APPLICATION OF THE EXTERNAL CONVERSION METHOD TO THE CASE OF A LENS TYPE SPECTROMETER

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The external conversion method of Hultberg for the determination of gamma ray intensities is extended to the case of a lens type beta ray spectrometer. The correction factors f for photoelectric angular distribution and for finite source and converter dimensions have been numerically calculated for lead converter and for various source-converter geometries. The results are presented graphically.

1. INTRODUCTION

The study of external conversion electrons with the high precision beta ray spectrometer is a powerful method of obtaining detailed information about the intensities of the gamma rays, in particular in cases of complex decay schemes.[1] Although the Ge(Li) detector developed quite recently has several distinct advantages of comparatively good resolution, high luminosity and simultaneous recording of all energies, the beta-spectrometric study of external conversion electrons is still attractive not only because of its higher resolution but also because it can be used for the absolute determination of the internal conversion coefficients when it is combined with the internal conversion study with the same spectrometer.[1-3]

The relative gamma ray intensity I_γ is related to the intensity A_{ex} of the K-shell external conversion electrons observed in a beta ray spectrometer by the relation

$$I_\gamma \propto \frac{A_{ex}}{\tau_K f}, \qquad (1)$$

where τ_K is the K-shell photoelectric cross section and f is the correction factor for photoelectric angular distribution and for finite source and converter dimensions. The relative gamma ray intensities can be determined provided the factor f has been determined for the instrument and tables of τ_K are at hand. The f-factor for flat type spectrometer is well known[1,4,5] and has been successfully used by many authors,[3,4,6] but the f-factor for lens type spectrometer has not yet been fully computed. Because of the higher gathering power of the lens type spectrometer it seems to be of importance to calculate the f-factor and to apply the external conversion method to the case of a lens type spectrometer.

In the present paper, a description of the numerical calculation of the f-factor for a DuMond type ring-focusing beta ray spectrometer is given. The results calculated for various source-converter geometries are presented graphically. Some suggestions for practical measurement are also given.

2. CALCULATION OF f-FACTOR

The experimental arrangement for the external conversion measurements is drawn schematically in Fig. 1 for a point source Q at a distance a from a circular converter of radius ρ. The converter is, of course, placed at the normal source position of the beta ray spectrometer. Let k_γ be the source strength in disintegrations per second, I_γ the number of gamma rays per disintegration per unit solid angle (gamma ray intensity), d the thickness of the converter in $mg \cdot cm^{-2}$, b a so-called dimension factor in $atoms \cdot cm^2 \cdot (mg \cdot barn)^{-1}$, $NJ(\theta)$ the K-shell photoelectric cross section per unit solid angle in the direction θ, N a factor containing all angle-independent quantities, and Ω_e the solid angle subtended by the slit at P. Then, the number of K photoelectrons through the slit is

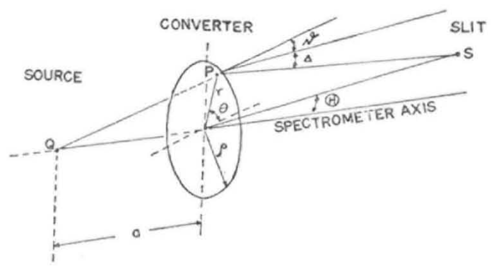

Fig. 1. Schematic arrangement for the external conversion measurement.

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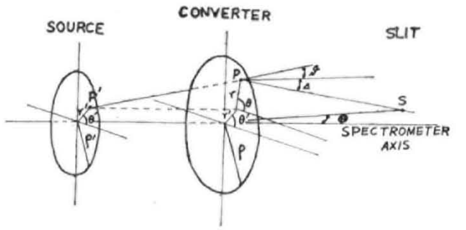

Fig. 2. Schematic arrangement for a circular source and a circular converter.

$$A_{ex}=I_r k_r dbN\Omega_e \int_c J(\vartheta+\Delta)\frac{dS}{r^2+a^2}$$

$$=K\Omega_e \int_c J(\vartheta+\Delta)\frac{dS}{r^2+a^2} \quad (2)$$

with $K=I_r k_r dbN.$ (2)′

The integral extends over the converter surface c. The angle Δ may be neglected when the slit S is sufficiently apart from the converter. In our DuMond type ring-focusing spectrometer with a Hubert slit,[7] the Hubert slit serves both as a defining and a ring-focusing slit. In such a case the defining slit S in Fig. 1 may be considered to be virtual.

