How selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencast intervention

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Abstract

This research project is an interventionist case study, oriented in the interpretive paradigm, which aims to investigate how selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in screencast interventions. The aim of my screencast intervention programme, which lies at the heart of this study, is to develop practices, *inter alia,* of how such devices and software may be "used to develop conceptual rather than procedural or decorative knowledge" (Larkin & Calder, 2015:1) in solving linear equations.

The planned intervention was delivered in the form of a series of screencasts: these take the form of audio-video lessons with an emphasis on the visual impact, and were recorded using an application called Explain Everything. The screencast interventions were delivered via Google Classroom and included animations supported by such conceptual explanations of early algebra as are relevant to Grade 7 students, and in line with the South African Curriculum and Assessment Policy Statements - Department of Education, 2011.

The fundamental components of an early algebraic equation that would be relevant to a Grade 7 student were considered and used to develop an analytic framework. This was based on a taxonomy designed according to four identified "clusters" in order to analyse the workings of the purposefully selected Grade 7 participants who were video recorded and questioned in a talk-aloud interview while they completed a post-intervention pencil-and-paper test.

What emerges from this research project is that there is a significant need for specific and concentrated technology-based techniques, such as the interventions undertaken here, and that exploration and development in the field could benefit the delivery of a pedagogy for algebra. The pedagogical methods implemented and studied in the form of screencasts proved to be successful and were well received by the learners particularly in relation to the conceptualisation of "symbol sense" and transformation in early algebra. The structure and design of the screencast interventions were important in supporting the acquisition of these concepts and were demonstrated to be worthwhile tools for an epistemological application in a classroom or teaching context.

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TABLE OF CONTENTS

APPENDICES

LIST OF ABBREVIATIONS

CAPS	Curriculum and Assessment Policy Statements
MKO	More Knowledgeable Other
ZPD	Zone of Proximal Development

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION AND CONTEXT

Do we have an explicit understanding of how children in their elementary years solve algebraic equations? One of the most prevalent misunderstandings is learners' failure to understand the mathematical referents of the equal sign.

Various influential texts on this topic, draw on notions of equations and equality in order to determine the cognitive processes that take place in the computation of equations by Grade 7 students. Wiggins (2014) refers to a definition extracted from 'The Common Core Standards in Mathematics' which stresses the importance of conceptual understanding as a key component of mathematical expertise. He argues, however, that the 'Standards' is contradictory in that it does not offer the tools for teachers to convey the conceptual understanding, and that understanding means being able to justify the use of procedures.

Given how children are required to solve basic arithmetic equations by way of bond, it is necessary to allow for a sophisticated method of thinking and of solving for unknowns on either side of the equal sign. Despite the inability of children to wrestle with this concept independently, it would be helpful to reassess teaching and learning strategies rather than to continue to misguide pupils.

Yerushalmy (2005) challenges the visually sensitive curriculum design and "draws on examples from algebra in order to highlight technological affordances with noticeable impact on the way we visualise and understand mathematical objects and mathematical actions, amongst others" (as cited in Nardi, 2014:195

1.2 MATHEMATICS AND ALGEBRA

According to Jennifer Taylor-Cox (2003:14) it is never too early to "start thinking in terms of algebra." Given the levels of complexity that learners attach to algebra, the calculations in the research are straightforward and will be restricted to linear equations which are also grade relevant for this point in the curriculum (Department of Basic Education, 2011).

Daniel Willingham's views (2009) with regard to conceptual understanding in mathematics are those of the three types of knowledge that students need; conceptual knowledge is the most difficult to acquire for the reason that concepts cannot be poured into a student's head but need to be built upon something they already know. This implies a process of scaffolding which, in the context of algebra, is difficult to recall as it is a rather novel concept introduced at Grade 7 level in the South African curriculum. This study argues that one of the main concerns that surfaces is that students fail to understand the true property of the equal sign and what it represents. Many students at this level believe the equal sign signifies that an answer must follow, and, further, that it implies that the equation must be completed. Much has been said about the problem that learners have in recognising the mathematical meaning of equations and the notion of equality. Since most signs are polyvalent, thus indexing more than a single thing, it is not always problematic for "=" to be read as indicating that the answer follows. Such an understanding of the equal sign need not interfere with the idea of an equation. By the time learners get to high school, they have had extensive computational experience with using most of the features of the addition and subtraction of, at least, natural and rational numbers. They certainly have a great deal of experience with the structures of addition and multiplication of rational numbers and of addition and multiplication of rational numbers once the integers are introduced. These structures are computational spaces involving the basic operations along with numerical order (Stewart and Tall, 2015). When algebra is introduced the computations performed are meant to be consistent with real number addition and multiplication, but it may be the case that learners (and even teachers) introduce computational resources that are auxiliary to addition and multiplication defined over the real numbers, entailing objects and operations that are not compatible with real number addition and multiplication (Davis, 2013). Da Rocha Falcao (1995) indicated that although straightforward arithmetic may be solved directly, algebraic problems require that they be "translated and written in formal representations first, after which they can be solved" (as cited in Van Amerom, 2003:66).

Oksuz's (2007) work suggests that algebraic thinking does begin to develop in students in the earlier grades when their knowledge of numbers and operations establishes a basis from which to learn algebra. In fact Linchevski (1995) suggested that pre-algebra should be viewed as a continuation of the basic arithmetic that asks different questions about numbers and about how children know and understand them.

1.3 GOALS & OBJECTIVES

> *...to ensure that children acquire and apply knowledge and skills in ways that are meaningful to their own lives....*
> (Curriculum and Assessments Policy Document, Department of Health, 2011:4)

In an attempt to answer the main research question articulated below, one of the intentions of this research project is for selected Grade 7 participants to be able to determine, specifically, the numerical value of an expression by substitution and by understanding the concept of the equal sign (“=”) and the unknown variable in an equation. According to Carraher (2016) the perceived complexity of algebra should not be justified when it is introduced to students by increasing the content load, but should consider an approach that allows them to understand it in “deeper, more challenging ways” (https://as.tufts.edu/education/earlyalgebra/about.asp) in order to develop a better conceptualisation of the topic.

The goal of this research project is to determine how animating algebraic equations has an effect on learners’ conceptualisation of solving early algebra by using screencasts as a result of an intervention programme; and how, as a result of these screencast interventions, learners solve algebraic equations.

The research question that guides this project is thus: How do selected Grade 7 participants solve algebraic equations as a result of participating in a screencast intervention?

1.4 THEORETICAL FRAMEWORK

The role of visualisation is central to this project which intends to challenge the argument that "mathematics learners tend to prefer to think algorithmically rather than visually" (Clements, 2014:181) because it is abundantly clear that Hilbert's sentiment rings true, that mathematics "requires crystallized logical relations in order to develop an intuitive understanding, especially through visual imagination" (as cited in Zimmerman & Cunningham, 1990:2). This is based on the construct of "concept image" which describes the notion of visualisation as "the total concept structure that is associated with the concept, and includes all the mental pictures and associated properties and processes” (Tall & Vinner, 1981:152), and is built on years of experience, of all kinds, changing as the individual meets new stimuli and matures. “Cognitive structure is a psychological construct that accounts for a form of human knowledge. Cognitive structures provide meaning and organisation to experiences and guides both the processing of new information and the retrieval of stored information” (https://doi.org/10.1007/978-1-4419-1428-6_2071). In an attempt to understand how students develop a conceptual understanding of how to solve algebraic equations, it is important to develop an understanding of the concept of equality and, consequently, the role of the unknown variable in an equation.

Central to this research project is how students build and construct knowledge, to which I have referred more specifically as social constructivism. Vygotsky’s two important constructs are: The More Knowledgeable Other (MKO) and the Zone of Proximal Development (ZPD). With regard to the social interactive perspective and to constructivism, they have played a significant role in the construction of the required knowledge. Vygotsky's (1978) suggestion is that language and communication have a crucial impact on cognitive development in students (and humans generally) which is supported by Wiggins's (2014) idea that "novices need clear instruction and simplified/scaffolded learning" (as cited in Willingham, 2015:5). This implies that students are not empty vessels into which one simply pours random concepts because knowledge is acquired by building onto a concept already known (Wiggins, 2014). Kent & Hedger (1980) argue, regarding language, in support of Vygotsky's (1978) claim that it has a large part to play "because imagery can be brought to life by the appropriate and relevant language and examples" (as cited in Clements, 1982:33). Bishop’s (1985) sentiment regarding the requirements of an authority to assist in solving complex problems is similar to Vygotsky’s “More Knowledgeable Other”: that screencast interventions provide an opportunity for the construction of meaning by representing the “More Knowledgeable Other”. In this instance his reference to "tools of intellectual development" (Amineh & Asl, 2015:13) is cemented by the effect of my screencast interventions which allowed the students to scaffold their basic mental functions, or prior knowledge of number sentences, "effectively/adaptively" (Amineh & Asl, 2015:13).

According to Bishop (1985), it is essential that the research methodology used to capture the data of the sharing and development of mathematical meaning, through screencast interventions, is done so accurately in order carefully to include the role visualisation played in concept acquisition. To capture the conceptualisation of the notion of equality and the role that is played by the unknown variable, it is necessary to understand the importance and relevance of the audio-visual combination that is provided through the screencast interventions and also that this component of the research was captured accurately.

1.5 METHODOLOGY

This research project is oriented in the interpretive paradigm, relying on both qualitative and quantitative data collection methods which lends itself to a mixed-method design, "enjoying the rewards of both numbers and words" (Glesne & Peshkin, 1992:8). According to Heale and Forbes (2017), this is an approach that facilitates triangulation "where two or methods are used" (2017:98), in this instance combining "both qualitative and quantitative methods to answer a specific research question, where converging results aim to increase the validity through verification, which can lead to better explanations for the phenomenon under investigation" (2017:98). The conceptualisation of a topic by a learner is based on comprehension of what is known already and on how existing knowledge is implemented; this research project attempts an understanding of how Grade 7 students solve early algebraic equations by employing a visualisation process through screencast interventions.

The research was conducted utilising seven Grade 7 students from an independent school in the Eastern Cape, South Africa. It is a single-sex, all-boys' school. They were sampled purposefully to ensure that a representative sample is taken of the group and to avoid bias and unnecessarily skewed results, because they will be studied "in-depth and studied as they naturally occur" (Denscombe, 1998:81). The aim of the project is to "gain depth in" (Denscombe, 1998:81) this particular area of algebra by analysing the methods of their calculations post the screencast intervention test. This is a method used "widely in qualitative research for the identification and selection of information-rich cases related to the phenomenon of interest" (Palinkas et al., 2013:1). The participants were of mixed-ability. A diagnostic test was conducted in order to establish the participants for this project. Based on the selection criteria, the intervention group would be subjected to the screencast interventions, through which the concepts of early algebra were explained using images, audio and video. The screencast interventions, designed on the basis of the participants' lack of algebraic knowledge from the diagnostic test, took place over a period of seventeen lessons (video clips).

This research took the form of an interventionist case study chosen because the research is contextualised, and it is also an up-close, in-depth, and detailed examination of a topic in mathematics. The data that was collected for analysis was from the diagnostic tests, video recordings of each of the participants' completion of their post-intervention test, talk-aloud interviews, and direct observations which, according to Yin (1994), defines the intentions of a case study. According to Gerring (2004) the research project is intended for the intensive study of specific components “for the purpose of understanding a larger class of (similar) units. In this instance, the unit of analysis is how the participants solve algebraic equations (as a result of a screencast intervention)” (2004:342). This mixed-method approach assisted the analysis of the data from different perspectives providing validity and reliability. The unit of analysis was how each selected participant interacted with algebraic equations.

According to the Curriculum and Assessment Policy Statement (CAPS) documents, these learners are expected to be able to complete certain tasks with respect to arithmetic, algebra, or equations, and the assessment will be based on grade and year relevant content. The screencast support took the form of short video recordings to which participating learners had access via their tablets and at any time over a ten-day period. These recordings or screencasts supported the visual delivery of the concept of early algebra. The programme took place in the afternoon, at the end of the school day.

According to Gibbs, “It is not sufficient to have an experience in order to learn. Without reflecting on this experience it may quickly be forgotten, or its learning potential lost” (1988:9). Another means of reflection was to establish the advantages and disadvantages of the screencast interventions from the learners’ perspective, a helpful guide in the process of determining how effective screencasts are or can be.

1.6 SIGNIFICANCE

At the heart of this research project was an intervention programme that used screencasts as a means of pedagogy with respect to algebraic equality. The screencast interventions took the form of various animations that reflected the notion of equivalence between two expressions, to the left and right of the equal sign (“=”). The significance of this research project is that it provided rich data that will be relevant to the epistemology of Algebra, and, more specifically, early algebra. The possibility of improving the pedagogy of early algebra is an area that arouses the curiosity of many teachers and is an area that can generate angst for students and teachers alike. The data will provide insight, and from a different perspective, by using visualisation through screencasts, into how students at Grade 7 level understand early algebra and how it can be taught, or delivered, to give meaning for students. Much of the research on this topic has been conducted in American schools and so it is advantageous to discover whether similar disparities can be drawn on from other areas of the world. In conjunction with the result(s) of this research, the intention is to determine an effective method of conveying the notion of equations and equality to students in pre-high school years.

1.7 OVERVIEW OF THESIS

Chapter 2 - Literature Review

The Literature Review provides a review of the past and current research in the development of early algebra in students in order to inform this project. Important issues are raised and discussed in this chapter.

Chapter 3 - Methodology

Theoretical elements relevant to the practical methodological approach to this research project are interrogated in this chapter. The choice of methodology and the methodological approaches are justified in the context of the theoretical framework.

Chapter 4 - Analysis and Discussion

The results of this research are presented to provide a perspective in relation to its overall research questions, goals, and objectives. From there, an in-depth analysis investigates various components of algebra in relation to the lack of fluidity in students at Grade 7 level. A meta-analysis of participants provides insight into the mechanisms of visualisation and the effect they have on the acquisition of knowledge.

Chapter 5 - Findings and Conclusions

The final chapter consolidates the analysis and findings of this research project in the context of the research question with reference to the adopted theoretical framework and against the methodological strategies that were implemented. In addition the limitations and significance of the research project are highlighted, and a few recommendations are suggested for further research.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

> *Education should be responsive to societal needs for it to be relevant and as students are changing, so should teachers be obliged to adapt their teaching styles to suit the new demographic.*
>
> Morris & Chikwa (2014:2)

Many would argue that we do not have an understanding of how children solve equations in the elementary years. Morris (2001), for instance, asserts that students routinely fail to understand the mathematical referents of the equal sign; and both Morris (2001) and Taylor-Cox (2003) suggest that the sooner students are introduced to algebra the better because it is a gatekeeper subject. This alludes to the fact that algebra is used widely with patterns and functions, with ways of representing mathematical relationships, in analysis, and it provides an opportunity to represent complex mathematical ideas succinctly.

The starting point of this research project is an area of mathematics that causes confusion, especially at a Grade 7 level. The existing literature on the Grade 7 level of understanding of the terms 'equations' and 'equals' suggests that the equal sign actually generates a misunderstanding of algebra (McNeil et al, 2006; Knut et al, 2006; McNeil, Fyfe & Dinwiddie, 2014). Because it has been interpreted to mean that an 'answer must follow' or that the equation 'must be completed' less contact time is spent on the relevant concepts so there is an eventual lack in the acquisition of the necessary and relevant mathematical skills (Kinnari-Korpela, 2014:68). The effect is seen in reduced motivation, a spiral of negativity, and anxiety about the subject.

This research topic was inspired by an interest in understanding how pupils learn to bridge what Herscovics and Linchevski (1994) describe as the cognitive gap. This is between solving basic arithmetic equations (linear equations in this case) and the substitution with a number of a letter symbol inherent in these equations; the use of visualisation will be used rather than the procedural, analytical rigour traditionally employed by teachers; the motivation for this is to counter the tendency to ignore and disallow in mathematics the "often visually based intuitive insight" (Nardi 2014:214). Yerushalmy (as cited in Nardi, 2014:214) "suggests that curricular research could benefit from systematic studies that re-examine visualisation as a cognitive challenge and as pedagogical preferences, especially those that concern the semiotic potential of technological tools, for teaching school algebra". Because children's understanding of equality is at the root of algebraic discovery and appropriation, it follows that the concept of an equation and its representation, namely, that the left-hand side is the same as the right-hand side, facilitates their need to construct to construct knowledge.

In order to engage visually with participants and the algebraic concepts, an Information and Communications Technology (ICT) intervention programme will be used to deliver the pedagogy. It is well documented that technology can assist with the acquisition of knowledge (Henrie, Halverson, Graham, 2015; Jordan, Loch, Lowe, Mestel, & Wilkins, 2012; Faherty & Faherty, Ahmad, Doheny & Harding, 2015), but it is my intention to determine how this occurs by using a screencasting intervention programme to solve early algebraic problems. The fundamental question is, how can technology bridge the gap between basic arithmetic and early algebra. The use of a series of screencasts, which is a visual approach that includes audio, reflects Vygotsky's social learning theory which argues that "learning is a necessary and universal aspect of the process of developing culturally organized, specifically human psychological function" (Vygotsky, 1978:90); this "explains human behaviour in terms of continuous reciprocal interaction between cognitive, behavioural, and environmental influences" (Mendoza et al., 2015:82). Bandura's theory expands on this, claiming that "most human behaviour is learned observationally through modelling: from observing others, one forms an idea of how new behaviours are performed, and later on, this coded information serves as a guide for action" (1977:16). Without losing focus on visual and auditory inputs Paivio's (1986) Dual Coding Theory (DCT) (1986) states that "meaningful learning occurs when students process information simultaneously through two discrete input channels, namely the visual and auditory channels" (as cited in Sugar et al., 2010:3). He refers to screencasts as being a "fusion of visual and audio elements, [they] support the way the human brain learns, which is by making associations by what is being seen (visual stimuli) and heard (auditory stimuli)" (Faherty et al., 2015:12), and adds that this is "what makes screencasts particularly beneficial to Maths learning" (Faherty et al., 2015:12).

2.2 ALGEBRA

2.2.1 The Notion of Equality

It is estimated that as many as 75% of American sixth-graders do not have an appropriate understanding of the concept of equality and many students still believe that an equal sign is merely a symbol that precedes an answer to a problem (Wiggins, 2014). According to Kilpatrick et al (2001:261-262) at "[e]lementary schools, arithmetic tends to be heavily answer-orientated and does not focus on the representation of relations." "The understanding of algebraic equations depends on the right conceptual understanding of the equal sign" (McNeil & Alibali, 2006:298) implying that the notion of equality is an important concept for teachers to teach accurately and effectively, allowing for an exploration of what the equal sign is intended to denote.

Filloy and Rejano (1989) offer an example, that "operating on an unknown requires a new notion of equality because in the transfer of a word problem (arithmetic) to an equation (algebraic), the meaning of the equal sign changes from announcing a result to stating equivalence" (as cited in Van Ameron, 2003:650). According to Stewart and Tall (2015) the problem for many students is recognising the appropriate mathematical meaning of equations and the notion of equality. This could be the case since the symbol used to represent "equality" or "equivalence" is polyvalent; indexing more than a single thing', it is sometimes problematic for "=" to be read as indicating that the "answer follows".

An example of this point is provided by Taylor-Cox (2003) who introduced early algebra to her first-graders by using visually stimulating manipulatives, such as a scale and six film canisters filled with varying levels of sand were used to balance each side of the scale. Woods (1993) draws from Wood, Cobb, Yackel (1990) that "a cognitive perspective on learning necessarily implies ways of teaching in which children are acknowledged as active constructors of knowledge" (Woods, 1993:16); by linking this to Vygotsky's (1978) notion of active knowledge construction, she uses visually stimulating concrete objects to connect developmentally appropriate and applicable ideas with regard to algebra for young children in areas such as (1) patterns, (2) mathematical situations and structures, (3) models for quantitative relationships, and (4) change.

In addition to Wiggins's (2014) statistics, Baroody and Ginsburg (1983) and Carpenter et al (2003) indicate that fewer than 10% of students in any grades, one to six, was able to provide the correct answer to the following problem: 8 + 4 = ___ + 5. They discovered that given the appropriate experiences, e.g. 5 = 5, even first-graders were capable of comprehending the equal sign as a relational symbol. This method of providing for the fact that preschoolers have the ability to compare expressions can be tied into the work of Taylor-Cox (2003) by using concrete and visual objects, for example scales and canisters. In the example above, 8 + 4 = __ + 5, Kilpatrick et al (2001:261-262) suggest that instead of providing 7 as the missing number students would typically state that the missing number is 12 because they have seen the "=" as a "separator between the problem and the solution, taking this as a signal to write the result of performing the operations to the left of the sign" and that "the '=' sign is often treated as a left-to-right operational signal." Hence, students are perplexed by an expression such as x + 3 or an equation as in these examples because they have little idea of what to do with them. According to Kibbe & Feigenson (2015) the operation 4 + 2 = x, is an algebraic equation that requires solving for an unknown by two known addends; 2 + x = 10 can be considered an algebraic equation which is one of the operations that seem to be difficult, even for high school children, because one of the addends is unknown, despite an "answer" seeming to have been provided. Van Ameron (2003) refers to what may be considered the difference between arithmetic and algebra in that the expression 2 + 5 in arithmetic means a problem which needs to be interpreted as a command to add 2 to 5 whereas in algebra the same expression means the number 7.

It is important for students to appreciate the different contexts in which the equal sign appears. McNeil and Alibali (2005a) identify four different contexts: (1) the operations equals answer context; (2) the operations on the right-hand side context; (3) the reflexive context; and (4) the operations on both sides context.

These can be represented as follows:

1. The equal sign is presented in the typical addition equation: 5 + 4 = 9;
2. The equal sign is presented in an equation in which the addends appear on the right-hand side of the equal sign: 9 = 5 + 4;
3. The equal sign is presented in a reflexive equation: 9 = 9;
4. The equal sign is presented in an equation with the operations on both sides of the equal sign: 5 + 4 = 6 + 3.

Borenson suggests that "generally, signs have more than one meaning, depending on the context" and has led many an educator to the statement that "it does not mean that the answer comes next" (2013:90). In fact, the necessity of the equal sign's role that is preceded by a "numerical result of the sequence of computations that precede it (the calculator use of the equal sign) is a valid and necessary use of this sign" (Ginsburg, 1996; Falkner, Levi, & Carpenter, 1999; Seog & Ginsburg, 2003:90). This can be referred to as the operational result of the equal sign, as opposed to "understanding the relational meaning of the equal sign, which is vital for success in mathematics and, particularly in algebra" (Borenson, 2013:90). With respect to algebra, it should be established that the equal sign exhibits "equivalence between two sets of expressions, each one of which includes one or more operations within it" (Borenson, 2013:90). These examples have been arranged to illustrate this statement while modifying them to include an unknown variable for algebraic purposes. These frameworks form the basis of the analytical instrument to analyse how participants solved different linear equations.

Table 2.1 Analytical Framework for various Arithmetic/Algebraic scenarios

Scenario 1 (Numeric only)	**Scenario 2 (Algebraic only)**
a. 5 + 1 = ___ + 4	a.7 + 10 = m + 11
b. 10 = 5 + ___	b. 10 = 6 + 2m
c. ___ = 7	c. m = 8
d. 3 + ___ = 4 + 4	d. 2 + 2m = 4 + 4

These examples would not be self-evident or intuitive to students nor is it "an understanding that naturally follows from knowing the operational meaning of the equal sign" (Borenson, 2013:91). To assist in determining how the equal sign is understood in this research, the examples identified by McNeil and Alibali (2005a) have been elaborated on as can be seen in Table 2.1 above. Examples are presented in arithmetic format in the first column as 'Scenario 1', whereas in the second column, are included similar examples, except that letters are included where the missing numbers were previously, with a letter that represents the unknown variable, or missing number. A significant motivation for the research is to determine how students interpret the equal sign and the letter, particularly in Scenario 2; and how much it affects their understanding in solving for the unknown variable. It is unfortunate that the equal sign has been used to "indicate both meanings, relational and operational, and that it never occurred to anyone to provide a different symbol for the two different meanings" (Borenson, 2013:91).

According to Gathercole (1999), by the age of thirteen-years-old, children are said to have a "mature working memory system" that should complement their ability to solve complex arithmetic problems (Hitch, 1978) and "process complex relations" (Halford, Wilson, & Philips, 1997) (as cited in McNeil et al., 2006:368) which, from a cognitive development perspective, should support their ability to comprehend a relational understanding (Skemp, 1976) of the notions of equations and the equal sign, or equality, which could, ultimately, lead to success in their conceptualisation of algebra. Herscovics and Linchevski (1994) claim that the difficulty of solving these types of problems is reduced when the problems are presented in a way that "taps" the children's intuition and that "children's well-known difficulty in mastering algebra may be influenced by their difficulty manipulating symbols using formal rules" (as cited in McNeil et al., 2006:368)

Given these notions of equality and equations, how do we bridge the "cognitive gap" (Herscovics and Linchevski, 1994) (as cited in Van Ameron, 2003:65) between solving basic arithmetic equations and linear algebraic equations and is the substitution of a letter symbol in conflict with a clear thought process related only to number? It may be that the answer to this question would benefit the way future generations of children are able to enjoy and excel at Mathematics.

2.2.2 Alphanumeracy and 'Symbol Sense'

> *.......a non-algorithmic feel for, a sound understanding of their nature and the nature of the operations, a need to examine the reasonableness of results, a sense of the relative effects of operating with numbers, a feel for orders of magnitude, and the freedom to reinvent ways of operating with numbers differently from the mechanical repetition of what was taught and memorized*
>
> (Arcavi, 1994:24)

With respect to this definition of number sense, should there not be a similar situation in algebra? That is, symbolic manipulations must be considered as one of the central issues in algebraic instruction (Arcavi, 1994). High school students struggle to make sense of literal symbols even though “they manage to handle the algebraic techniques successfully, [they] often fail to see algebra as a tool for understanding, expressing, and communicating generalizations, for revealing structure, and for establishing connections and formulating mathematical arguments (proofs)” (Arcavi, 1994:24); it would therefore “seem reasonable to attempt a description of a parallel notion to that of number sense in arithmetic: the idea of symbol sense”.

Arcavi (2005) asks the question, “how to extend the construct of number sense from the realm of school arithmetic to the realm of school algebra” (Arcavi, 2005:42) given the lack of sense-making with regard to the inclusion of symbols to arithmetic. He adds that “most students with a substantial background in algebra, do not resort to symbols as a tool to enable them to investigate problems in a general way” and “they are not invoked unless prompted to do so” (Arcavi, 2005:43). His wish is for students to have the confidence and understanding of the situation to generate enough sense to know when and when not to use them.

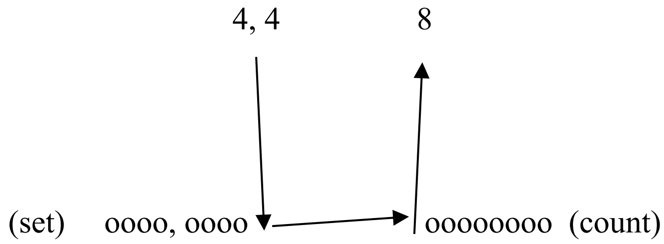
In my experience, the introduction of letters, or variables, into a mathematical equation as a symbol to represent a missing number leads to misunderstanding among many students. It is not enough to argue that the introduction of symbols makes the difference in the conceptualisation of solving for unknown variables in equations, but a notion derived from Dehaene’s research can be highlighted to illuminate this difference (Dehaene, 2011). The majority of his work is centred on the idea that a specific area of our brain “makes a special contribution to number processing” known as the intraparietal sulcus. Its relevance is that “it is activated consistently when subjects were asked to look at a number but did not show any reaction to letters or colours” (Dehaene, 2011:239). Could this have something to do with how children solve linear equations or the difficulty they have in doing so? The intention here is to find out how they develop the ability to manipulate symbols through space and how they conceptualise the process. From Arcavi’s paper (2005), concerns raised for discussion around symbol sense are:

1. The characterisation of symbol sense is not fully developed;
2. How do experts develop symbol sense?
3. What is the underlying knowledge required?

These "reflections and discussions" generate debate around how one teaches symbol sense and how one generates an understanding or makes "sense of the symbols and their mathematical actions" (Arcavi, 2005:43) and, thus, are paid special attention as potentially strong indicators of "symbol sense". Following on from the question about the extent to which symbol sense can be taught Arcavi (2005) pondered whether "symbol sense is something that only mathematically able people will develop by themselves, through, for instance, practice or insights, or can most (if not all) people develop it at least partially?" This is what being a mathematics educator entails: in part, to "design, implement and monitor interventions in order to maximize students' potential for learning" (Arcavi, 2005:43). His belief is that it can be nurtured, although "students tend to view graphs as operational tools for sense-making and tend to ignore the symbols" (Arcavi, 2005:44). The dichotomy is that this is not adequately served in traditional classroom settings and, unfortunately, classroom culture dictates how and what knowledge may be acquired (Arcavi, 2005). He claims that "for supporting the development of symbol sense, it may be best to ask students not to jump to symbols right away, but to make sense of the problem, to draw a graph or a picture, to encourage them to describe what they see and to reason about it" (Arcavi, 2005:44). These observations are significant for the topic of "visualisation" because, as Arcavi demonstrates, "if these activities are not experienced by students, or given some seal of approval, then, at best, spontaneous sense-making may be relegated to a lower priority, or at worst, it will not happen at all" (Arcavi, 2005:44).

Existing research, not confined to the following examples of Powell and Fuchs (2014, 106 - 116), Brizuela and Schliemann (2004, 33 - 40), Willingham (2009/2010), Hersocovics and Kieran (1980, 572 - 580), Kaput, Carpenter and Levi (2000, 1 - 20), Kieran (2004, 138 - 152), and Dehaene (2011), points to there being a lack of continuity and ability to solve basic arithmetic equations and algebraic equations (e.g., if $\square + 2 = 7$, then it is $5 + 2 = 7$ versus $p + 2 = 7$, then $p = 7 - 2$, $\therefore p = 5$). It is essential, as alluded to by Weinberg et al., that "students develop fluency with algebraic symbols in order to engage fully with the concepts and to prepare for further study in mathematics" (2015:70)

Practices common to the teaching of school algebra generate, unintentionally, a computational field/space that is internally inconsistent for children at this level. In other words, with arithmetic 4 + 4 is associated with the combination/merging of two sets, each having four elements, that is, oooo merged with oooo implies that 8 is the solution.



It is, therefore, not unreasonable for children to think of addition as an indication to merge two collections. In expressions such as x + y, both x and y have to be associated with an entire infinite set, like natural numbers or positive rational numbers. Teachers tend to use terminology incorrectly such as "one cannot add apples to oranges" helpfully to illustrate the explanation that $x + y \neq xy$.

With reference to sets, initial conceptions of addition are closely bound with the merging of collections, and, if entities like x and y cannot be thought of as numbers, then it is difficult to think of x and y as associated with collections of things, such as 2 and 5. On this basis, it is reasonable that x and y are the things to be merged as can be described by concatenation. This occurs when sets are brought into close proximity with each other (e.g., oooo, oooo → oooooooo) and so concatenating x and y to produce xy is not surprising or unusual for children at this level. Read in conjunction with how children interpret an expression such as "x + y" and that the result is not "xy" adds to the abstract that is created by the fact that children are being asked to "add" letters together and that not being able to do so will inhibit their algebraic knowledge and acuity.

Kibbe & Feigenson (2015) develop this idea and question why children struggle to solve for "x" when it is presented in a formal notation, but achieve success when the same problem is presented in a non-symbolic manner such as 5 + 3 = _ as opposed to 5 + x = 8. There is evidence that three-year-olds can "demonstrate some understanding of mathematical inversions (c + a - a = c) when presented with blocks (concrete, visual items), but they are unable to count" (Kibbe & Feigenson, 2015:3). Expressed in symbols this would mean that a + b is a command to add a and b together, whereas x = a + b is a value for x. "This is a switch that proves essential as letters occur in the formulae and a + b cannot easily be interpreted as a problem" (Freudenthal, 1962:35). Van Ameron and her colleagues used "pre-algebraic methods of reasoning and symbolizing as a way to facilitate the transition from arithmetic to an algebraic mode of problem-solving" (2003:73).

According to Fey (1990), "symbol sense" is not a formal term used to describe the ability to manipulate unknown variables in algebra, but a term to suggest an idea that could be elaborated, by trying to ease the process of algebra and its perceived level of complication. A crucial step in using symbols is described by Arcavi: "[w]hen we translate a situation into symbols, one of the first steps is to choose what to represent and how" (1994:28) as this choice will impact on the process of solving equations. A better definition of understanding symbols and their effective use in algebra is that "symbol sense" is a "complex and multifaceted 'feel' for symbols, a quick or accurate appreciation, understanding, or instinct regarding symbols" (Arcavi, 1994:31). Arcavi offers a list of the characteristics of the use of symbols which can be summarised as but not limited to:

- An understanding of and aesthetic feel for the power of symbols: understanding how and when symbols can and should be used in order to display relationships, generalizations, and proofs which otherwise are hidden and invisible.
- A feeling for when to abandon symbols in favour of other approaches in order to make progress with a problem, or in order to find an easier or more elegant solution or representation.
- An ability to manipulate and to 'read' symbolic expressions as two complementary aspects of solving algebraic problems. On the one hand, the detachment of meaning necessary for manipulation coupled with a global "gestalt" view of symbolic expressions makes symbolic-handling relatively quick and efficient. On the other hand, the reading of the symbolic expressions towards meaning can add layers of connection and reasonableness to the results.

Arcavi (1994:31)

The fact that "symbol sense" was not yet a fully-fledged idea and that Arcavi (1994), having suggested that it would be "presumptuous to describe/prescribe fully-fledged instructional implications" (Arcavi, 1994:32), now provides an opportunity for this notion to be explored more visually, the following implications are suggested in conjunction with Arcavi's (2005) views:

1. Symbol sense is a crucial component of algebra and understanding the manipulation rules at the heart of algebra is crucial. It would make sense if this was taught in the rich

context of visualisation to support the acquisition of the knowledge required of how and when to use these manipulations.

2. We follow by referring to the ways in which we can harness technology in the service of the development of tasks and problems which, in the hands of a skilful teacher, have the potential to foster symbol sense. These tasks should rely on the computational power which liberates mental resources for the development and enrichment of meanings and connections.
3. From these statements it can be noted that the intention is not to deviate from the curricula by being too novel and innovative with tasks or problems, but is to ensure that the activities that are created appropriately engage with the construction of symbol sense.
4. The symbolism we come across in algebra should be introduced at the beginning of situations and not left to display a formalised and meaningless structure with the introduction of these entities at a later stage: these only confuse students. Their inclusion will serve as powerful ways to solve and understand problems and improve ways to communicate them (Arcavi, 1994:33).
5. Classrooms are places wherein which questions need to be asked and boundaries need to be pushed, particularly "regarding the role of symbols and their rules" (Arcavi, 1994:34). For example, "can sense be made of the result of substituting for x and y to obtain, say 2 = m3 + b for the general expression, y = mx + b, for a linear function?" (Arcavi, 1994:34). The suggestion is that it would help students "to regard symbols as entities which can be the object of their constant reinspection, and not just governed by rules arbitrarily imposed on them from above" .

(Arcavi, 1994:34).

Interestingly, Arcavi's sentiment is that "appropriate practices of thought and discussion" will improve the idea of symbol sense, but that "collecting implications from observations from our own environments" (Arcavi, 1994:34) may assist the process of acquiring conceptual knowledge while also developing a sense of symbol which in turn will enhance the ability to solve algebraic equations.

2.2.3 Transformation

A concept highlighted in Wallis's work (2016) is that the human mind is constructed to explore causal relations as a default which is what children do when they cannot solve a problem or situation. It can be inferred that learning is based on correlations which draw on the premise that, with continual practice, a positive learning outcome should be expected. An example of this would be the transformation of equations and the "taking over and changing of signs", which requires the construction of a causal relation in order to try to explain a correlation. It involves the pattern of movement or symbols changing position by delocating and relocating across the "=" sign. A more efficient way of explaining this is found in Ngu et al. (2015) who imply that the impetus of equivalence, of the expressions on either side of an equation, is at the heart of solving equations, although doing so when the process involves more than one element can demand a high cognitive load (Ngu et al., 2015).

Ngu et al. (2015) point to two different methods used to solve equations, namely the balance method and the inverse method, and the cognitive load required from each. Cognitive load is a term used to explain the theory of our cognitive handicap, our "limited working memory" (Ngu et al., 2015:273). It explains that there is a restrictive impact on our cognitive calculations when engaging with a cognitive task as opposed to our long-term memory which operates in an antithetical manner with regard to its unlimited capacity to store a rather large amount of organised schema (Ngu et al., 2015). While the balance method requires an operation on both sides of the equal sign (equation), the inverse method requires an operation on one side of the equal sign (equation), only (Ngu et al., 2015). Children's understanding of equality is at the root of their algebraic discovery and appropriation and it is imperative that they understand the concept of balancing an equation (left-hand side = right-hand side) from as young an age as possible (Jennifer Taylor-Cox, 2003); Ngu et al (2015), however, elaborate on the difficulty of so doing, given the different circumstances of the required "cognitive load" that exists. While it is useful to suggest the use of film canisters, sand, and scales as a simple effective way to illustrate how to balance two "sides" of a scale, it is crucial also for the teacher to know just how to progress these types of concrete visual demonstrations towards equations at a later stage.

Although a slight advance from Taylor-Cox's methods, her notion is backed up by Cumali Oksuz's (2007) work in which he suggests that algebraic thinking does begin to develop in students in the earlier grades when their knowledge of numbers and operations establishes a basis from which to learn algebra and, according to Ballheim (1999), as cited in Ngu et al., 2015:271), "equation solving is a basic skill." Linchevski (1995) suggested that pre-algebra should be seen as an extension of basic arithmetic that asks children different questions about numbers as they hve come to know and understand them in arithmetic; in this way teachers can better assist students because to "learn equation solving efficiently is an important issue" (Ngu et al., 2015:271).

Considering the different techniques available for solving equations, it is notable that most textbooks recommend the balance method (Kalra & Stammell, 2005; Mock & Wade, 2004: Vincent, Price, Caruso, McNamara, & Tynan, 2011, as cited in Ngu et al., 2015). The inverse method, on the other hand, is preferred in some Asian countries (Cia et al., 2005, as cited in Ngu et al., 2015). That different cultures solve equations differently is revealing: Asian methods tend to emphasize "practice, training, and efficiency, whereas non-Asian countries emphasize exploration and flexibility more" (Imbo & LeFevre, 2009 as cited in Ngu et al., 2015:271). This is because, by comparison with the preferred inverse method, many Asian high school teachers find the balance method more "complicated, error-prone and inefficient, particularly for the more complicated equations involving multiple steps" (as cited in Ngu et al., 2015:271).

In this thesis the balance method has been implemented for the screencast interventions in view of the need to ensure the concept of the notion of equality. The types of equations that required solving were also, typically, one- or two-step calculations. This approach reflects the general consensus of what is considered to be the westernised "view" of solving equations as opposed to the "inverse method, which is regarded as change side, change sign which falls short of addressing the concept of balance in equation solving" (Ngu et al., 2015:272). According to Ngu et al. (2015), to solve an equation, for example $x + 3 = 5$ and using the balance method there "are a few key concepts that need to be considered such as (1) x is a variable that can be replaced by a number to preserve the equality of the equation; (2) the = sign describes a problem state or a relationship such that the left side equals to the right side; and (3) the goal is to isolate the variable" (Ngu et al., 2015:272). The balance method implies that the same operation is required on either side of the equation, "-3" (known as procedural knowledge; Star & Rittle-Johnson, 2008, as cited in Ngu et al., 2015) and although this changes the state of the equation it still "maintains the equality of the equation" (known as conceptual knowledge) (Star & Rittle-Johnson, 2008 as cited in Ngu et al., 2015:272).

As far as the inverse method goes, the learner will treat the addition of 3, in the equation $x + 3 = 5$, "as an inverse operation to subtraction, and moves the + 3 from one side to become - 3 on the other side (procedure knowledge) of the equation" (Ngu et al., 2015:272). Here again the equality of the equation is preserved even though the "step alters the problem state" (Ngu et al., 2015:272). To illustrate these methods, Ngu et al. (2015) propose the following steps:

1. **The balance method**

   Line 1 $x + 3 = 5$ (-3) on both sides

   Line 2 $-3 \quad -3$

   Line 3 $x = 2$

2. **The inverse method**

   Line 1 $x + 3 = 5$ (+ 3 becomes - 3)

   Line 2 $x = 5 - 3$

   Line 3 $x = 2$

The main difference between the two methods, as seen by Ngu et al., lies in the "critical procedural step (-3 on both sides vs. +3 becoming -3), which although "alters the problem state of the equation" (Ngu et al., 2015:272), the equality in both instances is preserved. The advantage of using the balance method is that by showing that you are -3 from both sides of the equation, "depicts the concept of balance in equation solving" (Ngu et al., 2015:272); this plays a specific role in the screencast interventions as compared to the inverse method, which "does not adequately address the concept of 'balancing' in equation solving" (Ngu et al., 2015:272).

The question to ponder is which method is better in the "acquisition of procedural and conceptual knowledge in equation solving" (Ngu et al., 2015:272), and, while it is argued that gains in one will lead to gains in the other (Rittle-Johnson & Star, 2007, 2009; Rittle-Johnson, Star, & Durkin, 2009, as cited in Ngu et al., 2015), there is also a strong argument for the development of "flexible procedural knowledge" (Rittle-Johnson & Star, 2007, 2009; Star & Rittle-Johnson, 2008, Star & Seifert, 2006 as cited in Ngu et al., 2015:273).

2.2.4 Visualisation in Algebra

The idea of visualising mathematics is not new but formal research of its processes has only recently gained traction. Yerushalmy (2005), a strong protagonist of visualisation and algebra, advocates for "meaningful integration of visualisation" (Nardi, 2014:193) by supporting the notion that concrete objects and interactive diagrams should be in use in conveying algebraic epistemology, much as does Taylor-Cox's use of the film canisters which can be progressed to an equation at a later, more abstract stage (2003). This ties into the “concrete to representational to abstract (CRA)” model mentioned in Sundling’s work (2012:17) in that once children have understood the manipulation of concrete tangible items, they can then progress on to the representation pictorially of the concrete, and then on to the abstract where mathematics becomes more formalised and symbolic with the substitution of images for number and letter symbols. Taylor-Cox (2003:18) believes this encourages her students to become “capable of masterful thinking”.

According to West (2004), “provision of ways to see, understand and extend mathematical ideas have been underdeveloped in many curricula and mathematics is still presented as an almost entirely numerical and abstract subject” (as cited in Boaler et al., 2016:1). It is also suggested by West (2004) that visual representation in learning mathematics may enlighten students in a way that will give them “access to deep and new understandings” (as cited in Boaler et al., 2016:1). The fact that mathematics consists of numerals and symbols implies that we sometimes miss opportunities to develop and exploit visual understandings in the subject.

