THE DEVELOPMENT OF GRADE ONE TEACHERS' MATHEMATICS AND PEDAGOGICAL CONTENT KNOWLEDGE THROUGH PARTICIPATION IN A COLLABORATIVE INTERVENTION

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Kedisaletse Stella Mutlane

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DECLARATION

I, Kedisaletse Stella Mutlane, hereby declare that the work in this thesis is my own and where ideas of other authors have been used, they have been acknowledged in full using referencing according to the Rhodes University Guide to References. I further declare that the work in this thesis has not been submitted to any university for degree purposes.

05 January 2021

Signature Date

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ABSTRACT

The “South African education system is grossly inefficient, severely underperforming and egregiously unfair” (Spaull, 2013, p.3). In particular, grave concerns with learner performance in mathematics in South Africa are well documented (e.g., Taylor, 2008; Spaull, 2013; Venkat & Spaull, 2015). There are various explanations for the poor state of learner performance in mathematics in South Africa. Two of the explanations that relate closely to my research interest are teachers’ insufficient mathematics content and pedagogical knowledge, and inappropriate professional development. This study aims to ascertain how a collaborative intervention can develop teachers’ mathematics and pedagogical content knowledge as they focus on developing learners’ foundational number sense.

Cultural Historical Activity Theory, together with Mathematics Knowledge for Teaching (Ball et al., 2008) and the Knowledge Quartet (Rowlands & Turner, 2007) frameworks, provide the explanatory and analytic tools for the research.

The research is a qualitative case study underpinned by an interpretivist orientation. The study was conducted at a township public primary school in the Northern Cape. Three Grade One teachers participated in the research. Data was collected through interviews, classroom observations, and videos of collaborative lesson planning and reflection sessions.

A key finding emerging from this research is that the teachers had the necessary mathematics content knowledge to teach Grade One mathematics. Despite this and in contrast to it, they lacked adequate pedagogical content knowledge required to develop learners’ number sense. To develop their pedagogical content knowledge, they required the intervention of a ‘more knowledgable other’ (Vygotsky, 2008).

Several contradictions and tensions emerged from the research. For example, the teachers expressed that the opportunity to work collaboratively was beneficial, but it was evident that they were familiar with and accomplished in planning and working together. The contradictions emerging from this research provide an opportunity and basis for expansive learning for future collaborative teacher endeavours.

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LIST OF ABBREVIATIONS

ANS	Approximate Number System
CAPS	Curriculum Assessment Policy Statement
CCK	Common Content Knowledge
CHAT	Cultural Historical Activity Theory
DBE	Department of Basic Education
FNS	Foundational Number Sense
HCK	Horizon Content Knowledge
KCC	Knowledge of Content and Curriculum
KCS	Knowledge Content and Student
KCT	Knowledge Content and Teaching
KQ	Knowledge Quartet
LoLT	Language of Learning and Teaching
LTSM	Learning and Teaching Support Material
MKfT	Mathematical Knowledge for Teaching
MKO	More Knowledgeable Other
OTS	Object Tracking System
PCK	Pedagogical Content Knowledge
SA.DBE	South Africa. Department of Basic Education
SCK	Specialised Content Knowledge
SMK	Subject Matter Knowledge

CHAPTER ONE: CONTEXT OF THE RESEARCH

1.1. STATE OF MATHEMATICS IN SOUTH AFRICA

A good education system is important for human development as it enables people to obtain high level skills that ultimately assist in alleviating poverty and inequality while promoting development and growth (Modisaotsile, 2012). There are two essential systemic aspects required to deliver these outcomes, namely good quality and equal access. The South African education system is currently lacking in both these respects since overall quality is poor and it is highly unequal. In the words of Spaull (2013), it “is grossly inefficient, severely underperforming and egregiously unfair” (p.3). For example, in 2007 only 16% of Grade Three learners performed at the grade level in the national systemic evaluations (Spaull & Kotze, 2015). The vast majority of the learners assessed were from low socio-economic status communities and schools. While there are many systemic reasons for learners’ poor performance, a dominant explanation suggests that teachers do not have the required knowledge to develop learners’ mathematical competence (Spaull, 2013). Linked to this are concerns about the quality of professional development programmes for in-service teachers and the extent to which teachers translate what they learn in these programmes into the classroom. My research is situated in this context.

1.2. NUMBER SENSE AS FOCUS OF RESEARCH

Klibanoff, Levine, Huttenlocher, Hedges and Vasilyewa (2006) highlight that children begin preschool with variable pre-numeracy foundational skills and knowledge. Those with weaker foundations (early ‘learning deficits’) are prone to falling further and further behind as they progress through the schooling system. Number sense is viewed as one of the most important concepts to be developed in early mathematics (McGuire, Kinzie & Berch, 2012). This research is focused on strengthening the pedagogical and mathematical content knowledge of Grade One teachers, with a particular focus on number sense development. It is important because number sense development at this stage is essential to the learners’ mathematical achievement throughout the schooling years (Berch, 2005). Where number sense skills are adequately developed, learner performance in mathematics should improve. The learners’ acquisition of foundational knowledge in mathematics through number sense development enables them to perform in mathematics throughout their schooling years (Sayer & Andrews, 2015).

1.3. EXPLANATIONS OF LEARNER UNDERPERFORMANCE IN SOUTH AFRICA

Concerns with learner performance in mathematics in South Africa is well documented (e.g., Taylor, 2008; Spaull, 2013; Venkat & Spaull, 2015). There are numerous explanations for this. Klibanoff, Levine, Huttenlocher, Hedges and Vasilyewa (2006) maintain that children begin preschool having differences in their mathematical knowledge. Moreover, many of these children enter a primary school that is unable to equip them with the necessary skills to succeed (Spaull & Kotze, 2015). As Feza (2012) notes, the South African policy aims to provide quality education, but this endeavour is undermined by the reality that many learners lack foundational knowledge. According to Spaull & Kotze (2015) this may be due to the fact that learners are only allowed to spend a maximum of four years in a phase. This means that learners can be promoted to the next grade or phase without mastering the requirements of the grade or phase they are in. In addition, there are limited opportunities for remediation and support for learners who experience ‘learning deficits’ and those who are repeating a grade.

While children may come into the schooling system without basic numeracy, this situation is exacerbated by teachers’ insufficient content knowledge and poor pedagogical practices. According to Feza (2012), teachers of learners in the reception years often believe that their learners are not ready to learn mathematics because of their age, a view influenced by the work of Piaget. Spaull and Kotze (2015) recommend that attention should be paid to Grade R by employing quality teachers with good mathematical and pedagogical knowledge, and knowledge of how children learn.

1.3.1. Teacher knowledge

Spaull (2013) maintains that “South Africa has some of the least knowledgeable primary school mathematics teachers in the sub-Saharan Africa” (p. 8). His analysis of the 2004 Grade Six SACMEQ II test shows that primary school mathematics teachers’ content knowledge is limited. The SACMEQ II test was given to 498 North West teachers. Out of the 401 teachers who wrote the test, only 32% performed at grade six level. Spaull (2013) and Motshekga (cited in African News Agency, 2016) point out explicitly that implies that these teachers could not answer questions that their learners are required to answer. Spaull (2013) notes that this lack of basic knowledge amongst teachers invariably curtails the mathematical development of their learners.

The quality of mathematics teaching in primary schools is not limited to teachers' mathematics content knowledge. The Foundation Phase Study on '*Improving classroom practice and learner performance*' (Hoadley 2010) link learner performance to the teachers' 'ability' to teach. Criticisms in relation to teachers' ineffective teaching practices include: curriculum coverage and interpretation; cognitive expectations of learners; use of teaching time; use of resources; teaching strategies; and assessment of learning (Hoadley, 2010; Pendlebury, 2009; Adler, Pournara, Taylor, Thorne, Moletsane, 2009). Adler (2005) suggests that learners are underperforming as some teachers are unable to unpack mathematical ideas in a manner that is accessible to their learners. She suggests that this may be a result of poor pre-service and in-service teacher education programmes and the curriculum they are required to implement in their classroom.

According to Chapman (2012) and Westaway and Graven (2018) teachers are central to the implementation and interpretation of the curriculum, therefore a teacher's attitudes and beliefs towards the curriculum can either facilitate or inhibit the role of learning. In other words, teachers' attitudes and beliefs in this regard can make the curriculum easier or more difficult for learners to understand and learn. For example, when a teacher believes that mathematics is difficult to learn, s/he is unlikely to look for or find methods to make the content easier for learners. De Clercq and Phiri (2013) note that although the Department of Basic Education (DBE) has tried to implement its vision through practical teacher development strategies, it seems that these have been unsuccessful. Chapman (2012) stresses that the impact of professional development is limited if participant teachers are not offered in-classroom support. He suggests that for professional development programs to create sustainable change teachers should be encouraged to participate in practice-based professional learning communities. This gives teachers increased agency over their own learning.

1.3.2. Teacher Professional Development

According to Bertram (2011), the key purpose of professional development initiatives is to develop teacher competence in organising systematic learning. Bertram (2011) argues that there are two perspectives that underpin teacher learning, namely the socio-cultural approach and the cognitive approach. The socio-cultural approach assumes that teacher learning takes place in a community of practice or professional learning community where teachers learn ways of knowing and thinking in the context in which they practise. Teacher learning can take place in a classroom where they practise their expertise with the support of their colleagues or educational specialists. On the other hand, the cognitive approach refers to teachers' knowledge (located in individual minds) and separates acquisition of knowledge, skills and understanding from their practice. In other words, individual teachers can learn about teaching outside the classroom and afterwards implement the knowledge effectively in class. Du Four (2004) exclaims that the separation of knowledge acquisition and practice creates a contrast between teachers' working (teaching) and learning. In South Africa, professional development sessions are often in the form of workshops or refresher courses which tend to be offered in a 'one size fits all' approach for implementation of curriculum. Ono and Ferreira (2010) complain that the workshops are decontextualized and isolated from real classroom situations. For them, professional development must be "based on constructivism, perceived as long term, take place in context and conceived as a collaborative process" (Ono & Ferreira, 2010, p. 62) rather than the one or two-day courses offered by district officials (Graven, 2014).

Bertram (2011) points out that after attending workshops there are, in most instances, no class visits by subject advisors to support and monitor curriculum implementation. This lack of support results in teachers not implementing the curriculum. Although workshops are important, they are sometimes ineffective because of the duration and misinterpretation of information from trainers who often lack confidence (Westaway & Graven, 2018). Thus, Bertram (2011) argues that while workshops can model new material and content, teacher learning needs to be supported through the mentoring and support of subject advisors and colleagues in the classrooms.

Chapman (2012) suggests that the best support for the implementation of curriculum resides within schools. Bertram (2011) explains that to develop competence and confidence, teachers need to practice new pedagogies and assessment methods in the environment where they have the support of their colleagues. This approach acknowledges that teachers learn by acquiring knowledge and developing their competence as part of a collective, for example, participation in a collaborative intervention. This is done by providing them with opportunities to engage in their own learning as a collective (Schoenfeld, 2002; Chapman, 2012). This suggests that collaboration has the potential to (re)develop teachers' knowledge in the contexts in which they work instead of trying to put into practice knowledge they acquire in workshops (Pyrko, Dorfler & Eden, 2017). In addition, if teachers are actively involved in their own learning, the advances are likely to be more sustainable and they may provide the platform for ongoing learning (Chapman, 2012).

1.4. THE UTILISATION OF COLLABORATION IN THE RESEARCH

According to De Clercq and Phiri (2013) "teacher development continues to be the most important challenge where the improvement of South African schooling is concerned" (p.77). The predominant form of teacher development is 'one size fits all' workshops where they are advised how to implement policy; this approach yields little or no transformation in teachers' practice (Bertram, 2011). For this research I engaged Grade One teachers in a collaborative intervention to encourage them to engage actively in their own learning with the support of their colleagues (Chapman, 2012).

During workshops teachers are treated as 'homogenous' in terms of their subject knowledge and pedagogical content knowledge which does not offer all of them a meaningful learning opportunity (Chapman, 2012). Even the 'cascade' model introduced by the Department of Education, which refers to training a group of teachers and then assuming that they will pass the knowledge on to their colleagues, has not been effective (Ono & Ferreira, 2010). Participant teachers have complained that "district trainers did not always understand the curriculum which resulted in the misinterpretation of crucial information” (Ono & Ferreira, 2010, p.59). Significantly, Bertram (2011) emphasises that teachers learn best as members of community of practice, where learning happens through experience and practice. When teachers interact collaboratively in the classroom, they bring their different experiences, knowledge and skills to the learning environment. This research considers an intervention where teachers engaged directly and were encouraged to learn in the classroom context in which they work.

1.5. PROBLEM STATEMENT, AIMS AND RESEARCH QUESTIONS

There are various explanations for the poor state of learner performance in mathematics in South Africa. Three that relate closely to my research interest are teachers’ insufficient knowledge of mathematics, poor pedagogical practice, and inappropriate professional development.

Based on these explanations, the study seeks to develop teachers’ knowledge and pedagogical practices to improve learner performance in ways that are sustainable. To do this, I conceptualised and established a collaborative intervention, where teachers were supported to take responsibility for their professional development, with the intention of improving their mathematics and pedagogical content knowledge. The collaborative intervention involved three Grade One teachers who were all attempting to develop their learners’ number sense.

1.6. AIMS OF THE RESEARCH

The main aim of this research is to strengthen the mathematics and pedagogical content knowledge of Grade One teachers through participation in a collaborative intervention in order to develop learners’ number sense. As such, I want to examine the mathematics and pedagogical content knowledge required to develop learners’ number sense and how a collaborative intervention can enable the development of the mathematics and pedagogical content knowledge required to develop learners’ number sense. Through the collaborative intervention, I hoped to facilitate opportunities for teachers to work and learn with each other to improve their practice. The success of this programme could inform teacher professional development programmes run by the District Office, such as the newly established Professional Learning Communities.

1.7. RESEARCH QUESTIONS

The following research questions will guide data generation for this research:

1.7.1. Main research question

How can Grade One teachers' mathematics and pedagogical content knowledge (specifically the knowledge needed to develop learners' number sense) be strengthened through a collaborative intervention?

1.7.2. Sub-questions

- What mathematical and pedagogical content knowledge do teachers use to develop learners' number sense?
  - What is the nature of the number sense activities promoted prior/during the collaborative intervention?
  - What is the nature of the teachers' mathematics and pedagogical content knowledge prior/during the collaborative intervention?

- How does a collaborative intervention enable and constrain the development of the mathematics and pedagogical content knowledge required to develop learners' number sense?

1.8. CHAPTER OVERVIEW

My research thesis consists of seven chapters structured as follows:

Chapter One provides the context and the background for my study. It presents the mathematics crisis in South Africa as reflected in both national and international research. The key reasons behind the crisis that are of relevance to this paper are teachers' insufficient mathematics knowledge, poor pedagogical practices and inefficient professional development models.

Chapter Two reviews the theoretical framework that underpins this study, that is, Cultural Historical Activity Theory (CHAT). Both first- and second-generation CHAT are elaborated. The research is framed by Leontiev's second generation CHAT, which focuses on collective action. This is of relevance in this study as the research examines an intervention in which a group of teachers work collaboratively. The data is analysed using the various components of CHAT (subject, object, outcome, tools, division of labour, rules and community). Contradictions are identified as these are regarded as key in transforming the activity.

Chapter Three reviews the object of the study, which is strengthening the mathematics and pedagogical content knowledge of Grade One teachers to develop their learners' number sense. It focuses on the three models related to the development of teachers' mathematicsand pedagogical content knowledge. These are: Shulman's 'Pedagogical Content Knowledge'; Ball et al.'s 'Mathematical Knowledge for Teaching' and Rowlands et al.'s 'Knowledge Quartet' (KQ).

Chapter Four reviews the intended outcome of the study, which is the development of learners' number sense. In this chapter, number sense is traced from its roots in cognitive psychology to what is loosely referred to as the mathematics education perspective, that is the number sense developed in school. Attention is given to bridging the gap between what is regarded as preverbal knowledge, that is the knowledge people are biologically endowed with, and school knowledge. In so doing, the focus shifts to the Foundational Number Sense Framework as proposed by Sayer and Andrews.

Chapter Five describes the methodology of this study. This study is a qualitative case study underpinned by an interpretivist orientation. Three Grade One teachers and I participated in a collaborative intervention. Data was collected through classroom observations, video recordings of planning and reflection sessions, and interviews. Data was coded through emic and etic coding.

Chapter Six presents the data from the individually planned lessons using the MKfT, Knowledge Quartet, and Foundational Number Sense frameworks to highlight the the mathematics and pedagogical content knowledge that the teacher employ to develop learners' number sense.

Chapter Seven presents the data from the collaborative planned lessons using the MKfT, Knowledge Quartet and Foundational Number Sense frameworks to highlight the the mathematics and pedagogical content knowledge that the teacher employ to develop learners' number sense.

Chapter Eight analyses the data from interviews and the collaborative planned lessons using the theoretical framework of CHAT. The interviews highlight the benefits and constraints of a collaborative intervention as a vehicle for professional development and the development of teachers' mathematics and pedagogical knowledge. CHAT is used to analyse the collaborative intervention as a whole. It enables and analysis of the interaction of all the components of the activity (collaborative intervention) towards the achievement of the object (teachers' mathematics and pedagogical content knowledge) and the outcome (learners' number sense).

Chapter Nine is the conclusion of the research thesis. I discuss the findings of my study, the key contributions and implications of the study, and the limitations and recommendations for further research.

CHAPTER TWO: EXPLANATION CULTURAL HISTORICAL ACTIVITY THEORY

2.1. INTRODUCTION

Cultural Historical Activity Theory (CHAT) is a theoretical and analytical framework that focuses on how individuals learn by engaging in social and cultural practices (Feldman & Weiss, 2010.; Wilson, 2014 & Hancock & Miller 2017). Activity theory investigates human activity in a specific human setting, for instance, in work or learning (Murphy, Rodriguez-Manzanarez, 2008). Yamazumi (2006) describes CHAT as an intervention that seeks to facilitate and support innovative collaborative learning. As such, it is a methodology that analyses and redesigns human learning as it occurs in the communication between collaborators.

CHAT focuses on "collective social practices and considers the complexity of real-life activity. It also emphasises action or intervention to develop practice and site of practice" (Wilson, 2014. p. 20). In other words, CHAT allows for a focus on the day-to-day classroom activity of teachers and it is particularly useful in illuminating where the collaboration of teachers might result in the transformation of practice (Murphy, Rodriguez-Manzanarez, 2008). When participants work collaboratively, they identify the challenges they experience and take action to find solutions and improve their practice. For instance, CHAT can be applied to understand how teacher participation in an intervention facilitates change or entrenches stasis in their practice. Again, the lens of CHAT can provide an understanding of the transformation and restructuring of teaching practice facilitated by the teachers' engagement in an intervention (Murphy et al., 2008 and Feldman & Weiss, 2010).

2.2. THE ONTOLOGICAL ORIENTATION OF CHAT

CHAT originated in late 1920s and early 1930s, and was developed by Russian scholars particularly Vygotsky. His main concern was the relationship between an organism, its environment and how this enabled learning (Engeström, 2001, Yamagata-Lynch, 2010). According to Yamagata-Lynch & Haudenschild (2009), CHAT conceptualises individuals and their environment as a holistic unit of analysis that assumes a non-dualistic and process ontology. A non-dualistic ontology suggests that the people and the practices that they engage in cannot be separated; they are two-sides of the same coin. In other words, people make the practice, and thus, practices do not exist independent of people. A non-dualistic ontology asserts that individuals learn through interaction with others or objects in their social-cultural environment. In other words, the environment influences the development of human consciousness and therefore an individual's development affects the social environment (Roth, Radford and La Croix, 2012).

Moreover, Hancock and Miller (2017) explain that CHAT is based on a process ontology. A process ontology maintains that processes are real and that these processes are always embedded within social, cultural and historical contexts. For example, learning is always situated within a specific context.

2.3. AN EXPLANATION OF CHAT

According to Roth, Radford and La Croix (2012), CHAT's main features are culture, history, and activity. Foot (2014) points out that everything that people do is shaped by and draws upon cultural values and resources grounded in history. Activity refers to what people do together and is reconstructed by cultural and historical positioning. For example, teachers' practices are influenced by their beliefs, backgrounds and personal experiences. Additionally, Foot (2014) stresses that CHAT centres on three core ideas, which are:

- humans act collectively and learn through their actions and interactions;
- humans make, use and adapt tools to learn and communicate; and
- community is central to learning, communication and action.

Therefore, an activity continually evolves through collective learning actions. Engeström (2001) expresses that there are three generations of CHAT, namely Vygotsky's mediated action (first generation), Leontiev's collective model of an activity system as a unit of analysis (second generation) and Engeström's multiple activity systems to develop conceptual tools, dialogue and multiple perspectives (third generation). Here I deal with first and second generation CHAT because this research has one activity system that is a collaborative intervention of Grade One teachers. Data will be analysed, focusing on second generation CHAT and the principles of an activity system.

2.3.1. First generation CHAT

Vygotsky (1978) initiated the concept of mediated action to explain the process where learning is enabled by the use of conceptual and technical artefacts (tools) and by interacting with social others in an environment. It is through these processes that people find meaning in the world (Yamagata-Lynch, 2010). Yamagata-Lynch & Haudenschild (2009) explain mediated action as a conceptualised human activity capturing interaction among subject, tools, signs and object. Signs are by-products of the interaction between individuals and the tools that mediate thought processes. They are "not represented in Vygotsky's mediated action triangle but are assumed to be an artefact" (Yamagata-Lynch, 2010. p16) (Figure 2.1).

Vygotsky created a graphical representation of mediated action, illustrating the interaction of the mediating artefacts, subject and object (Figure 2.1).

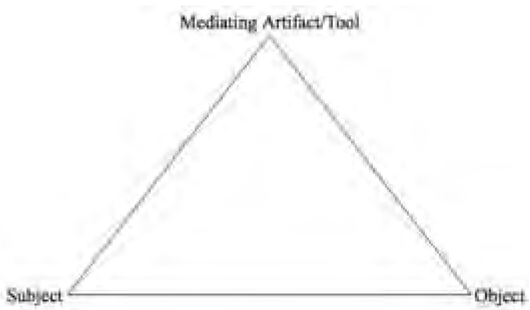

Figure 2.1: Vygotsky's mediated action diagram

The elements of the triangle include the:

- subject, which refers to the individuals involved in the activity;
- object that is the goal of the activity; and
- the mediating artefacts.

Hancock & Miller (2017) state that mediation occurs between object and subject. The object can be envisioned in project goals such as improved mathematics and pedagogical content knowledge. Yamagata-Lynch (2010) explains that tools mediate action. Tools can either be conceptual/psychological (e.g., language and knowledge) or practical/technical (e.g., lesson plans, observation schedules and policies). The subjects use the existing tools to mediate the object and, in the process, create new tools with which to engage, employ and attain the object (Foot, 2014).

The process of mediation has the potential to change not only individuals engaged in a practice but also culture and society (Wilson, 2014). The change can occur through the enhancement of the conceptual or cultural tools available to people as they work in the world to improve it. Therefore, Vygotsky initiated the concept of internalisation, which refers to how individuals process what they have learned through mediation to develop consciousness, that is, sense-making (Yamagata-Lynch, 2010). For example, a teacher (subject) uses tools to mediate action towards attaining an object (improved mathematics and pedagogical content knowledge). Practical tools could include policies, lesson plans, resources, books, interaction with other teachers etc. The teacher's prior knowledge and language could be conceptual tools that mediate the interaction.

Mediated action was criticised by a number of scholars for its being particularly individualistic (Engeström, 2001. & Yamagata-Lynch, 2010). Specifically they held that it over-emphasised the transformation experienced by individuals and under-emphasised the influence of this transformation on society and the social environment (Yamagata-Lynch, 2010). Consequently, Leontiev who was Vygotsky's colleague, developed second generation CHAT.

2.3.1.1. The role of tools in CHAT

Learning occurs in the form of interactions among signs, mediating tools and individuals (Yamagata-Lynch & Haudenschild, 2009). Tools mediate how individuals achieve an object and outcome. These tools include conceptual tools and practical tools (Hancock and Miller, 2017). Conceptual tools are psychological tools such as language and knowledge, while practical tools refer to concrete materials utilised by participants in an activity (e.g., a lesson plan). The object is the goal at which the activity is directed. Hancock and Miller (2017) suggest that appropriate tools contribute to understanding praxis (transformative action) by providing insight into how subjects refine their practice (Hancock & Miller, 2017). In this research CHAT will be used as a methodological framework to analyse and explain how an intervention can strengthen teachers' mathematics and pedagogical content knowledge and to analyse the changes that occurred (or did not) in the teachers' practices.

2.3.2. Second generation CHAT

Second generation CHAT moves beyond the individually focused analysis to a collective model of an activity system as a unit of analysis (Engeström, 2001). Leontiev extended CHAT to apply to a group of people instead of individuals and developed the activity system further (Wilson, 2014). Yamazumi (2006) suggests that "Leontiev's concept [of CHAT] associated with new components of division of labour and human cooperation, showed that activities motivated by purpose and objects are established …in a collective dimension" (p. 80). For Leontiev (cited in Yamazumi, 2006) learning occurs not only through collective activity within the activity system, but also through learning about the activity system.

Leontiev identified object-oriented activity, which involves interaction among subjects, object-action, socio historical context, consequences and activity (Yamagata-Lunch, 2010). In my research, subjects (teachers) are involved in object-actions such as lesson planning, observations in the classrooms in which they teach and reflections on their practices. Yamagata-Lunch (2010) refers to object-oriented activity as a mediational process in which individuals and groups of individuals participate. This process is driven by goals and motive which may lead to the creation of new cultural artefacts which make the activity robust (Yamagata-Lynch, 2010).

Yamagata-Lynch (2010) distinguishes between object-oriented activity and goal-directed action. Goal-directed action is temporary, individual and used as a step towards object- oriented activity. Engeström created a model representing an activity system as an entire unit of analysis (Yamazumi, 2006 & Yamagata-Lynch, 2010). This model recognises the importance of the rules, community and division of labour in an activity (Figure 2.2). According to Engeström (2001) object-oriented action is viewed as the key to understanding the human psyche. Wilson (2014) mentions that activity systems are not static. The development of activity theory aims at changing systems through provoking the collective to reinterpret the object and this results in the repositioning of the object and subject (Edwards, 2005). For example, in this research certain teachers realise whilst teaching number sense that there are some components of number sense (such as estimation) that they have not included in their lessons previously. This may result in improved knowledge of developing number sense, driving them closer to the object.

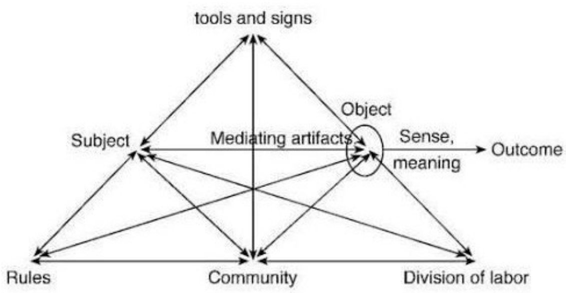

Figure 2.2: Engeström's structure of human activity system

Yamagata-Lynch (2010) explains that there are more components in an activity system than alluded to in first generation CHAT (Figure 2.2) and that these have the potential to mediate change. The components include: the rules that might constrain or enable the activity; the community that shares the interest and involvement in the object with the subjects, that is, the learners and the teachers; and the division of labour, which refers to the sharing of tasks oriented towards the object by the subjects (Yamagata-Lynch, 2010). In this research these components may include:

- ❖ Rules such as school times and regulations, participation in project activities, etc.;
- ❖ Community refers to learners, parents, SMT, teachers and district office; and
- ❖ Division of labour involves observations (two participants and myself), lesson presentations (one participant) and engaging in reflection and planning of new lesson sessions (all the participants and myself).

Engeström (2001) identified five principles of an activity system. Four of the principles relate directly to my research. The only principle that does not relate is expansive transformation. Although opportunities for expansive transformation are revealed through the contradictions in my research (Chapter 8), the activity system's duration did not give the subjects in my research the chance to act on them.

1. An activity system is a *unit of analysis* that enables the analysis of all components (mentioned above) of the activity system to understand what is happening in the activity. The activity system will be used to analyse the influence of all the components towards strengthening the mathematics and pedagogical content knowledge of Grade One teachers to develop learners' number sense.
2. Activity systems are *multi-voiced*. Individuals play multiple roles in an activity system. They are influenced by their own histories, traditions and beliefs. At the same time, participants in an activity system also share multiple points of view, traditions and interests. Accordingly, the teachers' age group, qualifications and experience will influence the way that they engage in their roles during the research, during the lesson planning, teaching and reflection sessions related to the intervention.
3. Activity systems *develop over time* and thus, history is important to understand the constraints and affordances. This suggests that I need to analyse the teachers' practices in relation to the curriculum, theories of learning and approaches applied in their teaching.
4. *Contradictions* in an activity system generate disturbances and conflicts. These have the potential to promote innovation and change. For example, teachers are now required to work collaboratively, having previously been accustomed to working in isolation.
5. *Expansive transformation* can occur within an activity system. This occurs when the object and motive of an activity are reinterpreted to realise more possibilities of learning. For instance, teachers engage in an activity to improve their mathematics and pedagogical content knowledge. However, during the activity they also learn strategies for making the lessons inclusive.

2.4. CONTRADICTIONS AS A SOURCE OF CHANGE AND DEVELOPMENT

Foot (2014) expresses that contradictions are present in every collective activity and indicate opportunities for activity development. Thus, Engeström (cited in Yamagata-Lynch & Haudeschild, 2009) developed an activity system model to allow researchers to identify inner contradictions that impose tensions on participants. Contradictions link a historically formulated activity with its "zone of proximal development" (Vygotsky, 1978). This refers to the area between the present activity system and the foreseeable future activity system (Engeström, 2001).

According to Yamagata-Lynch (2010), human activity may trigger tensions resulting in systemic contradictions. Hancock & Miller (2017) point out that these tensions may arise from any component of the activity. I give an example of a contradiction between the rules and the subjects of an activity. A group of teachers (subjects) set up a schedule for collaborative planning. However, an unexpected change to the scheduled sports calendar (rules) impacts on the teachers' planning schedule and they have to renegotiate alternative times to meet. Basically, contradictions are a source of development and change, and as such, they have the potential to cause the activity to succeed or fail (Feldman & Weiss, 2010. & Yamagata-Lynch, 2010). Consequently, the reason for this may be the subjects' willingness to adjust to the suggested innovative ways of development or persistence in 'sticking' to their historical and cultural ways of practice. Engeström (2001) identified four levels of contradiction. As this research will focus on the second generation of CHAT, the primary and secondary contradictions will be used for analysis of this research.

- *Primary contradictions* occur within each component of the activity. The tensions can occur between the subjects and are influenced by the other components of the activity (rules or division of labour). While subjects may share a common object, they may not agree with the division of labour necessary to work towards the object. In this scenario, it is likely that tensions will occur between the subjects.
- *Secondary contradictions* occur between the components of the activity. In other words, these are tensions that occur between the rules and the division of labour or any other component of the activity. The knowledge (tools) that the teachers apply to develop number sense may not support the learning of all the learners (community) in the classroom.

