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Effects of Attractive $K\bar{K}$ and Repulsive KK Interactions in $KK\bar{K}$ Three-Body Resonance

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$KK\bar{K}$ three-body resonance is discussed based on the coupled-channel complex-scaling method. We introduce three channels $KK\bar{K}$, $\pi\pi K$ and $\pi\eta K$ and determine the resonance energy and width using two-body KK, $K\bar{K}-\pi\pi$, $K\bar{K}-\pi\eta$ and πK potentials determined to fit two-body scattering properties. It is shown that the three-body resonance can be interpreted as $K(1460)$. The $\pi\pi K$ channel and the range of interaction make important effects on the resonance width and the $\pi\eta K$ channel and the repulsive KK interaction play essential roles in the resonance energy.

KEYWORDS: three-body meson resonance, complex scaling method, hadron molecule

1. Introduction

Recently, two-body and three-body hadronic resonances have been discussed intensively. Because the hadron number is not a good quantum number, the concept of *compositeness* of hadronic resonances is important to characterize these resonances [1]. $\Lambda(1405)$ has been interpreted as coupled-channel $\pi\Sigma$-$\bar{K}N$ resonance and $a_0(980)$ and $f_0(980)$ were described by coupled-channel two-meson $\pi\eta$-$K\bar{K}$ and $\pi\pi$-$K\bar{K}$ resonances, respectively.

Using the Faddeev equation and the variational calculation, A. M. Torres et al. [2] showed that a possible three-meson $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$ resonance can be interpreted as $K(1460)$. But they did not determine the resonance energy directly. In this paper, we use the complex-scaling method [5,6] in the semi-relativistic framework to determine the three-meson resonance $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$. Using simple local meson-meson potentials determined so as to reproduce meson-meson scattering properties, we obtain a three-body resonance and show that this resonance can be interpreted as $K(1460)$. We discuss the effects of the interaction range and repulsive KK interaction on the resonance energy and width.

2. Semi-relativistic coupled-channel complex-scaling method for three-meson system

To determine the three-body resonance in the coupled-channel $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$ system, we use the semi-relativistic coupled-channel complex-scaling method. The relativistic kinematics is essential because of very light pion mass. We start from the semi-relativistic Hamiltonian $H = \{H_{A,B}\}$ given by

$$H_{A,B} = \delta_{A,B}\left\{\sqrt{m_{A1}^2 + \boldsymbol{p}_1^2} + \sqrt{m_{A2}^2 + \boldsymbol{p}_2^2} + \sqrt{m_{A3}^2 + \boldsymbol{p}_3^2}\right\} + V_{A,B}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) \tag{1}$$

under the condition $\boldsymbol{p}_1 + \boldsymbol{p}_2 + \boldsymbol{p}_3 = \boldsymbol{0}$. $A, B = 1, 2, 3$ correspond to the channels $|KK\bar{K}\rangle$, $|\pi\pi K\rangle$ and $|\pi\eta K\rangle$, respectively. $V_{A,B}$ consists of the sum of meson-meson potentials (coupling potentials for $A \neq B$).

In the complex-scaling method, we introduce the rotation angle θ and define the complex-scaled Hamiltonian $H^\theta = \{H^\theta_{A,B}\}$ by the transformations $\boldsymbol{r}_i \to \boldsymbol{r}_i e^{i\theta}$ and $\boldsymbol{p}_i \to \boldsymbol{p}_i e^{-i\theta}$ $(i = 1, 2, 3)$ in Eq. (1).

Using suitable basic functions, we determine the eigenvalues of H^θ. In practice, we use the Gaussian basis functions (the *Gaussian expansion method*) for every Jacobi coordinates $\boldsymbol{\rho}_{Ai}$, $\boldsymbol{R}_{Ai}$ ($A = 1, 2, 3$, $i = 1, 2, 3$) and give the wave function in the form like as

$$\Psi = \Psi_1|KK\bar{K}\rangle + \Psi_2|\pi\pi K\rangle + \Psi_3|\pi\eta K\rangle, \tag{2}$$

$$\Psi_A = \Phi_{A1}(\boldsymbol{\rho}_{A1}, \boldsymbol{R}_{A1}) + \Phi_{A2}(\boldsymbol{\rho}_{A2}, \boldsymbol{R}_{A2}) + \Phi_{A3}(\boldsymbol{\rho}_{A3}, \boldsymbol{R}_{A3}), \tag{3}$$

$$\Phi_{Ai}(\boldsymbol{\rho}_{Ai}, \boldsymbol{R}_{Ai}) = \sum_{\alpha\beta} C_{A,i,\alpha,\beta} N_\alpha \exp(-\rho_{Ai}^2/\rho_\alpha^2) N_\beta \exp(-R_{Ai}^2/R_\beta^2). \tag{4}$$