2.3 VISUALISATION

While education changes at the speed of technology, content has, in general, been static. It is therefore important, when dealing with the twenty-first century student, that the traditional mathematical pedagogy, given the cognitive requirements of the subject, should take account of the assistance technology has been shown to provide for the successful acquisition of knowledge (Henrie, Halverson, Graham, 2015; Jordan, Loch, Lowe, Mestel & Wilkins, 2012; Faherty & Faherty, Ahmad, Doheny & Harding, 2015). If it is one thing to make algebra clear in this way, then the significant question remains, if algebra comes alive for students using various technologies, how is the gap bridged, cognitively, between basic arithmetic and early algebra.

There is a growing demand for teachers continually to expand their knowledge of what the possibilities are in terms of teaching resources, and that this should be playing an increasingly significant role in pedagogy. This will continue to drive the need to deliver a pedagogy that is more visual than the accustomed verbal/logico, algorithmic-based method. Almost all teaching requires a form of visualisation or a process thereof which is what we “see” in our mind’s eye, and this is a “personal process that assumes that the person involved is developing or using a mental image" (Clements, 2014:181). As for the questions surrounding the integrity of visualisation in mathematics, Albert Einstein remarked:

> *The words or the language, when they are written or spoken, do not seem to play any role in my mechanism of thought. The physical entities which seem to serve as elements in thought are certain signs and are more or less clear images which can be "voluntarily" reproduced and combined….The above-mentioned elements are, in my case, of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage…. In a stage when words intervene at all, they are, in my case, purely auditive, but they intervene only in a secondary stage as already mentioned.*
>
> (Hadamard, 1945:142).

The idea of formalising the introduction of visualisation into mainstream mathematics will need to be given impetus in terms of language and representation. Presmeg (2006a) calls for the teaching of visuality in order to introduce and cement visualisation into students' "mathematical custom" (Nardi, 2014:193). Importantly the "relationship between logical and visual thinking is not polarized, but orthogonal" (Nardi, 2014:193) and for teachers to be effective educators of visuality their "own preferences will need to be flexible and mixed" (Nardi, 2014:193). For Hershkowitz (1992) it is necessary to consider thinking more visually than is usual because it *is* a "mode of mathematical thinking and exists because of a group of signs and relationships (a 'language'), by which mathematical thinking, including the visual one, might be developed, limited, expressed and communicated to oneself and to others" (Nardi, 2014:195). There is an opportunity to establish "a language" able to express visual thinking because "when visual language does not represent a thought, it is just a group of signs without a meaning" (Nardi, 2014:198). For mathematics to be "expressed visually, it needs a language, visual or other: and visual language, to be meaningful, needs to be attached to some conceptual entity" (Nardi, 2014:193), all of which is key to the improved acquisition of mathematical concepts.

The argument of Klibanoff et al. (2006) is that "young children's mathematical knowledge is influenced by environmental input" which is supported by the claim made by Presmeg (as cited in Nardi, 2014) as well as Hershkowitz (as cited in Nardi, 2014) who would encourage teachers to "acknowledge visualisation as one of the languages of mathematics and as one of several ways of thinking mathematically" (Nardi, 2014:195). The construct of "concept image" (Presmeg, 1986a, 1986b, 2006 & Bishop, 1973, 1980) is a way of understanding a mathematical concept that describes the notion of visualisation as "the total concept structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (Tall &Vinner, 1981:152) and is built on years of experience, of all kinds, and changes and matures as the individual meets new stimuli.

There is a misconception that visual mathematics is for "lower level maths, younger or struggling students, and that students should only work visually as a prelude to more advanced or abstract mathematics" (Boaler et al, 2016:1). According to West (2004, as cited in Boaler et al., 2016:1) these thoughts have been held by academics for centuries, that words and mathematical symbols are for the real mathematicians, while images and diagrams are for children and the lay public. This paper seeks to dispel this myth by presenting compelling evidence as to why visual mathematics should be "integrated into curriculum materials and teaching ideas across the grades" (Boaler et al, 2016:1). It has been suggested that "good mathematics teachers typically use visuals, manipulatives and motion to enhance students' understanding of mathematical concepts" (Boaler et al, 2016:2), but the reality is that mathematics is still being presented as an "entirely numerical and symbolic subject, with a multitude of missed opportunities to develop visual understanding" (Boaler et al, 2016:2).

2.4 TECHNOLOGY IN EDUCATION

2.4.1 Screencasts

Incorporating screencasts as a medium to deliver this intervention programme provides an appropriate platform for scaffolding and visual processes. One of the advantages of modern technology is that it provides an opportunity to reflect and present our thinking as well as an insight into our imagination, and share it in a way that people can relate to. This can take many forms, from coding to a multitude of "apps" available on the internet and "the ubiquity of mobile devices has allowed students and lecturers to create, curate and view screencasts far more easily than ever before" (Galligan & Hobohm, 2018:1). The educational concept of Zone of Proximal Development (ZPD) relies on the More Knowledgeable Other (MKO) to deliver the pedagogy (Vygotsky, 1978). Scaffolding relies on the source as being knowledgeable and able to guide the student whose skills are not yet developed. According to Ketterlin-Geller (2008) and Fuchs et al., (2005) the scaffolding of learning increases student engagement and also that students respond efficiently to mathematics instruction. Screencasts vary in nature, but they are typically online tutorials, video lessons, or slideshow presentations (Patton, 2015) and, as Winterbottom (2007) points out, they are used "innovatively to support and enhance traditional teaching and learning" (Faherty et al., 2015:12). More accurately, "screencasts are a digital recording of a computer screen activity accompanied by a voice-over or concurrent audio commentary" (Faherty et al., 2015:12) which can be viewed on a mobile device (computer, tablet, or mobile phone) with the capacity for replay when convenient [or, 'at leisure'?] (Jordan, Loch, Lowe, Mestel, & Wilkins, 2012), which is a mindset to which teachers need to become more accustomed as the craft of teaching and technology progress. According to a student from a paper published by Jordan, Loch, Lowe, Mestel & Wilkins (2012), "screencasts are easier to understand, more engaging, and short and to the point" compared to textbooks that can be "sometimes dry" because "hearing someone talk through it with you helps you get it clear in your head" (Jordan, Loch, Lowe, Mestel & Wilkins, 2012:5).

The use of a screencast as an intervention would allow an increase of personalised teaching time (Kinnari-Korpela, 2014:68) and a visual approach that encourages the importance of the concept of the "relational" relationship of algebraic equations as opposed to simply calculating an "answer" to the right of the "=" symbol. Education is becoming increasingly dynamic in the sense that more and more resources are at the disposal of teachers. This particularly applies to the ubiquitous presence of technology. Although the content and general cognitive requirements of mathematics have remained relatively static, the pedagogy of teaching mathematics is constantly evolving and adapting to the ever-changing demands of the twenty-first-century student.

Ford (Ford et al., 2012) has demonstrated that there is not much literature to support the extent to which the effectiveness about screencasts as a pedagogical tool is known, or the extent of its effect from an intervention perspective. The opportunity to investigate the impact of screencasting as an innovative pedagogical tool and how effective it is for the students' learning experience of algebra is therefore a useful tool. Further to this, McLoughlin & Loch (2016) posit that with the continuous developments in eLearning "new opportunities for presenting and engaging students through dynamic visualisations are evident, and research to study the implications for learning with multi-representational resources is a flourishing area." The "majority of the studies conducted have concentrated on the face-to-face lectures"

(Kinnari-Korpela, 2014:69) while not much has been explored on the dissemination of short videos and how they benefit learning. According to Morris and Chikwa (2014) there is “little evidence around screencasts’ impact in terms of knowledge acquisition”. The concept of a screencast offers students an opportunity to “re-envisage some aspects of the mathematical learning experience and enhance students’ engagement and mathematical thinking” (Larkin & Calder, 2015:1).

With reference to Yerushalmy (2005), there is strong agreement with Presmeg’s sentiment regarding a gap in how technology is being used and “the important direction of the study of visualisation” (Nardi, 2014:205). Nardi (2014) argues that the appropriate pedagogical practices are developed so as to support and assist students in their quest to develop the ability to visualise mathematics (Nardi, 2014) and to be able to extract necessary information. The idea is to design a resource that will “focus on student engagement, and that more generally consider best practice in instructional design (Galligan et al., 2010) as the majority of research is geared to students’ perceptions and their use of the recordings” (McLoughlin & Loch, 2016).

Two questions that arise are:

1. do students perform better using screencasts?
2. how do students use these video materials?

It should be the intention of the visualisation process to “reshape the learning experience and influence engagement and understanding” (Larkin & Calder, 2015:1). According to McLoughlin & Loch (2016) visual representations, such as, in this instance, screencasts, have shown that “integrating visual and multimedia in teaching abstract concepts, such as algebra, can enhance learning.” Nardi (2008); this questions the reality of a new curriculum based on this ideal because it can be seen to be rather flimsy due to the fact that “students’ visualisation is unaccompanied by any explanation, or it may appear to be disconnected from the rest of the student’s writing” (Nardi, 2014:209). An essential point to take from these arguments is to ensure the most appropriate and relevant design of the screencast and that it should be effectively integrated as a method of intervention. Paivio (1986) suggests that meaningful learning occurs through the Dual Coding Theory which implies that students “process information simultaneously through two discrete channels, namely the visual and auditory channels” (as cited in Faherty, 2013:12). There is nevertheless a sense that there is not “a concomitant investment in developing practices regarding how such devices and software may be used to develop conceptual rather than procedural or decorative knowledge" (Larkin & Calder, 2015:1).

They have been referred to as "virtual learning environments (VLEs) in higher education" and are especially useful as "they allow the recording of handwritten step-by-step solutions of problems including specialist mathematical notation" (Jordan, Loch, Lowe, Mestel, & Wilkins, 2012). The convenience of this type of pedagogy is that the resources such as online tutorials, video lessons, or slideshow presentations, can be accessed at any point in time, watched any number of times, and can be replayed (Sugar et al., 2010). As per Patton (2015), "learning doesn't always take place in an academic setting".

2.5 INTERVENTION

At the heart of this study is an intervention programme that employed screencasts as a means of pedagogy with respect to algebraic equality. Screencast interventions took the form of various animations that reflected the notion of equivalence between two expressions, to the left and right of the equal sign ("="). According to Houssart's and Croucher's (2013:428) definition of intervention programmes, they are considered "as materials and instructions, usually for short- or medium-term use, aimed at raising selected pupils' attainment…"

A dictionary definition of 'intervention' explains it is a "systematic process of assessment and planning employed to remediate or prevent a social, educational, or developmental problem" (http://www.yourdictionary.com/browse/intervention). Sundling notes (2013) that although a multitude of mathematics interventions exist, "mathematics intervention programs are still in their infancy" (Sundling, 2013:5), and although teachers are easily able to identify students who struggle, teachers need a concrete way to establish how a concept such as *equality in early algebra* in this case is conceptualised. The intention of a good intervention is to improve pedagogy and thereby to improve an understanding of content.

Hulac, Dejong, & Benson (2012) argue that "fewer research-based options for mathematics interventions exist and teachers' time is already taxed due to identifying and conducting interventions in reading" (as cited in Sundling, 2012:6). Tools such as universal screening tools are implemented to provide schools with the necessary data to make decisions regarding the individual needs of its students (Sundling, 2012:11). This collected data is used to determine the most "appropriate curriculum and instructional processes" (Sundling, 2012:12) that may be required in order to tailor the “different instructional approaches or whether additional intensive interventions are required” (Sundling, 2012:12). One such intervention tool known as a graduated instructional sequence is most effective when it follows a process from concrete to representational to abstract (CRA) and it is highly recommended (Ketterlin-Geller et al., 2008). This allows for a “hands-on manipulation using concepts or procedures” which then progresses to “learning using pictorial representations” after which the more “abstract symbols” are implemented (Ketterlin-Geller et al., 2008). Ketterlin-Geller et al (2008) suggest that these scaffolding techniques are “components of effective mathematics instruction” all of which ties into the points raised by Boaler (2016) and her colleagues around the integration of visualisation into the mathematics curriculum sooner rather than later.

The lack of prior knowledge and an understanding of the notion of equality is a pertinent concern because it means that less time can be spent on acquiring new topics of content and that more time is required to revisit “prior” content (Kinnari-Korpela, 2014:68) such as: “what the notions of equivalence/equality are.” This, naturally, leads to a lack of motivation regarding the subject, which in turn leads to a spiral of negativity because less contact time has been spent on the relevant concepts and this results, therefore, in a lack of the necessary mathematical skills (Kinnari-Korpela, 2014:68). The intervention of screencasts would allow an increase of personalised teaching time (Kinnari-Korpela, 2014:68) because the time required to revise can be better used by progressing towards newly acquired knowledge.

The intervention took the form of a series of screencasts developed around the topic of equality. These screencasts were video recordings of my screen and my voice as I worked and spoke through the introduction of each component of early algebra. I modelled examples that were relevant to this level with the intention of keeping each screencast down to an average running time of approximately four to five minutes. I made use of Paivio’s (1986) Dual Coding Theory (DCT) which supports the use of “two discrete input channels, namely the visual and auditory channels” (as cited in Sugar et al., 2010:3) as a method of encouraging meaningful learning in students. The screencasts were designed in line with the initial diagnostic test results showing the students’ lack of understanding of the various concepts required for Grade 7 algebra as set out by the Department of Education (Department of Education, Curriculum and Assessments Policy, 2011). A total of sixteen screencasts were designed to be delivered to the participants via Google Classroom over a duration of ten days. The participants had access to them as and when they wished. I encouraged them to view and go through them in their own time without forcing them as the intention was not to over-emphasise or hinder the process with too much involvement. Included here is an example of a screencast intervention that encapsulated the introduction of the topic, and which required a series of two screencasts.

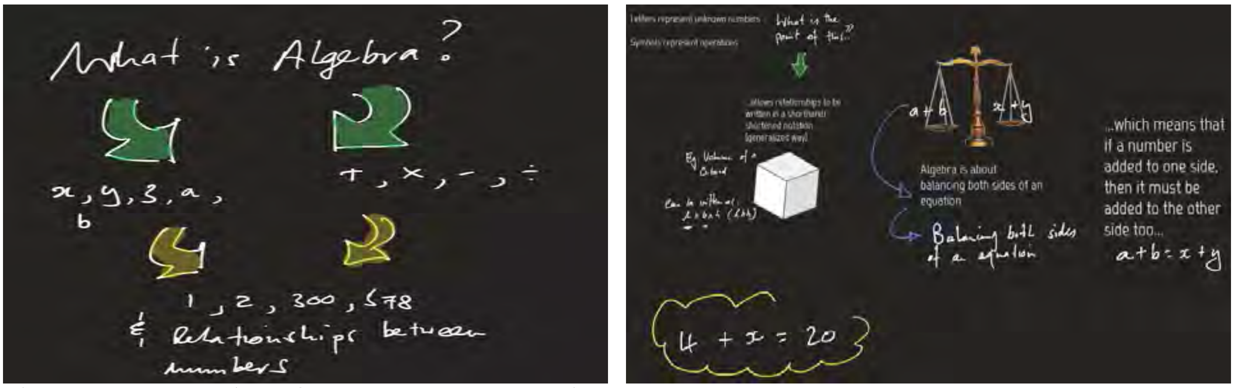


Figure 2.5 Examples of Screencast Interventions

2.6 THEORETICAL FRAMEWORK

2.6.1 Social Constructivism

The theoretical underpinnings of this research locate themselves in social constructivism as this project is based on how students build and construct knowledge, in this case an understanding of equality in an algebraic context. Extracted from Hilav (1990), cited in Erdem (2001), it was Socrates who claimed “teachers and students should talk with each other and interpret and construct the hidden knowledge by asking questions” (Amineh & Asl, 2015:9) and “that learning is a process of constructing meaning; it is how people make sense of their experience” (Merriam & Caffarella, 1990:260). According to Mvududu and Thiel-Burgess (2012) it is not only in the building of knowledge that constructivism plays a role but also its ability to “probe for children’s level of understanding and to show that that understanding can increase and change to a higher-level thinking” (Amineh & Asl, 2015:9). As constructivism is the thread that binds this research it also “refers to the how of learning and thinking” (Amineh & Asl, 2015:9) which forms the basis of the question posed by this topic.

The implementation of the screencasting intervention provided the scaffolding tool required to build on students’ prior knowledge. Through the construction of new knowledge and through building on prior knowledge Vygotsky (1978) places much emphasis on the role of language and communication in cognitive development. He refers to “tools of intellectual development (screencasting intervention in this instance), which allowed children to use basic mental functions more effectively/adaptively” (Amineh & Asl, 2015:13) and this in turn ties into his philosophy “that community (environment) plays a central role in the process of “making meaning” (Amineh & Asl, 2015:13).

Wiggins (2014) stresses the importance of conceptual understanding as a key component of mathematical expertise. His argument is that “novices need clear instruction and simplified/scaffolded learning” (Willingham, 2010:5) and adds to this by suggesting that “concepts cannot be poured into a student’s head but rather that they need to be built upon something that they already know” (Willingham, 2010:5). Vygotsky’s Social Development Theory, in particular, magnifies the point that “cognitive development stems from the social interactions from guided learning within the zone of proximal development as children and their partners co-construct knowledge” (Amineh & Asl, 2015:9). The screencasting intervention can be used as and when necessary, the benefit of this being that the pedagogy will stay the same with the added advantage of its being able to be played back. Amineh and Asl (2015:9) maintain that the environment to which children are exposed also has an impact on “how and what they think”. Vygotsky (1978) suggests that children are curious and actively involved in their own learning and the discovery and development of new understanding/schema, but his main emphasis for cognitive development relies upon the social contributions that are made to the process of learning.

Through this social interactive perspective, and with respect to social constructivism, Vygotsky’s two important constructs, namely the More Knowledgeable Other (MKO) and the Zone of Proximal Development, play a vital role. The ‘More Knowledgeable Other’ (MKO) states that “much important learning by the child occurs through social interaction with a skilful tutor”, to which he also refers “as cooperative and collaborative” (Vygotsky, 1978:86). The MKO is a person/teacher who is more knowledgeable or has a “better understanding or a higher ability level than the learner, with respect to a particular task, process, or concept” (Galloway, 2001) (as cited in the South African Journal of Education, 2013:6) . With respect to my use of electronic means to tutor and guide students via an intervention through the process of constructing knowledge, it is suggested that the “MKO must have (or be programmed with) more knowledge about the topic being learned than the learner does” (https://www.simplypsychology.org/vygotsky.html). This implies that the MKO need not be an adult or, necessarily, a teacher. Resources that students can relate to, such as electronic media or electronic devices, can also fulfil the requirements, provided these devices are programmed and designed to support the relevant epistemology.

Vygotsky's second principle, the Zone of Proximal Development, plays an integral part and "relates to what a child can achieve with guidance and encouragement from a skilled partner (teacher/tutor/peer)" (Vygotsky, 1978:86), which he summarises as "the area where the most sensitive instruction or guidance should be given - allowing the child to develop skills they will then use on their own - developing higher mental functions" (Vygotsky, 1978:86).

Given that children construct knowledge when they are actively and personally engaged in meaningful activities, as discussed by Garcai & Pacheco (2013) the use of visualisation in the screencasting allows the student to engage with the intervention on an interactive level which is supported by audio to teach the topic of early algebra and the notion of equality. The approach of employing both visual and audio is supported by research (Barron, 2003; Sugar, Brown, & Luterbach, 2010; Kizilcec, Papadopoulos, Sritanyaratana, 2014) in that the combination of the two is more advantageous than the visual approach alone. This allows the learner to see and hear what is being taught, and the advantage of referring back to a problem. The students who are exposed to the intervention will have the opportunity of engaging with one another through their social interactions by explaining and justifying their own thinking just as is suggested by Silver (1996). Through Vygotsky's sociocultural theory of learning, children come to know their world through various interactions that allow them to share knowledge and trigger reflection and assist in the process of acquiring "a certain level of understanding" (Vygotsky, 1978:86). Romberg (1993:102) add emphasis to the point that "students are seen as active constructors of knowledge in a social environment."

The significance of this research is for students to engage effectively with the transition from arithmetic to algebra. The intention is to understand what the disjuncture is and where it occurs in order to design a pedagogy that is able to deliver the epistemology of algebra to its relevant audience, without jeopardising the students' ability and confidence in mathematics.

2.6.2 Construction of Meaning

Berger (2015) poses the question, how do students, regardless of age or qualifications, "make personal meaning of a mathematical object presented in the form of a definition" and the extent to which this "is particularly relevant to the study of advanced mathematical thinking" Berger (2015:1). In this instance, the "advanced mathematical thinking" referred to includes the Grade 7 students for whom algebra is an advance upon their understanding at this. As cited in Berger (2015:15) Tall argues (1995) that "the learner is frequently expected to construct the properties of the object from the definition", and, "in many instances, neither diagrams nor exemplars of the mathematical object are presented alongside the definition; initial access to the mathematical object is through the various signs (such as symbols and words) of the mathematical definition."

At the Canadian Mathematics Education Study Group at the University of Waterloo in 1984 in a discussion he led, Alan Bishop referred to "a reworking of spatial ability that was made possible by analysing the distinction between the ability to Interpret Figural Information of the many figural forms we use in mathematics, and the ability for Visual Processing" (Bishop, 1983:177). From the perspective of visual processing, and there are many of these, some teachers and students prefer to do it and some do not (Krutetskii, 1976), and, wherever their preference lies, it certainly "links with ideas of intuition and imagery, and can also relate to the use of analogy and metaphor" (Bishop, 1985:2). He wonders, initially, how best imagery can be shared between teacher and student, and whether the use of diagrams and figures are important for this (Bishop, 1985). For Kent and Hedger (1980) language is an imperative and has a large part to play in this process "because imagery can be brought to life by the appropriate and relevant language and examples" (as cited in Clements, 1982:33). Bishop (1980) saw the classroom becoming a challenging space for assisting the process of constructing meaning, given its disruptive atmosphere which many believe has led to a decline in learning (Bishop, 1980). It was argued that classrooms need to be more controlled in order to ensure that appropriate learning is done, and should be led by authoritative and knowledgeable teachers. So, what constitutes the "ideal" lesson (Good & Gouws, 1979) and how is the meaning of a concept best constructed? Is it by means of individualised schemes based on programmed instruction? In this context and according to Vygotsky, this cannot be the answer as it "removes the teacher as a helper and an authority to that of administrator, marker and paper producer" (Bishop, 1985:2). According to Bishop (1985) the danger of this situation is that the higher the cognitive requirement in order to solve complex problems the more necessary an authority becomes, or as Vygotsky would name it The More Knowledgeable Other. Bishop's orientation of controlling the learning situation as a social construction has a number of features, a few of which are highlighted with specific reference to this project:

- It places the teacher and his or her knowledge in relation to the candidates;
- It emphasise the dynamic and interactive nature of teaching, utilising ICT;
- It assumes the interpersonal nature of teaching because the teacher is working with students providing the opportunity to revisit the concept at their convenience, and as

often as they choose;

- It recognises the "shared" idea of knowing and knowledge, reflecting both the content and context;
- It takes into account the student's existing knowledge, abilities, and feelings, emphasising a developmental rather than a learning theoretical approach;
- It emphasizes developing mathematical meaning as a general aim of mathematics teaching, including both cognitive and affective goals;
- It recognises the existence of many methods

Bishop (1985:3)

With reference to The Zone of Proximal Development, the teacher will deliver a new concept, the "meaning maker" (Bishop, 1985:26), which the student will interpret according to his or her existing knowledge. Bishop's argument is that "no two people will have the same sets of connections and meanings, and in particular teacher and student will have very different meanings associated with mathematics; however, the teacher - the More Knowledgeable Other - will know the ideas she is teaching in terms of the connections they make with the rest of the mathematical knowledge" (1985:3).

> *The real problem which confronts mathematics teaching is not that of rigour, but the problem of the development of meaning, of the existence of mathematical objects.*
>
> Thom (1973:202)

For Bishop the goal for education in mathematics is "that of sharing and developing mathematical meaning" (1985:3) and, to demonstrate his point he suggested three ideas on which his analysis is focused (1985:4), namely:

1. Mathematical activities

   This is to ensure that students "grapple and experience" mathematics so as to develop agility for numbers as opposed to the teacher's simply plying them with content only.
2. Communication

   Traditionally mathematics classes are where you "do" maths as opposed to the place in which meaning and understanding concepts are discussed. Exposing ideas allows them to be shared. Using symbolism is also important, as is the use of diagrams for conveying images, examples from different contexts, analogies, and metaphors.
3. Negotiation

> This is a means of developing meanings through the teacher, the More Knowledgeable Other, who has been given the authority to deliver the pedagogy; and also of how teachers can encourage students to play a role in the development of their own mathematical meanings.

It is natural to believe that a student's viewing of screencasts is not a participation in a social context as is ordinarily understood by the definition of "social" but Berger suggests that (2015:15) "it is important to note that a focus on the individual (possibly with a textbook or lecture notes) does not contradict the fundamental Vygotskian notion that social relations or relations among people genetically underlie all higher functions and their relationships. After all, a situation consisting of a learner with a text is necessarily social; the textbook or exercises have been written by an expert (and can be regarded as a reification of the expert's ideas); also, the text may have been prescribed by the lecturer with pedagogic intent. Thus, a focus on the individual does not in any way undermine the significance of the social" (Vygotsky, 1981:163). As the producer of the screencasts, I may be viewed as the "'expert'" as per Berger's proposition, while the "textbooks" or "lectures" to which she refers are the screencasts I have developed. Her argument against neo-Piagetian theories is "that they are rooted in a framework in which conceptual understanding is regarded as deriving from largely interiorized actions; the crucial role of language (or signs) and the role of social regulation and the social constitution of the body of mathematical knowledge is not integrated into the theoretical framework" (Berger, 2015:15). She maintains that mathematical learning is "by its very nature a social activity, mediated and constituted by language, signs and tools (i.e.: textbooks or screencasts in this instance)" (Berger, 2015:15) and, hence, "meaning, thinking and reasoning need to be seen as products of social activity" (as cited by Berger, 2015 from Lerman, 2000:23). This framework allows for an individual to construct a concept through "socially sanctioned knowledge (existing in the community of mathematicians and reified in textbooks, screencasts) to be linked. Van der Veer and Valsiner are cited in Berger as arguing that the majority of mathematics research that is based on the Vygotskian framework has focused on "groups of students or a dyad rather than the individual" (Veer &Valsiner, 1994, as cited in Berger (2015:3), that the use of his framework has been selective and, in fact, "the focus on the individual developing person has been persistently overlooked" (Veer &Valsiner, 1994:6).

Taken from Berger's understanding (2015) of how a student constructs a new mathematical concept, the construction of meaning is "based on Vygotsky's theory of how a child learns the meaning of a new word, which he regards as embodying a new concept" (Berger, 2015:3). It is suggested that the child uses new words without a real understanding or conception of these words which may "serve as meanings of communication long before they reach the level of concepts characteristic of fully developed thought" (Uznadze, cited in Vygotsky, 1934/1086:101). It may be similar for mathematics in that the description or sign that is used to refer to an object is made meaningful to the student through communication with peers, with teachers, or when writing, using new concepts of the new mathematical objects before he or she "has full comprehension of the mathematical object. It is this usage of the mathematical signs, with the accompanying communication, that gives initial access to the new object" (Berger, 2015:4). Her argument is that in "mathematics, a student is expected to construct a concept whose use and meaning is compatible with its use in the mathematics community" (Berger, 2015:4). This is achieved by the student by using "the mathematical signs in communication with more knowledgeable others", or, in this instance, screencasts, "which, through its usage, is socially regulated (via the interaction with a text or others, such as the teacher and screencasts), the meaning of a concept can evolve for a student in a way that is compatible with its culturally accepted usage" (Berger, 2004a, as cited in Berger, 2015:4). Dorfler (2000) describes this functional use argument by implying that to secure the acquisition of a a new mathematical object, the student must be willing to involve him or herself, through participation, "as-if" the discussion "is meaningful and coherent, even if he or she does not experience it as such" (Dorlfer, 2000, cited in Berger, 2015:4). This argument is supported by Pimm (1987) and his suggestion regarding "the importance of students' mathematical talk, no matter the impreciseness of this talk" (Pimm, 1987, as cited in Berger, 2015:4), for:

> *Once something is expressed, however haltingly and incompletely, then questions can be asked about the current formulation in order to encourage refinement, precision and clarity.*
>
> (Pimm, 1987:31)

2.6.3 Conceptual Understanding in Algebra

To counter the many texts that assert that "mathematics students tend to prefer to think algorithmically rather than visually" (Clements, 2014:181), one needs to consider the statements made by Hilbert who spoke of two tendencies in mathematics, to "crystallize logical relations and the need to develop an intuitive understanding, especially through 'visual imagination" (as cited in Zimmerman & Cunningham, 1990:2). The construct, "concept image", developed by Tall & Vinner (1981; 1983), as cited in Presmeg (2006:25), is a way of understanding a mathematical concept and describes the notion of visualisation as "the total concept structure that is associated with the concept, which includes all the mental pictures and associated properties and is built on years of experience, of all kinds, changing as the individual meets new stimuli and matures. Cognitive structures are the mental structures, tools and patterns of thought and the processes used to make sense of information" (Tall &Vinner, 1981:152).

Willingham (2009-2010) indicates that "concepts cannot be poured into a student's head but rather that they need to be built upon something that they already know". He mentions the fact that "the understanding of algebraic equations depends on the right conceptual understanding of the equal sign". The "equal" sign is an important concept for teachers at the kindergarten level to teach accurately and effectively, allowing for the exploration of to what the symbol is intended to refer. Children's understanding of equality is at the root of the algebraic discovery and appropriation and it is imperative that they understand the concept of an equation and its representation in symbolic language of the left-hand side = right-hand side from as early as possible.

2.7 SOUTH AFRICAN CURRICULUM: SENIOR PRIMARY AND SENIOR SECONDARY ALGEBRA

The current South African curriculum is prescribed by The National Curriculum Statement Grades R - 12 (2012). It represents a policy statement for learning and teaching in South African schools and comprises the following:

- Curriculum and Assessment Policy Statements (CAPS) for each approved school subject
- The policy document, National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R-12, and
- The policy document, National Protocol for Assessment Grades R-12 (January 2012).

The policy statements are comprehensive and have compared favourably in quality, breadth, and depth to mathematics curricula in countries all over the world. Skills that have been earmarked to do so are, but not limited to are:

- problem-solving;
- mathematical reasoning;

- logical reasoning, and;
- cognitive flexibility

The time and content of algebra have been highlighted in both the Senior Phase and the Further Education Training band (Grades 10 - 12) as an indicator of a student's mathematical journey through the South African school curriculum. More specifically, attention has been drawn to the importance of algebra in the curriculum and the prominent position it holds. The screencast interventions will be used to introduce the Grade 7 participants to early algebra, concentrating on linear algebra that will include solving for an unknown variable.

2.7.1 Senior Phase: Grades 7 - 9

Instructional Time

Mathematics in the Senior Phase (Grades 7 - 9) has been allocated the following instructional time and, for comparative purposes, other subjects are included in the Table below:

Table 2.2: Instructional Time per week (Grades 7 - 9)

SUBJECT	HOURS
Home Language	5
First Additional Language	4
Mathematics	4,5
Natural Sciences	3
Social Sciences	3
Technology	2
Economic Management Sciences	2
Life Orientation	2
Creative Arts	2
TOTAL	27,5

Focus of Content Areas

As can be seen from Table 2.2 Mathematics in the Senior Phase covers five main content areas. This table implies a general focus of each of the areas and how "each content area contributes towards the acquisition of specific skills" (CAPS document, 2012):

Table 2.3: Mathematics Content Knowledge (Grades 7 - 9)

MATHEMATICS CONTENT KNOWLEDGE		
Content area	**General content focus**	**Senior Phase specific content focus**
Numbers, Operations and Relationships	Development of number sense that includes: • the meaning of different kinds of numbers • relationship between different kinds of numbers • the relative size of different numbers • representation of numbers in various ways • the effect of operating with numbers • the ability to estimate and check solutions.	• Representation of numbers in a variety of ways and moving flexibly between representations • Recognising and using properties of operations with different number systems • Solving a variety of problems, using an increased range of numbers and the ability to perform multiple operations correctly and fluently
Patterns, Functions and Algebra	Algebra is the language for investigating and communicating most of Mathematics and can be extended to the study of functions and other relationships between variables. A central part of this content area is for the learner to achieve efficient manipulative skills in the use of algebra. It also focuses on the: • description of patterns and relationships through the use of symbolic expressions, graphs and tables; and • identification and analysis of regularities and change in patterns, and relationships that enable learners to make predictions and solve problems.	• Investigation of numerical and geometric patterns to establish the relationships between variables • Expressing rules governing patterns in algebraic language or symbols • Developing algebraic manipulative skills that recognize the equivalence between different representations of the same relationship • Analysis of situations in a variety of contexts in order to make sense of them • Representation and description of situations in algebraic language, formulae, expressions, equations and graphs
Space and Shape (Geometry)	The study of Space and Shape improves understanding and appreciation of the pattern, precision, achievement and beauty in natural and cultural forms. It focuses on the properties, relationships, orientations, positions and transformations of two-dimensional shapes and three-dimensional objects.	• Drawing and constructing a wide range of geometric figures and solids using appropriate geometric instruments • Developing an appreciation for the use of constructions to investigate the properties of geometric figures and solids • Developing clear and more precise descriptions and classification categories of geometric figures and solids • Solving a variety of geometric problems drawing on known properties of geometric figures and solids
Measurement	Measurement focuses on the selection and use of appropriate units, instruments and formulae to quantify characteristics of events, shapes, objects and the environment. It relates directly to the learner's scientific, technological and economic worlds, enabling the learner to • make sensible estimates; and • be alert to the reasonableness of measurements and results.	• Using formulae for measuring area, perimeter, surface area and volume of geometric figures and solids • Selecting and converting between appropriate units of measurement • Using the Theorem of Pythagoras to solve problems involving right-angled triangles
Data Handling	Data Handling involves asking questions and finding answers in order to describe events and the social, technological and economic environment. Through the study of data handling, the learner develops the skills to collect, organize, represent, Interpret, analyse and report data. The study of probability enables the learner to develop skills and techniques for making informed predictions, and describing randomness and uncertainty.	• Posing of questions for investigation • Collecting, summarizing, representing and critically analysing data in order to interpret, report and make predictions about situations • Probability of outcomes include both single and compound events and their relative frequency in simple experiments

Weighting of Content Areas

The two primary purposes for weighting these various areas in the Mathematics curriculum are:

1. Time needed adequately to cover the content in each area; and
2. The spread of context for assessment purposes (end-of-year summative assessment, particularly)

Table 2.4: Weighting of Content areas (Grades 7- 9)

WEIGHTING OF CONTENT AREAS			
Content Area	**Grade 7**	**Grade 8**	**Grade 9**
Number, Operations and Relations	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data Handling	10%	10%	10%
	100%	**100%**	**100%**

From Table 2.5(a), the following three tables, Grades 7 - 9, indicate more accurately what the percentage breakdown equates to in actual teaching time in hours per Term:

Table 2.5(a): Time Allocation per Topic for Grade 7

TIME ALLOCATION PER TOPIC: GRADE 7							
TERM 1		**TERM 2**		**TERM 3**		**TERM 4**	
Topic	**Time**	**Topic**	**Time**	**Topic**	**Time**	**Topic**	**Time**
Whole numbers	**9 hours**	Common fractions	**9 hours**	Numeric and geometric patterns	**6 hours**	Integers	**9 hours**
Exponents	**9 hours**	Decimal fractions	**9 hours**	Functions and relationships	**3 hours**	Numeric and geometric patterns	**3 hours**
Construction of Geometric figures	**10 hours**	Functions and relationships	**3 hours**	Algebraic expressions	**3 hours**	Functions and relationships	**3 hours**
Geometry of 2D shapes	**10 hours**	Area and perimeter of 2D shapes	**7 hours**	Algebraic equations	**3 hours**	Algebraic expressions	**3 hours**
Geometry of straight lines	**2 hours**	Surface area and Volume of 3D objects	**8 hours**	Graphs	**6 hours**	Algebraic equations	**4 hours**
				Transformation geometry	**9 hours**	Collect, organize and summarize data	**4 hours**
				Geometry of 3D objects	**9 hours**	Represent data	**3 hours**
						Interpret, analyse and report data	**3,5 hours**
						Probability	**4,5 hours**
Revision/ assessment	**5 hours**	**Revision/ assessment**	**9 hours**	**Revision/ assessment**	**6 hours**	**Revision/ assessment**	**8hours**
TOTAL: 45 hours		**TOTAL: 45 hours**		**TOTAL: 45 hours**		**TOTAL: 45 hours**	

Table 2.5(b): Time Allocation per Topic for Grade 8

TIME ALLOCATION PER TERM: GRADE 8							
TERM 1		TERM 2		TERM 3		TERM 4	
Topic	Time	Topic	Time	Topic	Time	Topic	Time
Whole numbers	6 hours	Algebraic expressions	9 hours	Common fractions	7 hours	Functions and relationships	6 hours
Integers	9 hours	Algebraic equations	3 hours	Decimal fractions	6 hours	Algebraic equations	3 hours
Exponents	9 hours	Construction of Geometric figures	8 hours	Theorem of Pythagoras	5 hours	Graphs	9 hours
Numeric and geometric patterns	4,5 hours	Geometry of 2D shapes	8 hours	Area and perimeter of 2D shapes	5 hours	Transformation geometry	6 hours
Functions and relationships	3 hours	Geometry of straight lines	9 hours	Surface area and volume of 3D objects	5 hours	Geometry of 3D objects	7 hours
Algebraic expressions	4,5 hours			Collect, organize and summarize data	4 hours	Probability	4,5 hours
Algebraic equations	3 hours			Represent data	3 hours		
				Interpret, analyse and report data	3,5 hours		
Revision/ Assessment	6 hours	**Revision/ Assessment**	8 hours	**Revision/ Assessment**	6,5 hours	**Revision/ Assessment**	9,5 hours
TOTAL: 45 hours		TOTAL: 45 hours		TOTAL: 45 hours		TOTAL: 45 hours	

Table 2.5(c): Time Allocation per Topic for Grade 9

TIME ALLOCATION PER TERM: GRADE 9							
TERM 1		TERM 2		TERM 3		TERM 4	
Topic	Time	Topic	Time	Topic	Time	Topic	Time
Whole numbers	4,5 hours	Construction of geometric figures	9 hours	Functions and relationships	5 hours	Transformation geometry	9 hours
Integers	4,5 hours	Geometry of 2D shapes	9 hours	Algebraic expressions	9 hours	Geometry of 3D objects	9 hours
Common fractions	4,5 hours	Geometry of straight-lines	9 hours	Algebraic equations	9 hours	Collect, organize and summarize data	4 hours
Decimal fractions	4,5 hours	Theorem of Pythagoras	5 hours	Graphs	12 hours	Represent data	3 hours
Exponents	5 hours	Area and Perimeter of 2D shapes	5 hours	Surface Area and volume of 3D objects	5 hours	Interpret, analyse and report data	3,5 hours
Numeric and geometric patterns	4,5 hours					Probability	4,5 hours
Functions and relationships	4 hours						
Algebraic expressions	4,5 hours						
Algebraic equations	4 hours						
Revision/ Assessment	5 hours	**Revision/ Assessment**	8 hours	**Revision/ Assessment**	5 hours	**Revision/ Assessment**	12 hours
TOTAL: 45 hours		TOTAL: 45 hours		TOTAL: 45 hours		TOTAL: 45 hours	

Specification of Content

The intention of the specification of content is to show the progression in terms of concepts and skills from Grades 7 to 9 for each of those areas. There is a similarity in the concepts and skills in a number of the topics across the three grades, so it is recommended that "The Clarification of Content" be read in conjunction with "The Specification of Content" so as to understand how progression should be followed in each grade.

Table 2.6: Algebraic Expressions (Grades 7 - 9

TOPICS	GRADE 7	GRADE 8	GRADE 9
2.3 **Algebraic expressions**	**Algebraic language** • Recognize and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae and/or equations	**Algebraic language** • Revise the following done in Grade 7: - recognize and interpret rules or relationships represented in symbolic form - identify variables and constants in given formulae and/or equations • Recognize and identify conventions for writing algebraic expressions • Identify and classify like and unlike terms in algebraic expressions • Recognize and identify coeficients and exponents in algebraic expressions **Expand and simplify algebraic expressions** Use commutative, associative and distributive laws for rational numbers and laws of exponents to: • add and subtract like terms in algebraic expressions • multiply integers and monomials by: - monomials - binomials - trinomials • divide the following by integers or monomials: - Monomials - Binomials - trinomials • simplify algebraic expressions involving the above operations • Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms • Determine the numerical value of algebraic expressions by substitution	**Algebraic language** • Revise the following done in Grade 8: - recognize and identify conventions for writing algebraic expressions - identify and classify like and unlike terms in algebraic expressions - recognize and identify coefficients and exponents in algebraic expressions • Recognize and differentiate between monomials, binomials and trinomials **Expand and simplify algebraic languages** • Revise the following done in Grade 8, using the commutative, associative and distributive laws for rational numbers and laws of exponents to: - add and subtract like terms in algebraic expressions - multiply integers and monomials by: ♦ monomials, binomials, trinomials - divide the following by integers or monomials: ♦ monomials, binomials, trinomials - simplify algebraic expressions involving the above operations - Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms - Determine the numerical value of algebraic expressions by substitution • Extend the above algebraic manipulations to include: - Multiply integers and monomials by polynomials - Divide polynomials by integers or monomials - The product of two binomials - The square of a binomial **Factorize algebraic expressions** • Factorize algebraic expressions that involve: - common factors - difference of two squares - trinomials of the form: ♦ $x^2 + bx + c$; $ax^2 + bx + c$, where a is a common factor. • Simplify algebraic expressions that involve the above factorisation processes. - Simplify algebraic fractions using factorisation.

Table 2.7: Algebraic Equations (Grades 7 - 9)

TOPICS	GRADE 7	GRADE 8	GRADE 9
2.4 **Algebraic equations**	**Number sentences** • Write number sentences to describe problem situations • Analyse and interpret number sentences that describe a given situation • Solve and complete number sentences by: - inspection - trial and improvement • Determine the numerical value of an expression by substitution. • Identify variables and constants in given formulae or equations	**Equations** • Revise the following done in Grade 7: - set up equations to describe problem situations - analyse and interpret equations that describe a given situation - solve equations by inspection - determine the numerical value of an expression by substitution. - identify variables and constants in given formulae or equations • Use substitution in equations to generate tables of ordered pairs • Extend solving equations to include: - using additive and multiplicative inverses - using laws of exponents	**Equations** • Revise the following done in Grade 8: - set up equations to describe problem situations - analyse and interpret equations that describe a given situation - solve equations by inspection - using additive and multiplicative inverses - using laws of exponents - determine the numerical value of an expression by substitution. - use substitution in equations to generate tables of ordered pairs • Extend solving equations to include: - using factorisation - equations of the form: a product of factors = 0

2.8 CONCLUSION

The topic of algebra has an enduring effect on students' confidence, more often than not, negatively so. Unfortunately, this occurs prior to their attempting to discover the real value of its contents and inner workings, and leads to a dislike and a lack of confidence in the subject of Mathematics. Research focusing on early algebra and specifically on Grade 7 is not readily accessible. The focus of this study is geared towards algebra as a general topic at Secondary and early Tertiary education levels. The key areas arising from the literature review include the notion of equality, the transformation of numbers and their operations across the equal sign, the use of letters as unknown variables which have been termed "alphanumeracy" and "symbol sense", the bringing of algebra to life through visualisation, Visualisation as a mathematical pedagogy, and the role of technology in education and mathematics, with a particular use of screencasts as a mode of intervention. The key areas that have been highlighted in the literature will be used to inform the focal point and methodology of the present study while providing a relevant understanding of the processes of analysis and interpretation.

