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Robust formation control of thrust-propelled vehicles under deterministic and stochastic topology

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ABSTRACT

Formation control of multiple thrust-propelled vehicles (TPVs) under deterministic and stochastic switching topologies and communication delay is addressed. Introducing a new version of variable structure control and based upon sliding mode technique, adaptive control and projection operator, we effectively handle the impact of uncertainties on the mass and inertia matrix and a set of time-varying disturbances affecting the translational and rotational dynamics. Global stability of the whole closed-loop system is guaranteed through Lyapunov stability theory. For the deterministic topology, sufficient condition in terms of LMIs is derived to achieve formation in the presence of jointly connected switching topology. In the case of stochastic topology, based on the concept of super-martingales, it is shown that if the probability of existing a connected topology is not zero, under some conditions, formation is almost surely solved in the network. Finally, numerical simulations verify the effectiveness of the proposed control framework.

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Formation control; robustness; disturbance rejection; stochastic topology; time-delay; under-actuated thrust-propelled vehicles

1. Introduction

Under-actuated vehicles, i.e. systems with less number of control inputs than the number of configuration variables, include a wide range of mechanical systems that have diverse applications. The study of these systems is motivated by the advantages they offer over the fully actuated counterparts such as lower weight, less complexity, more efficiency and reliability. Among such systems, thrust-propelled vehicles (TPVs), as an important class of under-actuated systems with pivotal importance to various applications such as surveillance and reconnaissance (Kopfstedt, Mukai, Fujita, & Ament, 2008; Pack, DeLima, Toussaint, & York, 2009), have gained significant attention in the research community.

In many applications, it is required and advantageous that multiple of these vehicles collaboratively pursue a common task. As a specific type of motion coordination, formation control holds particular promise for applications in areas such as deep space imaging and exploration, high-resolution seabed inspection, aerial coverage and reconnaissance, seabed mapping and so on (Das, Subudhi, & Pati, 2016). Despite the significant progress in the field of cooperation of fully actuated systems such as first- and second-order dynamics, general linear models and nonlinear systems under different conditions such as switching topology and time-delay (see for instance Atrianfar & Haeri, 2014; Ge, Park, Hua, & Guan, 2018; Komareji, Shang, & Bouffanais, 2018; Li, Hua, Liu, & Guan, 2018; Mateo, Horsevad, Hassani, Chamanbaz, & Bouffanais, 2019; Mirzaei, Atrianfar, Mehdipour, & Abdollahi, 2016; Rezaei, Kabiri, & Menhaj, 2018; Shen, Huo, Cao, & Huang, 2019; Shen, Wang, Xia, Park, & Wang, 2019; Wang, Ru, Xia, Wei, & Wang, 2019, just to cite a few), there is still much to be done to adopt strategies for motion control of complex, under-actuated systems with nonlinear coupling. As it will be explained throughout the paper, these results cannot be extended straightforwardly to our case due to input constraints on the translational input. For instance, in all of the aforementioned studies, the information of neighbouring agents was used in the designed control for each subsystem which renders the controller dependent on the network topology. As a result, switches between different topologies lead to discontinuous control input which is problematic when it comes to approaching the under-actuated systems.

As another aspect of practicality, designing a control framework endowed with robustness against inevitable disturbances as well as uncertainties is very crucial in the success of challenging missions. This is mainly because these vehicles usually operate in a medium (air, water, space) which are susceptible to unwanted external inputs

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such as wind, wave, atmospheric drag, solar radiation and aerodynamic torques. Moreover, in many modern control problems, it is not easy to obtain the exact dynamic model of systems or measure some system parameters accurately (like inertia matrix and mass due to e.g. fuel consumption, payload variation, changes in the overall spacecraft system configuration, the added-inertia and added-mass effect in underwater applications, etc.). In spite of the fruitful results in handling various kinds of disturbances, extending the reported results to these kinds of under-actuated systems is not a trivial task. The main challenge is that all of the works that considered the case of asymptotic tracking or regulation control used non-continuous controllers such as sliding mode control which cannot be exploited for control of TPVs (Cai, de Queiroz, & Dawson, 2006a; Chen, Shi, & Lim, 2015; Zou, Kumar, & de Ruiter, 2016). It is to be mentioned that although there are some efforts that introduced differentiable controllers to tackle the time-varying disturbances such as Xian, Dawson, DeQueiroz, and Chen (2004), Wen, Zhou, Liu, and Su (2011) and Lu and Xia (2013), these controllers just ensure ultimate boundedness of the error. In this paper, we aim at designing a controller such that asymptotic convergence of the error to zero is achieved.

Attitude control of such systems, which is fully actuated, has been extensively researched in both single and multi-agent contexts (see for instance Erdong & Zhaowei, 2009; Li, 2017; Wang, Hua, & Zong, 2019; Yin, Xia, Deng, & Huo, 2018). However, when the position is involved, the problem becomes more challenging due to the under-actuated nature of the translational dynamics. As a remedy to this difficulty, several classes of controllers have been undertaken to control the position of a single TPV such as back-stepping procedure (Cabecinhas, Cunha, & Silvestre, 2014), extraction method (Abdessameud & Tayebi, 2013; Kabiri, Atrianfar, & Menhaj, 2017b; Roberts & Tayebi, 2011; Roza & Maggiore, 2014), hybrid control theory (Casau, Sanfelice, Cunha, Cabecinhas, & Silvestre, 2015), sliding mode control (Xu & Özgüner, 2008) and singular perturbation theory (Bertrand, Guénard, Hamel, Piet-Lahanier, & Eck, 2011).

Some results in motion coordination of such under-actuated systems can be also found in Abdessameud, Polushin, and Tayebi (2015), Roza, Maggiore, and Scardovi (2014), Abdessameud and Tayebi (2011), Lee (2012), Børhaug, Pavlov, Panteley, and Pettersen (2011), Kabiri, Atrianfar, and Menhaj (2018) and Kabiri, Atrianfar, and Menhaj (2017a). The researches of Abdessameud and Tayebi (2011), Lee (2012) and Roza et al. (2014), based on separation method and back-stepping, investigated the cooperation of multiple TPVs, ignoring the effect of uncertainties and disturbances, and sharing the assumption that the communication graph characterising the interaction among the systems is fixed. In Kabiri et al. (2018), formation of VTOL aircraft with switching topology and constant disturbances was addressed. However, this study was limited to the constant disturbances and the effect of uncertainties and delays was neglected. Besides, the network topologies are required to contain a spanning tree at each time instant. Involving the problem of variations of the network topology, time-varying disturbances and uncertainties for under-actuated systems are critical, especially in 3-D space. For example, using relative position in control input results in a discontinuous controller, which renders the results reported in Lee (2012) and Kabiri et al. (2017b) non-extendable to the switching topology case; the approaches used in Abdessameud and Tayebi (2011), Abdessameud et al. (2015) and Kabiri et al. (2018) depend on the exact value of the mass and inertia matrix of the vehicle; the effect of disturbance in Kabiri et al. (2018) is counteracted by exploiting the zero derivatives of disturbances. In Kabiri et al. (2017a), the problem of stationary formation control for a group of TPVs was proposed in the presence of a pair of time-varying disturbances.

In this paper, the problem of formation control for a group of TPVs is solved where a hierarchical controller is adopted for each TPV in a team in such a way that all TPVs in a group attain a pre-specified formation globally with respect to the initial conditions, while all their linear velocities converge to a reference velocity signal. The mass and inertia matrix of each TPV are also considered to be unknown. The condition on network topology has been also made weaker, i.e. the network topology is required to be jointly connected.

Furthermore, due to the stochastic nature of communication channels and existing packet losses, we consider the topology that switches in a stochastic manner. Some studies have been focused on control of multi-agent systems with first-, second- and high-order dynamics under stochastic interconnections (Chen, Xie, & Yu, 2012; Liu, Lu, & Chen, 2011; Zhao, Park, & Zhang, 2014), but to the authors' best knowledge, it was not investigated for a team TPVs with nonlinear under-actuated dynamics so far. Hence, due to the drawbacks of existing researches, it is necessary to propose a systematic framework to design formation strategies for a group of TPVs with uncertain nonlinear under-actuated dynamics under deterministic and stochastic switching topologies, time delays and unknown time-varying disturbances. Compared with the relevant studies, Abdessameud and Tayebi (2011), Abdessameud et al. (2015), Lee (2012) and Kabiri et al. (2018), the proposed framework in this paper is robust against general uncertainties in the mass and inertia matrix of the systems, and a pair of unknown time-varying disturbances. In comparison with Kabiri et al. (2018), the condition which is considered here over network topology is more relaxed. Furthermore, the effect of delay, uncertainties and time-varying disturbances is included. It is relevant to mention that although just the effects of switching topology and constant delays are discussed in the manuscript, several cooperation problems, which have not been solved for this class of under-actuated systems, can be easily extended to our case. For example, the reported results in Yang, Bertozzi, and Wang (2008) can be easily extended to consensus of TPVs with nonuniform time-varying delays. The remainder of the paper is organised as follows. In the following section, preliminaries are given. The formulation of the problem is provided in Section 3. The procedure design for the position and attitude control is introduced in Section 4. The stability of the proposed control framework under both deterministic and stochastic switching topology is analysed in Section 5. Two illustrative simulation examples for both scenarios are given in Section 6 and the paper is finally concluded in Section 7.

2. Preliminaries

2.1. Notation

Throughout this paper, the subscript i in x_i denotes to x of the ith vehicle and superscript (i) in $x^{(i)}$ denotes the ith component of vector x. The symbol sign(.) stands for signum function and for vector x we define $\text{sign}(x) = (\text{sign}(x^{(1)}), \text{sign}(x^{(2)}), \text{sign}(x^{(3)}))^{\mathrm{T}}$. The Euclidean norm of a vector is denoted by $\|.\|$ and $|.|$ denotes the absolute value of a scalar. The identity matrix of order 3 is denoted by $I_3 \in \mathbb{R}^{3\times 3}$ and $^\times$ is an operator such that $x^\times = (0, -x_3, x_2; x_3, 0, -x_1; -x_2, x_1, 0)$. 'a.s.' stands for almost surely and $\mathbf{E}[.]$ and $\mathbf{Pr}[.]$ denote the expected value and probability of a stochastic variable, respectively. Let us define a smooth saturation function $\chi(x) = (\varrho(x^{(1)}), \varrho(x^{(2)}), \varrho(x^{(3)})) \to \mathbb{R}^3$ with $\varrho : \mathbb{R} \to \mathbb{R}$ and properties: (i) $d\varrho(x)/x$ is bounded for all x, (ii) $|\varrho(x)| < M$, for all x, (iii) $\varrho(0) = 0$ and $x\varrho(x) > 0$.

2.2. Interconnection communication topology

To represent the interconnection topology, we make a use of undirected graph $G(\mathcal{V}, \mathcal{E}, A)$ with a node set $\mathcal{V} = \{1, \ldots, n\}$, edge set $\mathcal{E} \in \{\mathcal{V} \times \mathcal{V}\}$ and weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ which is defined such that $a_{ii} = 0$ and $a_{ij} > 0$ if $(i,j) \in \mathcal{E}$. For the undirected graph, we assume $a_{ij} = a_{ji}$ for all $i \in \mathcal{V}$. To describe the time-varying topologies, we use a piecewise right-continuous switching function $\sigma(t)(\text{inshort}\sigma) : [0,\infty] \to \mathcal{P} = \{1, 2, \ldots, N\}$, where N is the total number of all possible communication graphs. $G^{\sigma(t)}$ denotes the communication graph at time t. A path is a sequence of adjacent edges of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots, (v_{i_{j-1}}, v_{i_j})$ where $i_j \in \mathcal{V}$. An undirected graph is called connected if every two distinct nodes are connected by a path. The union of a collection of graphs $G_1, \ldots, G_m$ with the node set $\mathcal{V}$ is a graph denoted by G_{1-m} with the same node set whose edge set is the union of edge sets of all graphs. A switching topology is said to be jointly connected if there exists an infinite sequence of nonempty, bounded and contiguous time intervals $[t_r, t_{r+1}]$, $r = 0, 1, \ldots$, with $t_0 = 0$ and $t_{r+1} - t_r \le T_1$ for some constant $T_1 > 0$ such that the union of undirected graphs across each time interval is connected. We assume that a complete graph is a graph in which each pair of graph vertices is connected by an edge. It is also assumed that there is a sequence of non-overlapping subintervals $[t_{r_0}, t_{r_1}, \ldots, [t_{r_j}, t_{r_{j+1}}), \ldots, [t_{r_{m_r-1}}, t_{r_{m_r}})$ where $t_r = t_{r_0}$, $t_{r+1} = t_{r_{m_r}}$ and $t_{r_{j+1}} - t_{r_j} \ge T_2$, $0 \le j < m_r - 1$ for some integer m_r and given constant T_2. The communication topology is supposed to be fixed on each time subinterval and switches at t_{r_j}.