In our calculations, we introduce 18 Gaussian basis functions for each Jacobi coordinates, that is, $\alpha,\beta = 1, \cdots, 18$ in Eq. (4). The coefficients $C_{A,i,\alpha,\beta}$ are given as the solutions of the generalized eigenvalue problem of the complex symmetric matrix H^θ:

$$\sum_{B,j,\alpha',\beta'} H^\theta_{A,i,\alpha,\beta;B,j,\alpha',\beta'} C_{B,j,\alpha',\beta'} = E \sum_{j,\alpha',\beta'} N_{A,i,\alpha,\beta;A,j,\alpha'\beta'} C_{A,j,\alpha',\beta'} \tag{5}$$

By symmetry consideration of boson system, the dimension of the matrix to be diagonalized is reduced to $2268 = 7 \times 18 \times 18$. The θ-independent eigenvalues are identified with the energies of resonances or bound states.

3. Meson-meson potentials

Among meson-meson potentials relevant to the coupled-channel $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$ system (with $I = 1/2$ and angular momentum $J = 0$), we assume temporarily $V_{\pi\eta-\pi\eta} = V_{\pi K-\eta K} = V_{\eta K-\eta K} = 0$ (we ignore the coupling between channels 2 and 3). For simplicity, we assume one-range Gaussian potentials with a common range r_G. We try some different ranges r_G = 0.3, 0.4, 0.5, 0.6 and 0.7 fm and discuss the r_G-dependence of the three-meson resonance. Potential strengths are determined to reproduce meson-meson scattering properties and their values for each r_G are listed in Table I.

For $V_{K\bar{K}-K\bar{K}}$, $V_{\pi\pi-K\bar{K}}$, $V_{\pi\pi-\pi\pi}$ ($I = 0$, $J = 0$) potentials, we have three potential strengths, V_{11}, V_{12} and V_{22}, that is,

$$V_{\pi\pi-\pi\pi} = V_{11}e^{-r^2/r_G^2}, \quad V_{\pi\pi-K\bar{K}} = V_{12}e^{-r^2/r_G^2}, \quad V_{K\bar{K}-K\bar{K}} = V_{22}e^{-r^2/r_G^2}. \tag{6}$$

These strengths are determined so as to reproduce f_0 resonance (M =980 MeV, $\Gamma/2$ =35MeV) and $\pi\pi$ phase shift ($\delta = 55^\circ$ at $\sqrt{s}$ =600MeV) given by experimental analysis. For $V_{K\bar{K}-K\bar{K}}$ and $V_{\pi\eta-K\bar{K}}$ ($I = 1$, $J = 0$) potentials ($V_{\pi\eta-\pi\eta} = 0$ is assumed), we have only two strengths parameters:

$$V_{\pi\eta-\pi\eta} = 0, \quad V_{\pi\eta-K\bar{K}} = V_{12}e^{-r^2/r_G^2}, \quad V_{K\bar{K}-K\bar{K}} = V_{22}e^{-r^2/r_G^2} \tag{7}$$

which are determined to reproduce a_0-resonance (M =980 MeV, $\Gamma/2$ =35 MeV). For $V_{\pi K}$ potentials ($I = 1/2$ and $3/2$), the strengths are determined to reproduce the phenomenological πK phase shifts ($\delta = 40^\circ$ at $\sqrt{s}$ =900 MeV for $I = 1/2$, and $\delta = -15^\circ$ at $\sqrt{s}$ =900 MeV for $I = 3/2$).

For KK ($I = 1$, $J = 0$) potential, experimental phase shifts are not yet available but have been predicted based on theoretical models [7,8]. These models predicted the repulsive phase shifts around -10° at $\sqrt{s}$ =1200 MeV. We employ this value and introduce an artificial factor f like as

$$V_{KK-KK} = f \times V_{11}e^{-r^2/r_G^2} \tag{8}$$

By changing f, we discuss the effect of the repulsive KK interaction on the three-meson resonance.