Central to this research is the notion of social constructivism which suggests how students build algebraic knowledge and construct meaning for the nuances involved in solving equations. In this case study the notion of equality, transformation, "alphanumeracy", and "symbol sense" is presented through a visual median intervention known as screencasts. The construction of these video clips was done with the intention of building knowledge by means of audio as language while enabling the simultaneous watching of the script as it unfolds.

Visualisation has been implemented in this research in order to determine how algebra can be better understood than it is at the most crucial stage of a learner's mathematical advancement. Although it could be deemed a challenge to deliver a visual pedagogy involving early algebra, this investigation presents the potential to broaden the opportunities that are available to assist the process of accurate and relevant delivery of the conceptual and procedural skills required at this level. From a theoretical perspective the methodology used to capture the processing capabilities of the students needed to be sensitive to the combination of their visual-logical/visual-pictorial and audio-visual abilities. It becomes imperative, therefore, that the methodologies involved in the data capture and data analysis are informed by the role of visualisation in early algebra.

The notion of equality, transformation, and "alphanumeracy" or "symbol sense" are integral to the conceptualisation of early algebra. To illustrate their relationship in the acquisition of early algebra and the understanding of the broader research topic, the activities included in this study are those of both a numeric and an algebraic nature. Their inclusion was purposeful so as to allow for a possible visual connection between arithmetic and early algebra, and the idea of equality by way of an animated scale. The focus of this study is underpinned by the idea that the construction of and building on prior knowledge, as well as the role of language and communication, will allow children to "consider everything alive and animate" (K Smith, 2008).

The ideas that have been presented in the literature review were used to inform both the topic and the methodology used in this study while providing the background to the processes of analysis and interpretation. The empirical evidence of how the theoretical framework is considered in the methodology of this study is described in the following chapter.

CHAPTER 3
METHODOLOGY

3.1 INTRODUCTION

> *Anyone who attempts to skip this problem, to jump over methodology in order to build some psychological science right away, will inevitably jump over his horse while trying to sit on it.*
>
> (Vygotsky, 1997:329)

The methodology of this research project is guided by Vygotsky's conviction that without a thoroughgoing methodology an undertaking such as this would "miss the horse". In order to expand on the methodology a mixed-method approach has been chosen after having given careful consideration to qualitative and quantitative methods. This allows for an effective process of triangulation so as to ensure reliable accuracy in terms of capturing and analysing the data most relevant to the research. It is essential that the essence of the screencast interventions be captured to demonstrate clearly how they support the acquisition of a conceptual understanding in the solving of algebraic tasks by a selected cohort of Grade 7 students.

3.2 ORIENTATION

The research is oriented towards the interpretive paradigm by using a predominantly qualitative approach; also included is an element of quantitative investigation with the provision of graphs and statistical representations. According to Enc (1999) "A 'paradigm' is a set of scientific and metaphysical beliefs that form a theoretical framework within which scientific theories can be tested, evaluated and if necessary, revised" (Photongsunan, 2010:1). Photongsunan (2010:2) does not see the social world as being detached from interaction but suggests, rather, that it is "constructed by human beings. There is a reliance or dependence on others in the field of the study", hence his belief that the researcher becomes a part of the research and interacts with his/her participants who would be attempting to construct meaning from the data. Cohen et al. (2001) refer to efforts to access the interior of the person to gain insight into his or her responses. The mixed-method design, based on a reliance on both the qualitative and quantitative collection approaches is useful. According to Heale and Forbes (2017), this is an approach that facilitates triangulation "where two or more methods are used" (2017:98), in this instance combining “both qualitative and quantitative methods to answer a specific research question, where converging results aim to increase the validity through verification, which can lead to better explanations for the phenomenon under investigation” (2017:98). In the context of this research and in an attempt to understand the participants’ concept knowledge of the topic, meaning needs to be made of what participants know and how their existing knowledge is implemented from within after the interventions have been experienced.

This project is based on how students build and construct knowledge, in this case, an understanding of equality in an algebraic context. For this reason this research's theoretical underpinnings locate themselves in social constructivism. A screencast intervention provides the scaffolding tool required to build on the students’ prior knowledge. The specific prior algebraic knowledge in this study instance would include the filling in of missing numbers in number sentences, a process which forms part of the Primary School curriculum (Department of Basic Education, Curriculum and Assessment Policy Statement, 2011:25). Through the use of screencast interventions, which Vygotsky (1978:86) would have referred to as the "tools of intellectual development" the role of the construction of new knowledge building on prior knowledge is undertaken. The intention of the pedagogy of the screencast interventions is to provide support for the cognitive development of the participants by signifying the role of language and communication, by means of audio and visual components of the screencasts. The intention of the screencast interventions is to build a strong understanding of the notion of algebraic equality which would be cemented through the role of the unknown variable in the algebraic calculations.

Of particular importance in the project is the analysis of the screencast interventions and the methods applied by the participants in order to solve for the unknown variables in each of the given algebraic equations so as to balance the equations. These solution strategies were important for the analysis of the data as providing details of the process of the epistemology and construction of the scaffolded knowledge.

3.3 METHODOLOGY

3.3.1 Case study

The form of the interventionist case study was chosen as being appropriate for the research which is contextualised and is up-close, in-depth, and detailed examination of a particular topic in mathematics. The data was collected from pre-tests, interviews, and talk-aloud tasks that the participants solved, as well as direct observations which according to Yin (1994) define, largely, what a case study intends. Gerring (2004:342) defines a case study as “an intensive study of a single unit for the purpose of understanding a larger class of (similar) units.” In this instance, the case was a cohort of Grade 7 students solving algebraic tasks and the unit of analysis was how the participants solved and interacted with algebraic equations (as a result of watching and engaging with my screencast interventions). The data collection tools generated depth and rich data. A mixed-method approach was used. The mixed-method approach assists in the analysis of data from different perspectives and provides the project with validity and reliability.

3.3.2 Participants

The research was conducted with seven Grade 7 students based at an independent school in the Eastern Cape, South Africa. It is a single-sex, all-boys’ primary school. The specific group was selected purposefully and was made up of three high, two high medium, and two low achievers, based on their diagnostic test performance (see 4.3.3. below). This method “is widely used in qualitative research for the identification and selection of information-rich cases related to the phenomenon of interest” (Palinkas et al., 2013:1). The phenomenon of interest, in this case, was to determine how algebraic tasks are solved by means of the introduction to screencast interventions, with the purpose of developing a conceptual understanding of the notion of equality. The participants were studied “in depth and studied as they naturally occur” (Denscombe, 1998:81) as the aim of the project was to “gain depth in” (Denscombe, 1998:81) this particular area of algebra when the methods of the participants’ calculations were analysed after the pre-intervention test. That students were a mixed-ability group would ensure a representative sample, and avoid bias and unnecessarily skewed results. Algebra is usually introduced at this level of the South African Curriculum and Assessment Policy Statements' (CAPS) Mathematics curriculum, with an emphasis on an increase in the algebraic content towards the latter half of the academic year. The policy at this particular primary school is not to introduce algebra formally in Grade 7, because many students only come to the school in the senior years, and arrive from schools from around the country in Grade 8; so, the formal teaching of algebra is delayed until that grade. Teachers do introduce the fundamentals of algebra in an informal way, but the selected group of participants were taught formal algebra when they took part in this project..

3.3.3 Research Design

The design of the research project followed three stages:

- Stage 1 – Pilot Study;
- Stage 2 - Diagnostic Test;
- Stage 3 - Screencast Interventions; and
- Stage 4 - Post-intervention Test

Stage 1 - Pilot Study

A pilot study was conducted prior to the collection of the formal data for the research project using seven mathematically mixed ability Grade 7 students. The intention of the pilot study was to ensure the integrity of the diagnostic test in terms of the flow of the test with regard to the clarity of the requirements and the time taken to complete the assessment. A full test paper was given to the participants to complete which meant that they had access to the numeric and algebraic equations. The pilot study was also used to address problem areas or potential problems that would have been unforeseen by testing the reliability of the research instruments. The participants who took part in the pilot study were not included in the research project.

Stage 2 - Diagnostic Test

A diagnostic pre-intervention pencil-and-paper test was conducted on solving basic algebraic equations in order to measure the levels of attainment of all Grade 7 classes in the school. Sixteen students were willing to participate and were tested. Questions were sourced from work that aligns with the Grade 7 curriculum and included basic arithmetic which then progressed into early linear algebra. The results of the pre-test, together with the results of the pilot study already conducted, were an indication of the material required to prepare the screencast interventions for the teaching of algebra. According to the Department of Basic Education's Curriculum and Assessment Policy Statements (2011), students are expected to be able to complete certain tasks on which the Diagnostic Test was based. The content used in this test was relevant to the grade and year.

Following on from the literature and the need to establish that the equal sign exhibits "equivalence between two sets of expressions, each one of which includes one or more operations within it" (Borenson, 2013:90), the test was designed to reflect "clusters" or a taxonomy of early algebraic equations, based on McNeil's and Alibali's (2005a) four different identified contexts (at Table 1). Each of the algebraic examples was arranged according to clusters (see page 7 of Chapter 2), modifying them from the numeric-only examples to include an unknown variable for algebraic purposes. These frameworks form the basis of the analytic instrument which would be used to analyse how the participants solve different linear equations in the clusters; this is because a large component of the research is to determine how students interpret the equal sign, particularly in Scenario 2 (see page 5), and the unknown variable; and how much this affects their understanding in solving for the unknown variable.

The test consists of two parts, Question 1 and Question 2. Question 1 includes examples used as a warm-up and consisting of numerical tasks only. They are reflected in Figure 4.1, below:

Question 1: Warm up

1. 20 + 40 = _____
2. 10 = _____ + 20 - 20
3. 300 ÷ 150 = _____
4. 7 + 15 = _____+ 12
5. _____+ 5 = 8 x 5
6. 6 + _____ = 72 ÷ 9

Figure 4.1: Question 1 Warm up

Question 2 (Refer to Appendix E for the Test paper) is the source of much of the analysis and the tasks reflect the various scenarios and clusters based on McNeil's and Alibali's (2005a) four different contexts. These scenarios were expanded upon to form the clusters in both the Numeric and Algebraic Tasks which informed the analytic framework. Each example from the clusters is arranged randomly in the test so as to avoid the potential identification of any patterns by the participants. These clusters were designed as depicted in Figure 4.2, in which are included actual questions from the test:

Observation Schedule to Question 2

NUMERIC TASKS	ALGEBRAIC TASKS
a) The operations equals answer context (eg. 5 + 4 = 9) ***Task 5*** Calculate: 65 - 44 = _____ ***Task 12*** Calculate: 25 x 4 =_____	a) The operations equals answer context (eg. 25 - b = 20) ***Task 1*** Find the value of c if 3c x 7 = 21 ***Task 6*** Find the value of a if 6a + 4 = 58
b) The operations on the right-hand side context (eg. 9 = 5 + 4) ***Task 14*** Calculate: _____ = 20 ÷ 20? ***Task 2*** Calculate: _____ = 9 + 91	b) The operations on the right-hand side context (eg. 25 = 2b + 1) ***Task 7*** Find the value of b if 28 = 3b + 4 ***Task 3*** Find the value of c if 125 = c + 77
c) The reflexive context (eg. 9 = 9) ***Task 15*** What is 9 = _____? ***Task 13*** Calculate: 15 + 10 = 10 + 15	c) The reflexive context (eg. b = b) ***Task 4*** Provide a real-life example that explains this scenario: a = a ***Task 10*** Provide a real-life example that best explains the following scenario: a + b = a + b
d) The operations on both sides context (eg. 5 + 4 = 6 + 3) ***Task 8*** Calculate: 10 x 3 = _____ ÷ 2 ***Task 11*** Calculate: _____ + 20 = 21 + 35	d) The operations on both sides context (eg. c + 10 = 20 - c) ***Task 9*** Find the value of a if 10 + 2a = 100 + 20 ***Task 16*** Find the value of b if 154 - 104 = 51b – 1

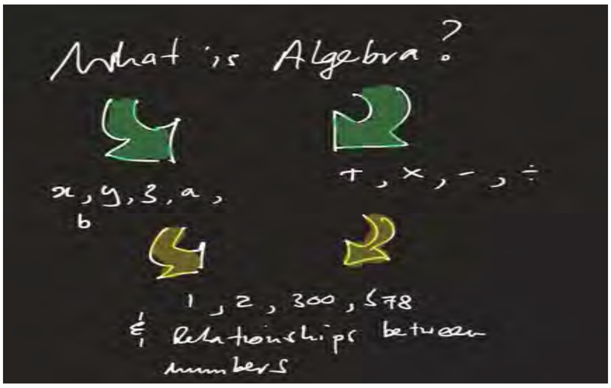
Figure 4.2: Clusters for Question 2

Stage 3 - Screencast Interventions

The visual engagement with algebraic concepts of participants was ensured through an Information and Communications Technology (ICT) intervention programme designed to deliver the pedagogy. It is well documented that technology can assist with the acquisition of knowledge (Henrie, Halverson, Graham, 2015; Jordan, Loch, Lowe, Meste l, & Wilkins, 2012; Faherty & Faherty, Ahmad, Doheny, & Harding, 2015), but the intention of this project is to determine not only that technology assists but how this occurs by means of a screencast intervention programme designed to solve early algebraic problems. The fundamental question of how Grade 7s solve algebraic equations as a result of this intervention is supported by the technology which is the means of delivery. A series of screencast interventions were implemented as in the form of a visual approach with the inclusion of audio; this reflects Vygotsky's social learning theory that "learning is a necessary and universal aspect of the process of developing culturally organized, specifically human psychological function" (1978:90) and is an explanation of "human behavior in terms of continuous reciprocal interaction between cognitive, behavioral, and environmental influences" (Mendoza et al., 2015:82). Bandura's theory expands on this arguing that "most human behaviour is learned observationally through modelling: from observing others, one forms an idea of how new behaviours are performed, and later on, this coded information serves as a guide for action" (1977:16). Screencast interventions provided the pedagogy for this research project and modelled the relevant examples. Without losing focus on the visual and auditory inputs Paivio's (1986) Dual Coding Theory (DCT) states that "meaningful learning occurs when students process information simultaneously through two discrete input channels, namely the visual and auditory channels" (as cited in Sugar et al., 2010:3). He refers to screencasts as being a "fusion of visual and audio elements" that "support the way the human brain learns, which is by making associations by what is being seen (visual stimuli) and heard (auditory stimuli)" (Faherty et al., 2015:12). This, he adds, is "what makes screencasts particularly beneficial to Maths learning" (Faherty et al., 2015:12).

Among the plethora of research around the topic and the learning progressions of early (core) algebra (Kaput et al., 2008) this research project concentrates on two of the five "big ideas" (Blanton et al., 2015:6) derived from Kaput's (2008) work "of generalizing, representing, justifying, and reasoning with mathematical relationships" (as cited in Blanton et al., 2015:6). The pre-test revealed that two of the "big ideas" that required reasonable focus included (a) equivalence, equations, and inequalities and (b) the variable. According to Kaput et al. (2008), equivalence, expressions, equations and inequalities refer, and are not limited to, inter alia, establishing a relational understanding of the equal sign, while the variable refers "to symbolic notation as a linguistic tool for representing mathematical ideas in succinct ways and includes the different roles variable plays in different mathematical contexts" (Blanton et al., 2015:6). Through the diagnostic (pre-intervention) test and pilot study, the areas that were highlighted as problematic were addressed so that the designs of the screencast interventions would include both basic algebraic skills and symbol sense.

The screencast interventions took place once the participants had been selected. The screencast support took the form of short video recordings that the participating students could access via their tablets or iPads and over the space of a week, the screencasts were released for them to watch in their own time. The tool of delivery was Google Classroom, where a 'classroom' was created for the participants, which enabled me to post the videos without any concerns about reliability. These recordings or screencasts support the visual delivery of the concept of algebra, equality, and the notion of equivalence, as well as the explanation of the unknown variable and its role. This programme was implemented over the course of sixteen screencasts which equated to a week, and took place in the afternoon after the completion of the normal school day. The screencasts were designed not to be too long, each being an average of between five and six minutes in length. Screenshots of each screencast intervention are shown in Figure 4.3 below with a brief description of each; they were designed, specifically, to follow a sequential and structured path showing how early algebra was unveiled.

<table>
<tr>
<td rowspan="2">Screencast 1 - Introduction to Algebra
The intention of this screencast was to introduce algebra to the participants by making them aware of the relationships between the operations, unknown variables, the notion of equality, and numbers. The animated scale indicates a balancing of the left- and right-hand sides of an equation, implying that what is done to one side is done to the other. Also included is an example of a basic equation.</td>
<td>
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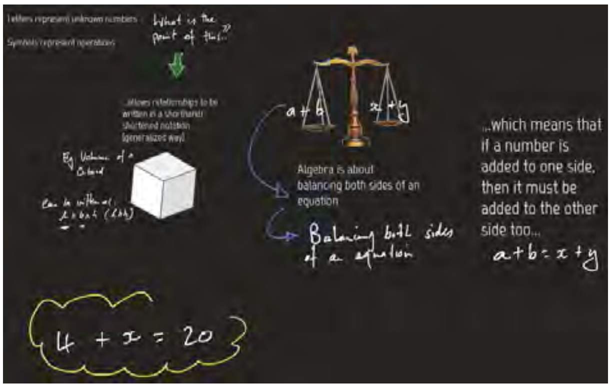
<td>
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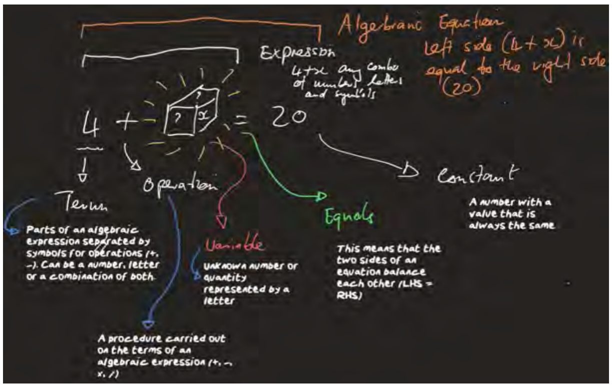
<tr>
<td>Screencast 2 - Deconstruction of an Algebraic Equation (1)
In this screencast the basic equation is used as an example in the first screencast and deconstructed to indicate each component's role in an equation.</td>
<td>
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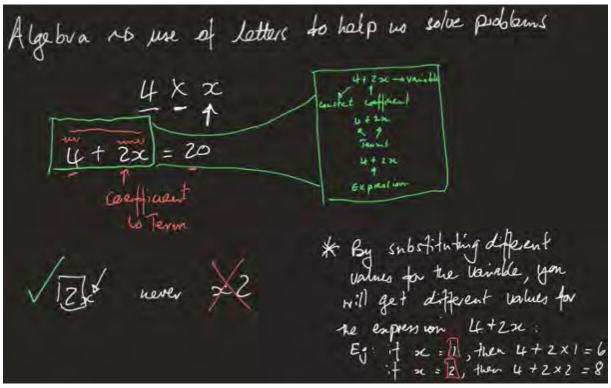
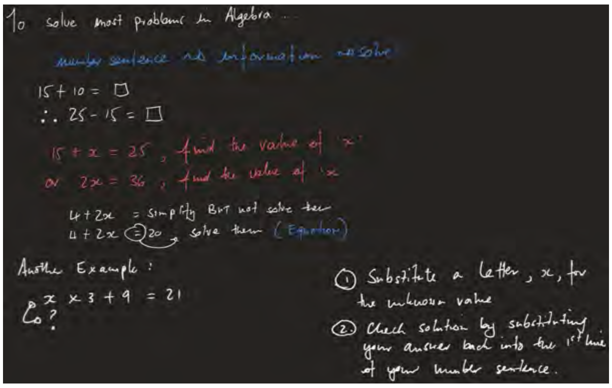
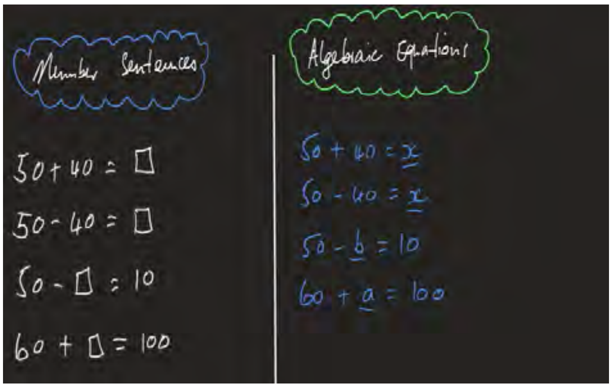
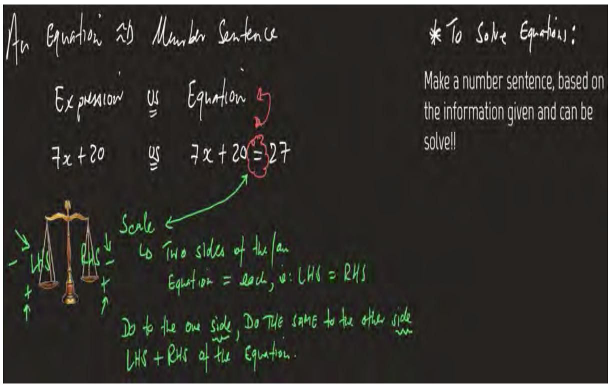
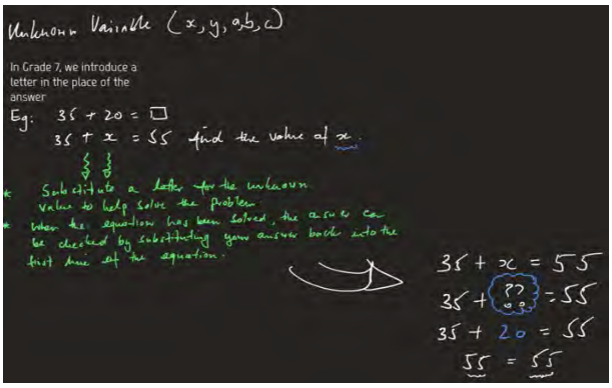
</table>

Screencast 3 - Deconstruction of an Algebraic Equation (2) As this is a continuation of the previous screencast it was necessary to elaborate on the letter as the unknown variable in the equation and the role it plays. It was discovered through the pilot study and pre-intervention test that the lack of symbol sense played as much of a significant part in the participants' limited knowledge of and confidence in algebra	
Screencast 4 - Introduction to Solving Algebraic Equations At this point, how one solves an equation is looked at, solving specifically for the letter, the unknown variable. A basic number sentence is introduced first as a way of easing participants into understanding the structure of algebraic equations and the fact that they are not dissimilar from numerical number sentences	
Screencast 5 - Difference between Number Sentences (Numeric only) and Early Algebraic Equations This screencast highlights the difference between numeric number sentences and early algebraic equations. This is an animated version similar to McNeil's and Alibali's (2005a) four contexts highlighted in Table 1.	
Screencast 6 - The Equal Sign It is at this point that I elaborate on the animated scale and the role that the equal sign ('=') plays in solving an equation. This is the first formal introduction to the notion of equality in the screencast interventions	

<table>
<tr>
<td>Screencast 7 - The Unknown Variable

As was discovered with the '=' sign and the limited understanding of the notion of equality in the more complex equations in this research project, so too is the participants' knowledge and understanding of the unknown variable. This screencast considers the unknown variable in depth and explains its role in equations</td>
<td>
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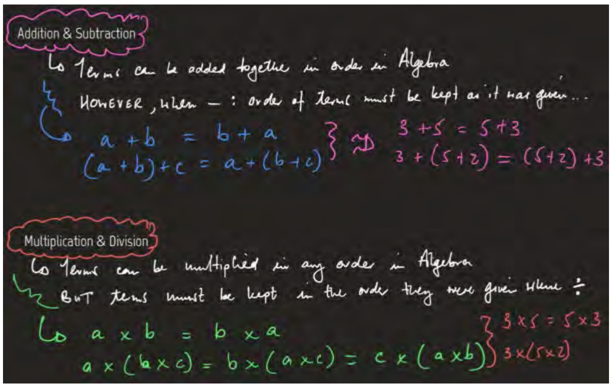
<tr>
<td>Screencast.8 - The Basic Rules of Algebra

This screencast elaborates on the rules of algebra, referring to the various operations (addition, subtraction, multiplication, and division). This was one of the longer screencasts, necessarily so to avoid the danger of providing too little information around the epistemology of the topic.</td>
<td>
</td>
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<tr>
<td rowspan="2">Screencast 9 - Changing the subject of an Equation

The following series of screencasts show how operations, numbers, and unknown variables are manipulated across the '=' sign in order to make the unknown variable the subject of the equation. The animated scale has been included to explain, visually, the balancing of the left-with the right-hand side of an equation.</td>
<td></td>
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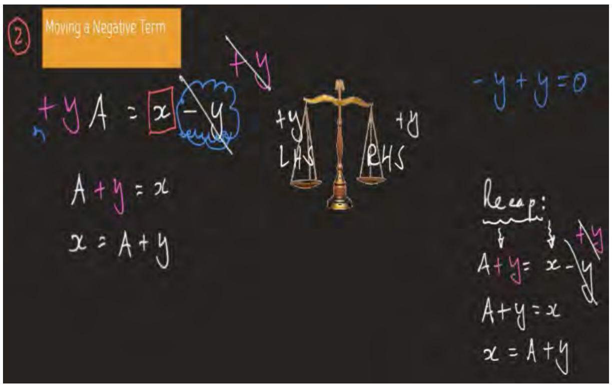
<td>
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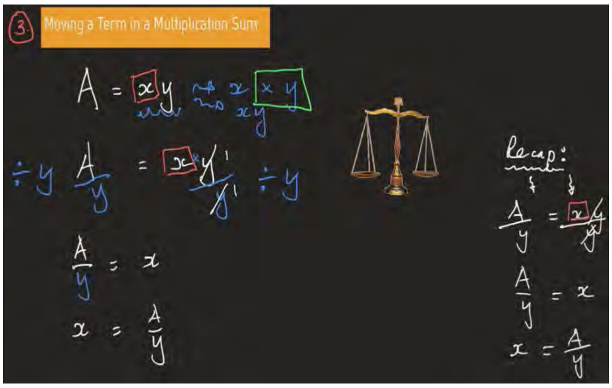
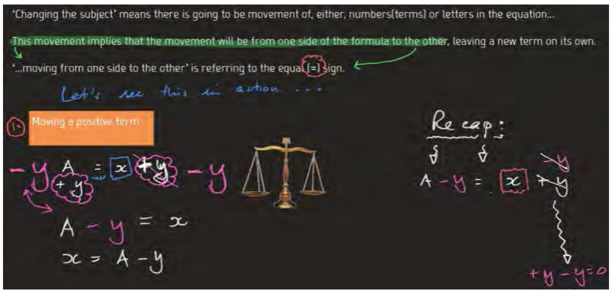
<tr><td></td><td>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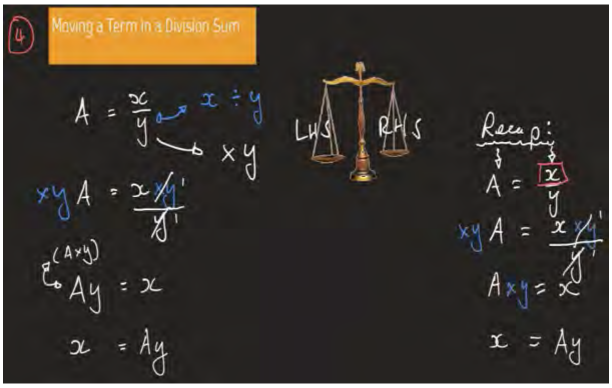
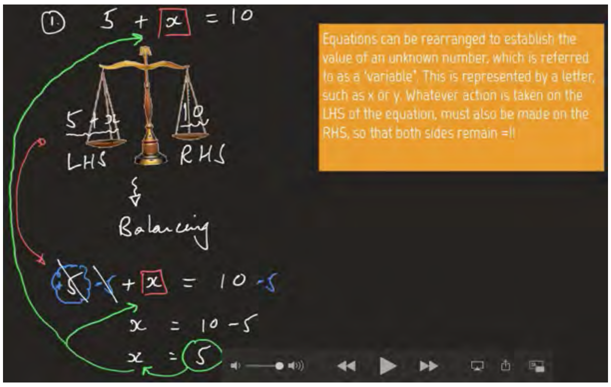
<tr><td></td><td>
</td></tr>
<tr><td>Screencast 13 - Examples of Solving Equations

In the following series of screencasts basic, early algebraic-type equations that are easy to follow serve as examples as they unfold for the participants on their devices. It covers what they would have seen in the previous series of screencasts and provides clarity when solving equations.</td><td>
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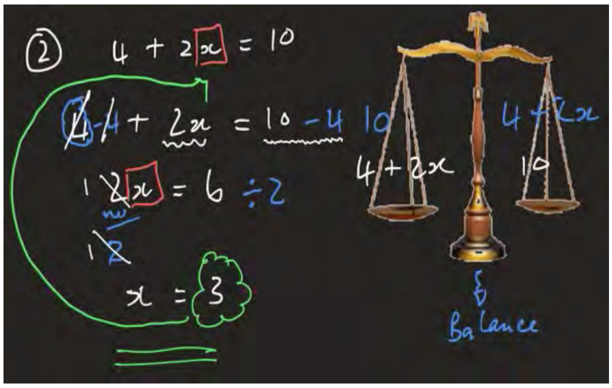
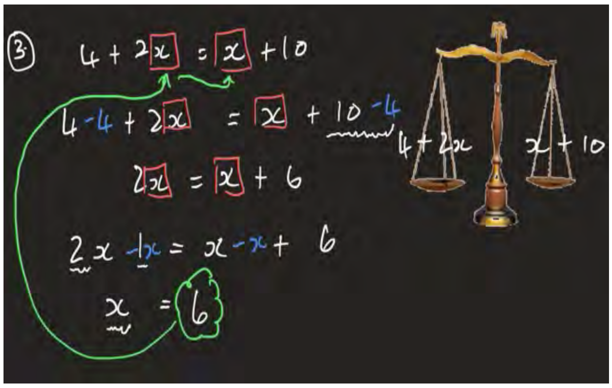
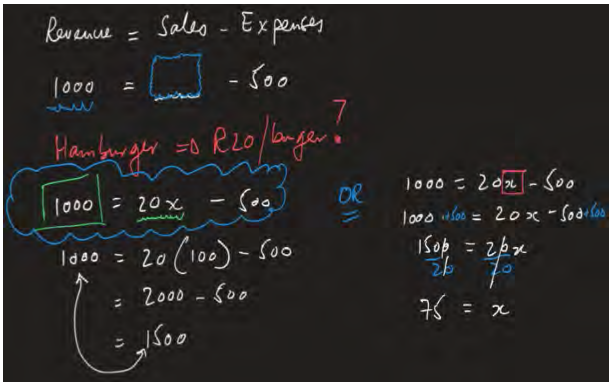
	
	
Screencast 14 - Creating an Equation In the final screencast an example of how to create an equation is provided by using a prosaic situation. The intention is to show the participants how algebra can be used by employing a relatively basic formula to work out revenues, sales, or expenses.	

Figure 4.3: Examples of the Screenshot Interventions

Stage 4 - Post-Intervention test

The main source of data for analysis purposes would take place at this stage of the project. The seven candidates were recorded individually via a video camera while completing the post-intervention test in my presence. In order to observe each of the participants' methods of calculation, a think-aloud interview was conducted with each participant to determine, among other questions, how each comprehended the meaning of the equal sign, and, secondly, how they calculated or manipulated the linear equations. The intention was to maintain a reflective journal that was to assist in "scrutinizing the experience, evaluating how to improve on it and linking theory with the reality of the exercise" (http://www.intranet.birmingham.ac.uk/as/libraryservices/library/asc/resources/a-short-guide-to-reflective-writing.aspx). Gibbs (1988:9) argues that "It is not sufficient to have an experience in order to learn. Without reflection on this experience, it may quickly be forgotten, or its learning potential lost." Another means of reflection would be to establish the advantages and disadvantages of the screencast interventions from the perspective of the student to assist in the process of determining the effectiveness of the screencast interventions and whether there is room for improvement.

3.3.4 Research Instruments

The main research instruments were the diagnostic test and observations in conjunction with a talk-aloud interview. Participants were video-recorded individually while completing the post-intervention test; they then engaged in individual semi-structured think-aloud interviews in order "to gain a deeper insight into how the participants' visualisation processes co-emerged with their reasoning processes" (Dongwi 2018:19). Over and above providing rich data, the different techniques sought to enhance the validity of the research for the purposes of triangulation, which is described by Cohen, Manion, and Morrison (2011:203) as "the use of two or more methods of data collection in the study of some aspect of human behaviour."

3.3.4.1 Observations

The seven participants were video-recorded individually, as they completed the "post-intervention test" which is the same as the initial diagnostic test of the linear algebraic problems. The video recorder was strategically positioned so as to capture the participants' hand working through the examples, and was positioned above and behind him to capture his engagement with the problems, both, aurally and physically. This maintained the anonymity of the participant.

Although the test followed a sequential numerical order, which was observed and video recorded, a structure to this research project was, advisedly, maintained by replicating the "clusters" which are explained in the previous chapter. Images are included under each of the clusters to display examples of various outcomes post the screencast intervention.

Cluster 1: Operations equals ('=') an answer context

For this, an "operations equals ('=') an answer context" was used for both the numeric and the algebraic tasks. Observation was concentrated, primarily, on the algebraic task in order to determine how participants solved for the unknown variables in Questions 1 and 6. In each of these questions participants were given the equation which included a subtle difference from the examples provided in the numeric tasks. Each of the equations contained a term that consisted of a coefficient and the unknown variable attached to it, namely: Question1, it was '3c' and Question 6, it was '6a'. For each scenario, these unknown variables needed to be solved in order to balance the equation. I included the pre- and post-intervention of the same participant for each question, as in Figure 4.4, in order to indicate the visual difference between their understanding once they had access to the screencast intervention.

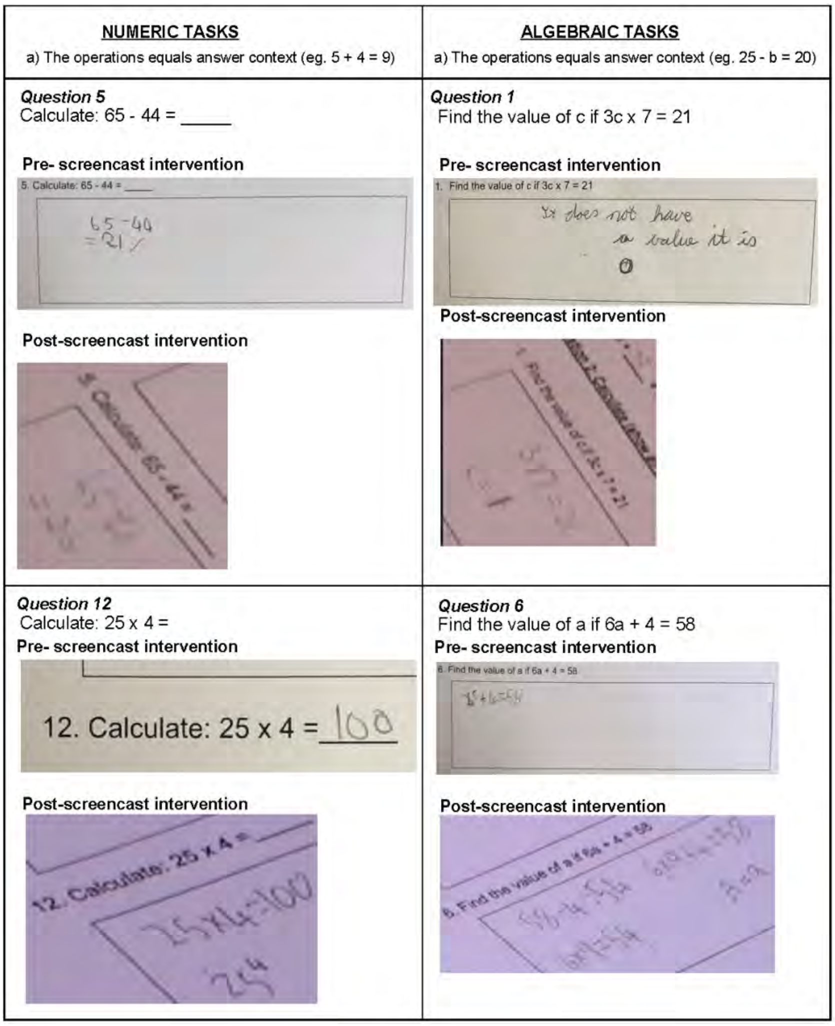


Figure 4.5: Cluster 1

Cluster 2: The operations on the right-hand side context

For this scenario an "operations on the right-hand side context" is used for both the numeric and the algebraic tasks. Observation was concentrated, primarily, on the algebraic task in order to determine how the participants solved for the unknown variables in Questions 7 and 3. For each question participants were given the equation which included a subtle difference from the examples provided in the numeric tasks. Each of the equations contained a term that consisted of a coefficient and the unknown variable attached to it, namely: Question 7, it was '3b' and Question 3, it was 'c'. For each scenario the unknown variables needed to be solved in order to balance the equation. The pre-and post-intervention of the same participant was included for each question, as in Figure 4.5, in order to indicate the visual difference between their understanding once they had access to the screencast intervention. Notably, in Question 3, due to no coefficient attached to the unknown variable, solving for it provided greater success even prior to the screencast intervention.

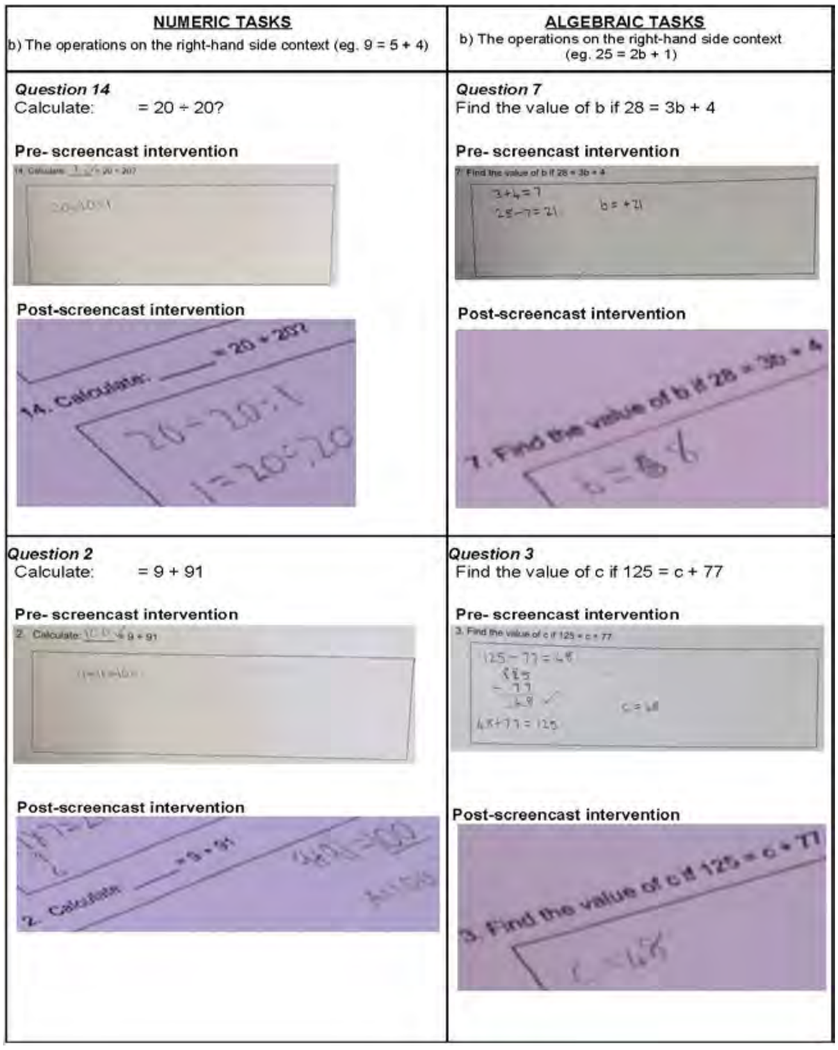


Figure 4.6: Cluster 2

Cluster 3: The reflexive context

For this a "reflexive context" is used for both the numeric and the algebraic tasks. Observation was concentrated on both the numeric and the algebraic task in order to determine how the participant solved for the missing number and unknown variables on either side of the '=' sign in each of the questions. These scenarios did not include any coefficients and participants were required to determine a way of balancing the left- and right-hand sides of the equations. Pre- and post-intervention of the same participant for each question was included, as per Figure 4.6, in order to indicate the visual difference between understanding once participants had access to the screencast intervention. An unknown variable on its own did not provide much difficulty in attaching a "value" to it. The numerical task did not prove to be problematic.

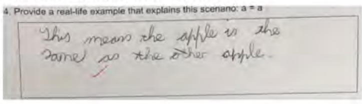
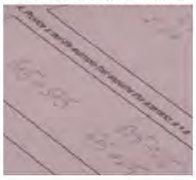
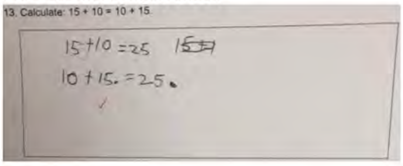
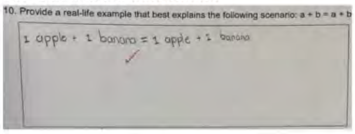
NUMERIC TASKS c) The reflexive context (eg. 9 = 9)	ALGEBRAIC TASKS c) The reflexive context (eg. b = b)
Question 15 What is 9 = ? **Pre- screencast intervention**  **Post-screencast intervention** 	*Question 4* Provide a real-life example that explains this scenario: a = a **Pre- screencast intervention**  **Post-screencast intervention** 
Question 13 Calculate: 15 + 10 = 10 + 15 **Pre- screencast intervention**  **Post-screencast intervention** 	*Question 10* Provide a real-life example that best explains the following scenario: a + b = a + b **Pre- screencast intervention**  **Post-screencast intervention** 

Figure 4.7: Cluster 3

Cluster 4: The operation on both sides' context

For this scenario an operation on both sides' context is employed for both the numeric and the algebraic tasks. Observation primarily concentrated on the algebraic task to determine how the participants solved for the unknown variables in Questions 9 and 16. In these questions participants were given the equation which included a subtle difference from the examples provided in the numeric tasks. Each equation contained a term that consisted of a coefficient and the unknown variable attached to it, namely: Question 9, it was ‘2a’ and Question 16, it was ‘51b’. For each scenario unknown variables needed to be solved in order to balance the equation. Pre-and post-intervention of the same participant was included for each question, as in Figure 4.8, to indicate the visual difference between their understanding once they had access to the screencast intervention.

