When r is positive, the demand curve shifts out at a constant rate over the period. By setting $r = 0$, it is possible to examine the case where the profit function is independent of time. Specifically, the supply and demand are, respectively,

$$p(s(t)) = p_0 + p_1 s(t) \tag{17}$$

$$P(b(t),t) = P_0 + rt - P_1 b(t) \tag{18}$$

Substituting these into the formulae from the previous section gives

$$\pi(x) = \frac{(P_0+rt-p_0)^2}{4(P_1+p_1)} - \frac{(P_0p_1+rtp_1+P_1p_0)}{P_1+p_1}.x - \frac{P_1 \cdot p_1}{P_1+p_1}.x^2 \tag{19}$$

$$\Rightarrow \qquad \pi_{xx} = -2.\frac{P_1 \cdot p_1}{P_1+p_1} \quad \text{and} \quad \pi_{xt} = -\frac{p_1 \cdot r}{P_1+p_1}$$

When inventories are held, the rate of change of net purchases is given as

$$\dot{x}(t) = \frac{\pi_{xt} + h}{\pi_{xx}} = \frac{(P_1+p_1)h - p_1 r}{2P_1 \cdot p_1}$$

$$\Rightarrow \qquad x(t) = \frac{(P_1+p_1)h - p_1 r}{2P_1 \cdot p_1}.t + x(0)$$

$$\Rightarrow \qquad I(t) = I_0 + \int_0^t x(t).dt = I_0 + \frac{(P_1+p_1)h - p_1 r}{4P_1 \cdot p_1}.t^2 + x(0).t$$

Define τ such that $I(\tau) = 0$, i.e. τ is the time at which inventories run out. This gives

$$I_0 + \frac{(P_1+p_1)h - p_1 r}{4P_1 \cdot p_1}.\tau^2 + x(0).\tau = 0$$

$$\Rightarrow \qquad x(0) = -\frac{I_0}{\tau} - \frac{(P_1+p_1)h - p_1 r}{4P_1 \cdot p_1}.\tau$$

$$\Rightarrow \qquad x(t) = \frac{(P_1+p_1)h - p_1 r}{2P_1 \cdot p_1}.t - \frac{I_0}{\tau} - \frac{(P_1+p_1)h - p_1 r}{4P_1 \cdot p_1}.\tau \tag{20}$$

and

$$I(t) = I_0(1 - \frac{t}{\tau}) + \frac{(P_1+p_1)h - p_1 r}{4P_1 \cdot p_1}.t.(t - \tau) \tag{21}$$

Consider first $\pi_{xt} > -h$. This is the case where inventories are gradually run down and for which $x(t) \leq 0$ for all t. Because the profit function is concave, the middleman will never wish for inventories to run out for $x(t) < 0$ since he could do better by smoothing slightly as the profit function is getting steeper as x increases. Therefore, τ is determined by $x(\tau) = 0$ if $\tau < T$ and by $\tau = T$ otherwise. The solution to the former is obtained from

$$\frac{(P_1+p_1)h - p_1 r}{4P_1 \cdot p_1}.\tau - \frac{I_0}{\tau} = 0$$

$$\Longrightarrow \qquad \tau = \text{Min}\left(T, \sqrt{I_0 \cdot \frac{4P_1 \cdot p_1}{(P_1+p_1)h - p_1 r}}\right) \qquad (22)$$

Inventories are exhausted before the end of the period if and only if

$$\sqrt{I_0 \cdot \frac{4P_1 \cdot p_1}{(P_1+p_1)h - p_1 r}} < T$$

This is more likely when the initial inventories are small, the holding cost is high, or the time horizon is large. A higher growth rate of demand has the effect of making inventories last longer because it increases the trade-off between current and future profit.

Consider next $\pi_{xt} < -h$. This is equivalent to $(P_1 + p_1).h < p_1.r$ which requires that the growth of the demand intercept be greater than the holding cost. The level of net purchases declines over the period. If $x(0) > 0$, then the middleman accumulates inventories earlier and depletes later. Otherwise, $x(0) \leq 0$ and there is no further accumulation of inventories, and the middleman runs down the initial inventories at an increasing rate. The former case is more likely to occur if I_0 is small or if the period is longer.