Table I. Meson-meson potentials used in the $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$ ($I = 1/2, J = 0$) calculations.

	$\pi\pi - K\bar{K}(I=0)$			$\pi\eta - K\bar{K}(I=1)$			$\pi K(I=1/2)$	$\pi K(I=3/2)$	$KK(I=1)$
r_G	V_{11}	V_{12}	V_{22}	V_{11}	V_{12}	V_{22}	V_{11}	V_{11}	V_{11}
0.3	−1877	−417	−2006	0	−1099	−1335	−1489	3694	903.0
0.4	−1164	−383	−1327	0	−686	−928	−895	1174	366.3
0.5	−800	−350	−969	0	−480	−695	−607	556	196.4
0.6	−590	−322	−763	0	−362	−550	−405	324	125.5
0.7	−456	−305	−635	0	−288	−456	−357	215	88.3
(fm)									(MeV)

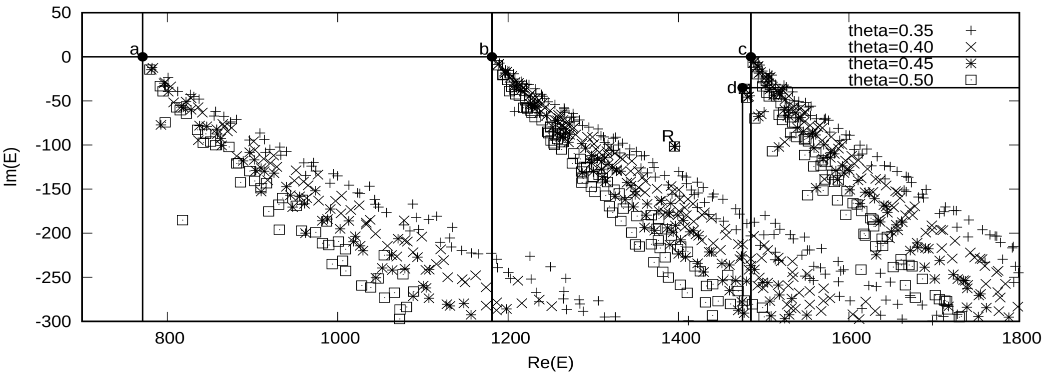

Fig. 1. An example of eigenvalues of complex-scaled Hamiltonian $H^{\theta}_{A,B}$ for $\theta = 0.35, 0.40, 0.45, 0.50$ radian ($r_G = 0.5$ fm). Points a, b, c and d are $\pi\pi K$, $\pi\eta K$, $KK\bar{K}$ and f_0K thresholds, respectively. A θ-independent eigenvalue is found at $E = 1395 - 102i$ MeV, which is labeled R.

4. Result

An example of the eigenvalues for the coupled-channel $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$ system is shown in Fig. 1. For each θ, we find three lines (not exactly linear due to relativisitic kinematics) of eigenvalues starting from the three channel-thresholds (points a, b and c in Fig. 1) and a line from f_0K threshold (point d at $1475 - 35i$ MeV). In addition, we find a θ-independent eigenvalue at E =$1395-102i$ MeV, which corresponds to the three-meson resonance with mass 1395 MeV and width 204 MeV.

For various ranges r_G, we find resonances as listed in Table II. In the two-channel $KK\bar{K}$-$\pi\pi K$ calculation, we obtain resonances with much higher mass and somewhat narrower width than three-channel calculation (no resonance is found for $r_G = 0.3$ and 0.7 fm). On the other hand, in the two-channel $KK\bar{K}$-$\pi\eta K$ calculation, we obtain the resonances with much narrower width than the three channel calculation. We find that the $\pi\pi K$ channel provides very important contribution to the width and the $\pi\eta K$ channel does to the mass of the resonance. This can be understood by strengths of channel coupling and differences between thresholds.

