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# Extracting Proprioceptive Information By Analyzing Rotating Range Sensors Induced Distortion

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**Abstract**—The increased autonomy of robots is directly linked to their capability to perceive their environment. Simultaneous Localization and Mapping (SLAM) techniques, which associate perception and movement, are particularly interesting because they provide advanced autonomy to vehicles in the field of Intelligent Transportation Systems (ITS). Such ITS are based on both proprioceptive sensors to estimate their dynamics and exteroceptive sensors in order to perceive the surrounding of the vehicle. This second class of sensor is dominated by camera and rotating range sensors such as LIDAR or RADAR. Indeed, the majority of intelligent vehicles uses today 2D/3D laser or panoramic radar to localize itself or detect and avoid obstacles. The use of a rotating range sensor, while moving at high speed, creates distortions in the collected data. Such an effect is, in the majority of studies, ignored or considered as noise and then corrected, based on additional proprioceptive sensors or localization systems. In this study, rather than considering distortion as a noise, we consider that it contains all the information about the vehicles displacement. We propose to extract this information from such distortion without any other information than the exteroceptive sensor data. The idea is to resort to velocimetry by only analyzing the distortion of the measurements. As a result, we propose a linear and angular velocities estimator of the mobile robot based on the distortion analysis.

**Index Terms**—Rotating range sensor, LIDAR, Distortion, Odometry, Localization, Dead-reckoning

## I. INTRODUCTION

In mobile robotics, it is commonly accepted that acquisition of telemetric data with a rotating range sensor is a collection of measurements taken from a single position of the vehicle. This assumption is acceptable while navigating at very low speed with a high rotating rate rotating sensor as it is the case in classical indoor robotics. Nevertheless, for road autonomous vehicle, considering the acquisition as instantaneous in regard to the dynamic of the vehicle can lead to unacceptable errors [9]. A strong data distortion phenomena appear and cannot be ignored anymore. For example in the case of radar mapping application [16], the sensor delivers radar images at 1 Hz. In such a case, while the vehicle moving at a slow speed of 5 m/s (18 kph), the panoramic images is subject to a 5 m distortion. In the case of a faster rotating LIDAR with a scanning frequency of 10 Hz the distortion should be around 50 cm. The fact is that in real traffic condition (50 to 130 kph) observed distortion are much bigger and cannot be ignored. Of course the rotation of the vehicle itself while scanning is another source of distortion that have to be taken into account.

The data distortion was in some studies totally ignored but in the majority of modern studies, distortion is taken into account but considered as a noise and then corrected [6]. Such correction is based on additional proprioceptive sensors or localization systems. Indeed, without such correction, due to the scanning nature of rotating sensor, data acquired during continuous vehicle motion presents geometric motion distortion and can subsequently result in poor metric odometry estimates. Such distortion is stronger as the distance augments but, even over short distances the use of such distorted data can cause localization failure if not taken into account. The contribution presented in this paper is to propose a full lidar-based odometry at high speed, which does not rely on any proprioceptive sensor. In this study we consider that distortion contains all the required information about the vehicles displacement. We propose a distortion formulation analysis to estimate the vehicle velocities.

## II. RELATED WORKS

Most ITS applications are based on proprioceptive data provided by odometer sensors, gyrometers, IMU or other positioning systems such as GPS. However, in order to estimate motion, some research works tried to use only exteroceptive data. Thus, [4], [8] and [11] proposed a visual odometry without proprioceptive data. [15] proposed to filter out moving objects before doing ego-motion. In such an approach, exteroceptive ego-motion is considered as intended to augment rather than replace classical proprioceptive sensors. Sometimes, classical displacement measurements are much more difficult and have limitations: inertial sensors are prone to drift, and wheel odometry is unreliable in rough terrain (wheels tend to slip and sink) and as a consequence visual odometric approaches are widely studied [5], [13], [17]. For example, in an underwater or naval environment classical ego-motion techniques are not suitable. In [3], Elkins *et al.* presented localization system for cooperative boats. In [7], Jenkin *et al.* proposed an ego-motion technique based on visual SLAM fused with IMU. In order to find displacement with exteroceptive sensors such as range finders, the scan matching method and ICP is commonly used [2], [12], [14] but each scan is corrected with proprioceptive sensors especially when the sensor is slow. In all scan matching work, distortion is

taken into account but considered as a disturbance and thus corrected.