This example illustrates some of the results of the previous section and gives an indication of the precise manner in which various factors such as the parameters of the demand and supply functions, the holding cost, the trend component in demand, and the length of the period affect the optimal inventory path.

5 Discussion

In summary, the main insights of this paper are as follows. If there are no initial inventories, the middleman will not accumulate any unless the marginal operating profit at $x = 0$ decreases substantially over the period or the operating profit function is convex at $x = 0$. In the former case, there exists an optimal inventory policy, characterised by gradual accumulation in earlier stages followed by gradual depletion. The linear example confirms that this is a well-behaved continuous process. In the latter, no optimal policy exists as the middleman wishes to accumulate and deplete infinitely fast. If there are initial inventories, these are run down at a decreasing rate, unless one of the two cases above (the marginal profit at $x = 0$ decreases over the period or the profit function is convex at $x = 0$.) arises. In that case, the inventory policy is as described, with the difference that there is relatively less accumulation of inventories over the period.

Thus we have characterised the optimal inventory policy, where such exists, shown that it depends on initial inventories, the level of the holding cost, and the rate of change of the marginal profit, and identified the circumstances under which no optimal policy exists.

The relevance of these results depends on whether the assumptions of the model are likely to be observed in practice. Obviously, the non-optimal chattering result exists only in a model of continuous time, since discrete time would impose an exogenous minimum on the length of time which x must rest at one position. Nevertheless, even in discrete time inventories would fluctuate, the only difference being that the optimum can be attained.

The model and results are not limited to the case of a monopolist middleman. For instance, the model could be modified to consider a (monopolist) producer who faces a demand curve that shifts out (or back) during the period. The inventory policy of such a producer could be analysed by defining a function analogous to $\pi(x)$ above and applying the results to this function. The cost function would play a role in determining the concavity of the profit function, and thereby in determining whether a solution exists.

The results may readily be extended to deal with the case where there are several quantity setting middlemen. The profit for a representative middleman i at any time t is

$$\pi_i = b_i.P(\sum_j b_j) - s_i.p(\sum_j s_j)$$

$$\Rightarrow \qquad \pi(x_i,t) = \mathrm{Max}\left\{b_i.P(\sum_j b_j) - s_i.p(\sum_j s_j) \;\middle|\; s_i - b_i = x_i,\ s_i,\ b_i \geq 0\right\}.$$

Continuing as in section 3 gives first order conditions

$$\lambda_i - h_i = \dot{\mu}_i;\quad \frac{\partial \Pi}{\partial x_i} = \mu_i;\quad \lambda(t).I(t) = 0.$$

The analysis proceeds as before. Even if the middlemen have different initial inventories, the aggregate path of inventories should have properties broadly the same as those of the monopolist middleman.

The absence of a discount factor, typical in models of this kind, does not materially affect the results. Including a discount factor would put a higher weight on profit in earlier periods, reduce the likelihood that inventories are held where there are no initial inventories, and increase the rate of depletion of any initial inventories which exist.

The holding cost was assumed to be linear and constant over time. If it varies over time, then the time path of disposal is altered: the precise effect may be obtained by analyzing equation 15 with $h(t)$ in place of h. If the inventory cost is non-linear, then the effect is more complicated and the model may not be solved explicitly as outlined above. In general, if the holding cost increases with the quantity held we would expect inventories to be depleted more quickly and chattering to be more likely to arise, owing to a bias in favour of holding smaller levels of inventory.

A limitation of the model is the absence of price expectations on the part of buyers and sellers. This might be justified by assuming either that there is a new generation of buyers and sellers at each instant or that only the middleman has access to or can afford the storage technology for an otherwise perishable good. Nevertheless, there are some situations in which agents may choose the period in which they trade, and the model as specified does not deal with this possibility. One obvious effect would be on the non-existence of an optimal path due to chattering. Recall that net purchases alternate between positive and negative values. This suggests that buying and selling prices will alternate rapidly between being low when x is positive and being higher when x is negative. Enabling agents to choose the period in which to trade would alter markedly the slope of the profit function at the margin and encourage the middleman to pursue an inventory policy which smoothes prices. Such a model would be technically more difficult to analyse, but we might expect the degree of price smoothing to be inversely related to the agents' costs of delaying or bringing forward purchases. For the case where there is a solution to the inventory problem, the model assumes elements of a durable good monopoly problem in continuous time, and this provides another interesting direction for further research.