2.5. LIMITATIONS AND POSSIBILITIES OF CHAT

Vygotsky's mediated action was developed into CHAT because the focus of transformation was on the individual, and to some extent, it ignored the influence of the social environment on transformation (Yamagata-Lynch, 2010). Yamagata-Lynch (2010) also points out that Vygotsky's concept of internalisation is based on dualistic language because it focuses on the individual consciousness which is contradictory to the explanation of mediated action. To clarify, internalisation focuses on the development of the individual and ignores the individual's influence on the environment (culture, tools and social others). Edwards (2005) explains that first and second generation CHAT are strong on mediation and culture however they cannot help us to deal with problems that have not yet been encountered. She further expresses that the main aim of CHAT is empowerment but that it is inherently conservative. In other words, although CHAT empowers participants by refining their practice, it also preserves the existing conditions, for instance, by taking into consideration regulations of an institution and finding solutions to work around them.

By engaging in reflection sessions subjects express their point of view and learn through discussion (Yamagata-Lynch & Haudenschild, 2009). They can further identify contradictions and discuss strategies to deal with contradictions. For example, in my research teachers struggled to make their lessons inclusive because there were learners who had 'learning deficits' and the teachers did not know how to assist them. During the reflection sessions they discussed strategies to plan inclusive lessons and deliver them as such. CHAT offers an opportunity to reflect on one's interpretations of an activity and stimulates new professional learning (Edwards, 2005). Edwards (2005) further suggests that new learning can lead to critical evaluation of current working and teaching practices, which may lead to recommendations for improvement and change.

2.6. SIGNIFICANCE OF CHAT TO THIS RESEARCH

Yamazumi (2006) describes CHAT as an intervention methodology to facilitate and support innovative, collaborative learning by practitioners, and a paradigm that analyses and redesigns human learning. I will use the principles of CHAT and Foot's (2014) core ideas to conduct this research and to answer my research questions.

How can Grade One teachers' mathematical and pedagogical content knowledge, required to develop learners' number sense, be strengthened through a collaborative intervention?

Subjects in this research will act collectively and learn through their actions and interactions. The subjects of the activity system engage in a collaborative intervention where they implement and observe lessons to develop learners' number sense in their classrooms. They will mediate conceptual and practical tools such as policies, knowledge, lesson plans, observation schedules and manipulatives to implement and observe lessons in the classroom. One subject will present the collaboratively planned lesson while the rest of the subjects (including me) observe and document our observations using observation schedules as mediating tools. Afterward, the subjects communicate their views, ideas and strategies during collaborative reflection and planning sessions. The purpose of the collaborative reflection is to discuss the strengths and challenges observed during the lesson and to share ideas on improving the next collaborative lesson.

What mathematics and pedagogical content knowledge is required to develop number sense?
The subjects implement and adapt the tools to learn and communicate in an activity system. The teachers use their existing knowledge to engage in the discussions while adapting what they know by listening to their colleagues' views and experiences. The existing and new knowledge will be adapted based on the subjects' understanding and employed during the lesson implementation and planning sessions. Therefore, the principle of historicity will be considered to answer this research question. The first phase of interviews and observation of first three lessons, which are individually planned by each of the teachers, will bring to light the different experiences, knowledge and beliefs evident in the subjects' practices. The teachers use their existing knowledge to engage in the discussions while adapting what they know by listening to their colleagues' views and experiences. The subjects will work collaboratively to develop a series of lessons to be implemented. The sharing of the subjects' diverse knowledge, experience, strategies and theories (even informal) of learning and teaching brings in the principle of 'multi-voicedness' into the research.

How does collaborative planning and reflection enable the development of the mathematics and pedagogical content knowledge to develop learners' number sense?
The community which refers to learners and teachers is central to learning, communication and action undertaken in the activity system. During collaborative planning and reflections, the discussions will focus on the implementation of the lesson and the impact on the learners,

teachers and the outcome of the activity. The development of the teachers' conceptual tools may improve learners' achievement and the attainment of the outcome. While contradictions may occur, the discussions during reflection sessions may allow teachers to negotiate new ways to 'act back' on the contradictions. As CHAT is an analytical tool, I will use it to identify the contradictions in the activity system and whether they encouraged learning and improved practice.

Foot (2014) highlights that contradictions are present in every collective activity and reveal opportunities for new ways of structuring and enacting activity by highlighting the area between present and future activities. Also, Foot stresses that an object is never fully accomplished; thus Roth et al. (2012) explain that an activity system is dynamic and can develop into other activity systems. Although this activity system is underpinned by second generation CHAT, the contradictions that arise will reveal opportunity for other activity systems and reinterpretation and restructuring of the object.

2.7. CONCLUSION

Cultural Historical Activity Theory (CHAT) is a theoretical and analytical framework that focuses on collective social practices and considers the complexity of real-life activity (Feldman & Weiss, 2010; Wilson, 2014 & Hancock & Miller 2017). The key features of CHAT are culture, history and activity (Foot, 2014). According to CHAT, everything that people do together is reconstructed by cultural beliefs, values, and tools grounded in history. According to Engeström (2001) there are three generations of CHAT that is Vygotsky's mediated action (first generation), Leontiev's collective model of an activity system as a unit of analysis (second generation) and Engeström's multiple activity systems (third generation). This research focuses on the second generation CHAT, which refers to understanding people's collective engagement in an activity and how they construct new tools to mediate the object. All the components of the activity system, that is, *the subject, object, tool, rules, community, division of labour and outcome* influence each other and are also influenced by various social, cultural and historical factors. The interaction between the components can result in contradictions, which are indicators of further activity development opportunities. These contradictions have the potential to facilitate change in the activity.

CHAPTER THREE: THE OBJECT OF THE RESEARCH: MATHEMATICAL AND PEDAGOGICAL CONTENT KNOWLEDGE

3.1. INTRODUCTION

Interest in the knowledge that teachers require for effective teaching arose out of an international comparison of learners' performance across counties in the 1980s. The comparison raised a number of concerns for countries such as the United Kingdom and United States. Teachers were regarded as part of the problem of poor learner performance (Petrou & Goulding, 2011). In the aftermath, Shulman (1986, 1987) began to engage with this concern. He maintained that prior to the 1980s, developing learner-teachers' content knowledge was the main focus of teacher education programmes. He identified a 'blind spot' between content knowledge and teaching competence. To clarify, Shulman (1987) argued that "possessing content knowledge and not the skills to teach it, is as useless as having skills without content" (p.8). He thus insisted that the teachers require a blend of content and pedagogical content knowledge in order to be effective in the classroom.

Drawing on CHAT framework (Chapter 2), the development of teachers' mathematics and pedagogical content knowledge is the object of my research. I review three dominant frameworks used to identify and develop the mathematics and pedagogical content knowledge that teachers draw on as they engage in the practice of teaching. These are Lee Shulman's 'Pedagogical Content Knowledge', Deborah Ball and colleagues' 'Mathematics Knowledge for Teaching Framework', and Tim Rowland and colleagues' 'Knowledge Quartet'.

3.2. SHULMAN'S PEDAGOGICAL CONTENT KNOWLEDGE

In his efforts to ascertain the knowledge that teachers require to do their work as teachers, Shulman (1986, 1987) identified seven categories of knowledge. The first four refer to aspects of teacher knowledge, as highlighted in Table 3.1. The last three refer to the missing- content dimensions of teacher knowledge (Ball et al, 2008 and Petrou & Goulding, 2011). These are Content Knowledge, Pedagogical Content Knowledge and Curricular Knowledge (Shulman, 1987).

Teacher knowledge	• General pedagogical knowledge
	• Knowledge of learners' characteristics
	• Knowledge of educational context
	• Knowledge of educational purpose and values
Pedagogical Content Knowledge (Missing Paradigm)	• Content Knowledge
	• Curriculum Knowledge
	• Pedagogical Content Knowledge

Table 3.1: Shulman's seven types of knowledge

Curricula and Content Knowledge refer to understanding a variety of ways in which the basic concepts, topics and skills are organised. The teacher needs to have substantive and syntactic knowledge. To clarify, the teacher needs to be able to define concepts and also have a deep comprehension and knowledge about the origin of the concepts, and how they came to be part of a particular discipline (Shulman, 1986).

Pedagogical Content Knowledge represents aspects of content that relate to teaching and understanding how learners learn. It incorporates the aspects of content with relevant and specific representations, examples and applications that teachers use in order to make subject matter comprehensible to students. Shulman (1986) stresses that teachers need to have a collection of alternative forms of representation so that they can overcome students' difficulties. Shulman (1986) recognises the importance of context by including Curricular Knowledge in his framework. Curricular Knowledge represents the full range of programmes, including materials and tools, designed for teaching the subject matter and topics at different grades. The teachers must know the content that they teach in different grades and levels, and they must know what students require to master the content at these respective levels.

3.2.1. Critique of Shulman's Pedagogical Content Knowledge

The focus of Shulman's research was the knowledge for teaching that secondary school teachers require. Ball, Thames & Phelps (2008) developed Shulman's ideas further by putting their energies into examining the knowledge that primary school teachers require. Unlike

Shulman whose research was not discipline specific, Ball et al. (2008) focused specifically on the teaching and learning of mathematics. As a result, they introduced the phrase ‘knowledge about mathematics’ which they later referred as ‘Mathematical Knowledge for Teaching’ to emphasise the specificity of the discipline.

Central to Ball’s (2000) research approach was the view that any attempt to ascertain the knowledge for teaching that teachers draw on, requires that one observes teachers in the act of teaching. Ball (2000) expressed that “knowing teaching is more than applying prior understandings. It also depends fundamentally on being able to know things in a situation” (p.90). Consequently, teachers need an explicit understanding of mathematics content in order to understand what makes learning a specific topic easy or difficult and the errors that learners are likely to make in a specific situation. In other words, the teacher needs to have different strategies of teaching specific topics to make them comprehensible to the learners. Furthermore, Ball, et al. (2008) point out that Shulman’s conceptualisation does not acknowledge the interaction between categories because of a lack of clear definitions and empirical testing. It is unclear how ideas in one subject relate in another subject and also how findings in the same subject relate to each other because it is not focused on a specific content area.

3.3. BALL’S MATHEMATICS KNOWLEDGE FOR TEACHING

Ball, Thames and Phelps (2008) refined Shulman’s seven categories by developing the Mathematical Knowledge for Teaching framework in order to clarify the distinctions between Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK).

Subject Matter Knowledge comprises three domains: Common Content Knowledge; Specialised Content Knowledge; and Horizon Content Knowledge. Common Content Knowledge (CCK) refers to the general knowledge that every citizen should have which includes an individual’s ability to calculate an answer and to solve mathematical problems correctly. Specialised Content Knowledge refers “to the knowledge and skill unique to the work of teaching” (Ball et.al., 2008, p.400). While the identification of learners’ errors is based on Common Content Knowledge, Specialised Content Knowledge (SCK) is the mathematical knowledge for teaching which is needed for the purpose of teaching (Ball et. al., 2008). It involves unpacking mathematics in a classroom setting. Therefore, Edwards, Hyde, O'Connor & Oldham (2015) further describe SCK as the knowledge required by teachers to know why a learner made an error and how to assist the learner in correcting the error. Horizon Content Knowledge (HCK) is the "mathematical knowledge that spans across the mathematical curriculum that helps the teacher to view mathematics as a whole" (Chikiwa, Graven & Westaway, 2019. p.2). HCK is evident when teachers understand the progression of the curriculum topics across grades and phases. According to Ball et al. (2008), this knowledge helps teachers to make appropriate decisions about teaching concepts, for example, linking counting and number bonds in Grade One to problem solving in Grade Three.

Pedagogical Content Knowledge comprises the following domains: Knowledge of Content and Students; Knowledge of Content and Teaching; and Knowledge of Content and Curriculum. Knowledge of Content and Students (KCS) enables teachers to help learners to access mathematics content such that their understanding of mathematics is enhanced. Teachers should show awareness of students' conceptions and misconceptions about mathematics topics. Teachers are required to predict what students may find interesting and motivating. Knowledge of Content and Teaching (KCT) enables teachers to choose appropriate teaching strategies, tasks, examples and representations to develop learners' understanding of mathematics. Ball et al. (2008) stress that teachers require an ability to link specific mathematical understanding with an understanding of pedagogical issues that affect student learning. Knowledge of Content and Curriculum (KCC) concerns knowledge of the curriculum requirements and the Learning and Teaching Support Materials (LTSM) that can be used and developed to teach particular content (Ball et al., 2008). The conceptualisation of Ball's MKfT is represented in Figure 3.1 below.

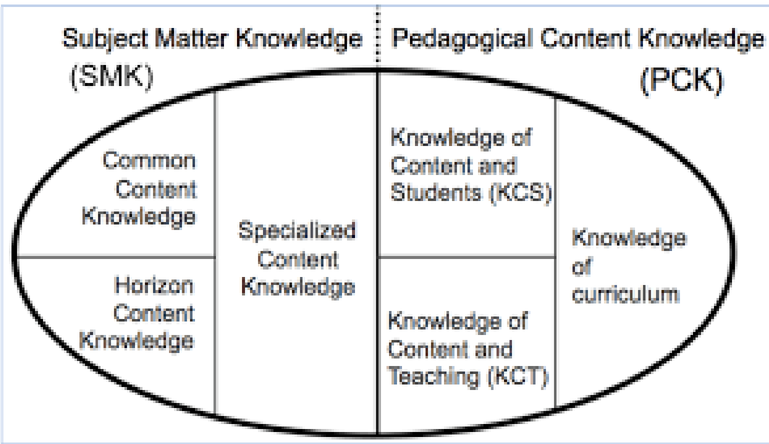

Figure 3.1: Dimensions of Mathematical Knowledge for Teaching (Ball, Thames & Phelps, 2008)

Edwards et al. (2015) exclaim that teachers need to be proficient in all six domains so that they can develop learners' understanding of mathematics.

3.3.1. Critique of Ball's conceptualisation of MKfT

Ball et.al. (2008) explain that the purpose of refining categories of subject matter and pedagogical content knowledge is to "ascertain whether there are aspects of teachers' content knowledge that may predict students' achievement more than others" (p.12). According to Petrou & Goulding (2011), Hill, Rowan and Ball's (2005) contribution to the conceptualisation of MKfT is showing that teachers' mathematical knowledge is related to student achievement and providing evidence through their research that teachers with weak knowledge transmit this to their students. This is elaborated in Chapter 1. They also developed measures of teachers' mathematical knowledge by refining Shulman's conceptualisation. However, Petrou & Goulding (2011) argue that Ball and colleagues' conceptualisation does not acknowledge the importance of teachers' beliefs in teaching mathematics. For example, if a teacher believes that mathematics is a subject of rules and routines to be remembered, this will inform his or her approach to teaching. They further highlight that there is no clear distinction between the definition of Specialised Content Knowledge and Pedagogical Content Knowledge because they both refer to the teacher having knowledge about the subject and being able to make it comprehensible to others. However, SCK is focused on mathematics whereas PCK is not focused on a specific subject.

Therefore, Rowlands & Turner (2007) proposed ‘the Knowledge Quartet’, which can be used in understanding the ways in which SMK and PCK are related and ’play out’ in the classroom and to identify opportunities to enhance knowledge.

3.4. THE KNOWLEDGE QUARTET

Rowlands & Turner’s (2007) Knowledge Quartet (KQ) refers to a comprehensive tool for thinking about the ways that subject knowledge comes into play in the classroom. The four types of knowledge that make up the quartet are: Foundation; Transformation; Connection; and Contingency. Foundation Knowledge is the knowledge, beliefs and understanding teachers acquire in preparation for their role in the classroom. Textbooks, articles and journals are the literature that informs this knowledge. Rowlands & Turner (2007) argue Foundation Knowledge informs teachers’ pedagogical choices and strategies. Transformation Knowledge refers to knowledge in action, as demonstrated both in planning to teach and in the act of teaching. It involves the examples, demonstration of procedures for concept development by the teachers as well as explanations and questions from students. Connection Knowledge enables the binding together of choices and decisions pertaining to mathematical content and learning procedures. It is the link made between different lessons, ideas and parts of the lesson as well as the linkage between concepts and topics in the mathematics curriculum. Connection Knowledge includes sequencing of activities for teaching and awareness of possible difficulties learners may experience with different tasks and topics. Contingency Knowledge refers to classroom activities that are almost impossible to plan, for example, a teachers’ readiness to respond to students’ questions. It refers to teachers’ ability to respond appropriately to students’ wrong answers and to deviate from their lesson plans. In other words, it requires an ability to ‘think on one’s feet’ (Rowland & Turner, 2007).

3.5. THE RELATIONSHIP BETWEEN THE DIFFERENT DOMAINS OF TEACHER KNOWLEDGE

Petrou & Goulding (2011) highlight that although researchers have proposed different domains of teacher knowledge, they all draw attention to the importance of understanding that knowledge should be integral to teaching. As shown in Figure 3.2 below, the relationships between the different frameworks are not limited to mathematics content knowledge.

Foundation Knowledge develops during the schooling, teacher education and professional teacher development experiences of the teachers. It includes content, pedagogical and curriculum knowledge, and beliefs (Rowland & Turner, 2007). This knowledge informs everything that a teacher does in the classroom, including planning for and delivery in the classroom (Transformation Knowledge), making connections across topics and concepts (Connection Knowledge) and responsive competence (Contingency Knowledge). As such, the Foundational Knowledge includes Shulman's PCK and all six domains of Ball's MKfT. I highlight the connections between the different frameworks in Figure 3.2 below.

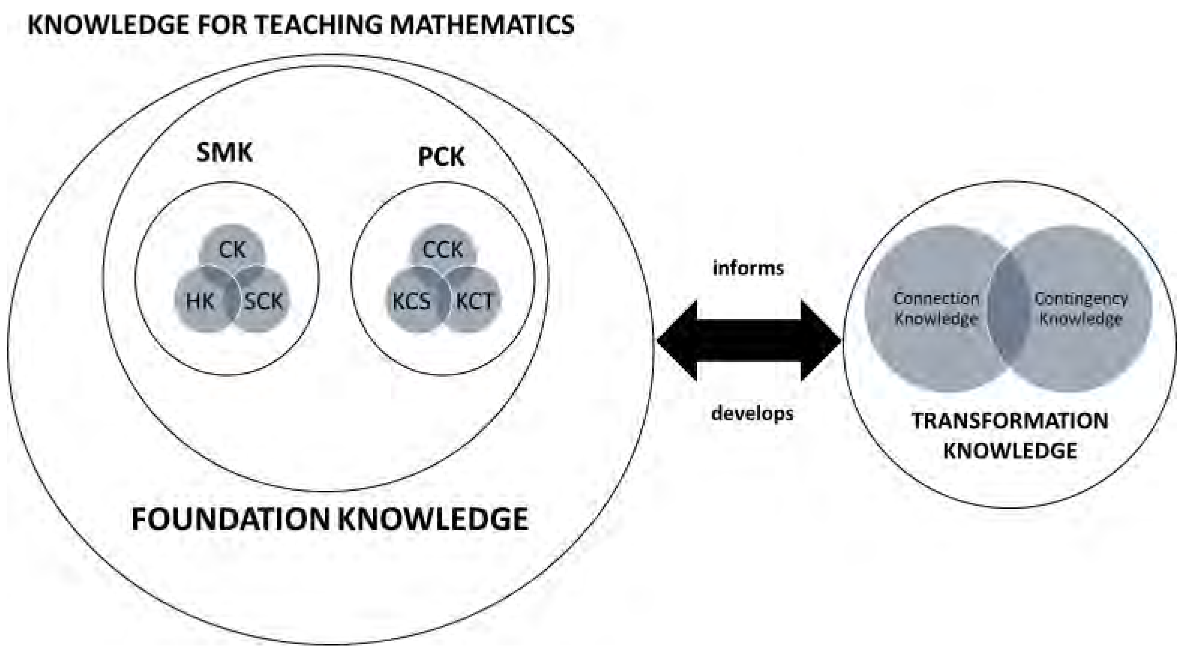

Figure 3.2: Synthesis of models on teacher mathematical knowledge

As mentioned above Foundation Knowledge incorporates the knowledge proposed by Shulman (PCK) and Ball and colleagues (SMK & PCK). This knowledge informs the manner in which teachers' plan and act in the classroom. This includes the manner in which they transform the Foundation Knowledge, the connections they make and the manner in which they respond to learner thinking, questions and errors. However, this is not a one-way direction. Transformation Knowledge (including Connection and Contingency Knowledge) also informs and develops Foundation Knowledge

3.6. CONCLUSION

Drawing on the explanation of learner underperformance being rooted in teachers' limited mathematics and pedagogical content knowledge (Chapter 1), this chapter examined three frameworks that focus explicitly on the knowledge teachers require to do their work as teachers. The frameworks were presented chronologically starting with Shulman's PCK, followed by Ball and colleagues' MKfT and finally Rowland and colleagues' KQ. After presenting each of the frameworks, a critique was offered with a view to providing a rationale for the choice of using MKfT and the KQ as analytic and explanatory tools. Both of these frameworks focus specifically on the teaching and learning of mathematics. While I have chosen the latter two frameworks, it should be emphasised that the conceptualisation of teacher knowledge proposed by Shuman, Ball and Rowland are not inconsistent with one another; rather, they build on each other.

CHAPTER FOUR: CONCEPTUALISING NUMBER SENSE

4.1. INTRODUCTION

Briand-Newman, Wong and Evans (2012) asserts the importance of number sense and maintains that the "acquisition of number sense has been recognised as a fundamental component of learning mathematics" (p.130). As highlighted in Chapter 1, learners' underperformance in mathematics in various national and international systemic evaluations has been partially attributed to poor number sense. Graven & Venkat (2017) explain that, in many classes, learners are required to progress from unit counting (or tallying) to the formal algorithm for calculating. The implication is that learners are not given the opportunity to develop a wide repertoire of strategies for calculating nor are they able to reason about the suitability and efficiency of different strategies for different problem-types.

Drawing on the CHAT framework (Chapter 2), the development of learners' number sense is the proposed outcome of my research. Thus I explore this concept in this chapter.

4.2. CONCEPTIONS OF NUMBER SENSE

The term number sense is contested, with little agreement as to what it entails. Berch (2005) highlights that number sense has been conceptualised from two different perspectives: cognitive science and mathematics pedagogy. The cognitive scientists argue that number sense is a competence that humans and non-human primates are born with (Dehaene, 2001). In other words, it is innate and applies to *all* humans. The mathematics education perspective refers to number sense as an acquired skill that develops through instruction.

From a mathematics education perspective, number sense is generally regarded as foundational knowledge needed by learners to understand and link quantities to our number system, numerical constructs and mathematical strategies (Briand-Newman, Wong and Evans, 2012). Howden (1989) describes number sense as involving good intuition about numbers and their relationships with one another. She further points out that number sense develops as learners visualise numbers in varying contexts and relate to them in ways that are not limited by traditional algorithms. McIntosh, Reys and Reys (1992) suggest that number sense exhibits itself in various ways as learners engage in mathematical thinking. They describe number sense as a person's general understanding of number and number operations as well as the ability and tendency to think in flexible ways and develop useful and appropriate strategies for calculating. Given the difficulty in defining number sense, McIntosch et al. (1992) suggest that researchers identify characteristics of number sense, specifically focusing on what it 'looks like' in the classroom. The characteristics which they highlight include:

- knowledge of and facility with numbers;
- knowledge of and facility with operations; and
- applying the above to computational settings (McIntosh et al., 1992, p.4).

Embedded in these characteristics is the view that children should be able to work effectively and flexibly with a range of calculation strategies. While the work of McIntosch et al. (1992) focuses on the development of number sense in school, Sayers and Andrews (2015) examine the link between children's innate number sense and the number sense required to be successful in school. They refer to this as Foundational Number Sense (FNS).

4.3. FOUNDATIONAL NUMBER SENSE

Sayers and Andrews (2015) have elaborated the characteristics of number sense as defined by McIntosch et al. (1992), but focus specifically on the early years. In developing characteristics particular to the early years, they acknowledge that there are three broad perspectives of number sense. These are referred to as preverbal, verbal and FNS.

4.3.1. Preverbal Number Sense

Preverbal number sense refers to the innate possession of number sense of all humans. It is grounded in the cognitive science perspective of number sense. This innate sense of numbers is referred to by Spelke (2000) as core knowledge. This knowledge is recognisable in early infancy. Spelke (2000) argues that the core knowledge system found in infants contributes to later cognitive functioning in two ways: it serves as building blocks for the development of new cognitive skills; and it continues to contribute to the development of number sense in older children and adults.

Core knowledge related to number sense includes the Approximate Number System (ANS) and the Object Tracking System (OTS). The ANS enables infants to compare quantities while the OTS assists in identifying changes in quantity. The OTS forms the basis of early arithmetic as infants are able to engage in preverbal addition and subtraction. In relation to OTS, there are limitations, for example, infants can only track up to three objects. The ANS enables infants to discriminate quantities through visual, spatial arrays of dots and auditory arrays of sounds. This core knowledge is the foundation for subitising and estimation (Spelke, 2000; Dehaene, 2001).

The significance of this contribution from cognitive science is that it supports the view that quantity is the basis for early mathematics rather than counting. The ANS system suggests that children have an innate capacity for subitising, and understanding ‘more’ and ‘less’. The OTS is the basis for simple arithmetic (addition and subtraction). The implication is that teachers should draw on these cognitive resources when teaching mathematics.

4.3.2. Verbal Number Sense

Verbal number sense develops as infants start interacting in the world and begin to develop the language of and for mathematics (e.g., the counting words, the terms for concepts such as more or less). Verbal number sense draws on the core knowledge of infants (Sayer & Andrews, 2015).

Spelke (2000) and Butterworth (2005) highlight that most children have a basic understanding of natural numbers before they go to school. Butterworth (2005) mentions that the key intersection or overlap between children’s number sense and language is counting. He defines counting as a “complex skill which involves learning the counting words in the correct order, coordinating the production of counting words with the identification of objects …and that each object in the set is counted only once” (p.6). Children understand that counting starts from one and continues by addition of one, that is, the successor principle (Spelke, 2000). According to Butterworth (2005) “counting makes the first bridge from the child’s innate capacity for numerosity” (p.7) because they start to relate the number of objects with the counting words. The language of number words and the counting routine allow young children to combine representations of objects by being able to announce the last counting word as the number of objects represented.

Moreover, Butterworth (2005) highlights that “children enter school with informal concepts of number and arithmetic based on their experience of counting and calculation” (p.10). This experience varies and may be as a result of the individual child’s family background (Sayer & Andrews, 2015). According to Sayers, Marschall, Petersson and Andrews (2019) parents are the initial teachers of children. Therefore, a parent’s attitude and experience of mathematics will influence a child’s initial experience of and exposure to number sense. Sayers et al. (2019) point out that the instruction that children receive at home may be formal or informal. Formal instruction refers to a parent offering a child the opportunity to practice ‘school based mathematics’ such as counting objects, practising number names and symbols and writing number names and symbol. Informal instruction refers to parents playing card games with children, involving them in house chores such as cooking and shopping, and other activities that develop simple early arithmetic competence. Sayer and Andrews (2015) refer to this verbal number sense that develops prior to school as Foundational Number Sense (FNS). Through their research with pre-school and Grade 1 teachers, parents and learners, they have developed a set of number sense characteristics that they refer to as FNS.

4.4. CHARACTERISTICS OF FOUNDATIONAL NUMBER SENSE

FNS is the number sense that starts to develop prior to school, but that needs to be consolidated through instruction during the first year of formal schooling. Sayer and Andrews (2015) have identified eight interrelated components of FNS:

1. *Number recognition, vocabulary and meaning* refers to an understanding of the relationship between a quantity, the appropriate number symbol and number name.
2. *Systematic counting* is the process of being able to count both forwards and backwards from 1, as well as from arbitrary starting points.

3. Relating numbers to quantity entails understanding the *one-to-one correspondence between number and the quantity* it represents.
4. *Quantity discrimination* refers to comparing magnitudes and using language like “bigger than” or “smaller than”.
5. *Using different forms of representation* (e.g., fingers, number lines) to make connections.
6. *Estimating* the size of a set or object.
7. Performing *simple arithmetic operations*, that is, addition and subtraction.
8. Recognition and extension of *number patterns* and, in particular, identifying a missing number.

These components are related because there is a link between learners’ knowledge of counting, magnitude of numbers, number representation and simple arithmetic competences (Sayers and Andrews, 2015). According to Sayers & Andrews (2015) if the components are not connected the child may be able to count but not understand the magnitude of numbers. Hence, teachers have to employ different teaching strategies to ensure that the components of number sense are included in their number-related lessons.

4.5. DEVELOPMENT OF FOUNDATIONAL NUMBER SENSE

Children acquire FNS through intentional instruction (Andrews & Sayers, 2015). Before learners begin formal schooling, parents can engage children in numerical games and activities in their daily lives to develop their number sense (Berch, 2005). Tsao & Lin (2012) stress that teachers play an important role in building number sense in the type of classroom they create, the teaching practices they employ and the activities they select. Andrews & Sayers (2015) highlight the importance of providing learners with different forms of representation, specifically, teachers should work with concrete materials (e.g., objects, ten- frames, calendars) to teach learners to count and explore numerical ideas, such as, more and less, and the composition and decomposition of numbers. Teachers should encourage learners to reason mathematically and share their thinking by asking open-ended questions, and encouraging learners to discuss their solution strategies, create alternative methods of calculation, and record their thinking.

4.6. FACTORS THAT INHIBIT THE DEVELOPMENT OF NUMBER SENSE?

Number sense is a teachable skill that develops through experience (Berch, 2005) however there are various factors that may inhibit its development (Aunio, Mononen, Ragpot and Tormanen, 2016). Brand-Newman et al. (2012) and Aunio et al. (2016) hold that teachers' knowledge of mathematics and pedagogy, and the provision of sufficient opportunities to learn, are key to the development of learners' number sense. According to Briand-Newman et al. (2012) learners commence school with varying degrees of number sense. Aunio et al. (2016) explain that children who enter school with an under-developed number sense are likely to remain behind their peers throughout their schooling career. Factors influencing the development of number sense in school include the Language of Learning and Teaching (LoLT). English learners often perform better than those who are taught in African languages in South Africa. However, this is often linked to the social-economic status of the environment in which the school is situated. For example, schools in more affluent areas have access to resources, have smaller classes, and are often better equipped to cater for learner diversity (Aunio et al. 2016).

4.7. CONCLUSION

According to Howden (1989), possessing the ability to relate to numbers in various ways prepares learners to study mathematics in higher grades. She mentions that possessing good number sense builds on the learners' natural insights and convinces them that mathematics makes sense and that it is not just a collection of rules to be applied. In developing number sense, learners gain confidence in their ability to do mathematics as they realise that there is more than one way to solve a mathematical calculation.

In this study I worked together with three Grade One teachers to develop the various components of number sense as determined by Andrews and Sayers (2015). These components of number sense were particularly important to this study since I explored the teachers' use and elaboration of the mathematical and pedagogical content knowledge required to develop children's FNS during the collaborative intervention.

CHAPTER FIVE: METHODOLOGY

5.1. INTRODUCTION

In this research, I engaged with teachers in a collaborative intervention in order to understand the mathematical and pedagogical content knowledge they apply to develop their learners' number sense. This research is a qualitative case study as it strives to understand the structure of knowledge that comes from the researcher interacting with the participants, spending time at the research site and probing to obtain more detail (Creswell, 2013).

5.2 RESEARCH ORIENTATION

This research is underpinned by the interpretivist orientation. Creswell (2013) and Bertram & Christiansen (2014) describe interpretivism as qualitative research. Specifically, the researcher's objective is to make sense of the meanings that others have about the world in the specific context in which they live and work. The purpose of interpretivist research is to understand the standpoint of an individual who is part of the ongoing action being investigated (Cohen, Manion & Morrisson, 2018). In this case I sought to understand the mathematics and pedagogical content knowledge that teachers apply when developing learners' number sense.