The only work dealing with distortion as a source of information used a rolling shutter specific camera. In [1], Ait-Aider *et al.* computed instantaneous 3D pose and velocity of fast moving objects using a single camera image but, in their context, prior knowledge about the observed object is required. In mobile robotics, we have no *a priori* about the surrounding environment of the robot. To the best of our knowledge, there is absolutely no work in the field of mobile robotics literature considering distortion as a source of information in an odometric purpose. This paper is an extension to higher rotating frequency of our previous work using a slow rotating panoramic radar sensor [16]. The originality of this proposition consists in considering and studying the distortion as a source of information rather than as a disturbance. In this work we focus on the use of a well-known rotating range sensor, a 3D LIDAR of type Velodyne. We demonstrate that even at low speed with a high frequency rotating range sensor, the measurement of the distortion can be used as a robust velocity estimator.

The paper is organized as follows: Section III describes the distortion phenomena and the assumptions made in classical approaches. The system overview and methodology used in our odometry framework is described in IV. The Section V presents experimental results. Finally, Section VI concludes.

### III. MORE ABOUT THE DISTORTION

#### A. Classical SLAM or Odometry approaches

Classical sequential SLAM is an estimation problem that can be expressed as  $p(x_t, M_t | u_t, z_t)$  where  $x_t$  and  $M_t$  are respectively the state of the vehicle and the map of the environment and  $u_t, z_t$  the proprioceptive and exteroceptive measurements.

$$p(x_t, M_t | u_{1:t}, z_{1:t}) \propto p(z_t | x_t, M_t) \dots$$

$$\int p(x_t | x_{t-1}, u_t) p(x_{t-1}, M_{t-1} | u_{1:t-1}, z_{1:t-1}) dx_{t-1}$$

In the case of a panoramic data provided by a LIDAR sensor, a measurement  $z_t$  is a collection of punctual measurements such as  $z_t = \{z_t^1, z_t^2, \dots, z_t^K\}$ . Considering the update step and if we suppose that all the acquisitions are independent:

$$p(x_t, M_t | u_{1:t}, z_{1:t}) \propto p(z_t | x_t, M_t) p(x_t, M_t | u_{1:t}, z_{1:t-1})$$

becomes:

$$p(x_t, M_t | u_{1:t}, z_{1:t}) \propto p(x_t, M_t | u_{1:t}, z_{1:t-1}) \prod_{j=1}^K p(z_t^j | x_t, M_t)$$

An other point is the asynchronous data acquisition. As the sensor is rotating, each beam is not taken at the same time, at a result  $z_t$  becomes:

$$z_t = \{z_{t-1+\delta t}^1, z_{t-1+2\delta t}^2, \dots, z_{t-\delta t}^{K-1}, z_t^K\}$$

While the vehicle is moving during the acquisition, the distortion phenomena occurs. Each punctual measurement has to be corrected by the displacement and is, as a consequence, correlated to the movement. At a result,  $z_t^j$  is a function of the previous state  $x_{t-1}$  and of proprioceptive data  $u_t$  :  $p(z_t^j | x_t, u_t)$ .

Classical approach is to assume that the detection in the sensor frame are not correlated:

$$p(x_t, M_t | u_{1:t}, z_{1:t}) \propto \dots \\ p(x_t, M_t | u_{1:t}, z_{1:t-1}) \prod_{j=1}^K \prod_{l=1}^N p(z_{t_l}^j | x_t, M_t)$$

Such assumption is not true anymore as each laser beam is a function of  $u_t$  and  $x_{t-1}$ . In such a case, all SLAM and VO approaches using rotating range sensor data and correcting the scan by using proprioceptive information break the assumption of sequential SLAM.