Appendix: Properties of $\pi(x)$

$$\pi(x(t),t) = \text{Max}\left\{ b(t).P(b(t),t) - s(t).p(s(t)) \;\middle|\; s(t) - b(t) = x(t),\ s,\ b \geq 0 \right\}$$

The first order conditions are

$$b.P' + P = s.p' + p \quad \text{and} \quad s = b + x \tag{23}$$

giving functions $b(x)$ and $s(x)$ and the reduced form

$$\pi(b(x),s(x)) = b(x).P(b(x),t)) - s(x).p(s(x),t) \tag{24}$$

Differentiating (23) with respect to x

$$(2P' + bP'')\frac{db}{dx} = (2p' + sp'')\frac{ds}{dx} \quad \text{and} \quad \frac{ds}{dx} = \frac{db}{dx} + 1$$

which solve to give

$$\frac{db}{dx} = \frac{r}{R - r} \qquad \text{where} \qquad r = 2p' + sp''$$

$$\frac{ds}{dx} = \frac{R}{R - r} \qquad\qquad\qquad R = 2P' + bP''$$

Differentiating again gives

$$\frac{d^2b}{dx^2} = \frac{d^2s}{dx^2} = \frac{R_b r^2 - r_s R^2}{(r - R)^3} \qquad \begin{aligned} R_b &= 3P'' + bP''' \\ r_s &= 3p'' + bp''' \end{aligned}$$

Differentiating (24)

$$\frac{d\pi}{dx} = (b.P' + P)\frac{db}{dx} - (s.p' + p)\frac{ds}{dx}$$

$$\frac{d^2\pi}{dx^2} = (2P' + bP'')\left(\frac{db}{dx}\right)^2 - (2p' + sp'')\left(\frac{ds}{dx}\right)^2, \text{ as } \frac{d^2b}{dx^2} = \frac{d^2s}{dx^2}$$

$$= \frac{r.R}{r - R}$$

There are three possible cases for $\pi(x)$ to be concave as follows:

a) $0 < r < R \Leftrightarrow \quad 0 < sp'' + 2p' < bP'' + 2P'$

b) $R < 0 < r \Leftrightarrow \quad bP'' + 2P' < 0 < sp'' + 2p'$

c) $r < R < 0 \Leftrightarrow \quad sp'' + 2p' < bP'' + 2P' < 0$

Note that $sp'' + 2p' > 0$ and $bP'' + 2P' < 0$ except when there is an unusually strong second order effect on either supply or demand. Thus for a large class of functions, $\pi(x)$ is concave in x. This class includes all concave demand functions and all convex supply functions. Case (b) includes many familiar examples (such as linear) which are given for supply and demand functions, and we would normally expect to find that $\pi(x)$ is concave. However, if there is an unusually strong negative second order effect on supply or an unusually strong positive second order effect on demand, then it is possible that $\pi(x)$ will be convex. Cases (a) and (c) above are more extreme, and we would not expect to encounter examples very often, unless the functions have irregular slopes - a curiosity which is not dealt with here. Linear supply and demand schedules very obviously give rise to a function which is concave in x. In general, the above conditions may be used on any demand and supply function to determine whether the optimal inventory policy exists, and to characterise it where it does.

Bibliography

Cantillon, R., 1755. *Essai sur la Nature du Commerce en General.* Edited with English Translation and Other Material by Henry Higgs, London: MacMillan (for the Royal Economic Society), 1931.

Irvine, F.O., 1981. "An Optimal Middleman Firm Price Adjustment Policy: The 'Short-run Inventory Based Pricing Policy'", *Economic Inquiry*, 19:245-260.

Fingleton, J.A., 1991. *Middlemen*, D.Phil. thesis, University of Oxford.

Romer, P. 1986. "Cake Eating, Chattering, and Jumps: Existence Results for Variational Problems", *Econometrica*, 54:897-908.