In the case of flat type spectrometer, the integral may be reduced to a single integral, since the slit S is on the axis of the instrument and the angle ϑ is a function of r only. But in the case of lens type spectrometer, the double integral must be numerically calculated, since the slit S is in a direction making an angle Θ with the spectrometer axis and the angle ϑ depends on both r and θ. The angle ϑ is given by

$$\cos\vartheta=\frac{r\cos\theta}{(r^2+a^2)^{1/2}}\sin\Theta+\frac{a}{(r^2+a^2)^{1/2}}\cos\Theta. \quad (3)$$

For the practical use of Eq. (2), it is convenient to insert the K-shell photoelectric cross section τ_K into Eq. (2). Then,

$$A_{ex}=I_r k_r dbC_r\tau_K f, \quad (4)$$

$$f=\frac{\int_c J(\vartheta+\Delta)\frac{dS}{r^2+a^2}}{2\pi\int_0^\pi J(\theta)\sin\theta\cdot d\theta}, \quad (5)$$

since τ_K is given by

$$\tau_K=2\pi N\int_0^\pi J(\theta)\sin\theta\cdot d\theta. \quad (6)$$

The f is the correction factor described in the introduction. In Eq. (4) the solid angle Ω_e has been replaced by the spectrometer transmission factor C_r. Thus the f-factor for a point source may be calculated by numerical integraion for the case of lens type spectrometer, though the calculation is rather tedious in comparison with the case of flat type spectrometer.

Fig. 2 illustrates the case of a circular source and a circular converter, which is the normal case for lens type spectrometer. In this case, the intensity of the K photoelectrons A_{ex} and the factor f are given by

$$A_{ex}=I_r bdC_r N\iint J(\vartheta)\frac{r\cdot dr\cdot d\theta}{r^2+a^2}\cdot\frac{k_r}{\pi\rho'^2}\cdot r'\cdot dr'\cdot d\theta' \quad (7)$$

$$=I_r k_r dbC_r\tau_K f, \quad (7)'$$

and
$$f=\frac{\frac{1}{\pi\rho'^2}\int J(\vartheta)\frac{r\cdot dr\cdot d\theta}{r^2+a^2}\cdot\int r'\cdot dr\cdot d\theta'}{2\pi\int_0^\pi J(\theta)\cdot\sin\theta\cdot d\theta}, \quad (8)$$

respectively. In Eq. (7)′ $k_r/(\pi\rho'^2)$ is the specific activity of the source. The angle ϑ is again given by Eq. (3). The angle ϑ, and hence the function $J(\vartheta)$, depends on r and θ only and not on r' and θ'.

Since P′ must be on the source and P on the converter, the following conditions must be satisfied:

(a) $r'\leq\rho'$,

(b) $r'^2+r^2+2rr'\cos(\theta-\theta')\leq\rho^2$.

If the values r and θ are fixed, the conditions (a) and (b) give the upper limits on the integral over

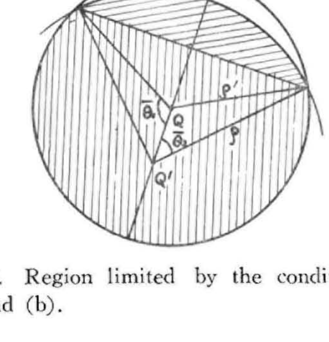

Fig. 3. Region limited by the conditions (a) and (b).