Nevertheless the formulation taking into account distortion should be:

$$p(x_t, M_t | u_{1:t}, z_{1:t}) \propto p(z_t | x_t, u_t, M_t) p(x_t, M_t | u_{1:t}, z_{1:t-1})$$

with both state and measurement depending on the proprioceptive information.

We can represent such link with the graphical model of SLAM (see Fig. 1). As we can see because the real measurement is not taken from the good position, the observed landmark has first to be corrected thanks to proprioceptive model, as a result the observations  $z_j^i$  during a scan  $i$  become correlated as they depend of the state and of the proprioceptive sensors integrated over time  $j$ .

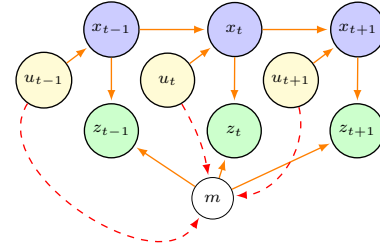


Fig. 1: Graphical model of classical SLAM approaches. Red dashed line are not part of the classical model but appears while correcting the distortion by using proprioceptive information.

The solution in the state of the art is to ignored this correlation and to suppose that all beams are independents. As a result an entire scan is corrected and used in scan-matching or classical ICP approaches. The main drawbacks of such approach is that, first, they require additional proprioceptive sensors, second, they break the probabilistic theoretical framework and often use twice the odometric information, once to predict the state, then to correct the observation. That can result in over-convergence of the algorithms as we consider odometry twice so more accurate than it really is.

We propose to revert the equation  $p(z_t|x_t, u_t, M_t)$  and to get information about  $u_t$  by using  $z_t$ :

$$p(u_t|x_t, z_t, M_t) = \frac{p(z_t|x_t, u_t, M_t)p(u_t)}{p(z_t, x_t, M_t)}$$

We need then to be able to estimate two pdf:  $p(u_t)$  and  $p(z_t, x_t, M_t)$ . The first one is obtained by assuming a displacement model such as a constant velocity model for example. The second one required to link both the map, the detection and the robot state so to parameterize the distortion.

### B. How to deal with distortion?

Data distortion results from combined sensor and vehicle movements. In fact, in a simplified 2D view, the displacement of the bearing sensor beam during an entire revolution can be compared to the movement of a bicycle valve in the case of a straight line movement (*cf.* Figure 2).

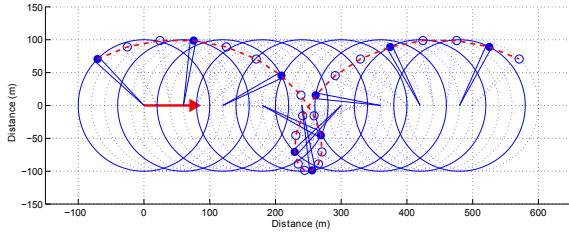


Fig. 2: Representation of a simple trochoïd described by the sensor beam in the case of a straight line movement.

As regards some other displacements of the center of rotation, the distortion equation in 2D can be represented by the parametric equation of a trochoïd. Indeed, at time  $t$ , the position of a detection done at range  $\rho$  is a function of the center pose  $(x_t^c, y_t^c, \phi_t^c)$  and of sensor bearing  $\theta_t$ .

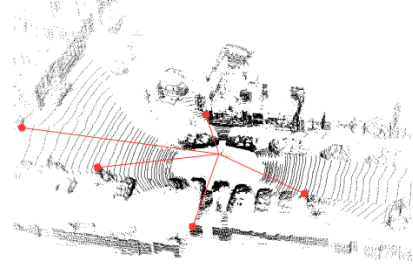
$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \underbrace{\begin{bmatrix} x_t^c \\ y_t^c \end{bmatrix}}_{\text{A) Center position}} + \underbrace{\begin{bmatrix} \cos(\phi_t^c) & -\sin(\phi_t^c) \\ \sin(\phi_t^c) & \cos(\phi_t^c) \end{bmatrix}}_{\text{B) Rotation}} \underbrace{\begin{bmatrix} \rho \cos(\theta_t) \\ \rho \sin(\theta_t) \end{bmatrix}}_{\text{C) Detection position at time } t} \quad (1)$$