Seierstad, A. and K. Sydsaeter, 1977. "Sufficient Conditions in Optimal Control Theory", *International Economic Review*, 18:367-391.

Stahl, D.O., 1988. "Bertrand Competition for Inputs and Walrasian Outcomes", *American Economic Review*, 78:190-201.

Takayama, A. 1974. *Mathematical Economics*, Illinois: The Dryden Press.

Zabel, E., 1972. "Multi-period Monopoly under Uncertainty", *Journal of Economic Theory*, 5:524-536.

EUI WORKING PAPERS

EUI Working Papers are published and distributed by the European University Institute, Florence

Copies can be obtained free of charge – depending on the availability of stocks – from:

The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy

Please use order form overleaf

Publications of the European University Institute

To

The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy

From

Name . . . .

Address . . . .

. . . .

. . . .

. . . .

. . . .

❒ Please send me a complete list of EUI Working Papers
❒ Please send me a complete list of EUI book publications
❒ Please send me the EUI brochure Academic Year 1992/93

Please send me the following EUI Working Paper(s):

No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .

Date . . . .

Signature . . . .

Working Papers of the Department of Economics Published since 1989

89/370
B. BENSAID/ R.J. GARY-BOBO/
S. FEDERBUSCH
The Strategic Aspects of Profit Sharing in the Industry

89/374
Francisco S. TORRES
Small Countries and Exogenous Policy Shocks

89/375
Renzo DAVIDDI
Rouble Convertibility: A Realistic Target

89/377
Elettra AGLIARDI
On the Robustness of Contestability Theory

89/378
Stephen MARTIN
The Welfare Consequences of Transaction Costs in Financial Markets

89/381
Susan SENIOR NELLO
Recent Developments in Relations Between the EC and Eastern Europe

89/382
Jean GABSZEWICZ/ Paolo GARELLA/
Charles NOLLET
Spatial Price Competition With Uninformed Buyers

89/383
Benedetto GUI
Beneficiary and Dominant Roles in Organizations: The Case of Nonprofits

89/384
Agustín MARAVALL/ Daniel PEÑA
Missing Observations, Additive Outliers and Inverse Autocorrelation Function

89/385
Stephen MARTIN
Product Differentiation and Market Performance in Oligopoly

89/386
Dalia MARIN
Is the Export-Led Growth Hypothesis Valid for Industrialized Countries?

89/387
Stephen MARTIN
Modeling Oligopolistic Interaction

89/388
Jean-Claude CHOURAQUI
The Conduct of Monetary Policy: What have we Learned From Recent Experience

89/390
Corrado BENASSI
Imperfect Information and Financial Markets: A General Equilibrium Model

89/394
Serge-Christophe KOLM
Adequacy, Equity and Fundamental Dominance: Unanimous and Comparable Allocations in Rational Social Choice, with Applications to Marriage and Wages

89/395
Daniel HEYMANN/ Axel LEIJONHUFVUD
On the Use of Currency Reform in Inflation Stabilization

89/400
Robert J. GARY-BOBO
On the Existence of Equilibrium Configurations in a Class of Asymmetric Market Entry Games *

89/402
Stephen MARTIN
Direct Foreign Investment in The United States

89/413
Francisco S. TORRES
Portugal, the EMS and 1992: Stabilization and Liberalization

89/416
Joerg MAYER
Reserve Switches and Exchange-Rate Variability: The Presumed Inherent Instability of the Multiple Reserve-Currency System

89/417
José P. ESPERANÇA/ Neil KAY
Foreign Direct Investment and Competition in the Advertising Sector: The Italian Case

* Working Paper out of print

89/418
Luigi BRIGHI/ Mario FORNI
Aggregation Across Agents in Demand Systems

89/420
Corrado BENASSI
A Competitive Model of Credit Intermediation

89/422
Marcus MILLER/ Mark SALMON
When does Coordination pay?