The interpretivist approach allowed me to understand the specific phenomenon of this research (Creswell, 2013) by focusing on the meaningful (inter)actions that the participants engaged in, during the planning sessions, as they taught in the classroom and in the reflection sessions that followed (Cohen et al, 2018). The purpose of the collaborative intervention was to strengthen their mathematical and pedagogical content knowledge. The (inter)action allowed for the expression of multiple perspectives from the participants and the subjective interpretation of the researcher (Creswell, 2013). Merriam (2009), Wahyuni (2012) and Bertram & Christiansen (2014) all maintain that interpretivist research is premised on the view that our knowledge of the world is based on our subjective experiences. In other words, there is no single reality or truth about the social world, but rather a set of realities or truths which are based on historical, local, specific experiences of individuals and groups. Given the subjective nature of our experiences, there is no objective, generalisable reality.

In my research, I was aware that the participants had their own interpretations and experiences of teaching and learning mathematics, and the development of learners' number sense. I had to be mindful of this, while at the same time being conscious that I was interpreting their (inter)actions from my subjective experiences and perceptions. Creswell (2013) refers to this as the 'double hermeneutic' meaning that interpretations are shaped by both the participants and researcher's experience and background. As such, it was necessary for me to ensure that I maintained, as far as possible, the validity of the data, and my interpretation thereof, through the processes of triangulation and member checking.

5.3 CASE STUDY

My research was a case study. Case study in an investigation into a 'bounded system' (Stake, 1995; Merriam, 2009; Yin 2009) and is used to enhance our understanding of communities and individuals and the contexts and deeper issues surrounding them (Hamilton & Corbett-Whittier, 2012). My research is 'bounded' by the fact that I focussed specifically on the application of Grade 1 teachers' mathematical and pedagogical content knowledge in developing learners' number sense. In this sense, my research was "an empirical inquiry that investigated a contemporary phenomenon in depth and with its real-life context" (Yin, 2009, p.31). Stake (1995) expands on this suggesting that case study considers a "particular and complex phenomenon… coming to understand its activity within important circumstances". I chose a group of Grade 1 teachers to work with and research, as many of the foundations of number sense are developed in this specific grade. Merriam (2009) adds that the case in case study research is explained "as a unit of analysis in a bounded context" (Merriam, 2009, p.54). The unit of analysis in my research is the application and development of Grade One teachers' mathematics and pedagogical content knowledge as they engage in a collaborative intervention.

Stake (1995) identified three types of case studies: intrinsic; instrumental; and collective. An intrinsic case study is undertaken when a researcher is interested in a particular case. An instrumental case study provides insight into an issue while a collective case study focuses on a number of cases. This research is an intrinsic-instrumental case study. My interest was understanding a single phenomenon with a select group of teachers with a view to providing insight into the mathematics and pedagogical content knowledge of the teachers as they develop learners' number sense. As such, this was small scale research study and thus the results are not generalisable (Merriam, 2009; Creswell, 2013 & Bertram & Christiansen, 2014).

5.4 SITE AND PARTICIPANTS

Purposive and convenient sampling were used to select both the site and participants for this research (Merriam, 2009). The research was conducted at a primary school in Botaleng, a township in the Northern Cape. The school is a no fee paying school and relies on the departmental subsidy and money raised from parents during fund-raising activities. The Language of Leaning and Teaching (LoLT) is Setswana in the Foundation Phase and English in the Intermediate and Senior Phase. Most of the learners speak Setswana mixed with Afrikaans at home. The school has 934 learners and starts from Grade R until Grade Seven. There are 29 teachers of whom five are on the school management team. There is a secretary, three general and auxiliary workers such as food handlers, interns who assist with administration and reading coaches. The school has resources such as textbooks, readers, mathematics and science manipulatives, and tablets and laptops. The latter were donated to the school. There is no library or science laboratory. Most of the learners are second generation learners at the school (their parents are former learners at the school).

My aim was to engage with Grade One teachers to facilitate opportunities for us to establish a collaborative intervention. Given that we are all teaching in the same school, we were able to meet on a regular basis. I selected these participants because the purpose of my research was to understand the necessary mathematics and pedagogical content knowledge of Grade One teachers to develop learners' number sense, and how this knowledge is developed (or not) in a collaborative intervention. I purposefully chose Grade One as this is where that the foundations of number sense (in school) are developed.

Three teachers (excluding myself) participated in my research. Doreen speaks Setswana. She has a Junior Primary Teachers Diploma and has been teaching in the Foundation Phase for 30 years. Lerato also speaks Setswana and has a Bachelor of Education in Foundation Phase. She has been teaching for three years. Gladys speaks Setswana and Afrikaans and as such, she can communicate easily with the learners who come from Afrikaans speaking households. Gladys has a Junior Primary Teachers Diploma and has been teaching in the Foundation Phase for 20 years. It was convenient working with the Grade One teachers where I work as I have already established working relations with the participants (they are my professional colleagues). We have shared aspects of our practice on numerous occasions over a protracted period and they previously participated in my Honours research project.

5.5 DATA COLLECTION METHODS AND TOOLS

When generating data on the collaborative intervention to develop teachers' mathematics and pedagogical content knowledge to support the number sense development of their learners, I utilised different data collection methods. These included interviews, observations and a reflective journal. The different data collection methods assisted in answering different questions as seen in Table 5.1.

Research Question	Data
What mathematics and pedagogical content knowledge is required to develop children's number sense?	Interviews Recordings of lesson planning and reflective sessions. Observation notes of the teachers My reflective journal
How does a collaborative intervention enable or constrain the development of the mathematics and pedagogical content knowledge required to develop learners' number sense?	Interviews My research journal

Table 5.1: Relationship between data collection methods and research questions.

5.5.1. Interviews

An interview is an interchange of views between two or more people on a topic of mutual interest (Cohen, Manion & Morrison, 2007). Kvale and Brinkman (2009) argues that interviews provide access to the lived world of the research participants who in their own words describe their activities, experiences and options. Interviews thus enable researchers to explore the social reality of the research participants' knowledge, understandings and interpretations (Mason, 2002).

According to Bertram and Christiansen (2014) there are three types of interviews: structured; semi-structured; and unstructured. In this research, I used semi-structured interviews as I wanted to ask follow-up questions and probe where necessary. The questions I asked were open-ended to enable the interviewee to talk freely about the topic. The interview schedule is presented in Appendix 1.

In this research, interviews were conducted in two phases. At the beginning of the research, I conducted an interview with each of the teachers in order to establish their teaching experiences, their knowledge of number sense and how they developed their learners' number sense. I decided to conduct individual interviews to afford the participants the opportunity to share their own experiences and practices (without possible peer pressure from their colleagues).

At the end of the intervention process, I interviewed the participants to ascertain their experiences of participating in the collaborative intervention, the mathematics and pedagogical content knowledge they developed, and the enhancement and constraints they experienced while participating in the intervention (Appendix 2). Again, I used individual interviews as I wanted to ascertain the views and experiences of each of the teachers. All of the interviews were audio-recorded. I transcribed all the interviews immediately after the interview. I chose to do this myself to begin engaging with my data without delay and to ensure the anonymity of the participants. Anonymity might be difficult to achieve if the participants' voices can be recognised by an external transcriber. While Cohen et al. (2007) argue that transcribing interviews is time-consuming, I did not find this to be the case. Having initially ensured that the interviews were not unnecessarily lengthy, I then transcribed each interview while it was still 'fresh in my mind'.

5.5.2. Observation

I conducted several types of observation in my research. I observed: teachers teaching in their classrooms; interactions while planning lessons as a group; and interactions while reflecting on the lessons. The latter two forms of observations occurred after the planning and reflection sessions respectively when I observed the video-recorded data.

Observations were a necessary component of my research as they enabled me to see the interactions, actions, behaviour and the way people interpret these actions (Mason, 2002). As Mason (2002) states, knowledge of the social world can be generated by observing or participating in natural real-life settings. Observations also enable the corroboration of interview data.

As noted above, I used the semi-structured observations. This was appropriate since I was clear about the specific phenomenon that I wanted to explore. For this method, the researcher is required or at least advised to develop a predetermined schedule (Appendices 3A & 3B) to gather data to illuminate the phenomenon (Cohen et al, 2018). I used a semi-structured observation schedule which I designed for the first three lessons as I wanted to find out how the teachers develop learners' number sense.

Creswell (2013) and Cohen et al. (2018) highlight that a researcher can participate in research in four ways, that is, as *complete participant, participant-as-observer, observer-as- participant and complete observer*. I was an observer-as-participant as I was involved in the group activities, for example, I was involved in the planning and reflection but not the lesson presentations. During the planning and reflection sessions I limited my participation as I wanted to hear the teachers' voices clearly.

At the beginning of the research process, after conducting the first interview, we (the participants and I) observed each of the three Grade One teachers presenting a lesson that aimed to develop learners' number sense. Together with the other two teachers in attendance, I was the observer whilst the third teacher presented the lesson. I did not participate in planning these individual lessons. Each teacher planned her own lesson following the national curriculum. Observation was focused on the eight components of number sense as explained in Chapter 2 and each observer had a schedule that they completed about the way the teacher developed the learners' number sense.

After observing the three individual lessons, we reflected on the lesson presentations as a collective. We planned a lesson (Appendix 4) that aimed to develop learners' number sense guided by the reflection notes. I realise that number sense development does not occur in one lesson but is rather an on-going process. Given that I was an observer-as-participant in the intervention, it was necessary for me to video-record the planning sessions so that I could 'observe' the session afterwards. It was difficult for me to observe and participate simultaneously.

One of the teachers volunteered to teach the collaboratively planned lesson to her class. The rest of us, that is, the other two teachers and me, observed the lesson using an observation schedule that we modified collaboratively from the one that I had developed initially (i.e., when we observed each other before the collaborative planning stage). After the lesson, we reflected on the extent to which the lesson promoted the development of the learners' number sense. Based on our reflections, we planned the next lesson. We agreed not to deviate from the teachers' normal termly planning as determined by the national curriculum. We planned three lessons; one for each teacher to teach.

We observed six lessons in total (three that the teachers planned individually and three that that we planned together). After every lesson presentation I collected the observation schedules and made copies for myself. The originals were given back to the teachers. Our notes in the observation schedules were used as prompts for the reflection sessions. Both the lessons and some phrases of the observation schedules were changed during reflection meetings to include the improvements discussed. Both the planning and reflective sessions were video-recorded. Given the significant amount of data generated during the planning and reflection sessions, I watched each of the videos a number of times and chose which sections of each video to transcribe. I transcribed the sections that focused on the planning of the lesson, resources and learner activities. In so doing, I needed to ensure that the sections I selected were representative of each session. I decided to exclude information where teachers were discussing issues that were not relevant to this research, for example, teachers talking about how lessons previously used to have introduction, presentation and conclusion sections or unrelated conversations that automatically take place when working as a group.

I took notes after every session I had with the participants in order to assist in formulating questions for the second interview and for the discussions during the planning and reflection sessions. The notes assisted me to reflect on the research process and to evaluate whether or not the strategies that I used were working. For example, during our first reflection after the three lessons I realised that the teachers were battling to express themselves. Therefore, I asked them to complete a reflection schedule to express their point of view. Then, I summarised the reflection schedules and the used the summary as a guide for our reflection (Appendix 5). Gradually, the teachers started to open up.

In Table 5.2, I provide a summary of my research process.

Week 1	Interview Lerato	Interview Doreen	Interview Gladys	
	Transcription of interviews			
Week 2	Lerato presented her individually planned lesson.	Doreen presented her individually planned lesson.	Gladys presented her individually planned lesson.	Reflected on the three lessons. These reflection sessions were video recorded.
Week 3	Transcribed reflection session			
Week 4	Planned a lesson collaboratively. This session was video recorded.	Lerato presented the lesson while the rest of us observed.	Reflected and re-planned the lesson.	
	Transcribed the planning and reflection sessions			
Week 5	Doreen presented the improved lesson while the rest of us observed.	Reflected on the improved lesson which was video recorded.	Planned a new lesson which was video recorded.	
Week 6	Gladys taught the new lesson while the rest of us observed	Reflected on the lesson presentation. This session was video recorded.		
	Transcribing of planning and reflection session			
Week 7	Second interview with	Second interview with	Second interview with	

	Lerato. This was audio recorded.	Doreen. This was audio recorded.	Gladys. This was audio recorded.	
Post data collection	Completion of all transcriptions and development of Excel spread-sheets to organise and collate the data.			

Table 5.2: Data collection process

5.6. POSITIONALITY

I realised that my position as a colleague and researcher may influence the teachers' decisions to participate in this research. I was also mindful of the fact that the teachers knew upfront that I am passionate about mathematics teaching and learning and that they may have felt intimidated or pressured into participating in this research. I needed to ensure that they felt comfortable to share their ideas during the interviews, planning and reflective sessions.

I explained to the teachers that the purpose of my research was not to evaluate their teaching. I clarified the aims and focus of the study at the beginning of the research and told the teachers that they could extricate or excuse themselves from the study at any time without any negative consequence. I was fortunate in that I had worked with this group of teachers before. They participated in my Honours research project. During that time, we also planned lessons collaboratively, observed each other teaching and reflected on the lessons afterwards. In many respects, this was an extension of that work, although the shared concern and my theoretical framing of the study were different. However, despite our experience working together in the past, I still reflected on my positionality throughout the research process in my reflective journal.

5.7. DATA ANALYSIS

In this research, I engaged with the process of reducing, selecting, describing and interpreting what the teachers and I had said and done as well as what was observed in order to generate the findings (Merriam, 1998). Bertram and Christiansen (2014) regard this as a three-stage (but iterative) process that includes data reduction, data display and conclusion drawing. These three stages are interrelated and interwoven.

In managing my data, I made an excel spread-sheet that included transcriptions of carefully chosen sections of the planning and reflection sessions (as explained above), transcribed observation schedules, and the transcriptions of the interviews. This meant that all my data was in a single excel document that had a number of sheets. Data was *reduced* by organising and sorting the data from transcripts (interviews and selecting pertinent sections from the videos of the planning and reflection sessions), and observation schedules. An example is indicated in Table 5.3.

Observation of Collaborative lessons **Lesson Presented by Lerato**			
Questions	**Researcher**	**Doreen**	**Gladys**
How did the learners respond to the introduction of the number 15?	Learners responded well by recognising the number, counting and estimating	Learners answered the questions asked by the teacher e.g., using problem solving methods : Give them dots cards , get them to count onto get 15 - Add	Educator used the number 14 to lead them to the number 15
How did learners respond to the teachers' explanation of how to use the number -line, counters and number cards for calculating?	They used counters for counting and making bonds. They used their number-lines for addition and subtraction		Learners were able to use a number-line and what it is used for

Table 5.3: Data reduction from observation schedule

The excel file was useful for data management and for other purposes. Since an excel spreadsheet has numerous cells, I was able to code and recode my data on each spreadsheet. Each time I recoded my data, I hid the columns of the previous codes that I generated. This enabled me to 'check' my initial coding. I used colours to code the similarities and differences that I found in the data. The analysis was thus based on inductive reasoning.

Thereafter, I began the process of classifying the data and developing categories by looking for similarities and differences in the data. Categories refer to a broad unit of information that consists of several codes aggregated to form a common idea. The purpose of coding inductively was to ensure that I did not miss anything that I may have done if I had applied the theoretical categories first. I then re-read the data with my theoretical categories in mind. I used the MKfT domains followed by the KQ dimensions to understand whether the mathematics and pedagogical content knowledge of the Grade One teachers was strengthened. The above data analysis explanation assisted me in answering the first research question, that is, what mathematics and pedagogical content knowledge is required to develop Grade One children's number sense. An example of my emic and etic coding appears in Table 5.4.

Observation of Collaborative lessons **Lesson Presented by Lerato**						
Questions	***Researcher***	**Doreen**	**Gladys**	**Comments**	**Comments**	**Codes**
				MKfT	**Knowledge Quartet**	
How did the learners respond to the introduction of the number 15?	Learners responded well by recognising the number, counting and estimating	Learners answered the questions asked by the teacher e.g., using problem solving methods: Give them dots cards , get them to count onto get 15 – Add	Educator used the number 14 to lead them to the number 15	Knowledge of Content and teaching- strategies and examples	Foundation and transformation- teachers choice of strategies andexample were understandable to the learners	Learner participation Teachers explanation
How did learners respond to the teachers' explanation of how to use the number -line, counters and number cards for calculating?	They used counters for counting and making bonds. They used their number-lines for addition and subtraction		Learners were able to use a number-line and what it is used for	Curruculum Content Knowledge - LTSM	Transformation - Explanation by the teacher	Usage of manipulatives

Table 5.4: Categories using theoretical framework

I used the CHAT components to analyse and explain the activity system that is the collaborative intervention. CHAT was particularly useful in assisting me to ascertain any contradictions emergent from the research data. Knowledge of the contradictions is also useful should the goal be to change practice (although this was not the focus of my research). CHAT was used to answer the second question: How does a collaborative intervention enable or constrain the development of the mathematics and pedagogical content knowledge required to develop learners' number sense? Figure 5.1 provides an overview of the analysis process.

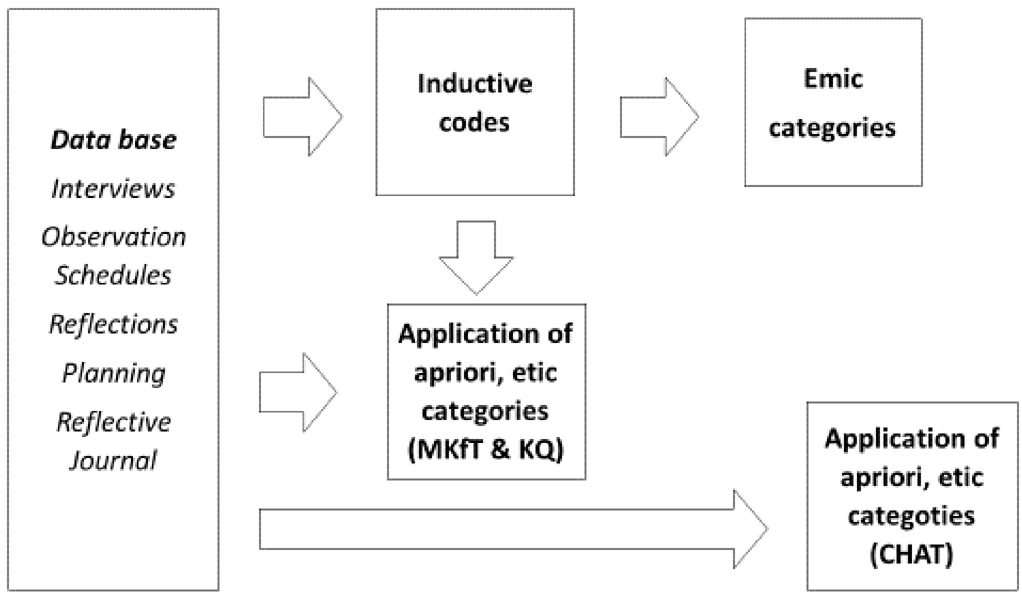

Figure 5.1: Reduction and classification of data

The next stage looked at *data display*. I identified quotes and pertinent information from the data. Identifying patterns and drawing conclusions from the data is the third stage in the data-analysis process. Themes were identified by abstracting the codes and the categories to the larger meaning of the data. I looked for correspondences between the categories from both the inductive codes and the apriori codes to answer the first question. In relation to the second question, the concept of 'thinking together' was utilised to explain how the collaborative intervention was formed and sustained. CHAT assisted me in analysing the intervention activity system in terms of the object, tools, subject, rules, community and division of labour. I discussed the tensions and contradictions experienced, focusing on the primary and secondary contradictions as this research is underpinned by second generation CHAT (Chapter 2). Identifying the contradictions assisted me in analysing the development of teachers' mathematics and pedagogical content knowledge as they participated in the intervention.

5.8. ETHICS

This research took several ethical principles into consideration; these included autonomy, non-maleficence and beneficence (Bertram & Christiansen, 2014). I asked the teachers to give themselves pseudonyms and they decided on Lerato, Doreen and Gladys to ensure that they remain anonymous throughout the research. These were the names that they chose to write on the observation sheets. We also decided on a pseudonym for the school, Botaleng Primary School.

The teachers decided on the sequence they were going to follow to present the lessons and they chose their own topics as per their quarterly planning. I discussed my research plan with the teachers but knowing that they have other commitments at school, I allowed them to choose the day and time that was convenient for them for lesson presentations and observations and the planning and reflection sessions. I ensured that during the lesson presentations, observations, planning and reflections, all participants (even the learners who we indirectly involved in the research) were treated with respect and not humiliated or undermined in any way. The teachers' perspectives and contributions during the planning and reflection sessions were always considered and I encouraged the teachers to give constructive criticism to inspire improvement.

The research was meant to strengthen the teachers' mathematics and pedagogical content knowledge so as to improve teaching practice and the development of learners' number sense. The teachers and I engaged in a collaborative intervention where we observed different methods of developing number sense. Afterwards, we engaged in reflection and planning sessions where teacher voiced out their opinions about the lessons, the teaching strategies they learned and improvements that can be made on that particular lesson. This process was meant to benefit the teachers by learning from each other and sharing ideas and experiences to strengthen their mathematics and pedagogical content knowledge for teaching.

The ethics application for this research was reviewed by the Rhodes University Ethics and Standard Committee through an online application and was approved on the 27 May 2019 (Appendix 6). I asked permission from the district director, circuit manager, principal, participant-teachers and the parents of the Grade One learners to conduct research in the school and classrooms of the Grade One teachers. These consent letters are in Appendices 7-11. The purpose and process of the research was thoroughly explained to the participants. In order, to further protect the identity of the participants I have kept the data in a locked drawer, and I am the only person in possession of the key. The transcriptions of interviews and spreadsheets of the observations schedules, video recordings of planning and reflection sessions are kept in a password locked file in my laptop. The audio recordings and video recordings are also stored under a password locked file as identity can also be recognised through the recordings (Brenner, 2006). The video recordings were only viewed by me for transcription purposes and my supervisor for guidance during analysis of data. As people can be recognised by their respective voices, I transcribed the interviews myself.

5.9. VALIDITY

In order to ensure validity of the research, I triangulated my data thought the use of three data collection methods, namely observations, interviews and the notes from the reflection and planning sessions (Merriam, 2009). This enabled me to cross-check the data from multiple sources and collection tools.

I gave the interview transcripts and the draft analysis to the participants to rule out any possible misinterpretations of what they said and "to confirm the researcher's interpretation of meaning and perceptions" of the participants (Brenner, 2006. p. 368). In other words, I continuously engaged with the process of member-checking. Being part of the intervention assisted in this process as the teachers could discuss anything they wished in relation to my interpretation of the research with me. The teachers clarified some of the statements they made when my interpretation was contrary to their meaning when reviewing the analysis draft. Finally, I engaged reflexivity throughout the process, specifically on my role in the research (Merriam, 2009). I particularly wanted to avoid bias and ensure that my position as a Grade 4 mathematics did not (as far as possible) interfere with my findings.

5.10. CONCLUSION

This chapter is summarised in the Table 5.5 below.

Research goals	❖ Strengthen the mathematics and pedagogical content knowledge of Grade One teachers through participation in an intervention to develop learner's number sense. ❖ Examine the mathematics and pedagogical content knowledge required to develop learner's number sense and how a collaborative intervention can enable the development of the mathematics and pedagogical content knowledge required to develop learners' number sense.
Key Research Question **Sub Questions**	❖ What mathematical and pedagogical content knowledge do teachers use to develop learners' number sense? ▪ What is the nature of the number sense activities promoted prior/during the collaborative intervention? ▪ What is the nature of the teachers' mathematics and pedagogical content knowledge prior/during the collaborative intervention? ❖ How does a collaborative intervention enable and constrain the development of the mathematics and pedagogical content knowledge required to develop learners' number sense?

<table>
<tr><td colspan="2">Research Design Study</td><td colspan="3">Case Study – Interpretivist</td></tr>
<tr><td colspan="2">Nature of Data Collected</td><td colspan="3">Qualitative</td></tr>
<tr><td colspan="2">Sampling</td><td colspan="2">Site:
Public Primary School in the Northern Cape</td><td>Participant:
Three Grade One teachers</td></tr>
<tr><td rowspan="3">Application Action</td><td>Data Collection Instruments</td><td>Observations
Lessons, Planning and Reflection sessions</td><td>Interviews
2 Phases:
1. Beginning of research
2. End of research</td><td>Document Analysis
Observation schedules
Reflection notes</td></tr>
<tr><td>Data Source</td><td>Lessons
Video-recordings</td><td>Audio-recordings and transcriptions</td><td>Observation schedules
Lesson plans</td></tr>
<tr><td>Data Analysis</td><td colspan="3">Emic coding – inductive reasoning to generate categories emerging from the data
Etic coding – MKfT, KQ and CHAT</td></tr>
<tr><td colspan="2">Ethical Considerations</td><td colspan="3">Confidentiality, anonymity, informed consent and data storage</td></tr>
</table>

Table 5.5: Summary of the research process

CHAPTER SIX: INDIVIDUALLY PLANNED LESSONS

6.1 INTRODUCTION

In this chapter, I focus on lessons that each of the three teachers who participated in the research developed. The teachers formulated and implemented individually planned lessons. This chapter examines two aspects of the collaborative intervention, that is, the outcome and the object. The outcome refers to the development of learners' number sense, and the object denotes the teachers' mathematics and pedagogical content knowledge. Essentially, this chapter responds to two questions:

- What is the nature of the number sense activities promoted prior to the collaborative intervention?
- What is the nature of the teachers' mathematics and pedagogical content knowledge prior to the collaborative intervention?

These questions assist me to answer the first sub-question in this research: *What mathematical and pedagogical content knowledge do Grade One teachers use to develop learners' number sense?*

6.2 THE TEACHERS' PERSPECTIVES ON NUMBER SENSE

During the first set of individual interviews, I asked the teachers to share their understanding of mathematics and number sense. Both Lerato (I1[1]) and Gladys explained that mathematics was required for everyday life. Lerato stated that "**mathematics is important to teach daily because we are using mathematics on daily basis, we use money, we count**". Lerato and Gladys's view of mathematics is similar to that of Sa'ad, Adamu and Sadiq (2014) who argue that numeracy is necessary to be an active citizen who is able to analyse everyday situations and problem-solve.

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[1] For ease of reading, I have not referenced every single comment made by the teachers. In this section, they are all from 'I1' or 'Interview 1', that is, the first interview I conducted with each of the teachers (Chapter 5).

The ability to become an active, numerate citizen depends to a large extent on the quality of mathematics education received at school.

The teachers all emphasised that ‘number sense’ is the ability to “**associate the numbers with the objects**” (Doreen), through “**one-to-one correspondence**” (Lerato). Lerato also emphasized the importance of being able to discriminate between different quantities (e.g., more, less, the same as). Lerato and Doreen thus view number sense as **“knowing how to count, knowing the one-to-one correspondence, knowing the same, more than, less than”** (Lerato) and being able to **“associate the number with the objects”** (Doreen)**.** This includes the following components, systematic counting, one-to-one correspondence, understanding quantity, and the link between quantity, number names and numerals.

While Doreen and Lerato foregrounded the importance of understanding ‘quantity’ in their explanations of number sense, Gladys’s view was seemingly limited to the number range within which the learners were working, for example **“if a learner can count from one to a hundred and is able to identify the number”** and **“if the learner doesn’t know the number, he will not be able to do Maths”** (Gladys). However, despite Gladys articulating this seemingly narrow perspective of number sense, her individually planned lesson (Lesson 6.3) shows evidence of systematic counting, identifying and comparing quantities, and matching number names and numerals. This view is consistent with that of Jordan (2007) who suggests that number sense includes “the ability to grasp and compare quantities (6 versus 8) and internalize counting principles where “the final number in a count indicates the quantity of a set” (p. 64).

In terms of developing learners’ number sense, Doreen mentioned that she developed learners’ number sense by employing the “**one to one correspondence strategies, matching numbers with pictures or dots**” while Gladys said she encouraged “**rhythmical counting and identification of numbers**”. The teachers all suggested that those learners who did not attend pre-school often experience challenges, therefore, they assist these learners by “**using objects**” (Lerato) as teaching aids, “**playing games**” (Gladys) and also teaching “**them aside [and using] group work so that they can understand**” (Doreen). Group work is not only used for learners with ‘learning deficits’ but also for those deemed more proficient at mathematics. Lerato explained that “**those who are ‘better’ need to be given attention**” too. Interestingly, despite simple calculations being a feature in all three teachers’ classrooms, none of the teachers mentioned this as a component of number sense in the interviews.

6.3 THE ENACTMENT OF NUMBER SENSE IN THE CLASSROOMS

While the teachers did not necessarily offer a full description of number sense, their classroom practice included many of the FNS characteristics as described by Sayers and Andrews (2015). While the first three lessons represented in this chapter were planned independently by the teachers, we all observed each other teaching. The focus of the observations during these lessons was to ascertain: (1) how each teacher developed their learners’ number sense; (2) which of the characteristics of FNS they promoted (Chapter 4); and (3) the Mathematics Knowledge for Teaching (Chapter 3) they drew on. In this section, I present a narrative of each of the three teachers’ lessons, an analysis of the extent to which they employed the characteristics of FNS and how they applied their MKfT to plan and implement the lessons. I focus the analysis on the teachers’ MKfT and the development of learners’ number sense, as these are the object and outcome of the activity system respectively.

The three teachers are Lerato, Doreen and Gladys (pseudonyms). Lerato’s lesson focused on the number ‘12’.

6.3.1 Lerato’s first lesson

Classroom layout Lerato’s class is arranged into ‘ability’ groups. There are six to eight learners seated in these groups. The groups are named according to colours. There are number grids (1 – 100) and number lines (1 – 20) stuck on the learner’s tables. Lerato has a table in front of the class where she puts the resources she uses in the lesson.

Mental Maths

The lesson starts with rote counting. The learners count forwards and backwards in:

1s up to 100;
2s up to 50;
5s up to 50; and
10s up to 100

The learners count in Setswana, which is the LOLT in the Foundation Phase. They point to the numbers on the number grid while they count. Lerato stands in front while the learners are counting. Most of the learners correctly point to the numbers on their number grids as they count.

Lerato asks the whole class to count using their double facts. The learners chorus "double 1 is 2, double 2 is 4 … until double 10 is 20'

Teaching and learning activities

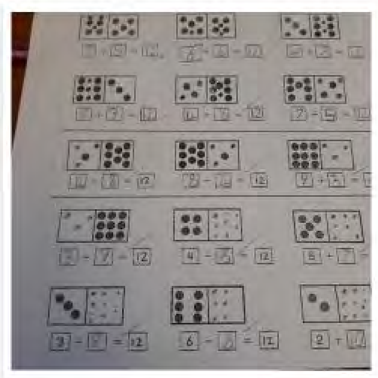
Lerato hands out counters to each of the learners. She asks the whole class for the number between 11 and 13. Learners raise their hands to answer individually. She calls one learner to the front to 'pick up' the number 12 and show the class. Lerato writes the number on the board and asks the learners to count up to 12. The learners have counters on their tables and they place a counter on each number on the number line as they count. Lerato shows the learners some tennis balls and asks them to count with her while she places the tennis balls one by one on the 'chalkboard railing'. They count 12 tennis balls. Lerato gives the learners a word problem orally: "*I have 12 sweets. If I eat 1, how many will I have left?"* The learners use their number lines to work out the answer. After using the tennis balls, Lerato uses the blocks on the chalkboard to demonstrate the solution to the problem confirming what the learners have done. She uses the same context for the next word problem, but this time, she focuses on addition: "*I have 12 sweets and my grandmother gave me 1 more. How many sweets do I have?* The learners work out the answer on their number lines. Lerato confirms the learners' answers with the blocks.