In order to obtain the center pose at time  $t$ , an evolution model taking into account the linear ( $V$ ) and angular ( $\omega$ ) velocities is formulated. Pose  $(x_t^c, y_t^c, \phi_t^c)$  is obtained as follows:

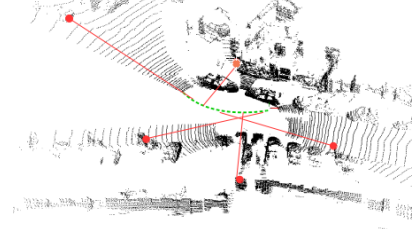
$$\begin{cases} \begin{bmatrix} x_t^c \\ y_t^c \end{bmatrix} = Vt \operatorname{sinc}\left(\frac{\omega t}{2\pi}\right) \begin{bmatrix} \cos\left(\frac{\omega t}{2}\right) \\ \sin\left(\frac{\omega t}{2}\right) \end{bmatrix} \\ \phi_t^c = \phi_0^c + \omega t. \end{cases} \quad (2)$$

Using this distortion equation, which is parameterized with respect to the linear and angular velocities  $V$  and  $\omega$ , both the distorted or undistorted measurements could be managed. With a prior estimate of these parameters, detection  $(\theta, \rho)$ , done at time  $t$  in the sensor frame, can be transformed into the world frame based on Equations (1) and (2) (*cf.* Figure 3).

Of course, this formulation can be extended to 6D navigation in such a way that the 3D detection taken at time  $t$  in



(a) LIDAR data obtained from a supposed unique position (no movement)



(b) Effect of the undistortion formulation for high speed. In green, the predicted trajectory, in red some corresponding landmarks

Fig. 3: Distortion formulation applied on real LIDAR data

the distorted frame denoted  $P_{3D}^s$  can be corrected in the world frame  $P_{3D}^w$  by using a 3D evolution model  $f(\cdot)$  and 3D rotation function  $\mathbf{R}_{3D}^t(\mathbf{w})$ . Thus,  $P_{3D}^w$  can be obtain by:

$$P_{3D}^w = \underbrace{f(\mathbf{X}_v^0, \mathbf{V}, \mathbf{w})}_{\text{Sensor pose estimation}} + \underbrace{\mathbf{R}_{3D}^t(\mathbf{w})}_{\text{Measurement mapping}} P_{3D}^s \quad (3)$$

As a result an entire 3D scan coming from a rotating range sensor (RADAR, LIDAR etc.) can be undistorted by using the estimated velocities  $\mathbf{V}$  and  $\mathbf{w}$ .

## IV. METHODOLOGY AND ODOMETRY FRAMEWORK

In order to extract the information from the distortion phenomenon using a rotating sensor without any knowledge of the environment shape, the required assumption is a local constraint on the movement of the vehicle, for example a constant velocity of the vehicle during two successive measurements. This assumption assume that the velocity is not changing too quickly between two LIDAR revolution. If the speed is changed, the estimated velocity would be the mean velocities. The pose of each measurement can then directly be linked to the observation pose and to the angle of observation. This observation pose can be expressed with the model of the vehicle and is only a function of the linear and angular speed of the robot. To simplify the problem, in the following, we choose to present equations for a planar movement. The 2D evolution model is then given by (4):

$$\begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \\ V(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} x(t-dt) + V * dt * \cos(\theta(t-dt)) \\ y(t-dt) + V * dt * \sin(\theta(t-dt)) \\ \theta(t-dt) + w(t-dt) * dt \\ V(t-dt) \\ w(t-dt) \end{bmatrix} \quad (4)$$

So at each time step  $\delta t_i$ , the current pose of the vehicle is estimated. As the sensor is rotating at a given frequency (or because we have the measured timestamp), we know exactly the time  $\delta t_{i,k}$  of acquisition of a sensor beam  $k$  and the bearing of the sensor. As a result the LIDAR data is composed of  $K$  simultaneous measurements (depending the number of LIDAR layers) taken at time  $\delta t_{i,k}$ ,  $k$  representing each angle of view.