3. System model and problem formulation

For attitude and position representation, two coordinate systems for each TPV are considered. The inertial frame which is attached to the earth centre and the body frame which is fixed to each TPV body defined to be at the centre of the gravity of the vehicle. Considering n as the number of vehicles, the equations characterising the motion of the ith TPV are given by

$$\begin{aligned} \dot{p}_i &= v_i, \\ \dot{v}_i &= g\hat{z} - \frac{\mathcal{T}_i}{m_i} R(\boldsymbol{q}_i)^{\mathrm{T}} \hat{z} + w_i(t), \end{aligned} \tag{1a}$$

$$\begin{aligned} \dot{\boldsymbol{q}}_i &= \frac{1}{2} \begin{pmatrix} \eta_i I_3 + q_i^\times \\ -q_i^{\mathrm{T}} \end{pmatrix} \omega_i, \\ J_i \dot{\omega}_i &= \Gamma_i - \omega_i^\times J_i \omega_i + d_i(t), \end{aligned} \tag{1b}$$

where $p_i \in \mathbb{R}^3$ and $v_i \in \mathbb{R}^3$ are, respectively, the position and the linear velocity of the centre of the mass of the ith TPV coordinated in the inertial frame, $\hat{z} = (0, 0, 1)^{\mathrm{T}}$, g is the gravitational acceleration, m_i is the unknown total mass of the ith TPV, $w_i(t)$ and $d_i(t)$ are, respectively, the translational and rotational lumped disturbances of vehicle i resulting from external disturbance, aerodynamic uncertainties and unmodelled dynamics, $J_i \in \mathbb{R}^{3\times 3}$ is the ith TPV positive definite inertia matrix with respect to its body-fixed frame (which is supposed to be unknown), the scalar $\mathcal{T}_i$ and $\Gamma_i \in \mathbb{R}^3$ are, respectively, the thrust and torque input for the ith vehicle, ω_i denotes the body-referenced angular velocity of the ith vehicle. The unit quaternion $\boldsymbol{q}_i = (q_i^{\mathrm{T}}, \eta_i)^{\mathrm{T}}$ is the attitude of the ith vehicle with respect to the inertial frame, which is composed of the vector part $q_i \in \mathbb{R}^3$ and the scalar part η_i, and satisfies the constraint $q_i^{\mathrm{T}} q_i + \eta_i^2 = 1$ (Diebel, 2006). The inverse of unit quaternion $\boldsymbol{q}_i$ is defined as $\boldsymbol{q}_i^{-1} = (-q_i^{\mathrm{T}}, \eta_i)^{\mathrm{T}}$ with the quaternion identity given by $\boldsymbol{q}_I = (0, 0, 0, 1)^{\mathrm{T}}$. The unit quaternion multiplication is defined by $\boldsymbol{q}_i \odot \boldsymbol{q}_j = ((q_i^{\mathrm{T}}\eta_j + q_j^{\mathrm{T}}\eta_i + q_i^{\times} q_j)^{\mathrm{T}}, \eta_i\eta_j - q_i^{\mathrm{T}} q_j)^{\mathrm{T}}$ which is also a unit quaternion. The rotation matrix $R(\boldsymbol{q}_i)$ which brings the inertial frame into the body frame is obtained by $R(\boldsymbol{q}_i) = (\eta_i^2 - q_i^{\mathrm{T}} q_i) I_3 + 2q_i q_i^{\mathrm{T}} - 2\eta_i q_i^{\times}$.

Our objective is to design a distributed controller in such a way that all TPVs in a group reach and maintain a desired formation pattern prescribed by the offset vectors while each vehicle tracks a reference velocity signal available to all vehicles. In other words $v_i(t) \to v_d(t)$ and $p_i - p_j \to \delta_{ij}$, where $\delta_{ij} = \delta_i - \delta_j$ and δ_i is the offset vector corresponding to the ith TPV whereby the position of the vehicle i with respect to the centre of the formation is determined. We assume that each vehicle can merely communicate with its neighbour(s) with a constant time-delay. Moreover, the objective is to be realised under two unknown time-varying disturbances and without any knowledge about the mass and inertia matrix. We show that under some condition on time-delay if the communication topology is jointly connected across contiguous time intervals of arbitrary but finite length then formation can be acquired. Note that the condition that is imposed on the network topology is regarded as a weak condition that allows for the isolation of some or even all vehicles in the group at some moment. In the rest, we make the following assumptions.

Assumption 3.1: *We assume that the mass of each vehicle has a known lower bound m_{L_i} and a known upper bound m_{U_i}. Hence without loss of generality, it is assumed that $1/m = \varphi_{0_i} + \bar{\varphi}_i$, where φ_{0_i} is $(1/m_{L_i} + 1/m_{U_i})/2$ which is known, and $\bar{\varphi}_i$ is an unknown constant which has the property that $|\bar{\varphi}_i| \leq (1/m_{L_i} - 1/m_{U_i})/2$.*

By Assumption 3.1, we decompose the mass of the vehicle into two parts: the known part and the unknown part. The unknown part is bounded by a symmetric bound. As it is explained later, this symmetric property allows us to use the projection operator which keeps the estimation within a-priori bound and hence we will assure that our estimation is always positive ($\hat{\varphi} = 1/\hat{m} > 0$).

Assumption 3.2: *It is assumed that the reference velocity signal and its first-, second- and third-order time derivatives are bounded. It is also assumed that there exist constants $\bar{w}_i$ and $\bar{v}_d$ such that $\sup_{t>0} \|w_i\| < \bar{w}_i$, $\sup_{t>0} \|\dot{v}_d(t)\| < \bar{v}_d$, $\bar{w}_i + \bar{v}_d < g$.*

Assumption 3.3: *Norm of the inertia matrix J_i is bounded by an unknown constant $\|J_i\| < \bar{J}_i$. We also assume that the torque disturbance $d_i(t)$ is bounded and it has an unknown upper bound by $\sup_{t>0} \|d_i\| = \bar{d}_i$.*

4. Control design strategy

For simplicity of presentation, we take $\varphi_i = 1/m_i$. Let add and subtract the terms $\hat{\varphi}_i \mathcal{T}_i R(\boldsymbol{q}_i)^{\mathrm{T}} \hat{z}$ and $\hat{\varphi}_i \mathcal{T}_i R(\boldsymbol{q}_{d_i})^{\mathrm{T}} \hat{z}$ to the second equation in (1a). Then we can put the linear acceleration equation in the following advantageous form:

$$\dot{v}_i = F_i + \tilde{F}_i - \tilde{\varphi}_i \mathcal{T}_i R(\boldsymbol{q}_i)^{\mathrm{T}} \hat{z} + w_i(t), \tag{2}$$

$$F_i = g\hat{z} - \hat{\varphi}_i \mathcal{T}_i R(\boldsymbol{q}_{d_i})^{\mathrm{T}} \hat{z}, \tag{3}$$

$$\tilde{F}_i = \hat{\varphi}_i \mathcal{T}_i \left(R(\boldsymbol{q}_{d_i})^{\mathrm{T}} - R(\boldsymbol{q}_i)^{\mathrm{T}} \right) \hat{z}, \tag{4}$$

where $\hat{\varphi}_i = \varphi_{0_i} + \hat{\bar{\varphi}}_i$ is the estimation of φ_i in which φ_{0_i} is known and $\hat{\bar{\varphi}}_i$ is the estimation of $\bar{\varphi}_i$, $\tilde{\varphi}_i = \varphi_i - \hat{\varphi}_i = \bar{\varphi}_i - \hat{\bar{\varphi}}_i$. The variable F_i is the virtual controller for the translational dynamics, and $\tilde{F}_i$ is the under-actuation error.

At this stage, the term $\tilde{F}_i$ can be seen as a perturbation which will be driven to zero by a suitable design of attitude controller. The desired orientation $\boldsymbol{q}_{d_i}$ is then extracted from F_i to be tracked.

For any F_i satisfying $F_i \neq g\hat{z}$, the input thrust $\mathcal{T}_i$ and the desired attitude can be acquired by the following equations which are taken from Abdessameud and Tayebi (2013)

$$\mathcal{T} = \hat{m}\|F - g\hat{z}\| = \frac{1}{\hat{\varphi}}\|F - g\hat{z}\|, \tag{5}$$

$$\eta_d = \sqrt{\frac{1}{2} + \frac{g - F^{(3)}}{2\|F - g\hat{z}\|}},$$

$$q_d = \frac{1}{2\|F - g\hat{z}\|\eta_d} \begin{pmatrix} F^{(2)} \\ -F^{(1)} \\ 0 \end{pmatrix}. \tag{6}$$

This is called the extraction method. The desired angular velocity ω_{d_i} and its derivative $\dot{\omega}_{d_i}$ can be obtained by the following expressions:

$$\omega_d = \Xi(F)\dot{F}, \tag{7}$$

$$\dot{\omega}_d = \dot{\Xi}(F, \dot{F})\dot{F} + \Xi(F)\ddot{F}, \tag{8}$$

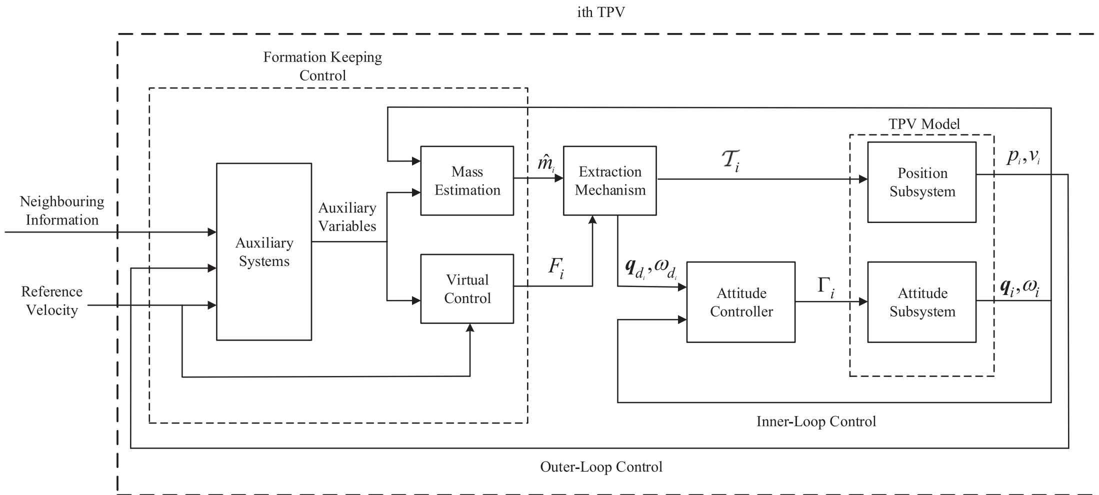

Figure 1. Overall control architecture for the ith agent.

with

$$\Xi(F) = \frac{1}{\ell_1^2\ell_2}\begin{pmatrix} -F^{(1)}F^{(2)} & -F^{(2)^2}+\ell_1\ell_2 & -F^{(2)}\ell_2 \\ F^{(1)^2}-\ell_1\ell_2 & F^{(1)}F^{(2)} & -F^{(1)}\ell_2 \\ F^{(2)}\ell_1 & -F^{(1)}\ell_1 & 0 \end{pmatrix},$$

where $\ell_1 = \|F - g\hat{z}\|$, $\ell_2 = \ell_1 + (g - F^{(3)})$ and $\dot{\Xi}(F, \dot{F})$ is the time derivative of $\Xi(F)$ and the subscript i is omitted for notational simplicity.