89/423
Marcus MILLER/ Mark SALMON/
Alan SUTHERLAND
Time Consistency, Discounting and the Returns to Cooperation

89/424
Frank CRITCHLEY/ Paul MARRIOTT/
Mark SALMON
On the Differential Geometry of the Wald Test with Nonlinear Restrictions

89/425
Peter J. HAMMOND
On the Impossibility of Perfect Capital Markets

89/426
Peter J. HAMMOND
Perfected Option Markets in Economies with Adverse Selection

89/427
Peter J. HAMMOND
Irreducibility, Resource Relatedness, and Survival with Individual Non-Convexities

* * *

ECO No. 90/1[**]
Tamer BAŞAR and Mark SALMON
Credibility and the Value of Information Transmission in a Model of Monetary Policy and Inflation

ECO No. 90/2
Horst UNGERER
The EMS – The First Ten Years
Policies – Developments – Evolution

ECO No. 90/3
Peter J. HAMMOND
Interpersonal Comparisons of Utility: Why and how they are and should be made

ECO No. 90/4
Peter J. HAMMOND
A Revelation Principle for (Boundedly) Bayesian Rationalizable Strategies

ECO No. 90/5
Peter J. HAMMOND
Independence of Irrelevant Interpersonal Comparisons

ECO No. 90/6
Hal R. VARIAN
A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed

ECO No. 90/7
Hal R. VARIAN
Sequential Provision of Public Goods

ECO No. 90/8
T. BRIANZA, L. PHLIPS and J.F. RICHARD
Futures Markets, Speculation and Monopoly Pricing

ECO No. 90/9
Anthony B. ATKINSON/ John MICKLEWRIGHT
Unemployment Compensation and Labour Market Transition: A Critical Review

ECO No. 90/10
Peter J. HAMMOND
The Role of Information in Economics

ECO No. 90/11
Nicos M. CHRISTODOULAKIS
Debt Dynamics in a Small Open Economy

ECO No. 90/12
Stephen C. SMITH
On the Economic Rationale for Codetermination Law

ECO No. 90/13
Elettra AGLIARDI
Learning by Doing and Market Structures

ECO No. 90/14
Peter J. HAMMOND
Intertemporal Objectives

ECO No. 90/15
Andrew EVANS/Stephen MARTIN
Socially Acceptable Distortion of Competition: EC Policy on State Aid

---

[**] Please note: As from January 1990, the EUI Working Papers Series is divided into six sub-series, each series will be numbered individually (e.g. EUI Working Paper LAW No. 90/1).

* Working Paper out of print

ECO No. 90/16
Stephen MARTIN
Fringe Size and Cartel Stability

ECO No. 90/17
John MICKLEWRIGHT
Why Do Less Than a Quarter of the Unemployed in Britain Receive Unemployment Insurance?

ECO No. 90/18
Mrudula A. PATEL
Optimal Life Cycle Saving With Borrowing Constraints: A Graphical Solution

ECO No. 90/19
Peter J. HAMMOND
Money Metric Measures of Individual and Social Welfare Allowing for Environmental Externalities

ECO No. 90/20
Louis PHLIPS/
Ronald M. HARSTAD
Oligopolistic Manipulation of Spot Markets and the Timing of Futures Market Speculation

ECO No. 90/21
Christian DUSTMANN
Earnings Adjustment of Temporary Migrants

ECO No. 90/22
John MICKLEWRIGHT
The Reform of Unemployment Compensation: Choices for East and West

ECO No. 90/23
Joerg MAYER
U. S. Dollar and Deutschmark as Reserve Assets

ECO No. 90/24
Sheila MARNIE
Labour Market Reform in the USSR: Fact or Fiction?

ECO No. 90/25
Peter JENSEN/
Niels WESTERGÅRD-NIELSEN
Temporary Layoffs and the Duration of Unemployment: An Empirical Analysis

ECO No. 90/26
Stephan L. KALB
Market-Led Approaches to European Monetary Union in the Light of a Legal Restrictions Theory of Money

ECO No. 90/27
Robert J. WALDMANN
Implausible Results or Implausible Data? Anomalies in the Construction of Value Added Data and Implications for Estimates of Price-Cost Markups

ECO No. 90/28
Stephen MARTIN
Periodic Model Changes in Oligopoly

ECO No. 90/29
Nicos CHRISTODOULAKIS/
Martin WEALE
Imperfect Competition in an Open Economy

* * *

ECO No. 91/30
Steve ALPERN/Dennis J. SNOWER
Unemployment Through 'Learning From Experience'