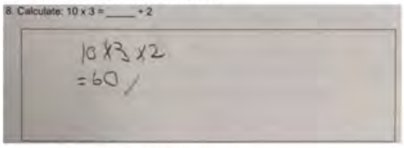
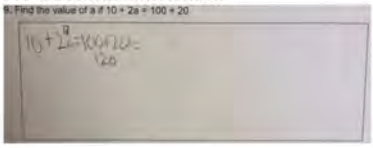
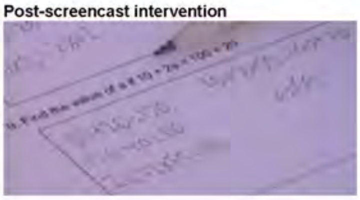
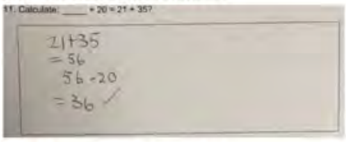
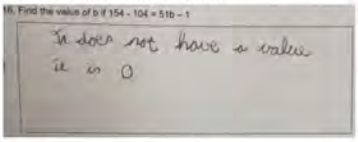
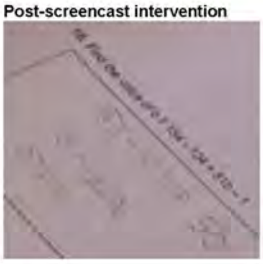
NUMERIC TASKS *d) The operations on both sides context (eg. 5 + 4 = 6 + 3)*	**ALGEBRAIC TASKS** *d) The operations on both sides context (eg. c + 10 = 20 - c)*
Question 8 Calculate: 10 x 3 = ÷ 2 **Pre- screencast intervention**  **Post-screencast intervention** 	***Question 9*** Find the value of a if 10 + 2a = 100 + 20 **Pre- screencast intervention**  **Post-screencast intervention** 
Question 11 Calculate: + 20 = 21 + 35 **Pre- screencast intervention**  **Post-screencast intervention** 	***Question 16*** Find the value of b if 154 - 104 = 51b – 1 **Pre- screencast intervention**  **Post-screencast intervention** 

Figure 4.8: Cluster 4

3.3.4.2 Think-aloud Interview

In conjunction with the completion of the post-intervention test with each of the seven participants, a think-aloud interview of a semi-structured nature (Cohen, 2001) was conducted and was necessary to establish how students comprehend the meaning of the equal (‘=’) sign, and how they calculate or manipulate linear algebraic equations. A significant insight into what participants' processes are, and the nature and extent of visualisation as an influence, was the motivation for this project; the screencast interventions demonstrated the conceptual understanding of the concomitant concern in this project and of how this intervention would facilitate an understanding of equality in solving algebraic equations. This took place as the post-intervention test was completed. These individual semi-structured think-aloud interviews provided "a deeper insight into how the participants' visualisation processes co-emerged with their reasoning processes" (Dongwi 2018:19). The questions were structured as follows:

1. "Briefly explain how you solved the algebraic equations."
2. "Why did you use the particular method you used to solve the equations in Question __?"
3. "Has the screencast assisted you in solving the equation?"
4. "What does the equal ('=') sign mean to you?"
5. "Why do we use letters in this equation?"
6. "Can you elaborate on and describe the advantages and disadvantages of the screencasts?"

These are similar questions to those used in the previous work examining students' understanding of the equal sign by McNeil and Alibali (2000, 2005a); and Rittle-Johnson and Alibali (1999). These questions enabled the extraction of relevant and rich data, reported in the "Analysis" component of this project. The advantages and disadvantages were established, thereby extracting the specific relevance and role of the screencast interventions from the students' perspectives; this supports the process of determining how effective the screencast interventions were, or could be.

Alongside the research process keeping a reflective journal assisted in the scrutiny of "scrutinizing the experience, evaluating how to improve on it and linking theory with the reality of the exercise" (http://www.intranet.birmingham.ac.uk/as/libraryservices/library/asc/resources/a-short-guide-to-reflective-writing.aspx). "It is not sufficient to have an experience in order to learn. Without reflecting on this experience, it may quickly be forgotten, or its learning potential lost" (Gibbs, 1988:0).

Other than providing rich data, the exploitation of different techniques enhanced the validity of the research and also informed triangulation, which according to Cohen, Manion, and

Morrison (2011:203) “is the use of two or more methods of data collection in the study of some aspect of human behaviour.”

3.4 ANALYSIS

The intention of the project was to establish how the core concept of the equation was understood and whether a student conceptualised the notion of equality. Skemp (1976) defined this as “knowing both what to do and why” (Brodie, 2004:68) and could be determined by the way the expressions to the left- and right-hand side of the equal sign (‘=’) of an equation are “balanced” with each other. A similar method to that used by Levi (2009) was implemented as is shown in the following explanation:

1. An example of what an equation may look like may have been presented as follows: 397 + 248 = 396 + t. It was hoped that the participant realised that adding the expression on the left-hand side of the equation will produce a solution of 645. Levi (2009) wanted students to look at the expression on the right-hand side and realise that 396 is 1 less than 397 and therefore all they had to do was to add 1 to 248 to discover that the unknown variable was, in fact, 249;
2. It was important to engage participants in a discussion about what they felt were the advantages and disadvantages of screencast interventions.

Specifically, I analysed the data qualitatively and quantitatively. The diagnostic test was analysed quantitatively using descriptive statistical methods while the video recordings and think-aloud interviews were analysed qualitatively. The basis of the analytical framework was taken from the structure represented in Figure 4.9 and was expanded upon. This structure informed the analysis to be developed and finalised once insights from the results of the diagnostic test and the roll-out of screencast interventions were known.

Scenario 1 (Numeric only)	**Scenario2 (Algebraic)**
a. 5 +___ = 9	a.25 – b = 20
b. 9 = 5 + ___	b. 25 = 2b + 1
c. ___ = 9	c. b = b
d. 5 + ___ = 6 + 3	d. c + 10 = 20 - c

Figure 4.9: Analytical Framework

The interrogation of how the selected Grade 7 participants solved the given equations, and an analysis of the think-aloud video recordings which consisted of the participants' grouped solution strategies (with specific reference to equality) according to the scenarios as illustrated above, endorsed the clarification of questions such as:

- How has the '=' been interpreted?
- What is understood by the symbol '='?
- Is the position of the '=' sign interpreted differently when it is either on the LHS or RHS of the addends (ie. 3 + 2 = ___ ; ___ = 3 + 2)?
- Do the different operations in the equation matter?
- What is the role of the unknown variable? What does it represent?

3.5 VALIDITY AND RELIABILITY

Using triangulation (Heale & Forbes, 2017) added validity to the data collected, while audio-video recordings and open-ended interviews were used. The credibility of the interviews were enhanced by including audio recordings to ensure an accurate account of the interview process. It was essential that the aims and objectives of the project were explained to all the participants in order to make sure of their understanding of the research. Cohen et al. (2011:179) state that in "qualitative research, validity should be addressed through honesty, depth and scope of the data achieved." "Validity determines whether the research truly measures that which it was intended to measure or how truthful the research results are. In other words, does the research instrument allow you to hit the 'bull's eye' of your research object? Researchers generally determine validity by asking a series of questions and will often look for the answers in the research of others" (Joppe, 2000:1). Further to the analysis presented, the diagnostic test was piloted with a few Grade 7 students from a school at which I had taught previously to ensure it was unambiguous and clear. The opportunity to pilot the positioning of the video recorder to ensure effective and productive filming during the research project was used so that, in the end, the data analysed using qualitative and quantitative methods strengthened the notion of triangulation and, ultimately, the validity of the data.

3.6 ETHICS

Because the research was conducted with 13-year-old students who are minors it was necessary that consent, and also assent, was sought for and obtained at several levels to protect the ethical position of the project. The stakeholders whose assent and consent was required are:

- The School, which was obtained through the headmaster (Appendix A);

- The parents/guardians of the individual participants (Appendix B); and
- The participants themselves (Appendix C)

Firstly, formal consent was requested from the headmaster. The research was explained and the details outlined in a letter to ensure transparency. It was also made clear that the school's and the participants' anonymity would be assured and safeguarded. At this point permission was requested to access the cohort of forty-two boys in Grade 7 to participate in an initial diagnostic test, after which I would select participants. Similarly Parent(s)/Guardian(s)were approached with a written letter of request and consent that was delivered or emailed. The letter described my details as well as the details of the research to provide as much background as possible in order to be transparent. At this stage permission for their child to participate in the diagnostic test was requested so that selection of participants for the intervention programme could be conducted. Finally, a written letter of request and consent was hand delivered outlining my details and details of the planned research to provide background information that would ensure transparency. The request was for the whole Grade 7 cohort to participate in the diagnostic test, and it was clear that only seven participants, based on their performances in the test, would be selected. In all three letters, it was clear that the participants could:

- withdraw from the research project at any time if they so wished;
- remain anonymous; and
- that their participation was voluntary

For the research project to have continued, the signed consent was required in all three instances to guarantee further consideration of the project.

Although there was no direct conflict of interest, possible power relations could not be denied due to my position as a senior teacher, and that contenders had reasonable and sufficient knowledge of me, including my background and location; I had been employed as a Grade 7 teacher at the school for two-and-a-half years. They were also made aware of my research intentions, given my employment history at the school from where the research would be conducted. I ensured that the students were aware that their answer and/or papers would not be collected for marks purposes but were strictly for my research. Once the intervention programme and the post-intervention test had been completed, the seven participants were assured again that I was not so much interested in correct or incorrect answers as the processes of calculation that they had implemented.

3.7 CONCLUSION

This research project, introduced through screencast interventions, attempted to understand how students conceptualise early algebra. A cohort of sixteen students wrote the initial diagnostic test as a pencil-and-paper test, from which 7 participants were chosen. Purposeful sampling selected participants who took part in the screencast intervention research to determine the benefits to be derived from this method of visualisation. The screencasts consisted of video related lessons in the introduction of early algebra, as determined by the South African Grade 7 curriculum. The video-audio screencasts were introduced over a ten-day period and participants were expected to familiarise themselves with the content upon receipt. After the period of intervention participants rewrote the test.

During the pencil-and-paper rewrite of the test participants were observed by means of a video camera positioned to avoid exposing identity during a think-aloud interview. The intention was to ascertain thought processes as each example was completed which would determine how they interpreted the interventions, why they used the techniques they did, and whether there was any benefit to the screencast interventions. With a mixed-method approach, the pre-intervention test was analysed using a quantitative tool while the rest of the analysis used qualitative tools. These methods allowed not only for triangulation, but for a collection of rich data from which to analyse each participant's thought processes and conceptualisations of early algebra via the teachings of screencast interventions.

CHAPTER 4
ANALYSIS AND DISCUSSION

4.1 INTRODUCTION

The equal sign occurs in several different contexts each of which students need to appreciate, so four different contexts for the sign are provided in the screencast intervention progamme guided by the work of McNeil and Alibali (2005a): (1) the operations equals answer context; (2) the operations on the right-hand side context; (3) the reflexive context; and (4) the operations on both sides context. These can be represented as follows:

1. The equal sign is presented in the typical addition equation: 5 + 4 = 9;
2. The equal sign is presented in an equation in which the addends appear on the right-hand side of the equal sign: 9 = 5 + 4;
3. The equal sign is presented in a reflexive equation: 9 = 9; and
4. The equal sign is presented in an equation with the operations on both sides of the equal sign: 5 + 4 = 6 + 3

These examples form the framework on which is based the analytic instrument and could not be self-evident or intuitive for learners nor is it "an understanding that naturally follows from knowing the operational meaning of the equal sign" (Borenson, 2013:91).

Given the quality and quantity of information collected, vertical and horizontal analyses of the candidates have been provided. The scope of this thesis has been limited to providing detailed vertical analyses of only three of the participants, and the other four are summarised in the Appendices (Appendix D). The information of the three participants who were chosen provided the richest data for the extraction of analysis. The analysis for each participant is structured as follows:

- Pre-screencasting intervention diagnostic test;
- Post-screencasting intervention diagnostic test; and
- Analysis per cluster of questions

The horizontal analysis was performed across the cohort of seven candidates who afforded the opportunity to consider any patterns presented in the data. The analysis shown here undertakes a general consideration of the data derived from the seven candidates.

4.2 VERTICAL ANALYSIS OF PARTICIPANTS

4.2.1 Candidate - NS

Pre-screencast intervention Diagnostic Test

Question 1: Warm-up

NS exhibited an understanding of all the contexts of the '=' sign in the initial component of the test. The contexts have been outlined in the Literature Review under the section "Notion of Equality" highlighted by McNeil and Alibali (2005a). Their definitions of the different contexts described the position of the equal sign relative to the expression(s) on the left- and right-hand side of the equal sign in an equation. These definitions are represented in test examples with values assigned to their meanings in order to arrange them according to "Numeric only" in the warm-up component. This arrangement underpinned the framework for the analysis and accounted for half of the research.

NS solved the equations with ease balancing each side of the '=' sign thereby indicating that his notion of equality may be sound. The speed with which he completed this initial exercise demonstrates a strong mental mathematics showing the potential for managing the advanced calcultions that would be required of him at later levels in the study of mathematics..

Post-screencast intervention Diagnostic Test

Question 1: Warm-up

The same conclusions apply to NS's post-intervention screencast interventions when answering this component of the test. He displayed a strong aptitude for mental mathematics answering questions quickly and accurately. The speed with which with he managed indicates his clarity in conceptualisation of the notion of equality and an understanding of balancing the left-hand side with the right-hand side of the '=' sign.

Clusters of Calculations (Numeric vs Algebraic)

a) The operations equals answer context [eg. (N) 5 + 4 = 9; (A) 25 - b = 20]

Question 5 (Calculate: 65 - 44 = ___)

Pre- screencast intervention

NS calculated the solution to this problem by using a vertical subtraction method and was successful in his calculation. This example was straightforward enough for him to complete without his having to rely too heavily on additional cognitive requirements.

Post-screencast intervention

NS completed this calculation by using a vertical subtraction method identical to his initial diagnostic test. He did not feel that the screencasts assisted as he had no need to recall this type of example, but indicated that he prefers numbers only, and finds them easy to manage. NS felt that the '=' sign referred to the need to find an answer for the expression to the left of the '=' sign based on its location in this equation, so, for him, there was no advantage to the screencasts.

Question 12 (Calculate: 25 x 4 = ___)

Pre-intervention screencast

NS had no problem with calculating the solution, by using a vertical multiplication method successfully, and demonstrated a fair grasp of numbers.

Post-intervention screencast

Although NS used a vertical multiplication method to complete the solution to this problem, his calculation was incorrect, which indicates either that he is not strong with numbers, or that it could be a careless error. He felt no need to recall information from the screencasts nor that there was advantage to this example; he concluded that the '=' sign required an answer for the expression to the left of the '=' sign as based on its location in this equation, so saw no advantage in the screencasts.

Question 1 (Find the value of c if 3c x 7 = 21)

Pre-intervention screencast

NS was unable to complete this equation and did not attempt an answer.

Post-intervention screencast

NS could not recall the explanations and screencasts that dealt with this type of example so could not solve the equation nor did he have any idea of how to manipulate the numbers and letters across the ‘=’ sign. This would have a lot to do with his understanding that the ‘=’ sign means that an answer must follow or that it is the answer to a sum. Encouragingly, he understands that the letter ‘c’ in an equation such as this one represents an unknown number. Unable to recall the screencasts, he gained no advantage by having had access to them.

Question 6 (Find the value of a if 6a + 4 = 58)

Pre-intervention screencast

NS could not complete this equation successfully but substituted ‘4’ for the letter ‘a’ and attached ‘4’ to ‘6’ to create the number ‘64’ adding the constant (or term) ‘4’ to ‘64’ to produce ‘68’. He assumed that ‘58’ on the right-hand side of the ‘=’ sign was incorrect and seemed satisfied with his solution of ‘68’, thereby showing he believes the ‘=’ sign symbolises that an answer must follow.

Post-intervention screencast

NS could not complete this problem so left it blank but wrote “Can’t Do” in the calculations space. He did, however, circle the term ‘6a’ in the question line to imply his understanding that it holds the key to solving the equation but had no idea of how to start solving the equation because he was unable to recall explanations from the screencast interventions. His conviction that an answer must follow the ‘=’ sign would have confused him although his limited knowledge of the letters included in number sentences means he realises he is dealing with algebra. Unable to recall the screencasts means he did not benefit from access to them.

b) The operations on the right-hand side context (eg. (N) 9 = 5 + 4; (A) 25 = 2b + 1)

Question 2 (Calculate: ____ = 9 + 91)

Pre-intervention screencast

NS solved this equation, successfully. Interestingly he did not change the context of the equation to the format of “the operations equals answer context” but solved it by writing it as follows: '100 = 9 + 91'. In the space provided for notes he used a vertical addition method. In the warm-up questions Question 1.4 is answered incorrectly by implying that an answer follows the '=' sign (ie.: '7 + 15 = 22 +12'). It is surprising that he balanced the left-hand side with the right-hand side of the '=' sign by providing '100' as a solution showing that he is comfortable with the notion of equality, at this stage, because many children at this level would write an expression to the left of the '=' sign to indicate that '9' is the result (ie., '8 + 1 = 9 + 91'), a common error when the '=' sign is misunderstood. NS has contradicted his understanding of the meaning of the '=' sign between questions only three examples apart.

Post-intervention screencast

NS answered this equation successfully by balancing the left-hand side with the right-hand side. by filling in '100' in the space provided in the equation. He saw no need to recall the screencasting interventions as he felt comfortable with calculating numbers only. Although this candidate is comfortable with the notion of equality, particularly post the screencast interventions (screencast numbers five and six), he could not escape his understanding of the meaning of the '=' sign, which he believed symbolised that an answer should follow. The possibility of his getting it correct without changing the context of the equation could be due to there being no other terms to the left of the '=' sign, which he interpreted to imply that only the *answer* is required. He has indicated that the '=' sign means that an answer must follow it, or that an answer is required, and did not feel the need to recall information from the screencasts so there was no advantage drawn from them in this instance.

Question 14 (Calculate: ____ = 20 ÷ 20)
Pre-intervention screencast

NS solved this equation successfully. Once again the context of the equation to the format, "the operations equals answer context" are unchanged, so that, in solving it, he wrote it as: '1 = 20 '. In the space provided for notes, he used a vertical addition method; interestingly in the warm-up questions he answered Question 1.4 and 1.6 incorrectly by implying that an answer follows the '=' sign (ie.: '7 + 15 = 22 +12' and '6 + 8 = 72 ÷ 9') so his balancing of the left-hand side with the right-hand side of the '=' sign, by providing '1' as a solution is surprising. He seems to be comfortable with the notion of equality because a large proportion of children at this level would write an expression to the left of the '=' sign to indicate that '20' is the result (ie., '6 + 66 = 72 ÷ 9'), which is a common error when the '=' sign is misunderstood. There is a contradiction in his understanding of the meaning of the '=' sign between questions only three examples apart.

Post-intervention screencast

NS answered this equation successfully by balancing the left-hand side with the right-hand side. He filled in '1' in the space provided and saw no need to recall the screencast interventions but felt comfortable calculating numbers only. Although this candidate is comfortable with the notion of equality, particularly post the screencast interventions (screencast numbers five and six) he could not get away from his understanding of the meaning of the '=' sign as symbolising that an answer should follow. A possibility for him to get it correct without changing the context of the equation could be because there are no other terms to the left of the '=' sign which he has interpreted to imply that only the answer is required. He has indicated that the '=' sign means that an answer must follow or is required. He did not feel the need to recall information from the screencasts so there was no advantage drawn from them in this case.

Question 3 (Find the value of c if 125 = c + 77)

Pre-intervention screencast

NS completed this equation accurately and successfully. He used vertical addition to add '77 + 48' to produce '125', thereby solving for 'c'. He deduced that the difference between '125' and '77' was '48' as no other calculation indicated another way to work out this solution. He then wrote '= 48 + 77' next to his vertical addition, symbolising that he had just balanced the two sides of the equation thereby displaying an understanding of the '=' sign to indicate the need to balance the left-hand side with the right-hand side.

Post-intervention screencast

NS used a similar method to solve this equation as he did in the initial diagnostic test. As a standalone unknown variable the letter was easily solvable as he worked it out to be the difference between the other two known terms '125' and '77'. He performed vertical subtraction by rewriting an expression '125 - 77' which he proceeded to solve. Prior to this, he rewrote the equation with what could be seen to be the answer to the right of the '=' sign and the expression with the unknown variable to the left of the '=' sign. This is probably a structural habit because as we read from left to right in western societies this made sense to him. He understood that the letter 'c' represented an unknown variable so would complete the equation and he felt that the screencast interventions assisted his introduction to algebra, but remains of the opinion that the '=' sign precedes an answer. He is aware that the introduction of a letter into a number sentence implies that he is dealing with algebra but his understanding of the notion of equality has not been confirmed by the screencast numbers three, five, and seven which would have assisted his interpretation of the letter, 'c', and its role in this equation. This implies that he feels the '=' sign precedes an answer probably because of his having been indoctrinated from an early age into the belief that an "answer" must follow the '=' sign; this is more out of habit, or habitual structure, as opposed to his declaring that an answer follows the '=' sign. He displayed some understanding of the '=' sign in his method of solving the equation by reversing the operation and subtracting '77' from '125' indicating that the screencast interventions assisted his progress in understanding equations and algebra, and he listed the following advantages of the screencasts:

- The combination of the visual/audio;
- They were clear and easy to follow

He would have recalled screencast numbers eight, nine, ten, eleven, and twelve demonstrating how to manipulate terms and operations across the '=' sign in order to solve for the unknown variable. He was not deterred by the location of the expression in relation to the '=' sign and the fact that the '=' sign was to the left of it. This positioning of the '=' sign can create misunderstanding among students who have not conceptualised the notion of equality, and who may look for an expression to the left of the '=' sign to satisfy the need for an answer to follow the '=' sign.

Question 7 (Find the value of b if 28 = 3b + 4)

Pre-intervention screencast

NS could not complete this equation successfully and substituted the number ‘4’ for the letter ‘b’. He attached ‘4’ to ‘3’ to create the number, ‘34’ and added the constant, or term, ‘4’ to ‘34’ to produce ‘38’. He has assumed that ‘28’ on the left-hand side of the ‘=’ sign is incorrect and seems satisfied with the solution of ‘38’ so leaves it at that. He went a step further and rewrote the equation '3b + 4 = 38' which indicated that he, with many other students at this level, believes that the ‘=’ sign symbolizes that an answer must follow.

Post-intervention screencast

NS could not complete this problem and left it blank. He went as far as stating “Can’t Do” in the space provided for calculations. He did, however, draw a reversible arrow from the term '3b' to the term '28' across the '=' sign implying a kind of thought process. He understands that it holds the key to solving the equation but has no idea where to start solving the equation as he cannot recall the explanations in the screencasting interventions. He believes an answer must follow the ‘=’ sign, which was confusing although his limited knowledge of letters included in these types of number sentences helped him to realise that he is dealing with algebra. That he was unable to recall the screencasts meant he gained no advantage from them.

c) The reflexive context (eg. (N) 9 = 9; (A) b = b)

Question 13 (Calculate: 15 + 10 = 10 + 15)

Pre-intervention screencast

NS calculated this solution mentally. He seems to have solved each expression on either side of the ‘=’ sign and pronounced that “they both equal 30”. He, therefore, miscalculated each expression or attached an alternative meaning to what was required.

Post-intervention screencast

NS solved this example without hesitation, and his method differed quite significantly from that in the initial test. He rewrote the equation in the answer box, then, below this line, wrote the solution to each side namely '25'. He wrote '25' below each expression but failed to write the '=' sign between the two terms which would have confirmed a balancing of the two sides of the equation. In his mind, he provided a solution that reflected an answer more than it reflected an equivalence between the left- and right-hand sides of the equation, and felt that there was no need for him to recall the screencast interventions as he was only dealing with numbers which were easy enough to manipulate. In this example he believed that the '=' sign is an indication that an answer must be provided for both sides of the '=' sign.

Question 15 (What is 9 = ___)

Pre-intervention screencast

NS was unable to provide any solution to this problem and left it blank. This indicates that he has a weak notion of equality and is unable to understand the need to equate an expression or value on the right-hand side of the equation that will indicate an equivalence to '9'.

Post-intervention screencast

NS did not solve this problem leaving it blank. He believed that the question was asking for something more complicated than providing an equivalence to the value, '9', so he wrote “Can’t Do” in the space provided for calculations. Once again he believes that an answer must follow the ‘=’ sign which would have confused him because he would be looking for an expression on the left-hand side of the '=' sign, but has been presented with an individual term or constant '9'. Because he tries to solve and provide a value that would be an answer to '9' his cognitive processing has been disrupted. Given his inability to recall the screencasts he felt no advantage from having had access to them.

Question 4 (Provide a real-life example that explains this scenario: a = a)

Pre-intervention screencast

NS reflected a conceptual understanding of the scenario indicating that '138 = 138'. In this test he used numbers to display his understanding so his answer implied an understanding of equivalence and what the letter represents, particularly in a scenario such as this.

Post-intervention screencast

NS substituted numbers for the letters 'a = a' to indicate that the left-hand side is '=' to the right-hand side and reads '9 = 9'but then seemed to overcomplicate this by including the statement "would be the meaning of sum = answers" to justify or try to prove complexity. This statement reflects his wholehearted belief that a "sum" should precede the '=' sign, which should be followed by an answer. In this case the screencasts would have benefited NS's introduction to algebra and the unknown variable which according to him indicates algebra and gives a clue to a solution. He would have benefitted from screencast numbers four, five, six, seven, and eight which would have provided the confidence to approach these types of problems. Unfortunately, he does not have a conceptual understanding of the left-hand side being '=' to the right-hand side of this reflexive equation so justified his meaning-making, of letters, only, by substituting number values for 'a'. NS listed the following as the main advantages to the screencasts:

- An introduction to algebra;
- The combination of the visual/audio; and
- They were clear and easy to follow

He believes they are beneficial and would certainly assist him. This positioning of the '=' sign was not fully understood given his statement next to his answer so its location is irrelevant in this context.

Question 10 (Provide a real-life example that explains the following scenario: a + b = a + b)

Pre-intervention screencast

NS did not complete this example accurately although he did try to balance the two expressions on either side of the '=' sign with each other. He indicated that 'a + b = a + b is the same as saying 5 x 2 = 2 ÷ 10' which is interesting because he is making it clear that he understands the '=' sign to mean that an answer must follow an interpretation following on from the way he has written out the equation, particularly the expression on the right-hand side of the '=' sign. His presentation indicates that he has read it from right to left followed by the '=' sign inferring that the answer is the solution to the expression on the left-hand side of the '=' sign. This expression which is to the left, has been written out correctly as a reflection of the opposite. He misunderstood the necessity of substituting the same numbers for the same letters, in order to ensure that that the values substituted for the letters are consistent and maintain their same value throughout.

Post-intervention screencast

NS recalls that the letters represent unknown numbers but that in this instance will be the same value on either side of the '=' sign as he has pointed out. He referred to it as the same answer although with different numbers with the example '1 + 2 = 1 + 2'. Initially he wrote this equation out as '1 + 2 = 2 + 1' which would have been inaccurate because he substituted the incorrect numbers for the relevant letters according to the values he attached to the letters in the expression on the left-hand side of the '=' sign. He realised his error and corrected himself by displaying the equation accurately and hinting at an understanding of what the '=' sign represents. He has understood that this example includes the letter 'b' which has not deterred his reasoning in balancing the equation. I have used the word "balancing" explicitly here because he mentioned that he believes that the '=' sign explains in a roundabout way that there is a need to balance the left-hand side with the right-hand side. There seems to be a glimmer of understanding of the notion of equality, but without confirmation at this stage of his development. NS listed the following as the main advantages to the screencasts:

- As an introduction to algebra;
- The combination of the visual/audio; and
- They were clear and easy to follow

He does believe that they are beneficial and certainly will assist him, but the positioning of the '=' sign was not fully understood given his statement next to his answer, so its location is irrelevant in this context.

d) The operations on both sides context (eg. (N) 5 + 4 = 6 + 3; (A) c + 10 = 20 - c)

Question 8 (Calculate: 10 x 3 = ____ ÷ 2)
Pre-intervention screencast

NS completed the question by falling into the trap that so many students tend to do at this level. He provided an answer to '10 x 3', a common mistake at this level, as children tend to provide an answer when presented with the '=' sign located in this position. He has discarded the fact that he needs to '÷ 2' and has left the equation as it is (ie., '10 x 3 = 30 ÷ 2'). This indicates his discomfort with the notion of equality and his not comprehending the need to balance the expression on the left-hand side of the '=' sign with the expression on the right of it.

Post-intervention screencast

NS used a different method of arriving at a solution, that he has miscalculated the answer. He followed the intended procedure and calculated the left-hand side of the equation first in order to determine what was required on the right-hand side to make the equation true or balanced. He did this by multiplying the two terms to each, '10 x 3' to produce '30', which he wrote in the space provided for calculations. He then continued and divided '30' by '2' to produce a solution of '15', which he entered as his answer. His calculation looked like this, '10 x 3 = 30 ÷ 2 = 15'revealing that he has no comprehension of the notion of equality. The indication that the '=' sign symbolises the need to provide an answer fails to understand the role of the '=' sign in this context, which is to balance the two expressions on either side of it. He admits that he prefers to work with numbers as the letters confuse him although he does understand them to imply algebra. He did not feel he needed to recall information from the screencasts so derived no advantage from them.

Question 11 (Calculate: ____ + 20 = 21 + 35)

Pre-intervention screencast

NS calculated this equation accurately by filling in the missing value, thereby completing the expression. He used mental mathematics to calculate the missing value, and seems to have balanced the equation. This is due to him having fallen into the category of students who tend to believe that the '=' sign precedes an answer, as his answers have revealed to date. He does not have a strong notion of equality and does not always understand the need to balance the left- and right-hand sides of an equation with each other. The common error expected at this level is to assume '21' as the answer which implies that they will use '1' as the missing number, for example: '1 + 20 = 21 +35'.

Post-intervention screencast

Once again, NS had little difficulty completing this question. He calculated the missing value using mental mathematics and filled the space with the missing value. Although this may point to a conceptualisation of the notion of equality in how he solved the sum, having avoided falling into the trap of providing '21' as an answer to the expression on the left-hand side of the equation, for example: '___ + 20 = 21 +35', it is clear that he does not have a strong notion of equality in that he does not always understand the need to balance the left- and right-hand sides of an equation with each other. He has indicated he prefers to work with numbers only and does not feel the need to recall information from the screencasts from which no real advantage could be drawn.

Question 9 (Find the value of a if 10 + 2a = 100 + 20)

Pre-intervention screencast

NS could not complete this equation successfully but stated that the value of 'a' would be '0'. This does not have any other meaning as his interpretation means that '2 x 0 = 0' which when added to '10' is nowhere close to the value on the right-hand side of the '=' sign.

Post-intervention screencast

NS could not complete this problem and left it blank stating "Can't Do" in the space provided for calculations. He has no idea of how to start solving the equation or how to manipulate operations and terms across the '=' sign as he cannot recall the explanations from the screencasting interventions. Contrary to his thoughts thus far he indicated that the two expressions on either side of the '=' sign need to equal each, but is confused by the term '2a'. He understands that the inclusion of letters in equations indicates that he is dealing with algebra but given his inability to recall the screencasts, he gained no advantage from having had access to them. This is unfortunate because there is some recollection in his mentioning his thoughts about the two expressions on either side of the '=' sign needing to equal each or be the same value.

Question 16 (Find the value of b 154 - 104 = 51b - 1)

Pre-intervention screencast

NS was unable to complete this equation and left it out.

Post-intervention screencast

NS could not complete this problem and left it blank stating "Can't Do" in the space provided for calculations. He made no attempt to manipulate the terms or operations across the '=' sign and had no recollection of how to do so from the screencast interventions. He could not remember the use of an animated scale throughout the screencast interventions which has led me to believe that his viewing of the screencasts was limited or that he did not bother. He understands that it holds the key to solving the equation but his indication that the two expressions need to '=' each other, means he is confused by the term '51b' and how to solve this problem. He has understood that he needs to balance each side so the expression on the left-hand side of the '=' sign is the same value as the right-hand side. Even with his limited knowledge of the letters included in these types of number sentences he was aware that he was dealing with algebra. His inability to recall the screencasts means he gained no advantage from his access to them.

4.2.2 Candidate - JR

Pre-screencast intervention Diagnostic Test

Question 1: Warm-up

JR exhibited an understanding of all the contexts of the '=' sign in the initial component of the test. The contexts referred to have been outlined in the Literature Review under the section "Notion of Equality" which were highlighted by McNeil and Alibali (2005a). Their definitions of the different contexts describe the position of the equal sign relative to the expression(s) on the left- and right-hand side of the equal sign in an equation. These definitions are represented as test examples by values being assigned to their meanings and arranging them according to "Numeric only" in the warm-up component. This arrangement, accounting for half of the research, underpinned the framework for analysis.

JR solved the equations with ease, balancing each side of the '=' sign, indicating a sound notion of equality. The speed with which he completed this initial exercise suggests strong mental mathematics capabilities, promising ease with the complicated mathematical calculations expected of him later in his school career.

Post-screencast intervention Diagnostic Test

Question 1: Warm-up

The same applied to JR's post-intervention screencast interventions when answering this component. He displayed an aptitude for mental mathematics answering the questions quickly and accurately. The ease and speed with which he worked indicated clarity in conceptualisation of the notion of equality and an understanding of balancing the left-hand side with the right-hand side of the '=' sign. He did miscalculate the last question, which was carelessness more than misunderstanding.

Clusters of Calculations (Numeric vs Algebraic)

a) The operations equals answer context [e.g. (N) 5 + 4 = 9; (A) 25 - b = 20]

Question 5 (Calculate: 65 - 44 = ___)

Pre- screencast intervention

This calculation was completed accurately using the vertical subtraction method. The candidate did not seem to think about the calculation but wrote it down mechanically. He is proficient in mental mathematics.

Post-screencast intervention

That JR completed this calculation without pausing displayed strong mental mathematics capabilities. He filled in the solution by completing the sum with the missing number to make the equation true showing that the '=' sign referred to the need to find an answer for the expression to the left of the '=' sign, based on its location in this equation. He did not feel it was necessary to recall any screencast interventions given the simplicity of the example; even though dealing with numbers only, he is comfortable with the notion of equality and the balancing of the left- and right-hand sides of an equation or sum.

Question 12 (Calculate: 25 x 4 = ___)

Pre-intervention screencast

JR had no problem in calculating the solution to this equation. Although he provided an answer immediately and followed up with setting it down in a linear format, by calculating with a vertical multiplication method as he would have been taught, he demonstrated considerable mental mathematics capacity.

Post-intervention screencast

JR had no problem answering this question given his mental mathematics processing abilities. He used a vertical multiplication method by multiplying '25' to '4' to produce the solution of '100' which he filled in in the number sentence. Although he did not feel the need to recall information from the screencasts, JR drew on his knowledge, which he indicates was confirmed by the screencasts, to indicate that the '=' sign referred to the fact that the left-hand side of an equation must '=' the right-hand side of an equation. This would imply that the location of the '=' sign does not matter in relation to the expression(s), at this point. He conveyed no sense of being advantaged by the screencasts.

Question 1 (Find the value of c if 3c x 7 = 21)

Pre-intervention screencast

JR is competent enough at mathematics to comprehend that '3 x 7 = 21', though the inclusion of the letter in this equation led him to argue that ‘c’ does not have a value and, in fact, it is ‘= nothing’ so was unable to solve this equation.

Post-intervention screencast

JR provided the accurate answer only, implementing mental maths to calculate the correct value for ‘c’, ‘1’. He recalled the balancing of the animated scale, which symbolises the role of the ‘=’ sign in equations from the screencasts. Screencast numbers three, five, and seven would have assisted his interpretation of the letter and what it represents in the equation. This allowed him to 'see' that '3' multiplied by 'something', represented by 'c' in this example, and then multiplied by '7' would provide a solution to the equation. He understands that the letter represents an unknown number/unknown the unknown variable, recalling basic arithmetic used in earlier grades where an empty block represents a missing number, but now comprehends that the letter has replaced the more rudimentary empty block as he would have seen from screencast number seven. Because he felt confident about his mental mathematics ability he did not feel the equation was difficult enough to display the physical manipulation of the numbers and operations across the ‘=’ sign. He understands the notion of equality and this knowledge would have been endorsed by screencast number six which describes the '=' sign. The advantages of the screencast interventions highlighted here include the relevance of the method of “balancing” an equation by manipulating the numbers and operations across the '=' sign, although not required in this example. He listed the following advantages to having had access to the screencasts:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

Although his calculation method was slightly different from those in the screencasts, he was able to incorporate them into his understanding of how to arrive at a solution for 'c'. The only disadvantage was that of the lack of interaction if assistance had been required.

Question 6 (Find the value of a if 6a + 4 = 58)

Pre-intervention screencast

JR could not complete this equation successfully and discarded the letter ‘a’. He added ‘6’ to ‘4’ and subtracted the result of ‘10’ from '58'. With this number he added '6' and '48' to each other to produce '54' which is not relevant to this equation. He has implied a basic understanding of reversing the operation across the ‘=’ sign by subtracting ‘10’ from ‘58’ although these actions could be speculation and not genuine comprehension.

Post-intervention screencast

JR implemented mental mathematics to solve for the unknown variable 'a' in this equation. He completed solving for 'a' accurately by drawing on the image of the animated scale from the screencast interventions as a reference. He performed opposite operations and manipulated the equation in order to balance the left-hand side with the right-hand side thereby solving for 'a'. The screencast interventions assisted his understanding of how to manipulate the operations and numbers across the '=' sign in order to balance the equation, again incorporating the animated scale. His understanding would have been assisted after watching screencast numbers three, five, and seven, which would have assisted his interpretation of the letter 'a' and its role in this equation. These screencasts would have allowed him to 'see' that '6' multiplied by an unknown number represented by 'a' and then added to '4' will balance the equation by this solution by equalling '58'. His understanding of the letter ‘c’ was extended by its representation as an unknown variable that needed to be solved to complete the equation and balancr it. In terms of balancing the equation his notion of equality was confirmed by a number of the screencasts particularly numbers two, four, and six referring specifically to the '=' sign and its meaning. He conceptualised this to mean that the expression on the left of the ‘=’ sign needs to equal the expression on the right so was able to reflect back on the processes from the screencast interventions in the manipulation of the numbers and operations across the '=' sign. The explanation(s) of manipulating terms and operations across the '=' sign was described in screencast numbers nine, ten, eleven, and twelve to dealt with this topic. His explanation showed how he calculated this equation and understood the concept of equality/equivalence. Clearly, his understanding of what the '=' sign represents means he is not pressured into the idea that an answer must follow the '=' sign but is comfortable with his own understanding that its location in an equation is not important because of his reasoning between the left- and right-hand sides of equations and the need to balance these two sides with each other. The advantages of the screencast interventions highlighted here include the relevance of the method of 'balancing' an equation by manipulating the numbers and operations across the '=' sign although not required in this example. JR indicated that the screencast interventions assisted his progress in conceptualising the ‘=’ sign and the letter, ‘a’ in this instance. He listed the following advantages to having had access to the screencasts:

- The ability to revisit the screencasts at his own leisure;

- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

Although his calculation method was slightly different from those from the screencasts he incorporated it into his understanding of how to arrive at a solution for 'a'. The only disadvantage was that of the lack of interaction if assistance was required.

b) The operations on the right-hand side context (e.g. (N) 9 = 5 + 4; (A) 25 = 2b + 1)

Question 2 (Calculate: ____ = 9 + 91)

Pre-intervention screencast

JR solved this equation successfully. Notably he changed the context of the equation and rewrote it in the format “the operations equals answer context” so that he solved it by writing it: '9 + 91 = 100’, indicating a more comfortable and familiar structure or one he has been taught. It is encouraging that he is comfortable with the notion of equality because many children at this level would write an expression to the left of the '=' sign to indicate that '9' is the result (i.e. '7 + 2 = 9 + 91'), which is a common error for when the ‘=’ is misunderstood.

Post-intervention screencast

JR answered this equation successfully by balancing the left-hand side with the right-hand side. He wrote a solution in the space provided in the number sentence so this problem was solved mentally. He thought there was no need to recall the screencast interventions as he felt comfortable calculating the numbers only. This candidate is comfortable with the notion of equality at this level particularly post the screencast interventions (screencast numbers five and six). Evidently he is comfortable with the ‘answer’ to the left of the ‘=’ sign context when provided with numbers only.

Question 14 (Calculate: ____ = 20 ÷ 20)

Pre-intervention screencast

Given JR’s ability and flexibility with numbers, he was able to solve this equation immediately without having to think about it. It is significant that he changed the context of the equation and rewrote it in the format of “the operations equals answer context”. In other words when he solved it he wrote it as: '20 ÷ 20 = 1'. This implies, as with many students at this level of mathematics, that he feels an answer should follow to the right of an expression. For this level he has a sound notion of equality although many children at this level would write an expression to the left of the '=' sign to indicate that '20' is the result (i.e. '10 + 10 = 20 ÷ 20') which is a common error when the '=' is misunderstood. This could be a case of 'habitual structure' more than that the answer follows the '=' sign.

Post-intervention screencast

Again, JR answered this equation successfully by balancing the left-hand side with the right-hand side. He wrote a solution in the space provided in the number sentence so solved this problem mentally. He did indicate that you could also say '10 ÷ 10' to yield the same result/solution. He felt no need to recall screencast interventions as he felt comfortable calculating the numbers only; he is comfortable with the notion of equality particularly post the screencast interventions (screencast numbers five and six). It is evident that he is now comfortable with the 'answer' to the left of the '=' sign context when provided with numbers only.

Question 3 (Find the value of c if 125 = c + 77)

Pre-intervention screencast

JR completed this equation accurately and successfully. He reversed the operation, '+ 77', and subtracted it from '125' thereby solving for 'c' and producing a result of '48' which he wrote out in a linear manner. I would assume this is for the sake of formality but he rewrote the calculation vertically to illustrate his calculations and concluded that 'c = 48' thus displaying an understanding of the '=' sign and the need to balance the left-hand side with the right-hand side.

Post-intervention screencast

For JR this was not a particularly complicated equation given that there was no number (coefficient) attached to the letter. This example was simple enough for him to realise that the letter was similar to the 'empty box' familiar from Foundation and early Intermediate Phase teaching of arithmetic. As a standalone unknown variable the letter was easily solvable as the difference between the other two known terms '125' and '77'. He did not show any form of working out or calculation but solved for the unknown variable mentally. Although he felt there was no need to recall details from the screencast interventions as this example was straightforward, his understanding of the notion of equality would have been assured by the screencast numbers three, five, and seven to assist his interpretation of the letter 'c' and its role in this equation. He specifically mentioned that they assisted him in making better sense of letters in equations, or, in this instance, 'c', to represent the unknown variable. He demonstrated a conceptual understanding of the '=' sign when he solved this equation during the pre-intervention test, although this was confirmed by his indicating that the '=' sign represents the need to balance the left-hand side with the right-hand side of an equation having recalled screencast numbers two, four, and six. Although they assisted in his garnering a better understanding of equations and equivalence his method of calculation was slightly different from those from the screencasts, though he could still incorporate them into his understanding of how to arrive at a solution for 'c', even though he calculated it mentally.