It is to be mentioned that the extraction algorithm is not unique and several strategies can be found in the literature (see for instance Roberts & Tayebi, 2011; Roza & Maggiore, 2014).

In the second step, the torque input is designed such that the extracted attitude $\boldsymbol{q}_{d_i}$ is tracked by the rotational dynamics and consequently perturbation term $\tilde{F}_i$ is pushed to zero. In fact, F_i can be viewed as the control input for the fully actuated system which can be fulfilled through proper design of attitude dynamics. Figure 1 depicts the overall structure of the control design for each aircraft .

4.1. Position control

In this section, the virtual control input is designed. Since $\boldsymbol{q}_{d_i}$, as a function of the translational controller F_i, should be tracked by the rotational dynamics, some requirements on F_i in the design process should be considered. (1) To satisfy the feasibility of the extraction algorithm, it is required that the condition $|F_i^{(j)}| < g$ holds for $j = 1, \ldots, 3$. (2) It should be at least twice differentiable such that ω_{d_i} and $\dot{\omega}_{d_i}$ exist and its time derivative should be known such that ω_{d_i} is available to be used in the trajectory tracking control design for rotational dynamics. The difficulty with this approach is the above-mentioned constraints on F_i. For example, a typical approach in motion coordination is to use relative positions and linear velocities in the control input which leads to a non-continuous controller in the case of switching topology. Hence because of the requirement (2), in designing virtual input F_i, we are not allowed to use the relative position of the agents either in F_i or its time derivative. Moreover, we need the time derivative of the input to be used in attitude control. Thus using the typical control scheme for fully actuated systems necessitates that each agent has access to the acceleration information of its neighbours which is not practical.

Regarding the above constraints, we define the error variable $z_i = p_i - \theta_i - \alpha_i - \xi_i$, where θ_i, α_i and ξ_i are the auxiliary variables to be properly designed. In view of (2), we can have

$$\ddot{z}_i = F_i + \tilde{F}_i - \tilde{\varphi}_i \mathcal{T}_i R(\boldsymbol{q}_i)^{\mathrm{T}}\hat{z} + w_i(t) - \ddot{\theta}_i - \ddot{\alpha}_i - \ddot{\xi}_i. \quad (9)$$

Now we define

$$F_i = \dot{v}_d - k_{\dot{\theta}_i}\chi(\dot{\theta}_i) - k_{\theta_i}\chi(\theta_i) + u_{1_i}, \quad (10)$$

$$\ddot{\theta}_i = -k_{\dot{\theta}_i}\chi(\dot{\theta}_i) - k_{\theta_i}\chi(\theta_i) + k_{\dot{\alpha}_i}\dot{\alpha}_i + k_{\alpha_i}\alpha_i, \quad (11)$$

$$\ddot{\alpha}_i = -k_{\dot{\alpha}_i}\dot{\alpha}_i - k_{\alpha_i}\alpha_i + k_1\left(\dot{\xi}_i - v_d\right) \quad (12)$$

$$+ \sum_{j\in\mathcal{N}_i(t)} a_{ij}^{\sigma(t)}\xi_{ij} - u_{2_i}, \quad (13)$$

$$\ddot{\xi}_i = \dot{v}_d - k_1\left(\dot{\xi}_i - v_d\right) - \sum_{j\in\mathcal{N}_i(t)} a_{ij}^{\sigma(t)}\xi_{ij}, \quad (14)$$

where (11)–(14) are the auxiliary systems with arbitrarily initial conditions, $\xi_{ij} = \xi_i(t-\tau) - \xi_j(t-\tau) - \delta_{ij}$, $\delta_{ij} = \delta_i - \delta_j$ in which τ denotes the constant time delay, δ_i is the desired position of the vehicle i from the centre of the formation, a_{ij} is the entry of the adjacency matrix, and $\mathcal{N}_i(t)$ is the neighbour set of the ith TPV at time t and $k_{\dot{\theta}_i}$,

$k_{\theta_i}, k_{\dot{\alpha}_i}, k_{\dot{\alpha}_i}, k_1$ are strictly positive scalar gains, u_{1_i} and u_{2_i} are to be designed.

Assumption 4.1: *Assume that for each subinterval $[t_{r_j}, t_{r_{j+1}}]$, there exist a constant γ and $Q^i_\sigma \in \mathbb{R}^{d^i_\sigma \times d^i_\sigma}$, $i = 1, \ldots, l_\sigma$ such that $\bar{H}^i_\sigma \Psi^i_\sigma(Q^i_\sigma, \tau, \gamma)\bar{H}^i_\sigma < 0$, where l_σ is the number of connected components in each subinterval and $Q^\sigma_i \in R^{d^i_\delta \times d^i_\delta}$ for $i = 1, \ldots, l_\sigma$ is a matrix that satisfies the conditions $0 \le Q^\sigma_i = Q^{\sigma\,T}_i$, $\mathrm{Rank}(Q^\sigma_i) = d^i_\delta - 1$ and $Q^\sigma_i 1 = 0$. The matrix $\bar{H}^i_\sigma = \mathrm{diag}(\bar{U}_{2d^i_\sigma}, \bar{U}_{d^i_\sigma}, I_{2d^i_\sigma})$ in which $\bar{U}_n$ is the first n−1 columns of U_n and U_n is the $n \times n$ orthogonal matrix such that $U_n^{\mathrm{T}} C_n U_n = \mathrm{diag}(nI_n - 1, 0)$ with $C_n = nI_n - 11^{\mathrm{T}}$ and the last column of U_n is $1/\sqrt{n}$. The equation for matrix $\Psi^i_\sigma(Q^i_\sigma, \tau, \gamma)$ is defined in Equation* (10) *in* Lin and Jia (2010). *For detailed information, the readers are referred to* Lin and Jia (2010).

With the above structure, (9) can be written as

$$\ddot{z}_i = u_{1_i} + u_{2_i} + \tilde{F}_i - \tilde{\varphi}_i \mathcal{T}_i R(\boldsymbol{q}_i)^{\mathrm{T}} \hat{z} + w_i(t). \tag{15}$$

In the proposed structure, ξ_i and $\dot{\xi}_i$ play the role of respectively the positions and velocities of virtual agents corresponding to each TPV which are expected to reach a predefined formation and simultaneously track a reference velocity signal. In this framework, the only signal that is needed to be transmitted between vehicles is $\xi_i - \delta_i$. The idea is to design u_{1_i} and u_{2_i} such that $z_i, \dot{z}_i \to 0$ and then by pushing the auxiliary variables θ_i, $\dot{\theta}_i$, α_i, $\dot{\alpha}_i$ to zero, we can easily drive the position and velocity of each vehicle to those of its corresponding virtual vehicle. It is to be noticed that in the design of u_{1_i} and u_{2_i} some considerations must also be kept in mind. Since u_{1_i} is involved in F_i, it must meet the requirements mentioned for F_i. The variable u_{2_i} should be designed in such a way that as z_i and $\dot{z}_i$ go to zero, it also converges to zero so that the convergence of α_i and $\dot{\alpha}_i$ can be realisable. Toward this end, borrowed from back-stepping technique and variable structure control, we design u_{1_i}, u_{2_i} and the adaptive law for estimation of $\hat{\varphi}_i = \varphi_{0_i} + \hat{\bar{\varphi}}_i$. Taking $e_i = \dot{z}_i + c_{1i} z_i$ with $c_{1i} > 0$, we define

$$\dot{\hat{\bar{\varphi}}}_i = \lambda_{\varphi i} \mathrm{proj}(\Theta_i, \hat{\bar{\varphi}}_i), \quad \Theta_i = -e_i^{\mathrm{T}} \mathcal{T}_i R(\boldsymbol{q}_i)^{\mathrm{T}} \hat{z}, \quad \lambda_{\varphi_i} > 0, \tag{16}$$

$$u_{1_i} = \frac{-u_{m_i} z_i}{\sqrt{\|z_i\|^2 + (\kappa_i \sigma_i)^2}},$$
$$u_{2_i} = -c_{1_i}\dot{z}_i - c_{2_i} e_i, \quad \sigma_i,\ c_{2_i},\ u_{m_i} > 0, \tag{17}$$

where the variable $\kappa_i(t)$ is adjusted by

$$\dot{\kappa}_i = -\frac{\lambda_{\kappa_i}}{\kappa_i} u_{mi} \|e_i\| \left(1 + \frac{\|z_i\|}{\sqrt{\|z_i\|^2 + (\kappa_i \sigma_i)^2}}\right), \quad \lambda_{\kappa_i} > 0. \tag{18}$$

The symbol $\mathrm{proj}(\Theta, \hat{\bar{\varphi}})$ in (16) is the projection operator and is defined by Cai, de Queiroz, and Dawson (2006b)

$$proj\left(\Theta, \hat{\bar{\varphi}}\right) = \Theta - \frac{\varpi_1 \varpi_2}{2(\varepsilon^2 + 2\varepsilon B)^{n+1} B^2} \hat{\bar{\varphi}},$$

with $\varpi_2 = \hat{\bar{\varphi}}\Theta + ((\hat{\bar{\varphi}}\Theta)^2 + \bar{\delta}^2)^{1/2}$ and

$$\varpi_1 = \begin{cases} (\hat{\bar{\varphi}}^2 - B^2)^2 & \hat{\bar{\varphi}}^2 > B^2, \\ 0 & \text{otherwise,} \end{cases}$$

where ε and $\bar{\delta}$ are arbitrary positive constants, $\hat{\bar{\varphi}}$ is the estimation of $\bar{\varphi}$, $\tilde{\varphi} = \varphi - \hat{\varphi}$, and $B > 0$ is the bound on the estimation. It can be easily shown by the discussion in Cai et al. (2006b) that by design of the adaptive law as

$$\dot{\hat{\bar{\varphi}}} = \gamma_\varphi \mathrm{proj}\left(\Theta, \hat{\bar{\varphi}}\right) = \gamma_\varphi \left(\Theta - \frac{\varpi_1 \varpi_2}{2(\varepsilon^2 + 2\varepsilon B)^{n+1} B^2} \hat{\bar{\varphi}}\right),$$

the following properties hold: (p1) $|\hat{\bar{\varphi}}| \le B + \varepsilon,\ \forall\ t > 0$; (p2) $\tilde{\varphi}\mathrm{proj}(\Theta, \hat{\bar{\varphi}}) \ge \tilde{\varphi}^{\mathrm{T}}\Theta$; (p3) $\mathrm{proj}(\Theta, \hat{\bar{\varphi}}) \in \mathcal{C}^n$, where the subscript i was omitted for clarity of presentation.

Remark 4.1: The projection operator in (16) is used to keep our estimation within a-priori bounded set by the property (p1). Since from Assumption 3.1, we know that $|\bar{\varphi}_i| \le (1/m_{L_i} - 1/m_{U_i})/2$, we can choose the parameter B_i of the projection operator as $B_i < (1/m_{L_i} - 1/m_{U_i})/2 - \varepsilon$, where B and ε are defined earlier. By this selection, one can guarantee that the estimations $\hat{\bar{\varphi}}_i, \hat{\varphi}_i$ are always bounded a priori and $\hat{\varphi}_i$ never touches zero. This avoids the possible singularity in the extraction algorithm (i.e. $\hat{\varphi}_i \neq 0$).