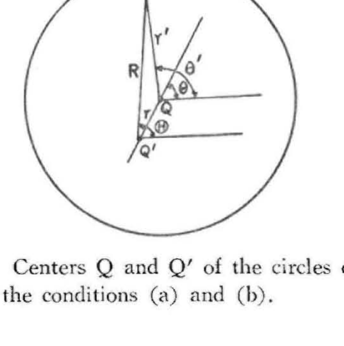

Fig. 4. Centers Q and Q′ of the circles defined by the conditions (a) and (b).

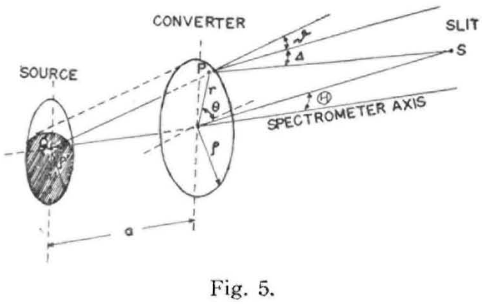

Fig. 5.

the source. Thus, the integral $\int r'\mathrm{d}r'\mathrm{d}\theta'$ gives the area included between two circles $r'=\rho'$ and $r'^2+r^2+2rr'\cos(\theta-\theta')=r'^2$ on the source plane. The shaded portion in Fig. 3 shows this area. The center of the latter circle is at the point Q′ whose polar coordinates are $(r, \theta+\pi)$, as shown in Fig. 4. Using the notations in Fig. 3, the area of the shaded portion σ is given by

$$\sigma=\rho'^2(\bar{\theta}_1-\cos\bar{\theta}_1\sin\bar{\theta})+\rho^2(\bar{\theta}_2-\cos\bar{\theta}_2\sin\bar{\theta}_2), \quad (9)$$

where $$\bar{\theta}_1=\arccos\frac{\rho'^2-\rho^2+r^2}{2\rho' r}, \quad (10)$$

and $$\bar{\theta}_2=\arccos\frac{\rho^2-\rho'^2+r^2}{2\rho r}. \quad (11)$$

It must be noted that the area σ depends on r only and not on θ. Thus, the f-factor for an extended source is

$$f=\frac{\frac{1}{\pi\rho'^2}\int J(\vartheta)\sigma(r)\frac{r\cdot\mathrm{d}r\cdot\mathrm{d}\theta}{r^2+a^2}}{2\pi\int_0^\pi J(\theta)\sin\theta\cdot\mathrm{d}\theta}, \quad (12)$$

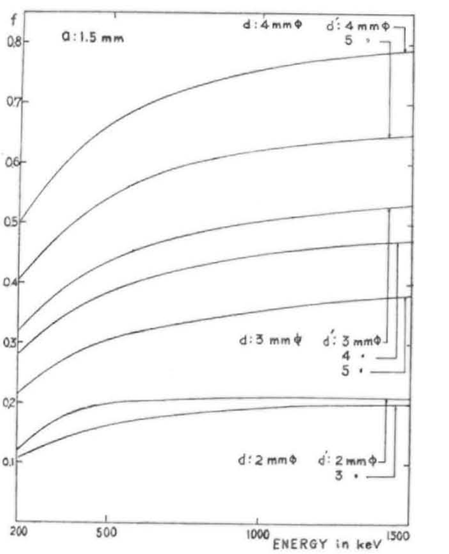

Fig. 6. f values as a function of gamma ray energy. Source-converter distance a: 1.5 mm. d: Converter diameter, d': Source diameter.

where $\sigma(r)=0$ when $r\geq\rho+\rho'$.

A comparison between Eqs. (12) and (5) shows that an extended source may be regarded as a point source with the reduced source strength $k_r\sigma(r)/(\pi\rho'^2)$. A more simple consideration leads to the same conclusion. In Fig. 5 the shaded portion of the source is a projection of the converter parallel to a certain direction PQ. All the angles ϑ and hence all the values $J(\vartheta)$ are the same for the gamma rays emitted in the direction parallel to PQ from all the points of the shaded portion of the source. The gamma rays emitted in the same direction from the unshaded portion of the source do not hit the converter and no photoelectrons are emitted. Thus, the same conclusion described above is obtained.