Lerato asks them what makes up the number 12 and learners respond in unison by saying '10 and 2'. She uses the blocks to show learners that 12 can be represented by 10 and 2 or 3 groups of 4.

In their groups, the learners are given an A3 piece of paper with dots on one half of the paper (like dominoes). The learners place counters on the other side of the paper to make up the number 12. Each group is given one sum to complete with a different number of dots. Lerato moves around each group explaining how to count on to make the bonds of 12. Some groups managed to complete the sum quickly while the teacher is explaining to the groups of learners who appear to be struggling. Once all the groups are finished Lerato writes some sums (e.g., 9 + □ = 12; 5 + □ = 12 etc.) for each group on the board and asks individual learners from each group to come to the board to insert the correct numbers to complete the sums.

Consolidation
Lerato gives the learners two worksheets:
- Worksheet 1 requires the learners to calculate numbers up to 12.
- Worksheet 2 requires learners to show different ways to make the number 12.

WORKSHEET 1



WORKSHEET 2

The learners work on their own to complete the worksheets.

Lesson 6.1: Lerato's individual lesson

6.3.1.1. *Number sense development in Lerato's lesson*

Lerato incorporated five of the eight FNS characteristics in her lesson. Lerato encouraged systematic counting in 1s throughout the lesson as the learners had to repeatedly count out 12 objects or 12 on their numberline. However, during the mental mathematics session the learners counted in 1s, 2s, 5s and 10s. Each time the learners were asked to count in the mental mathematics session, they would start from the lowest multiple of that number (i.e., 1, 2, 5 and 10). To introduce the number '12', Lerato asked a learner to identify the number from a group of numbers written on card. She also required the learners to count out the required number of counters. In this way, she wanted the learners to make the connection between the number and the quantity. In addition, she asked the learners to explain 'what makes 12' in order to promote quantity discrimination. Numbers were represented using a variety of different resources such as counters, number lines, tennis balls, wooden blocks and dominoes. Learners also engaged with simple arithmetic when they answered the two word problems, and completed the group dominoes activity and the two worksheets. Table 6.1 shows the activities of the learners and the teacher that incorporated the FNS characteristics in each stage of the lesson: mental mathematics; whole class or group teaching and learning; and consolidation activities.

FNS Characteristic	**Mental Maths Activities**	**Teaching and Learning Activities**	**Consolidation Activities**
Number recognition	Pointing to numbers on number grids while counting	Placing counters on the numbers on the number line while counting	
Systematic counting	Counting backwards and forwards in given intervals Counting in doubles of 1 up to 10	Counting on the number line up to 12 Counting tennis balls while teacher places them on the chalkboard railing	
Relating number to quantity		One-to-one correspondence (Relating the number of tennis balls to the numbers on the number line while counting) Placing counters on paper to complete the dominoes and make 12	Drawing pictures to make up the bonds of 12 Relating numbers to the dominoes to make the bonds of 12
Different representations		Using different resources (e.g., number grids, number lines, counters, tennis balls, blocks) to count, represent numbers and solve problems	Pictures and dominoes to make the bonds of 12 (worksheet 1 and 2)
Simple arithmetic		Solving problems using number lines Completing the dominoes activity in groups Completing sums written on	Simple addition sums to complete bonds of 12 (worksheet 2)

		the board	

Table 6.1: FNS activities in Lerato's class

6.3.1.2. Lerato's MKfT

Lerato is familiar with the Subject Matter Knowledge that she is required to teach.

In her lesson, Lerato, displays evidence of Knowledge of Content and Curriculum (KCC) as she is aware of how to structure her lesson in accordance with the CAPS requirements (SA.DBE, 2011) and knows the expectations of Grade 1 learners at this stage of the year. The CAPS requires "mathematics lessons to include mental mathematics activities, whole class and small group teaching where concepts and problem solving activities take place and independent work to reinforce and consolidate concepts" (SA. DBE, 2011, pp.10-11). Lerato chooses a variety of resources suited to both the lesson and the learners. These include number lines, number grids, tennis balls and blocks. She knows which resources are appropriate for Grade One (e.g., counters), but also strives to push her learners' mathematics engagement, by having number grids (or 100 squares) and number lines stuck onto each table. This allows her learners to count with visual support.

Lerato uses word problems as a strategy to support learning, linking the abstract mathematics to everyday life. She promotes active learning to maintain the learner interest and motivation and relates the content to the students by using different resources. These are examples of her Knowledge of Content and Students (KCS). Lerato demonstrates her Knowledge of Content and Teaching (KCT) when she uses different strategies to promote sense-making. She makes use of practical materials (e.g., counting the tennis balls with the learners), iconic demonstrations (e.g., writing the numbers on the board) and the symbolic through the use of worksheets. She uses a variety of teaching strategies, such as questioning, practical demonstrations, whole class teaching, group work and independent work. The MKfT evident in Lerato's lesson is summaried in Table 6.2.

Domain	**Indicators**
KCS	• Learners are place in 'ability' groups so that she can cater for learners' needs • Uses the Home Language of the learners to promote their learning • Resources are placed on the learners' desks (number lines and number grids) for ease of access and use during the lesson • Builds on 'real-life' examples that the learners are familiar with (e.g., using a word problem to explain a sum) • Knows that learners should be actively involved with the different activities (e.g., getting the learners to work with the various resources)
KCT	• Uses a variety of resources to develop learners understanding of the number '12' (e.g., number grids, number lines, counters, tennis balls, blocks) • Uses different teaching strategies to develop an understanding of the number '12' (chorusing, questioning, practical demonstrations with resources, problem-solving) • Builds on the learners' thinking as the lesson progresses (identify the number '12', counting 12 objects, 1 more and 1 less, making 12) • Teaches the whole class and groups, and creates opportunities for learners to work independently as individuals • Supports learners who appear to be struggling (e.g., re-explaining

	the dominoes worksheet)
KCC	• Knows the curriculum requirements for Grade One (i.e., the number range and curriculum topics) • Knows the structure of a mathematics lesson as described in CAPS • Chooses a variety of resources (e.g., number lines)

Table 6.2: Lerato's MKfT

6.3.2 Doreen's first lesson

In her lesson, Doreen focuses on 'building up and breaking down numbers'.

Classroom layout

The learners sit in three groups of eight and two groups of six. These are organised according to abilities. There are number grids stuck on the learners' tables and the teacher has wooden blocks, with the numbers 1 to 20 written on them, placed on the 'chalkboard railing'. This is her number line. Each group has a small container with beans and each pair has an abacus on their table. Doreen has a table in front of the class near the chalkboard where she puts the resources that she will be using during the lesson. The LOLT is Setswana as this is the Home Language of the learners

Mental Maths

Doreen asks the learners to count forward in 10s from 10 to 80 and backwards from 80 to10. As the learners count, they point to the numbers on number grids that are stuck on their tables. She asks if they recognise the pattern. The learners respond in unison that all numbers end with a zero.

Doreen continues with the counting activities and asks the learners to count in 1s from 31 to 50, 70 to 80, 23 to 19, and 45 to 35. The learners point to the numbers on their number grids when counting and Doreen moves around to check if they point to the correct numbers. When she realises a learner is not pointing to the correct number, she shows the learner the correct number and observes if the learner is able to continue without her support.

Doreen holds up flashcards with two-digit numbers on them. She asks the learners to put a bean on the number on their number grid and give her the number name orally. The learners raise their hands to answer. She shows them five more numbers. Two learners make errors: saying 17 instead of 70 and 90 instead of 19. Doreen asks the other learners to assist. Doreen demonstrates to the learners the difference between the number 17 and 70. She shows the learners cards with the numbers written on them. She 'packs out' 10 counters and then 7 counters to show them 17. She explains that '17' has one ten and '70' has 7 tens.

Teaching and learning activities

Doreen then gives the groups worksheets with three numbers written on them. She asks the learners to colour the blocks according to the number given.

Doreen gives each group between 11 and 15 small wooden blocks. She asks the learners to count the blocks on the tables as a group. Then, she asks the groups to put 10 blocks aside and write the numeral '10' on a piece of paper. She asks the groups to complete the number sentence by adding the remaining number of blocks to 10. One learner from each group represents their sum to the class: 10 + 1 = 11; 10 + 2 = 12 etc.

Doreen reminds the learners that the '1' in numbers like 12 stands for tens and the remaining blocks represent the units. She holds up number cards with two-digit numbers and asks the learners to 'break down' the numbers on the cards into tens and units: 23 = 20 + 3; 39 = 30 + 9 etc. The learners respond in unison.

Consolidation

Doreen asks the learners to take out their classwork books and complete the sums written on the board.

Tlhakanya (Add)	**Ntsha (Minus)**
10 + 1 = ______	**11 - 1 = ______**
10 + 2 = ______	**13 - 3 = ______**
10 + 5 = ______	**14 - 4 = ______**

She reads the instructions with the learners before they start to write. A few learners need her assistance using abacuses to work out their solutions. Doreen demonstrates how to use the abacus for one addition and one subtraction sum.

Lesson 6.3: Doreen's individual lesson

6.3.2.1. *Number sense development in Doreen's lesson*

Doreen incorporated seven FNS characteristics throughout the three sections of her lesson. During the mental mathematics session, the learners were engaged in systematic counting in different intervals. When counting in 10s, the learners were asked to identify the pattern (i.e., all the numbers end with '0'). Unlike Lerato, the learners in Doreen's class did not only count from one, they also counted to and from arbitrary numbers. Opportunities for systematic counting also occurred during the lesson as the learners counted blocks on the worksheet and actual blocks on their desks.

Doreen flashed two digit numbers and asked the learners to place beans on the corresponding number on their number grids. Thereafter, she asked them to give the name of the number on their number grid. The focus in this activity was number recognition. Simple arithmetic was incorporated when wooden blocks were used for teaching place value by recognizing the tens and adding units to the tens to make a two- digit number, and also during the classwork activity when learners copied and calculated addition and subtraction sums from the board. Doreen made use of different resources to represent numbers that is wooden blocks, number grids and abaci. In order to differentiate between quantities and relate number to quantity, learners gave number names, learned the difference of the value between '17' and '70' and the difference between tens and units. Estimation was the only characteristic not incorporated in this lesson. A summary of the FNS developed during Doreen's lesson appears in Table 6.3.

FNS Characteristic	**Mental Maths Activities**	**Teaching and Learning Activities**	**Consolidation Activities**
Number recognition	Pointing to numbers on number grids while counting Recognising two-digit numbers by giving number names	Colouring the quantity of given number Breaking down numbers into tens and units	
Systematic counting	Counting forward in 10s and then 1s from arbitrary numbers Counting out 10 and 7 counters to show 17 and 7 groups of 10 counters to show 70	Counting wooden blocks as instructed by the teacher Colouring in the correct number of blocks on the worksheets Counting out the blocks to perform calculations	
Relating number to quantity		Colouring blocks on a worksheet Using blocks to break down numbers Using the abacus to complete the sums	
Quantity discrimination	Differentiating between 17 and 70	Breaking down numbers into tens and units	
Different representations	Using counters to differentiate between 17 and 70	Colouring-in the correct number of squares on the worksheet Using wooden blocks to add tens and units	Using the abaci to perform addition and subtraction calculations
Simple arithmetic		Adding tens and units (calculations from the groups using wooden blocks)	Calculating addition and subtraction sums on the board.

		Breaking down of two-digit numbers	
Number patterns	Recognising the pattern when counting in 10s		

Table 6.3: FNS activities in Doreen's class

6.3.2.2 *Doreen's MKfT*

Doreen is familiar with the Subject Matter Knowledge that she is required to teach.

Doreen is aware of the lesson structure and expectations of CAPS in Grade 1 at this time of the year. She chose resources that are appropriate for developing an understanding of the concepts and skills for Grade 1. This shows evidence of her KCC. She organises her learners in 'ability' groups so that she can give them attention in accordance with their needs. She involves the learners in a range of activities and assists them in the process of sense-making by utilizing a range of resources. She includes the learners in explaining the difference between '17' and '70' to their peers. The above providing evidence of her KCS. She draws on her KCT by using a variety of teaching strategies and modes of representation. She gives individualized attention to learners who need more assistance completing their activity. This is an example of her KCT and KCS. A summary of the MKfT evident in Doreen's lesson appears in Table 6.4.

Domains	**Indicators**
KCS	• Learners are placed in 'ability' groups so that she can cater for learners' needs • Uses the Home Language of the learners to promote their learning • Resources placed on learners' desks (number grids) for the learners to use

	• Involves learners in different activities (colouring-in squares, counting, and writing in classwork books) • Involves learners in providing explanations for peer errors (17 and 70) • Gives attention to learners who needs more assistance
KCT	• Uses a variety of resources and modes of representation (wooden blocks, number cards, chalkboard) • Knows how to address student errors (17 and 70) • Uses a variety of teaching strategies (chorusing, practical demonstration using resources, questioning) • Teaches whole class, groups and provides opportunities for independent work
KCC	• Chooses a variety of appropriate resources and modes of representations (wooden blocks, number cards, chalkboards) • Knows how to structure a lesson in accordance with the CAPS requirements • Knows Grade One curriculum requirements (curriculum topics, number range)

Table 6.4: Doreen's MKfT

6.3.3 Gladys's first lesson

The last individually planned lesson that we observed was that of Gladys. Gladys was teaching addition and subtraction.

Classrooom 'set-up'
The 'set-up' in Gladys's classroom is the same as Lerato's. The learners sit in groups of six or eight arranged according to 'ability'. The learners have number grids on their

tables. Each pair also has an abacus to share. There is a table in front of the classroom where Gladys puts the resources she will use in the lesson. Teaching is in Setswana, the Home Language of the learners.

Mental Maths

Gladys places an incomplete number line on the board and number cards with numbers written on them on the table in front of the class. She asks individual learners to come to the front, choose a number card and place it on the correct place on the number line. Some learners make mistakes and Gladys asks the rest of the learners to assist them.
Gladys asks the learners to count forward and backwards in:
1s up to 50;
2s up to 30; and
10s up to 100.
Most learners count without pointing to the number grid.

Teaching and learning activities

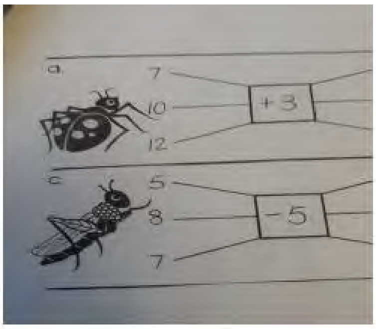
Gladys places picture cards and number cards randomly on the board. She asks the learners to match the picture cards with the number cards. Then, she asks learners to write the number names of the numbers on the board. Some learners make mistakes with the number names. Gladys encourages them to correct their mistakes.

Gladys places three cards with pictures of a fish, triangle and ball, in three different sizes each on the board and asks the learners to circle the big fish, small triangle and a big ball. Learners come individually to the board and they use chalk to draw circles around the different sizes of the picture

Gladys gives each group bottle tops to work with. The learners work in groups of fours. She explains that they will be doing addition and subtraction sums. She revises the meaning of the addition and subtraction symbols using hand signals. She demonstrates that when you make a cross with your arms you bring things together and you add. Then, when you put your arm straight across your body, you move the objects away and that means take away. Each group uses bottle tops to complete the addition and subtractions sums.

Consolidation

Gladys gives each learner a worksheet and encourages them to use the bottle tops and abaci to solve the problem. Learners work individually and use the resources to calculate. All the learners complete the same worksheet.



The teacher walks around to assist learners and marks the work of those who are finished.

Lesson 6.3: Gladys's individual lesson

6.3.3.1. *Number sense development in Gladys's lesson*

Gladys presented the third independently planned lesson. She incorporated six of the FNS characteristics. Learners counted systematically in 1s, 2s and 10s. She suggests that they use bottle tops or abaci when calculating. The latter also develops their knowledge of the relationship between quantity and number. Matching the picture cards with the number cards further develops this relationship. The learners recognize numbers by completing the number lines, and matching picture cards with their respective number. The learners completed a pattern by filling in the missing numbers on a number line. Gladys provided a variety of resources to promote the learning of addition and subtraction. Table 6.5 provides a summary of the FNS developed in the lesson.

FNS Characteristic	Mental Maths Activities	Teaching and Learning Activities	Consolidation Activities
Number recognition	Pointing to numbers on the number grid while counting Filling in missing numbers on a number	Writing the number names of the numbers of the pictures on the board	

	line		
Systematic counting	Counting forwards and backwards in 1s, 2s and 10s		Counting while completing the worksheet
Relating number to quantity		Matching picture cards with number cards	
Different representations	Completing the number sequence on a number line	Using bottle tops to calculate given sums in groups	Using bottle tops to calculate and complete individual worksheets
Simple arithmetic		Adding and subtracting in groups	Simple addition and subtraction (individual worksheet)
Number patterns	Identifying missing numbers in a number sequence (number line)		

Table 6.5: FNS activities in Gladys's class

6.3.3.2. Gladys's MKfT

Gladys is familiar with the Subject Matter Knowledge that she is required to teach.

Like Lerato and Doreen, Gladys exhibits evidence of KCC in her lesson as she is aware how to structure her lesson, knows what is expected in the third term of Grade 1 and chooses resources necessary to support the content learners are expected to learn.

Gladys draws on her KCS in a variety of ways. She organises her learners into 'ability' groups and she uses the Home Language of the learners for teaching and learning. She places the resources on the learners' desks to assist them while counting. She recaps what the learners have learned about addition and subtraction by using the body to explain 'add' and 'take away'. In doing this, she acknowledges that many learners are kinesthetic learners. The learners are involved in a number of different activities and she affords her learners the opportunity to assist each other when they make errors.

Gladys teaches the whole class, groups and creates opportunities for learners to work individually. She uses a variety of teaching strategies and different modes of representation to support the learners' understanding of number, and addition and subtraction. These are all aspects of her KCT. Table 6.6 provides a summary of Glady's MKfT.

Domains	**Indicators**
KCS	• Learners are placed in 'ability' groups • Uses the Home Language of the learners to promote their learning • Places resources on learners' desks (number grids) for ease of access and use • Learners are involved in a number of activities to reinforce number concept (e.g., counting, filling in missing numbers on the numberline, matching numerals to number names) • Recaps what the learners have already learned (the meaning of the addition and subtraction symbols) • Affords learners opportunities to correct the mistakes of other learners
KCT	• Uses different modes of representation to reinforce concepts (number cards, pictures, number lines, symbols) • Uses different teaching strategies (chorusing, questioning, practical demonstration) • Teaches whole class, groups and provides opportunities for independent work
KCC	• Knows Grade One curriculum requirements (curriculum

	topics, and number range) • Knows the structure of the mathematics lesson as explained in CAPS • Uses various resources (number line, pictures, bottle tops)

Table 6.6: Gladys's MKfT

6.4 CONCLUSION

In this chapter, I have analysed the methods that the three Grade One teachers used to attempt to develop learners' number sense in their classrooms. This exercise has required me to engage with one of the thesis sub-questions, namely: *What mathematical and pedagogical content knowledge do Grade One teachers use to develop learners' number sense?* Drawing on CHAT, and as noted earlier, this chapter examines two aspects of the collaborative intervention: the outcome and the object. The outcome refers to the development of learners' number sense, and the object is the teachers' mathematics and pedagogical content knowledge. Essentially, this chapter responds to the sub-question with it's two supporting questions:

- What mathematical and pedagogical content knowledge do teachers use to develop children's number sense?
  - What is the nature of the number sense activities promoted prior to the collaborative intervention?
  - What is the nature of the teachers' mathematics and pedagogical content knowledge prior to the collaborative intervention?

All three teachers appeared to have a limited conception of number sense and how to develop learners' number sense when interviewed at the beginning of the research process. This stood in stark contrast to the extent to which they developed learners' number sense during their lessons. In analysing their lessons using the FNS framework, Lerato and Gladys both incorporated six FNS characteristics in their lessons, while

Doreen incorporated seven. The FNS characteristics that were evident in all three teachers' lessons were number recognition, systematic counting, relating number to quantity, quantity discrimination, using different representations of number and simple arithmetic. The only characteristic that none of the teachers promoted in their individually planned lessons was estimation.

It was clear in all the lessons that the teachers had the necessary subject matter knowledge to teach Grade 1 mathematics. The PCK that the teachers demonstrated included all three domains, that is, KCC, KCS and KCT. As evidence of the KCC, all of the teachers knew the curriculum expectations for Grade 1, how to structure a lesson and were able to select appropriate resources to support the development of number sense. Drawing on their KCS, the teachers all organised their learners into 'ability' groups to meet their learners' needs and taught in Setswana. They included a variety of activities in their lessons and drew on a wide variety of resources, thereby exposing the learners to different forms of representation. The teachers also varied their teaching strategies and exposed the learners to different forms of representation to support the development of learners' number sense. Thus the teachers provided evidence of their KCT.

In the next chapter, I examine the collaboratively planned lessons of the teachers as they participated in the collaborative intervention.

CHAPTER SEVEN: THE COLLABORATIVELY PLANNED LESSONS

7.1. INTRODUCTION

In Chapter 6, I presented and analysed the individually planned lessons that each of the teachers had planned prior to the collaborative intervention. I used the FNS of Andrews and Sayer (2015) and the MKfT framework of Ball et al. (2008) to analyse the teachers' lessons as enacted in their respective classrooms. In this chapter, I focus specifically on the lessons that we developed collaboratively. Similarly, to Chapter 6, here I examine two aspects of the collaborative intervention, that is, the outcome and the object. The outcome refers to the development of learners' number sense, and the object is the teachers' mathematics and pedagogical content knowledge. Whereas Chapter 6 focused on the object and outcome of the research prior to the collaborative intervention, this chapter examines the object and outcome during the collaborative intervention. Essentially, this chapter responds to two questions:

- ❖ What is the nature of the number sense activities promoted during to the collaborative intervention?
- ❖ What is the nature of the teachers' mathematics and pedagogical content knowledge during to the collaborative intervention?

These questions support me in answering the first sub-question in this research: What mathematical and pedagogical content knowledge do Grade One teachers use to develop learners' number sense?

Having observed each other teach a lesson, we decided to start planning our lessons collaboratively. As a group of teachers, we shared ideas and strategies about the planning and implementation of the lesson. We planned three lessons, thus giving each of the Grade One teachers an opportunity to teach a lesson. After the implementation of each lesson, we met to reflect on the lesson. We focused on the:

- strategies the teacher employed to teach the concept;
- learners' participation and opportunities for sense-making;
- challenges experienced by the teachers during the lessons;
- difficulties experienced by learners; and
- strategies for improving the lessons to include all the learners.

7.2. COLLABORATIVELY PLANNED LESSONS

The three teachers selected their own topics for the lesson based on the curriculum. Lerato volunteered to teach the first lesson which was about the 'number 15'. Lerato chose a topic that was similar to her individual lesson (Chapter Six). During our lesson planning session, we focused on improving the points highlighted during reflection of initial lesson.

Before highlighting the planning phase for the first collaborative lesson, I describe the lesson as it occurred in the classroom. I have chosen to do this for ease of reference to the events that took place in the classroom as a result of the planning and reflection discussions.

7.2.1. Collaboratively planned lesson 1

Classroom layout
The learners are seated in groups according to their 'abilities'. There are six to eight learners seated in these groups. The groups are named according to colours. There are number grids (1-100) and number lines (1-20) stuck on the learners' tables. Lerato has a table in front of the class where she puts the resources she uses in the lesson. Setswana, the Home Language of the learners in the class, is the LoLT.

Mental Maths
Lerato starts the lesson by telling the learners to count forwards and backwards from a given number. The whole class starts counting. Lerato stops the class and gives each group a turn to continue counting. For example, the yellow group counts from 12 to 37,

the green group carries on and counts from 37 to 52 and finally, the blue group counts from 52 to 72. The learners point to the numbers on the number grids while they count.

Lerato has flashcards with simple calculations on them (e.g., 10 + 4 =, 14 – 4 =, 12 + ____ = 15). She shows these to the learners one-by-one and the learners give her the answer to each sum. She continues with the mental maths and asks the learners to give the answers for double 2 and 5, and half of 14.

Lerato gives the blue and the yellow group each a polystyrene cup with a few ‘disks’ inside and asks them to estimate the number of disks in the cup. Learners give their responses and she writes these on the board. One learner from each group counts the disks in the cup. Lerato asks the class which estimate was nearest to the correct answer, and which of the two numbers is bigger (i.e., the estimate or the actual number of disks).

Teaching and learning activities

Lerato questions the learners: *How many steps do we take from 14 to 15?* The learners raise their hands and one of the learners responds ‘1’. *What is the number between 14 and 16? Point to that number on your number cards.* She asks one learner to pick up the number from the number cards displayed on the board. The learner shows the number ‘15’ to the class. The rest of the learners point to ‘15’ on their number grids.

Lerato takes out a container with tennis balls in it and asks the learners to estimate the number of tennis balls in the container. She writes the estimates on the board. She asks one learner to count them. She asks the following questions:

If I throw away one tennis ball, how many will be left?

If I get two more tennis balls, how many do I have?

If I lose two tennis balls, how many will I have left?

The learners come to the board one by one to perform the calculations practically, in other words, they use the balls to work out each word problem.

The groups each receive a ‘dominoes worksheet’ with a single sum that requires them to make the bonds of 15. Each group has to complete a different sum. On the one side of the worksheet, Lerato has stuck on some dots. The learners must count the dots and then decide how many more they need to make 15. The learners draw the number of dots needed to make 15. The groups take turns to report to the class. Lerato writes the learners’ calculations on the board as an addition sum.

Lerato returns to the flashcards and shows the learners different sums one-by-one (e.g., 6 +_ = 15, 1 +__= 15) and places them on the board. The learners use the number lines that are stuck onto their desks to solve the calculations. After which, one learner (selected by the teacher) goes to the board and chooses a number card to complete the number sentence.

Consolidation
Lerato gives learners two worksheets.
-Worksheet 1 requires learners to add on pictures to make the number 15 (similar to the dominoes activity).
-Worksheet 2 requires learners to complete addition and subtraction calculations up to 15

Collaborative lesson 7.1 (presented by Lerato)

7.2.1.1. Collaborative talk during the planning of the 'Number 15' lesson

During the planning of this lesson, Lerato and Gladys dominated the conversation. Doreen remained quiet, simply listening to the other two teachers. The teachers planned the lesson according to the lesson structure recommended in CAPS that is mental maths, teaching and learning activities, and consolidation.

Lerato suggests that counting should be done in different learning groups whereby **"one group will count from 1 up to 11 and the next group will start from 12 to 30 and the same will be done when counting in 2's"** (Lerato, CL1[2]). The reason for the change in the counting procedure was that Stella pointed out during the reflection of the

[2] For ease of reading, I have not referenced every single comment made by the teachers. In this section, they are all from 'CL1' or 'Collaborative Lesson 1' (Chapter 5).

individually planned lesson that **"learners were counting as a whole class and some were just pointing to the numbers and they always counted starting from the beginning".** Stella also suggested that they move from always starting at '1' when counting on, to starting at arbitrary numbers (i.e., any number) as Lerato had done in her initial lesson. In relation to the counting activity that occurred in Lerato's individual lesson where the learners were required to count the tennis balls, Gladys expressed the view that an **"educator should do less and [the] learners do more during the lesson"** because Lerato was handling the tennis balls instead of giving the task to a learner**.**

Stella highlighted that while checking the observation schedules, she **"realised that the estimation is one aspect that was not included in the lessons. As a Grade Four teacher I know that estimation is important because it prepares learners for concepts like rounding off".** Lerato then suggested that they do an estimation activity **"I will give them counters in a container and ask them how many do they have in front of them"**.

The teachers suggested that they use a variety of resources to teach the number '15', such as **"dominoes, number line and ten-frames"** (Lerato). Lerato expressed that **"groups will be given 'dominoes worksheets' to build the number fifteen"**. She further added that she will **"give them sums in the form of a story"**. Gladys suggested that there should be **"one to one correspondence activities; they must pick up the number and match with the circles"** that the learners add on the dominoes worksheet to make up 15**.**

For classwork activities, Gladys suggested a **"worksheet with addition and subtraction sums to make bonds of 15"** and Lerato added, **"they can complete a worksheet where they draw pictures to make the number 15".**

After the lesson, the four of us met to reflect on the lesson and to see how to improve it.

7.2.1.2. Reflection on the 'Number 15' lesson

The observation and reflection of this lesson focused on the presentation of the lesson, the participation of the learners, challenges experienced by the teacher, difficulties experienced by the learners and strategies to improve the lesson.

Gladys suggested that they should **"not prolong the lesson by doing a lot of counting activities"** in future. Doreen concurred and suggested that the counting activities could also include examples where the teacher **"asks questions [for example] what comes before, after and the number between".** However, having said that, she proposed that next time, the teacher should not **"ask too many questions in one area",** for example, she thought that they planned too much for the mental mathematics and that this made **"the lesson becomes too long".** Doreen suggested that there be greater variability in the mental mathematics, but with fewer examples in each area.

Gladys highlighted that Lerato drew on the learners' prior knowledge when she "**used the number 14 to lead them to the number 15"**. While Lerato did the same in her individually planned lesson, Gladys felt that this was an important, positive aspect of the lesson, and necessary to emphasise.

Gladys remarked that the learners' participated well as they used various concrete and semi-concrete materials to support their learning. "**They used counters for counting and making bonds. They used their number-lines for addition and subtraction"**. Doreen added that the use of resources, such as the number line, developed the skill of **"counting on using the number line and the number chart".** She also mentioned that the peer work is important so that the learners could check their mistakes and "**correct each other".**

7.2.1.3. *Number sense development in collaboratively planned lesson 1*

The teachers and I incorporated seven FNS characteristics in the first collaboratively planned lesson. Learners counted systematically in groups from arbitrary numbers and from 1 during the various activities (e.g., counting the tennis balls). In so doing, the learners showed that they recognised the numbers by pointing to each number on their number grids. Estimation was included in this lesson, as the learners had to estimate the number of disks and tennis balls. After the estimation activity, the learners were required to discriminate between quantities by comparing their estimates with the actual number of disks. The learners were required to relate number to quantity using different forms of representation. These include the use of concrete objects (disks) and semi-concrete (drawing pictures) and simple calculations represented symbolically. Number patterns did not form part of this lesson. Table 7.1 provides a summary of the FNS developed in the collaborative lesson.

FNS Characteristics	**Mental Maths**	**Teaching and learning activities**	**Consolidation**
Number recognition	Pointing to numbers on the number grid while counting	Showing the number 15 to the class Pointing to the number 15 on their number grids	
Systematic counting	Counting from arbitrary numbers Counting the disks after estimating	Counting the tennis balls after estimation Counting the dots in the group activity to make the number 15	
Relating number to quantity	Giving the number of disks in the cup after counting Exploring more and less in relation to the number 15	Completing the dominoes worksheet (bonds of 15)	Drawing missing pictures to make 15

Quantity discrimination	Comparing numbers after estimation activity		
Different representations	Estimating and counting the disks for estimation	Drawing dots to complete dominoes worksheet Using tennis balls to solve word problems Using number lines to calculate	Drawing the missing pictures to make 15
Estimation	Estimating the number of disks in a cup	Estimating the number of tennis balls	
Simple arithmetic	Simple addition and subtraction (flashcards)	Word sums to add and subtract Making 15 using addition (dominoes worksheet) Calculating sums using the number line	Drawing the missing pictures to make bonds of 15 Addition and subtraction sums

Table 7.1: FNS characteristics in the first collaborative lesson

7.2.1.4. *The MKfT evident in collaboratively planned lesson 1*

As the teachers engaged in the planning and reflection stages of the lesson, it was clear that they knew the subject matter knowledge required to teach the number '15'. The focus of the interactions was PCK. Key aspects of the pedagogical content knowledge were KCS, KCT and KCC. Table 7.2 shows the PCK that we shared with each other.