It is to be noted that from the property of saturation function and (17), each component of F_i is bounded in advance by $|F_i^{(j)}| < (k_{\theta_i} + k_{\dot{\theta}_i})M + u_{m_i} + \bar{v}_d$; hence viewing Assumption 3.2, by proper selection of the gains the feasibility condition of the extraction algorithm, defined in requirement 1, can be met. Moreover, boundedness of F_i results in a bounded $\tilde{F}_i$ which benefits us in handling the perturbation term in the overall Lyapunov function. From (4), the extraction equations and utilising the inequality $\|(R(\boldsymbol{q}_{d_i}) - R(\boldsymbol{q}_i))^{\mathrm{T}}\hat{z}\| < 2\sqrt{2}\|\tilde{q}_i\|$ (inequality (33) in Abdessameud & Tayebi, 2009), we have

$$\|\tilde{F}_i\| = \|F_i - g\hat{z}\| \left\| \left(R(\boldsymbol{q}_{d_i}) - R(\boldsymbol{q}_i)\right)^{\mathrm{T}} \hat{z} \right\| \le 2\Upsilon_i \|\tilde{q}_i\|, \tag{19}$$

with

$$\Upsilon_i = \sqrt{2}\left(g + \sqrt{3}M\left(k_{\theta_i} + k_{\dot{\theta}_i}\right) + \sqrt{3}u_{m_i} + \bar{v}_d\right). \tag{20}$$

Let us define a positive definite function for the ith TPV

$$V_{1i} = \frac{1}{2} e_i^{\mathrm{T}} e_i + \frac{\kappa_i^2}{2\lambda_{\kappa_i}} + \frac{1}{2\lambda_{\varphi_i}} \tilde{\varphi}_i^2. \tag{21}$$

Taking the time derivative of V_{1i} along (15), we get

$$\dot{V}_{1i} = -c_{2_i}\|e_i\|^2 + e_i^{\mathrm{T}}\tilde{F}_i + e_i^{\mathrm{T}}u_{1_i} + e_i^{\mathrm{T}}w(t) + \kappa_i\dot{\kappa}_i/\lambda_{\kappa_i}. \tag{22}$$

Upon using Assumption 3.2, the property (p2) of the projection operator and along with the following inequality:

$$\frac{e_i^{\mathrm{T}}z_i}{\sqrt{\|z_i\|^2 + (\kappa_i\sigma_i)^2}} \geq \|e_i\|\left(1 - \left(1 + \frac{\|z_i\|}{\sqrt{\|z_i\|^2 + (\kappa_i\sigma_i)^2}}\right)\right),$$

we get

$$\dot{V}_{1i} \leq -\left(c_{2_i} - \epsilon_i\right)\|e_i\|^2 - \|e_i\|\left(u_{m_i} - \bar{w}_i\right) + \frac{\Upsilon_i^2}{\epsilon_i}\|\tilde{q}_i\|^2, \tag{23}$$

for some positive scalar ϵ_i, where (19), and $2ab < \epsilon a^2 + b^2/\epsilon$ were used.

4.2. Attitude control

Taking the attitude error and angular velocity error, respectively, by $\tilde{\boldsymbol{q}}_i = \boldsymbol{q}_{d_i}^{-1} \odot \boldsymbol{q}_i$ and $\tilde{\omega}_i = \omega_i - R(\tilde{\boldsymbol{q}}_i)\omega_{d_i}$, the attitude error dynamics can be written as

$$\dot{\tilde{q}}_i = \tfrac{1}{2}\left(\tilde{\eta}_i I_3 + \tilde{q}_i^{\times}\right)\tilde{\omega}_i, \quad \dot{\tilde{\eta}}_i = -\tfrac{1}{2}\tilde{q}_i^{\mathrm{T}}\tilde{\omega}_i, \tag{24}$$

$$J_i\dot{\tilde{\omega}}_i = -\omega_i^{\times}J_i\omega_i + J_i\left(\tilde{\omega}_i^{\times}R(\tilde{\boldsymbol{q}}_i)\omega_{d_i} - R(\tilde{\boldsymbol{q}}_i)\dot{\omega}_{d_i}\right) + \Gamma_i + d_i(t). \tag{25}$$

Consider the following transformation for the ith TPV

$$\Omega_i = \tilde{\omega}_i + c_{3_i}\tilde{q}_i, \quad c_{3_i} > 0. \tag{26}$$

We design the torque controller for each vehicle by

$$\Gamma_i = -c_{4_i}\Omega_i - k_{qi}\tilde{q}_i - \nu_i, \quad c_{4_i}, k_{qi} > 0, \tag{27}$$

$$\nu_i = \begin{cases} \left(\hat{\mu}_{1_i} + \hat{\mu}_{2_i}\|\tilde{\omega}_i\|\right)\dfrac{\Omega_i}{\|\Omega_i\|} & \|\Omega_i\| \neq 0, \\ 0 & \|\Omega_i\| = 0, \end{cases} \tag{28}$$

$$\dot{\hat{\mu}}_{1_i} = k_{\mu_{1i}}\|\Omega_i\|, \quad k_{\mu_{1i}}, \ \hat{\mu}_{1_i}(0) > 0, \tag{29}$$

$$\dot{\hat{\mu}}_{2_i} = k_{\mu_{2i}}\|\tilde{\omega}_i\|\|\Omega_i\|, \quad k_{\mu_{2i}}, \ \hat{\mu}_{2_i}(0) > 0, \tag{30}$$

where $\hat{\mu}_{1_i}$ and $\hat{\mu}_{2_i}$ are, respectively, the estimation of the unknown constants μ_{1_i} and μ_{2_i} defined as follows:

$$\bar{J}_i\left(c_{3_i}\|\omega_{d_i}\| + \|\omega_{d_i}\|^2 + \frac{\bar{d}_i}{\bar{J}_i} + \|\dot{\omega}_{d_i}\|\right) \leq \mu_{1_i}, \tag{31}$$

$$\bar{J}_i\left(c_{3_i} + 2\|\omega_{d_i}\| + \frac{c_{3_i}}{2}\right) \leq \mu_{2_i}. \tag{32}$$

Note that taking (31)–(32) is sensible if ω_{d_i} and $\dot{\omega}_{d_i}$ are bounded which can be proven similar to Kabiri et al. (2017a). Now consider the following positive definite function as

$$V_{2i} = \frac{1}{2}\Omega_i^{\mathrm{T}}J_i\Omega_i + 2k_{qi}\left(1 - \tilde{\eta}_i\right) + \frac{1}{2k_{\mu_{2_i}}}\tilde{\mu}_{1_i}^2 + \frac{1}{2k_{\mu_{1_i}}}\tilde{\mu}_{2_i}^2, \tag{33}$$

with $\tilde{\mu}_{1_i} = \mu_{1_i} - \hat{\mu}_{1_i}, \tilde{\mu}_{2_i} = \mu_{2_i} - \hat{\mu}_{2_i}$. Using $\omega_i = \tilde{\omega}_i + R(\tilde{\boldsymbol{q}}_i)\omega_{d_i} = -\Omega_i - c_{3_i}\tilde{q}_i + R(\tilde{\boldsymbol{q}}_i)\omega_{d_i}$, the derivative of (33) along (24)–(25) is obtained by

$$\begin{aligned}\dot{V}_{2i} = &-\Omega_i^{\mathrm{T}}\left(-\Omega_i - c_{3_i}\tilde{q}_i + R(\tilde{\boldsymbol{q}}_i)\omega_{d_i}\right)^{\times} \\ &\times J_i\left(\tilde{\omega}_i + R(\tilde{\boldsymbol{q}}_i)\omega_{d_i}\right) \\ &+ \Omega_i^{\mathrm{T}}J_i\left(\tilde{\omega}_i^{\times}R(\tilde{\boldsymbol{q}}_i)\omega_{d_i} - R(\tilde{\boldsymbol{q}}_i)\dot{\omega}_{d_i}\right) \\ &+ \frac{c_{3_i}}{2}\Omega_i^{\mathrm{T}}J\left(\tilde{q}_i^{\times} + \tilde{\eta}_i\right)\tilde{\omega}_i + \Omega_i^{\mathrm{T}}\Gamma_i + \Omega_i^{\mathrm{T}}d_i(t) \\ &+ k_{qi}\tilde{q}_i^{\mathrm{T}}\tilde{\omega}_i - \tilde{\mu}_{1_i}\|\Omega_i\| - \tilde{\mu}_{2_i}\|\Omega_i\|\|\tilde{\omega}_i\|. \end{aligned} \tag{34}$$

Upon exploiting the cross-product property that $\Omega^{\mathrm{T}}\Omega^{\times} = 0$, along with $\|\boldsymbol{q}\| < 1$, the fact the rotation matrix R(.) does not change the norm of a vector, and using Γ_i given by (27), one could get

$$\begin{aligned}\dot{V}_{2i} \leq &\|\Omega_i\|\left(\mu_{1_i} + \mu_{2_i}\|\tilde{\omega}_i\|\right) - c_{3_i}k_{qi}\|\tilde{q}_i\|^2 - c_{4_i}\Omega_i^{\mathrm{T}}\Omega_i \\ &- \|\Omega_i\|\left(\hat{\mu}_{1_i} + \hat{\mu}_{2_i}\|\tilde{\omega}_i\|\right) - \tilde{\mu}_{1_i}\|\Omega_i\| \\ &- \tilde{\mu}_{2_i}\|\Omega_i\|\,\|\tilde{\omega}_i\|,\end{aligned}$$

which leads to

$$\dot{V}_{2i} \leq -c_{4_i}\|\Omega_i\|^2 - c_{3_i}k_{qi}\|\tilde{q}_i\|^2. \tag{35}$$

The positive definite function proposed here is used in stability analysis of the overall closed-loop system given in the next section.

5. Stability analysis

5.1. Stability analysis in deterministic networks

Let summarise our main result in the following theorem.

Theorem 5.1: *Consider formation of a network of multiple TPVs under jointly connected switching topology with time-delay which satisfies the condition given in Assumption 4.1. Let the model of each TPV given by* (1) *and the desired velocity* $v_d(t)$ *satisfies Assumptions* 3.2–3.3 *and gains are chosen such that*

$$M\left(k_{\theta_i} + k_{\dot{\theta}_i}\right) + u_{m_i} + \bar{v}_d < g, \quad u_{m_i} > \bar{w}_i,$$

$$c_{2_i} > \epsilon_i, \quad c_{3_i}k_{qi} > \frac{\Upsilon_i^2}{\epsilon_i}, \tag{36}$$

with Υ_i *given in* (20) *and* ϵ_i *is some arbitrary positive constant. Let the thrust input* $\mathcal{T}_i$ *be given by* (5) *along with*

(16), *the intermediate control input* F_i (10), *and Equations* (11)–(14), (17)–(18). *Consider the torque input* Γ_i *be given in* (27) *with* (28)–(30). *Then the formation problem is globally asymptotically solved, i.e. for each* $i, j \in [1, \ldots, n]$, *we have* $v_i \to v_d(t)$ *and* $p_i - p_j - \delta_{ij} \to 0$.