3. RESULTS

The f values for various source-converter geometries have been numerically calculated on the NEAC-2230 computer at the Kobe University of Commerce. Since lead converters are easily obtainable as thin foils by vacuum evaporation and have been generally used in our laboratory, the f-factors have been calculated for lead converter ($Z=82$). The angle Θ was taken to be 45° which is the emission angle of the DuMond type ring-focusing spectrometer. For the K photoelectric angular distribution function J_K, the values of Pratt *et al.*[8] were used.

The results are shown in Figs. 6–23. In Figs. 6–9, the calculated values of f are plotted as a func-

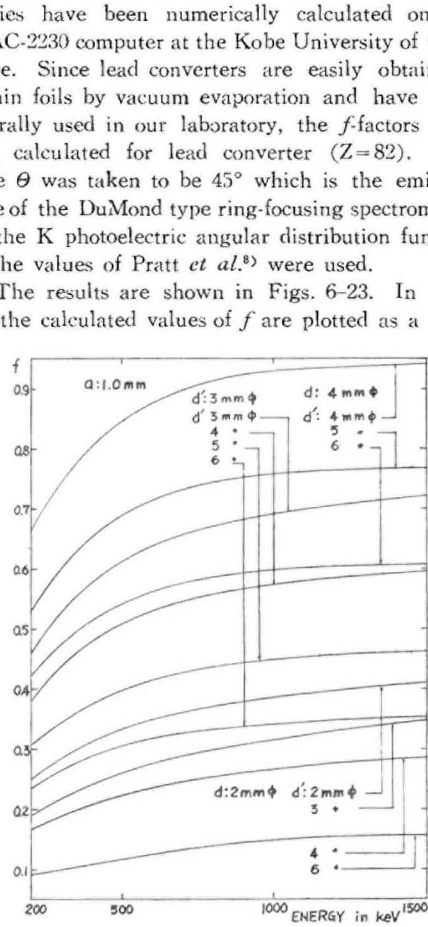

Fig. 7. f-values as a function of gamma ray energy. Source-converter distance a: 1.0 mm. d: Converter diameter, d': Source diameter.

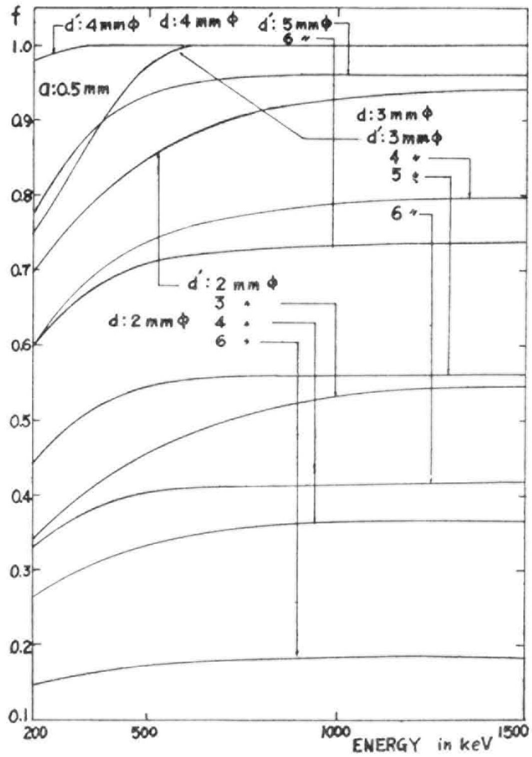

Fig. 8. *f*-values as a function of gamma ray energy. Source-converter distance *a*: 0.5 mm. *d*: Converter diameter, *d'*: Source diameter.