Question 7 (Find the value of b if 28 = 3b + 4)

Pre-intervention screencast

JR could not complete this equation successfully and discarded the letter, ‘b’. and added ‘3’ to ‘4’ and subtracted ‘7’ from '28'. He implied a basic understanding of reversing the operation across the ‘=’ sign by subtracting ‘7’ from ‘28’ although his actions could have been speculation.

Post-intervention screencast

JR used a process of deduction once he manipulated the '+ 4' across the ‘=’ sign. He then used '24' to determine that the missing variable must be '8' which is multiplied by '3' to give him ‘24’. With mental maths he provided the solution for 'b' immediately. and explained that he used the animated scale from the screencast interventions as a reference. He performed opposite operations and manipulated the equation in order to balance the left-hand side with the right-hand side of the equation, thereby solving for 'b'. He confirmed that the screencasts improved and increased his knowledge of algebra by helping him understand how to manipulate the operations and numbers across the '=' sign in order to balance the equation incorporating the animated scale again, and with that, a better conceptual understanding of the '=' sign. The screencast numbers that would have assisted his interpretation of the '=' sign would have been two, three, four, and six, while numbers four, five, and seven would have assisted his interpretation of the letter 'b', the unknown variable. Given his notion of equality and deep understanding of the unknown variable, he felt well equipped and comfortable solving the equation after viewing the post-intervention screencasts. He cited the following advantages of having had access to the screencast intervention: as beneficial:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

Although they assisted in a good understanding of equations and equivalence his method of calculation was not shown because he solved the equation mentally. He was still able to incorporate them into his understanding of how to arrive at a solution for 'b' and would have recalled screencast numbers eight, nine, ten, eleven, and twelve which explicitly indicated how to manipulate terms and operations across the '=' sign in order to solve for the unknown variable undeterred by the location of the expression in relation to the '=' sign and the fact that the '=' sign was to the left of it. This positioning of the '=' sign can create a misunderstanding among students who have not conceptualised the notion of equality and who may be prone to look for an expression to the left of the '=' sign to satisfy the need for an answer to follow the '=' sign.

c) The reflexive context (e.g. (N) 9 = 9; (A) b = b)

Question 13 (Calculate: 15 + 10 = 10 + 15)

Pre-intervention screencast

JR calculated this solution without hesitation. He solved each expression on either side of the '=' sign to read '35'and solved the first expression on the first line, which was the initial expression to the left of the '=' sign. Unfortunately he miscalculated the solution to be '35' following this calculation by stating that '35 = 35' in the following line; this is interesting given his usual capacity and fluency with numbers; also interesting is that he only wrote out the expression to the left of the '=' sign, ignoring the expression to its left possibly implying that he could be uncomfortable with the expression to the right of an '=' sign.

Post-intervention screencast

JR solved this example without hesitation and his method differed quite significantly from the first time when applying mental maths. He wrote out the solutions '25 = 25', only, post the intervention screencasts. Having indicated that they were already balanced, he felt it necessary to simplify the expressions, providing, therefore, a solution that reflected an equivalence between the left- and right-hand sides of the equation. He felt that the visual imagery of the animated scale balanced the left-hand side with the right-hand side of an equation which went a long way in assisting and confirming his knowledge and understanding of the '=' sign and the role it plays in equations. The screencasts that he would have recalled were numbers four, five, and six, and also number thirteen which would have assisted this process. Given that he was dealing with numbers only, he did not consider there was significant advantage gained from the screencasts. The positioning of the '=' sign is understood to indicate that the solution to each of the expressions, on either side of the '=' sign, is the same value and, given that there are no letters in this equation it is straightforward enough for JR to have solved.

Question 15 (What is 9 = ___)
Pre-intervention screencast

JR provided a solution that indicated an equivalence or notion of equality. He completed the example by writing ‘9’ in the space provided for calculations. He calculated this example without uncertainty and conveyed a strong sense of numbers and a strong notion of equality in terms of balancing the left- with the right-hand side of the equation.

Post-intervention screencast

As he did in his initial diagnostic test JR filled in what he deemed to be the solution only '9'. Encouragingly he mentioned that there are many options to reflect '9' as the value on the right-hand side of the '=' sign, an accurate reflection of what was asked for. He has interpreted correctly the most important element of what was tested by showing to what ‘9’ could be equivalent on the right-hand side of the '=' sign, without rewriting it in a different context; this indicated that, for him, this reflective context was straightforward enough to not have to reflect on the screencasts. He is comfortable with the notion of equality and therefore with the need to balance the left- and right-hand sides of an equation. The screencast numbers that would have reinforced this thought process would have been three, four, five, and six. All this shows a sound concept of the role of the '=' sign. The context of the '=' sign was understood to indicate that the solution to each of the expressions, on either side of the '=' sign, is the same value and given that there were no letters included in this equation, it was straightforward enough for JR to solve.

Question 4 (Provide a real-life example that explains this scenario: a = a)
Pre-intervention screencast

JR reflected a conceptual understanding of the scenario indicating that “1 apple is the same as another apple” and wrote it out as: '1 apple = 1 apple'.; he used a tangible object to demonstrate his understanding. His answer implied an understanding of equivalence and the representation of the letter which could be anything or any value particularly in such a scenario.

Post-intervention screencast

JR did not budge from his initial answer in the diagnostic test but stated that “2 apples = 2 apples” secure in his understanding that, given the parameters of this example, he could use any concrete items to indicate that the letter 'a' to the left of the '=' sign is the same as the one to the right. Although he cannot fathom why letters only have been used, he is able to use his knowledge of the '=' sign and reason to justify an answer, but in this case the screencasts that would have benefited JR's interpretation of the '=' sign and the unknown variable would have been numbers four, five, six, seven, and eight. These would have given him the confidence to approach these types of problems, and reason a solution. His conceptual understanding of the left-hand side being '=' to the right-hand side of this reflexive equation is firm. JR did not pinpoint the screencasts that assisted his reasoning process, feeling no advantage in the screencasts in this example which was straightforward enough to work out for himself, regardless of assistance. Given his bemusement at the randomness of the letters such as ‘a’ being equal to each other and then being able to translate that into a concept that makes meaning for him he justified this meaning-making by substituting concrete *values* for 'a'. The positioning of the '=' sign would indicate that the terms on either side of the '=' sign were the same value and, because the problem only contained letters, although the same, it was straightforward enough for JR to solve this time after the screencast interventions.

Question 10 (Provide a real-life example that best explains the following scenario: a + b = a + b)

Pre-intervention screencast

JR reflected a conceptual understanding of this example by using a practical scenario an apple and a banana Arguing that “1 apple and 1 banana = 1 apple and 1 banana”. His answer implied an understanding of equivalence and the representation of a letter as an unknown variable or, in this instance, two unknown variables which could be anything or any value particularly in such a scenario.

Post-intervention screencast

JR has pointed out that letters represent unknown numbers but here will be the same value on either side of the ‘=’ sign. He understands this example includes the letter 'b' and does not waver in his reasoning, so he substituted 'a' and 'b' for an 'apple' and a 'bat', respectively, thereby balancing the equation accurately. His knowledge and understanding of the letters and the introduction to algebra enables his realisation that 'a' and 'b' are different in this context but that each carries the same value in order to make the equation true that the “left-hand side = the right-hand side”. Although he finds it logical, the scale in the screencast would have ensured his prior knowledge confirmed as a definite advantage as it *animated* the scale. This provided a balance for the left- and right-hand sides of an equation. He also indicated that it reinforced his understanding of letters in number sentences in that they represent an unknown variable. He found no advantage in these screencast interventions as he felt it was straightforward enough to work out for himself without assistance.

d) The operations on both sides context (e.g. (N) 5 + 4 = 6 + 3; (A) c + 10 = 20 - c)

Question 8 (Calculate: 10 x 3 = ____ ÷ 2)
Pre-intervention screencast

JR calculated this equation by balancing the expressions on either side of the '=' sign by working out the left-hand side of the equation first to determine what was necessary for the equation to be true. He is comfortable at this level with the notion of equality and understands what it represents in this context. He has avoided the temptation of providing an *answer* to '10 x 3', a common mistake at this level, as children tend to provide an answer when presented with the '=' sign in this position. Although he completed this task accurately, he calculated each expression below one another in the "operation on the left" context implying that an answer must follow. This could be due to a structure he has become used to throughout junior school but it is surprising given his understanding of the concept of equality. He certainly filled in the correct missing number in the number sentence.

Post-intervention screencast

JR filled in the answer only by applying his mental mathematics skills. He did this cognitively by multiplying the two terms to each '10 x 3' to produce '30', then reversed the operation on the right-hand side of the equation by multiplying '30' to '2' giving him '60' which divided by '2' would provide the result of '30', which then balanced the two sides of the equation. At ease working with and manipulating numbers he did not feel the need to recall any particular screencasts to assist. The position of the '=' sign was to indicate that the terms on either side of the '=' sign would produce the same value when simplified and would balance the left-hand side with the right-hand side of the equation. This conceptualisation was evident in his manner of solving the sum, avoiding the trap of providing an 'answer' to '10 x 3' (i.e.: 10 x 3 = 30 ÷ 2) as so many students do at this level.

Question 11 (Calculate: ____ + 20 = 21 + 35)

Pre-intervention screencasts

JR calculated this equation accurately by completing the expression on the right-hand side of the '=' sign, first, and then subtracting '20' from that solution. He has a good notion of equality and understands the need to balance the left- and right-hand sides of the equation with each other as is evident through his manipulation of the operation '+ 20' where he used the expression to calculate the difference between them. Although he completed this task accurately he calculated each expression below one another in the “operation on the left” context implying that an answer must follow. This could be due to a structure that he used throughout junior school, but is surprising given his understanding of the concept of equality. He did, however, fill in the correct missing number in the number sentence. A common error at this level is for students to assume '21' to be the answer which implies that they will use '1' as the missing number as in: '1 + 20 = 21 +35'.

Post-intervention screencasts

Once again, JR had little difficulty completing this question. First he simplified the right-hand side of the equation and only wrote down the simplified solution '56', in the space provided for calculations then accurately filled in the missing number in the number sentence before calculating the expression on the right-hand of the ‘=’ sign. This enabled him to determine what was required to make the equation true, or to balance the two expressions on either side of the ‘=’ sign. He is at ease with manipulating numbers and did not feel the need to recall particular screencasts for assistance.. This positioning of the '=' sign was understood to indicate that the terms on either side of the '=' sign would produce the same value when simplified thereby balancing the left-hand side with the right-hand side of the equation. This conceptualisation of the notion of equality was evident in how he solved the sum avoiding the trap of providing '21' as an answer to the expression on the left-hand side of the equation: '___ + 20 = 21 +35'. He did not feel the screencasts provided an additional advantage as the example, being numbers only, was straightforward enough to complete.

Question 9 (Find the value of a if 10 + 2a = 100 + 20)

Pre-intervention screencasts

JR could not complete this equation successfully and, discarding the letter ‘a’, worked out the expression on the right-hand side of the equation first which provided '120'. Interestingly, he wrote this calculator out in the “operations on the left” context following the '=' sign with what he thought was an answer. In the line below this linear calculation he added ‘10’ to ‘2’ and left it at that. To the right of these two lines of calculation, he wrote 'a = x 10' which does not make much sense if he is implying that something is multiplied to '10'. It would seem he had no sense of where to go at that point.

Post-intervention screencasts

JR accurately manipulated the numbers and operations across the '=' sign reflecting an understanding of the need to balance the left-hand side with the right-hand side of the equation. In this example there is an expression on either side of the '=' sign which did not deter his thought process even though he is required, in this problem, to handle a letter representing the unknown variable. He used mental maths to deduce the unknown variable 'a' between the two sides of the equation, first solved what he knew, the right-hand side of the equation and used this information to work out that he had to multiply '55' to '2' to produce a number in order to make the equation true. Here is a thorough understanding of the notion of equivalence as, in his process of deduction, he calculated the expression on the right-hand side of the '=' sign subtracting '10' from '120' and dividing by '2'. This indicates a clear conceptual understanding of the role of the '=' sign and the notion of equality, as well as the letter 'a' used to represent the unknown variable. The solution or an "answer" in the space allowed for calculations, concludes that 'a = 55'. He confirmed that the screencast interventions were an advantage as they animated the scale, strengthened his understanding of letters and their role, and the balanced the left-hand side with the right-hand side of the equation. Screencast interventions also assisted progress in conceptualising the '=' sign and the letter, the unknown variable. The specific screencast numbers that would have assisted him to understand these concepts would have been one, four, five, six, seven, eight, nine, and eleven as they show one how to manipulate terms and operations across the '=' sign in order to solve for the unknown variable. He listed the following as the main advantages of having had access to the screencasts:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio helped (specifically reminding him that the term, '2a' implies that you multiply the coefficient to the letter); and
- The ability to stop them at any point, rewind, and replay

Although they assisted in gaining a better understanding of equations and equivalence, his method of calculation was slightly different from those of the screencasts, yet he was able to incorporate them into his understanding of how to arrive at a solution for 'a'. He was not deterred by the location of the two expressions either side of the '=' sign or the fact that there did not seem to be an answer. The position of the '=' sign in this context, together with the inclusion of the letter, can create misunderstanding among students who have not conceptualised the notion of equality, nor the idea of symbol sense and the manipulation of terms across the '=' sign. They may be prone to look for an expression to the left of the '=' sign to satisfy the need for an answer to follow the said '=' sign. Over and above these advantages JR found it useful that examples and explanations were explained succinctly step-by-step as they unfolded in the screencasts.

Question 16 (Find the value of b 154 - 104 = 51b - 1)

Pre-intervention screencasts

JR could not complete this equation successfully and discarded the letter ‘b’. As in previous and similar examples that presented unknown variables he attempted to manipulate the equation by simplifying each expression on either side of the '=' sign. This was correct for the expression on the left-hand side of the equation but not so for the expression on the right-hand side. He did so by discarding the letter 'b' and subtracting '1' from '51' to arrive at '50' and indicated that ‘b = nothing’. This could be due to the complicated context, that the letter is presented in an equation with an expression on either side of the '=' sign and as a coefficient. At this level, an example such as this will be beyond the understanding of most students as they would not have been exposed to algebra yet. Even then, an example such as this normally produces confusion due to a term such as '51b' for which the conceptualisation of equivalence and symbol sense would have to be sound.

Post-intervention screencasts

JR used his mental mathematics abilities to deduce the unknown variable 'b'. He first simplified the expression he knew, the expression on the left-hand side of the equation, to produce a solution of '50', then this information helped him work out that he had to multiply '51' to '1' to produce a number in order to make the equation true which would happen by subtracting ‘1’ from ‘51’. This indicates conceptual understanding of the ‘=’ sign and the notion of equality and also the letter ‘b’ which has been used to represent the unknown variable. He found the screencast interventions e an advantage as they animated the scale, and reinforced his understanding, of letters and their role, and the balancing of the left-hand side with the right-hand side of the equation. JR indicated too that the screencast interventions assisted his progress in conceptualising the ‘=’ sign and the letter (the unknown variable). The specific screencast numbers that would have assisted him to understand these concepts would have been one, four, five, six, seven, eight, nine, and eleven: they show clearly how to manipulate terms and operations across the '=' sign in order to solve for the unknown variable. Listed here are his main advantages of having had access to the screencasts:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

Although they assisted in garnering an understanding of equations and equivalence his method of calculation was slightly different from those on the screencasts yet he was able to incorporate them into his understanding of how to arrive at a solution for 'b'. He was not deterred by the location of the two expressions on either side of the '=' sign or the fact that there did not seem to be an answer. The position of the '=' sign in this context, together with the inclusion of the letter, can create misunderstanding among students who have not conceptualised the notion of equality, nor the idea of symbol sense and the manipulation of terms across the '=' sign. They may be prone to look for an expression to the left of the '=' sign to satisfy the need for an answer to follow the said '=' sign. He enjoyed the video/audio combination particularly because of useful explanations of examples, step-by-step, as they unfolded. The screencasts also helped him remember that the term '51b' implies that you multiply the coefficient to the letter.

4.2.3 Candidate - EVDM

Pre-screencast intervention Diagnostic Test

Question 1: Warm-up

EVDM exhibited an understanding of all the contexts of the ‘=’ sign in the initial component of the test. The contexts referred to have been outlined in the Literature Review under the section “Notion of Equality” highlighted by McNeil and Alibali (2005a). Their definitions of the different contexts described the position of the equal sign relative to the expression(s) on the left- and right-hand side of the equal sign in an equation. These definitions are represented as test examples by assigning values to their meanings and arranging them according to “Numeric only” in the warm-up component. This arrangement accounted for half of the research and underpinned the framework for analysis.

EVDM solved the equations with ease, balancing each side of the ‘=’ sign indicating thereby that his notion of equality may be sound. The speed with which he completed this initial exercise indicates strong mental mathematics capacity which frees up space to attempt the complicated mathematical calculations expected of him in his later school career.

Post-screencast intervention Diagnostic Test

Question 1: Warm-up

The same applied to EVDM’s post-intervention screencast interventions when answering this component of the test. His strong aptitude for mental mathematics meant he answered questions quickly and accurately; he raced through the questions means an understanding and conceptualisation of the notion of equality, and balancing the left-hand side with the right-hand side of the ‘=’ sign.

<u>Clusters of Calculations (Numeric vs Algebraic)</u>

a) The operations equals answer context [e.g. (N) 5 + 4 = 9; (A) 25 - b = 20]

Question 5 (Calculate: 65 - 44 = ___)

Pre-intervention screencasts

This calculation was completed accurately using mental mathematics. The problem was solved with the vertical subtraction method and a solution written down immediately with proficient mental mathematics abilities.

Post-intervention screencasts

EVDM completed this calculation without pause displaying strong mental mathematics capabilities, and using linear subtraction. He felt that the screencasts assisted his understanding of the '=' sign and the role it plays in equations, and that it means the left-hand side must balance with the right-hand side of the equation. There was no real advantage in having had access to the screencasts in this example as it consisted of numbers only and the requirement was easy for him to comprehend.

Question 12 (Calculate: 25 x 4 = ___)

Pre-intervention screencasts

EVDM had no problem in calculating the solution to this equation. His skill in mental mathematics provided an answer immediately, and he wrote it down in a linear format, with the '=' signfollowed by the solution.

Post-intervention screencasts

EVDM had no problem with this question either, demonstrating his capacity for processing mental mathematics. He used a linear multiplication method providing the solution to '25 x 4' as '100'. In this instance he did not feel he need to recall information from the screencasts and drew no real advantage from them. He did, however, indicate that the '=' sign referred to the fact that the left-hand side of an equation must '=' the right-hand side of an equation. This would imply that the location of the '=' sign does not matter in relation to the expression(s).

Question 1 (Find the value of c if 3c x 7 = 21)

Pre-intervention screencasts

EVDM is competent enough at mathematics to comprehend that '3 x 7 = 21' but the inclusion of the letter in this equation has left him uncertain as to what the role of the letter 'c' is'. He left it out of the equation and rewrote it as '3. X 7 = 21' so was unable to solve this equation successfully.

Post-intervention screencasts

EVDM used linear multiplication to complete this equation implementing mental maths to calculate the correct value for 'c', '1'. He recalled the balancing of the animated scale which symbolises the role of the '=' sign in equations from the screencasts, but now has a better understanding of the letters and their role in equations. Screencast numbers three, five, and seven would have assisted his interpretation of the letter and what it represents in the equation. A process of elimination was chosen to manipulate numbers across the '=' sign and this example was straightforward enough to calculate mentally so he could 'see' that '3' multiplied by 'something', represented by 'c' in this example, and then multiplied by '7' would provide a solution to the equation. He has understood the letter 'c' to represent the unknown variable/number which he would have seen from screencast number seven. His strong understanding of the notion of equality was confirmed by screencast number six describing the '=' sign in detail. EVDM indicated that the screencasting interventions assisted his conceptualisation of the '=' sign and the letter 'c' in this instance. H listing the following advantages to access to the screencasts:

- The ability to revisit the screencasts at his own leisure;
- The description and meaning of the letter(s);
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

In addition he felt they assisted in garnering an understanding of equations and equivalence. Although his calculation method was slightly different from those in the screencasts he could incorporate them into his understanding of how to arrive at a solution for 'c'.

Question 6 (Find the value of a if 6a + 4 = 58)

Pre-intervention screencasts

EVDM could not complete this equation successfully and discarded the letter 'a'. He wrote '6 to the power of 5 + 4 = 58' but failed to continue the equation and left it there making it difficult to determine whether he had understood or was able to solve equations.

Post-intervention screencasts

EVDM implemented mental mathematics to solve for the unknown variable 'a' in this equation which he solved in a linear manner. He made the effort to isolate the letter 'a' but realised he should multiply something to '6' to produce a solution of '54' to make the equation true/balanced. His understanding would have been assisted in interpreting the letter ‘a’ and its roles after watching screencast numbers three, five. and seven. The screencasts would have allowed him to “see” that '6' multiplied by an unknown number, represented by 'a', and then added to '4', will balance the equation by this solution by equalling '58' and, therefore, the right-hand side of the equation. His understanding of the letter ‘c’ was extended by its representation as an unknown variable that needed to be solved; this would complete the equation and balance it. In terms of balance his notion of equality was informed by a number of the screencasts in particular numbers two, four, and six referring to the '=' sign and its meaning. He has grasped the concept that the expression on the left of the ‘=’ sign needs to equal the expression on the right so was able to reflect back on the processes from the screencasting interventions in the manipulation of the numbers and operations across the '=' sign. The explanation(s) of manipulating terms and operations across the '=' sign was described in screencast numbers nine, ten, eleven, and twelve which dealt with this topic. His explanation of how he calculated this equation shows that he understands the concept of equality/equivalence, and what the '=' sign represents. Not pressured into the idea that an answer must follow the '=' sign, he is comfortable with his understanding that its location in an equation is not important because of his reasoning between the left- and right-hand sides of equations, and the need to balance these two sides with each other. EVDM indicated that the screencasting interventions enabled progress in conceptualising the ‘=’ sign and, in this instance, the letter ‘a’. The following advantages of having had access to the screencasts are:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

In addition he felt they assisted his understanding of equations and equivalence. Although his calculation method was slightly different from those on the screencasts he was able to incorporate them into his understanding of how to arrive at a solution for 'a'.

b) The operations on the right-hand side context (e.g. (N) 9 = 5 + 4; (A) 25 = 2b + 1)

Question 2 (Calculate: ____ = 9 + 91)

Pre-intervention screencasts

EVDM solved this equation. He changed the context of the equation and rewrote it in the format of "the operations equals answer context" and when he solved it, he wrote it as: '9 + 91 = 100'. This may indicate that with which he is most familiar and comfortable, or with how he has been taught. It is encouraging that he is comfortable with the notion of equality because a large proportion of children at this level would write an expression to the left of the '=' sign to indicate that '9' is the result (i.e., '8 + 1 = 9 + 91'), a common error occurring when the '=' sign is misunderstood.

Post-intervention screencasts

EVDM answered this equation successfully by balancing the left-hand side with the right-hand side. He wrote down the expression from the right-hand side of the '=' sign as '9 + 91' and below it, '= 100'. He saw no need to recall screencast interventions at this stage but was at ease with only calculating the numbers.. Although this candidate is comfortable with the notion of equality, particularly post the screencast interventions numbers five and six, he still felt it necessary to rearrange the equation, although rather subtly. He could indicate the notion of equality by including the value that was equivalent to '9 + 91' on the left-hand side of the '=' sign. Evidently he, along with the other participants, feels more comfortable with the "answer" to the right of the '=' sign when provided with numbers only. This may be due to a traditional approach to how mathematics is taught that an answer follows the '=' sign, and as western societies read from left to right, this could be a reason why students are uncomfortable with another context.

Question 14 (Calculate: ____ = 20 ÷ 20)

Pre-intervention screencasts

Because of an ability and flexibility with numbers EVDM solved this equation immediately without having to "think" about it: he rewrote the equation with '20 ÷ 20' in the first line and '= 1' in the line below it. So many students at this level of their mathematical journey feel an answer should follow to the right of an expression after the '=' sign.

Post-intervention screencasts

Once again EVDM calculated the solution without seeming to "think" about it but wrote it down in a linear format, a testament to his fluency and agility with numbers. Interestingly as with his initial diagnostic test he rewrote the equation with ‘20 ÷ 20’ in the first line, followed by ‘= 1’ meaning that, given his conceptualisation of the ‘=’ sign in the more complicated examples, he would have realised that it is not necessary for an answer to follow an ‘=’ sign, particularly to the right of the said '=' sign. He has a sound notion of equality so this could be a case of “habitual structure” as opposed to the assumption that an answer follows the '=' sign. His concept of the '=' sign is proved by his not developing an expression to the left of the '=' sign to satisfy an “answer” of '20' as would have been the case if he misunderstood the '=' sign; this suggests that his solution to the equation may have resembled an equation such as: '10 + 10= 20 ÷ 20'. There was no need to recall any screencast interventions as he felt at ease with the simplicity of the example.

Question 3 (Find the value of c if 125 = c + 77)

Pre-intervention screencasts

EVDM completed this equation accurately and successfully. He reversed the operation ‘+ 77’ and subtracted it from ‘125’ solving for ‘c’. The unknown number was calculated with vertical subtraction; he did not follow through but implied that ‘48’ is the answer, disregarding the fact that 'c = 48' having an understanding of the ‘=’ sign and the need to balance the left-hand side with the right-hand side.

Post-intervention screencasts

EVDM used the same method to solve this equation as he did in the initial diagnostic test. This example was simple enough for him to realise that the letter was similar to how an 'empty box' would have been represented from Foundation and early Intermediate Phase arithmetic and teaching. As a standalone unknown variable, the letter was easily solved as it was the difference between the other two known terms '125' and '77'. He performed a vertical subtraction with the expression '125 - 77', which he solved with a traditional “borrowing” method he would have been taught according to South African curriculum. He followed up this calculation by formally stating that ‘c = 48’ in the space next to his vertical calculation. This was a straightforward calculation for him as there were no numbers or coefficient attached to the letter; he completed the equation by filling in the missing amount and solving for the unknown variable. The notion of equality would have been confirmed by the screencast numbers three, five, and seven to assist his interpretation of the letter 'c' and its role in this equation. He mentioned their help in his making sense of letters in equations, in this instance ‘c’, to represent the unknown variable. A conceptual understanding of the ‘=’ sign was clear when he solved this equation during the pre-intervention test, and confirmed this by indicating that the '=' sign represents the need to balance the left-hand side with the right-hand side of an equation. He would have recalled screencast numbers two, four, and six understanding it to mean the notion of equality or equivalence. The screencast interventions assisted his progress in conceptualising the '=' sign and the letter listing the following advantages to having access to the screencasts:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

Although they assisted in garnering a better understanding of equations and equivalence, and his method of calculation was slightly different from those from the screencasts, he was able to incorporate them into his understanding of how to arrive at a solution for 'c'. He would have recalled screencast numbers eight, nine, ten, eleven, and twelve to indicate exactly how to manipulate terms and operations across the '=' sign in order to solve for the unknown variable. He was not deterred by the location of the expression in relation to the '=' sign and the fact that the '=' sign was to the left of it. This positioning of the '=' sign can create misunderstanding among students who have not conceptualised the notion of equality and may be prone to look for an expression to the left of the '=' sign to satisfy the need for an answer to follow the '=' sign.

Question 7 (Find the value of b if 28 = 3b + 4)

Pre-intervention screencasts

EVDM could not complete this equation and discarded the letter 'b'. He rewrote the equation to reflect that '28 = 3^3 + '4' and left it at that. It is difficult to interpret information from his initial diagnostic test as he did not provide anything else from which to draw.

Post-intervention screencasts

EVDM used a process of deduction once he manipulated the '+ 4' across the '=' sign. He used '24' to determine that the missing variable must be '8', multiplied by '3' to give him '24'. He recognised that this example is similar to the previous question so his calculation and reasoning remained. He confirmed that he should multiply the letter 'b' to the number '3' in the term '3b' confirming that the screencasts improved and increased his knowledge of algebra and a better conceptual understanding of the '=' sign. The screencast numbers that would have assisted his interpretation of the '=' sign would have been two, three, four, and six, while numbers four, five, and seven would have assisted interpretation of the letter 'b' as the unknown variable. Given his notion of equality and understanding of the unknown variable, he felt better equipped and more comfortable in solving the equation after viewing the post-intervention screencasts citing the following advantages of having had access to the screencast interventions:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

Although they assisted in understanding equations and equivalence, his method of calculation was slightly different from the examples provided in the screencasts, and he had incorporated them into his understanding of how to arrive at a solution for 'b'. He would have recalled screencast numbers eight, nine, ten, eleven, and twelve, which showed how to manipulate terms and operations across the '=' sign in order to solve for the unknown variable. He was not deterred by the location of the expression in relation to the '=' sign and that the '=' sign was to the left. This positioning of the '=' sign can create misunderstanding among students who have not conceptualised the notion of equality, and may be prone to look for an expression to the left of the '=' sign to satisfy the need for an answer to follow.

c) The reflexive context (e.g. (N) 9 = 9; (A) b = b)

Question 13 (Calculate: 15 + 10 = 10 + 15)

Pre-intervention screencasts

EVDM calculated this solution without difficulty. He rewrote the original equation and followed it with '= 25' and suggested this meant that both expressions, on either side of the '=' sign, have the same value which is '25'. Below this line of calculation he wrote '25' which was random appearing to occupy "space", but was written below the expression on the left-hand side of the '=' sign.

Post-intervention screencasts

EVDM solved this example without hesitation with a method differing significantly from the first attempt. He rewrote the expressions below each other and determined each solution as '25'. In other words, he wrote the left-hand side expression as '15 + 10 = 25', while following in the next line with the expression from the right-hand side with '10 + 15 = 25'. It can be said that he provided a solution that reflected an equivalence between the left- and right-hand sides of the equation. He felt that the visual imagery of the animated scale balancing the left-hand side with the right-hand side of an equation went a long way in assisting, and confirming, his knowledge and understanding of the '=' sign and the role it plays in equations. The screencasts that he would have recalled were numbers four, five, and six, and number thirteen would have assisted. A few main advantages EVDM highlighted from the screencasts were:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

This positioning of the '=' sign was understood to indicate that the solution to each expression on either side of the '=' sign was the same value and, with no letters included in this equation, it was straightforward enough for EVDM to solve.

Question 15 (What is 9 = ___)
Pre-intervention screencasts

EVDM provided a solution that indicated an equivalence or notion of equality. He completed the example by writing ‘3^2’ having calculated this example without hesitation, implying a strong sense of numbers and a strong notion of equality in terms of balancing the left with the right-hand side of the equation. The simplicity of this context often confuses students as they try to solve an operation to the left of the '=' sign to provide an “answer” to the right of an '=' sign.

Post-intervention screencasts

EVDM provided an operation that reflected an equivalence between the left- and right-hand side of the '=' sign. The location of the term and the '=' sign were interpreted accurately by completing the equation with an expression on the right-hand side of the '=' sign. He solved the example by indicating that '9 = 3 x 3', followed by writing '3^2' next to it, an accurate reflection of what was asked in the question. He has interpreted correctly the most important element of what was tested by showing to what ‘9’ could be equivalent on the right-hand side of the '=' sign without rewriting it in a different context. He made reference to the visual imagery of the animated scale in the screencasts which assisted in his balancing the two sides of the ‘=’ sign of an equation. The screencast numbers that would have strengthened this thought process would have been three, four, five, and six. EVDM conceptualised the notion of equality, to which he made specific mention, and highlighted the following main advantages:

- Easy to understand;
- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

This positioning of the '=' sign was understood to indicate that the solution to the problem, on either side of the '=' sign, was the same value and, given that there were no letters included in this equation, it was straightforward enough to solve.

Question 4 (Provide a real-life example that explains this scenario: a = a)

Pre-intervention screencasts

EVDM could not comprehend what this scenario meant and could not articulate how to understand, or, indeed, misunderstand, it, so he wrote '28' in the answer box without reference to anything and, because he left it with no explanation, it was difficult to interpret his meaning.

Post-intervention screencasts

EVDM substituted numbers for the letters 'a = a' to indicate that the left-hand side is '=' to the right-hand side which he simplified to read '2 = 2', a turnaround from his initial diagnostic test. He was able to recall that the letters represent an unknown number, but in this instance the letter 'a' is substituted by '2' to display his conceptual understanding of this type of example. He did recall a specific example like this from the screencasts although the screencasts that would have benefitted EVDM's interpretation of the '=' sign and the unknown variable would have been numbers four, five, six, seven, and eight. These would have provided the clarity and confidence to approach problems in these contexts. EVDM pointed out that the screencasts assisted in the concept of random letters such as 'a' being equal to each other and then being able to translate that into clear meaning. He justified this meaning-making by substituting number values for 'a', and listed the following as advantages to the screencasts:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

This positioning of the '=' sign was understood to indicate that the terms on either side of the '=' sign were the same value and, given that the problem only contained letters, although the same letters, was straightforward enough for EVDM to solve post the screencast interventions.

Question 10 (Provide a real-life example that best explains the following scenario: a + b = a + b)

Pre-intervention screencasts

EVDM did not even attempt this example.

Post-intervention screencasts

EVDM recalls that letters represent unknown numbers but in this instance will be the same value on either side of the '=' sign as he pointed out. He has understood that this example includes the letter 'b' and does not waver in his reasoning; he substituted 'a' and 'b' for '2' and '3', respectively, thereby balancing the equation, but not solving the expressions once he had substituted the numbers in and, instead, left the equation to reflect the following: '2 + 3 = 3 +

2'. He did not substitute and place the numbers for the letters which reflected on the right-hand side of the equation but swapped the letters and numbers around. In the first expression on the left-hand side of the '=' sign, for 'a + b' he wrote '2 + 3', but followed this on the right-hand side of the '=' sign with '3 + 2'. This implies that he has swapped the '3' and '2' for 'a' and 'b', which is counter to what he did on the left-hand side expression. It is clear he grasps the notion of equality and has balanced the equation, but that his implementation of the accurate substitution of the letters for the numbers was careless.

EVDM confirmed that the screencast interventions were an advantage as they animated the scale which indicated a balancing of the left- and right-hand sides of an equation. He also felt that it galvanised his understanding of letters in number sentences and the role they play in representing an unknown variable. The additional benefit of the screencasts was in allowing him to revisit and replay them when necessary; the video/audio combination was useful as it talked through the examples step-by-step, while unfolding on the page.

d) The operations on both sides context (e.g. (N) 5 + 4 = 6 + 3; (A) c + 10 = 20 - c)

Question 8 (Calculate: 10 x 3 = ____ ÷ 2)
Pre-intervention screencasts

EVDM calculated this equation accurately by balancing the expressions on either side of the '=' sign. He is comfortable with the notion of equality, understands what it represents, and has avoided the trap of providing an "answer" to '10 x 3', a common mistake at this level as children tend to provide an 'answer' when presented with the '=' sign located in this position. Although he completed this task accurately, he did write the result, the missing number, in the line below his calculation as follows: '10 x 3 = 30' then realising he needed to balance the left- and right-hand sides of the equation, he corrected it by writing '= 60 ÷ 2' in the following line, so was able to balance the equation.

Post-intervention screencasts

EVDM calculated the left-hand side of the equation first in order to determine what was required on the right-hand side to make the equation true, or balanced. He did this by multiplying the two terms to each '10 x 3' to produce '30'. He then reversed the operation on the right-hand side of the equation by subtracting '2' from '30' (he mistook the operations so I treated it as being how he answered it), giving him '28', which he reasoned when added back to '2' would provide him with the outcome of '30', so balancing the two sides of the equation.

At ease working with and manipulating numbers, he prefers numbers only, versus equations that include letters, the unknown variables. He cited as beneficial the visual imagery of the animated scale of balancing the left-hand side with the right-hand side of an equation in the screencast interventions. The screencasts assisted him in developing a notion of equality which would have come through strongly from numbers four, five, and six. EVDM listed the following main advantages to the screencasts, even when solving equations that did not require solving for an unknown variable:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

This positioning of the '=' sign was understood to indicate that the terms on either side of the '=' sign would produce the same value when simplified thereby balancing the left-hand side with the right-hand side of the equation. This conceptualisation was evident in how he solved the sum avoiding the trap of providing an "answer" to '10 x 3'.

Question 11 (Calculate: ____ + 20 = 21 + 35)

Pre-intervention screencasts

EVDM calculated this equation accurately by completing the equation using mental mathematics skills. He has a strong notion of equality and understands the need to balance the left- and right-hand sides of the equation. as is evident through his mental manipulation of the operation '+ 20' when using the expression to calculate the difference between the two sides of the '=' sign. A common error at this level is for students to assume '21' is the answer which means they will use '1' as the missing number for example: '1 + 20 = 21 +35'.

Post-intervention screencasts

Once again, EVDM had little difficulty completing this question. He first calculated the expression on the right-hand of the '=' sign which enabled him to determine what was required to make the equation true and to balance the two expressions on either side of the '=' sign. He cited the visual imagery of the animated scale balancing the two sides of an equation as useful confirming his thought process regarding the '=' sign and the notion of equality. The screencasts have assisted in developing the idea of equality from numbers four, five, and six. EVDM listed the main advantages to the screencasts even when solving equations that did not require solving for an unknown variable:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and

- The ability to stop them at any point, rewind, and replay

The positioning of the '=' sign in this question was understood to indicate that the terms on either side of the '=' sign would produce the same value when simplified to balance the left-hand side with the right-hand side of the equation. This notion of equality was evident in how he solved the sum without falling into the trap of providing '21' as an answer to the expression on the left-hand side of the equation, for example: '___ + 20 = 21 +35'. Upon solving each of the expressions EVDM wrote the answer below each of the expressions, on a separate line as an '='s the answer' So, when he calculated the expression '35 + 21' he followed it with '= 56'. In the line below this calculation, he wrote '56 - 20 = 36'. He concluded with '36 + 20 = 31', which is incorrect and confusing because he has a strong sense of numbers. This could be careless error, because immediately alongside his solution he wrote 'A = 36' but did not end the calculation with a balancing of the two sides of the equation: he calculated the solution to the problem. Given his sound knowledge of equivalence, this seems habit and not misunderstanding or what he believes should be.

Question 9 (Find the value of a if 10 + 2a = 100 + 20)

Pre-intervention screencasts

EVDM could not complete this equation going as far as placing a "box" above and in between the coefficient '2' and the unknown variable 'a'. He would have done this drawing upon Foundation Phase mathematics. He then added '100 and 20' to each other, the result of '120' written below his initial statement. He left the calculation here without a further attempt at solution.

Post-intervention screencasts

EVDM accurately manipulated the numbers and operations across the '=' sign reflecting an understanding of the need to balance the left-hand side with the right-hand side of the equation. There is an expression on either side of the '=' sign in this example which did not deter his thought process, even though this problem requires the handling of a letter representing the unknown variable. He calculated the expression on the right-hand side of the '=' sign, providing a solution of '120 following the '=' sign then continued by subtracting '10' from '120' in the next line. By a process of deduction he was able to use his mental mathematics abilities to conclude that '2a = 2 x 55 = 110'. Finally, next to his general working out and scribbling, he concluded that 'a = 55'. This indicates conceptual understanding of the role of the '=' sign and the notion of equality, and also the letter 'a' used to represent the unknown variable. While performing the calculation he excluded the term '2a' from the equation only manipulating the numbers. Again, as he has done throughout, he isolated the expression(s) and performed the calculations on the left-hand side of the '=' sign. He then provided the solution or an 'answer' to the right of the '=' sign. Even though they may have an absolute understanding of equivalence, this reflects a common occurrence among students at this level. This is more out of habit as opposed to believing that an answer must follow the '=' sign, particularly in students who display a notion of equality. He confirmed that the screencast interventions were an advantage as they animated the scale, strengthened his understanding of letters and their role, and the balancing of the left-hand side with the right-hand side of the equation. EVDM indicated that screencast interventions assisted his progress in conceptualising the ‘=’ sign and the letter (the unknown variable). The specific screencast numbers that would have assisted in understanding these concepts would have been one, four, five, six, seven, eight, nine, and eleven as they show how to manipulate terms and operations across the '=' sign in order to solve for the unknown variable. He listed the main advantages of having had access to the screencasts:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

Although they assisted in garnering an understanding of equations and equivalence, his method of calculation was slightly different from those of the screencasts, yet he was still able to incorporate them into his understanding of how to arrive at a solution for 'a'. He was not deterred by the location of the two expressions on either side of the '=' sign, nor the fact that there did not seem to be an answer. The position of the '=' sign in this context, together with the inclusion of the letter, can create misunderstanding for students who have not conceptualised the notion of equality nor the idea of symbol sense and the manipulation of terms across the '=' sign. They may be prone to look for an expression to the left of the '=' sign to satisfy the need for an answer to follow the said '=' sign. Over and above these advantages EVDM found it useful that examples/explanations were explained succinctly step-by-step as they unfolded in the screencasts.

Question 16 (Find the value of b 154 - 104 = 51b - 1)

Pre-intervention screencasts

EVDM could not complete this equation successfully. He has not had success with equations that have terms in which a number and a letter are joined (i.e.: ‘51b’). This could be due to the complicated context of this equation, in that the letter is presented in an equation with an expression on either side of the '=' sign. At this level, an example such as this will be beyond the understanding of students who would not have been exposed to algebra, requiring numbers and letters to be manipulated across the ‘=’ sign. Even then, an example such as this brings confusion from a term such as '51b', requiring a sound concept of equivalence and symbol. As a possible solution for ‘b’ in his mind, EVDM wrote ‘$b = 51^1$’, seemingly a random solution to indicate that he has set something down just for the sake of it. This could be that he understands that ‘51 to the power of 1’ is still ‘51’, which is correct, if that is what was required, but this does not satisfy the equation and the context of the problem.

Post-intervention screencasts

EVDM used his mental mathematics abilities to deduce the unknown variable 'b'. He first solved the expression he knew, on the left-hand side of the equation then used this information to work out that he had to multiply '51' to '1' to produce a number in order to make the equation true, which would happen by subtracting ‘1’ from ‘51’. Below the line of calculation, ‘154 - 104 = 50’, he wrote ’51b = 51 x 1’; then, to the right he completed the equation displaying the number ‘1’ which he substituted for the letter ‘b’. It read as follows: ‘154 - 104 = 51 x 1 -1’. This indicates a clear understanding of the ‘=’ sign and the notion of equality and also the letter ‘b’ which has been used to represent the unknown variable. He has shown a balancing of the equation, by balancing the left-hand side with the right-hand side recalling similar examples from the screencast interventions and confirming a definite advantage in that they animated the scale and his understanding of letters and their role in balancing the left-hand side with the right-hand side of the equation. EVDM also indicated that the screencast interventions assisted his progress in conceptualising the ‘=’ sign and the letter, the unknown variable. The specific screencast numbers that would have assisted his understanding of these concepts would have been one, four, five, six, seven, eight, nine, and eleven as they are explicit about how to manipulate terms and operations across the '=' sign in order to solve for the unknown variable. He listed the following as the main advantages of access to the screencasts:

- The ability to revisit the screencasts at his own leisure;
- The combination of the visual/audio; and
- The ability to stop them at any point, rewind, and replay

Although they assisted a better understanding of equations and equivalence, his method of calculation was slightly different from those of the screencasts though he incorporated them into his understanding of how to arrive at a solution for 'b'. He was not deterred by the location of the two expressions on either side of the '=' sign, nor that there did not seem to be an “answer”. The position of the '=' sign in this context, together with the inclusion of the letter, can create misunderstanding among students who have not conceptualised equality, nor the idea of symbol sense and the manipulation of terms across the '=' sign. They may look for an expression to the left of the '=' sign to satisfy the need for an answer to follow the said '=' sign. He enjoyed the combination of video and audio which was useful in this respect as it explained examples, step-by-step, as they unfolded on the screen. The screencasts were also beneficial in that they helped him remember that the term '51b' implies that you multiply the coefficient to the letter.