Proof: The condition (36) guarantees that the extraction of the desired attitude is always possible. Let introduce the following Lyapunov function for the complete system as $V = \sum_{i=1}^{n} V_i$, with $V_i = (V_{1i} + V_{2i})$ and V_{1i} and V_{2i} are given, respectively, in (21) and (33). Differentiating V along (9), (24) and (25) leads to

$$\dot{V} \leq \sum_{i=1}^{n} \Bigg[-\left(c_{2_i} - \epsilon_i\right) \|e_i\|^2 - \|e_i\| \left(u_{m_i} - \bar{w}_i\right) - \left(c_{3_i} k_{qi} - \frac{\Upsilon_i^2}{\epsilon_i}\right) \|\tilde{q}_i^2\| - c_{4_i} \|\Omega_i\|^2 \Bigg], \tag{37}$$

which is negative semi-definite, if condition (36) is satisfied. Hence, we can conclude that signals e_i, κ_i, $\tilde{\varphi}_i$, $\tilde{\mu}_{1_i}$, $\tilde{\mu}_{2_i}$ and Ω_i are bounded for $i = 1, \ldots, n$. Invoking Barbalat's Lemma (Slotine et al., 1991), convergence of e_i, $\tilde{q}_i$ and Ω_i to zero is concluded which is followed by $\tilde{q}_i \to 0$ and $\tilde{\omega}_i \to 0$. From boundedness and convergence of e_i to zero, we can prove that z_i, $\dot{z}_i$ are also bounded and converge to zero. Now we show that both $(\dot{\xi}_i - v_d)$ and $(\xi_i - \xi_j - \delta_{ij})$ are bounded and converge to zero. Making the transformation $x_i = \xi_i - \int_0^t v_d(\tau)\,\mathrm{d}\tau - \delta_i$ and $\upsilon_i = \dot{\xi}_i - v_d(t)$, (14) can be rewritten as

$$\dot{x}_i = \upsilon_i, \tag{38}$$

$$\dot{\upsilon}_i = -k_1 \upsilon_i - \sum_{j \in \mathcal{N}_i} a_{ij}^{\sigma} (x_i(t-\tau) - x_j(t-\tau)), \tag{39}$$

which is the same as the model (1) with consensus protocol (2) in Lin and Jia (2010). Therefore applying Theorem 1 in Lin and Jia (2010), it is proved that if the condition given in Assumption 4.1 is met, x_i and υ_i are bounded and the average consensus is achieved for (38)–(39), and finally, we have $\upsilon_i \to 0$, $x_i \to x_j$ for $i, j = 1, \ldots, n$ which is followed by $\dot{\xi} \to v_d(t)$ and $\xi_i - \xi_j \to \delta_{ij}$ for $i, j = 1, \ldots, n$. From this and boundedness of z_i and $\dot{z}_i$ which results in bounded u_{2_i} that converges to zero, we can easily see by Lemma 1 in Abdessameud and Tayebi (2011) that α_i and $\dot{\alpha}_i$ are bounded and converge to zero (u_{2_i} is a vanishing perturbation for (13)). The same fact also applies to prove that θ_i and $\dot{\theta}_i$ are bounded and converge to zero. Finally, regarding $z_i = p_i - \theta_i - \alpha_i - \xi_i$, we obtain $v_i \to v_d(t)$ and $p_i - p_j - \delta_{ij} \to 0$. ■

As it can be observed from (5) and (17), the proposed structure suffers from potential converging of κ_i to zero. Taking the similar procedure as in Kabiri et al. (2017a), it can be shown that one can keep κ_i away from zero by the following bound:

$$\kappa_i(t) \geq \sqrt{\kappa_i^2(0) - 4 u_{m_i} \lambda_{\kappa_i} \rho_i}. \tag{40}$$

Hence, the potential singularity can be avoided by proper selection of initial values $\kappa_i(0)$ and the gains λ_{κ_i}.

Remark 5.1: It is to be noted that the proposed control scheme can be applied for trajectory tracking of the desired path $p_d(t)$ for a single TPV with disturbances and uncertainties provided that $\sup_{t>0}(\|\ddot{p}_d(t)\| + \|w(t)\|) < g$. In that case, the need for the system in (14) is obviated and the terms with $z(t)$ and $\dot{z}(t)$ in u_1 and u_2 should be substituted with respectively $(z(t) - p_d(t))$ and $(\dot{z}(t) - \dot{p}(t))$.

Remark 5.2: The proposed control scheme employing the virtual agents completely separates the design into two parts: coordination design and tracking design. Hence, other investigations on cooperation of second-order systems of the form $\ddot{\xi} = u_i$ can be directly adopted to our case provided that the designed u_i is globally bounded and converges to zero, for example, our result can be easily extended to switching topology information exchange and non-uniform time-varying delays case using the results in Yang et al. (2008).

Remark 5.3: The proposed control can be used for cooperative control for other types of under-actuated systems with little adaptation such as UGVs in Almayyahi, Wang, Hussein, and Birch (2017) and the under-actuated aircraft in Olfati-Saber (2002). Moreover, our structure can be beneficial whenever the robust cooperative differentiable controller is in demand.

5.2. Stability analysis in stochastic networks

In this section, due to the stochastic nature of communication channels and existing packet losses, we consider the topology that switch in a stochastic manner. In each switching period, the existence of a link between two agents in the communication topology of the network is probabilistic. This kind of topology can be modelled by a random graph in which the existence of the edge between the ith and jth nodes is a stochastic variable with probability $p_{ij} = p_{ji} \in [0, 1]$. In other words, the probability matrix P for the stochastic network topology represents the probability of existence a link between agents. For instance, by $P_{12} = 0.3$, we mean that at each switching cycle, the agents 1 and 2 would be connected with the probability of 0.3. Suppose that the sample space of all possible random graphs on n nodes associated with the edge probability matrix $P = [p_{ij}]$ is defined by $\mathbf{G}(n, P)$. Therefore, it can be obtained that a random graph is, in general, a switching graph. In this case, the switching function is a stochastic process and is shown by $\vartheta(t) : [0, \infty] \to \left\{1, 2, \ldots, 2^{\binom{n}{2}/2}\right\}$. In this condition, the adjacency matrix associated with the MAS communication topology is considered as $A^{\vartheta(t)}(t) : [a_{ij}^{\vartheta(t)}]$, where

$$a_{ij}^{\vartheta(t)} = \begin{cases} a_{ij} & \text{with probability } p_{ij}, \\ 0 & \text{with probability } 1 - p_{ij}. \end{cases}$$

Hence, similar to the deterministic switching topology, there exists a sequence of non-overlapping bounded subintervals $[t_{r_j}, t_{r_{j+1}}]$ on which the communication topology is fixed and it switches at t_{r_j}. We define $\pi = \mathbf{Pr}\{G \in \mathbf{G}(n, P)\}$ as the probability of existing at least one connected graph in $\mathbf{G}(n, P)$.

Before presentation of the main results on reaching formation in a network of TPVs under stochastic topology, let us introduce some preliminaries in stochastic process which our subsequent results are based on. Let $(\Omega, \mathcal{F}, \mu)$ be a probability space, where Ω is the space of events, $\mathcal{F}$ is the Borel-algebra of Ω and μ is a probability measure defined on Ω (Williams, 1991). A collection $\{\mathcal{F}_t\}_{t\geqslant 0}$ of sub σ-algebras is called filtration if, for every $s \leqslant t$, we have $\mathcal{F}_s \subseteq \mathcal{F}_t$. The stochastic process $X = \{X(t),\ t \geq 0\}$ is called adapted to the filtration $\mathcal{F}_t$ if, for every t, $X(t)$ is measurable with respect to $\mathcal{F}_t$.

Definition 5.1 (Williams, 1991): A stochastic process X is a martingale relative to $(\{\mathcal{F}_t\}_{t\geqslant 0}, \mu)$, if the following conditions hold:

(i) $X(t)$ is $\{\mathcal{F}_t\}_{t\geqslant 0}$-adapted,
(ii) $\mathbf{E}[|X(t)|] < \infty$ for all $t > 0$,
(iii) $\mathbf{E}[X(t)|\mathcal{F}_s] \leqslant X(s),\ t > s$.

Definition 5.2 (Tempo, Calafiore, & Dabbene, 2012): (Almost sure convergence or convergence with probability 1) A sequence of random variables $X(t)$ is said to converge almost surely or with probability 1 (denoted by a.s. or w.p. 1) to X_∞ if

$$\mathbf{Pr}[\lim_{t\to\infty} X(t) = X_\infty] = 1.$$

In other words, almost sure convergence forces the random variables $X(t)$ not to converge on a set of zero measure. Now, we are in the position to state the super-martingales convergence theorem as follows:

Theorem 5.2 (Mahmoud, Jiang, & Zhang, 2003): *Suppose that $\{X(t),\ t \geqslant 0\}$ is a nonnegative super-martingale with respect to $(\{\mathcal{F}_t\}_{t\geqslant 0}, \mu)$. Then there exists a random variable X_∞, measurable with respect to $\mathcal{F}_\infty$, such that*

$$\lim_{t\to\infty} X(t) \xrightarrow{a.s.} X_\infty.$$

Now, we investigate the consensus for second-order MASs under time-delayed stochastic communications. Consider a network of second-order agents described by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{41}$$

with the following consensus protocol under time-delay τ and a stochastic topology $G^{\vartheta(t)} \in \mathbf{G}(n, P)$

$$u_i = -k_1 v_i(t) - \sum_{j\in N_i(t)} a_{ij}^{\vartheta(t)}(x_i(t-\tau) - x_j(t-\tau)). \tag{42}$$

Definition 5.3: The network of agents (41) is said to asymptotically achieve almost sure second-order consensus, i.e. $x(t) \xrightarrow{a.s.} span\{\mathbf{1}\}$ and $v(t) \xrightarrow{a.s.} \mathbf{0}$ if

$$\mathbf{Pr}[\lim_{t\to\infty} X(t) \in \text{span}\{\mathbf{1}\}\ \&\ \lim_{t\to\infty} V(t) = \mathbf{0}] = 1,$$

where $x(t) = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}]^{\mathrm{T}}$, $v(t) = [v_1^{\mathrm{T}}, v_2^{\mathrm{T}}, \ldots, v_n^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{Pr}$ indicates probability. This means that the events other than reaching consensus in the network, on both position and velocity variables, have zero probabilities.

Assumption 5.1: *Assume that network topology is a random graph with $\pi = \mathbf{Pr}[G \in \mathbf{G}(n, P)\text{isconnected}] = 1$. In other words, there exists a connected graph G in $\mathbf{G}(n, P)$ with non-zero probability.*

Lemma 5.1: *Suppose that the stochastic topology satisfies Assumption* 5.1 *and time-delay satisfies the condition given in Assumption* 4.1. *Then, the consensus protocol* (42) *guarantees the asymptotic almost sure second-order consensus in the stochastic multi-agent system* (41).

Proof: Using a variable transformation $\bar{v}_i = 2v_i/k_1 + x_i$ and by considering $\eta = [x_1^{\mathrm{T}}, \bar{v}_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \bar{v}_2^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}, \bar{v}_n^{\mathrm{T}}]$, the closed-loop dynamics of the whole MAS (41) under control strategy (42) can be restated as

$$\dot{\eta}(t) = (I_n \otimes A)\eta(t) - (\mathcal{L}_{\vartheta(t)} \otimes B)\eta(t-\tau) \tag{43}$$

with

$$A = \begin{bmatrix} -\frac{k_1}{2}I_3 & \frac{k_1}{2}I_3 \\ \frac{k_1}{2}I_3 & -\frac{k_1}{2}I_3, \end{bmatrix}, \quad B = \begin{bmatrix} \mathbf{0}_3 & \mathbf{0}_3 \\ \frac{2}{k_1}I_3 & \mathbf{0}_3 \end{bmatrix}.$$

It can be easily verified that $\alpha = (1/2n)\sum_{i=1}^{n} \bar{v}(t) + x_i(t)$ is an invariant quantity under above dynamics. Therefore, by defining disagreement vector as $\delta(t) = \eta(t) - \alpha\mathbf{1}$, (43) can be restated as follows:

$$\dot{\delta}(t) = (I_n \otimes A)\delta(t) - \left(\mathcal{L}_{\vartheta(t)} \otimes B\right)\delta(t-\tau). \tag{44}$$