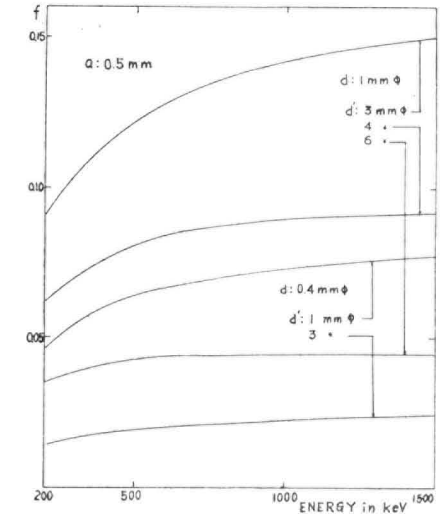

Fig. 9. *f*-values as a function of gamma ray energy. Source-converter distance *a*: 0.5 mm. *d*: Converter diameter, *d'* Source diameter.

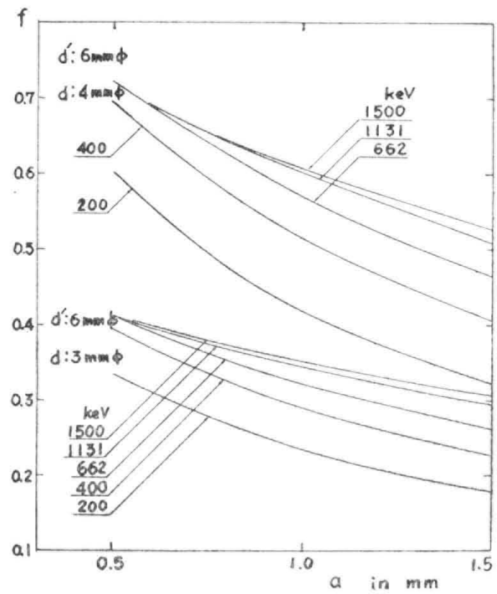

Fig. 10. *f*-values as a function of source-converter distance *a*. Source dimeter *d'*: 6 mm. *d*: Converter diameter.

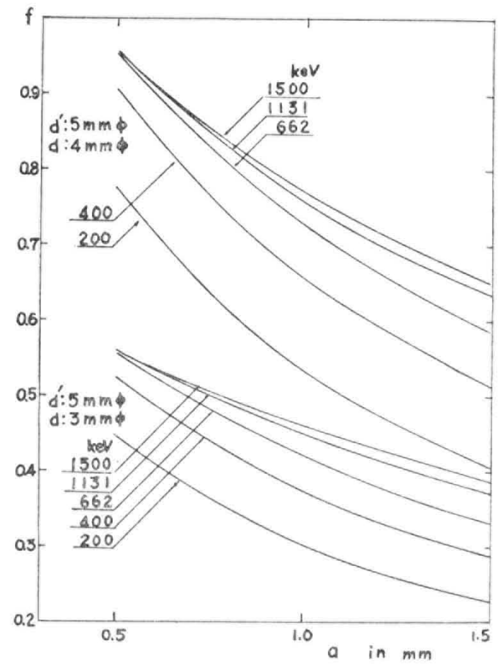

Fig. 11. *f*-values as a function of source-converter distance *a*. Source diameter *d'*: 5mm. *d*: Converter diameter.

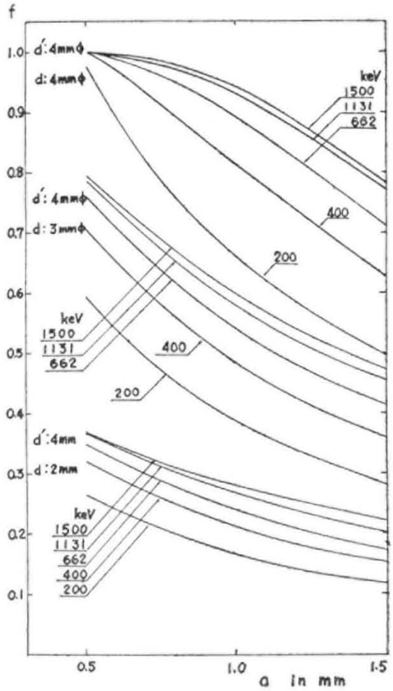

Fig. 12. f-values as a function of source-converter distance a. Source diamater d': 4 mm. d: Converter diameter.