The teachers' knowledge of KCC was evident in the manner that they structured their mathematics lessons, the choice of suitable resources, and the focus of the lesson, which was appropriate for the third term in Grade 1.

The teachers planned the lesson sequentially with a view that the learners need to first identify where ‘15’ is on the number sequence and the numbers that come before and after, prior to making 15 using addition. They drew on a variety of teaching strategies (e.g., questioning, practical demonstration with resources). Questions were used to draw on learners’ prior knowledge to introduce the number 15 and to motivate learners to engage in the lesson. The teachers planned the activities carefully and chose modes of representation that were appropriate for the task (e.g., disks and tennis balls for estimation). The teachers ensured that there was a combination of whole class teaching, group work and individual tasks.

The teachers’ demonstrated their KCS by ensuring resources were on the learners’ tables for them to use as they counted or calculated. They promoted active involvement by encouraging the learners to engage with a wide variety of activities. The lesson drew on the learners’ prior knowledge as the learners had already learned the number ‘14’.

Domain	**Indicators**
KCS	• Learners are placed in ‘ability’ groups and teaching takes place in Setswana, the Home Language of the learners. • Resources are placed on the learners’ desks (number lines and number grids) for ease of access and use during the lesson • Draws on the learners’ prior knowledge and uses this to teach new concepts (e.g., moving from the number 14 to 15) • Knows that learners should be actively involved with the different activities (e.g., getting the learners to work with the various resources) • Knows what will assist learners in developing an understanding of mathematics (e.g., using tennis balls to demonstrate the concept of more and less)

KCT	• Uses a variety of resources to develop learners understanding of the number '15' (e.g., number grids, number lines, counters, tennis balls, blocks) • Uses different teaching strategies to develop an understanding of the number '15' (chorusing, questioning, practical demonstration with resources) • Builds on the learners' thinking as the lesson progresses (identify the number '15', before and after, counting 15 objects, making 15) • Chooses modes of representation that are appropriate for the activities (e.g., drawing pictures when making 15) • Teaches the whole class and groups, and creates opportunities for learners to work independently as individuals
KCC	• Knows the curriculum requirements for Grade One (i.e., the number range and curriculum topics) • Extended their knowledge of curriculum (i.e., by including estimation in the lessons) • Selects a variety of resources (e.g., number grids, worksheets counters) to develop the learners' understanding of the number '15' • Knows how to structure a mathematics lesson in accordance with the CAPS requirements

Table 7.2: Mathematics Knowledge for Teaching in Collaborative lesson 1

Given that collaboratively planned lesson 1 is similar to the Lerato's initial lesson, it is not surprising that the MKfT drawn on in the planning and presentation of this lesson is primarily the same. Emergent from the discussion in the planning session was the suggestion to get the learners more actively involved in the lesson and that estimation activities be included, thereby extending the teachers' knowledge of the curriculum. What differs between Lerato's initial lesson and the first collaboratively planned lesson is the FNS characteristics. In this lesson, the learners participated in an estimation activity and, emerging from that activity, they were required to discriminate between different quantities. In other words, the learners had to compare their estimates with the exact number of disks in their cups.

7.2.2. Collaboratively planned lesson 2

Doreen volunteered to present the second collaboratively planned lesson. As such, the lesson was based on the topic she was teaching her class. In her individually planned lesson, she focused on 'breaking down and building up' numbers. In this lesson, she chose to focus on addition and subtraction up to 15. 'Breaking down and building up' numbers is often used as a strategy for solving addition, and in some instances, subtraction sums.

Classroom layout

The learners sit in three groups of eight and two groups of six according to their 'abilities'. There are number grids (1-100) stuck on the learners' tables for counting and performing calculations. The teacher places wooden blocks with numbers 1 to 20 on the chalkboard railing, which she uses as a number line. Each pair of learners has an abacus on their table. Doreen has a table in front of the class near the chalkboard where she puts the resources that she will be using during the lesson. Teaching occurs in the Home Language of the learners

Mental Maths

Doreen asks the whole class to count in 1s from 1 to 20. As the learners count, they point to the numbers on the number grids that have been stuck on their tables. She asks the learners to count in 10s from 10 to 100. As the learners count, they point to the numbers on their number grids. Doreen moves around the class to check if the learners are pointing the correct numbers.

Teaching and learning activities

She shows the learners a worksheet that requires them to match the calculation in column A with the answer in column B. She points to a sum in column A and asks a learner to choose the answer from column B. The learners seem confused by the instructions, but some attempt to answer. She draws their attention to a calculation in column A: *What number minus five equals four*? She gives the learners an opportunity to think about the answer. When there are no responses, she turns the sum into a word problem. She asks: *"How many marbles do you have altogether, when you put 5 in your pocket and you are left with 4 in your hand?* She demonstrates this by pretending to put some marbles in her pocket and hold some in her hand. The next question is '7 -__= 2'. This time she immediately asks it in the form of a word problem: "G*randmother has 7 biscuits and gives you some and then she is left with 2. How many biscuits did she give you?* Doreen says the word sum while pointing to the sum in column A of the worksheet. She then points to column B indicating that the learners must choose an answer from column B.

Doreen gives each group a piece of paper and a small bowl with counters inside. Each group has a different type of counter (e.g., bottle tops, Cuisenaire rods, disks and cubes). She asks the learners to estimate the number of counters they have and write the answer down on the piece of paper. While she is repeating the instructions, the learners count the counters in their bowls instead of estimating them. When she asks for the estimates, the learners respond with the exact number of counters in the bowl.

Doreen asks the learners to take out ten counters from the bowl. She asks them to place the counters on their desks in a straight line and count them. Some groups are not sure how they are supposed to organise the counters. Doreen goes to the learners and assists them by showing them how to place counters in ones in a straight line. Once the learners understood the instruction, she asks them to place 10 counters on the table (she does the same with the numbers 12, 8 and 15). Doreen moves around to each group to check if learners understand her instruction.

Doreen holds up the worksheet with sums on it. She asks the learners to use their counters to calculate (e.g., 10 + 5 =_; 7 + 4 =_;8 + 3 =_). The learners raise their hands to give their answers.

Doreen puts the learners who are at tables of six and eight into groups of three and four respectively. In their groups, she asks them to take 15 counters from the bowl and put them on their desk. Then she asks them to remove 5 counters. She asks which operation they will use. An individual learner responds that it is ‘subtraction’. She asks how many counters are left and learners respond individually by raising hands. Doreen asks individual learners to answer. She asks the learners to count 14 counters in their groups. She asks the learners to remove 2 counters. As she moves around the class, she asks how many are left. She realises one group is struggling and she does the activity practically with them. She counts with these learners and shows them how to ‘take away’. Then she counts what is left with the learners. She uses the phrase “move aside” as she moves the counters away.

Consolidation

The learners complete a calculation worksheet and draw pictures that match the sum. Learners draw the number of pictures that corresponds with the numbers given in the number sentence. Doreen shows learners an example on the board before letting them complete the activity.

Example: 6 + 8 = 14

Collaboratively planned lesson 7.2 (presented by Doreen)

7.2.2.1. Planning of collaborative lesson 2

After deciding on the topic for the second collaborative lesson, Doreen drafted the lesson plan prior to our planning session.

While Doreen made the point during the planning of Lerato’s lesson that they need to develop their collaboration skills, she dominated the planning of the second collaborative lesson. Doreen presented a page with activities that she suggested should be included in the lesson. She indicated that she wanted to use a range of resources, which include, **“counters, abacus, numbers, number lines to complete and they will also use DBE books for extra work**” **(**Doreen, CL2) [3]**.**

During the reflection of the first collaborative lesson, Doreen pointed out that mental maths activities have to be minimised. She suggested that for mental maths activities the learners will **“count forward and backwards in 10s between 0 and 70”.** Lerato wanted to know if **“we only count in tens”** during the mental maths activity. Gladys suggested that “**there is also addition and subtraction”.** In other words, Gladys maintained that there should be the opportunity for the learners to engage with some mental calculation after counting. She suggested that simple calculations be written onto card that the teacher ‘flashes’ to the learners. Lerato reminded Gladys that **“we have spoken about time before”** and Doreen added that, “**I do not want to make it long’.** Lerato comments, **“I think at the beginning… I don't know if you included estimation, but you are going to give them counters. Ask them at that time, how may do they think there are”.** Then Doreen expressed that **“Then I must include it in my daily activities”**, that is, during the teaching and learning stage of the lesson.

The teacher suggested that the learners be given counters to arrange and count to develop their understanding of quantity. **“Then ask them to arrange the counters that they show 10, 12, 15. You want to see that they know the numbers” (Doreen).** Thereafter, Doreen suggested that the learners will use the counters to solve addition and subtraction calculations “**Then from that 10, you can ask them to take 5 and add 5 to it, or pose it as a problem and say maybe if you have 10 sweets someone gives you 5 more. How many sweets will you have?”** (Doreen). Focusing on the issue of time again, Gladys suggested that Doreen must reduce the number of calculations to **“two addition and two subtraction”**. Doreen suggested that to reinforce the learners’

[3] For ease ease of reading, I have not referenced every single comment made by the teachers. In this section, they are all from ‘CL2’ or ‘Collaborative Lesson 1’ (Chapter 5).

understanding she will ask them to draw their solutions on paper. "**They can also draw pictures to show their number sentences".**

In order, to develop the learners' calculation skills, Doreen suggested that she will include sums with two operations (+ and -) for the learners to solve "**Then here they will be doing the 15…same time…I want them to get into this number … (She wrote on a piece of paper what she was trying to say) 15 + 5 - 10. The numbers increase and decrease".** As Doreen was dominating this planning, I asked questions such as **"are the resources enough… are we okay with them?", "Is there anything else?"** and **"do we need to add or, is there something else we can assist with?"** so that the other teachers could engage more with the planning and preparation of the lesson. Doreen responded that **"the number sentences were not typed with the correct font".** She asked the team if they could fix that for her. Finally, the teachers agreed that the lesson was well planned.

7.2.2.2. Reflections on collaboratively planned lesson 2

During the reflection of the lesson, the teachers were concerned about making their lessons inclusive of all the different abilities in their classrooms and Doreen commented that "**we give learners the same work and others work fast and get bored"**. Doreen moved around a lot to assist struggling learners with the counting and calculations and suggested that they must "**make cards, single cards for them because when they point, you are still not convinced that they know 5"**. The teachers explained that their challenge with 'struggling learners' was caused by **"those who do not come from pre-school, I have those who do not come from pre-school and it is worse with them"** (Lerato). As the teachers were aware of the challenge with 'struggling learners', I wanted to find out if they have plans on how to assist them. **"When you look at those learners, what is it that you can do for them at the beginning of the year?"** (Stella). Doreen responded that they can give learners activities that include **"the one with the dots, one to one correspondence and use more manipulatives"**. However, she highlighted that **"we are not letting them play with the manipulatives…I have seen with myself ... because we do not want them to make noise"**. Gladys added that "**we are too formal when we work with them and they actually learn through playing"** and suggested that they must include "**games like Ludo, snakes and ladders"** Doreen suggested that **"we should sit together and make our own teaching aids"** in order to cater for all learners in their classrooms.

Despite the concern with 'struggling learners' the teachers expressed that "**they (the 'struggling learners') responded positively because most of them were actively involved in the lesson"** (Lerato). The **"learners managed to use abacus and counters on their own"** (Lerato) as they were performing calculations in their class activities. While most of the learners managed to work on their own "**the teacher moved around and assisted learners who were struggling"** (Gladys). The learners were unclear about Doreen's instructions when they had to match the answer to number sentence on the worksheet therefore **"teacher used word problems to make learners understand the question"** (Stella). This was the strategy used by Lerato during her lesson, and the learners in this instance responded well because the word problems assisted in contextualising the symbolic calculations.

The teachers highlighted some of the issues that they need to improve. Doreen commented that **"you need to look at your time because mental maths is 5 minutes and introduction of the new concept is 10 minutes and that is only 15 minutes"** as the lesson took longer than anticipated**.** While Doreen was moving around and assisting other learners the other groups were making a noise and Doreen suggested that "**we can have group rules like classroom rules and read the rules regularly with the learners so that they get used to them"** as a strategy to manage the group work**.** Lerato also added that the teachers can **"let the learners share responsibilities and rotate leadership"**. The teachers were also concerned about the quality of the teaching aids because during the matching activity in this lesson the numbers on the flashcards were not sufficiently legible for the learners and Doreen expressed that **“they must be bigger (she demonstrates the size with her finger). They must be big enough for learners to hold”.**

7.2.2.3. The FNS evident in collaboratively planned lesson 2

The teachers integrated six of the FNS characteristics into the lesson. The learners counted systematically in ‘1s’ and ‘10s’ and pointed to numbers while counting. In this way, they demonstrated their recognition of the numbers. Counting continued throughout the lesson as the learners used counters to assist them with the various calculations. Bowls with counters in them were given to the learners. The intention was for the learners to estimate the number of counters in the bowls. While this FNS was included in the lesson the learners were not familiar with the concept and so they counted the objects before Doreen could explain it properly. The learners were required to work with counters, draw pictures and respond to symbolic calculations during the lesson. The exposed the learners to different forms of representation and assisted in developing an understanding of the relationship between numbers and quantity. The learners engaged in simple arithmetic activities by matching the sum to the answer, using counters to perform addition and subtraction calculations and by drawing pictures to represent the sums in the consolidation activity. Quantity discrimination and number patterns were not incorporated in this lesson. The activities relating to FON appear in Table 7.3.

FNS Characteristics	**Mental Maths**	**Teaching and learning activities**	**Consolidation**
Number recognition	Pointing to the numbers on the grid while counting	Writing down the estimated number in their groups	
Systematic counting	Counting forward in 1s and 10s	Counting a given number of counters in a bowl Counting the counters to calculate given sums	

Relating number to quantity		Relating numbers written on the paper to estimated counters in the cup Arranging counters according to a given number	Drawing pictures to match the sums
Different representations		Placing counters on the table to match a specific number Symbolic representations of sums in the matching activity	Drawing pictures to match the sums
Estimation[4]		Estimating the number of counters in a bowl	
Simple arithmetic		Solving word sums to match the sums in column A with the answers in column B Using counters to solve problems Addition and subtraction using counters	Addition and subtraction (worksheet)

Table 7.3: FNS activities in the collaborative lesson 2

7.2.2.4. The MKfT evident in collaboratively planned lesson 2

The pedagogical content knowledge that the teachers drew on as they planned and reflected on their lessons includes knowledge of teaching, learners and curriculum. The teachers' KCC was evident in the planning and reflection of the lesson, as they were knowledgeable of the CAPS requirements, the structure of the mathematics lesson, and

[4] I have included estimation, as it was part of the planned lesson even if it was not successful in the lesson.

chose resources that were suitable for Grade 1 learners, and for learning addition and subtraction.

Their KCS is reflected in the manner in which they they drew on the learners' prior knowledge and ensured that the learners were actively involved in the lesson. Some of the resources were placed on the learners' desks to encourage the learners to use them. During the lesson Doreen realised that the learners did not understand the sums in the matching activity. Thinking on her feet, she immediately switched to using word problems based on real-life contexts, to assist the learners. She also supported the learners who required additional help during the lesson.

The teachers' KCT appears through their decisions to use a variety of different teaching strategies, multiple forms of representation and resources that support the development of number sense. Their MKfT is presented in Table 7.4.

Domain	**Descriptions**
KCS	• Learners are placed in 'ability' groups and teaching takes place in Setswana, the Home Language of the learners. • Resources are placed on learners' desks (number grids) for ease of access and use during the lesson • Builds on 'real-life' examples that the learners are familiar with (using marbles and biscuits as contexts for solving word problems) • Draws on learners' prior knowledge and uses this to teach a new concept • Knows that learners should be actively involved with the different activities (i.e., oral problem solving, practical calculations, written calculations)

	• Gives attention to learners who needs more assistance • Knows what will assist learners in developing an understanding of mathematics (e.g., word problems to contextualise the mathematics)
KCT	• Uses a variety of resources to develop learners understanding (e.g., counters, flashcards, worksheets) • Uses different teaching strategies to develop the learners' understanding (word problems, practical demonstration, questioning) • Uses different forms of representation (e.g., concrete, iconic and symbolic) • Teaches the whole class and groups, and creates opportunities for learners to work independently as individuals
KCC	• Knows the curriculum requirements for Grade One (topics, number range, lesson structure and duration) • Extends knowledge of curriculum (including estimation in the lesson) • Knows how to structure a mathematics lesson in accordance with the CAPS requirements • Selects a variety of resources (counters, flashcards, number grids, worksheets)

Table 7.4: Mathematics Knowledge for Teaching for collaborative lesson 2

Doreen had a very clear idea of what she wanted to achieve in this lesson and thus dominated the planning stage of the collaborative intervention. The other teachers made suggestions of how to improve the lesson based on the previous reflections. The FNS is similar to the previous lessons. However, the teachers drew on their new knowledge of estimation and included this in the lesson. The estimation activity was not successful. This shows that the FNS characteristics one intends to include in the lesson, are not necessarily developed in the lesson. Despite this new knowledge reflecting as part of their KCC, Doreen was not able to ensure that her learners developed their knowledge of estimation

7.2.3. Collaboratively planned lesson 3

Gladys taught the last collaboratively planned lesson. This lesson focused on doubling and halving.

Classroom layout
The learners are seated in 'ability' groups of six and eight. They have number grids stuck on their tables for counting and calculations. Each pair has an abacus to share. There is a table in front of the class where Gladys puts the resources she will use in the lesson.

Mental Maths
Gladys shows the learners a transparent container with coloured bears inside and asks learners to estimate how many bears are inside. She writes the estimates from three learners on the board. She empties the container and counts the coloured bears with the learners. She asks which estimation is closest to the number of bears in the container. She shows the class a second container again with coloured bears inside and asks the learners to estimate the number of bears inside the container. She writes three estimations from the learners on the board and asks which estimation is close to the exact number of bears in the container.

Gladys holds up a number (18) written on card and gives the learners the opportunity to tell her what the number is. For each number that she holds up (18, 12, 16, 15), she asks them to tell her what number is one more or one less.

Teaching and learning activities
Gladys tells the learners a story about a man who has 5 ducks. She puts pictures of 5 ducks on the board. The man has to feed the ducks a certain amount of food every day. She puts up a picture of a man showing his two hands which he uses to feed the ducks. The ducks eat 3 bags of food every day. Each bag is made from 4 bundles of hay. The ducks go to the pond and each finds a friend. Gladys places a picture of 5 more ducks underneath the initial 5 ducks. She asks learners to count all the ducks. They do this in unison and answer '10'. She writes the number 10 on the board. She explains that 5 ducks and another 5 ducks makes 10 ducks. She moves on to explain that extra hands are needed to feed the ducks and they also need extra food. The man brings his brother to help him. She says there are 2 men. As she says this, she puts up a picture of another

man. She asks the learners to double 1. She does the same with 4 hands, 6 bags of food and 8 bundles of hay. The learners count after each set of pictures is placed on the board. As the learners tell her the answer to the sum, Gladys writes the answer on the board. During the lesson Gladys asks various questions related to doubling:

"We had 1 man and to double the number how many men will be there"?

"One man has 2 hands therefore how many hands will 2 men have? etc.

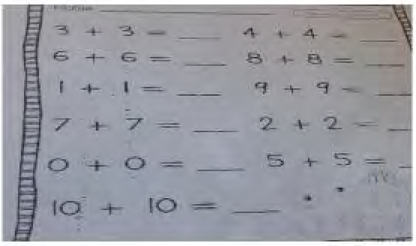
After introducing the concept of doubling with the use of the story, Gladys continues with the story, but shifts the focus to halving. In this situation, half the ducks move back to the dam, and she asks the class how many ducks are left. Learners raise their hands and she selects one learner to respond. She explains the concept of halving with an example. She tells the learners that there were 10 ducks, 5 moved away and so there were 5 left. She tells them that half of 10 ducks is 5 ducks. The man's brother has to go as only one man is needed to feed the 5 ducks, so there are now only 2 hands. She explains that the food is also halved from 6 bags to 3 and the 8 bundles of hay becomes 4 bundles. Gladys demonstrate this by removing the pictures to provide the learners with a visual image of the process of 'halving'. When she asks learners to half the numbers without pictures (e.g., what is half of 4) they take a long time to respond. Gladys repeats the questions and shows learners the pictures from the story to clarify the questions.

To reinforce the concept of doubling and halving, Gladys takes the learners outside. She places 8 hoola-hoops on the ground in two rows of 4 (Row A and Row B). In Row A, she places one learner in the first hoola-hoop, two learners in the second, three learners in the third and four learners in the fourth. She wants double the number of children in each of the hoops in Row B. She uses the hoola-hoops to demonstrate doubling by getting the learners physically active.

Gladys shifts the focus to halving. She calls out "*Half of 6, half of 4*" etc. She expects the 3 learners in the second row of hoola-hoop to leave so that only half the number of children are left However, Gladys' instructions are not clear and the learners are confused. Gladys explains by pointing to the hoola-hoops and telling the learners that when she says half of four, some of the learners in the second row must get out of the hoola-hoop so that there are only two left. She explains that half of four is two. After realising that the learners are confused with halving especially the hoola- hoop activity she expressed that she will repeat the activity the next day.

Assessment

Learners complete a doubling worksheet and Gladys encourages them to use the abaci to do the calculations.



3 + 3 = ___ 4 + 4 = ___
6 + 6 = ___ 8 + 8 = ___
1 + 1 = ___ 9 + 9 = ___
7 + 7 = ___ 2 + 2 = ___
0 + 0 = ___ 5 + 5 = ___
10 + 10 = ___

Collaborative lesson 7.7 presented by Lerato

Unlike Lerato and Doreen, Gladys did not come to the planning session with a lesson plan or a list of ideas. Nevertheless, whilst the team planned the lesson together, Gladys led the discussion. Interestingly, having planned the lesson collaboratively, Gladys changed part of it when she presented it. Instead of using the mirrors and apples, she developed a story to teach doubling and halving.

7.2.3.1. Planning of collaborative lesson 3

During the planning of this lesson, it was decided that Gladys would present the lesson. Gladys (CL3[5]) indicated that she would "**need mirrors to do doubling and halving"**.

[5] For ease of reading, I have not referenced every single comment made by the teachers. In this section, they are all from 'CL3' or 'Collaborative Lesson 3' (Chapter 5).

In thinking about how to present the notion of doubling and halving to the learners, Gladys also added that "**we can also take an apple, cut it in half and have two pieces"** or use an activity **"where you give them (the learners) a picture and they have to do the same thing and complete the picture"** (Doreen). Gladys shared a lesson idea that her granddaughter told her. **"My granddaughter's teacher teaches it in a fun way using hoola-hoops, one learner in the hoola-hoop then double that another learner gets in another hoola-hoop"**. Lerato added, "**it can be done outside, where learners are free to run".** The team was clearly interested in the idea and agreed to try it for teaching doubling and halving

For the mental maths activities, Gladys said that she would show the learners a number. She intended to ask the learners to tell her what number it is and then to double it. **"You can flash a number and they can double it, then flash again then they can half the number"** (Gladys). Lerato added that **"they can do the rote counting of doubles … Double 1 is 2, Double 2 is 4 etc…"** The discussion moved onto estimation and Doreen suggested that they vary the resources for estimation. She added that for **"estimation you can use balls on a string** and Gladys offered that **"you can also use smarties and the one who is closest gets the smarties".**

Doreen suggested that Gladys **"use the one in the DBE books"** for class activity. Stella added that they should consider including "**a picture worksheet where learners draw double the number of pictures given"**.

7.2.3.2. Reflection of collaboratively planned lesson 3

The reflection and observation of this lesson focused on two aspects, namely the story and the hoola-hoop activity. The teachers thought that the storytelling strategy Gladys used to teach doubling and halving worked well. "**The story captured the attention of the learners. The learners were listening attentively**" (Lerato). Lerato added, "**they (the learners) enjoyed the story, the pictures and using hoola –hoops"**.

While the teachers were positive about the use of storytelling, Lerato suggested that **"you see with the ducks… they already see the first 5 ducks … she could have continued and not go back to 1" (Lerato).** In other words, learners have to be encouraged to count on from the existing number instead of always starting at '1', for example 5 ducks count on from '6' to '12'.

It was evident that learners struggled to follow the hoola-hoop instructions. Doreen explained that she **"also liked the one of the hoola hoops, it is just that learners needed more time to practice**". Lerato then added that Gladys can **"start by having numbers in front of every hoola hoop so that learners can see them and participate".** Doreen mentioned that she liked the way the doubling was done, but that she was not sure that the halving was that successful. **"I liked the doubling but could see that the halving was confusing to the learners"**. Gladys agreed saying **"I also feel that I did not explain halving well so that the learners can understand".**

After both teaching activities (story and hoola-hoop), Gladys realised that learners were struggling with some of her instructions. She "**calmly corrected the learners and showed them again how to double and halve"** (Lerato) by using the pictures from the story, and also the learners and hoola-hoops. As a result of learners having challenges with the teachers' instruction while teaching halving, Gladys only gave learners a doubling classwork activity and the learners "**used apparatus well as they are familiar with using them in class"** (Stella).

7.2.3.3. The FNS evident in collaboratively planned lesson 3

There are seven FNS characteristics incorporated in all three sections of this lesson. Learners recognised numbers when the teacher flashed numbers to them during mental maths. Learners estimated the number of bears in a container and compared their responses with the exact number to show which number was nearest to the exact number. They related numbers to pictures in the story and numbers were represented using numerals and pictures. The teacher incorporated simple arithmetic of addition and subtraction during the mental maths (one more or one less) session and during the doubling and halving activities in both the lesson development and consolidation. Number patterns were not incorporated in this lesson. Table 7.5. indicates the activities used to develop FNS in this lesson.

FNS Characteristic	Mental Maths Activities	Teaching and Learning Activities	Consolidations
Systematic counting	Counting the bears after estimating.	Counting the ducks, hands, bags of food, bundles of hay and men in the story	
Number recognition	Recognise the numbers shown by the teacher		
Relating number to quantity	Relating the number to the estimated quantity and to the actual quantity	Matching the pictures in the story with the numbers Identifying the number of learners in the hoola-hoops	
Quantity discrimination	Comparing learners estimates to the exact number of coloured bears in the container Identifying one more and one less than a given number	Comparing quantities in the story during the process of doubling and halving	
Different representations	Using coloured bears for estimation and then counting the exact number	Representing doubling and halving using pictures and learners in hoola hoops	Using the abaci to double
Estimation	Estimating the number of coloured bears in a container (uses a referent)		
Simple arithmetic	One more or one less than a given number	The relationship between addition, and doubling and subtraction and halving	Addition sums that required the learners to double (worksheet)

Table 7.5: FNS activities in the collaborative lesson 3

7.2.3.4. The MKfT evident in collaboratively planned lesson 3

The pedagogical content knowledge that the teachers employed during the reflection and planning sessions are the KCS, KCT and KCC as illustrated in Table 7.6. Gladys did not come to the planning session with a draft or pre-planned lesson. The group planned the lesson together. During the implementation of this lesson, Gladys deviated from some of the suggestions made during the collaborative planning session (e.g., Gladys used a story to teach doubling and halving). In order, to draw on the learners' prior knowledge the group suggested counting in doubles and using cards with numbers on that the learners have to double and halve. Together with the teachers, Gladys planned to engage the learners in the lesson by using resources that are interesting to the learners. This is evidence of their KCS.

The teachers' KCT enabled the selection of different and appropriate methods and strategies to develop the concept of doubling and halving. Added to the practical demonstration, Gladys used storytelling to develop the concept. The teachers exhibited awareness of different resources that can be utilised to develop the concept of doubling and halving.

The teachers also exhibited the curriculum expectations in terms of the content to be taught and the organisation of the mathematics lesson. In addition, they chose resources that contributed to the lesson. In her decision to include a story, Gladys made sure that she used pictures to develop the learners understanding of doubling and halving. In addition, Gladys made the link between doubling and addition and halving and subtraction which is evident of her knowledge of the curriculum, but also that learners learn mathematics by making connections.

The teachers' SMK was assumed during the planning and reflection of their lesson as this was not included in our discussions.

Domain	Descriptions
KCS	• Learners are placed in 'ability' groups and teaching takes place in Setswana, the Home Language of the learners. • Resources are placed on learners' desks (e.g., number grids, abaci) for ease of access and use during the lesson • Knows that learners should be actively involved with the different activities (e.g., participating in the story and hoola hoops) • Knows what will assist learners in developing an understanding of mathematics (e.g., the story)
KCT	• Uses different teaching strategies to develop the learners' understanding of new concepts (e.g., story-telling, questioning and answer, practical demonstration) • Introduces one concept at a time to the learners (doubling then halving) • Uses different modes of representation (concrete, iconic and symbolic) • Links mathematical concepts (e.g., addition with doubling and take away with halving) • Teaches the whole class and creates opportunities for learners to work independently as individuals
KCC	• Knows the curriculum requirements for Grade One (topics, number range, lesson structure and duration) • Extends knowledge of curriculum (including estimation in the lesson)

	• Knows how to structure a mathematics lesson in accordance with the CAPS requirements • Selects a variety of resources (counters, number cards, cards, pictures hoola hoops, worksheets)

Table 7.6: Mathematics Knowledge for Teaching presented in collaborative lesson 3

Despite Gladys working collaboratively during the planning phase with her colleagues to develop the third collaboratively planned lesson, she decided to change an aspect of the lesson prior to teaching it. She included a story to teach doubling and halving that was not part of the discussion during the planning session.

7.5. THE KNOWLEDGE QUARTET: DIFFERENCES BETWEEN THE INTIAL AND COLLABORATIVELY PLANNED LESSONS

In terms of the MKfT framework, the focus during the collaborative planning and reflection stages of the research were on Pedagogical Content Knowledge, that is KCC, KCT and KCS. One can assume that teachers' have the Subject Matter Knowledge, that is, the CCK and SCK required for teaching in Grade 1. Whether these teachers have the necessary HK is a moot point as it did not emerge during the lesson discussions. Talk about the concepts taught remained at the Grade 1 level.

As explained in Chapter 3, I suggest that all of the MKfT domains are part of Foundation Knowledge. Foundation knowledge refers to the knowledge, beliefs and understanding acquired during schooling, pre- and in-service teacher education, professional development and in the act of teaching itself. Rowlands & Turner (2007) maintain that Foundation Knowledge has five components: (1) knowledge and understanding of mathematics content; (2) the knowledge of significant tracts of the literature on teaching and learning mathematics; (3) beliefs concerning the nature of mathematics; (4) knowledge of the purpose of mathematics education; and (5) the conditions under which pupils best learn. Ball et al’s. (2008) MKfT is embedded in each of these.