From the definition of α and δ, it can be easily seen that $\mathbf{1}^{\mathrm{T}}\delta = 0$. Define a Lyapunov–Krasovskii function $\tilde{V}(t)$ for system (44) as what is defined in (14) of Lin and Jia (2010). Suppose that the communication topology $G^{\vartheta(t)}$ has $l_{\vartheta(t)} \geq 1$ connected subgraphs at time t and the number of agents in each subgraph is defined by $d^i_{\vartheta(t)}$, $i = 1, 2, \ldots, l_{\vartheta(t)}$. Pursuing the same line of proof of Theorem 1 in Lin and Jia (2010), under the condition of Assumption 4.1, we obtain that the derivative of $\tilde{V}(t)$ satisfies

$$\dot{\tilde{V}}(t) \leq \lambda_{\max} \sum_{i=1}^{l_{\vartheta(t)}} \|\delta^i_{\vartheta(t)} - \varphi^i_t \mathbf{1}\|^2 \leq 0, \tag{45}$$

where $\delta^i_{\vartheta(t)}$, $i = 1, \ldots, l_\vartheta$, is the disagreement vector corresponding to the ith connected component of $G^{\vartheta(t)}$. $\lambda_{\max}$ is a negative value calculated based on the sequence of switching topology and $\varphi^i_t = \mathbf{1}^{\mathrm{T}}\delta^i_{\varphi(t)}/2d^i_{\vartheta(t)}$ is an invariant quantity of network between two consequent switches of the topology. By some calculations, one can get

$$\left(\delta^i_{\vartheta(t)} - \varphi^i_t\mathbf{1}\right)^{\mathrm{T}}\left(\delta^i_{\vartheta(t)} - \varphi^i_t\mathbf{1}\right) = \delta^{i^{\mathrm{T}}}_{\vartheta(t)}\delta^i_{\vartheta(t)} - \frac{\left(\mathbf{1}^{\mathrm{T}}\delta^i_{\vartheta(t)}\right)^2}{2d^i_{\vartheta(t)}}$$
$$= \frac{1}{2d^i_{\vartheta(t)}}\delta^{i^{\mathrm{T}}}_{\vartheta(t)}\left(2d^i_{\vartheta(t)}I_{2d^i_{\vartheta(t)}} - \mathbf{1}\mathbf{1}^{\mathrm{T}}\right)\delta^i_{\vartheta(t)}.$$

Since the system (44) satisfies the conditions of the extended invariance principle for non-autonomous systems, Theorem 2.11 of Barkana (2014), it can be obtained from (45) that

$$\lim_{t\to\infty}\sum_{i=1}^{l_{\vartheta(t)}}\frac{1}{2d^i_{\vartheta(t)}}\delta^{i^{\mathrm{T}}}_{\vartheta(t)}\left(2d^i_{\vartheta(t)}I_{2d^i_{\vartheta(t)}} - \mathbf{1}\mathbf{1}^{\mathrm{T}}\right)\delta^i_{\vartheta(t)} = 0. \tag{46}$$

Since $\tilde{V}(t)$ is a stochastic variable at time t, (46) yields that the evolution of the quantity $\mathbf{E}[\dot{\tilde{V}}(t) \mid \delta(t)]$ according to the trajectories of the stochastic dynamical system (44) converges to zero too, i.e.

$$\lim_{t\to\infty}\sum_{i=1}^{l_{\vartheta(t)}}\frac{1}{2d^i_{\vartheta(t)}}\delta^{i^{\mathrm{T}}}_{\vartheta(t)}\mathbf{E}\left[2d^i_{\vartheta(t)}I_{2d^i_{\vartheta(t)}} - \mathbf{1}\mathbf{1}^{\mathrm{T}}\right]\delta^i_{\vartheta(t)} = 0. \tag{47}$$

Now, by defining

$$\hat{\mathcal{L}}^i_{\vartheta(t)} = 2d^i_{\vartheta(t)}I_{2d^i_{\vartheta(t)}} - \mathbf{1}\mathbf{1}^{\mathrm{T}}, \quad i = 1, 2, \ldots, l_{\vartheta(t)},$$

$\hat{\mathcal{L}}^i_{\vartheta(t)}$ can be considered as the Laplacian of a complete subgraph on $2d^i_{\vartheta(t)}$ nodes. Define the collective disagreement vector of the network as $\delta(t) = [\delta^{1^{\mathrm{T}}}_{\vartheta(t)}, \delta^{2^{\mathrm{T}}}_{\vartheta(t)}, \ldots, \delta^{l^{\mathrm{T}}_{\vartheta(t)}}_{\vartheta(t)}]^{\mathrm{T}}$, Equation (47) can be restated as

$$\lim_{t\to\infty}\delta^{\mathrm{T}}(t)\mathbf{E}[\hat{\mathcal{L}}_{\theta(t)}]\delta(t) = 0, \tag{48}$$

where $\hat{\mathcal{L}}_{\vartheta(t)} = \mathrm{diag}(\mathcal{L}^1_{\vartheta(t)}, \ldots, \mathcal{L}^{l_{\vartheta(t)}}_{\vartheta(t)})$. $\hat{\mathcal{L}}_{\vartheta(t)}$ is associated with a graph $\hat{G}_{\vartheta(t)}$ that consists of $l_{\vartheta(t)} \geq 1$ disjoint complete subgraphs each with $2d^i_{\vartheta(t)}, i = 1, 2, \ldots, l_{\vartheta(t)}$ nodes. Note that $l_{\vartheta(t)}$ and $d^i_{\vartheta(t)}$ are characteristics of $G^{\vartheta(t)}$. Hence, we can say that, inspired by the main network topology, $\hat{G}_{\vartheta(t)}$ is constructed and the number of disjoint subgraphs of $\hat{G}_{\vartheta(t)}$ is the same as $G_{\vartheta(t)}$. Now, we will show that if $\pi = \mathbf{Pr}[G \in \mathbf{G}(N, P)$isconnected$] = 1$, the only solution of (48) is $\delta(t) = 0$. Note that

$$\mathbf{E}[\hat{\mathcal{L}}_{\vartheta(t)}] = \sum_{i=1}^{2^{\binom{n}{2}/2}} p_i\hat{\mathcal{L}}_i,$$

where $\hat{\mathcal{L}}_i$ represents the Laplacian corresponding to graph $\hat{G}_{\vartheta(t)}$, $\vartheta(t) \in \{1, 2, \ldots, 2^{\binom{n}{2}/2}\}$ and p_i is the probability of occurrence of $\hat{G}_{\vartheta(t)}$. Since for all $i \in \{1, 2, \ldots, 2^{\binom{n}{2}/2}\}$, the Laplacian $\hat{\mathcal{L}}_i$ is positive semi-definite and $p_i > 0$, one must have for all $\hat{\mathcal{L}}_i$, $i \in \{1, 2, \ldots, 2^{\binom{n}{2}/2}\}$:

$$\lim_{t\to\infty}\delta^{\mathrm{T}}(t)\hat{\mathcal{L}}_i\delta(t) = 0. \tag{49}$$

Given the non-zero probability of existing a connected graph G in $\mathbf{G}(n, P)$, it can be obtained that the probability of existence of a complete graph on $2n$ nodes in the set of possible $\hat{\mathcal{L}}_i$ is non-zero too. Therefore, based on Theorem 1 of Olfati-Saber and Murray (2004), Equation (49) for such a complete graph yields that $\delta(t) \in \mathrm{span}\{1\}$. On the other hand, we have $\mathbf{1}^{\mathrm{T}}\delta(t) = 0$ and as a result, the only solution of (48) is $\delta(t) = 0$. By considering all the above-mentioned arguments, if $\pi = \mathbf{Pr}[G \in$

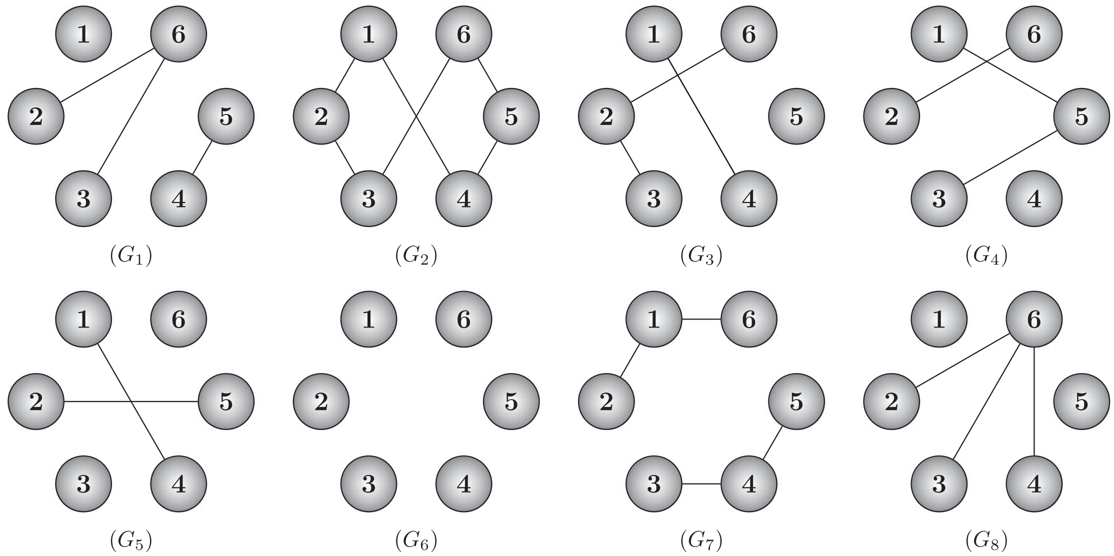

Figure 2. Eight network topologies for the deterministic switching topology case.

$\mathbf{G}(N, P)$isconnected$] = 1$, for any $\varepsilon > 0$, we have

$$\lim_{t\to\infty} \mathbf{Pr}[\|\delta(t)\| \geqslant \varepsilon] = 0. \tag{50}$$

By considering $\dot{\tilde{V}}(t) \leqslant 0$ from (45), one can conclude that $\tilde{V}(t)$ satisfies the conditions (ii) and (iii) of supermartingales given in Definition 5.1. Let the filtration $\mathcal{F}_t$ be defined as follows:

$$\mathcal{F}_t = \{\delta(s),\ 0 \leqslant s \leqslant t\}.$$

Hence $\tilde{V}(t)$ satisfies all the conditions of supermartingales given in Definition 5.1, and therefore, Theorem 5.2 guarantees that

$$\lim_{t\to\infty} \tilde{V}(t) \xrightarrow{a.s.} \tilde{V}_\infty.$$

Moreover, it can be easily verified from (50) that $\tilde{V}_\infty = 0$ and finally, we obtain that

$$\lim_{t\to\infty} \delta(t) \xrightarrow{a.s.} 0$$

and consequently $\eta(t)$ almost surely converges to $\alpha = (1/2n)\sum_{i=1}^{2n} \bar{v}_i(0) + x_i(0)$. Therefore, from the definition of $\eta(t)$ one can observe that $x_i(t) \xrightarrow{a.s.} \alpha$ and $v_i(t) \xrightarrow{a.s.} 0$. Thus almost sure second-order consensus is achieved in the MAS and the proof is completed. ■

Next we show that by invoking Lemma 5.1 and under some LMI conditions, second-order consensus can be achieved almost surely in the system (41)–(42).

Theorem 5.3: *Consider the formation of a network of multiple TPVs under a stochastic topology satisfying Assumption 5.1 and time delay which satisfies the LMI conditions given in Assumption 4.1. Let the model of each TPV given by (1) and the desired velocity* $v_d(t)$

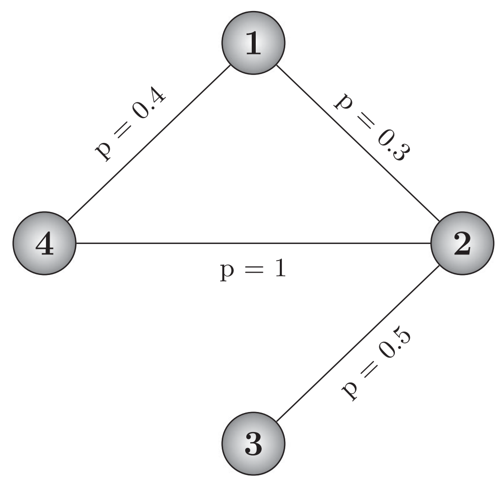

Figure 3. The network topologies for the stochastic switching topology case.

Table 1. Simulation parameters.