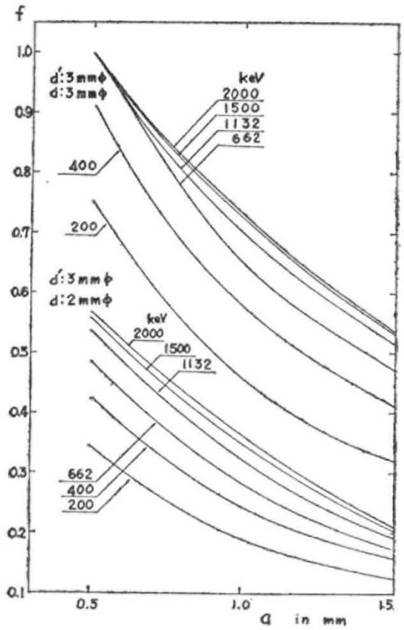

Fig. 13. f-values as a function of source-converter distance a. Source diameter d': 3 mm. d: Converter diameter.

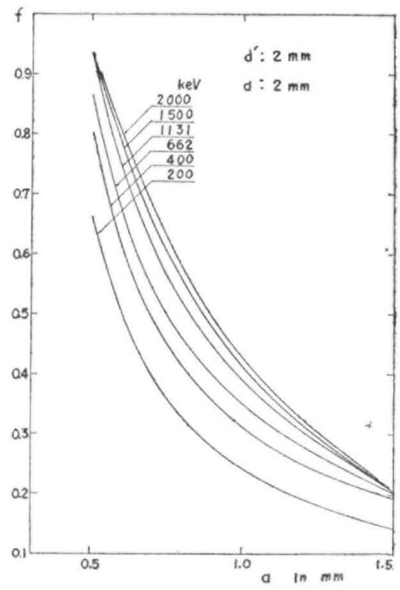

Fig. 14. f-values as a function of source-converter distance a. Source diameter d': 2 mm. Converter diameter d: 2mm.

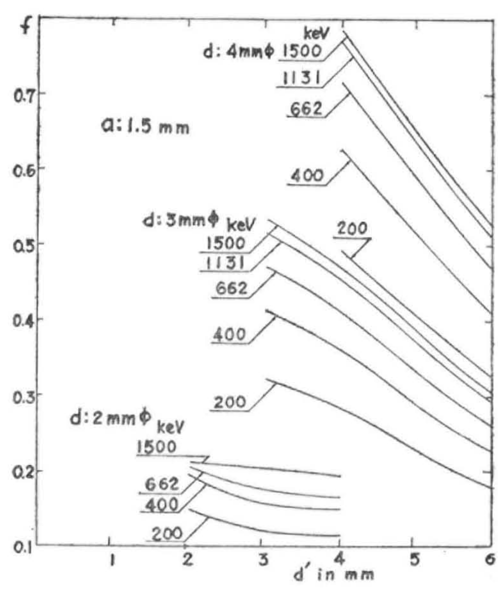

Fig. 15. f-values as a function of source diameter d'. Source-converter distance a: 1.5 mm. d: Converter diameter.