Considering the collected data, the LIDAR data is represented as a spherical matrix in which rows represent the elevation angle, columns the azimuth angle and the value of each pixel the distance of the laser impact. Such matrix can easily be converted to a classical 3D point cloud using Spherical to Cartesian coordinates conversion.



Fig. 4: LIDAR data representation as a spherical image. Row represent the elevation, Columns the azimuth and the value of each pixel represent the depth of the detection.

The distortion based odometry follows this simple process:

- 1) Get two successive sensor acquisitions (two full panoramic scans)
- 2) Suppose the vehicle motion is locally constraint (for example constant velocity) so you can predict the vehicle trajectory during the two acquisitions
- 3) Given an estimated  $\hat{V}$  and  $\hat{w}$ , apply distortion on both sensor acquisition
- 4) Minimize the measurements differences with respect to the estimated velocity values.

#### A. Image based approaches

We propose to analyze the distortion of the LIDAR data by considering directly the entire spherical image (see Fig. 4). Such methodology is inspired from Direct SLAM Methods that compare the entire successive images to get both the pose of the camera and the shape of the environment.

We are using a direct minimization of the photometric error representing in our case the distance to the scene. As a result we are trying to minimize the 3D reconstruction error between two successive acquisitions. The error is given by:

$$\epsilon(\xi) = (I_2(\mathcal{U}(\mathbf{x}_i, \mathbf{p}_s, \xi)) - I_1(\mathbf{x}_i))^2$$

where  $\mathcal{U}()$  is a warp function that maps each point  $\mathbf{x}_i \in \Omega_1$ , in the reference spherical image  $I_1$  to the respective point  $\mathcal{U}(\mathbf{x}_i, d_i, \xi) \in \Omega_2$  in the new spherical image  $I_2$ . In our case,  $\Omega$  represents the spherical coordinate ensemble described by  $(\rho, \theta, \phi)$ . As input it requires the estimated velocities of the LIDAR sensor  $\xi \in \mathbb{R}^2$ ,  $\xi = (V, w)$  and uses the LIDAR sensor parameters such as the acquisition frequency and the calibration  $\mathbf{p}_s$ . Note that no information with respect to  $I_2$  is required.

Minimizing this error  $\epsilon(\xi)$  can be interpreted as computing the maximum likelihood estimator for  $\xi$ , assuming independent noise on the LIDAR detection values.

The warp function  $\mathcal{U}()$  is directly related to the distortion formulation given by equation (3). It has to be noted that in order to get the spherical image  $I_2$  after the application of the distortion warping function, an interpolation is required between the different spherical points.

As a results the velocities  $V$  and  $w$  are provided by minimizing:

$$\underset{\xi}{\operatorname{argmin}} (I_2(\mathcal{U}(\mathbf{x}_i, \mathbf{p}_s, \xi)) - I_1(\mathbf{x}_i))^2$$

In order to minimize this cost function, we used stochastic gradient descent (SGD), also known as incremental gradient descent. SGD is an iterative method for optimizing a differentiable objective function, a stochastic approximation of gradient descent optimization. In such approach, the gradient of  $\epsilon(\xi)$  is approximated by a gradient at a single example:

$$\xi := \xi - \eta \nabla \epsilon(\xi)$$

In our case we propose to estimate the gradient in different direction  $\theta_i$  with different steps  $\delta_i$  in order to guide the convergence of the  $\epsilon(\xi)$  minimization process. We propose to find the best starting point  $\xi_0$  thanks to random test and then to test the gradient in the different directions with a step length  $\lambda$  depending of the iteration  $n$  such as:  $\lambda = 2^{-n}$

Such method is less sensitive than for example Levenberg-Marquardt Algorithm to local discontinuation of the cost function but of course the accuracy of the solution is function of the number of iteration allowed to the algorithm.

## V. EXPERIMENTS

The presented approach in section IV have been tested on the public data-set "Velodyne SLAM-Dataset" [10]. The results are presented in Fig. 5.