Knowledge of mathematics content equates with CCK. Knowledge of teaching and learning mathematics incorporates all of the PCK domains, that is, KCT, KCS and KCC, and SCK. Teachers do not only need to know how learners learn mathematics (KCS), and how to teach mathematics (KCT), they also need to know the curriculum and the LTSM available that are suitable for teaching and learning (KCC), and how to recognise and address learner errors (SCK). Beliefs about the nature of mathematics underpins all that teachers do and say in the classroom, and thus informs PCK. If a teacher believes that mathematics is objective and abstract, they are more likely to teach it in a manner that provides little opportunity for sense making (Westaway & Graven, 2018). Knowledge of the purpose of mathematics education would include KCT, KCC, KCS and HK. While Ball et al. (2008) regard HK as the knowledge teachers require to understand how the concepts, they teach in a specific grade contribute to later mathematics development, Jankvist, Mosvold and Clark (2016) suggest that HK also includes knowledge of the history of mathematical concepts. Knowledge of the history of mathematical concepts requires an understanding of how and why these concepts emerged. This requires an understanding the importance (and necessity) of mathematics in the world throughout the ages and into the present. Knowing the conditions under which learners learn best, includes all the domains of PCK. Figure 3.1 (Chapter 3) provides an overview of the inclusion of the six MKfT domains as part of Foundation Knowledge.

Transformation Knowledge, Connection Knowledge and Contingency Knowledge are all forms of knowledge that teachers draw on in-action, that is, as they teach. These forms of knowledge were not the focus of this thesis. Rather this research attempted to understand the knowledge that teachers draw on as they plan and reflect on their teaching. This Rowland (2007) refers to as Foundation Knowledge.

Leutzinger, Rathnell and Urbatsch (1986) argue that primary school learners are capable of learning estimation skills. All three lessons included an estimation activity. This emerged after I raised the importance of estimation during the initial lesson reflection. As a Grade 4 teacher, I explained that estimation supports learners with their rounding off activities in higher grades. Furthermore, Tsao (2004) maintains that estimation is a skill that develops learners to make connections between different mathematical concepts, and it also, enables learners to judge the reasonableness of the answers to calculations.

The collaboratively planned lesson that Doreen taught included estimation, but it was not successful as the learners counted the actual number of counters rather than estimating them. Tsao (2004) adds that when learners attempt to give an exact number instead of approximate number, it is likely that either they do not understand the concept or they have poor estimation skills. In this instance, the teachers' instructions were not clear. Pizarro, Gorgorió and Albarracín (2015) maintain that learners should be given a referent on which to base their estimations. While one could argue that the estimation activities in the first two collaboratively planned lessons were examples of a 'guesstimation', the third collaboratively planned lesson that Gladys taught included a referent, making it an estimation activity.

The recognition and extension of number patterns were omitted in all of the collaboratively planned lessons, despite both Doreen and Gladys including patterns in their initial lessons. While the learners were asked to identify where to place 'X' on the number lines, the learners' attention was not drawn to the fact that as the numbers progress along the number line, 'we add one'. Furthermore, we did not ask any process questions (Tsao & Lin, 2012) as one of the strategies suggested for developing number sense. For instance, after a learner places a number on a number line the teacher could ask how they chose that number.

The collaborative planning and reflection sessions led to a shift in the nature of the mental mathematics activities used in the lessons. While systematic counting featured in all the lessons, it was only in the collaboratively planned lessons that the shift was made from counting solely as a whole class, to counting in groups, with each group following on from the previous group.

It was clear in all the lessons that the teachers had the necessary subject matter knowledge to teach Grade 1 mathematics. SMK did not emerge in the planning or reflection stages of the research. Across the initial and collaboratively planned lessons, teachers knew the content that they were required to teach.

The PCK that the teachers demonstrated included all three domains, that is, KCC, KCS and KCT. Ball, Thames and Phelps (2008) refer to Knowledge of Content and Curricular (KCC) as a range of programs for teaching of a particular subject and topics at a particular grade. As evidence of the KCC, all of the teachers knew the curriculum expectations for Grade 1, how to structure a lesson and were able to select appropriate resources to support the development of number sense.

The PCK across the lessons was primarily the same. As with the initial lessons, the teachers all organised their learners into 'ability' groups to meet their learners' needs and taught in Setswana. Drawing on their KCS, they included a variety of activities in their lessons and drew on a wide variety of resources, thereby exposing the learners to different forms of representation. The teachers also varied their teaching strategies and exposed the learners to different forms of representation to support the development of learners' number sense. Thus, providing evidence of their KCT. The only difference was the incorporation of estimation in the lessons, which signals a shift in their knowledge of the curriculum expectations.

7.6. CONCLUSION

In this analysis, I have shown the opportunities the three Grade One teachers used to developed learners' number sense in their classrooms. I decided to engage with the sub-question: *What mathematical and pedagogical content knowledge do Grade One teachers use to develop learners' number sense?* Drawing on CHAT, and as noted in the introduction to this chapter, this chapter examines two aspects of the collaborative intervention: the outcome and the object. The outcome refers to the development of learners' number sense, and the object is the teachers' mathematics and pedagogical content knowledge. Essentially, this chapter responds to two questions:

- ❖ What is the nature of the number sense activities promoted during the collaborative intervention?
- ❖ What is the nature of the teachers' mathematics and pedagogical content knowledge during the collaborative intervention?

As highlighted in Chapter One, many authors are critical of professional development programmes that focus on workshops. Bertram (2011), Chapman (2012) and Pyrko, Dorfler & Eden, (2017) all argue that teachers learn to implement new pedagogies and assessment methods when they have the opportunity to work as a collective. Schoenfeld (2002) maintains that such collaboration enables teachers to develop their knowledge within the context of the classroom. This was not the case in the collaborative intervention that formed the focus of my research. Across both the initial and collaboratively planned lessons, teachers knew the content they are required to teach. Apart from the inclusion of estimation as an example of their KCC, developed during the collaborative intervention, the teachers' PCK did not shift. It appears to me that without the intervention of a 'more knowledgeable other' (Vygotsky, 1978), teachers' PCK is unlikely to improve.

The Department of Basic Education are promoting the development of Professional Learning Committees in schools with the view to teachers taking responsibility for their own professional development. This leaves me with the question: *How do we ensure that teachers are able to develop the required knowledge to change their practices in order to improve learning and teaching in the classroom?*

In this chapter, it is evident that the teachers included estimation in the lessons. Estimation activities were included after I signalled the importance thereof for learning and teaching in the higher grades. Estimation is a characteristic of FNS. In terms of the FNS characteristics, patterns were included in two of the initial individually planned lessons and it was omitted entirely from the collaboratively planned lessons.

While there were some difference as to the components of FNS developed in the collaboratively planned lessons when compared to the initial lessons, it appears, on reflection, that we made the assumption that the learners were indeed developing an understanding of these components, and were in fact, developing their number sense. According to Andrews & Sayers (2015), FNS is the number sense that starts to develop prior to school, but that needs to be consolidated through instruction during the first year of formal schooling. Tsao & Lin (2012) stress that teachers play an important role in building learners' number sense. While this is not the focus of the thesis, I am left with the question: *Does number sense develop based on the implementation of the different components of FNS in the lesson(s) or does it need to be explicitly taught?*

CHAPTER EIGHT: THE COLLABORATIVE INTERVENTION

8.1 INTRODUCTION

During this collaborative intervention, the teachers shared ideas and strategies about planning and implementation of the lessons. After each lesson, we reflected on the lessons to discuss the employment of teaching strategies, learner participation, challenges experienced by teachers and learners and ideas to improve the lessons. This chapter responds to the question: *How did the collaborative activities strengthen the teachers' mathematics and pedagogical content knowledge?*

The object of this research is to strengthen the teachers' mathematics and pedagogical content knowledge, meaning that teachers are to acquire new knowledge in the classroom context where they teach (Sawyer, 2002). We established a collaborative intervention in order to engage in an evolutionary relationship of openness, trust, and support (Sawyer, 2001). The collaboration is meant to break teacher isolation and begin to establish a collaborative culture which affords teachers the opportunity to gain new knowledge and critique their existing knowledge (Sawyer, 2002) in developing learners' number sense. To answer the research question, I focused the analysis in this chapter on the teachers' reflections on the collaborative intervention. Since this research is underpinned by CHAT, I also moved on to analyse data using activity system as a unit of analysis. I identify the activity system by focusing on the interaction of the components of the activity system and how they impact on the achievement of the object. Then I examine the appropriation of the conceptual and practical tools on mediating the object of the research. I move on to reveal the contradictions that occurred and the opportunities they reveal. I end this chapter with a summary of the principles of CHAT and how they pertain to my research.

8.2. THE TEACHERS' PERSPECTIVE ON COLLABORATION AND PCK

After the implementation of the collaboratively planned lessons, I conducted individual interviews with the three Grade One teachers to ascertain their views about the following aspects:

- the benefits and challenges of engaging in a collaborative intervention;
- improvements to be made to sustain the collaborative intervention;
- teaching strategies developed that relate to the development of learners' number sense; and the
- learning that occurred through engaging in a collaborative intervention.

Four themes emerged during the individual interviews. These were collegial development, building and sustaining professional learning communities, pedagogical transformation, and developing inclusive lessons.

8.2.1. Collegial Development

According to Sawyer (2001), collaboration in teacher development is based on relationships teachers make between personal meanings, their work and their acquisition of new ways of teaching. The teachers expressed that collaboration was beneficial to them because they learned different methods and strategies. Lerato expressed that '**it has helped me to become a better teacher, who is always willing to learn from others"** and Gladys added that "**you can exchange methods of teaching because usually, the methods that we are using are not the same".** Doreen illustrated that by contributing that "**the one that I very impressed with was how to teach doubling, so the storytelling has also helped me a lot to use in future".** In addition to the benefits mentioned, the teachers also raised a few challenges they experienced by engaging in a collaborative intervention. Lerato as a novice teacher mentioned that "**I was very**

nervous, in front of very experienced teachers. I thought they have twenty years, thirty years and I only have three years".

8.2.2. Building and sustaining a professional learning community

Johnston (2009) expresses that teachers can learn professionally in sustained and meaningful ways when they can work together. Although there were challenges that the teachers experienced, they perceived the collaborative intervention as a model that can be utilised for teacher development. Gladys highlighted that "**it can be included as part of in-service training and encourage teachers to always work together"** and Lerato suggested that it can be implemented "**by having regular meetings maybe as a grade or as a phase to discuss how lessons are being presented, how others are helping learners who are experiencing learning difficulties in their classes".** Hence, Sawyer (2002) highlights that collaboration is an essential aspect of a supportive community to break teacher isolation and establish a more problem-solving and collaborative school culture. In order to establish and sustain the collaborative intervention in the school, the teachers suggested that "**when we have our phase meetings and then we discuss this also with the other teachers, what it has done for us"** (Gladys) and Lerato added that they can **meet regularly to reflect on the lessons** and "**continue assisting each other and giving each other guidance and new ideas".** Gladys also requested my involvement in sustaining the collaborative intervention by visiting them and asking, "**are you still going on with what we have done or was it only for that (research)"**.

8.2.3. Pedagogical transformation

This research aimed to strengthen the teachers' mathematics and pedagogical content knowledge through collaborative intervention; therefore, I wanted to find out if there was learning that the teachers experienced through collaboration. Consequently, Minnett (2003) mentions that shared reflection involves being aware of and evaluating our activities, inviting others' perspectives and engaging in dialogues about what we do, how we do it, and how we might improve it. Focusing on the development of number sense, Lerato pointed out that "**I have learned the importance of counting in groups and also the importance of doing estimation activities because really I was not doing it before".** Doreen added that "**I was struggling with teaching learners' multiples at least from the other class I saw them doing it the other way, so I copied that and I teach my children the same way".** Gladys focused on the pedagogical knowledge and mentioned that "**one of the educators advised me to use stories when giving learners problems. The next lesson that is what I did and I could see that storytelling way of teaching learners assisted a lot".**

This research focuses on developing learners' number sense; I wanted to find out the teaching methods and strategies that the teachers applied to develop number sense. They mentioned different methods such as **"solving problems by using number line"** (Lerato), "**group work"** (Doreen) and "**when learners are hands-on"** (Gladys). Lerato highlighted that lesson planning and presenting are her strengths. Gladys emphasised the importance of setting a conducive learning atmosphere, where learning takes place and the learners are engaged.

8.2.1. Developing inclusive lessons

All three teachers expressed that they battled to cater to struggling learners; this is the central challenge of inclusive education. Teachers highlighted strategies that they might employ to assist these learners, such as "**play more mathematical games with them, maybe it can help them"** (Lerato) and "**call them individually to the table and work with them individually"** (Doreen). In addition to these strategies, Gladys expressed that they can seek additional assistance from "**colleagues, try to find out how they handle it, my immediate seniors and from the officials when we have workshops".** Sawyer (2001) claims that professional development aims to build conditions for continuous growth by providing ongoing opportunities for an inquiry into practice. The teachers realised that working with 'struggling learners' was an area they needed to collaborate with in the future.

The four themes that emerged from the teacher interviews after the intervention suggest that the teachers found the opportunity to work collaboratively beneficial to themselves, their collegiality, and the development of their pedagogical content knowledge and practices. They suggested that they would require support in developing their pedagogical content knowledge to support learners who 'struggle' with mathematics. This seems like a useful starting point for further collaboration. The teachers did express concern with the sustainability of working collaboratively. It appears to me that making use of this opportunity to consider how to support 'struggling' learners may be a useful way to continue with the collaboration, and hopefully, in the process, develop more sustainable collaborative practices.

In bringing the research process together, I now analyse the activity system of the research, that is, the collaborative intervention.

8.1. THE ACTIVITY SYSTEM

The activity system is the collective engagement of individuals in an activity mediated by tools and driven by goals and motives to create new psychological tools (Yamagata-Lynch, 2010). All the components of an activity system influence each other and are also influenced by social, cultural and historical factors such as backgrounds, knowledge, beliefs and availability of tools (Koszalka & Wu, 2005). Figure 8.1 below, provides an example of how the different components of this activity system, that is the collaborative intervention, interact with each other towards the object and ultimately the outcome of the research.

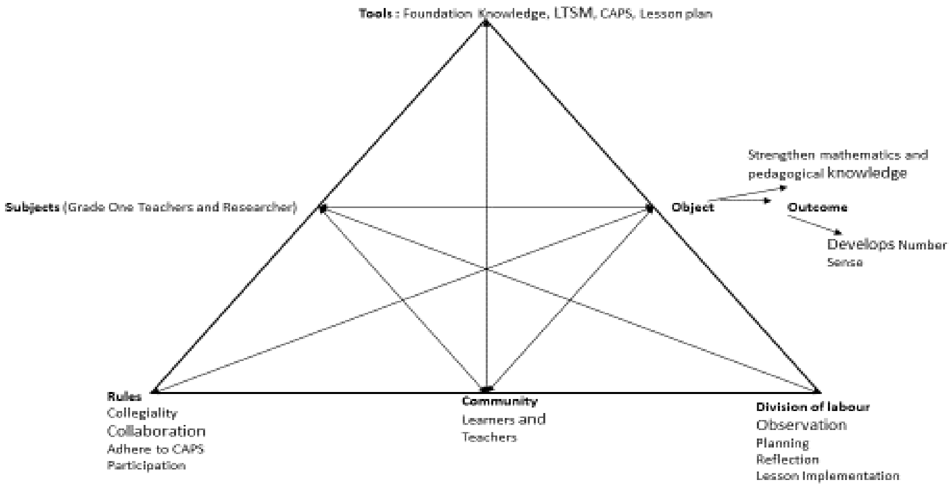


Figure 8.1: Activity System (Engeström, 2001)

According to Engeström's (2001), there are five principles of the activity system (Chapter 2) namely *activity system as a unit of analysis, multi-voicedness, historicity, contradictions and expansive transformation*. Engeström (2001) further suggests four questions (*who is learning, what do they learn, why do they learn and how do they learn*) linked to the principles that researchers should ask when analysing a project. The first four principles apply to this activity system because it has a single activity system and expansive learning applies to multiple activity system where the object is restructured and reinterpreted. The linkage of the activity system's principles and the four questions in this research are summarised in Table 8.2. showing how they underpinned the activity system.

Approaching the analysis of the activity system of this research, I used Hancock and Miller (2017) three CHAT-related elements of analysis, namely:

- ❖ Identifying the activity system
- ❖ Appropriation of tools

- Discovery of contradictions

The three elements above also incorporate the principles of the activity system; the activity system is identified using it as a unit of analysis to understand what is happening in the activity. Appropriation of tools examines how conceptual and practical tools mediate the achievement of the object while looking at the subjects' application of tools and the effect of history in mediating the tools. The discovery of contradictions and their exhibition of success or failure of the activity reveal opportunities for creative new ways of structuring and enacting the activity.

8.3.1. Identifying the activity system

In Chapter 2, the activity system components are named as the following: *subjects*, *mediating tools*, *community*, *rules division of labour*, *object and the outcome*. This research's activity system is a collaborative intervention that is multi-voiced because it consists of a group of participants called *subjects*. The subjects of this activity are three Grade One teachers and myself. We undertake different roles in the activity system, such as enacting a lesson, observing a lesson and participating in collaborative discussions where are shared. The subjects work collaboratively towards the *object* of the activity, which is developing the Grade One teachers' mathematics and pedagogical content knowledge, and are influenced by their personal experiences and Foundation Knowledge. We mediated the object by using conceptual and practical tools (Chapter 2). The conceptual tools included language and the Foundation Knowledge (Chapter 7) of the teachers. The practical tools used in the intervention were the CAPS document, lesson plans, observation schedules, and teaching and learning materials.

The *community* that is the teachers and learners engaged in the lesson by mediating practical tools to develop number sense. The subjects mediated conceptual tools individually to engage in lesson implementation and observations, and collaboratively in the planning and reflection activities. The activities of the subjects orientated the activity towards the object and transforming their practice. The actions of the community in the classroom are directed towards developing number sense, which is the *outcome* of the activity system. The subjects adhered to *rules*, such as collegiality, where they exhibited a good working relationship. However, collaboration is a skill that still requires development. The other rules that subjects followed are adhering to CAPS by selecting topics relevant to the grade and school term. The subjects cooperated and participated well in all the actions and activities of the activity system.

According to Roth et al. (2012) and Foot (2014) an activity system is a dynamic learning process. The actions and activities that the community and subjects were engaged in gave them opportunities to improve on the classroom strategies and develop the subjects' historical and cultural tools mediated towards the object and outcome. Foot (2014) points out that an object is never fully accomplished; this is unpacked below in the discussion of the contradictions that create opportunities to reinterpret the object. I now move on to examine the appropriation of tools and how they mediated the accomplishment of the object and the outcome.

8.3.2. Opportunities for improved practice

According to Foot (2014) the essential task of a CHAT analysis is to grasp the systemic whole of an activity, the components and the interaction of components at a particular time, and the transformation of the activity system over time**.** Identifying the activity system allowed me to examine the teachers' practices relating to the activity system's components. Gladys expressed that by engaging in the collaborative intervention, they can "**exchange methods of teaching because usually, the methods that we are using are not the same.** According to Foot (2014) community is central to the process of learning, and interaction in the activity system. Hence Lerato suggested that learners should be encouraged to learn by **"engaging them more in lessons despite them making mistakes because they tend to make mistakes, so I must not concentrate more on mistakes because they learn from it"**. And Doreen shared a strategy she uses in her classroom and mentioned that **"I call them individually on the table and**

work with them individually". The teachers expressed that they had learned and grown by engaging in the collaborative intervention. Murphy et al. (2008) and Feldman and Weiss (2010) state that the lens of CHAT can provide an understanding of transformation and restructuring of teaching practice facilitated by the teachers' engagement in an intervention. Gladys claimed that the advice of using the "**storytelling way of teaching learners assisted a lot"** while Lerato expressed that she learned "**the importance of counting in groups".** I move on to examine the appropriation of tools and how they mediated the accomplishment of the object and the outcome.

8.3.3. Appropriation of tools

Vygotsky initiated the concept of mediated action to explain the process where learning is enabled by: (1) the use of conceptual and practical artefacts (tools); and (2) interacting with others in an environment. Tools mediate how individuals achieve an object and outcome (Hancock & Miller, 2017). Botha (2012) stresses that tools "improve people's senses and empower them by allowing mediated access to the social context" (p.61). The subjects in this research mediated conceptual and practical tools to engage in a collaborative intervention to achieve the object that is the development of mathematics and pedagogical content knowledge. The appropriation of tools is used to examine the teachers' application of mathematics and pedagogic knowledge to develop learner number sense and opportunities for teachers to achieve the object.

8.3.4. Appropriation of conceptual and practical tools in the teachers' lessons

The teachers' lesson planning, implementation, observations and reflections were all informed by their Foundation Knowledge. This includes subject content knowledge, pedagogical content knowledge, beliefs, and general knowledge related to teaching that teachers acquire during their pre- and in-service training and various professional teacher development programmes (Chapter 3). In Chapter 3, I argued that Foundation Knowledge includes Ball et al's. (2008) Mathematics Knowledge for Teaching. Rowlands (2005) suggests that knowledge and understanding of mathematics content and pedagogy are critical aspects of Foundation Knowledge. In this research, the teachers were familiar with the subject-matter knowledge that they required to teach. During the planning and reflection sessions, no reference was made to any of the SMK domains: CCK, SCK and HK.

8.3.5. Practical tools supporting conceptual development

According to Boggan, Harper and Whitmire (2001) "when students manipulate objects they are taking a first step towards understanding math process and procedure" (p.4). The subjects select manipulatives during the planning session and share how the learners will use them throughout the lesson. Learners were using counters, number lines, number grids, wooden blocks and worksheets to learn and consolidate number sense. Thus, Lerato mentions that **"my learners understand number line well and they even join in to explain the method of using number line to others".** Manipulatives can also be used by teachers to examine the conceptual understanding of the learners. All three teachers mentioned that they experience a challenge with catering for 'struggling' learners. For example, Doreen pointed out that "**I have those learners who struggles with numbers, I always ask them to show me the numbers in the number card, because if the child cannot add or subtract it means he doesn't have a clear understanding of number sense".** Moreover, Gladys suggested "**that having lots of aids, simple things that can help them to be able to learn things that they don't understand".** Teachers mediated practical tools to enact the lesson and engage learners in various activities that develop their understanding of number.

During the collaborative intervention, the participants engaged in different 'transformative actions' (Hancock & Miller, 2017) to improve their practice. Through the 'transformative actions', the participants experienced contradictions. According to Foot (2014), contradictions are not points of failure or deficit in an activity system but indicators of further activity development opportunities.

8.4. CONTRADICTIONS THAT EMERGED FROM THE COLLABORATIVELY PLANNED LESSONS

Chapter 2 noted four levels of contradictions: primary contradictions, secondary contradictions, tertiary contradictions, and quaternary contradictions. Two levels of contradictions occurred during the activity in my research, that is, primary contradictions and secondary contradictions. Tertiary contradictions are triggered by the introduction of a new object and this activity had one unaltered object throughout (that is, developing teachers' mathematics and pedagogical content knowledge to develop learners' number sense). Quaternary contradictions arise between multiple activity systems. In my research, the focus was on a single activity system (that is, the collaborative intervention).

8.4.1. Primary contradiction – conceptual tools

Primary contradictions are tensions that occur within a component of an activity system. They arise from the dual construction of the components of the activity system. During collaboratively planned lessons 1 and 2, Lerato and Doreen, who had volunteered to teach those lessons, had lessons in mind before our planning meeting. It is these lessons that they chose to implement in their classrooms. The contradiction occurs amongst the subjects in relation to the collaborative engagement in an activity.

As noted in Chapter 7, the team members made a few suggestions in the planning sessions for collaboratively planned lessons 1 and 2. In collaboratively planned lesson 1, for example, it was suggested that Lerato limit the counting activities during mental maths due to time constraints and include an estimation activity as an introduction to concept development. However, Lerato included several counting activities that did not decrease the time spent on mental maths. In the collaboratively planned lesson 2, Doreen diverted from the suggestion that counting should be done in groups and from arbitrary numbers. The contradiction is between the conceptual tools suggested collaboratively and conceptual tools implemented by subjects individually during lesson implementation. Each teacher taught the lesson they had planned and included some of the additional strategies and ideas suggested by colleagues in the collaborative planning session. This shows that although teachers perceived collaboration as beneficial, they stick to what they know because collaboration is a skill that requires frequent practice.

The primary contradictions occurred within the subjects where the subjects pre-planned collaborative and also within tools by sticking to what they know and omitting to utilise collaboratively planned strategies. These primary contradictions indicated opportunities for developing collaborative engagement of the subjects in planning the lessons and implementing the conceptual tools as discussed collaboratively to successfully achieve the object.

8.4.2. Secondary contradictions

In addition to the primary contradictions of the activity system, there were also secondary contradictions. Secondary contradictions refer to tensions between two components of the activity system. Contradictions occurred in all three collaborative lessons between the subjects and object, the conceptual tools and the object, and the community and outcome.

8.4.2.1. Subjects and object

This research aims to strengthen teachers' mathematics and pedagogical content knowledge utilising a collaborative intervention. The achievement of the object is oriented by collaborative planning and reflection activities. However, Doreen and Lerato chose to pre-plan their lessons. This limited the contribution of the other subjects to the lesson. This contradiction is not only a primary contradiction, as highlighted above. It is also a secondary contradiction because it occurred between the subjects and the object.

The subjects' actions, that is, the limited contribution of views and strategies by the subjects affected the achievement of the object because all subjects were not collectively engaged in the planning process. In addition to Doreen and Lerato, Gladys also added to this contradiction. She diverted from the collaboratively planned lesson and added her ideas after we had developed the lesson together.

8.4.2.2. Conceptual tools and community

The learners' seating in 'ability' groups reflects that teachers are aware of different learning abilities in their classrooms. Taking this into consideration, during the reflection on her lesson, Lerato expressed that she needs assistance with catering for 'struggling' learners. Doreen suggested that teachers can "**call them individually to the table and work with them individually"** and Gladys suggested "**that having lots of aids, simple things that can help them to be able to learn things that they don't understand".** However, these suggestions were not incorporated into Lerato's lesson. This contradiction affected the community because the teachers possessed some knowledge to cater for 'struggling' learners in their classroom, but the advice given from these teachers was not taken into consideration. All the learners were involved in the same learning activities and the' struggling' learners were seemingly left unsupported.

The contradiction between the conceptual tools and the community also affected the achievement of the object of the activity. If differentiated activities had been included in the lesson plan and implementation thereof, the strategies' success or failure could have been considered during the reflection sessions. These actions could have given Lerato some experience in catering for 'struggling' learners and developing her mathematics and pedagogical content knowledge to support her 'struggling' learners. Edwards (2005) explains that the development of activity theory aims to change systems by provoking the collective to reinterpret the object, which results in repositioning the object and subjects. Gladys suggested that "**officials"** can assist them in lessons that cater for 'struggling' learners. Notably, the composition of the subjects needs to include a 'More

Knowledgeable Other' (Vygotsky, 1978) to assist in planning inclusive lessons. This will result in the transformation of some of the components for instance the planning process where the teachers will "**plan according to the 'abilities'"** (Doreen) of the learners.

8.4.2.3. Conceptual tools and outcome

In Chapter 3, I explained that Foundation Knowledge informs everything that a teacher does in the classroom, including planning for and delivering in the classroom (Transformation Knowledge), making connections across topics and concepts (Connection Knowledge) and responsive competence (Contingency Knowledge). However, during all three lessons, there was seemingly a lack of implementation of Contingency Knowledge. The teachers stuck to the lesson plan and missed opportunities to deviate and allow learners to explain their solutions to the tasks or ask questions. This would have allowed learners to identify their mistakes and misconceptions and exhibit their conceptual understanding. In so doing, the teachers would have the opportunity to examine whether learning is taking place and identify areas in their planning that need further attention.

The secondary contradictions revealed that subjects need opportunities to engage in collaborative activities to develop their collaborative skills. Also, they need to engage in 'transformative actions' where they effectively implement the knowledge they already possess while, at the same time, examining the components of the activity through reflection. In this way, they can find new creative ways to redefine and reconfigure the object of the activity system.

Table 8.1 provides a summary of the primary and secondary contradictions that emerged during the collaborative intervention.

	Collaboratively planned lesson 1	**Collaboratively planned lesson 2**	**Collaboratively planned lesson 3**
Primary Contradictions	Subjects • Collaborative engagement- Pre-planned lesson limits collaboration	Subjects • Collaborative engagement-Pre-planned lesson limits collaboration	
Secondary Contradictions	Subjects and object • Limited implementation of reflection suggestions (limited collaboration skill by pre-planning lesson) Conceptual tools and outcome and community • Lack of Contingency Knowledge employed in the lessons	Subjects and object • Limited implementation of reflection suggestions (limiting collaboration inputs by pre-planning lessons) Conceptual tools and outcome • Lack of including activities for 'struggling learners Conceptual tools and outcome and community • Lack of Contingency Knowledge employed in the lessons	Subjects and object • Deviating from the collaboratively planned lesson (Gladys' collaborative lesson) Conceptual tools and outcome • Lack of inclusive activities for 'struggling learners Conceptual tools and community • Limited employment of Contingency Knowledge to develop concepts

Table 8.1: Summary of primary and secondary contradictions

According to Hancock and Miller (2017), contradictions provides an opportunity to design purposeful change. Figure 8.2 summarizes the contradictions identified above, shown by the bold lines on the diagram.

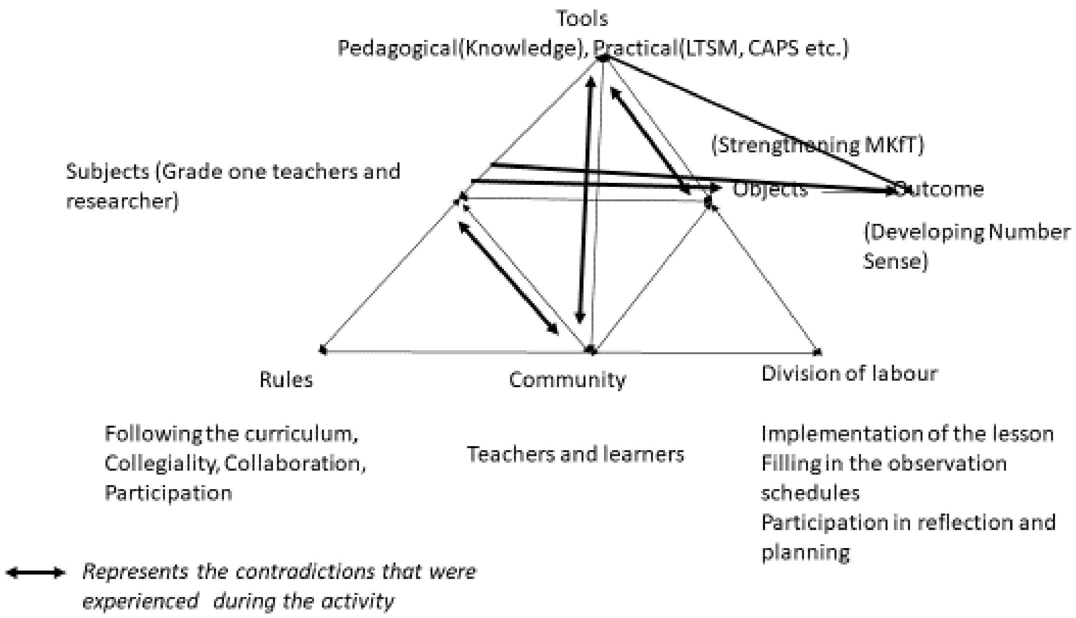


Figure 8.2: Representation of contradictions within the activity system

8.5. SUMMARY OF THE CHAT PRINCIPLES IN RELATION TO MY RESEARCH

I summarised the linkage between the four principles of the activity system using Engeström's (2001) four questions of project analysis in the Table 8.2. below.