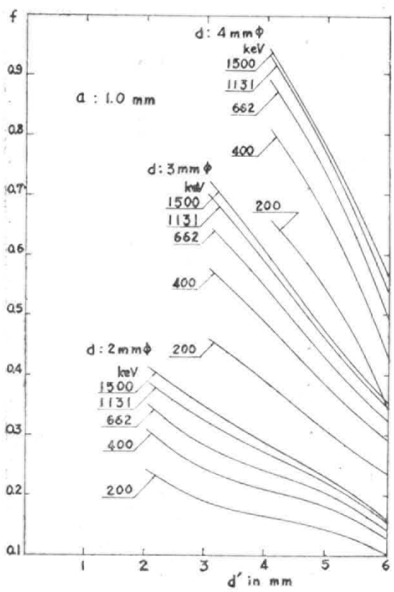

Fig. 16. *f*-value as a function of source diameter d'. Source-converter distance a: 1.0 mm. d: Converter diameter.

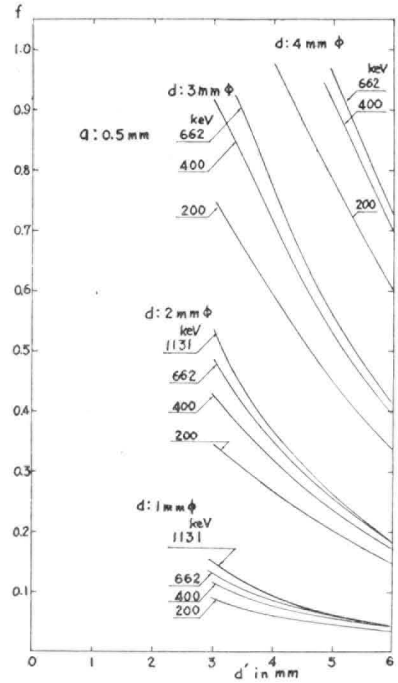

Fig. 17. *f*-values as a function of source diameter d'. Source-converter distance a: 0.5 mm. d: Converter diameter.

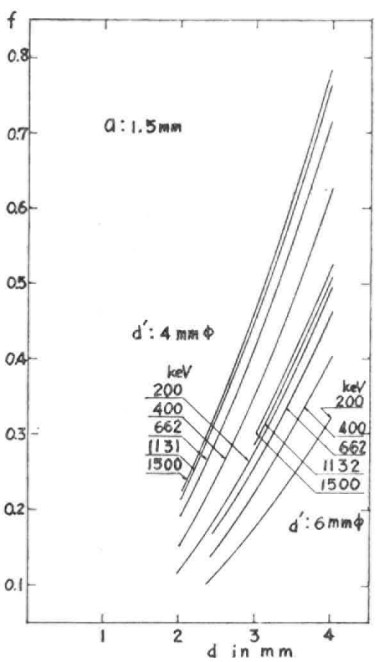

Fig. 18. *f*-values as a function of converter diameter d. Source-converter distance a: 1.5 mm. d': Source diameter.

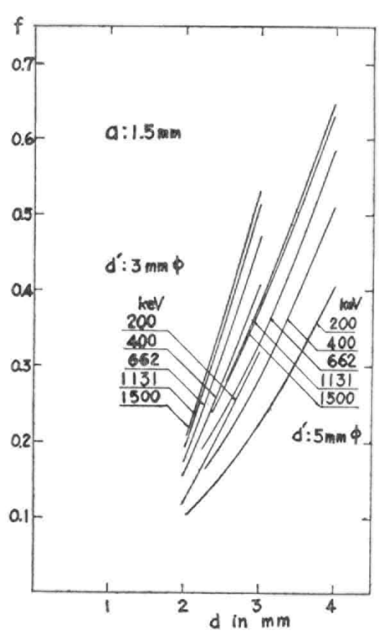

Fig. 19. *f*-values as a function of converter diameter d. Source-converter distance a: 1.5 mm. d': Source diameter.