ECO No. 91/31
David M. PRESCOTT/Thanasis STENGOS
Testing for Forecastible Nonlinear Dependence in Weekly Gold Rates of Return

ECO No. 91/32
Peter J. HAMMOND
Harsanyi's Utilitarian Theorem: A Simpler Proof and Some Ethical Connotations

ECO No. 91/33
Anthony B. ATKINSON/
John MICKLEWRIGHT
Economic Transformation in Eastern Europe and the Distribution of Income

ECO No. 91/34
Svend ALBAEK
On Nash and Stackelberg Equilibria when Costs are Private Information

ECO No. 91/35
Stephen MARTIN
Private and Social Incentives to Form R & D Joint Ventures

ECO No. 91/36
Louis PHLIPS
Manipulation of Crude Oil Futures

ECO No. 91/37
Xavier CALSAMIGLIA/Alan KIRMAN
A Unique Informationally Efficient and Decentralized Mechanism With Fair Outcomes

* Working Paper out of print

ECO No. 91/38
George S. ALOGOSKOUFIS/
Thanasis STENGOS
Testing for Nonlinear Dynamics in Historical Unemployment Series

ECO No. 91/39
Peter J. HAMMOND
The Moral Status of Profits and Other Rewards: A Perspective From Modern Welfare Economics

ECO No. 91/40
Vincent BROUSSEAU/Alan KIRMAN
The Dynamics of Learning in Mis-Specified Models

ECO No. 91/41
Robert James WALDMANN
Assessing the Relative Sizes of Industry- and Nation Specific Shocks to Output

ECO No. 91/42
Thorsten HENS/Alan KIRMAN/Louis PHLIPS
Exchange Rates and Oligopoly

ECO No. 91/43
Peter J. HAMMOND
Consequentialist Decision Theory and Utilitarian Ethics

ECO No. 91/44
Stephen MARTIN
Endogenous Firm Efficiency in a Cournot Principal-Agent Model

ECO No. 91/45
Svend ALBAEK
Upstream or Downstream Information Sharing?

ECO No. 91/46
Thomas H. McCURDY/
Thanasis STENGOS
A Comparison of Risk-Premium Forecasts Implied by Parametric Versus Nonparametric Conditional Mean Estimators

ECO No. 91/47
Christian DUSTMANN
Temporary Migration and the Investment into Human Capital

ECO No. 91/48
Jean-Daniel GUIGOU
Should Bankruptcy Proceedings be Initiated by a Mixed Creditor/Shareholder?

ECO No. 91/49
Nick VRIEND
Market-Making and Decentralized Trade

ECO No. 91/50
Jeffrey L. COLES/Peter J. HAMMOND
Walrasian Equilibrium without Survival: Existence, Efficiency, and Remedial Policy

ECO No. 91/51
Frank CRITCHLEY/Paul MARRIOTT/
Mark SALMON
Preferred Point Geometry and Statistical Manifolds

ECO No. 91/52
Costanza TORRICELLI
The Influence of Futures on Spot Price Volatility in a Model for a Storable Commodity

ECO No. 91/53
Frank CRITCHLEY/Paul MARRIOTT/
Mark SALMON
Preferred Point Geometry and the Local Differential Geometry of the Kullback-Leibler Divergence

ECO No. 91/54
Peter MØLLGAARD/
Louis PHLIPS
Oil Futures and Strategic Stocks at Sea

ECO No. 91/55
Christian DUSTMANN/
John MICKLEWRIGHT
Benefits, Incentives and Uncertainty

ECO No. 91/56
John MICKLEWRIGHT/
Gianna GIANNELLI
Why do Women Married to Unemployed Men have Low Participation Rates?

ECO No. 91/57
John MICKLEWRIGHT
Income Support for the Unemployed in Hungary

ECO No. 91/58
Fabio CANOVA
Detrending and Business Cycle Facts

ECO No. 91/59
Fabio CANOVA/
Jane MARRINAN
Reconciling the Term Structure of Interest Rates with the Consumption Based ICAP Model

ECO No. 91/60
John FINGLETON
Inventory Holdings by a Monopolist Middleman