$w_i(t) = (\sin(0.1t), -1.5\sin(0.1t), 0.7\cos(0.2t))^{\mathrm{T}}$ (m/s^2)
$d_i(t) = (2\sin(t), 3\cos(.2t), \sin(0.3t))^{\mathrm{T}}$ (N.m)
$k_1 = 10, \varphi_{0_i} = 10, B_i = 5, \varepsilon = 0.5, \bar{\delta} = 0.01$
$\hat{\hat{\varphi}}_1(0) = -2, \hat{\hat{\varphi}}_2(0) = -1, \hat{\hat{\varphi}}_3(0) = 1, \hat{\hat{\varphi}}_4(0) = 2$
$\boldsymbol{q}_i(0) = [0, 0, 0, 1]^{\mathrm{T}}, \omega_i(0) = [0, 0, 0]^{\mathrm{T}}$
$k_{\theta_i} = 3, k_{\dot{\theta}_i} = 2, k_{\alpha_i} = 5, k_{\dot{\alpha}_i} = 2, c_{1_i} = 4, c_{2_i} = 1$
$u_{m_i} = 3, \lambda_{\kappa_i} = 0.001, \delta_i = 0.001, \lambda_{\varphi_i} = 4, k_{\mu_{2_i}} = 0.01$
$k_{\mu_{1_i}} = 0.1, c_{3_i} = 10, c_{4_i} = 15, k_{qi} = 20$
$\theta_i = \dot{\theta}_i = \alpha_i = \dot{\alpha}_i = \xi_i = \dot{\xi}_i = 0 \quad$ for $i = 1, \ldots, 4$
$\kappa_1(0) = 2, \kappa_2(0) = 2.5, \kappa_1(0) = 3, \kappa_4(0) = 3.5$
$\hat{\mu}_{1_1}(0) = 4, \hat{\mu}_{1_2}(0) = 1, \hat{\mu}_{1_3}(0) = 3, \hat{\mu}_{1_4}(0) = 2$
$\hat{\mu}_{2_1}(0) = 0.4, \hat{\mu}_{2_2}(0) = 0.2, \hat{\mu}_{2_3}(0) = 0.3, \hat{\mu}_{2_4}(0) = 0.1$

satisfies Assumptions 3.2–3.3 and gains are chosen such as Theorem 5.1. Consider the thrust input $\mathcal{T}_i$ defined as (5) *together with* (16), *the intermediate control input F_i* (10), *as well as Equations* (11)–(14) *and* (17)–(18). *Let the torque input Γ_i be given in* (27) *with* (28)–(30). *Then the formation problem is globally asymptotically almost surely solved, i.e. for each $i, j \in [1, \ldots, n]$, we have $v_i(t) \xrightarrow{a.s.} v_d(t)$ and $p_i - p_j - \delta_{ij} \xrightarrow{a.s.} 0$.*

Proof: The sketch of the proof is similar to Theorem 5.1, except Equations (38)–(39) which are changed to the same form as (41)–(42), due to the stochastic topology of the MAS. Therefore, according to Lemma 5.1, it is proved that if the LMI conditions given in Assumption 4.1 are satisfied and there exists a connected graph G in $\mathbf{G}(n, P)$ with non-zero probability, almost sure second-order consensus is achieved for (41)–(42), and finally pursuing the same lines of proof as in Theorem 5.1, we have $v_i(t) \xrightarrow{a.s.} v_d(t)$ and $p_i - p_j - \delta_{ij} \xrightarrow{a.s.} 0$. ■

6. Simulation results

Numerical simulations are given under both deterministic and stochastic switching topologies to test the proposed scheme. For both cases, we consider all TPVs have the identical mass and inertia matrix as $m = 0.1\,\mathrm{kg}$ and $J = \mathrm{diag}(0.3, 0.4, 0.25)\,\mathrm{kg.m}^2$. We also choose $g = 9.8\,\mathrm{m/s}^2$. For the deterministic case, we consider six TPVs to construct a six-sided polygon formation while tracking the reference velocity $v_d(t) = (\cos(t/5\pi), \sin(t/5\pi), 3\,\mathrm{e}^{-0.3t})^{\mathrm{T}}$. For this purpose, the offsets are selected to be $\delta_i = 2(\cos((i-1)\pi/3), \sin((i-1)\pi/3), 0)$ for $i = 1, \ldots, 6$. For the stochastic switching topologies, we simulated four agents to reach a rectangular formation defined by offsets $\delta_1 = (2, 0, 0)^{\mathrm{T}}$, $\delta_2 = (0, 2, 0)^{\mathrm{T}}$, $\delta_3 = (0, -2, 0)^{\mathrm{T}}$ and $\delta_4 = (-2, 0, 0)^{\mathrm{T}}$ while tracking the reference velocity $v_d(t) = (\cos(t/5\pi), \sin(t/5\pi), 0.1)^{\mathrm{T}}$. The interaction topology between agents is assumed to switch every 0.02 s. For the deterministic case, in each switching period, one of the eight graphs shown in Figure 2 is selected randomly. The probability matrix for the stochastic network is supposed to be

$$P = \begin{pmatrix} 0 & 0.3 & 0 & 0.4 \\ 0.3 & 0 & 0.5 & 1 \\ 0 & 0.5 & 0 & 0 \\ 0.4 & 1 & 0 & 0 \end{pmatrix},$$

where each entry of the matrix represents the probability of existence a link between two agents. The Network topology for the stochastic case is also shown in Figure 3.

The communication delay is assumed to be $\tau = 0.1\,\mathrm{s}$ and $\tanh(.)$ is selected as the smooth saturation function $\varrho(.)$ with $M = 1$. The initial positions and linear velocities for each TPV are selected randomly in $[-5, 5] \times [-5, 5] \times [-5, 5]$. Other gains, parameters and initial

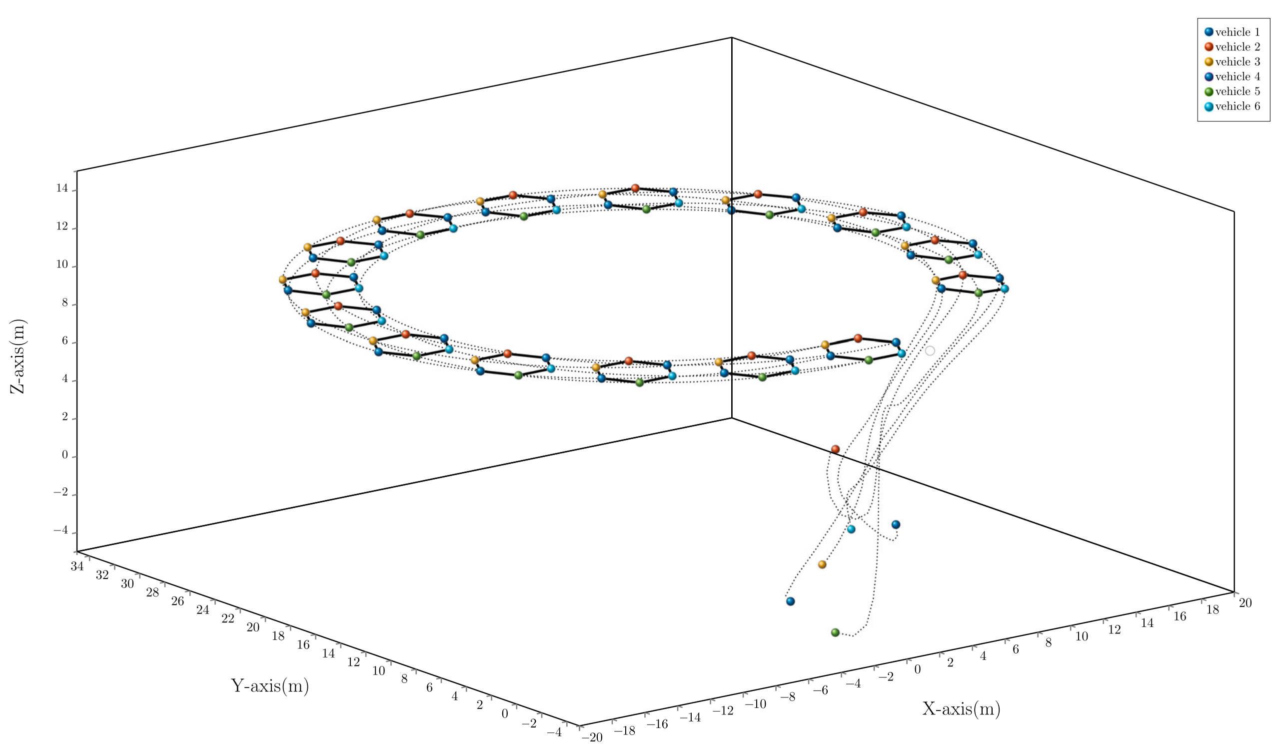

Figure 4. Formation of TPVs for the deterministic switching topology case.

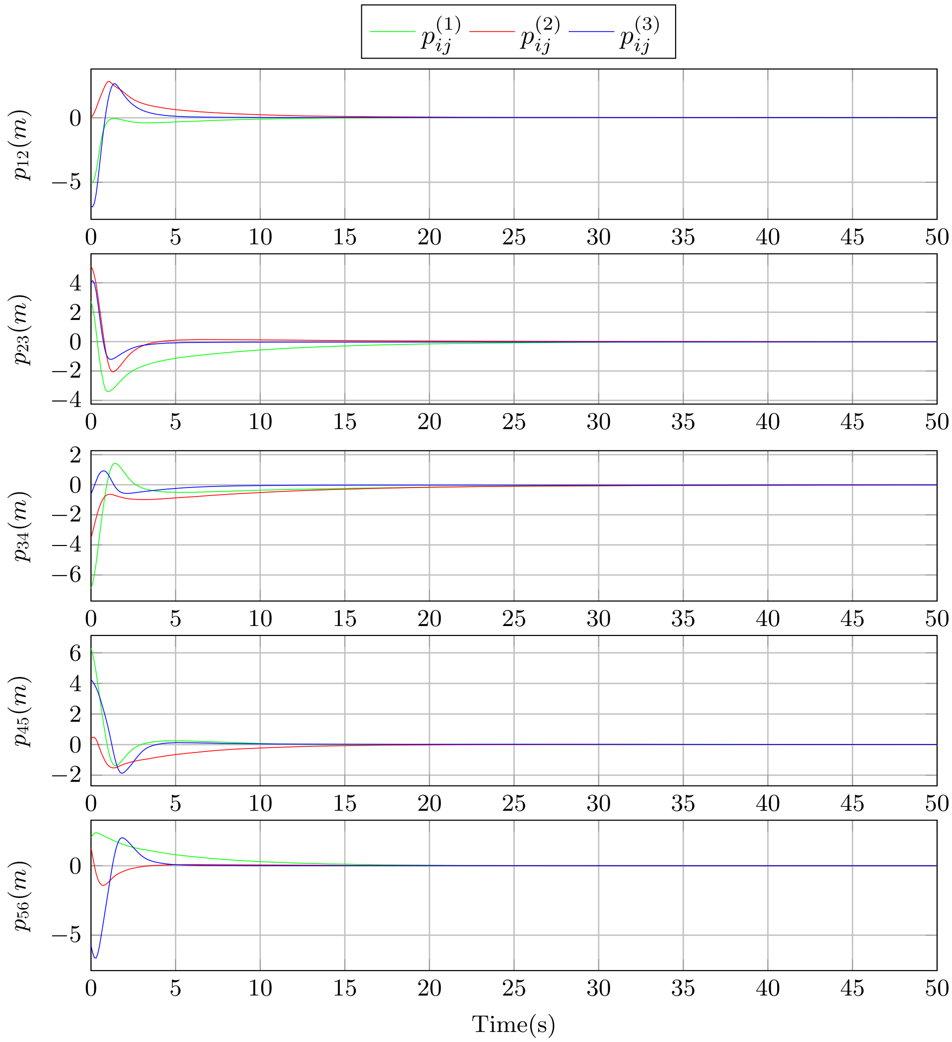

Figure 5. Formation errors for the deterministic switching topology case $p_{ij} = p_i - p_j - \delta_{ij} = (p_{ij}^{(1)}, p_{ij}^{(2)}, p_{ij}^{(3)})^{\mathrm{T}}$.