	Activity system as a unit of analysis	**Multi-voicedness**	**Historicity**	**Contradictions**
Who is learning?	The Grade One teachers and I are the *subjects* (i.e., the participants in the research). Learners and teachers as the *community*	The voices of the Grade One teachers and me	The three Grade One teachers with teaching experiences that range from 3 -30 years (See Chapter 5)	The teachers by implementing suggestions of reflection and planning sessions (object) Learners through explicit explanations and inclusive activities (outcome)

Why do they learn?	The intention is to achieve the *object*, that is, strengthening our mathematics and pedagogical content knowledge	To develop interpersonal collaboration to improve our knowledge and practice	To improve lesson planning and implementation thereof to develop learners' number sense. Teachers pre-plan lessons because they prefer to 'stick' to what they know	Discard working in isolation and plan lessons collaboratively to strengthen mathematics and pedagogical content knowledge Improve practice through a collaborative intervention Develop learner number sense
What do they learn?	To improve our knowledge and practice on how to develop number sense	They engage in a collaborative intervention to plan, implement and reflect on their practice	Collaboration can promote a change in our practice	Limited application of collectively initiated strategies to make lessons fun and inclusive while meaningful learning is taking place Strategies to incorporate FNS characteristics in their lessons Develop learners' number sense
How do they learn?	We learn through the process of observation, and discussion during the reflection and planning stages of the collaborative intervention.	This emerges through the collaborative planning and reflection discussions	Engaging in a collaborative intervention where we plan, implement, observe and reflect on lessons	Implementing suggestions from reflection sessions in our lessons Planning collaboratively by sharing teaching and learning strategies and activities

Table 8.2: A summary of the principles of CHAT as reflected in this research

8.6. CONCLUSION

This chapter focused on the research question: *How does a collaborative intervention enable or constrain the development of mathematics and pedagogical content knowledge required to develop learners' number sense?* I began this chapter with an analysis of the teachers' perspectives of engaging in a collaborative intervention. Four themes emerged: collegial development; building and sustaining a professional learning community; pedagogical transformation; and developing inclusive lessons. These themes highlight the benefits and the challenges the teachers experienced during the collaborative intervention.

As this research is underpinned by second generation CHAT, I moved on to analyse the activity system, that is, the collaborative intervention. I used Hancock and Miller's (2017) three CHAT-related elements of analysis the activity system, namely identifying the activity system, appropriation of tools and discovery of contradictions. I also explained how these elements incorporate the principles of the activity system by Engeström (2001) which I summarised in Table 8.2. above.

Having presented and analysed the data, it is evident that there are some benefits that the teachers experienced by engaging in the collaborative intervention. There was the development of conceptual tools applied during the lesson implementations, observations, planning and reflection sessions. The teachers observed and shared teaching strategies during these sessions and further developed their existing mathematics and pedagogical content knowledge. Through collaborative reflection the teachers realised that catering for 'struggling' learners is a challenge for them. They suggested a few strategies as solutions to the challenge, and further realised the need for external support in this matter. The identification of the challenges creates opportunities for further learning.

The analysis of the activity system as a unit highlighted the interaction of the components of the activity system and how they impacted on the attainment of the object and the outcome. The subjects mediated conceptual and practical tools to achieve the object. The subjects adhered to the rules of the activity system while they took part different activities, such as planning, implementing, observing and reflecting. Analysing the activity system revealed some of the shortcomings that impacted on the attainment of the object and outcome. These shortcomings are the primary and secondary contradictions that occurred.

Several contradictions emerged during the activity system. These contradictions occurred in various components of the activity system such as subjects, conceptual tools, object, outcome, community and rules. The teachers pre-planned and diverted from the collaborative lessons and this limited the contribution of the subjects, the results of collaborative engagement and also impacted on the object of the activity. The lack of implementation of the teachers’ suggestions on how to cater for struggling learners impacted on the engagement of some learners in the lesson. The lack of implementation of Contingency Knowledge and catering for "struggling" learners constrained the achievement of the outcome. Even though the contradiction constrained the object and outcome of the activity, they also created an opportunity for expansive learning by developing multiple activity systems.

CHAPTER NINE: CONCLUSION

9.1. OVERVIEW OF THE RESEARCH

In Chapter One, I locate my research within the context of leaner underperformance. I show that both national and international benchmarking assessments attest to the fact that learners are underperforming in mathematics. I draw on Spaull (2013) and Hoadley (2012) to argue that teachers do not have the required content and pedagogical content knowledge to teach mathematics. One of the explanations for this is that the teacher education system is not preparing teachers with the required knowledge to teach mathematics in a manner that promotes the achievement of the necessary mathematics learning outcomes. Based on research, I show that professional development practices for in-service teachers in South Africa are primarily based on workshops that offer a one-size-fits-all approach and suggest an approach that gives teachers agency of their own professional development. In so doing, I argue that a professional development approach based on a collaborative intervention may support teachers in developing the content and pedagogical content knowledge required to teach mathematics.

In Chapter Two, I illustrate that my research is underpinned by second generation CHAT. I show that CHAT can be used to facilitate and support collaborative learning and also to analyse the complexities of classroom activities. Research (Johnston, 2009; Ono & Ferreira, 2010 and Bertram, 2011) shows that although teachers attend in-service training workshops, at school they tend to work in isolation. I draw on Leont'iev and Engeström's second generation CHAT to argue that it is a challenge for teachers to develop their mathematics and pedagogical content knowledge when working in isolation. I therefore propose that a collective model of an activity system be utilised. I argue teacher's engagement in a collaborative intervention affords them opportunity to learn collaboratively about their practice.

In Chapter Three, I looked at the object of my research, that is, the knowledge Grade One teachers need to teach mathematics. Research suggests that teachers' lack of content and pedagogical content knowledge is one of the factors the contributes to poor learner achievement. Drawing on Shulman, (1986, 1987) Ball et al. (2008) and Rowland et al. (2007, 2011), I showed how research on teachers' mathematics and pedagogical content knowledge has shifted over the past three decades and how each of the aforementioned researchers build on the work of the other(s). In so doing, I highlighted the theoretical framework for my research.

In Chapter Four, I turned to the outcome of the research, that is, the development of number sense. According to research, many learners in South Africa lack foundational knowledge. I drew on Andrews and Sayers (2015), who argued that children with weaker foundations are prone to perform poorly in mathematics as they progress through the schooling system. An explanation for this is that, children begin school with varying degrees of foundational numeracy. Based on this research I suggested that by engaging in a collaborative intervention, the participants in my research can develop their mathematics and pedagogical knowledge necessary to develop learners number sense.

In Chapter Five, I focused on the design of the research. This is a qualitative case study underpinned by the interpretivist orientation. I discussed the participants, site and my positionality as a participant and researcher. To collect data, I used interviews, observations and collaborative reflections. Ethical procedures are followed from the university, district director, principal, teachers and parents. Validity is ensured by using three methods of data collection and member checking is also applied. The data was analysed using both emic and etic approaches.

In Chapters Six and Seven, I presented and analysed the data. Chaper 6 focused on the data generated prior to the collaborative intervention, that is, the initial interview and individually planned lessons. The initial interview focused on the teachers' understanding of number sense. Each of the teachers' individually planned lessons are then analysed using Ball et al's. (2008) MKfT, particularly PCK (the object of the research), and Sayers and Andrews (2015) characteristics of FNS (the outcome of the research). The focus of Chapter Seven was the collaboratively planned lessons. I showed how the teachers engaged in the collaborative process of planning, teaching and observing, and reflecting on each of the lessons taught. The analysis focused on the characteristics of FNS and MKfT. At the end of the chapter, I drew on Rowland et al's. (2007, 2011) Knowledge Quartet to analyse the knowledge demonstrated in teaching in both the individually and collaboratively planned lessons.

The focus in Chapter Eight is the teachers' perspectives of the collaborative intervention. I used CHAT as framework to analyse the activity system as a unit, the appropriation of both practical and conceptual tools and the contradictions experienced.

9.2. KEY FINDINGS

In Chapter Three, I argued that Foundation Knowledge incorporates the knowledge proposed by Shulman (PCK) and Ball and colleagues (SMK & PCK). This knowledge informs the manner in which teachers plan and act in the classroom. It includes content knowledge, pedagogical content knowledge, curriculum knowledge and beliefs (Rowland & Turner, 2007). This research exhibited the following findings.

Teachers appeared to have the required subject matter knowledge

While SMK is important, it was never an issue in this research, probably because the work was at Grade 1 level. The focus was always (in the planning and reflections) on PCK.

Techers had limited knowledge of number sense and how to develop learners' numbers sense

It was evident in the initial interview that teachers had a limited knowledge on number sense and the strategies to develop number sense. Their collective responses to the question on number sense focused on the number range learners were required to work with, linking quantity to number through on-to-one correspondence and systematic counting. Despite the teachers articulating a narrow conception of number sense during the interview, their planned lessons were indicative of a broader conception of number sense. It might be that the teachers were not familiar with the components of number sense. Likewise, the strategies for developing learners' number sense seemed restricted in the initial interview when compared with their lessons. The teachers focused on using manipulatives, playing games and working closely with learners who need support.

Teachers found the opportunity to work collaboratively beneficial

Focusing on CHAT, we established a collaborative intervention where we engaged in collaborative planning and reflection to develop mathematics and pedagogical content knowledge. In the beginning of the research the teachers experienced challenges with expressing their views during collaborative planning and reflection. My finding is that teachers' refection skills needs to be developed. Lerato and Doreen pre-planned their collaborative lessons and dominated the collaborative planning session. This showed that teachers tend to stick to what they know because of always working in isolation. During the planning of the third collaborative lesson, the teachers started to open up and collaborate in planning the lesson. Therefore, collaboration needs to be afforded time and included in the school programme for teachers to engage and develop by learning from each other.

Teachers' found it difficult to deviate from the planned lesson

The lack of implementation of Contingency Knowledge inhibited the development of number sense. The teachers incorporated most of the Foundational Number Sense characteristics (Andrews & Sayers, 2015). Teachers did not divert from the lesson plan and make use of opportunities to ask questions and enable learners to explain their reasoning and share their understanding.

Teachers' Foundation Knowledge did not develop as anticipated during the collaborative intervention

Apart from the inclusion of estimation in the collaboratively planned lessons, there was no real shift in teachers' Foundation Knowledge during the collaborative intervention. The shift to including estimation activities during the collaboration was prompted by me in the first collaboratively planned intervention. I explained the value of estimation in the Intermediate Phase to the teachers. This begs the question as to whether such collaborative interventions require a 'More Knowledgeable Other' (Vygotsky, 1978) to assist in shifting teachers' knowledge and practices.

The teachers raised concerns that could lead to 3rd generation CHAT

The research is underpinned by the second generation CHAT because it focused on a single activity system. However, it emerged that the research can develop into third generation CHAT with multiple activity systems. Contradictions that were experienced and inhibited the successful achievement of the object and the outcome of the activity can be used to develop multiple activity systems with reinterpreted objects. For example, teachers mentioned that they found it difficult to cater for 'struggling' learners. The collaborative intervention can lead to another activity system with a reinterpreted object that focuses on employing inclusive strategies to develop learners' number sense. The development to third generation needs extension of the research duration and might result in expansive learning which will strengthen the teachers' mathematics and pedagogical content knowledge to develop number sense.

9.3. KEY INSIGHTS

It was evident in my research that teachers can improve their practice when they engage in collaborative interventions, where they observe each other teaching, reflect on their observations and share different teaching strategies, methods and resources. However, the teachers are reliant on their collective Foundational Knowledge to plan, teach and reflect. To develop the teachers Foundation Knowledge further may require a ‘More Knowledgeable Other’ (Vygotsky, 1978) to be included as a member of the collaborative intervention. For example, it was only when I noted that the learners should be given an opportunity to count from arbitrary numbers that changes to their practice began to emerge. These changes however, where limited to the suggestions I made.

Teachers (in service and preservice) also need to be thoroughly trained to teach in groups and differentiate the lessons. While the learners in this research were seated in ‘ability’ groups, the teaching and learning activities were the same for learners and teachers did not even implement some of the strategies that they knew. During the phase meetings and district workshops, reflection on practice should be included on the agenda so that teachers can learn to share challenges and solutions to the challenges they experience when teaching mathematics.

While there were some difference as to the components of FNS developed in the collaboratively planned lessons when compared to the initial lessons, it appears, on reflection, that we made some assumptions that the learners were indeed developing an understanding of these components, and were in fact, developing their number sense. According to Andrews & Sayers (2015), FNS is the number sense that starts to develop prior to school, but that needs to be consolidated through instruction during the first year of formal schooling. Tsao & Lin (2012) stress that teachers play an important role in building number sense in the type of classroom they create, the teaching practices they employ and the activities they select. While this is not the focus of the thesis, I am left with the question: Does number sense develop based on the implementation of the different components of FNS in the lesson(s) or does it need to be explicitly taught?

9.4. LIMITATIONS

The findings of this research have to be seen in the light of some limitations.

Teachers are not familiar with planning collaboratively

The first limitation is the limited collaborative knowledge exhibited by the teachers. We established a collaborative intervention to plan and reflect collaboratively about each other's lessons. However, during the reflection session after the individually planned lessons, the teachers found it difficult to express their views and opinions about the lessons. During the collaborative intervention, two of the teachers pre-planned their lessons ahead of the collaborative planning sessions thereby limiting the other members' inputs in the lesson plan. Also, the third collaborative lesson was changed, as a result the teachers could not realise whether the strategies they suggested were plausible or not.

Teachers' limited transformation, connection and contingency knowledge

The second limitation is the teachers' limited Foundational Knowledge which informs the Transformation, Connection and Contingency Knowledge. The limited Foundation Knowledge seems to suggest that teachers need support of a "More Knowledgeable Other" to assist them with strategies of catering for 'struggling learners'. The teachers showed lack of Contingency Knowledge which limited the learners input and questions during the lesson and affected the achievement of the outcome. Moreover, the lack of Contingency Knowledge limited the incorporation of FNS characteristics while inhibiting the development of number sense.

Time for the collaborative intervention and research

The third limitation of the research was time. The duration of our lessons should have been one hour, but they took longer. As a result, we were not able to observe how learners performed in the consolidation activities. In addition, the duration of the research was limited to show the achievement of the object. During the planning of the third collaborative lesson teachers started to be comfortable with collaboration and had made some changes in their lessons. However, this does not fully reveal the strengthening of the mathematical and pedagogical content knowledge.

9.5. RECOMMENDATIONS

After engaging in this research I recommend that there needs to be more research of teacher's mathematics and pedagogical content knowledge in the foundation phase in South Africa. The research needs to be conducted in collaboration with both pre- and in-service teachers, district officials and subject specialists and teacher educators.

Professional Learning Communities (PLC) should be created where teachers are afforded opportunities to collaboratively share challenges, strategies, and with permission, observe each other's practice. Instead of focusing mostly on policy during subject meetings in schools, subject heads should encourage collaborative talk on lesson planning, implementation and learner performance.

9.6. SHORT REFLECTIVE PIECE

This research has transformed me in many respects. It has made me aware that transformation in practice is possible, however it needs time, planning and consistency. Observing the three Grade One teachers in their classroom, the knowledge, methods and strategies they apply and the challenges they encounter, has brought light to the challenges I have experienced with my learners as a Grade Three teacher previously, and currently as a Grade Four mathematics teacher. Using CHAT to conduct this research, I have learned that it takes the development of more than one activity system and time to bring transformation in practice. As a newly appointed Head of Department and leader in the PLC at my school, I believe that using collaborative intervention can be an effective model for development to utilise in my subject and within the school.

REFERENCES

Adler, J. (2005). Mathematics for teaching: What is it and why is it important we talk about it? *Pythagoras,* 2-11.

Adler, J., Pournara, C., Taylor, D., Thorne, B., & Moletsane, G. (2009). Mathematics and science teacher education in South Africa: a review of research, policy and practice in times of change. *African Journal of Research in Mathematics, Science and Technology Education*, 13 (Special Issue), 28-46.

Andrews, P., & Sayers, J. (2015). Identifying opportunities for grade one children to acquire foundational number sense: Developing a framework for cross classroom analyses. *Early Childhood Education Journal*, 43,1-17.

Andrews, P., Sayers, J., & Marschall, G. (2015). Developing Foundational Number Sense: Number line examples from Poland and Russia. In Krainer, K., & Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education,* (pp.1681-1687). Prague: Charles University, Faculty of Education and ERME

Aunio, P., Monone, R., Ragpot, L., & Tormanen, M. (2016). Early numeracy performance of South African school beginners. *South African Journal of Childhood Education*, 6(1), 1-8.

Ball, D. L. (2000). Bridging practices. Intertwining content and pedagogy in teaching and learning to teach. *Journal of Teacher Education*. 51(3), 241-247.

Ball, D. L.; Thames, M. H., & Phelps, (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.

Berch, D. B. (2005). Making sense of number sense: implications for children with mathematical disabilities. *Journal of Learning Disabilities*, 38(4), 333-339.

Bertram, C. (2011). What does research say about teacher learning and teacher knowledge? Implications for professional development in South Africa. *Journal of Education,* 52, 3-26.

Bertram, C. & Christiansen, I. (2014). *Understanding research: an introduction to reading research*. Pretoria: Van Schaik

Boggan, M., Harper, S., & Whitmire, A. (2010). Using manipulatives to teach elementary mathematics. *Journal of Instructional Pedagogies,* 3, 1-6.

Botha, L. R. (2012). Using expansive learning to include indigenous knowledge. *International Journal of Inclusive Education,* 16(1), 57-70.

Brenner, M. E. (2006). Handbook of complementary methods in education: interviewing in Educational Research. London: Lawrence Erlbaum Associates

Briand-Newman, H., Wong, M., & Evans, D. (2012). Teacher subject matter knowledge of number Sense. In Dindyal, L., Cheng, L. P., & Ng, S. F. (Eds.), Mathematics Education: Expanding horizons. *Proceedings of the 35th Annual Conference of the Mathematics Education Research Group of Australia* (pp.130-137). Singapore: Mathematics Education Research Group of Australasia

Butterworth, B. (2005). The development of arithmetic disabilities. *The Journal of Child Psychology and Psychiatry,* 46(1), 3-18.

Chapman, O. (2012). Challenges in mathematics teacher education. *Journal Mathematics Teacher Education*, 15, 263-270.

Chikiwa, S., Westaway, L., & Graven, M. (2017). Learning from practice: What mathematics knowledge is needed for developing number sense? In T. Penlington & C. Chikiwa (Eds.), *Proceedings of the 23rd Annual National Congress of the Association for Mathematics Education of South Africa* (pp.55-67). Port Elizabeth: AMESA

Chikiwa, S., Westaway, L., & Graven, M. (2019). What mathematics knowledge for teaching is used by a Grade 2 teacher when teaching counting? *South African Journal of Childhood Education*, 9(1), a567. https://doiorg/104102.sajce.v9i1.567

Creswell, J.W. (2013). Qualitative inquiry and research design: choosing among five approaches. Los Angeles: SAGE

Cohen, L., Manion, L. & Morrison, K. (2007). *Research Methods in Education* (6th ed). London: Routledge

Cohen, L. Manion, L. & Morrison, K. (2018). *Research Methods in Education* (8th ed). London: Routledge

De Clercq, F & Phiri, R. (2013). The challenges of school-based teacher development initiatives in South Africa and the potential of cluster teaching. *Perspectives in Education*, 31(1), 77-86.

Du Four, R. (2004). Leading edge: The best staff development is in a workplace not in a workshop. *Journal of Staff Development*, (25)2, 1-3

Dehaene, S. (2001). Precis of the Number Sense. *Mind and Language*, 16(1), 16-36.

Edwards, A. (2005, November). Cultural Historical Activity Theory and learning, a relational turn. *Paper presented at the 6th Annual conference of the Teaching and Learning Research Programme*. University of Warwick: Warwick

Edwards, R., Hyde, R., O'Connor, M., & Oldham, J. (2015). The importance of subject knowledge for mathematics teaching: An analysis of feedback from subject knowledge enhancement courses. In Adams, G, (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 35(3), 37-42.

Engeström, Y. (2001). Expansive learning at work: towards an activity theoretical reconceptualisation. *Journal of Education and Work*, 14(1), 133-156.

Feldman, A., & Weiss, T. (2010). Understanding change in teachers' ways of being collaborative action research: A cultural historical activity theory analysis. *Educational Action Research*, 18(1), 29- 55.

Feza N. N. (2012). Can we afford to wait any longer? Pre schoolchildren are ready to learn Mathematics. *Early Childhood Education Journal*, 2(2), 58-7

Foot, K. A. (2014). Cultural-Historical Activity Theory: Exploring a Theory to Inform Practice and Research. *The Journal of Human Behaviour in Social Environments,* 24(3), 329-347.

Graven, M. H. (2014). Poverty, inequality and mathematics performance: the case of South Africa's post-apartheid context. *ZDM Mathematics Education.* 46, 1039-1049

Graven, M.H., & Venkat, H. (2017). Advocating linked research and development in the primary mathematics landscape in contexts of poverty. In M. Graven & H, Venkat (Eds.), *Improving primary mathematics education, teaching and learning* (pp.11-23). London, Palgrave Macmillan.

Hamilton, L., & Corbett-Whittier, C. (2012). *Using case study in education research*. London: SAGE

Hancock, C.L., & Miller, A.L. (2017). Using cultural historical activity theory to uncover praxis for inclusive education. *International Journal of Inclusive Education,* 22(9), 937 -953.

Hill, H.C., Rowan, B. & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research*, 42(2), 371-406.

Hoadley, U. (2010). What do we know about teaching and learning in primary schools in South Africa? A review of the classroom-based research literature. Cape Town: WCED

Howden, H. (1989). Teaching Number Sense. *National Council of Mathematics Teachers,* 36(6), 6-11.

Jankvist, U.T., Mosvold, R., & Clark, K. (2016). *Mathematical knowledge for teaching teachers: the case of history in mathematics education*. Retrieved on 08 December 2019 from https://hal.archives-ouvertes.fr/hal-01349261/document

Johnston, B. (2009). Collaborative teacher development. In A. Burns & J.C. Richards, J. C. (Eds.), *The Cambridge guide to second language teacher education* (pp. 241-249). Cambridge: Cambridge University Press

Jordan, N. (2007). The need for number sense. Journal of the Department of Supervision and Curriculum Development, 65(2), 63-66.

Klibanoff, R.S., Levine, S.C., Huttenlocher, J., Vasilyeva, M., & Hedges, L.V. (2006). Preschool children's mathematical knowledge: The effect of "Math Talk." *Developmental Psychology*, 42(1), 59-69.

Koszalka, T. A. & Wu, C. P. (June, 2005). A Cultural Historical Activity Theory (CHAT). Analysis of Technology Integration: Case Study of two teachers. *Paper presented at Association for Educational Communication and Technology Annual Meeting*. Retrieved 23 June 2020 from https://wwwlearntechlib.org/76944/.

Kvale, S., & Brinkman, S. (2009). Interviews: Learning the craft of qualitative research interviewing. Thousand Oaks, California: SAGE

Leutzinger, L. P., Rathnell, E. C., & Urbatsch, T.D. (1986). Developing estimation skills in the primary grade. *National Council of Teachers of Mathematics*. Reston: NCTM

Mason, J. (2002). *Qualitative Researching* (2^{nd} ed.). London: SAGE

McGuire, P., Kinzie, M.B., & Berch, D.B. (2012). Developing number sense in Pre-K with five frames. *Early Childhood Education*, 40, 213-

McIntosh, A.; Reys, B. J. & Reys, R. E. (1992). A proposed framework for examining basic Number Sense. *For the Learning Mathematics*, 12(3), 2-8,44.

Modisaotsile, B. M. (2012*). The failing standard of basic education in South Africa.* (Policy Briefing no.72). Pretoria: African Institute of South Africa

Merriam, S. (1998). Qualitative research and case study applications in education. San Francisco: Jossey-Bass

Merriam, S. B. (2009). Qualitative research: a guide to design and implementation. San Francisco: Jossey-Bass

Minnett, A. M. (2003). Collaboration and shared reflections in the classroom. *Teachers and Teaching Theory and Practice*, 9(3), 279-285.

Ono, Y. & Ferreira, J. (2010). A case study of continuing teacher professional development through lesson study in South Africa. *South African Journal of Education*, 30, 59-74.

Pendlebury, S. (2009). Meaningful access to basic education. In S. Pendlebury, L. Lake & C. Smith (Eds.). *South African Child Gauge 2008/2009* (pp. 24-29). Cape Town: Children's Institute.

Petrou, M. & Goulding, M. (2011). Conceptualising teachers' mathematical knowledge in teaching. In: Rowlands, T. & Ruthven, K. (eds). *Mathematical Knowledge in Teaching*. Dordrecht: Springer

Pyrko, I., Dorfler, V. & Eden, C. (2017). What makes Communities of Practice work? *Human Relations*, 70(4), 389-409.

"Quality of maths teaching is poor," says Motshekga (2016, December 12). *The Citizen.* Retrieved on 8 August 2018 from https://www.google.com/amps/citizen.co.za/news/south-africa/1373191/maths-teaching-poor-quality-says/amp/

Roth, W.M., Radford, L. & La Croix, L. (2012). Working with Cultural Historical Activity Theory. *Forum: Qualitative Social Research*, 13(2), 1-20, Art 23

Roth, W. M. (2012). Cultural-historical activity theory: Vygotsky's forgotten and suppressed legacy and its implication for mathematics education. *Mathematics Education Research Journal*, 24, 87-104.

Rowlands, T. & Turner, F. (2007). Developing and using the knowledge: a framework for the observation of the mathematics teaching. *The Mathematics Educator*, 10(1), 107-123.

Rowland, T. & Ruthven, K. (2011*). Mathematical Knowledge in Teaching.* Springer: Dordrecht

Sa'ad, T. U., Adamu, A., Sadiq, A.M. (2014). The causes of poor performance in mathematics among public senior secondary students in Azare Metropolis of Bauchi State, Nigeria. *Journal of Research and Method in Education*, 4(6), 32-40.

Sawyer, R. D. (2001). Teacher decision-making as fulcrum for teacher development exploring structures of growth. *Teacher Development*, 5(1), 39-58.

Sawyers, R. D. (2002). Situating teacher development: The views from two teachers' perspectives. *Journal of Educational Research*, 37, 733-753.

Sayers, J. & Andrews, P. (2015). Foundational number sense: Summarising the development of an analytical framework. *Proceedings of the 9th Congress of the European Society for Research in Mathematics Education*. Prague (pp. 361-367). Czech Republic: HAL

Sayers, J. and Andrews, P. (2015). Foundational number sense: The basis for whole number arithmetic competences. *ICMI Topic Study Group 23: Primary study on whole numbers,* Macao, China: ICMI.

Sayers, J., Marschall, G., Joran, P & Andrews, P. (2019). English and Swedish teacher's perspectives on the role of parents in year one children's learning of number manifestation of culturally conditioned norms. *Early Child Development and Care,* 1-14. DOI: 10.1080/03004430.2019.1646741

Schoenfeld, A.H. (2002). Making mathematics work for all children: issues of standards, testing and equity. *Educational Researcher*, 31(1), 13-25.

Shulman, L. S. (1986). Those who understand: knowledge growth in teaching. *Educational Research*, 15(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: foundations of the new reform. *Harvard Educational Review*, 57(1), 1-22.

South African. Department of Basic Education (SA.DBE). (2011). *Curriculum and Assessment Standards: Mathematics (Grade 1-3*). Pretoria. DBE

Spaull, N. (2013). South Africa's Education crisis: the quality of education in South Africa 1994-2011. Johannesburg: Centre for Development and Enterprise

Spaull, N. and Kotze, J. (2015). Starting behind and staying behind in South Africa: The case of insurmountable learning deficits in mathematics. *Journal of Educational Development*, 41, 13-24.

Spelke, E. S. (2000). Core knowledge. *American Psychologist*, 55(11), 1233-1243.

Stake, R. E. (1995*). The art of case study research.* Thousand Oaks, California. SAGE

Taylor, N. (2008). *What's wrong with South African schools?* Johannesburg: JET Education Services

Tsao, Y. & Lin, Y. (2012). Elementary school teachers understanding towards related knowledge of number sense. *US-China Education Review,* 17-30.

Van der Berg, S., Taylor, S., Gustafsson, M., Spaull, N., & Armstrong, P. (2011). *Improving Education Quality in South Africa*. Report for National Planning Commission, RESEP, University of Stellenbosch. Stellenbosch

Venkat, H. & Spaull, N. (2015). What do we know about primary teachers'mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Education Development,* 121-130.

Vygotsky, L. (1978). Mind in society: the development of higher psychological processes. Massachusetts: Harvard University Press.

Wahyuni, D. (2012). The research design maze: understanding paradigms, case, methods and methodologies. *Journal of Applied Management of Accounting Research,* 10(1), 69-80.

Westaway, L. & Graven, M. (2019). Exploring grade 3 teachers' resistance to 'take up' progressive mathematics teaching roles. *Mathematics Education Research Journal* 31(1), 27-46.

Wilson, V. (2014). Examining teacher education through cultural historical activity theory. *Teacher Education Advancement,* 6(1), 20 - 29.

Yamagata-Lynch, L.C., & Haudenschild, M.T. (2009). Using activity system analysis to identify inner contradictions in teacher professional development. *Teaching and Teacher Education,* 25, 507- 517.

Yamagata-Lynch, L.C. (2010). Activity system analysis methods: understanding complex. learning environments. In *Activity System Analysis Methods*. (pp.13-26). Boston.: Springer

Yamazumi, K. (2006). Activity theory and the transformation of pedagogic practices. *Educational Studies in Japan,* 1, 77 - 90.

Yin, R. K. (2009). *Case study research design and methods*. Thousand Oaks, California: SAGE

APPENDICES

APPENDIX 1

INTERVIEW SCHEDULE

These interviews were conducted before the classroom lesson presentation and the following questions were asked.

Phase 1 (Before classroom observation and lesson presentation)

1. How long have you been a teacher, particularly mathematics teacher?
2. What are your views about mathematics as a school subject?
3. What is your understanding of number sense?
4. What are the methods you might use to develop learners' number sense?
5. How do you adjust your lessons to clarify learners' confusion about number sense?

APPENDIX 2

INTERVIEW SCHEDULE

Phase 2 These interviews followed the procedure after the classroom observations, lesson presentations and reflections. The following questions were asked:

1. Tell me what being a member of the collaborative intervention has meant for you?
2. What are your teaching strengths? How do you think you can build on that?
3. What could be your weakness and how might you improve on it?
4. How do you find new ideas for lesson?
6. Which methods do you think works well for your learners?
7. What benefits or challenges, have you experienced by engaging in the collaborative intervention?
8. What did you learn about developing learners' number sense through your involvement in a collaborative intervention?
9. How do you determine if learners have learned?
10. How would you engage learners who are experiencing mathematical difficulties?
11. Based on your experience, how do you think collaborative intervention can be utilised to develop teacher knowledge?
12. Are there any improvements we can make to sustain our participation in a collaborative intervention?

APPENDIX 3A

Foundational Number Sense Framework as analytical tool

FNS characteristics	Teachers encourage learners to:	Observers comments
Number recognition	❖ Identify a particular number symbol from a collection of number symbols and name when shown its symbol	
Systematic counting	❖ Count systematically both forwards and backwards and from arbitrary starting points between 1-100. Count in 2s, 5s, 10s	
Relating number to quantity	❖ Understands the one-to - one correspondence between a number's name and the quantity it represents	
Quantity discrimination	❖ Compare magnitudes and deploy language like bigger than or smaller than	
Different representation	❖ Recognise, work with and make connection between different representation of number, using cards, circles and numbers	

Estimation	❖ Estimate whether it be the size of a set or an object	
Simple arithmetic	❖ Perform simple addition and subtraction operations to make the bonds and solve problems	
Number patterns	❖ Recognises and extend number patterns and, in particular, identify missing number	

(Adapted from Andrews and Sayers, 2015, p.7)

APPENDIX 3B

Observation Schedule

Observation schedules were adapted according to each of the collaborative lesson.