4.3 HORIZONTAL ANALYSIS OF PARTICIPANTS

4.3.1 Solving Algebraic Equations: Operations equal answer context

a) The operations equals answer context [Q1: 3c x 7 = 21; Q6: 6a + 4 = 58]

In this first cluster, which included Questions 1 and 2, all but two of the candidates solved the equations by using a method that replicated what they would have seen in the screencast interventions. This method would involve the transformation of operations, unknown variables, and terms across the '=' sign that allow for the solving of the unknown variable in each of the equations essentially by isolating the letters 'c' and 'a'. After watching the screencast interventions five of the seven candidates were able to interpret the meaning of the letters, the unknown variables, and the notion of equality effectively. In this cluster of questions the coefficient attached to the unknown variable was understood to mean that they were multiplied by each other (i.e.: '3c = 3 x c', '6a = 6 x a') by these five candidates. Four of the five candidates solved for the unknown variables using mental mathematics and arrived at the solution via deduction. By looking at the equation, they recognised 'a number multiplied by an unknown number' and either added or multiplied to another term (a constant) was an expression that would yield a value that was the same as a value on the right-hand side of the '=' sign. Only one of the candidates solved these equations with a more formal method comprised of writing out his workings of the manipulation of terms and operations across the '=' sign until each of the unknown variables 'c' and 'a' were solved.

The two candidates who were unsuccessful in solving this cluster of algebraic equations either ignored the question or discarded the letters and only manipulated the numbers. The discarding of the unknown variable was a method used by the majority of the candidates prior to the screencast interventions.

4.3.1.1 Advantages of the Screencasts

The five candidates who were successful in solving the algebraic equations indicated that they had benefitted from the experience of having had access to the screencast interventions. Although their methods of calculation were not exact replicas of what they would have seen in the screencasts, it is evident in their reasoning and how they arrived at a solution that they had studied the screencast interventions. They did not feel the equations were over-complicated hence their use of mental deduction to solve for the unknown variables. They all shared the same sentiment, separately, that the screencast interventions assisted their understanding of an equation presented in this context, in the following, varied, ways:

- Assisted with a better understanding of the structure of equations;
- An understanding of the notion of equality/equivalence;
- An understanding of the idea of 'symbol sense';
- Easy to understand and follow;
- Audio/visual combination was significant in that the example unfolded on screen as it was being explained;
- The ability to replay, revisit, and recall the particular screencast;
- More engaged with the content and what needs to be understood;
- The manipulation of an equation;
- Made algebra seem easier than they had deemed it to be;
- The balancing of an equation in algebra; and
- The animated scale which explained the point above.

The two candidates that were unsuccessful in their attempt to solve for the unknown variables indicated that they could not recall the screencasts or that there is an advantage, but that they could “just not pinpoint them”.

4.3.1.2 Disadvantages of the Screencasts

Six of the seven candidates did not feel that there were any disadvantages to the screencasts, although one of these six candidates could not recall any of the screencast interventions relating to Questions 1 and 6. One of the candidates, answering Question 1, noted that he feels there would be a lack of interaction between student and teacher if the student needed further assistance. This disadvantage was highlighted again by the same candidate in Question 6 although he was comfortable with how he was to solve the equation and the assistance from the screencasts. He felt that in more complicated examples where further assistance may be required the lack of human intervention and assistance may be a disadvantage.

4.3.2 Solving Algebraic Equations: Operations on the right-hand side context

***b) The operations on the right-hand side context (Q3:* 125 = c + 77*; Q7:* 28 = 3b + 4**

This cluster of questions included two variations:

1. Question 3 with an unknown variable 'c' as a standalone; and
2. Question 7 with an unknown variable 'b' attached to a number, as a coefficient, '3', implying that the two are multiplied to each other.

In the first variation, all seven candidates were successful in calculating and solving for the unknown variable. It was understood that the unknown variable, 'c' in this case, would complete the equation and balance the left- and right-hand sides of the '=' sign. Six of the seven applied mental mathematics and only wrote in the answer while one followed a more formal approach by indicating the transformation of terms and operations across the '=' sign showing that what you do to the one side you do to the other because of the '=' sign.

In the second variation four of the candidates were able to solve for 'b' successfully and without hesitation. Three of the successful candidates did so by using a method that involved a form of mental mathematics. They marked a few calculations down on the page and realised what number was missing in order to make the equation true by balancing the left- and right-hand sides. It is clear that they understand what is required in terms of the mechanics of algebra. Although they did not write their calculations out as they would have seen in the screencast interventions, they performed calculations that implied their conceptualisation of the process as having made meaning of the screencast interventions to assist them in arriving at the correct value for 'b'. One of the successful candidates was more formal and articulate in his approach when calculating the missing number and solving for 'b'. He followed the procedures from the screencast interventions by showing his working out step-by-step as he manipulated the terms and operations across the '=' sign. He has shown that any action to the one side of the equation ('=' sign) must be met with the same action on the other side of the '=' sign, implying a strong notion of equality.

The second variation proved more challenging for two of the candidates in that they were uncertain as to how to manipulate the terms and operations in order to solve for the unknown variable 'b'. It is clear that there is no comprehension of what the letters represent as confirmed by their answers to this specific question in the interviews. One response was that it meant that we are "dealing with algebra", while the other was confused by the role of the letter. Both candidates failed to understand the meaning and role of the '=' sign in an equation believing that an answer follows the '=' sign and, hence, and grappled with the more complicated problems with which they were presented. One candidate wrestled with this idea, and realised that there is more required when presented with an equation of this nature. He verbalised the need to follow the required procedures to solve the equation, but does not know how or where to start the process.

4.3.2.1 Advantages of the Screencasts

The five candidates who were successful in solving the algebraic equations indicated that they had benefitted from having had access to the screencast interventions, particularly for Question 7. All seven suggested that Question 3 was straightforward enough not to have had to rely on the screencast interventions. Although their methods of calculation were not exact replicas of what they would have seen in the screencasts, it is evident in how they reasoned and arrived at a solution that they had studied the screencast interventions. They did not feel that these equations were over-complicated, hence, their use of mental deduction. They all shared the same sentiment, separately, that the screencasts assisted their understanding of an equation presented in this context in the following, varied, ways:

- Assisted with a better understanding of the structure of equations;
- An understanding of the notion of equality/equivalence;
- An understanding of the idea of "symbol sense";
- Easy to understand and follow;
- Audio/visual combination was significant in that the example unfolded on screen as it was being explained;
- The ability to replay, revisit, and recall the particular screencast;
- More engaged with the content and what needs to be understood;
- The manipulation of an equation;
- Made algebra seem easier than they had deemed it to be;
- The balancing of an equation in algebra; and
- The animated scale which explained the point above.

The two candidates who were unsuccessful in their attempt to solve for the unknown variables in Question 7 indicated that they could not recall the screencasts so could not draw any advantage from them.

4.3.2.2 Disadvantages of the Screencasts

Six of the seven candidates did not feel there were any disadvantages to the screencasts. One of the candidates described his sense that there would be a lack of interaction between student and teacher if the student needed assistance. This disadvantage was highlighted by the same candidate in questions 3 and 7. He was comfortable with how he had to solve the equation and the assistance from the screencasts and felt that in more complicated examples, when further assistance may be required the lack of human intervention may be a disadvantage.

4.3.3 Solving Algebraic Equations: The Reflexive context

c) The reflexive context (Q4: a = a; Q10: a + b = a + b)

All seven candidates could demonstrate a solution to Questions 4 and 10. Five candidates were comfortable in their recognition of the need to ensure a balance was maintained between the left- and right-hand sides of each equation. A few of these candidates also indicated that they understood that they could have used any item(s) to reflect an equivalence between the two sides, but because they were dealing with mathematics they thought it best to use numbers. One or two used everyday items, such as fruit and other random objects, while the rest used numbers. The two candidates, having grappled with the notion of equality and the unknown variable, sought for an answer and struggled to see the question for what it was: what numbers or everyday objects can be used to balance the left- and right-hand side of an equation? One of the five successful candidates did mix up numbers in Question 10, but this was careless error seeing that he recognised the letters and substituted the same values on either side, mixing them only as would have been seen in his vertical analysis. Encouragingly five of the seven candidates were resolute in their understanding of the role and the meaning of the '=' sign and the unknown variable and completed the questions unhesitatingly and with confidence.

4.3.3.1 Advantages of the Screencasts

Five of the candidates indicated that they had benefitted from access to the screencast interventions particularly for Question 10. All seven implied that Question 4 was straightforward enough not to have had to rely on the screencast interventions although they confirmed that access was useful. No working out or calculations were required for either of the two questions, but it is evident in their reasoning and how they arrived at interpretation that they had studied the screencast interventions. They did not feel that Question 4 was over-complicated so responses were split in terms of whether the screencast interventions were beneficial or not. From Question 10, only one candidate saw no advantage because the questions were difficult enough to have to reflect back to the relevant screencasts. All but one candidate shared the same sentiment, separately, that the screencasts assisted their understanding of an equation presented in this context, in the following varied ways:

- Assisted with a better understanding of the structure of equations;
- An understanding of the notion of equality/equivalence;
- An understanding of the idea of “symbol sense”;
- Easy to understand and follow;
- Audio/visual combination was significant in that the example unfolded on screen as it was being explained;
- The ability to replay, revisit, and recall the particular screencast;
- More engaged with the content and what needs to be understood;
- The manipulation of an equation;
- Made algebra seem easier than they had deemed it to be;
- The balancing of an equation in algebra; and
- The animated scale, which explained the point above.

Two of the candidates felt that they did not need to draw on the screencast interventions for these two examples so did not highlight any advantages. In addition to these two, one other candidate felt no advantage in the screencast interventions specifically for Question 4.

4.3.3.2 Disadvantages of the Screencasts

No disadvantages were highlighted.

4.3.4 Solving Algebraic Equations: Operations on both sides context

d) Operations on both sides context (Q9:* 10 + 2a = 100 + 20*; Q16:* 154 - 104 = 51b - 1*)

In this last cluster which included Questions 9 and 16, all but two of the candidates solved the equations with a method that replicated what they would have seen in the screencasting interventions. This would involve the transformation of operations, unknown variables, and terms across the '=' sign that would allow for the solving of the unknown variable in each of the equations essentially by isolating the letters 'a' and 'b'. After watching the screencasting interventions five of the seven candidates improved their ability to interpret the meaning of the letters, the unknown variables, and the notion of equality. In this cluster the coefficient attached to the unknown variable was understood to mean that they were multiplied by each other (i.e.: '2a = 2 x a', '51b = 51 x b') by the five candidates. Four of the five solved for the unknown variables using mental mathematics, first simplifying the relevant expression to produce a value, either on the left- or right-hand side of the '=' sign, which assisted their deduction of the solution. By looking at the equation they recognised that "a number multiplied by an unknown number" and either added to or subtracted to/from another term, a constant, was an expression that would yield a value that was the same as a value on the right-hand side of the '=' sign. Only one of the candidates solved these equations by using a more formal method which consisted of writing out the manipulation of terms and operations across the '=' sign until each of the unknown variables, 'a' and 'b', were solved.

The two candidates who were unsuccessful in solving this cluster of algebraic equations either ignored the question completely and left it out or discarded the letters and manipulated the numbers only. The discarding of the unknown variable was used by the majority of the candidates prior to the screencast interventions. One of these two candidates simplified the expression to the left or right of the '=' sign, but was then uncertain as to how to proceed beyond that point.

4.3.4.1 Advantages of the Screencasts

The five candidates who were successful in solving the algebraic equations indicated that they had benefitted from their experience of having had access to the screencast interventions. Although their methods of calculation were not exact replicas of what they would have seen in the screencasts it is evident in how they reasoned and arrived at a solution that they had studied the screencast interventions. Three of these five candidates displayed a complete turnaround from their initial diagnostic test in solving these equations feeling that the screencast interventions were a significant advantage. The other two candidates realised the solutions with success in their initial diagnostic test and post the screencast interventions. All five shared the same sentiment, separately, that the screencasts assisted their understanding of an equation presented in this context, in the following, varied, ways:

- Assisted with a better understanding of the structure of equations;
- An understanding of the notion of equality/equivalence;

- An understanding of the idea of “symbol sense”;
- Easy to understand and follow;
- Audio/visual combination was significant in that the example unfolded on screen as it was being explained;
- The ability to replay, revisit, and recall the particular screencast;
- More engaged with the content and what needs to be understood;
- The manipulation of an equation;
- Made algebra seem easier than they had deemed it to be;
- The balancing of an equation in algebra; and
- The animated scale which explained the point above.

The two candidates who were unsuccessful in their attempt to solve for the unknown variables indicated that they could not either recall the screencasts or that there is an advantage, but could “just not pinpoint them”.

4.3.4.2 Disadvantages of the Screencasts

Six of the seven candidates did not feel disadvantages to the screencasts, although one could not recall any of the screencasts or examples relating to Questions 9 and 16. One of the candidates recorded his sense that there would be a lack of interaction between student and teacher if the student needed further assistance. This disadvantage was highlighted in both questions, although he was comfortable with how he had to solve the equations and the assistance from the screencasts but felt that in more complicated examples, where further assistance may be required, the lack of human intervention and assistance may be a disadvantage.

4.4 CONCLUSION

In conclusion, it became evident that five of the seven participants completed the screencast intervention course with improved success in the post-intervention test. The two participants who were not as successful confirmed that they had not completed watching all seventeen screencasts and this was apparent in their answers during the think-aloud interviews and in the way they answered the post-intervention test tasks. There was evidence that the animations in the screencast interventions were met with a favourable response, with the five participants who had success in the post-intervention test citing the animated scale as a factor in assisting and/or enforcing their conceptualisation of equivalence. The details of the various components outlined in the Literature Review for consideration will be elaborated on in the following chapter, where the findings of this research project have been consolidated.

CHAPTER 5

FINDINGS AND CONCLUSIONS

5.1 INTRODUCTION

In this chapter findings from the research are consolidated with respect to the original research question and with reference to the theoretical framework and methodological approach.

> *Choosing a research problem through the professional or personal experience route may seem more hazardous than the suggested [by faculty] or literature routes. This is not necessarily true. The touchstone of your own experience may be more valuable an indicator for you of a potentially successful research endeavour.*
>
> (Strauss and Corbin, 1990: 35 - 36)

Both the limitations and the significance of the research are included with recommendations for future research.

5.2 FINDINGS OF THE STUDY

The fundamental components of an early algebraic equation have been considered as relevant to a Grade 7 student in order to answer the questions of this project:

(1) How do selected Grade 7 participants solve algebraic equations as a result of participating in a screencast intervention;

(2) What are the advantages and disadvantages of using the screencast approach in teaching early algebra.

Each component has been analysed using an analytic framework based on a taxonomy designed according to the four identified "clusters".

5.2.1 Cluster 1: Operations equal ('=') an answer context

The Notion of Equality

The participants had no problem providing solutions to the "Numeric-only" Task 5, although two of the seven participants indicated that the equal sign '=' implies that an answer must follow the operation(s), while two of the participants who displayed a strong notion of equality, indicated that they believed that an answer must follow the equal sign ('='). The balance of participants indicated a strong notion of equality, stating that they understood the equal sign

('=') to refer to a balancing of the left- with the right-hand side of an equation. All seven participants provided the correct solution, with four implementing a vertical subtraction method, while the other three completed the number sentence by filling in the solution in the space provided.

The participants had no problem providing solutions to the “Numeric-only” Task 12, although two of the seven participants indicated that the equal sign '=' implies that an answer must follow the operation(s). The two participants who displayed a strong notion of equality, and who indicated that they believed that an answer must follow the equal sign ('=') in Task 5, changed their minds while performing this task recognising that the equal sign ('=') referred to equivalence. Five of the participants indicated, therefore, a strong notion of equality, stating that they understood the equal sign ('=') to refer to a balancing of the left- with the right-hand side of an equation. All seven participants provided the correct solution to the problem, three of them implementing a vertical multiplication method, while the other three completed the number sentence by using varying methods of calculation.

There was a definite difference between the success in the participants' pre- and post-intervention tests. In Task 1 and Task 6 five participants were comfortable with the equal sign ('=') representing equivalence between the left- and right-hand side of an equation, and indicated the need to balance the two sides with each other. The location of the equal sign ('=') did not interfere with the participants' thought processes. Two participants indicated that it "probably means that an answer must follow" but they failed to solve the equation as their conceptual understanding of the equal sign ('=') was misguided and they failed to understand the representation of symbols like the unknown variable. The other five participants solved the equations in both tasks, 1 and 6, successfully. By Task 6, one of the two unsuccessful participants began to realise that there was more to the equal sign ('=') than he had initially believed.

Transformation

For the “numeric-only” tasks, this was not a major factor in solving for the missing number in the equation as the participants indicated a preference for working with numbers only; because they could see the answer immediately there was no need to manipulate any terms or unknown variables across the equal sign ('=').

The five participants who were successful in solving for the unknown variable used various methods; all agreed that to solve for the unknown number manipulation across the equal sign ('=') would be necessary. Given the location of the equal sign ('=') and the structure of the equations, they felt they could solve Tasks 1 and 6 mentally. Even when solving these tasks mentally, they would have incorporated concept images in their mind's eye to produce the solution. They felt that the animated scale assisted meaning-making of the algebraic problems and indicated the need to transform what you do to the one side of an equation must be replicated on the other side of the equation.

Alphanumeracy and "Symbol Sense"

The concept of the unknown variable and "symbol sense" prior to the screencast interventions turned out to be a mystery to all but one participant who had learned about this at his previous school. The post-intervention test indicated that six of the participants understood the unknown variable to represent an unknown number and, more specifically, that it would need to be multiplied to its coefficient when necessary. One of the participants implied that he was not convinced by what it represented, that it "*probably* represents an unknown number" and that it provided some sort of clue to an answer. Five participants provided the correct solution for the unknown variable while two could not solve the equation.

5.2.2 Cluster 2: Operations on the right-hand side of the equal sign ('=') context

The Notion of Equality

The "numeric-only" Tasks, Tasks 2 and 14, were completed successfully by all participants. Four of them solved the expressions on the right-hand side first before providing a solution to the left-hand side of the equation. Interestingly, they rewrote the expression to the left of the equal sign ('=') deeming an answer should follow the equal sign ('='); This could be because we read from left to write so the structure makes sense for younger inexperienced students. The balance of the participants merely filled in the missing number in the space provided. The same five participants from the previous cluster understand the notion of equality and the need to balance equations, while two still understand that an answer must follow an expression. Given the location of the equal sign ('='), that there was only one expression evident in the equation, and that no other numbers were available to the left of the equal sign ('='), it did not seem misplaced for these two participants to read the equation "backwards".

There was a definite difference between the success in the participants' pre- and post-intervention tests. In Task 3 and Task 14 five participants were comfortable with the fact that the equal sign ('=') represents equivalence between the left- and right-hand side of an equation indicating the need to balance the two sides with each other. The location of the equal sign ('=') did not interfere with the participants' thought processs which provides further evidence that a strong sense of equivalence has developed for these participants as the context of equation can be confusing as it may seem to be "inverted" or" "back-to-front". Two participants indicated that it symbolises that an answer must follow but for Task 14 one of the two participants began to reason as to whether the equal sign ('=') changes the operations and, therefore, he thinks it changes the equations in some respect. Both participants failed to solve the equation as their conceptual understanding of the equal sign ('=') was misguided, coupled with their failure to understand the representation of symbols like the unknown variable. The other five participants successfully solved the equations in Task 3 and Task 14.

Transformation

For the "numeric-only" tasks this was not a major factor in solving for the missing number in the equation as the participants indicated a preference in working with numbers only because they could "see" the answer immediately so there was no need to manipulate any terms or unknown variables across the equal sign ('=').

All seven participants were successful in solving for the unknown variable and used mental maths to do so. All agreed that in order to solve for the unknown number, manipulation across the equal sign ('=') would be necessary in complicated circumstances. Even though the location of the equal sign ('=') and the structure of the equations were slightly different from that to which they had been exposed prior to the screencast interventions they felt comfortable solving Task 3 mentally. Even in solving this Task they would have incorporated concept images in their mind's eye to produce their solution. The participant who has had previous encounters with algebra solved this equation step-by-step by systematically following the steps he would have encountered in the screencast interventions. Five participants solved Task 7 successfully and understood the requirement of isolating the unknown variable so they could manipulate operations and terms across the equal sign ('=') where necessary. They found the animated scale assisted their meaning-making of the algebraic problems indicating the need to transform what you do to the one side of an equation must be replicated on the other side of the equation. This assisted their conceptualisation of a term such as '3b' and its structure. Two participants could not comprehend the concept of the term, '3b'and, which meant that he could not make meaning of, or progress towards, solving this equation.

Alphanumeracy and "Symbol Sense"

Prior to the screencast interventions, the concept of the unknown variable and "symbol sense" were a mystery to all but one participant and he had knowledge of this from his previous school. The post-intervention test, on the other hand, indicated that six of the participants understood the unknown variable to represent an unknown number and, more specifically, that it would need to be multiplied to its coefficient when necessary. One of the participants implied that it "*probably* represents an unknown number" and provided some sort of clue to an answer. One of the participants believed it to be a representation of algebra, and five participants provided the correct solution for the unknown variable being able to manipulate the equation in order to isolate it. This indicates a conceptualisation of the role of the unknown variable in this instance as it had a coefficient attached to it, which is not always easily understood. Two of the participants could not solve the equation suggesting their having missed the conceptual meaning of what it represents.

5.2.3 Cluster 3: Reflexive context

The Notion of Equality

All seven participants completed Task 13 with success having a solid conceptual understanding of the notion of equality; by delving into their understanding of the concept of the notion of equality, however, it was evident that only five of the participants completely understood what the equal sign ('=') represents. The two participants who were able to complete the equation and provide a simplified solution for each side of the equal sign ('=') indicated that the equal sign ('=') symbolises that an answer must follow or that it is a "replica in a different language". The fact that two addends were present on either side of the equal sign ('=') guided them to a solution to Task 13 because their struggle to provide a solution to Task 15 indicates a lack of conceptualisation of the notion of equality. The balance of the participants could provide a solution to Task 15 by using varying methods to indicate how to balance the left- with the right-hand side of the equation. They recalled the animated scale symbolising the balancing of the two sides of an equation which assisted and endorsed their conceptual understanding of the notion of equality.

The five participants provided suitable examples that represent the notion of equality on either side of the equal sign ('=') implying a thorough understanding of the concept for Tasks 4 and 10. The two participants who failed to comprehend the notion of equality provided some explanation; for Task 4 they both indicate that the equal sign ('=') must be preceded by a sum and that they need to find an answer to satisfy their understanding of the equal sign ('='). For Task 10 the same participants explained that the equal sign ('=') is used to balance the equation, although one of their responses implies that they are still searching for an "answer" to satisfy their understanding of what the equal sign ('=') represents. Five participants revealed conceptual understanding of the equal sign ('=') as representing the notion of equality which they credited the animated scale in doing.

Transformation

Transformation did not play a significant role in the two 'numeric-only' tasks as they were straight forward enough not to have to manipulate terms across the equal sign ('=').

The same applied to the algebraic tasks as there was no need to manipulate terms or operations across the equal sign ('='). Mechanically both tasks were straight forward enough requiring only a brief explanation as opposed to providing solutions.

Alphanumeracy and "Symbol Sense"

Six of the participants indicated that the letter represented the unknown variable which they substituted for numbers or other objects implying a conceptualisation of what the letter represents and its role in an equation. Two participants substituted inaccuracy between the left- and right-hand, although this was probably more the result of careless error than a misunderstanding of the concept, although the same values were used. One of the participants stated that the letters are what makes algebra, algebra and that it "gives us a clue."

5.2.4 Cluster 4: Operations on both sides context

The Notion of Equality

Six of the participants completed Task 8 successfully, while the one participant who did not displayed the typical misunderstanding of the equal sign ('='). His solution showed his belief that an answer must follow the equal sign ('='), avoiding the need to balance the equation, and meaning that he has misunderstood the concept of the notion of equality. Five of the participants indicated a strong notion of equality and have conceptualised what the equal sign

('=') represents, while two stated that an answer must follow the equal sign ('='), which is interesting in that only one of these two participants fell into the trap of providing an answer to '3 x 10 = ___ ÷ 2'.

Five of the participants were successful in completing and balancing the left- with the right-hand side of the equations in Tasks 9 and 16. This provided excellent insight into their comprehension and conceptualisation of the notion of equality, given the complexity of the equations. Two of the participants were unsuccessful in their attempt to balance the two sides with each other though they were beginning to reason that the two expressions on either side of the equal sign ("=") may need to "=" each other in some way. They could not get past the comprehension of the terms in each of the tasks that had a coefficient attached to an unknown variable. All five participants who conceptualised the notion of equality cited the animated scale as an aid in confirming their understanding of what the equal sign ('=') truly represents and assisting in their solving of the complex equations.

Transformation

Transformation did not play a significant role in the two "'numeric-only" tasks as they were straightforward enough not to have to manipulate terms across the equal sign ('=').

Five of the participants cited the animated scale as having assisted their understanding of the need to balance an equation and considering the complexity of the two tasks it would have been necessary to do so. Six of the seven participants used manipulation of terms and operations across the equal sign ('='), which indicated their conceptualisation of the "transformation" process, while one participant could visualise the transformation in his mind's eye so could provide a solution to the equation. Five participants could manipulate the equation and solve for the unknown variable by applying what they had access to by way of the screencast interventions. They were able to recall the examples from these interventions and applied them with success. The two unsuccessful participants could not apply what they had seen in the screencast interventions and could not recall any examples of the like. They also implied that they did not remember coming across any animated scale(s) in the screencast interventions. This meant they were unable to manipulate the terms and operations in Tasks 9 and 16 so were at a loss to try to solve for the unknown variable.

Alphanumeracy and "Symbol Sense"

In Task 9, six of the participants have understood the role of the unknown variable and what it represents. They have conceptualised its meaning and know it is multiplied to the coefficient, once the unknown variable is solved, to produce a solution that will satisfy the equation and balance the left- with the right-hand side. I am not convinced that one of these participants fully comprehends the meaning of the unknown variable as he has struggled to grasp the concept throughout the test. This suspicion was confirmed in Task 16 as he had no concept of the letter and ignored it in the calculation. The other participant who has not conceptualised the role of the letter stated that it "represents algebra". The five participants who conceptualised the unknown variable's meaning all indicated that it represents an unknown number which needs to be solved, and that is done through the mechanics and manipulation of terms and operations across the equal sign ('='). These five were successful in solving for the unknown variable while the other two did not complete either Tasks 9 or 16.

5.2.5 Advantages Of The Screencast Interventions

The five participants who were successful in their solving of the equations and who were comfortable with the notion of equality, "symbol sense," and the transformation of the equations, specifically with those that carried unknown variables, cited that they benefitted from having had access to the screencast interventions. Their turnaround from the initial diagnostic test was significant and the screencast interventions can be credited for that. All five separately shared the same beliefs, that the screencast interventions assisted their concept of the mechanics of an equation presented in this context, in the following, varied, ways:

- Understanding of the structure of an equation;
- Understanding of the notion of equality/equivalence and the balancing of an equation;
- Understanding of the idea of "symbol sense";
- Easy to understand and follow;
- The significance of the audio-visual combination in each screencast and the fact the examples unfolded on screen as it was being explained;
- Ability to replay, revisit, and recall specific screencasts;
- More engaged with the content and what needs to be understood;
- The manipulation of an equation in order to isolate the unknown variable;
- The animated scale, which put a lot of algebra into perspective;
- Made it seem less complicated and easier than what they had believed.

5.2.6 Disadvantages Of The Screencast Interventions

The only disadvantage that was listed was brought up by one of the more successful participants who indicated that there may be a lack of interaction between student and teacher if the student needed additional assistance, and he was concerned least this could apply when complex calculations were being attempted arguing that this is a lack if human interaction is not available.

5.2.7 Mechanisms of Visualisation

Analysis of answers to the various questions proved interesting with specific reference to the algebraic tasks. Given the cognitive load required to solve for an unknown variable, a presentation of a pictorial explanation via images only, would not have elicited the same positive results. While it is documented that technology does assist in the acquisition of knowledge (Henrie et al., 2015; Jordan et al., 2012; Faherty & Faherty et al., 2015), how this assistance is brought about needs to be understood. The experience of this research demonstrated that the mode of delivery of the pedagogy is crucial to the attainment of a concept and while this may be obvious, students do often forget, a few hours later, an important topic covered in classrooms.

With this said, it became abundantly clear that the participants "latched" onto specific aspects of the screencast interventions, that of the animated scale. The animated manipulation of operations and terms, which were accompanied by an audio explanation in the screencast interventions significantly enhanced the ability of participants to "see" the concept unfold on the digital page. Boaler et al., (2016) published a paper that would dispel the myth that visualisation caters for "lower level" mathematics by demonstrating compelling brain evidence proposing that visual mathematics should be "integrated into curriculum materials and teaching ideas across the grades" (Boaler et al., 2016:1); she adds: "good mathematics teachers use visuals, manipulatives and motion to enhance students' understanding of mathematical concepts" (Boaler et al., 2016:2).

The aspect of 'relational' relationship in algebra was easily understood and the conceptualisation thereof was evident. The location of the equal sign ('=') which so often plays an integral role in the conceptualisation of algebra did not deter those participants who completed the screencast intervention programme. Once again, the animated manipulation of the terms and operations, combined with the analogy of the scale, cemented the participants' conception of how to solve early algebraic equations.

5.3 LIMITATIONS

This research project was conducted as an interventionist case study as it was contextualised, being an up-close, in-depth, and detailed examination of a topic in mathematics. The data was collected using tools for a mixed method approach to provide depth and rich data, which, according to Yin (1994) largely defines what a case study intends. The research participants were chosen using purposeful sampling, because they were considered "information-rich" (Patton, 1990:169); all at Grade 7, they ranged between three high, two high medium, and two low achievers, based on their diagnostic performance. According to Creswell (2002), the more diverse the range of individuals allow for many perspectives to be presented that highlights the uniqueness of the individual, but also the intricacies of the world.

Although the purposeful sampling provided potentially rich data and in view of the parameters that make a case study effective, the size of the sample was small in relation to the size of the Grade. Ideally, a wider range of participants would have allowed for a larger perspective, as per Creswell (2002).

The scope of the research and the requirements for participation implied that technology would be necessary to benefit from the screencast interventions. This will prove to be difficult for students who do not have appropriate access to technology. To add to the difficulty of access, the necessary component of delivery is access to the internet. The screencast interventions were "pushed" to the participants via an online educational portal, Google Classroom, a convenient, efficient, and effective system but this does necessitate that this type of programme can only be implemented at a resource-rich school.

In addition to this requirement is the need for a level of maturity among participants. Unfortunately, the two participants who struggled through the post-intervention test confirmed that they did not complete viewing the course of screencast interventions "posted" to them. This suspicion was confirmed by them post-analysis. This research relied on the participants to exercise a fair amount of responsibility and intentionally, I did not spot-check their viewing progress as this may have been deemed to be interference.

5.4 SIGNIFICANCE

> *Algebra is the language for investigating and communicating most of Mathematics...*
>
> (Curriculum and Assessment Policy Statement, Department of Basic Education, 2011:10)

According to CAPS (2011), a central component to Algebra is for students to "achieve efficient manipulative skills that recognize the equivalence between different representations of the same relationship" (CAPS, 2011:10); this is achieved by:

- recognising and interpreting rules or relationships represented in symbolic form; and
- Identifying variables and constants in given formulae and/or equations

There is a specific mention of the “Concepts and Skills” to be developed, which is referred to as “Equivalent Forms” for Algebra at Grade 7 level. It is required, as per the documents (CAPS, 2011), that the students should be able to “determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented in various formats” CAPS, 2011:53), one of which is a number sentence and one of the formats on which are based screencast interventions. This format allows for the manipulation across the equal sign (‘=’) and one can see it happening visually whether using a pencil and paper or via a screencast.

The results of this research project support the idea that the notion of equality, “symbol sense,” and transformation are areas that play a significant role in Grade 7 students’ misunderstanding of how to solve algebraic problems. Of particular significance to their success or failure is the structure of the equation.

5.4.1 The Notion of Equality

Although the majority of the participants were clear in their understanding of the equal sign (‘=’) and its specific role in a number sentence or an equation it was notable that out of this small sample, two participants retained the opinion that it precedes an answer. As previously mentioned there could be an argument that the equal sign (‘=’) may be viewed as being polyvalent, and that it can be seen to precede an answer; this is not the case for algebra from a theoretical perspective, as it is essential for it to be understood to refer to an equivalence between the left- and right-hand side of an equation. The use of the images from the screencast interventions, particularly the animated scale, which symbolised the equal sign (‘=’), balancing the two sides of an equation proved to be highly successful in affirming participants conceptualisation of the notion of equality. For such a diverse group of students, the visualisation strategies proved to have a positive pedagogical effect whether physically present in the classroom or not.

5.4.2 Alphanumeracy and “Symbol Sense”

This notion, as an imperative component to better the understanding of algebra, is not given enough attention, as supported by Arcavi’s (1994) sentiment which asserts that symbolic manipulations must be one of the central issues in algebraic instruction. He poses the question of how students are meant to improve their understanding of algebra, given “the vast lack of sense-making with regard to the inclusion of symbols to arithmetic” (Arcavi, 1994:24). He adds that “most students with a background in algebra, do not resort to symbols as a tool…” and “they are not invoked unless prompted to do so” (Arcavi, 1994:24). Key to his wish is for students to have the confidence and understanding of the situation so they can implement their use effectively. With reference to the unknown variable and what it represents in an equation, the majority of the participants were firm in the conceptualisation of its role and what it represented. Solving for the “standalone” unknown variable proved to be easy enough for all the participants, while the term that had a coefficient attached to the unknown variable needed assistance, which they found to be useful in its mode of explanation via the screencast interventions. The relationships between the different expressions and the rules that apply to ensure their equivalence was observed more effectively with the introduction of the screencast interventions, particularly with the need to isolate the unknown variable from its constant. This proved to be the fundamental hurdle for the majority of the participants’ success prior to the interventions. The use of the animated scale and the audio component of the screencast as the examples and explanations unfolded on the screen improved conceptualisation of the role of the unknown variable and its manipulation across the equal sign (‘=’) when relevant. The visualisation strategies demonstrated a positive pedagogical influence on the discourse regarding “symbol sense”.

5.4.3 Transformation

Referring to Tayor-Cox's (2003) feeling regarding the need for children to be exposed to algebra as early as possible and to be reliant on their understanding of equality as it is at the root of their algebraic discovery and appropriation. This project implemented the balance method which, according to Ngu et al. (2015), requires an operation on either side of the equal sign ('=') or equation. This is explained by what is considered to be at the heart of solving equations and requires a high cognitive load as it involves more than one element when relocating and delocating operations and terms across the equal sign ('=') (Ngu et al., 2015). The animated scale played an important role in assisting the participants' processing capacity with regard to manipulating terms and operations in the equations by reducing the cognitive load. The mental image that they were able to recall conceptualised the process of the need to balance the equations fortifying their understanding that what you do to the one side, should be done to the other. Although the majority of the successful participants completed the equations using a mental mathematics approach, they all indicated the need to isolate and move terms and operations across the equal sign ('=') to which they referred as the scale, in order to balance each side of an equation. The video-audio aspect of the screencast interventions were clear and concise and an aid to their understanding of the concept and the skill required to complete the equation. The visualisation strategies process revealed a positive pedagogical effect on the complicated process of manipulation or the balancing of equations with regard to transformation.

5.5 RECOMMENDATIONS

5.5.1 General

The research project focused on a small sample of the Grade 7 cohort from a resource-rich independent school. It would be valuable to repeat the research with a cohort of students from a school that does not have access to the same resources and students and who are lower achievers, but who would be provided with the necessary tools for the project.

5.5.2 Further Research

There is much scope for the investigation of equations and their structure, with a more thorough approach to alphanumeracy or “symbol sense” and from an earlier age, based on the findings of this research project, and also using visualisation. To this end, and more specifically, a useful project would be to determine how the use of letter symbols conflict with a clear thought process related to numbers. Papadopolous (2019) uses the term “Friendliness with Symbols” (Papadopolous, 2019:214), which describes the way relationships are displayed, generalised, and proved. Through my research project, it was evident, prior to the screencast interventions, that the participants were uneasy with the role of the unknown variables in this context, and how they would manipulate the equations, and, "despite symbols being available as a tool they are not used unless students are prompted to do so..." and "so, it is important for symbols to be readily available and for students to have confidence that these are appropriate tools" (Papadopolous, 2019:214). According to Blanton et al. (2017), a long-term fostering of "variable and variable notation from early schooling might be a way to facilitate symbol sense" (Papadopolous, 2019:214); this supports Arcavi's (2005) call for the "nurturing the search for symbol meaning….before one automatically starts using symbols" (Arcavi, 2005:47).

5.6 CONCLUDING COMMENTS/REFLECTION

Algebra is not given the attention in early grades that is required of a topic that is considered so meaningful to mathematics. What stands suggests that access is deliberately blocked to puzzled pupils. There is also a misconception regarding the shift from arithmetic to algebra as pointed out by Kieran's work (1981, 1989, 1992). Of particular interest are the following points:

- The focus of algebra is on relations rather than calculations;
- It is important to understand inverses;
- An ability to express some situations algebraically in order to solve them;
- Letters and numbers are used together so that numbers may have to be used as symbols, and not evaluated;
- The equal sign has an expanded meaning; in arithmetic it often means to 'calculate' (polyvalent) but in algebra it more often means 'is equal to' or even 'is equivalent to'.

(Watson, 2009:9)

Algebra should not be seen as a "standalone" component of mathematics, or as an add-on for the sake of completing a curriculum, nor should a "top-down" approach be how it is taught. It is a topic that generates fear in students as well as teachers, given its perceived complexity. It should, instead, be seen for the part it plays in problem-solving.

The results from this research project strongly support the need for a very specific and concentrated technique with regard to delivering an algebraic pedagogy. The methods of visualisation implemented in this research demonstrate that they were well received and were successful, particularly regarding "symbol sense" and transformation. The structure of the screencast interventions was important for the acquisition of these concepts and are worthwhile tools to consider as an epistemological application in a classroom or teaching context.

It is my wish that this research project will shed light on the opportunities presented by the innovative ways that knowledge can be built and scaffolded, and maybe we can consider Gaskin's (2016) significant words:

> *Turn each equation into a little story. The mental picture helps your child reason out the relationships between the numbers and symbols.*
>
> (https://denisegaskins.com/2016/06/03/faq-trouble-with-worksheets)

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Appendix A – School Consent form

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ACCESS LETTER REQUESTING PERMISSION TO CONDUCT RESEARCH

George Wienekus
P.O. Box 187
Grahamstown
South Africa
31 March 2019

The School's Principal

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

My name is George Wienekus and I am writing to you as a Master's degree student at Rhodes University (RU) in Grahamstown, South Africa. I will be conducting a research project entitled: '*How selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention'*. The research I wish to conduct for my Master's full thesis requires me to observe, interview and video-record ten Grade 7 students engaging with selected algebraic tasks. The aim of my screencasting intervention programme, which lies at the heart of this study, is to *inter alia* develop practices of how such devices and software may be used to assist the development of conceptual knowledge rather than procedural knowledge in solving algebraic equations. The participation of ten selected Grade 7 students in this study is important as they will provide me with the source of my data. This research will be conducted under the supervision of Professor Marc Schäfer. My intention is to initially engage all the Grade 7 boys in a diagnostic test that will provide me with important information about their level of algebraic knowledge. This information will be used for planning my intervention programme and also to select ten boys to participate in the intervention. The ten boys will span the spectrum of achievement levels of the diagnostic test. The intervention will consist of fifteen lessons on algebraic equations in conjunction with screencasting support that I will produce and make available to the boys at any time they wish.

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This letter serves to seek formal consent from you that I may:

1. engage all the Grade 7 boys in a diagnostic test that will determine the boys' knowledge of algebraic equations. The results of this test will provide me with important information that I need to design and plan my screencasting intervention, and will enable me to select ten participants spanning the entire achievement spectrum of that particular test.
2. Upon completion of the diagnostic test, I will be in a position to select ten participants to partake in the screencasting intervention programme. This intervention program consists of fifteen contact sessions which will take place in the afternoons, so as not to interfere with normal school hours. Integral to the teaching programme I will use screencastings to support my teaching of algebra. The boys will have access to these screencastings on their tablets at any time they wish. Once the intervention programme is complete I wish to engage each of the ten participants in a talk-aloud post-intervention test where the boys will talk me through how they solved each of the algebraic problems they were given. The engagement of the boys with the tasks will be video recorded for analysis purposes. The identity of the ten boys will not be revealed as the camera will be placed behind the boys as I am only interested in their solution strategies and their explanations of these strategies. Their faces will thus not be visible. The real names of the boys will also not be revealed as I intend to use pseudonyms.

I attach a copy of my research proposal which includes copies of the consent and assent forms to be used in the research process. I have received ethical clearance from Rhodes University, a copy of which I have attached to this request. As part of this, I will undertake to seek written permission from individuals whom I wish to record. At no stage during the research will the identity or location of the school, the identities of any of its staff and the identities of any learners be identified. The school and any research subjects referred to will be given pseudonyms. All the material I collect as part of the research will be accessible only to myself and my supervisor. The school may withdraw permission for conducting the research at any time. I would be happy to answer any questions relating to the proposed research project and to address the Senior Management Team if necessary.

The anticipated benefits that will accrue to the participants is that they will experience a new method of teaching that incorporates technology that will assist them to acquire algebraic skills at a more

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conceptual level. The anticipated benefits to the school will be the opportunity to experience a new method of teaching, which can be shared with my colleagues and other teachers. It also provides the school with the status that it welcomes research in education, and being an institution that encourages a growth mindset.

Upon completion of the study, I undertake to provide you and the teachers with access to the research findings. If you require any further information, please do not hesitate to contact me at the details provided below.

Thank you for your time and consideration in this matter. Your permission to conduct this study will be greatly appreciated.

Yours sincerely,

MEd Student and Researcher
Mobile: 083 781 7290
g98W6478@campus.ru.ac.za
Student Number: g98w6478
Rhodes University

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The signature below grants permission for the above mentioned research to be carried out at this school.