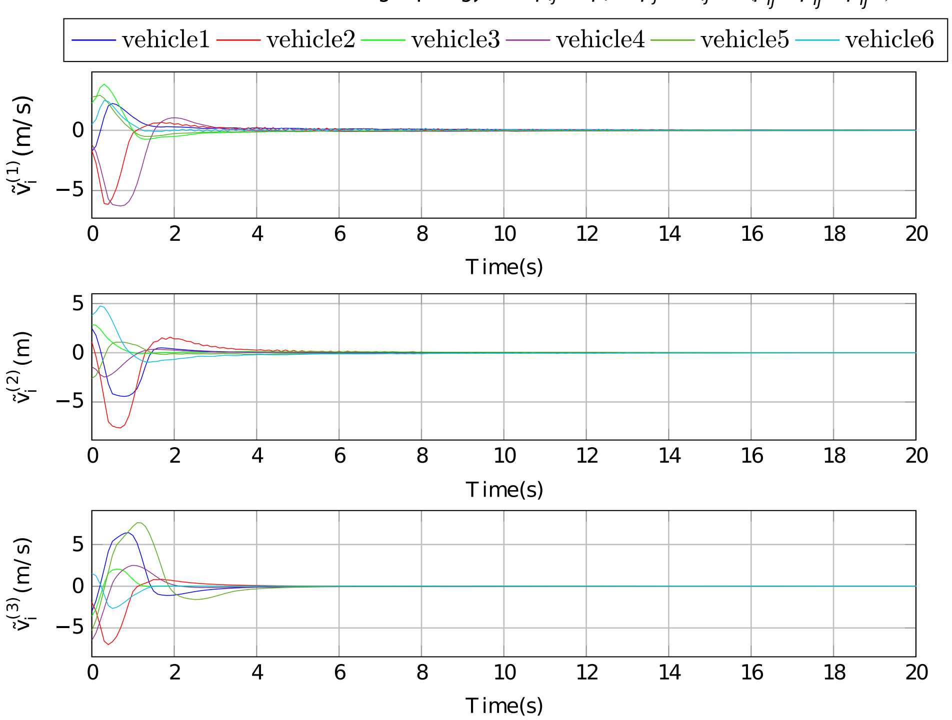

Figure 6. Linear velocity errors for the deterministic switching topology case $\tilde{v}_i = (\tilde{v}_i^{(1)}, \tilde{v}_i^{(2)}, \tilde{v}_i^{(3)})^{\mathrm{T}}$.

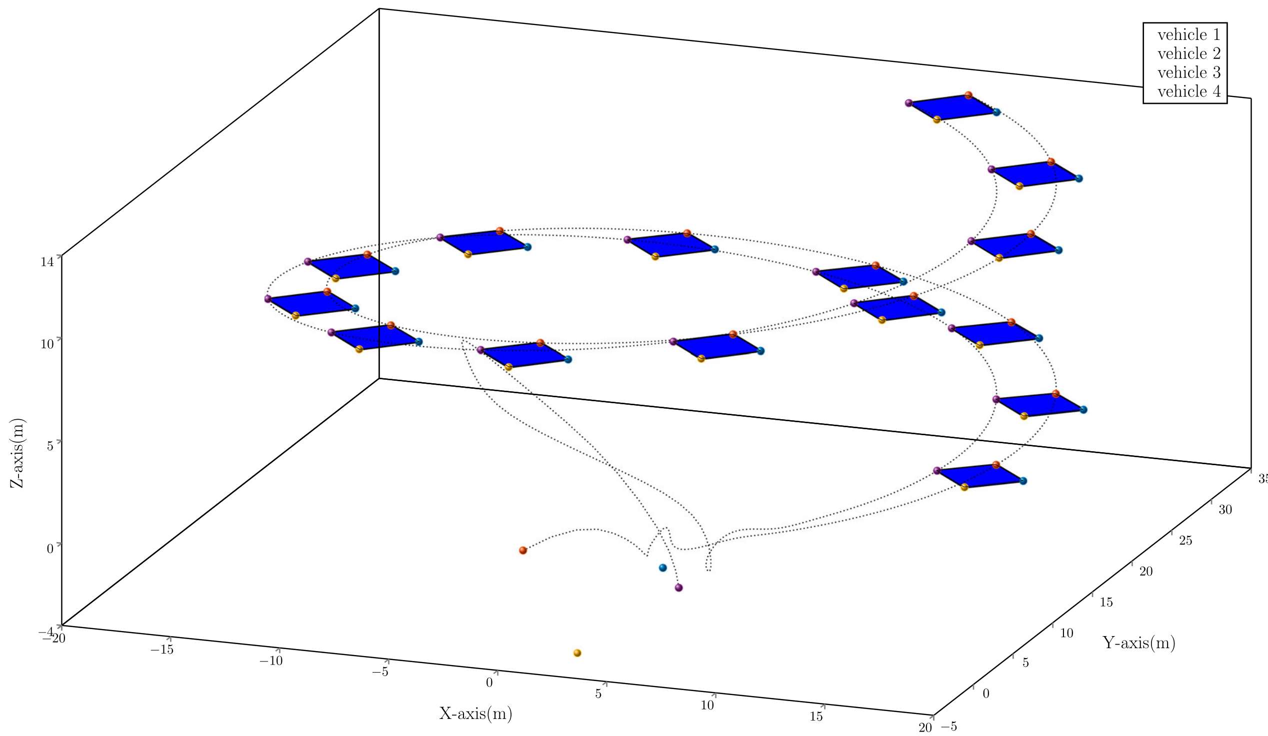

Figure 7. Formation of TPVs for the stochastic switching topology case.

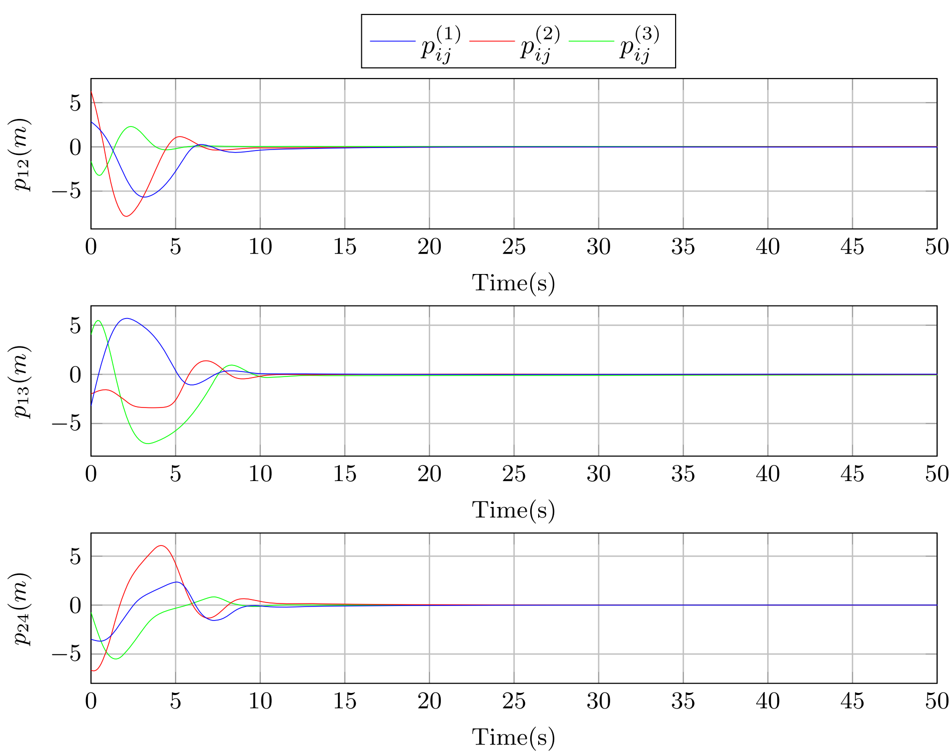

Figure 8. Formation errors for the stochastic switching topology case $p_{ij} = p_i - p_j - \delta_{ij} = (p_{ij}^{(1)}, p_{ij}^{(2)}, p_{ij}^{(3)})^{\mathsf{T}}$.

values for both scenarios, i.e. stochastic and deterministic network topologies, are given in Table 1.

The results for the deterministic switching topology are shown in Figures 4–6. In Figure 4, 3D plot of the motion of the six TPVs is shown in which all vehicles reach the desired geometric polygon shape from an arbitrary initial condition, meanwhile, they track the velocity signal. The errors of formation are depicted in Figure 5.

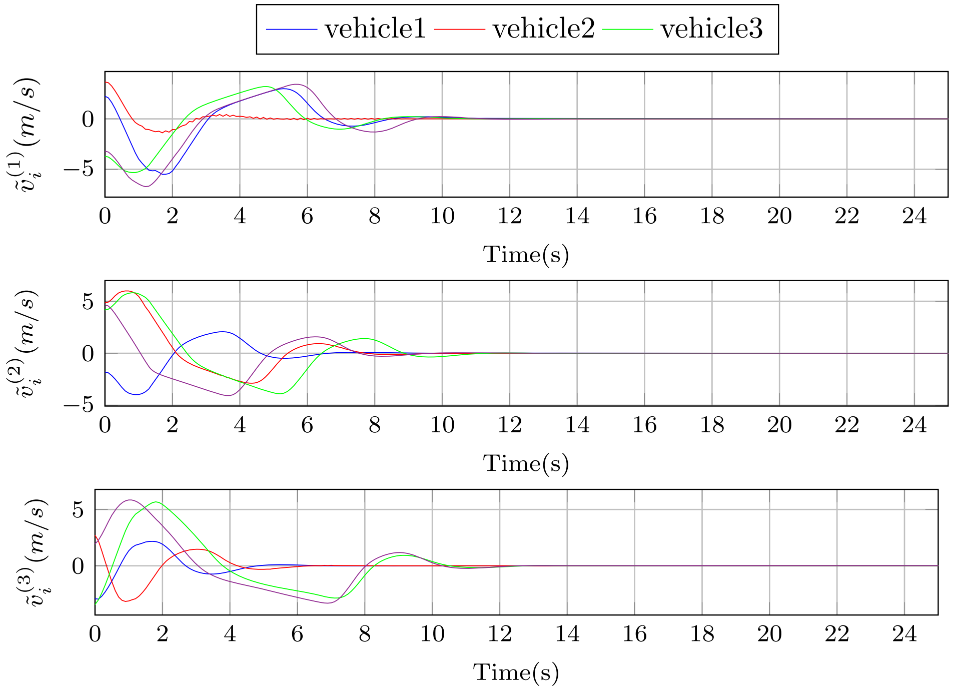

Figure 9. Linear velocity errors for the stochastic switching topology case $\tilde{v}_i = (\tilde{v}_i^{(1)}, \tilde{v}_i^{(2)}, \tilde{v}_i^{(3)})^{\mathsf{T}}$.

The tracking errors of the reference velocity signal are presented in Figure 6. The corresponding results for the stochastic switching topology are also illustrated in Figures 7–9 showing that the formation is achieved almost surely in the presence of stochastic topology, time delays, time-varying disturbances and model uncertainties.

7. Conclusion

Formation control of a team of multiple TPVs under deterministic and stochastic switching interaction topology and communication delay with robustness against a pair of disturbances and uncertainties has been investigated. A hierarchical control strategy, which separates the design for the translational and rotational dynamics, has been developed in two stages to tackle the underactuation of such systems. The stability of the overall system has been analysed through Lyapunov technique. For the deterministic topology, sufficient condition in terms of LMIs is derived to achieve formation in the presence of jointly connected switching topology. In the case of the stochastic topology, based on the concept of super-martingales, it is shown that if the probability of existing a connected topology is not zero, under some conditions, formation is almost surely solved in the network. Numerical simulations have been included to show the practical efficiency of the presented controller. It is worth mentioning that this paper does not consider the potential collision of agents and this needs to be further researched.

Disclosure statement

No potential conflict of interest was reported by the authors.

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