	Observation Notes
How did the learners respond to the introduction of the number 15 activities?	
How did the learners respond to the teachers' explanations of how to use the number-line, coloured circles and dominoes for building 15?	
How did the learners use the number-line, coloured circles and number grids when calculating on their own?	
How did the teacher's response to learner errors support the learners use of the number-line, coloured circles and number grids?	
How did the teacher adapt the lesson to support the development of learners' use of the number-line, coloured circles and number grids to calculate and build the number 15?	

APPENDIX 4

Examples of the lesson plans

These are the samples of lesson plans that were planned and taught by the teachers. The first lesson plan was individually planned by the teachers and second was planned collaboratively.

Botaleng Primary School: Lerato's individually planned lesson

<table>
<tr><td colspan="2">GRADE: 1
Date: 20 July 2019</td><td>Presenter: Lerato
Duration: 1 hour</td></tr>
<tr><td colspan="3">Curriculum and Assessment Policy Statement</td></tr>
<tr><td>Content Area</td><td>Topic(s)</td><td>Concepts and skills</td></tr>
<tr><td>Number operations and relationships</td><td>Number 12</td><td>Counting, Addition, subtraction</td></tr>
<tr><td colspan="3">Teaching and Learning Strategies</td></tr>
<tr><td colspan="3">Problem solving</td></tr>
<tr><td colspan="3">Cooperative learning</td></tr>
<tr><td colspan="3">Resources / Teaching material
Number charts, Number cards, Number line, 2 colour circles., tennis balls, Dominoes (Worksheets), Blocks</td></tr>
<tr><td colspan="3">Teaching and Learning Activities</td></tr>
<tr><td colspan="3">Mental Maths:
• Counting forwards and backwards in 1s, 2s, 5s and 10s.
• Doubling and halving between (1 – 20)
Activities:
• Ask learners what number comes between 11 &13.
• Ask one learner to pick number 12 from mixed numbers pasted on the chalkboard. Learner show others number 12.
• Teacher writes number on the board and asks learners to count using circles and number lines. They put circles on the number line as they count.Using tennis balls the teacher counts with the learners until 12.
• Teacher presents a word problem using the tennis balls for learners to perform simple addition and subtraction sums e.g 12 + 1 = 12 -1 =</td></tr>
</table>

<table>
<tr><td colspan="2">Ask learners to break down 12 10 2
Using blocks show learners that 12 can be presented in many different ways eg.
Explain to learners that even though there are so many ways to present 12 (12 will always remain 12)
Ask one learner to write number name on the chalkboard.</td></tr>
<tr><td>Assessment</td><td>Group: Using dominoes ask learners to find the missing half of the domino to equal 12 (using 2colour circles)
Individual: Learners complete a domino activity</td></tr>
</table>

Botaleng Primary School: Collaboratively planned lesson 1

<table>
<tr><td colspan="2">GRADE: 1
Date: 20 August 2019</td><td colspan="2">Presenter: Lerato
Duration: 1 hour</td></tr>
<tr><td colspan="4">Curriculum and Assessment Policy Statement</td></tr>
<tr><td>Content Area</td><td colspan="2">Topic(s)</td><td>Concepts and skills</td></tr>
<tr><td>Number operations and relationships</td><td colspan="2">Number 15</td><td>Counting, Addition, subtraction</td></tr>
<tr><td colspan="4">Teaching and Learning Strategies</td></tr>
<tr><td colspan="2">Problem solving</td><td colspan="2">Cognitively guided instructions</td></tr>
<tr><td colspan="4">Cooperative learning</td></tr>
<tr><td colspan="4">Investigative Approach</td></tr>
<tr><td colspan="4">Resources / Teaching material
Number boards, Flash cards, Number line , 2 colour circles, tennis balls, Worksheets</td></tr>
<tr><td colspan="4">Teaching and Learning Activities

Mental Maths:
Using number charts learners count forwards and backwards from any given number from 1-100. They count in groups e.g., Yellow group from 14-25, and green group continues from where the yellow group ended etc.
Simple addition and subtraction sums, doubling and halving word problems. (Revision of 14 e.g., 10 + 4 =, 14 – 4 =, 12 +____= etc.
Activities:
Estimation : Give 2 learners 2 cups with 2 coloured circles in them , ask each learner to estimate, how many circles, are in a cup . Ask them to count the circles to find out whether their estimation was close or not.
Ask learners who has more/less circles between 2 learners
How many steps from 14 to 15
Give the number between 14 & 16, point the number on the number charts and pick the number 15 from the chalkboard and show it to the other learners.
Using tennis balls ask learners to estimate again before counting them. Perform simple addition and subtraction sums using tennis balls.</td></tr>
</table>

• Ask one learner to write number name of 15 on the chalkboard. • Give learners worksheets to make bonds of 15 using 2 colour circles (working in groups). Groups report on their number and teacher write their findings on the chalkboard. Use flashcards to find the unknown. Teacher flashes the cards and learners uses number line to find the unknown correct answer is placed on the flash card. Learners show others how to use number line on the chalkboards.	
Assessment	Classwork: Worksheets- Draw missing picture to make 15 Complete number sentences

APPENDIX 5

Reflection guide: questions for the teacher participants

The following guide was followed in order to get discussions to take place during reflections.

1. What did you learn during the lesson?

2. Do you know what you need to do in order to further improve your teaching?

3. Which aspects of the teaching worked well for you?

4. Which aspect of the teaching was challenging for you?

5. How do you think the lesson could be improved? Why?

APPENDIX 6

RHODES UNIVERSITY
Where leaders learn

Human Ethics subcommittee
Rhodes University Ethical Standards Committee
PO Box 94, Grahamstown, 6140, South Africa
t: +27 (0) 46 603 8055
f: +27 (0) 46 603 8822
e: ethics-committee@ru.ac.za

www.ru.ac.za/research/research/ethics
NHREC Registration no. REC-241114-045

27 May 2019

Dr Lise Westaway

Email: l.westaway@ru.ac.za

Dear Dr Lise Westaway

Re: Strengthening Grade One teachers Mathematical Knowledge for Teaching .Strengthening Grade One teachers Mathematical Knowledge for Teaching , 0430 , May : 2019

Principal Investigator: Dr Lise Westaway

Collaborators: stella mutlane , Mrs Kedisaletse Stella Mutlane , ,

This letter confirms that the above research proposal has been reviewed by the Rhodes University Ethical Standards Committee (RUESC) – Human Ethics (HE) sub-committee and **PROVISIONALLY APPROVED PENDING GATEKEEPER PERMISSION**.

Gatekeeper permission is required from:

The Department of Education, Northern Cape, Kimberly and relevant school/s (if public schools are involved). Because your research involves participants from a school population (teachers)

Once the Gatekeeper permission letter/s have been received please forward to the Ethics Coordinator, (s.manqele@ru.ac.za) in order to finalise your ethics approval.

Sincerely

Prof Joanna Dames

Chair: Human Ethics sub-committee, RUESC- HE

APPENDIX 7

RHODES UNIVERSITY
Where leaders learn

The District Director (Frances Baard)

Mr L. Monyera

9 Hayston Road

Hadison Park

8301

Dear Sir

Request for permission to conduct research in a school

My name is Kedisaletse Stella Mutlane. I am currently registered for my Masters in Education (Mathematics Education) at Rhodes University in Grahamstown. I'm writing to request permission to conduct research for my thesis. The title of my thesis is 'Strengthen the mathematical and pedagogical knowledge of Grade One teachers to develop the learners' number sense '. This research will be conducted under supervision of Dr Lise Westaway from Rhodes University. I hereby seek your consent to conduct research with the Grade One teachers at Kim Kgolo Primary School.

The research aims to develop Grade One teachers' mathematical and pedagogical knowledge so as to improve learner performance. I plan to work with the Grade One teachers at the school as this is where the mathematics foundations are laid.

They participation of the teachers will be voluntarily, meaning that they can withdraw from the research at any time. In the interests of anonymity and confidentiality, pseudonyms will be used for the participating teachers and the school. While the research focuses on the teachers, we will observe their mathematics lessons in their classrooms. This means that the learners in the Grade One classrooms will be indirectly involved in the research. I will send a letter to all the parents/guardians of the Grade One learners requesting consent to observe in their teachers' classrooms.

The research should be of benefit to both the school and the teachers involved in the research.

My intention is to improve teachers' mathematical and pedagogical content knowledge through collaborative planning and reflection on research lessons. The benefits thereof should extend to the learners.

Included with this letter is my research proposal which includes examples of interview questions and observation schedules to be used in the research process, as well as a copy of the approval letter which I have received from Rhodes University Ethical Standards Committee (RUESC). The ethics approval number is 0430. Should there be any complaint of ethical misconduct executed by me during this research, you can report the matter to Mr. Siyanda Manqele at s.manqele@ru.ac.za.

Upon completion of the study, I undertake to provide the Northern Cape Department of Education with a copy of the research report. If you require any further any information, please do not hesitate to contact me on: 073 213 0736 or 053 871 1033 or kedisaletsemutlane@gmail.com

Thank you for your time and consideration of this matter.

Yours sincerely

Kedisaletse Stella Mutlane
(Rhodes University)

Dr. Lise Westaway
(Rhodes University)

INFORMED CONSENT FORM

District Director: Frances Baard District (Department of Education)

I L. MONYERA ______ give permission for Kedisaletse Stella Mutlane to conduct research with Grade One teacher on: Strengthening Grade One teachers mathematical and pedagogical knowledge to develop learners' number sense.

I have read the research information and understand that:

- The role of the school and teachers is voluntary.
- The school and teachers may be withdrawn from participation at any time.
- All information obtained will be treated with confidentiality.
- Participants and school names will not be written in the study (Pseudonyms will be used).
- A report of the findings will be made available to myself and the school.
- I may contact Kedisaletse Stella Mutlane on 073 213 0736 or kedisaletsemutlane@gmail.com

District Director's signature)

______

(Date)

2019/07/02.

APPENDIX 8

Northern Cape
Department of Education

FRANCES BAARD DISTRICT

9 Hayston Road
Hadison Park
Kimberley 8301

Private Bag X5041
Kimberley 8300
Republic of South Africa

Tel: 053 830 1600
Fax: 053 830 1629

DEPARTMENT OF EDUCATION

Enquiries:	L. Monyera
Contact No:	053 830 1602
Ref No:	L4.3.4
Date:	02 July 2019

Ms. K. S. Mutlane
4353 Kgosi Street
Vergenoeg
KIMBERLEY
8345

SUBJECT: REQUESTING PERMISSION TO CONDUCT RESEARCH ON GRADE 1 TEACHERS AT KIM KGOLO PRIMARY SCHOOL

The Northern Cape Department of Education encourages research, which is in the best interest of education and will consider any meaningful research project in this regard. The Department therefore supports the conducting of high quality research that enables the Department to make evidence based policy decisions, and to enhance delivery of quality education to our learners.

When preparing your questionnaires, you must take the sensitivity of the contents, learners, since respondents such as the Northern Cape Department of Education, educators, learners, governing bodies and parents may not be offended or embarrassed by them.

You must obtain consent from participant categories, such as Principals, parents, teachers and learners. After approval has been granted by the Northern Cape Department of Education, the following conditions would be applicable.

1. There must not be any financial implications for the Northern Cape Department of Education.
2. Institutions and respondents must not be identifiable in any way from the result of the investigation.
3. The researcher must make all the arrangements concerning his/her investigation.
4. Prospective researchers must present a copy of the written approval of the Northern Cape Department of Education to the head of the institution concerned before any research may be undertaken.
5. In case of some research projects it will be necessary for the applicant to obtain the written permission of the parents or legal guardians concerned personally before learners/ learners are involved.
6. Research may not be conducted during official contact time, as educator programmes should not be interrupted.
7. The research may not be conducted during the fourth term.
8. The research will be limited to those schools or institutions for which approval has been granted.

9. A copy of the completed report, dissertation or thesis, accompanied by a separate synopsis (maximum 2-3 typed pages) of the most important findings and recommendations if it does not already contain a synopsis, must be provided to the Frances Baard District Director.

This letter herewith provides you with permission for the research project to be conducted on Grade 1 teachers at Kim Kgolo Primary School within the Frances Baard District in the Northern Cape Province on condition the above are adhered to.

Yours sincerely

2019/07/02.

L. MONYERA
DISTRICT DIRECTOR: FRANCES BAARD DISTRICT

APPENDIX 9

RHODES UNIVERSITY
Where leaders learn

4353 Kgosi Street

Vergenoeg

Kimberley

8345

05 July 2019

Ms. [redacted]

[redacted] Primary School

[redacted]

Mankurwane

8345

Dear [redacted]

Re: Permission to conduct research

My name is Kedisaletse Stella Mutlane, a Masters of Education student at Rhodes University. I am conducting research in the Mathematics Education field under the supervision of Dr. Lise Westaway. The title of my thesis is *Strengthening Grade One teachers' mathematical and pedagogical knowledge through collaborative participation*. This research has met the requirements of the Rhodes University Ethical Standards Committee (RUESC) and the approval number is 0430. The provincial Department of Education has given me permission to approach your school for my research. Copies of both documents are attached to this letter.

The research aims to develop teachers' knowledge and pedagogical practices to improve learner performance in ways that are sustainable. I intend to work collaboratively with the Grade One teachers where the Grade One teachers take responsibility for their professional development and work collaboratively to improve their mathematics and pedagogical knowledge.

The teachers are expected to present mathematics lessons developing the learners number sense. One teacher will present the lesson while the rest of us observe using observation schedules. Learners are not the focus of the research however it is important for them to participate during lesson presentations. The purpose of lesson observations is to observe the knowledge the teachers apply to develop the learners number sense and learners participation and responses. There will be no video or audio recordings of the classroom activities. After the lessons I will have reflection and planning sessions with the teachers where we will discuss the lessons and plan improved lessons. I might request to videotape some of the teachers' lesson planning and reflection sessions for the purposes of observation as a researcher.and will conducted in a duration of one hour.

Data will be collected through observations, interviews and document analysis. The interviews will be audio-recorded for accuracy when transcribing and transcriptions will be shared with the teachers to corroborate. I will ask for permission from the Grade One teachers and the parents of the learners in the Grade One classes. The research will involve collaborative lesson planning, observation and reflections. This research is intended to benefit teachers by strengthening the Mathematical Knowledge for Teaching to develop number sense and to improve learner performance in mathematics.

Data collected will be treated with confidentiality and pseudonyms will be used for the participants and school. The video and audio recordings of the planning and reflection sessions willonly be used by me and will be kept in my computer in a password controlled file. The teachers will participate voluntarily and may withdraw from the research at any time. The role of the school is also voluntary, and the principal may withdraw the school participation at any time.

Once I have received the permission to approach the Grade One teachers to participate in the study, I will clearly inform them about the research. I will also arrange time with the school for data collection to take place. After the research, a copy of the research report will be made available to the school. Once the teachers have consented, I will approach the parents and request their consent.

Should you require further information you can contact me at 073 213 0736 or kedisaletsemutlane@gamail.com.

Should I execute any ethical misconduct, you can report the matter to Mr. Siyanda Manqele at s.manqele@ru.ac.za.

If you grant me permission to conduct research, please complete and return the attached form.

Thank you for taking time to read this information.

Yours Sincerely

Kedisaletse Stella Mutlane	Dr. Lise Westaway
______________________	___________________
(Rhodes University)	(Rhodes University)

INFORMED CONSENT FORM

School Principal

I give permission for you to conduct your research with Grade One teacher on: Strengthening Grade One teachers mathematical and pedagogical knowledge to develop learners' number sense.

I have read the research information explaining the purpose of the research and understand that:

- The role of the school is voluntary
- I may decide to withdraw the school's participation at any time
- All information obtained will be treated with confidentiality
- Participants and school names will not be written in the study (Pseudonyms will be used)
- A report of the findings will be made available to the school
- I may contact Kedisaletse Stella Mutlane on 073 213 0736 or kedisaletsemutlane@gmail.com

[redacted] [redacted]

Principal Signature

09/07/2019

Date

APPENDIX 10

RHODES UNIVERSITY
Where leaders learn

Ms.________________(Participant's Name)

Rhodes University

Grahamstown.

6139

Dear Madam

Re: Invitation to participate in a research study

You are invited to participate in a research study entitled *'Strengthening Grade One teachers' mathematics and pedagogical knowledge for developing learners' number sense'*. The research aims to develop teachers' knowledge and pedagogical practices in ways that are sustainable in order to improve learner performance. I propose the establishment of a collaborative working environment where teachers' take responsibility for their professional development to improve their mathematics and pedagogical knowledge. I have been granted permission to conduct the research by the suprintendent general and the principal of the school. I have included all the documents with this letter.

Your participation in this research will involve a request for you to present a lesson while the rest of us observe using observation schedules and they will not be video or audio recorded. We will also conduct planning and reflection sessions which I might request to video record for my observation as a researcher. You will be expected to present two lessons and observe four over the period of four weeks. The lesson presentation duration will be 30 minutes and the planning and reflection sessions will be an hour after school. The focus of this research is the teachers however learners are expected to participate during lesson observation.

This research is intended to benefit teachers by strengthening their Mathematical Knowledge for Teaching to develop learners number sense and improve learner achievement in mathematics.

I will collect data through observations, interviews and document analysis. The interviews will be audio-recorded for accuracy during transcriptions and the transcriptions will be shared with you for corroboration. There will be collaborative lesson planning, observation and reflections. Data collected will be treated with confidentiality and pseudonyms will be used for the names of participants and school. The video and audio recordings will only be used by me and stored safely in a password controlled file in my computer.

If you agree to participate, I will explain in more detail what would be expected of you and provide you with the information you need to understand the research prior to the implementation thereof.

These guidelines would include potential risks, benefits, and your rights as a participant. Attached, please find the approval from Rhodes University Ethical Standards Committee (RUESC) and the approval number is 0430.

Participation in this research is voluntary and a positive response to this letter of invitation does not oblige you to take part in this research. To participate, you will be asked to sign a consent form to confirm that you understand and agree to the conditions, prior to collection of data. Please note that you have the right to withdraw at any given time during the study.

Thank you for your time and I hope that you will respond favourably to my request.

Yours sincerely,

Student name: Kedisaletse Stella Mutlane Signature:____________________

Supervisor name: Dr. Lise Westaway Signature:_____________________

RHODES UNIVERSITY
Where leaders learn

INFORMED CONSENT FORM

Research Project Title:	Strengthening Grade One teachers' mathematics knowledge
Principal Investigator:	Kedisaletse Stella Mutlane

Participation Information

- I understand the purpose of the research study and my involvement in it.
- I understand the risks and benefits of participating in this research study.
- I understand that I may withdraw from the research study at any stage without any penalty.
- I understand that participation in this research study is done on a voluntary basis.
- I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name..
- I understand that audio and video recording may be used.
- I understand and agree that the interviews will be recorded electronically.
- I understand that I will be given the opportunity to read and comment on the transcribed interview notes.
- I confirm that I am not participating in this study for financial gain.

Information Explanation

The above information was explained to me by: Kedisaletse Stella Mutlane

The above information was explained to me in English and I am in command of this language.

Voluntary Consent

I, [redacted]
hereby voluntarily consent to participate in the above-mentioned research.

[redacted]	Date: 10/7/2019

Investigator Declaration

I, Kedisaletse Stella Mutlane, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.

Signature:	Date: 10/7/2019

RHODES UNIVERSITY
Where leaders learn

INFORMED CONSENT FORM

Research Project Title:	Strengthening Grade One teachers' mathematics knowledge
Principal Investigator:	Kedisaletse Stella Mutlane

Participation Information

- I understand the purpose of the research study and my involvement in it.
- I understand the risks and benefits of participating in this research study.
- I understand that I may withdraw from the research study at any stage without any penalty.
- I understand that participation in this research study is done on a voluntary basis.
- I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name..
- I understand that audio and video recording may be used.
- I understand and agree that the interviews will be recorded electronically.
- I understand that I will be given the opportunity to read and comment on the transcribed interview notes.
- I confirm that I am not participating in this study for financial gain.

Information Explanation

The above information was explained to me by: Kedisaletse Stella Mutlane

The above information was explained to me in English and I am in command of this language.

Voluntary Consent

I, [redacted]
hereby voluntarily consent to participate in the above-mentioned research.

[redacted]	Date: 10 / 7 / 2019

Investigator Declaration

I, Kedisaletse Stella Mutlane, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.

Signature:	Date: 10 / 7 / 2019

RHODES UNIVERSITY
Where leaders learn

INFORMED CONSENT FORM

Research Project Title:	Strengthening Grade One teachers' mathematics knowledge
Principal Investigator:	Kedisaletse Stella Mutlane
Participation Information	
• I understand the purpose of the research study and my involvement in it. • I understand the risks and benefits of participating in this research study. • I understand that I may withdraw from the research study at any stage without any penalty. • I understand that participation in this research study is done on a voluntary basis. • I understand that while information gained during the study may be published, I will remain anonymous and no reference will be made to me by name.. • I understand that audio and video recording may be used. • I understand and agree that the interviews will be recorded electronically. • I understand that I will be given the opportunity to read and comment on the transcribed interview notes. • I confirm that I am not participating in this study for financial gain.	
Information Explanation	
The above information was explained to me by: Kedisaletse Stella Mutlane	
The above information was explained to me in English and I am in command of this language.	
Voluntary Consent	
I, [redacted] . hereby voluntarily consent to participate in the above-mentioned research.	
[redacted]	Date: 10 / 07 / 2019
Investigator Declaration	
I, Kedisaletse Stella Mutlane, declare that I have explained all the participant information to the participant and have truthfully answered all questions ask me by the participant.	
Signature:	Date: 10 / 07 / 2019

APPENDIX 11

RHODES UNIVERSITY
Where leaders learn

THIS LETTER WILL BE TRANSLATED INTO THE HOME LANGUAGE OF THE LEARNERS

Mr/ Ms_________________________(Parent/Guardian's name)

Rhodes University

Grahamstown

6139

Dear Sir/ Madam

Permission for your child to participate in research

My name is Kedisaletse Stella Mutlane, a registered Masters in Education student at Rhodes university. The purpose of the research is to *Strengthen the Mathematical and Pedagogical Knowledge of Grade One teachers through collaborative participation to develop the learners number sense.* This study was approved by the Rhodes University Ethical Standard Committee (RUESC) and the approval number is 0430 .I ask permission for your child/ward to take part in the research.

This research focuses on developing the Mathematical Knowledge for Teaching of the Grade One teachers however it is important to have learners in the classroom during lesson observations. Your child/ward will participate in the lessons that are presented by the Grade One teachers in the classroom. The purpose of the lesson observation is to observe how the knowledge the teachers apply to develop the learners number sense and the response of the lesson. Six lessons will be observed over a four weeks period. The duration of the each lesson will be 30 minutes. Your child/ward's involvement in this study does not involve any physical or emotional risk.

The intended benefit of this research is to strengthen the Mathematical Knowledge for Teaching of the Grade One teachers to develop the learners number sense and improve learner achievement in mathematics.

I will use pseudonyms to protect the participants in this study. The classroom lesson observations will not be recorded audio nor video. Participation of your child/ward in this study is voluntary, therefore you can withdraw him/her anytime you wish. I will make the copy of the report available to the school.

If you have any questions you may contact:

Kedisaletse Stella Mutlane at 073 213 0736 or kedisaletsemutlane@gmail.com

Should your child experience any ethical misconduct executed by me during this research, you can report the matter to Mr. Siyanda Manqele at s.manqele@ru.ac.za.

If you grant permission for your child to participate in the research, please complete and return the attached form.

Thank you for taking time to read this information.

Yours Sincerely

Kedisaletse Stella Mutlane	Dr. Lise Westaway
______________________	____________________
(Rhodes University)	(Rhodes University)

INFORMED CONSENT FORM

	Strengthening Grade One teachers' mathematics knowledge
Principal Investigator:	Kedisaletse Stella Mutlane
Participation Information	

- I understand the purpose of the research study and my child's involvement in it.
- I understand the risks and benefits of participating in this research study.
- I understand that I may withdraw my child from the research study at any stage without any penalty.
- I understand that participation in this research study is done on a voluntary basis.
- I understand that while information gained during the study may be published, my child's will remain anonymous and no reference will be made to my child.

Information Explanation	
The above information was explained to me by: Kedisaletse Stella Mutlane	
The above information was explained to me in English and I am in command of this language.	
Voluntary Consent	
I,____parent/guardian of ____________________________ hereby voluntarily consent for my child/ward to participate in the above-mentioned research.	
Parent/ Guardian's Signature:	Date: / /
Investigator Declaration	
I, Kedisaletse Stella Mutlane, declare that I have explained all the participant information to the parent and have truthfully answered all questions ask me by the parent.	
Researcher's Signature:	Date: / /

THE LETTER TRANSLATED INTO THE HOME LANGUAGE OF THE LEARNERS (SETSWANA)

Rre/Mme_________________________(Leina la motsadi/motlhokomedi)

Rhodes University

Grahamstown

6139

Madume Rre/ Mme

Kopo ya tetla gore ngwana wa gago a tseye karolo mo tlhotlhomisong

Leina la me ke Kedisaletse Stella Mutlane, morutabana wa Mophato wa Bone mo sekolong sa ngwana wa gago. Ke ikwadiseditse lokwalo Masters in Education kwa Rhodes University. Maikaelelo a tlhotlhomiso ya me ke *Go tiisa kitso ya dipalo le go ruta ya barutbana ba Kereiti ya ntlha go ba dira mmogo go tokafatsa tiriso ya dinomoro mo baneng.*

Ke tlhoka go dira tlhotlhomiso mo sekolong sa ngwana wa gago gore ke kgone go falola dithuto tsa me tsa Masters. Ke kopa tetla ya gore ngwana wa gago a tseye karolo mo tlhotlhomisong e. Ngwana wa gago o tla itemogela dirutwana tse pedi, di rutiwa ke morutabana wa gagwe mo bekeng dil le nne. Serutwana sengwe le sengwe se tla tsaya metsotso e masome a mararo. Fa morutabana wa ngwana wa gago a ruta, nna le barutabana ba bangwe re tla lebelela serutwana. Ga go kitla go nna le kgatiso ya video kgotsa ya theetso. Ditebelelo tsa dirutwana di tla direlwa mo diforomong tsa tebelelo tse ke tla di dirang le barutabana. Maina a bana ga a ye go kwala mo diforomong tsa tebelelo. Morago ga dirutwana barutabana ba tla kopano go buisana ka diphitlhelelo tsa bona.

Tlhotlhomiso e e lebagnye thata le go tokafatsa kitso ya barutabana fela go botlhokwa gore bana ba nne teng mo phapusing ka nako ya dithuto. Re tla lebelela kitso e morutabana a e dirisang go ruta bana dinomoro le gore bana ba tsaya karolo jang ebile ba amogela jang dithuto tse. Ngwana wa gago o tla tsaya karolo mo dithutong tse di tla neelwang ke barutabana mo phapusing. Go tsaya karolo ga ngwana wa gago mo tlhotlhomisong e, ga go akaretse tekeletso epe ya go dira ka mmele kgotsa maikutlo.

Tlhotlhomiso e e ikaelela go maatlafatsa kitso ya barutabana ya go go aga kitso ya dinomoro mo baneng le go tokafatsa dipholo tsa bana mo serutweng sa dipalo.

Ke tla dirisa maina a e seng a nnete a batsayakarolo. Ga go kitla go nna le kgatiso ya video kgotsa ya theetso mo dirutwaneng tsa phapusi. Ngwana wa gago o tla tsaya ka go ithaopa, ka jalo o ka nna mo ntsha nako nngwe le nngwe fa o ikutlwa jalo. Ke tla dira kgatiso ya pegelo ya tlhotlhomiso e nne teng kwa sekolong.

Lebisa dipotso tsa tshedimotso kwa go Kedisaletse Stella Mutlane kwa 0732130736 kgotsa or kedisaletsemutlane@gmail.com

Tlhotlhomiso e e amogetswe ke Rhodes University Ethical Standards Committee (RUESC), nomoro ya kamogelo ke________. Fa o ka lemoga tiragatso ya tlolo ya matseo a mantle e dirwa ke nna, o ka bega seo kwa go Rre Siyanda Manqele kwa s.manqele@ru,ac,za.

Fa o nneela tletla gore ngwana wa gago a tseye karolo mo tlhotlhomisong e, ke kopa o tlatse foromo e fa morago mme o e buse.

Ke lebogela nako ya gago ya go buisa tshedimosetso e.

Weno

Kedisaletse Stella Mutlane	Dr. Lise Westaway
______________________	__________________
(Rhodes University)	(Rhodes University)

Foromo ya kitso ya tetlelelo

<table>
<tr><td>Leina la Porojeke ya tlhotlhomiso</td><td colspan="2">Maatlafatso a kitso ya go ruta dipalo a barutabana ba Kreiti ya ntlha</td></tr>
<tr><td>Motlhotlhomisi:</td><td colspan="2">Kedisaletse Stella Mutlane</td></tr>
<tr><td colspan="3">Tshedimosetso ya go tsaya karolo</td></tr>
<tr><td colspan="3">

Ke tlhaloganya maikaelelo a tlhotlhomiso le karolo ya ngwanake mo go yone.
Ke tlhaloganya ditekeletso le mosola wa go tsaya karolo mo tlhotlhomisong .
Ke tlhaloganya gore ngwana wa me o tsaya karolo ka boithaopo.
Ke tlhaloganya gore ke kgona go ntsha ngwanake mo tlhotlhomisong nako nngwe le nngwe mo tlhotlhomisong kwa ntle ga kotlhao .
Ke tlhaloganya gore tlhotlhomiso ka ga tlhotlhomiso e e ka phasaladiwa, mme leina la ngwanake ga nkitla le kitla le tlhagelela ebile ga kitla a umakiwa gope.

</td></tr>
<tr><td colspan="3">Tlhaloso ya tshedimosetso</td></tr>
<tr><td colspan="3">Tshedimosetso e e fa godimo ke e tlhaloseditswe ke: Kedisaletse Stella Mutlane</td></tr>
<tr><td colspan="3">Tshedimosetso e e fa godimo e tlhalositswe ka Setswana, gonne ke na le taolo ya puo e.</td></tr>
<tr><td colspan="3">Tetla ya boithaopo</td></tr>
<tr><td colspan="3">Nna,____motsadi/motlhokomedi wa ga __________________

ke neelana ka tetla ya boithapo gore ngwanake a tseye karolo mo tlhotlhomisong e e boletsweng fa

godimo.</td></tr>
<tr><td colspan="2">Mosaeno wa motsadi/motlhokomedi:</td><td>Letlha: / /</td></tr>
<tr><td colspan="3">Kitsiso ya phatlhalatso ya motlhotlhomisi</td></tr>
<tr><td colspan="3">Nna , Kedisaletse Stella Mutlane, ke itsise phatlhalatsa gore ke tlhaloseditse batsadi ba batsayakarolo botlhe tshedimosetso le go araba dipotso tsa batsadi ka bonnete.</td></tr>
<tr><td colspan="2">Mosaeno wa motlhotlhomisi:</td><td>Letlha: / /</td></tr>
</table>