In Fig.2, we show the r_G- and f-dependence of the resonance position in the three channel calculation. We find that the width ($-2\mathrm{Im}(E)$) depends strongly on the potential range r_G. This can be explained by the overlap between the coupling potential and the resonance wave function. On the other hand, the resonance mass ($\mathrm{Re}(E)$) depends strongly on f (the strength of repulsive KK potential). This means that the main component of resonance is $KK\bar{K}$, that is, the resonance is almost the quasi-bound state of $KK\bar{K}$, which seems to have f_0K structure. From Fig.2, we can say that this resonance is interpreted as $K(1460)$ for the range $r_G = 0.5 - 0.6$ fm and the repulsive KK interaction.

Table II. Resonance positions in three-channel $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$ and two-channel $KK\bar{K}$-$\pi\pi K$ and $KK\bar{K}$-$\pi\eta K$ calculations. Hyphens mean no resonance is found.

	$KK\bar{K}$-$\pi\pi K$-$\pi\eta K$		$KK\bar{K}$-$\pi\pi K$		$KK\bar{K}$-$\pi\eta K$	
r_G	Re(E)	Im(E)	Re(E)	Im(E)	Re(E)	Im(E)
0.3	1436.0	−57.1	-	-	1445.5	−12.9
0.4	1406.7	−74.4	1464.7	−48.7	1432.4	−14.0
0.5	1395.2	−101.9	1457.7	−72.0	1424.0	−14.3
0.6	1396.1	−133.0	1465.3	−87.3	1414.5	−14.6
0.7	-	-	-	-	1404.0	−16.2
(fm)						(MeV)

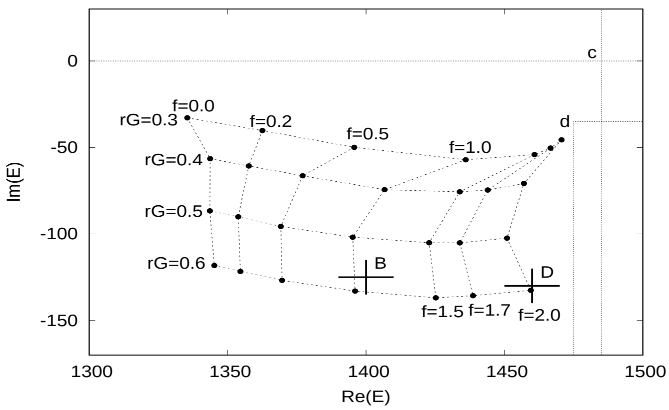

Fig. 2. r_G- and f-dependence of the $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$ resonance. Points c and d are the same with those in Fig. 1. Crosses B and D are experimental resonance position of $K(1460)$ [3,4] (their sizes are arbitrary).

5. Summary

Using the complex-scaling method, we determined the three-meson $KK\bar{K}$-$\pi\pi K$-$\pi\eta K$ resonance. This resonance can be interpreted as $K(1460)$ resonance discovered in two SLAC experiments. For this interpretation, the interaction range $r_G \sim 0.5 - 0.6$ fm is supported and the repulsive KK interaction is necessary. In addition, the coupling to the $\pi\pi K$ channel is essential to reproduce the large width of the resonance and the coupling to $\pi\eta K$ channel makes large contribution to the mass of the resonance. We need some refinements, for example, to use more realistic two-body potentials, to include p-wave components and to include the $\eta\eta K$ channel. In addition, to confirm our interpretation, we must calculate the partial decay widths for ϵK, $K^*\pi$ and ρK modes observed in experiments.

References

[1] T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C**85**, 015201 (2012).
[2] A. M. Torres, D. Jido, and Y. Kanada-Enyo, Phys. Rev. C **83**, 065205 (2001).
[3] G. W. Brandenburg, et al. Phys. Rev. Lett. **36**, 1239(1976).
[4] C. Daum, et al. Nucl. Phys. **B187**, 1 (1981).
[5] T. Myo, Y. Kikuchi, H. Masui, and K. Kato, Prog. Part. Nucl. Phys. **79**. 1 (2014).
[6] A. Dote, T. Inoue, and T. Myo, Prog. Theor. Exp. Phys. **2015**, 043D02 (2015).
[7] N. T. H. Xiem, and S. Shinmura, Prog. Theor. Exp. Phys. **2014**, 023D04 (2014).
[8] D. Lohse, J. W. Durso, K. Holinde, and J. Speth, Nucl. Phys. **A516**, 513 (1990).