I, ______________________ (full name) give permission for my school______________________ to participate in the research project.	
Signature of School Principal: ______________________ Witness: ______________________	Date: ______________ Place: ______________________

STAMP:

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ACCESS LETTER REQUESTING PERMISSION TO CONDUCT RESEARCH

Rhodes University
Drostdy Road
Grahamstown
6139

The School Headmaster
[redacted]

31 March 2019

Dear Mr [redacted]

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

I am a registered Master's student in the Department of Education at the Rhodes University. My supervisor is Professor Marc Schäfer

The proposed topic of my research is: '*How selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention*'.

The objectives of the study are:

(a) To determine and to *inter alia* develop practices, by utilizing a screencasting intervention programme, to establish how such devices and software may be "used to develop conceptual rather than procedural or decorative knowledge" (Larkin & Calder, 2015:1) in solving linear equations.

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I am hereby seeking your consent to conduct this research at your school. To assist you in reaching a decision, I have attached to this letter:

(a) A copy of an ethical clearance certificate issued by the University

(b) A copy the research instruments which I intend using in my research

Should you require any further information, please do not hesitate to contact me or my supervisor. Our contact details are as follows:

Mr. George Wienekus	Professor Marc Schäfer	RUESC
MEd student and researcher	Supervisor	Rhodes University, Research Office, Ethics
Mobile: 083 781 7290	Mobile: 083 409 5580	T: +27 (0) 46 603 7727 F: +27 (0) 86 616 7707
g98W6478@campus.ru.ac.za	m.schafer@ru.ac.za	ethics-commtee@ru.ac.za
Student Number: g98w6478		Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139
Rhodes University	Rhodes University	

Upon completion of the study, I undertake to provide you with a feedback and access to the research findings, if you so desire.

Your permission to conduct this study will be greatly appreciated.

Yours sincerely,

George Wienekus

Appendix B – Parent/Guardian Consent form

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ACCESS LETTER REQUESTING PERMISSION TO CONDUCT RESEARCH

George Wienekus
P.O. Box 187
Grahamstown
South Africa
31 March 2019

Dear Parent/Guardian

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

My name is George Wienekus, a mathematics teacher at your son's school. I am writing to you as a Master's degree student at Rhodes University (RU) in Grahamstown, South Africa. I will be conducting a research project entitled: '*How selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention*'. The research I wish to conduct for my Master's full thesis requires me to observe, interview and video-record Grade 7 boys interacting with selected algebraic tasks. This research will be conducted under the supervision of Professor Marc Schäfer.

This letter serves to seek formal consent from you, as a parent of one of the Grade 7 boys that I may:

1. engage all the Grade 7 boys in a diagnostic test that will determine the boys' knowledge of algebraic equations. The results of this test will provide me with important information that I need to design and plan my screencasting intervention, and will enable me to select ten participants spanning the entire achievement spectrum of that particular test.
2. in the event of your son being one of the ten selected participants, involve him in the screencasting intervention programme. This intervention program consists of fifteen contact sessions which will take place in the afternoons, so as not to interfere with normal school hours. Integral to the teaching programme I will use screencastings to support my teaching of algebra. The boys will have access to these screencastings on their tablets at any time they wish. Once

the intervention programme is complete I wish to engage each of the ten participants in a talk-aloud post-intervention test where the boys will talk me through how they solved each of the algebraic problems they were given. The engagement of the boys with the tasks will be video recorded for analysis purposes. The identity of the ten boys will not be revealed as the camera will be placed behind the boys as I am only interested in their solution strategies and their explanations of these strategies. Their faces will thus not be visible. The real names of the boys will also not be revealed as I intend to use pseudonyms.

Could you please sign the consent below if you grant permission for your son to participate in my research project. Participation in all activities is entirely voluntary and will not take place without written permission from the learner. If you do not wish your child to participate, he will not be disadvantaged in any way. If you do allow your child to participate, he may withdraw any point.

The data that will be collected from this study will be used primarily for my thesis writing, but it may also be used for conference presentations, journal writings or a book chapter. The data from the intervention and the observations that will be carried out will be stored on a password protected computer for five years. All the videos that will be taken during the research process will only be used for the purposes of this research project.

The study will benefit your son by engaging him in a topic that is, traditionally, difficult to grasp at this level and give him an opportunity to experience an intervention that may assist him to develop a better conceptual understanding of the topic as a precursor to what he will experience at high school. From an ethical perspective, I undertake to offer a similar intervention to all the Grade 7 boys who were not selected to take part in the screencasting intervention research project, after the thesis is completed.

Upon completion of the study, I undertake to provide you and the school with access to the research findings, if you so desire. If you require any further information, please do not hesitate to contact me at the details provided below. In parallel to the above request I have also sought consent from the headmaster of the school.

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Yours sincerely,

George Wienekus
MEd student and researcher
Mobile: 083 781 7290
g98W6478@campus.ru.ac.za
Student Number: g98w6478
Rhodes University

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PARENT AND GUARDIAN'S INFORMED CONSENT

INFORMED CONSENT DECLARATION
(Parent or Guardian)

Project Title: *'How selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention.'*

George Wienekus from the Department of Education, Rhodes University has requested my permission to allow my child/ ward to participate in the above-mentioned research project.

The nature and the purpose of the research project, and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to determine how selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention.

2. The Rhodes University has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate. [**Certificate number**]

3. By participating in this research project my child/ward will be contributing towards a better understanding of the notions of equivalence or equality. The study will benefit my son by engaging him in a topic that is, traditionally, difficult to grasp at this level and give him an opportunity to experience an intervention that may assist him to develop a better conceptual understanding of the topic of algebra as a precursor to what he will experience at high school.

I give consent that my son may:

4. (a) engage in a diagnostic test that will determine his knowledge of algebraic equations. The results of this test will provide Mr Wienekus with important information that he needs to design and plan a screencasting intervention and will enable him to select ten participants spanning the entire achievement spectrum of that particular test.

   (b) be involved in a screencasting intervention programme. This intervention program consists of fifteen contact sessions which will take place in the afternoons, so as not to interfere with normal school hours. Integral to the teaching programme Mr Wienekus will use screencastings to support his teaching of algebra. My son will have access to these screencastings on his tablet at any time he wishes. Once the intervention programme is complete Mr Wienekus I wishes to engage each of the ten participants in a talk-aloud post-intervention test where the boys will talk through how they solved each of the algebraic problems they were given. The engagement of my son with the tasks will be video recorded for analysis purposes. The identity of my son will not be revealed as the camera will be placed behind him as Mr Wienekus is only interested in my son's solution strategies and his explanations of these strategies. His face will thus not be visible. The real name of my son will also not be revealed as Mr Wienekus intends to use pseudonyms.

5. My child's participation is entirely voluntary and if my child/ward is older than seven (7) years, s/he must also agree to participate.

6. Should I, or my child, at any stage wish to withdraw my child from participating further, we may do so without any negative consequences.

7. My child may be asked to withdraw from the research before it has finished if the researcher or any other appropriate person feels it is in my child's best interests, or if my child does not follow instructions.

8. Neither my child nor I will be compensated for participating in the research.

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9. There are no anticipated risks associated with my child's participation in the project.

10. I am aware that Mr Wienekus intends to publish the research results in the form of a thesis. I am also aware that he may wish to publish the data in conference presentations, journal writings or book chapters.

11. I will receive feedback in the form of a presentation regarding the results obtained during the study, if I request this.

12. Any further questions that I might have concerning the research or my son's participation will be answered by:

Mr. George Wienekus	Professor Marc Schäfer	RUESC
MEd student and researcher	Supervisor	Rhodes University, Research Office, Ethics
Mobile: 083 781 7290	Mobile: 083 409 5580	T: +27 (0) 46 603 7727 F: +27 (0) 86 616 7707
g98W6478@campus.ru.ac.za	m.schafer@ru.ac.za	ethics-commtee@ru.ac.za
Student Number: g98w6478		Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139
Rhodes University	Rhodes University	

13. By signing this informed consent declaration, I am not waiving any legal claims, rights or remedies that I or my child/ward may have.

14. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

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I, ... have read the details of Mr Wienekus' research project and confirm that the information has been explained to me in a language that I understand and I am aware of the document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of my child during the research.

I have not been pressurised in any way to let my child take part. By signing below, I voluntarily agree that my son may participate in Mr Wienekus' research project.

I, ______________________________ (full name) give permission for my child________________________ to participate in: 1. the diagnostic test; and. ☐ 2. if selected, to participate in the screencasting intervention programme. ☐	
Signature of Parent/Guardian: _________________________	Date: ___________________
Witness: _________________________	Place: _________________________________

Appendix C – Participant Consent form

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ACCESS LETTER REQUESTING PERMISSION TO CONDUCT RESEARCH

George Wienekus
P.O. Box 187
Grahamstown
South Africa
31 March 2019

Dear Learner

REQUEST FOR PERMISSION TO CONDUCT RESEARCH

My name is George Wienekus and am writing to you as a Master's degree student at Rhodes University (RU) in Grahamstown, South Africa. I will be conducting a research project entitled: '*How selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention*'. I would like to invite you to participate in this research project conducted by me. The research I wish to conduct for my Master's full thesis requires me to observe, interview and video-record selected Grade 7 boys. This research will be conducted under the supervision of Professor Marc Schäfer.

This letter serves to seek formal consent from you that I may:

1. include you as part of my research. It is my intention to engage all the Grade 7 boys in a diagnostic test that will determine the boys' knowledge of algebraic equations. The results of this test will provide me with important information that I need to design and plan my screencasting intervention, and will enable me to select ten participants spanning the entire achievement spectrum of that particular test.
2. in the event of you being one of the ten selected participants, involve you in the screencasting intervention programme. This intervention program consists of fifteen contact sessions which will take place in the afternoons, so as not to interfere with normal school hours. Integral to the teaching programme I will use screencastings to support my teaching of algebra. The

participants will have access to these screencastings on their tablets at any time they wish. Once the intervention programme is complete I wish to engage each of the ten participants in a talk-aloud post-intervention test where you will talk me through how you solved each of the algebraic problems you were given. The engagement of the participants with the tasks will be video recorded for analysis purposes. The identity of the ten boys will not be revealed as the camera will be placed behind the you as I am only interested in your solution strategies and your explanations of these strategies. Your faces will thus not be visible. The real names of the participants will also not be revealed as I intend to use pseudonyms.

The data that will be collected from this study will be used primarily for my thesis writing, but it may also be used for conference presentations, journal writings or a book chapter. The data from the intervention and the observations that will be carried out will be stored on my password protected computer, on my supervisor's password protected computer and in the cloud for a period of five years. All the videos that will be taken during the research process will only be used for the purposes of this research project.

If you agree to participate, we will explain in further detail what would be expected of you, and provide you with the information you need to understand the research. These guidelines would include potential risks, benefits and your rights as a participant. I attach my letter for ethical approval from the Ethics Committee of the Faculty of Education.

Participation in this research is voluntary and you can decide that you do not want to take part at any point by informing your parent/guardian, teacher, principal or me at your school. You will not be disadvantaged in any way if you do not want to take part. Your name will not be recorded in any way.

To participate in this research, you will be asked to sign a consent form to confirm that you understand and agree to the conditions, prior to any data collection tools detailed here. Attached to this letter is the form where you agree to participate in the activity or not.

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Thank you for your time and I hope that you will respond favourably to my request.

Yours sincerely,

George Wienekus
MEd student and researcher
Mobile: 083 781 7290
g98W6478@campus.ru.ac.za
Student Number: g98w6478
Rhodes University

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PARTICIPANT INFORMED CONSENT

INFORMED CONSENT DECLARATION

(Participant)

Project Title: *'How selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention.'*

George Wienekus from the Department of Education, Rhodes University has requested my permission to participate in the above-mentioned research project.

The nature and the purpose of the research project and of this informed consent declaration have been explained to me in a language that I understand.

I am aware that:

1. The purpose of the research project is to determine how selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention.

2. The Rhodes University has given ethical clearance to this research project and I have seen/ may request to see the clearance certificate.

3. By participating in this research project I will be contributing towards a better understanding of the notions of equivalence or equality. The study will benefit you by engaging you in a topic that is, traditionally, difficult to grasp at this level and give you an opportunity to experience an intervention that may assist to develop a better conceptual understanding of the topic of algebra as a precursor to what you will experience at high school.

4. I will participate in the project by being selected as one of ten Grade 7 learners that may:

(a) engage in a diagnostic test that will determine my knowledge of algebraic equations. The results of this test will provide the researcher important information that will enable him to select ten participants spanning the entire achievement spectrum of that particular test and to also plan a screencasting intervention.

(b) be involved in a screencasting intervention programme. I have been informed that this intervention program consists of fifteen contact sessions which will take place in the afternoons, so as not to interfere with normal school hours. I am aware that we will have access to these screencastings on our tablets at any time we wish during this process. I am aware that once the intervention programme is complete I will be engaged in a talk-aloud post-intervention test where I will discuss how I solved each of the algebraic problems. I have also been made aware that the tasks will be video recorded for analysis purposes. My identity will not be revealed. I understand that my real names will not be revealed as the researcher intends to use pseudonyms.

5. My participation is entirely voluntary and should I at any stage wish to withdraw from participating further, I may do so without any negative consequences.

6. There may be risks associated with my participation in the project. I am aware that

   a. the following risks are associated with participation:

      Conflict of Interest/Positionality of the Teacher:

      Although there is no direct conflict of interest, I am aware of the possible power relation of the researcher due to his position as a senior teacher of the school. I have reasonable and sufficient knowledge about him, including his background and location as I have been attending the same school while he has been here for the past two and a half years. The intentions of the research have been explained to me. I currently attend the school at which the research will be conducted. The intentions of the research have been made clear in all the letters of consent that are being used for these purposes.

   b. the following steps have been taken to prevent the risks: the researcher has explained the fact that the answers will not be collected for mark purposes but are strictly for research purposes. It has been emphasized again that once the intervention is complete and the post-intervention test has to be completed, that the researcher is not so much interested about the correct/incorrect answer(s) as he is about the process/method of calculation I have implemented. I have been reminded that participation in all activities is entirely voluntary and will not take place without written permission from my parent(s)/guardian. If I do not wish to participate, I will not be disadvantaged in any way. I am also aware that I may withdraw any point.

c. there is a 0% chance of the risk materialising

7. The researcher intends publishing the research results in the form of a thesis, but it may also be used for conference presentations, journal writings or a book chapter. The data from the intervention and the observations that will be carried out will be stored on a password protected computer for five years. All the videos that will be taken during the research process will only be used for the purposes of this research project. However, confidentiality and anonymity of records will be maintained and that my or my child's/ward's name and identity will not be revealed to anyone who has not been involved in the conduct of the research.

8. I will receive feedback in the form of a presentation regarding the results obtained during the study.

9. Any further questions that I might have concerning the research or my participation will be answered by:

Mr. George Wienekus	Professor Marc Schäfer	RUESC
MEd student and researcher	Supervisor	Rhodes University, Research Office, Ethics
Mobile: 083 781 7290	Mobile: 083 409 5580	T: +27 (0) 46 603 7727 F: +27 (0) 86 616 7707
g98W6478@campus.ru.ac.za	m.schafer@ru.ac.za	ethics-commtee@ru.ac.za
Student Number: g98w6478		Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139
Rhodes University	Rhodes University	

10. By signing this informed consent declaration I am not waiving any legal claims, rights or remedies.

11. A copy of this informed consent declaration will be given to me, and the original will be kept on record.

12. Request to take pictures, video and voice recording for this study will take place.

I, ... have read the above information / confirm that the above information has been explained to me in a language that I understand and I am aware of this document's contents. I have asked all questions that I wished to ask and these have been answered to my satisfaction. I fully understand what is expected of me during the research.

I have not been pressurised in any way and I voluntarily agree to participate in the above-mentioned project.

....	....	....
Participants signature	**Witness**	**Date**

Rhodes University, Research Office, Ethics
Ethics Coordinator: ethics-commitee@ru.ac.za
t: +27 (0) 46 603 7727 f: +27 (0) 86 616 7707
Room 220, Main Admin Building, Drostdy Road, Grahamstown, 6139

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CHILD PARTICIPANT'S ASSENT FORM

INFORMED CONSENT DECLARATION
(Child participant)

Project Title: *'How selected Grade 7 participants develop conceptual understanding in solving algebraic problems as a result of participating in a screencasting intervention.'*

Researcher's name: George Wienekus

Name of participant: ....

1. Are you willing to take part in the research?

   YES NO

2. Has the researcher explained what s/he will be doing and wants you to do?

   YES NO

3. Has the researcher explained why s/he wants you to take part?

   YES NO

4. Do you understand what the research wants to do

   YES NO

5. Do you know if anything good or bad can happen to you during the research?

   YES NO

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6. Do you know that your name and what you say will be kept a secret from other people?

YES NO

7. Did you ask the researcher any questions about the research?

YES NO

8. Has the researcher answered all your questions?

YES NO

9. Do you understand that you can refuse to participate if you do not want to take part and that nothing will happen to you if you refuse?

YES NO

10. Do you understand that you may pull out of the study at any time if you no longer want to continue?

YES NO

11. Do you know who to talk to if you are worried or have any other questions to ask?

YES NO

12. Has anyone forced or put pressure on you to take part in this research?

YES NO

13. Have you been made aware that you will be completing a Diagnostic Test

YES NO

14. Have you been made aware that you will be included in a video-recording and that your identity will not be revealed?

YES NO

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15. Have you been made aware that you will be interviewed as part of the data collection process?

YES NO

______________________ ______________________
Signature of Child **Date**

I, ______________________ (full name) hereby consent to participate in **(PLEASE TICK RELEVANT BOX)**:

1. the diagnostic test ONLY ☐
2. the diagnostic test AND the screencasting intervention programme (IF SELECTED). ☐

Signature of Participant: ______________________	Date: ______________________
Witness: ______________________	Place: ______________________

Appendix D – DN Analysis: Algebraic Summary

Operations equals Answer Context (25 - b = 20)	DN	
	Q1	Q6
Explain how you solved the equations	linear multiplication, mebtal maths, deduction	He used a bit of mental maths as a method, although he made an effort to isolate the letter 'a', but then realised he need to multiply something to '6' to get '54' in order to make the eqtaion true and balance the LHS with the RHS.
How has the Screencasting helped you solve the equation	Recalls the balancing of the scale, but not really needed, this is merely mental maths	He was able to reflect back on the processes from the screencastings in the manipulation of the numbers and operations across the '=' sign so as to balance the equation and ultimately solve for the missing number/unknown variable. It is clear from his explanation and how he has calculated this sum that he understands the concept of equality/equivalence.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	SC's assisted him and were an advantage to him garnering a better understanding of equations and equivalence. Although his calculation method was slightly different to those from the SCs, he was still able to incorporate them into his undertsanding of how to arrive at a result for 'c'	The ability to revisit the video clips and also hearing the audio descriptions were a great help in his case.
Disadvantages of the SC	none	none

Operations on RHS Context (25 = 2b + 1)	DN	
	Q3	Q7
Explain how you solved the equations	linear subtraction of '125 - 77', mental maths, took away and added back.	Process of deduction once he manipulted the '+ 4', he used '24' to deduce that the missing variable must be '8' multiplied by '3'.
How has the Screencasting helped you solve the equation	The screencasts improved and increased his knnowledge of algebra, an understanding of the '=' sign sign and the unkown variable represented by the letter. He could also revisit and replay the screencasts, and the video/audio combination was a significant advantage.	The screencasts improved and increased his knnowledge of algebra, an understanding of the '=' sign sign and the unkown variable represented by the letter. He could also revisit and replay the screencasts, and the video/audio combination was a significant advantage.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	Revisit, Replay, Increased knowledge of algebra, cemented understanding of the '=' sign, video/audio combination was a significant advantage	Revisit, Replay, Increased knowledge of algebra, cemented understanding of the '=' sign, video/audio combination was a significant advantage
Disadvantages of the SC	none	none

Reflexive Context (b = b)	DN	
	Q4	Q10
Explain how you solved the equations	provided an example including numbers to reflect both sides '=' each other, '5 x 5 = 5 x 5', which he simplified to '25 = 25'.	Still recalls that letters represent unkown numbers, but in this instance they will be the same value as he points out, even including the letter 'b' does not waiver his reasoning. In this example he has substituted 'a' and 'b' for '2' and '5', thereby balancing the equation. However, he did not reflect the same entirely accurately on the RHS and instead swopped the letters and values.
How has the Screencasting helped you solve the equation	Screencasts highlighted as a means of assistance, it helped him better understand the concept of random letters, such as 'a' equallling each other and being able to translate that into a concept that makes meaning for him.	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	Revisit, Replay, Increased knowledge of algebra, cemented understanding of the '=' sign, video/audio combination was a significant advantage	Revisit, Replay, Increased knowledge of algebra, cemented understanding of the '=' sign, video/audio combination was a significant advantage
Disadvantages of the SC	none	none

Operations on both sides context (c + 10 = 20 - c)	DN	
	Q9	Q16
Explain how you solved the equations	He has accurately manipulated the numbers and operations across the '=' sign refelcting an understanding of the need to balance the LHS with the RHS of the equation. He has indicated a thorough understanding in this instance by calculating the expression on the RHS first, subtracting '10' and, finally, dividing by 2.	He used mental maths to deduce the unkown variable, 'b', between the two sides of the equation/'=' sign. He did first solve what he knew, the LHS of the equation. He could then use this information to work our that he had to multiply '51' to '1' to produce a number in ordr to make the equation true. In his mind he was able to reverse the operation in order to calculate a value for 'b'.
How has the Screencasting helped you solve the equation	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly.	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly. Also helped him remember that the term, '51b' implies that you multiply the coefficient to the letter.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	Revisit, Replay, Increased knowledge of algebra, cemented understanding of the '=' sign, video/audio combination was a significant advantage	Revisit, Replay, Increased knowledge of algebra, cemented understanding of the '=' sign, video/audio combination was a significant advantage
Disadvantages of the SC	none	none

Appendix D – DN Analysis: Numeric Summary

Operations equals Answer Context (5 + 4 = 9)	DN	
	Q5	Q12
Explain how you solved the equations	linear subtraction, mental maths	linear multiplication, using 10s/whole numbers/breaking up numbers
How has the Screencasting helped you solve the equation	not needed to recall	not needed to recall
What does the '=' mean to you	answer only in this case	equivalent/notion of equality
Why do we use letters in this	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

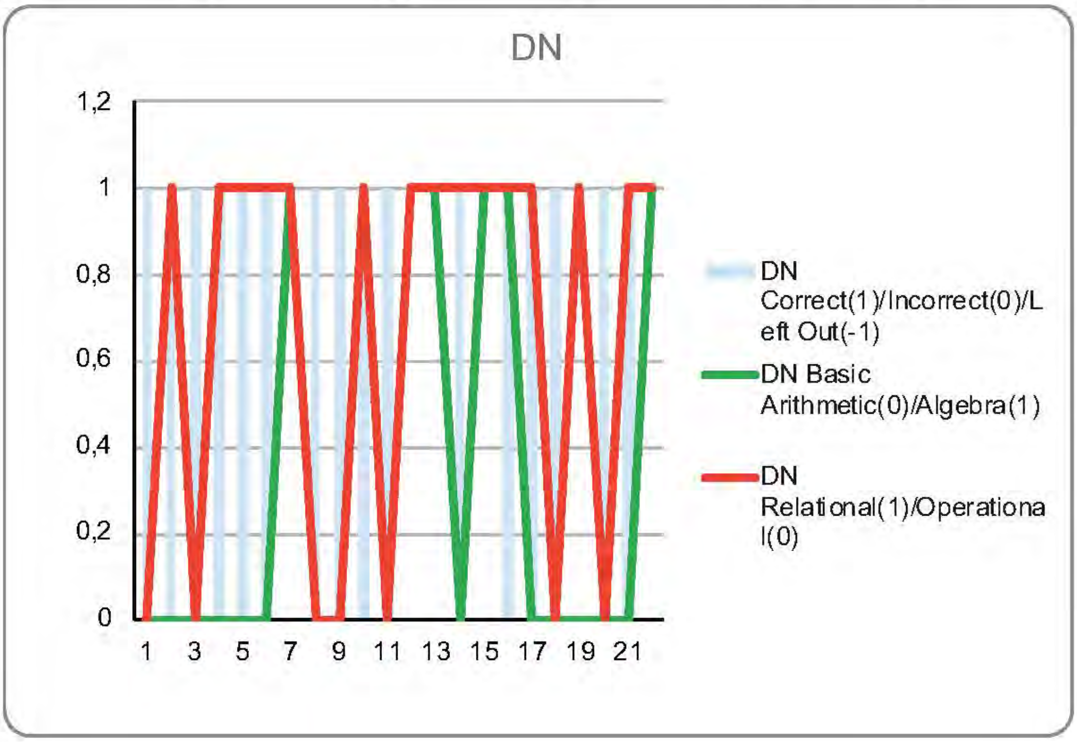
Operations on RHS Context (9 = 5 + 4)	DN	
	Q2	Q14
Explain how you solved the	linear addition, mental maths	linear division, mentl maths
How has the Screencasting helped you solve the equation	not needed to recall	not needed to recall
What does the '=' mean to you	equivalent/notion of equality	equivalent/notion of equality
Why do we use letters in this	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (9 = 9)	DN	
	Q13	Q15
Explain how you solved the equations	solved each expression, on either side of the '=' sign	provided an operation that reflected an equivalence between the LHS and RHS of the '=' sign
How has the Screencasting helped you solve the equation	the visual imagery of the animated scale balancing the LHS with the RHS of an equation	the visual imagery of the animated scale balancing the LHS with the RHS of an equation
What does the '=' mean to you	equivalent/notion of equality	equivalent/notion of equality
Why do we use letters in this	n/a	n/a
Advantages of the SC	the explanation using the scale; of balance	use of scale to indicate the balance
Disadvantages of the SC	none at this stage	none at this stage

Operations on both sides context (5 + 4 = 6 + 3)	DN	
	Q8	Q11
Explain how you solved the equations	Calculates LHS first, determines what is required to make the equation true	Calculates RHS first, determines what is required to make the equation true.
How has the Screencasting helped you solve the equation	the visual imagery of the animated scale balancing the LHS with the RHS of an equation	the visual imagery of the animated scale balancing the LHS with the RHS of an equation
What does the '=' mean to you	equivalent/notion of equality	equivalent/notion of equality
Why do we use letters in this	n/a	n/a
Advantages of the SC	It improves his knowledge and refreshes his thinking; the explanation of balance using the animated scale	use of scale to indicate the balance
Disadvantages of the SC	none at this stage	none at this stage

Appendix D – DN Quantitative Analysis

	DN		
Question	Correct(1)/Incorrect(0)/Left Out(-1)	Basic Arithmetic(0)/Algebra(1)	Relational(1)/Operational(0)
1	1	0	0
2	1	0	1
3	1	0	0
4	1	0	1
5	1	0	1
6	1	0	1
7	0	1	1
8	1	0	0
9	1	0	0
10	1	1	1
11	1	0	0
12	0	1	1
13	0	1	1
14	1	0	1
15	0	1	1
16	1	1	1
17	1	0	1
18	1	0	0
19	0	0	1
20	1	0	0
21	1	0	1
22	0	1	1



Appendix D – EVDM Analysis: Algebraic Summary

Operations equals Answer Context (25 - b = 20)	**EVDM**	
	Q1	**Q6**
Explain how you solved the equations	Process of deducation as opposed to manipulating numbers across the '=' sign, straight forward enough to work it out mentally.	He used a bit of mental maths as a method, although he made an effort to isolate the letter 'a', but then realised he needed to multiply something to '6' to get '54' in order to make the eqtaion true and balance the LHS with the RHS. Describes solving for 'a' by isolating 'a' and manipulating numbers and their operations across the '=' sign, reversed operations
How has the Screencasting helped you solve the equation	The SCs made him aware of what the letters in an equation may represent, the unknown variable. He now has a better idea of their purpose.	Understands to multiply the letter 'a' to the number '6' in the term, '6a'
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	There is an advantage to the SCs as he understood the description and its meaning and purpose better, also for all rthe reasons he has listed, easy to understand and follow, video/audio combination a significant advantage, revisit/replay	Explaining the benefits of the screencastings and the role they played in his understanding of how to solve for the unkown variable, 'a', in this instance. The advantages of screencastings including the audio as the problem is solved and spoken through is a significant advantage.
Disadvantages of the SC	none	none

Operations on RHS Context (25 = 2b + 1)	**EVDM**	
	Q3	**Q7**
Explain how you solved the equations	For him, this was not complicated as he simply used the difference between '125' and '77' to calculate for 'c'. Very straight forward as there were no numbers (coefficient) attached to the letter, he simply completed the equation by filling in the missing amount and, ultimately, solving for the unkown variable.	isolated 'b by manipulating operations and numbers across the '=' sign, which assisted him by realising that he needed to multiply the unkown number by '3' to give him '24'. Also recognises that it is very similar to the previous question and so his calculation and reasoning stays the same
How has the Screencasting helped you solve the equation	made the undertsanding of the letter, 'c', clearer in terms of what role it fulfills in algebra and, particularly, in equations.	Understands to multiply the letter 'b' to the number '3' in the term, '3b'
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	Easy to undertsand, can be revisited, replayed, and seeing the video as the audio explains it makes it a great help.	Revisit, Replay, cemented understanding of the '=' sign, video/audio combination was a significant advantage
Disadvantages of the SC	none	none

Reflexive Context (b = b)	**EVDM**	
	Q4	**Q10**
Explain how you solved the equations	recalls that letters represent unkown numbers, but in this instance they will be the same value as he points out, uses numbers, '2 = 2', to indicate how he understands the question, there was hesitation at first	Still recalls that letters represent unkown numbers, but in this instance they will be the same value as he points out, even including the letter 'b' does not waiver his reasoning. In this example he has substituted 'a' and 'b' for '2' and '3', thereby balancing the equation. However, he did not reflect the same entirely accurately on the RHS and instead swopped the letters and values.
How has the Screencasting helped you solve the equation	Cannot recall this particular kind of example, so did not find the screencasts to be particulalry helpful in this instance, however, the explanation using the animated scale would have provided him with clarity.	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation, it also allowed him to revisit and replay when necessary, the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	not on this case	Revisit, Replay, cemented understanding of the '=' sign, video/audio combination was a significant advantage
Disadvantages of the SC	none	none

Operations on both sides context (c + 10 = 20 - c)	**EVDM**	
	Q9	**Q16**
Explain how you solved the equations	His reference to two expressions either side of the '=' sign. He has accurately manipulated the numbers and operations across the '=' sign, He has indicated a thorough understanding in this instance by calculating the expression on the RHS first, subtracting '10' and, finally, dividing by 2.	He solves both sides of the equation, for each expression; He has solved the LHS of the equation first, to determine a value he will need to in order to balance the equation. In other words, he will use this in his calculation when he needs to solve for 'b'.
How has the Screencasting helped you solve the equation	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation, it also allowed him to revisit and replay when necessary, the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly.	He recalls the examples and explanations from the screencastings, he screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation, it also allowed him to revisit and replay when necessary, the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	Revisit, Replay, cemented understanding of the '=' sign, video/audio combination was a significant advantage	Revisit, Replay, cemented understanding of the '=' sign, video/audio combination was a significant advantage
Disadvantages of the SC	none	none

Appendix D – EVDM Analysis: Numeric Summary

Operations equals Answer Context (5 + 4 = 9)	EVDM Q5	EVDM Q12
Explain how you solved the equations	linear subtraction, mental maths	linear multiplication, mental maths
How has the Screencasting helped you solve the equation	A better understanding of what the '=' sign represents.	not needed to recall, mental maths
What does the '=' mean to you	equivalent/notion of equality	equivalent/notion of equality
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

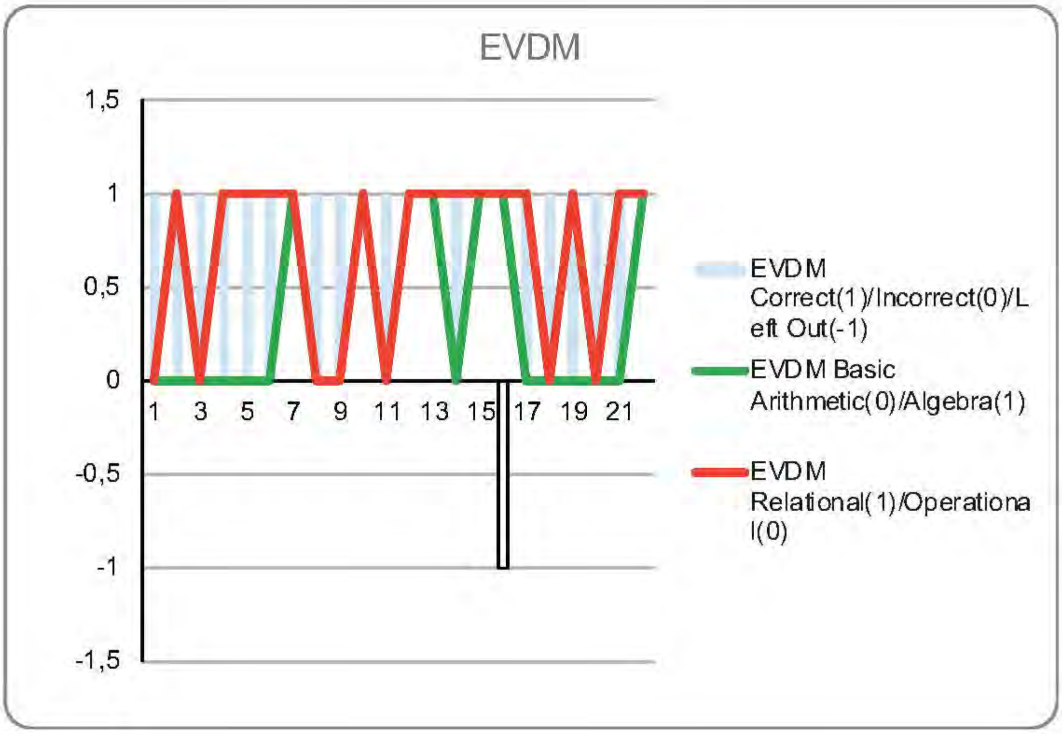
Operations on RHS Context (9 = 5 + 4)	EVDM Q2	EVDM Q14
Explain how you solved the equations	linear addition, mental maths	linear division, mentl maths
How has the Screencasting helped you solve the equation	not needed to recall	not needed to recall
What does the '=' mean to you	still early, but implied a notion of equality/equivalence	Couldnt' think of writing it another way, but understands the notion of equality/equivalence
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (9 = 9)	EVDM Q13	EVDM Q15
Explain how you solved the equations	solved each expression, on either side of the '=' sign	made 9 = 3 x 3
How has the Screencasting helped you solve the equation	numbers only, no need to recall, mental maths	the visual imagery of the animated scale balancing the LHS with the RHS of an equation, helped him understand or explain it better
What does the '=' mean to you	equivalent/notion of equality	equivalent/notion of equality
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	not in this paticular example	easy to understand, visual/audio combination assisted, revisit and replay
Disadvantages of the SC	not in this paticular example	none at this stage

Operations on both sides context (5 + 4 = 6 + 3)	EVDM Q8	EVDM Q11
Explain how you solved the equations	Calculates LHS first, determines what is required to make the equation true, reverse operation, use of the '=' sign	Calculates RHS first, determines what is required to make the equation true.
How has the Screencasting helped you solve the equation	the visual imagery of the animated scale balancing the LHS with the RHS of an equation, movement of terms and operations across the '=' sign	the visual imagery of the animated scale balancing the LHS with the RHS of an equation, movement of terms and operations across the '=' sign
What does the '=' mean to you	equivalent/notion of equality	equivalent/notion of equality
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	easy to understand; visual/audio combination assisted; revisitt and replay	easy to understand, visual/audio combination assisted, revisitt and replay
Disadvantages of the SC	not in this paticular example	none at this stage

Appendix D – EVDM Quantitative Analysis

	EVDM		
Question	Correct(1)/Incorrect(0)/Left Out(-1)	Basic Arithmetic(0)/Algebra(1)	Relational(1)/Operational(0)
1	1	0	0
2	1	0	1
3	1	0	0
4	1	0	1
5	1	0	1
6	1	0	1
7	0	1	1
8	1	0	0
9	1	0	0
10	0	1	1
11	1	0	0
12	0	1	1
13	0	1	1
14	1	0	1
15	0	1	1
16	-1	1	1
17	1	0	1
18	1	0	0
19	1	0	1
20	1	0	0
21	1	0	1
22	0	1	1



Appendix D – JR Analysis: Algebraic Summary

Operations equals Answer Context (25 - b = 20)	JR	
	Q1	Q6
Explain how you solved the equations	provided the accurate answer only, mental maths	Completed solving for 'a' accurately as per his explanation. Using the animated scale from the screencastings as a reference, he performed opposite operations and so manipulated the equation in order to balance the LHS with the RHS and thereby solving for 'a'.
How has the Screencasting helped you solve the equation	He quotes the screencasting examples and the fact that I used a scale to indicate/animate a balancing of the LHS with the RHS.	The screencastings assisted his undertsanding of how to manipulate the operations and numbers across the '=' sign in oder to balance the equation, again, incorporating the animated scale.
What does the '=' mean to you	Equality/equivalence/balance; He realises the necessity to balance the LHS expression with the RHS expression, using the '=' to symbolise it.	Equality/Equivalence/Balance
Why do we use letters in this equation?	the unkown variable; recalling basic arithmetic used in ealrier grades where an empty block represents a missing number, but now comprehends that the letter has replaced the more rudimentary empty block. He understands that the letter represents an unknown number/unknown variable.	the unkown variable
Advantages of the SC	The advantages of the screencastings highlighted here include the relevance of the mehtod of 'balancing' an equation by manipulating the numbers and operations across the '=' sign, although not required in this example, however, the visuals and adio of the animated scale indicated the necessity to balance the LHS with the RHS was valuable for him; the video clip woth the auditory explanation was helpful and an advantage; the abilityto be able to revisit the screencastings whenever necessary is a significant advantage	Recall, and most convenient; the ability to revisit the screencastings/explanations including the audio and being able to stop the video and replay it back is a significant advantage.
Disadvantages of the SC	No immediate interaction if the he is unsure of something.	No real disadvantage in this instance as he claims to have understood the xample pretty well. This could also be due to it being the sixth example abd he may have warmed up by now, but also his recollection of the examples may have assisted the process; cannot engage when there is immeidate assistance required.

Operations on RHS Context (25 = 2b + 1)	JR	
	Q3	Q7
Explain how you solved the equations	For him, this was not complicated as he simply used the differance between '125' and '77' to calculate for 'c'. Very straight forward as there were no numbers (coefficient) attached to the letter.	Provided the solution for 'b' immediately, mentaal maths; Completed solviing for 'b' accurately as per his explanation. Using the animated scale from the screencastings as a reference, he performed opposite operations and so manipulated the equation in order to balance the LHS with the RHS and thereby solving for 'b'.
How has the Screencasting helped you solve the equation	No need to recall details from the screencastings as this example was straight forward.	The screencastings assisted his undertsanding of how to manipulate the operations and numbers across the '=' sign in oder to balance the equation, again, incorporating the animated scale.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	the unkown variable	the unkown variable
Advantages of the SC	No real advantage from the screencasts as the example was straight forward, in other words there were letters attached to numbers and so it was a case of simply working out the missing number to make the equation true, LHS = RHS.	the ability to revisist the screencastings/explanations including the audio and being able to stop the video and replay it back is a significant advantage, and a better understanding of the what the letter represents.
Disadvantages of the SC	Cannot engage with the tecaher immediately	No real disadvantage in this instance as he claims to have understood the example pretty well. This could also be due to it being the seventh example and he may have warmed up by now, but also his recollection of the examples may have assisted the process; cannot engage when there is immeidate assistance required.