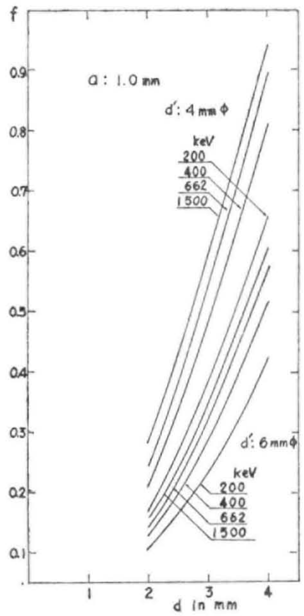

Fig. 20. f-values as a function of converter diameter d. Source-converter distance a: 1.0 mm. d': Source diameter.

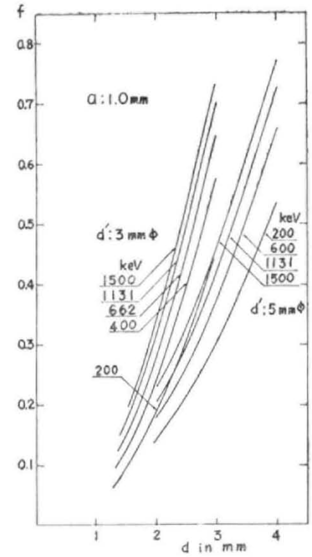

Fig. 21. f-values as a function of converter diameter d. Source-converter distance a: 1.0 mm. d': Source diameter.

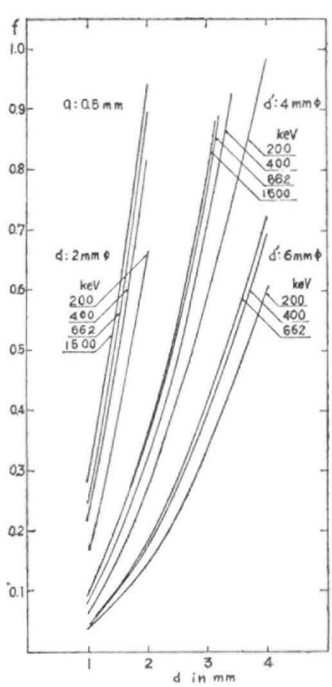

Fig. 22. f-values as a function of converter diameter d. Source-converter distance a: 0.5 mm. d': Source diameter.

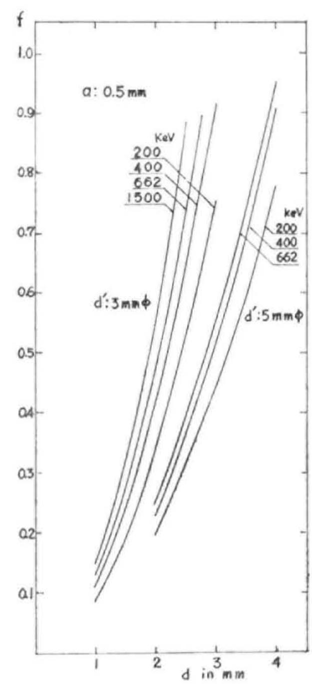

Fig. 23. f-values as a function of converter diameter d. Source-converter distance a: 0.5 mm. d': Source diameter.

tion of gamma ray energy. As expected, the f values increase with energy in the low energy region, and are nearly constant for higher energy. Figs. 10-14 show the variations in the f values with the source-converter distance a. In Figs. 15-17, the f values are plotted as a function of source diameter, and in Figs. 18-23, the values as a function of converter diameter. From these figures the necessary accuracy of the source-converter system may be estimated. In order to obtain the accurate values of f, it is advisable to use the larger source and smaller converter and to keep the source far from the converter. Although the "absolute" values of f vary largely with the source and converter dimension, the variations in the "relative" values for various gamma ray energies are not so large. Thus, it is rather easy to obtain the accurate relative values of f and hence to determine the accurate relative intensities of gamma rays with the external conversion method. In comparison with the source dimension, the converter dimension has a larger influence on the f values. Fortunately, the difficulty is not so serious, because the lead converter with an accurate diameter may easily be prepared by vacuum evaporation. It should be emphasized that the source diameter, the converter diameter and the source to converter distance must be carefully determined to obtain the accurate values of f.

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