In this experiment, the vehicle is driving in a urban environment at a velocity varying from 0 to 8 meter per second ( $\approx 30 \text{ kph}$ ) which is relatively slow for the ITS field. In such a context, the distortion of the LIDAR data scans are also small. Nevertheless, we can see that the estimation of the linear and angular velocity are quite accurate whatever the speed all along the trajectory. The velocities estimation mean error and covariance are provided in Table. I results.

	Mean error	$\sigma$
Linear velocity estimator	0.08 m/s	0.64 m/s
Angular velocity estimator	-0.0022 rad ( $\approx 0.13 \text{ deg}$ )	0.023 rad ( $\approx 1.3 \text{ deg}$ )
SBG Ellipse 2 velocity (IMU+GPS)	0.1m/s	0.03m/s

TABLE I: Quantitative evaluation of the distortion based odometry. For comparison, as there is no work on such estimation in the community, the given precision given in the datasheet of a SBG Ellipse2 (IMU/GNSS) are provided (Under good GNSS availability).

Such results are promising and prove that the distortion contains all the proprioceptive information and that we can extract

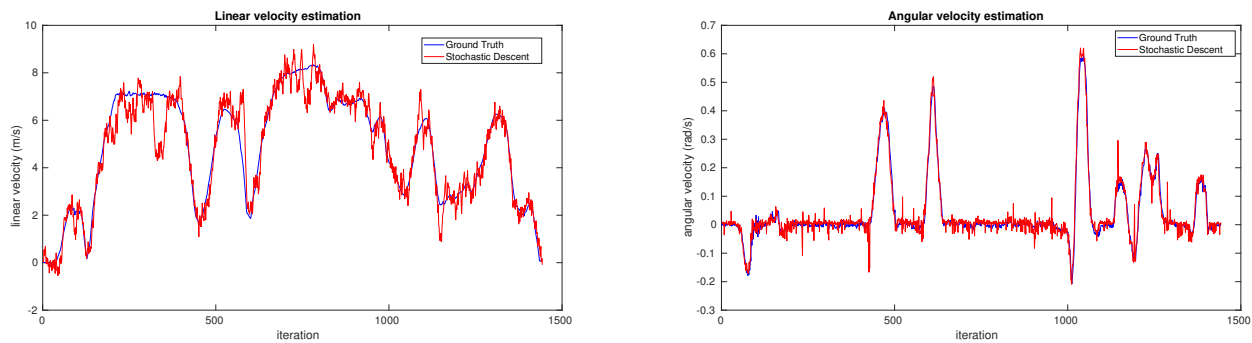


Fig. 5: Distortion based velocities estimation results. Red: Distortion based SGD. Blue ground truth (IMU+DGPS fusion).

a very correct odometry from such data without any other source of information compared to commercial IMU/GNSS<sup>1</sup>. Of course, because our estimations are not smooth by any GNSS information, our standard deviation is higher but we achieve a comparable mean error. At this step, let's note that the very simple constant velocity assumption have been used. Such assumption is very strong considering two complete successive scans. Such assumption could be extended to other less strict constraint like a model with constant acceleration or constant jerk that better fit a car motion.

## VI. CONCLUSION

In this paper, an original method for finding the instantaneous velocity of a mobile robot equipped with a 3D rotating range LIDAR was presented. Such velocimetry is only based on the consideration of the data distortion involved by the scanning mechanism of the LIDAR. The distortion formulation due to the displacement of the sensor was established. Comparison techniques between successive LIDAR scans were applied to obtain the robots angular and linear velocity parameters. Even under the assumption of constant velocity, the algorithm is robust at moderate velocity variations. The approach was evaluated on real LIDAR data from a public dataset showing its feasibility and reliability at  $\approx 30$  *kph* speed. The main novelties of the proposed approach include considering distortion effect as sources of information rather than as disturbances, using no other sensor than the LIDAR sensor, and working without any knowledge of the environment.

## VII. ACKNOWLEDGEMENTS

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<sup>1</sup>[https://www.sbg-systems.com/wp-content/uploads/2018/09/Ellipse\\_Series\\_Leaflet.pdf](https://www.sbg-systems.com/wp-content/uploads/2018/09/Ellipse_Series_Leaflet.pdf)