Reflexive Context (b = b)	JR	
	Q4	Q10
Explain how you solved the equations	knows that given the parameters of this example, he could use any items to indicate that they'=' each other.	His knowledge and understanding of the letters and the introduction to algebra allows to relaise that 'a' and 'b' are different in this context, but each carries the same value in order to make the equation true, LHS = RHS
How has the Screencasting helped you solve the equation	Although he cannot fathom why I have used letters only, he is still able to use his knowledge of the '=' sign and reason and justify an answer.	he finds it logical, although the scale in the screencasting would have cemented his prior knowledge.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	the unkown variable	the unkown variable
Advantages of the SC	No real advantage of the screencastsings in this instance as he felt it was straight forrward enough to work out for himself regardless of assistance.	No real advantage of the screencastsings in this instance as he felt it was straight forward enough to work out for himself regardless of assistance.
Disadvantages of the SC	none	None

Operations on both sides context (c + 10 = 20 - c)	JR	
	Q9	Q16
Explain how you solved the equations	He used mental maths to deduce the unkown variable, 'a', between the two sides of the equation/'=' sign. He did first solve what he knew, the RHS of the equation. He could then use this information to work our that he had to multiply '55' to '2' to produce a number in ordr to make the equation true; he has accurately manipulated the numbers and operations across the '=' sign refelcting an understanding of the need to balance the LHS with the RHS of the equation. He has indicated a thorough understanding in this instance.	He used mental maths to deduce the unkown variable, 'b', between the two sides of the equation/'=' sign. He did first solve what he knew, the LHS of the equation. He could then use this information to work our that he had to multiply '51' to '1' to produce a number in ordr to make the equation true; he has accurately manipulated the numbers and operations across the '=' sign refelcting an understanding of the need to balance the LHS with the RHS of the equation. He has indicated a thorough understanding in this instance.
How has the Screencasting helped you solve the equation	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly. Also helped him remember that the term, '2a' implies that you multiply the coefficient to the letter.	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly. Also helped him remember that the term, '51b' implies that you multiply the coefficient to the letter.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	the unkown variable	the unkown variable
Advantages of the SC	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly. Also helped him remember that the term, '2a' implies that you multiply the coefficient to the letter.	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly. Also helped him remember that the term, '51b' implies that you multiply the coefficient to the letter.
Disadvantages of the SC	Cannot engage with the tecaher immediately	Cannot engage with the tecaher immediately

Appendix D – JR Analysis: Numeric Summary

Operations equals Answer Context (5 + 4 = 9)	JR	
	Q5	Q12
Explain how you solved the equations	answer only, mental maths	vertial multiplication, mental maths
How has the Screencasting helped you solve the equation	not needed to recall	not needed to recall
What does the '=' mean to you	equivalent/notion of equality/balance LHS and RHS	equivalent/notion of equality/balance LHS with RHS
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

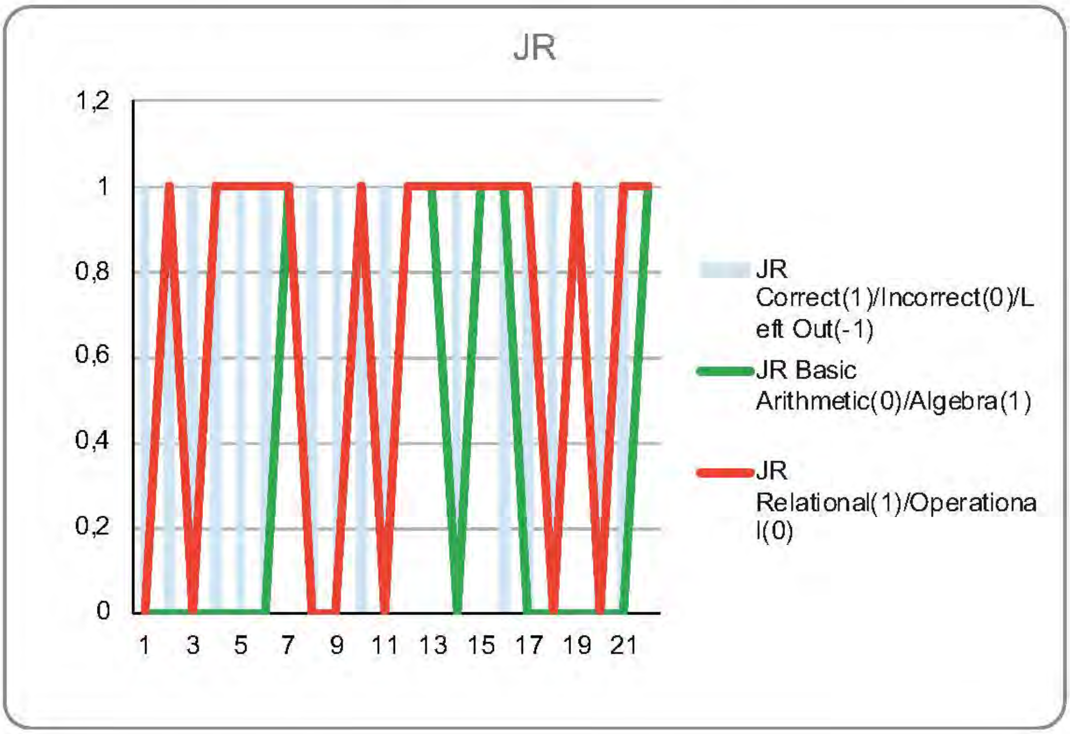
Operations on RHS Context (9 = 5 + 4)	JR	
	Q2	Q14
Explain how you solved the equations	filled in answer only	filled in answer only
How has the Screencasting helped you solve the equation	not needed to recall, numbers only can see it mentally.	not needed to recall, numbers only, can see it mentally, did indicate that you could also say '10 divided by 10'.
What does the '=' mean to you	equivalent/notion of equality/balance LHS and RHS	equivalent/notion of equality/balance LHS with RHS
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (9 = 9)	JR	
	Q13	Q15
Explain how you solved the equations	solved each expression, on either side of the '=' sign, already balanced, just simplified	filled in answer only, '9'; mentioned there are lots of options to reflect '9' as the value on the RHS.
How has the Screencasting helped you solve the equation	to understand numbers and the notion of equality better	straight forward so not needed to reflect on sreencasts
What does the '=' mean to you	equivalent/notion of equality/balance LHS and RHS	equivalent/notion of equality/balance LHS with RHS
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Operations on both sides context (5 + 4 = 6 + 3)	JR	
	Q8	Q11
Explain how you solved the equations	filled in answer only, mental maths	simplified RHS of equation, then filled in the missing number only
How has the Screencasting helped you solve the equation	mental maths for him, not needed to recall/reflect on any of the screencastings	straight forward so not needed to reflect on sreencasts
What does the '=' mean to you	equivalent/notion of equality/balance LHS and RHS	equivalent/notion of equality/balance LHS with RHS
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Appendix D – JR Quantitative Analysis

	JR		
Question	Correct(1)/Incorrect(0)/Left Out(-1)	Basic Arithmetic(0)/Algebra(1)	Relational(1)/Operational(0)
1	1	0	0
2	1	0	1
3	1	0	0
4	1	0	1
5	1	0	1
6	1	0	1
7	0	1	1
8	1	0	0
9	1	0	0
10	1	1	1
11	1	0	0
12	0	1	1
13	0	1	1
14	1	0	1
15	0	1	1
16	1	1	1
17	1	0	1
18	1	0	0
19	0	0	1
20	1	0	0
21	1	0	1
22	0	1	1



Appendix D – NF Analysis: Algebraic Summary

Operations equals Answer Context (25 - b = 20)	NS	
	Q1	Q6
Explain how you solved the equations	Could not solve it, did not know how to manipulate the equation.	Could not solve it, did not know how to manipulate the equation.
How has the Screencasting helped you solve the equation	Cannot recall the explanations in the various sreencastings	Cannot recall the explanations in the various screencastings
What does the '=' mean to you	symbolises an answer to a sum.	the answer
Why do we use letters in this equation?	to represent an unknown number	to represent an unkown number
Advantages of the SC	none (could not recall the screencasts)	none (could not recall the screencasts)
Disadvantages of the SC	none (could not recall the screencasts)	none (could not recall the screencasts)

Operations on RHS Context (25 = 2b + 1)	NS	
	Q3	Q7
Explain how you solved the equations	Calculated the difference between the two given terms, understood that 'c' was the missing number to complete the equation (sum).	Could not solve it, did not know how to manipulate the equation.
How has the Screencasting helped you solve the equation	It assisted his introduction to algebra	Cannot recall the explanations in the various screencastings
What does the '=' mean to you	symbolises an answer to a sum.	that an answer must follow
Why do we use letters in this equation?	to represent that it is algebra.	Means that is algebra
Advantages of the SC	The video/audio combination helped him, was clear and he was able to follow their instructions clearly.	Cannot see the advantages in this instance as he cannot recall them.
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (b = b)	NS	
	Q4	Q10
Explain how you solved the equations	He used a number to indicate the equality of '9 = 9', but then explained that it would be a sum = abswer.	Refers to it as the same answer even with different numbers, provides an example '1 + 2 = 1 + 2'.
How has the Screencasting helped you solve the equation	It assisted his introduction to algebra, the visual/audio combination is good	It assisted his introduction to algebra, the visual/audio combination is good
What does the '=' mean to you	the answer, usually preceded by a sum.	Explains in a roundabout way that there is a need to balance the LHS with the RHS/equality.
Why do we use letters in this equation?	for algebra, it gives you a clue.	letters make algebra what algebra is.
Advantages of the SC	Believes that the screencastings are still beneficial and will assist him	Believes that the screencastings are still beneficial and will assist him
Disadvantages of the SC	none at this stage	none at this stage

Operations on both sides context (c + 10 = 20 - c)	NS	
	Q9	Q16
Explain how you solved the equations	Could not solve it, did not know how to manipulate the equation.	Could not solve it, did not know how to manipulate the equation.
How has the Screencasting helped you solve the equation	He cannot recall any examples from the screencastings for this particular context.	He has no recollection of how to manipulate the equation, using the visuals from the screencastings. He cannot remember the use of the animated scale.
What does the '=' mean to you	Quite contradictory to his thoughtsthis far, but has indicated tht the two expressions need to '=' each other, but is confused by the term '2a'.	He has indiicated that the two expressions need to '=' each other, but he is completely confused by the term, '51b' and how to solve this problem. In a sense, he has understood that he needs to balance each side so that LHS = RHS, but the term '51b' is in his way at the moment.
Why do we use letters in this equation?	represents algebra	represents algebra
Advantages of the SC	none in this instance	none in this instance
Disadvantages of the SC	none	none

Appendix D – NF Analysis: Numeric Summary

Operations equals Answer Context (5 + 4 = 9)	NF	
	Q5	Q12
Explain how you solved the equations	Vertical subtraction, mental maths	added 25 4 times, linear addition, mental maths
How has the Screencasting helped you solve the equation	not needed to recall	not needed to recall
What does the '=' mean to you	equivalent/notion of equality/balance LHS and RHS	equivalent/notion of equality/balance LHS with RHS
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

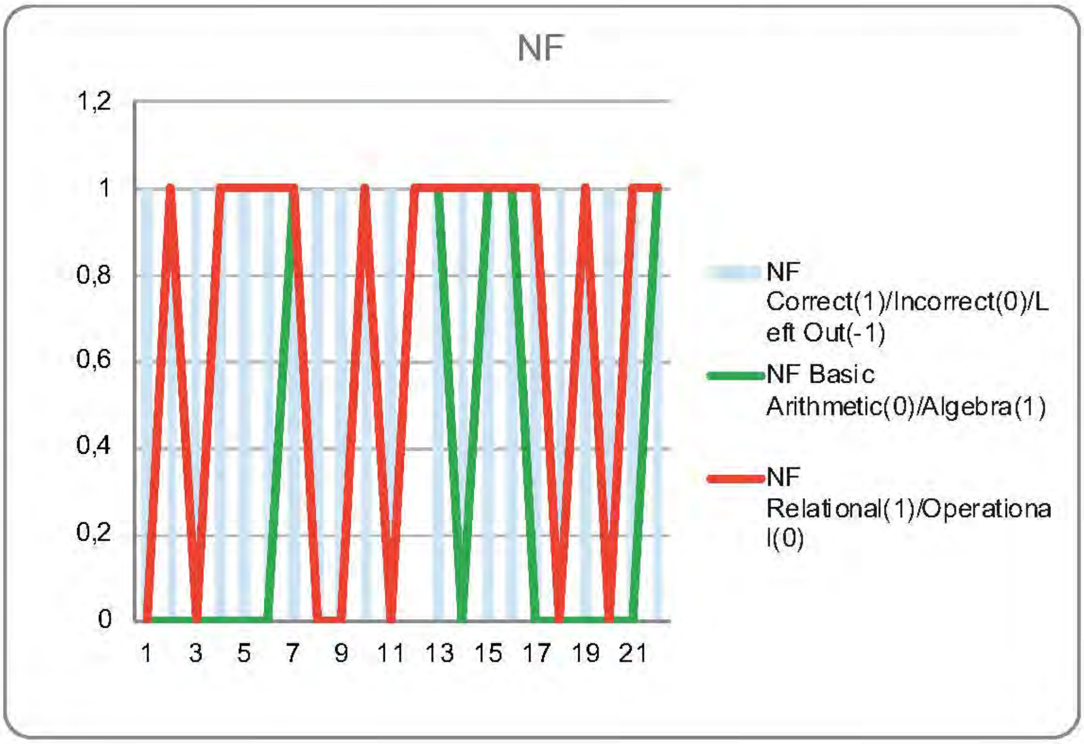
Operations on RHS Context (9 = 5 + 4)	NF	
	Q2	Q14
Explain how you solved the equations	calculated expression on the RHS, to determine the missing value on the LHS, balanced the two sides with each other.	calculated expression on the RHS, to determine the missing value on the LHS, balanced the two sides with other
How has the Screencasting helped you solve the equation	not needed to recall, numbers only can see it mentally.	not needed to recall, numbers only, can see it mentally.
What does the '=' mean to you	equivalent/notion of equality/balance LHS and RHS	equivalent/notion of equality/balance LHS with RHS
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (9 = 9)	NF	
	Q13	Q15
Explain how you solved the equations	solved each expression, on either side of the '=' sign	provided an operation that reflected an equivalence between the LHS and RHS of the '=' sign, 9 = 3 x 3
How has the Screencasting helped you solve the equation	showed calculation, worked out either side.	made more aware of the meaning of the '=' sign
What does the '=' mean to you	equivalent/notion of equality/balance LHS and RHS	equivalent/notion of equality/balance LHS with RHS
Why do we use letters in this equation?	n/a	you could use letters to represent the unknown number(s), an awareness of what they represent.
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Operations on both sides context (5 + 4 = 6 + 3)	NF	
	Q8	Q11
Explain how you solved the equations	Calculates LHS first, determines what is required to make the equation true, reverse operation, use of the '=' sign	Calculates RHS first, determines what is required to make the equation true, indicated performing opposite operations by moving the numbers and operations across the '=' sign, used the letter 'x' to represent the missing number and proceeded to calculate mentally.
How has the Screencasting helped you solve the equation	mental maths for him, not needed to recall/reflect on any of the screencastings	the visual imagery of the animated scale balancing the LHS with the RHS of an equation, movement of terms and operations across the '=' sign, use of a letter to represent missing number
What does the '=' mean to you	equivalent/notion of equality/balance LHS and RHS	equivalent/notion of equality/balance LHS with RHS
Why do we use letters in this equation?	n/a	you could use letters to represent the unknown number(s), an awareness of what they represent.
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Appendix D – NF Quantitative Analysis

	NF		
Question	Correct(1)/Incorrect(0)/Left Out(-1)	Basic Arithmetic(0)/Algebra(1)	Relational(1)/Operational(0)
1	1	0	0
2	1	0	1
3	1	0	0
4	1	0	1
5	1	0	1
6	1	0	1
7	1	1	1
8	1	0	0
9	1	0	0
10	1	1	1
11	1	0	0
12	0	1	1
13	1	1	1
14	1	0	1
15	1	1	1
16	1	1	1
17	1	0	1
18	1	0	0
19	1	0	1
20	1	0	0
21	1	0	1
22	1	1	1



Appendix D – NS Analysis: Algebraic Summary

Operations equals Answer Context (25 - b = 20)	NS	
	Q1	Q6
Explain how you solved the equations	Could not solve it, did not know how to manipulate the equation.	Could not solve it, did not know how to manipulate the equation.
How has the Screencasting helped you solve the equation	Cannot recall the explanations in the various sreencastings	Cannot recall the explanations in the various screencastings
What does the '=' mean to you	symbolises an answer to a sum.	the answer
Why do we use letters in this equation?	to represent an unknown number	to represent an unkown number
Advantages of the SC	none (could not recall the screencasts)	none (could not recall the screencasts)
Disadvantages of the SC	none (could not recall the screencasts)	none (could not recall the screencasts)

Operations on RHS Context (25 = 2b + 1)	NS	
	Q3	Q7
Explain how you solved the equations	Calculated the difference between the two given terms, understood that 'c' was the missing number to complete the equation (sum).	Could not solve it, did not know how to manipulate the equation.
How has the Screencasting helped you solve the equation	It assisted his introduction to algebra	Cannot recall the explanations in the various screencastings
What does the '=' mean to you	symbolises an answer to a sum.	that an answer must follow
Why do we use letters in this equation?	to represent that it is algebra.	Means that is algebra
Advantages of the SC	The video/audio combination helped him, was clear and he was able to follow their instructions clearly.	Cannot see the advantages in this instance as he cannot recall them.
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (b = b)	NS	
	Q4	Q10
Explain how you solved the equations	He used a number to indicate the equality of '9 = 9', but then explained that it would be a sum = abswer.	Refers to it as the same answer even with different numbers, provides an example '1 + 2 = 1 + 2'.
How has the Screencasting helped you solve the equation	It assisted his introduction to algebra, the visual/audio combination is good	It assisted his introduction to algebra, the visual/audio combination is good
What does the '=' mean to you	the answer, usually preceded by a sum.	Explains in a roundabout way that there is a need to balance the LHS with the RHS/equality.
Why do we use letters in this equation?	for algebra, it gives you a clue.	letters make algebra what algebra is.
Advantages of the SC	Believes that the screencastings are still beneficial and will assist him	Believes that the screencastings are still beneficial and will assist him
Disadvantages of the SC	none at this stage	none at this stage

Operations on both sides context (c + 10 = 20 - c)	NS	
	Q9	Q16
Explain how you solved the equations	Could not solve it, did not know how to manipulate the equation.	Could not solve it, did not know how to manipulate the equation.
How has the Screencasting helped you solve the equation	He cannot recall any examples from the screencastings for this particular context.	He has no recollection of how to manipulate the equation, using the visuals from the screencastings. He cannot remember the use of the animated scale.
What does the '=' mean to you	Quite contradictory to his thoughtsthis far, but has indicated tht the two expressions need to '=' each other, but is confused by the term '2a'.	He has indiicated that the two expressions need to '=' each other, but he is completely confused by the term, '51b' and how to solve this problem. In a sense, he has understood that he needs to balance each side so that LHS = RHS, but the term '51b' is in his way at the moment.
Why do we use letters in this equation?	represents algebra	represents algebra
Advantages of the SC	none in this instance	none in this instance
Disadvantages of the SC	none	none

Appendix D – NS Analysis: Numeric Summary

Operations equals Answer Context (5 + 4 = 9)	NS	
	Q5	Q12
Explain how you solved the equations	vertical subtraction of the numbers	Vertical multiplication of the numbers
How has the Screencasting helped you solve the equation	N/A, easy (prefers numbers only)	N/A, easy (prefers numbers only)
What does the '=' mean to you	answer follows	answer follows
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

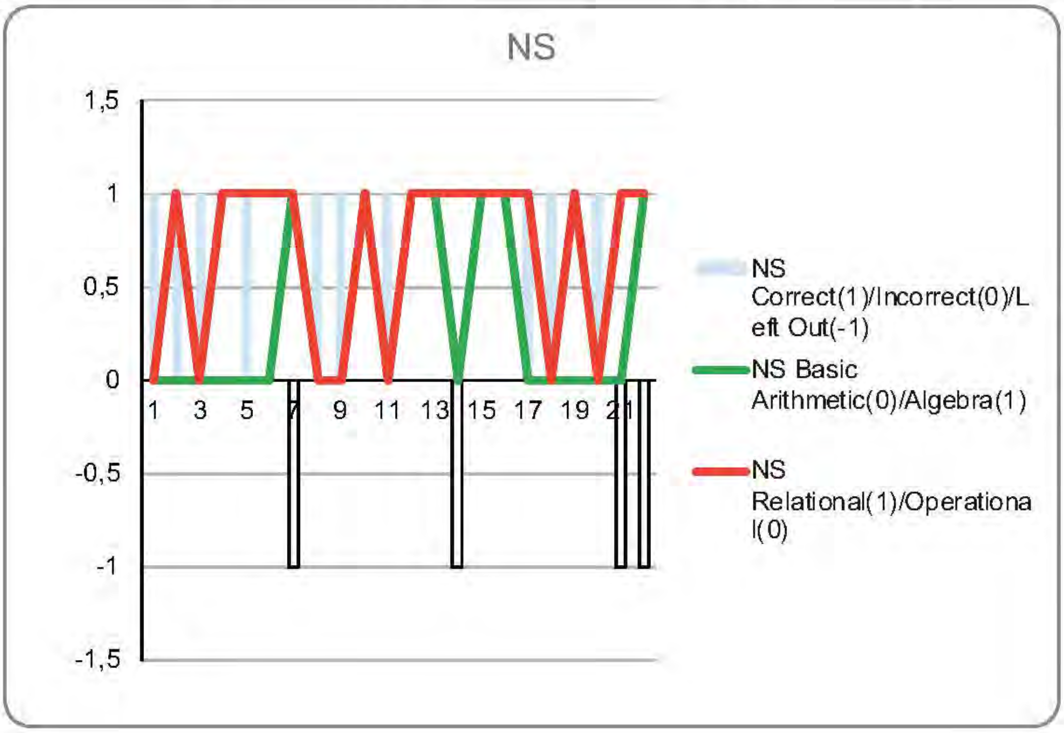
Operations on RHS Context (9 = 5 + 4)	NS	
	Q2	Q14
Explain how you solved the equations	filled in answer only	filled in answer only
How has the Screencasting helped you solve the equation	N/A, easy (prefers numbers only)	N/A, easy (prefers numbers only)
What does the '=' mean to you	answer follows	answer follows
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (9 = 9)	NS	
	Q13	Q15
Explain how you solved the equations	adds two expressions in space provided	did not coplete (cannot do it)
How has the Screencasting helped you solve the equation	N/A, easy (prefers numbers only)	did not recall
What does the '=' mean to you	the answer, the same thing in this case	the answer
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	did not recall
Disadvantages of the SC	none at this stage	none at this stage

Operations on both sides context (5 + 4 = 6 + 3)	NS	
	Q8	Q11
Explain how you solved the equations	multiplied the 2 terms on the LHS and divided the product by 2	filled in the missing number, mental maths
How has the Screencasting helped you solve the equation	N/A, easy (prefers numbers only)	N/A, easy (prefers numbers only)
What does the '=' mean to you	the answer	the answer
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Appendix D – NS Quantitative Analysis

	NS		
Question	Correct(1)/Incorrect(0)/Left Out(-1)	Basic Arithmetic(0)/Algebra(1)	Relational(1)/Operational(0)
1	1	0	0
2	1	0	1
3	1	0	0
4	0	0	1
5	1	0	1
6	0	0	1
7	-1	1	1
8	1	0	0
9	1	0	0
10	0	1	1
11	1	0	0
12	0	1	1
13	0	1	1
14	-1	0	1
15	0	1	1
16	0	1	1
17	1	0	1
18	1	0	0
19	0	0	1
20	1	0	0
21	-1	0	1
22	-1	1	1



Appendix D – RB Analysis: Algebraic Summary

Operations equals Answer Context (25 - b = 20)	RB	
	Q1	Q6
Explain how you solved the equations	He solved for 'c' by manipulating numbers and operations across the '=' sign, reflecting what you do to one side, you do to the other because of the '=' sign, he solved for 'c'.	He knows that 'a' needs to be multiplied to '6', he understands the process of needing to isolate the letter so as to solve for the unknown number, manipulates numbers and operations across the '=' sign, reflecting what you do to one side, you do to the other because of the '=' sign, he solved for 'a'.
How has the Screencasting helped you solve the equation	The video clip of the working through the example as well as the audio describing each step assisted this candidate in terms of seeing the image/visual and been spoken to as the example is completed. It also served as a reminder of what he may have forgotten from what e had previously learned and given that it was accessible at any point, he could revisit the screencasting at his convenience.	He knows from the scale that moving the operations and numbers across the '=' sign creates an opposite operation, which is necessary to balance the equation, LHS = RHS.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	the unkown variable	the unkown variable
Advantages of the SC	enjoys/ prefers the visual as opposed to a 'talk-and-talk' scenario, he feels more engaged with what he needs to understand and conceptualise.	enjoys/ prefers the visual as opposed to a 'talk-and-talk' scenario, he feels more engaged with what he needs to understand and conceptualise.
Disadvantages of the SC	none	none

Operations on RHS Context (25 = 2b + 1)	RB	
	Q3	Q7
Explain how you solved the equations	This candidate worked the example out step for step, using the '=' sign to indicate a balnacing of the two sides, LHS and RHS of the equation. He performed opposite operations when manipulating/moving operations and numbers across the '=' sign.	No problem in solving for the unkown variabl, he used both sides of the equation to do so, manipulating numbers and operations across the '=' sign thereby solving for 'b'.
How has the Screencasting helped you solve the equation	Referred to the screencastings and the reference to the animated scale and how it represents the '=' sign and that it balances the euation, LHS with the RHS.	He knows from the scale that moving the operations and numbers across the '=' sign creates an opposite operation, which is necessary to balance the equation, LHS = RHS.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	the unkown variable	the unkown variable
Advantages of the SC	Only advantages for him, Revisist and replay, the video clip and audio explanation assisted his understanding	There is an advantage to screencastings, you can revisit and replay the clips and the fact that the audio is attached as the explanation plays through is a key feature.
Disadvantages of the SC	none	none

Reflexive Context (b = b)	RB	
	Q4	Q10
Explain how you solved the equations	He was confident enough to use his own example to portray his understanding of the reflexive context, he explined that 'a = a' represents the fact that the same value is '=' to itself	His knowledge and understanding of the letters and the introduction to algebra allows to relaise that 'a' and 'b' are different in this context, but each carries the same value in order to make the equation true, LHS = RHS
How has the Screencasting helped you solve the equation	Referred to the screencastings and the reference to the animated scale and how it represents the '=' sign and that it balances the euation, LHS with the RHS.	He knows from the scale that moving the operations and numbers across the '=' sign creates an opposite operation, which is necessary to balance the equation, LHS = RHS.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	the unkown variable	the unkown variable
Advantages of the SC	The advantages of the screencastings in this example will have provided a better understanding/confirmation of the '=' sign, balancing the LHS with RHS using the animated scale as a reference.	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly.
Disadvantages of the SC	none	none

Operations on both sides context (c + 10 = 20 - c)	RB	
	Q9	Q16
Explain how you solved the equations	He has accurately manipulated the numbers and operations across the '=' sign refelcting an understanding of the need to balance the LHS with the RHS of the equation. He has indicated a thorough understanding in this instance.	He solves both sides of the equation, for each expression, he used a method that feels natural to him to calculate the unkown variable, 'b', between the two sides of the equation/'=' sign. He solved what he knew first in terms of what he was given, the LHS of the equation. He could then use this information to work out that he had to then divide '50' by each side to produce the value for 'b', '1', to make the equation true.
How has the Screencasting helped you solve the equation	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly.	he visual image of the scale helped his understanding and guided his thoughts in solving tihs particular example.
What does the '=' mean to you	Equality/equivalence/balance	Equality/Equivalence/Balance
Why do we use letters in this equation?	Represents an unknown number	Represents an unknown variable
Advantages of the SC	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly.	The screencatsing were a definite advantage as they animated the scale and cemented his understanding of letters and the balancing of the LHS with RHS of an equation; it also allowed him to revisit and replay when necessary; the video/audio combination was useful in this respect as it talked through the examples step-by-step, succinctly. Also helped him remember that the term, '51b' implies that you multiply the coefficient to the letter.
Disadvantages of the SC	none	none

Appendix D – RB Analysis: Numeric Summary

Operations equals Answer Context (5 + 4 = 9)	RB	
	Q5	Q12
Explain how you solved the equations	vertical subtraction	vertial multiplication
How has the Screencasting helped you solve the equation	not needed to recall	not needed to recall
What does the '=' mean to you	answer only in this case	equivalent/notion of equality
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

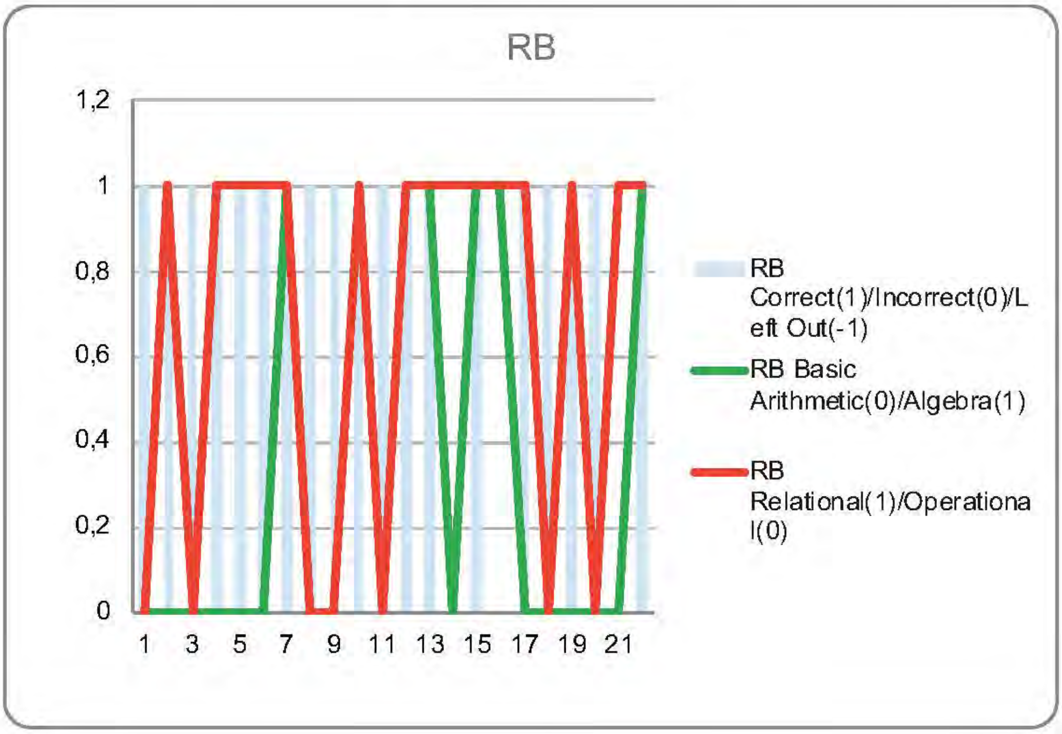
Operations on RHS Context (9 = 5 + 4)	RB	
	Q2	Q14
Explain how you solved the equations	calculated expression on the RHS, to determine the missing value on the LHS, balanced the two sides with each other.	calculated expression on the RHS, to determine the missing value on the LHS, balanced the two sides with other
How has the Screencasting helped you solve the equation	not needed to recall, numbers only can see it mentally.	not needed to recall, numbers only, can see it mentally.
What does the '=' mean to you	equivalent/notion of equality	equivalent/notion of equality, what happens on the one side must happen on the other.
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (9 = 9)	RB	
	Q13	Q15
Explain how you solved the equations	solved each expression, on either side of the '=' sign	provided an operation that reflected an equivalence between the LHS and RHS of the '=' sign, 9 = 3 x 3
How has the Screencasting helped you solve the equation	showed calculation, worked out either side.	Refreshed his memory with this particular example
What does the '=' mean to you	equivalent/notion of equality	equivalent/notion of equality/balance the LHS with the RHS
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	easy to understand; visual/audio combination assisted; revisit and replay
Disadvantages of the SC	none at this stage	none at this stage

Operations on both sides context (5 + 4 = 6 + 3)	RB	
	Q8	Q11
Explain how you solved the equations	Calculates LHS first, determines what is required to make the equation true, reverse operation, use of the '=' sign	Calculates RHS first, determines what is required to make the equation true, indicated performing opposite operations by moving the numbers and operations across the '=' sign
How has the Screencasting helped you solve the equation	the visual imagery of the animated scale balancing the LHS with the RHS of an equation, movement of terms and operations across the '=' sign	the visual imagery of the animated scale balancing the LHS with the RHS of an equation, movement of terms and operations across the '=' sign
What does the '=' mean to you	equivalent/notion of equality/balance the LHS with the RHS	equivalent/notion of equality/balance the LHS with the RHS
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	easy to understand; visual/audio combination assisted; revisitt and replay	easy to understand; visual/audio combination assisted; revisit and replay
Disadvantages of the SC	none at this stage	none at this stage

Appendix D – RB Quantitative Analysis

	RB		
Question	**Correct(1)/Incorrect(0)/Left Out(-1)**	**Basic Arithmetic(0)/Algebra(1)**	**Relational(1)/Operational(0)**
1	1	0	0
2	1	0	1
3	1	0	0
4	1	0	1
5	1	0	1
6	1	0	1
7	1	1	1
8	1	0	0
9	1	0	0
10	1	1	1
11	1	0	0
12	1	1	1
13	1	1	1
14	0	0	1
15	1	1	1
16	0	1	1
17	1	0	1
18	1	0	0
19	1	0	1
20	1	0	0
21	0	0	1
22	1	1	1



Appendix D – RVZ Analysis: Algebraic Summary

Operations equals Answer Context (25 - b = 20)	RVZ	
	Q1	Q6
Explain how you solved the equations	linear multiplication, ignored the letter, did not solve for 'c'	Added '6' and '4', discarded and ignored the letter, worked out that the diffrence of '10' was the answer. He is unaware of the fact that the term '6a' reflects/means that you multiply '6' to 'a', in this particular case; he wants to simply attache another number to '6' to replace the 'a', but realises he cannot do so because this will be a gretaer value than 58 (RHS), which must indicate that he has some kind of understanding that the '=' does not necessarily refelct an answer only.
How has the Screencasting helped you solve the equation	could not recall	He does not recll the clips nor the the explanations, so he is struggling to understand the concept of the notion of equality/equivalence and, therefore, he cannot manipulate the numbers across the '=' sign, but he now knows he has missed out on a bit of information and is thinking hard about it (and the other questions he has already completed).
What does the '=' mean to you	the answer probably follows the '=' sign	Beginning to realise that the '=' sign does not only precede an answer, and that there could be more to it.
Why do we use letters in this equation?	probably represents an unknown number	He believes that it provides some sort of clue to an answer.
Advantages of the SC	Increase and improve his knowledge his algebraic knowledge	He believes there are, just cannot pinpoint it.
Disadvantages of the SC	none	none

Operations on RHS Context (25 = 2b + 1)	RVZ	
	Q3	Q7
Explain how you solved the equations	For him, this was not complicated as he simply used the difference between '125' and '77' to calculate for 'c'. Very straight forward as there were no numbers (coefficient) attached to the letter, he simply completed the equation by filling in the missing amount and, ultimately, solving for the unkown variable.	Because he is confused by the letters and what they represent, he cannot solve for 'b' by isolating it in the equation, but he tries to reason that you 'attach' a value to it rather and 'plaster it to '3', but he realises that it does not make the equation true, which makes him grapple with it. He knows his calculations are incorrect, but cannot calculate the correct answer because of his lack of understanding of the letters.
How has the Screencasting helped you solve the equation	It can improve and increase algebaric knowledge; straight forward with no coefficient attached to the letter representing the unkown variable.	Cannot recall the examples or explanations (for that matter) from the screencastings.
What does the '=' mean to you	the answer	He begins to reason the meaning of how the '=' sign changes the equation, but is referring to how it actaully changes the operations and that you may have to manipulate the operations and numbers to do so, which he confirms below. Confirms that it could reverse the operation; He is beginning to rethink his thoughts on the '=' sign, he believes it changes the equation in some respect.
Why do we use letters in this equation?	probably an unkown number	very confused by the role of the letter
Advantages of the SC	Straight forward, no need to really refelct back to thhe screencasts	none
Disadvantages of the SC	none	none

Reflexive Context (b = b)	RVZ	
	Q4	Q10
Explain how you solved the equations	He is trying to get his head around the fact that you are not necessarily performing any operations, but yet there is a letter that '=' another letter, which in his reasoning should actually be an answer. But, when I pose the question slightly differently he gets it and substitutes the number for the letter because he can see that the letter 'a' on the LHS of the '=' sign will represent the same value on the RHS of the '=' sign.	Interestingly, he is reflecting on the idea that the LHS '=' RHS. He is more comfortable with the idea of operations and actual expressions, even though they are the same thing, on either side of the '=' sign; he wrote 'object + subject = object + subject'
How has the Screencasting helped you solve the equation	Does not recall a similar type of example in the screencasting video clips	Does not recall using any method from the screencastings to assist him.
What does the '=' mean to you	He is of the belief that the '=' sign indicates that an answer must follow, his thought process of how the '=' sign works is very mechanical in that it signifies that he must simply find an 'answer'.	He confirms that he using the '=' sign to balance the equation with the two expressions either side of the '=' sign; he also confirms that what happens to the LHS happens to the RHS of the '=' sign.
Why do we use letters in this equation?	could represent an unkown number	to represent an unknown number
Advantages of the SC	He does not feel that the screencatsings were an advatntage to his understanding of this concept/type of example.	Does not recall an example of this type from the screencatsing clips and so cannot suggest any advantage he would have gained from them.
Disadvantages of the SC	none	none

Operations on both sides context (c + 10 = 20 - c)	RVZ	
	Q9	Q16
Explain how you solved the equations	He simplified the expression on the RHS of the equation, but the divided both sides by '20'.	He only simplifies the expression on the LHS of the equation, but then does not know how to progress.
How has the Screencasting helped you solve the equation	Struggling to find an advantage to the screencasts as he cannot recall the examples from the clips that was shared with him. Given his reasoning and thought process, I believe he has not spent time on the screencastings.	Struggling to find an advantage to the screencasts as he cannot recall the examples from the clips that was shared with him. Given his reasoning and thought process, I believe he has not spent time on the screencastings.
What does the '=' mean to you	And so he ralises that the '=' sign my need to balance the equation, or least make the expressions equal the same result/answer; he has gone as far as saying that the '=' sign changes the operation to make the two sides of the '=' sign end up with the same answer. Unfortunatley, he has no ide of how to manipulate the numbers and operations in order to balance the equation.	At this stage, he has realised that the '=' sign means more than just the answer to a sum, as far as commenting that he will have to reqrite the equation; and so he ralises that the '=' sign my need to balance the equation, or least make the expressions equal the same result/answer; he has gone as far as saying that the '=' sign changes the operation to make the two sides of the '=' sign end up with the same answer. Unfortunatley, he has no ide of how to manipulate the numbers and operations in order to balance the equation.
Why do we use letters in this equation?	represents another digit.	To this candidate, the letter 'b' could represent '0'; e does not understand the role of the letter in algebra, to the extent that he ignores it completely when calculating the equation.
Advantages of the SC	Struggling to find an advantage to the screencasts as he cannot recall the examples from the clips that was shared with him. Given his reasoning and thought process, I believe he has not spent time on the screencastings.	Struggling to find an advantage to the screencasts as he cannot recall the examples from the clips that was shared with him. Given his reasoning and thought process, I believe he has not spent time on the screencastings.
Disadvantages of the SC	none	none

Appendix D – RVZ Analysis: Numeric Summary

Operations equals Answer Context (5 + 4 = 9)	RVZ	
	Q5	Q12
Explain how you solved the equations	Vertical subtraction	answer only, mental maths
How has the Screencasting helped you solve the equation	N/A, easy (prefers numbers only)	N/A, easy (prefers numbers only)
What does the '=' mean to you	answer follows	answer follows
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

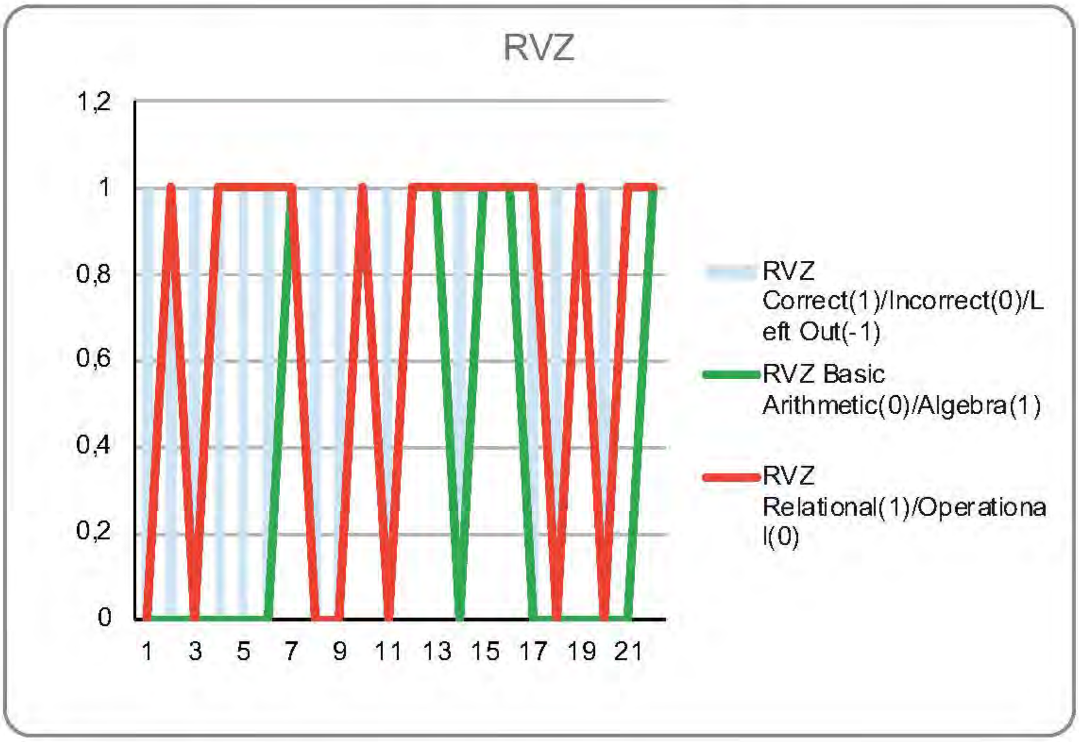
Operations on RHS Context (9 = 5 + 4)	RVZ	
	Q2	Q14
Explain how you solved the equations	filled in answer only	mistook division for addition, vertical addition
How has the Screencasting helped you solve the equation	N/A, easy (prefers numbers only)	N/A, easy (prefers numbers only)
What does the '=' mean to you	answer follows	answer follows
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Reflexive Context (9 = 9)	RVZ	
	Q13	Q15
Explain how you solved the equations	solved each expression, on either side of the '=' sign	Left it out, unsure
How has the Screencasting helped you solve the equation	N/A, easy (prefers numbers only)	cannot recall this example or explanation
What does the '=' mean to you	answer follows	the replica in a different language
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	did not recall
Disadvantages of the SC	none at this stage	none at this stage

Operations on both sides context (5 + 4 = 6 + 3)	RVZ	
	Q8	Q11
Explain how you solved the equations	Calculates LHS first, determines what is required to make the equation true, reverse operation, use of the '=' sign	Calculates RHS first, determines what is required to make the equation true.
How has the Screencasting helped you solve the equation	not really	N/A, easy (prefers numbers only)
What does the '=' mean to you	the answer	the answer
Why do we use letters in this equation?	n/a	n/a
Advantages of the SC	none at this stage	none at this stage
Disadvantages of the SC	none at this stage	none at this stage

Appendix D – RVZ Quantitative Analysis

	RVZ		
Question	Correct(1)/Incorrect(0)/Left Out(-1)	Basic Arithmetic(0)/Algebra(1)	Relational(1)/Operational(0)
1	1	0	0
2	1	0	1
3	1	0	0
4	1	0	1
5	1	0	1
6	1	0	1
7	0	1	1
8	1	0	0
9	1	0	0
10	0	1	1
11	1	0	0
12	0	1	1
13	0	1	1
14	1	0	1
15	0	1	1
16	0	1	1
17	1	0	1
18	1	0	0
19	0	0	1
20	1	0	0
21	0	0	1
22	0	1	1



Appendix E – Pre- and Post-intervention Test

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ALGEBRAIC EQUALITY TASKS

Instructions:

1. **PLEASE NOTE: this task will not be used for mark recording purposes of any kind.**
2. **Complete the questions on the paper provided.**
3. **Show your calculations in the space (block) provided below each question.**
4. **You may illustrate, use an image or sketch the answer as a means to explain your meaning in the box provided.**
5. **Request additional paper if needed.**
6. **NO calculators may be used.**
7. **Enjoy and thank you for participating in this task.**

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ALGEBRAIC ACTIVITY TASKS

Question 1: Warm up

1. 20 + 40 = _____
2. 10 = _____ + 20 - 20
3. 300 ÷ 150 = _____
4. 7 + 15 = _____+ 12
5. _____+ 5 = 8 x 5
6. 6 + _____ = 72 ÷ 9

Question 2: Calculate (show all of your working for all the tasks below)

1. Find the value of c if 3c x 7 = 21

2. Calculate: _____ = 9 + 91

3. Find the value of c if 125 = c + 77

4. Provide a real-life example that explains this scenario: a = a

5. Calculate: 65 - 44 = _____

6. Find the value of a if 6a + 4 = 58

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7. Find the value of b if $28 = 3b + 4$

8. Calculate: $10 \times 3 =$ ______ $\div 2$

9. Find the value of a if $10 + 2a = 100 + 20$

10. Provide a real-life example that best explains the following scenario: a + b = a + b

11. Calculate: _____ + 20 = 21 + 35?

12. Calculate: 25 x 4 =_____

13. Calculate: 15 + 10 = 10 + 15.

14. Calculate: _____ = 20 ÷ 20?

15. What is 9 = _____?

16. Find the value of b if 154 - 104 = 51b – 1