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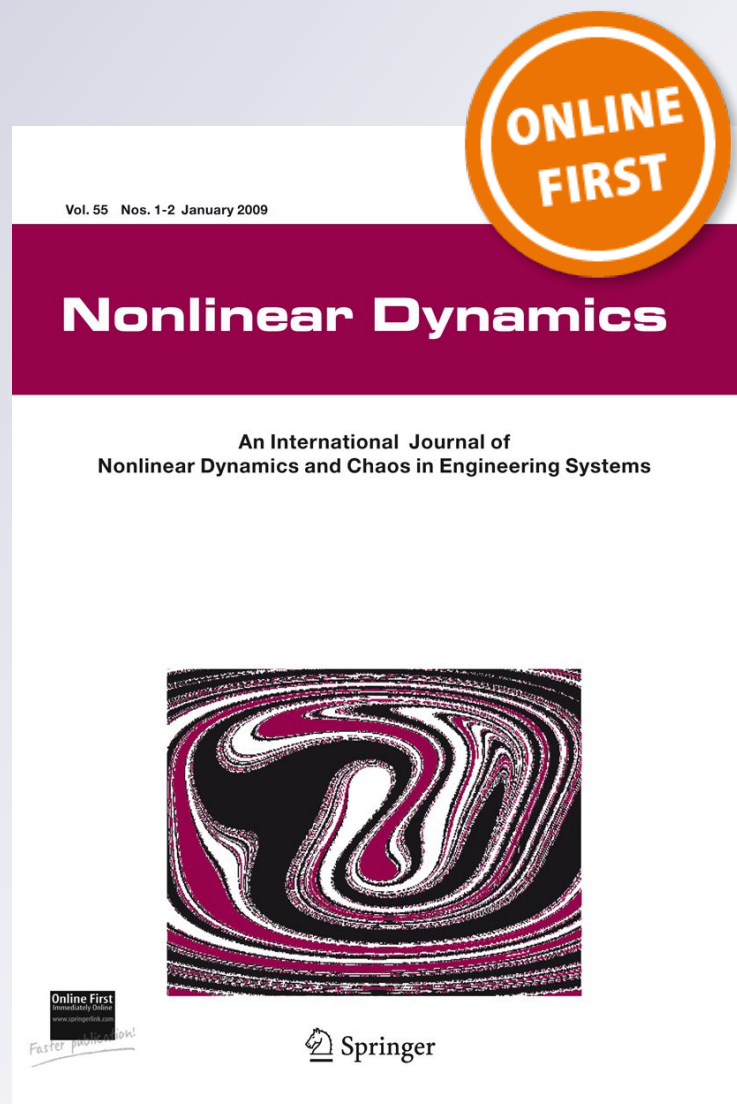
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Trajectory tracking of a class of under-actuated thrust-propelled vehicle with uncertainties and unknown disturbances

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Abstract This paper deals with the problem of designing a controller for a thrust-propelled vehicle which steers the vehicle to track a 3D spatial path, while effective compensation for both time-varying disturbances and uncertainties is achieved as well. Taking advantage of extraction algorithm, we separate the design for the translational and rotational dynamics. A back-stepping-based controller and a sliding mode controller are, respectively, designed for the translational and rotational dynamics in succession. The stability of the control framework is established through Lyapunov analysis. A numerical simulation is also included in the paper to render the effectiveness of the proposed control scheme.

Keywords Disturbance rejection · Robustness · Thrust-propelled vehicle · Trajectory tracking

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1 Introduction

Recently, motion control of thrust-propelled vehicles (TPVs) as an important class of under-actuated vehicles has attracted too much attention. The under-actuation of these systems lies in their translational dynamics in which a propulsive thrust force accelerates the vehicle along one body-fixed axis. A vectored torque input with independent components is used to modify the direction of the thrust force. Typical examples of TPVs are vertical takeoff and landing unmanned aerial vehicles (VTOL-UAVs) [15] and autonomous underwater vehicles (AUVs) [5].

The motivation of the paper is trajectory tracking control of a thrust-propelled vehicle evolving in SE(3) in which the vehicle tracks a geometric path parameterized by time. This problem for fully actuated systems is well studied. Yet, despite the tremendous effort [13,17–19,21], trajectory tracking control for the under-actuated mechanical systems are still an active topic of research.

Attitude control of rigid bodies, as a part of the TPV dynamics, has been extensively researched [6,25, 26,29]. However, when the position is involved, the problem becomes more complicated especially when asymptotic stability in the presence of disturbances and uncertainties is to be achieved. Up to now, position control of this type of systems has been the focus of several researches. In [4], exploiting singular perturbation theory a hierarchical controller for VTOL UAVs was employed for stabilization of a hovering VTOL. In [1],

relying on extraction algorithm, a control framework was proposed to solve global trajectory tracking of a single VTOL UAV. In [14], a similar control framework was implemented for a class of under-actuated systems with the difference that instead of using the orientation of the system as an intermediary control input to stabilize the position, the angular velocity was used.

In our study, we aim to include the impact of model uncertainties and time-varying disturbances, which are unavoidable in practical applications, in the procedure design. Despite the practical significance of this issue, most of the studies devoted to this problem either do not consider the disturbance impact or the disturbances are assumed to be fixed in the inertial frame. For instance, [7,22] investigated merely existence of constant disturbances in the model where a set of estimators are included to cope with the disturbances; however, in the presence of time-varying disturbance, these estimators can no longer be implemented. In [20], a controller for position stabilization of a ducted fan aircraft with a constant crosswind was introduced. Yet global stability was not ensured and just stabilization around a desired position was considered. In [10], a hybrid controller is proposed for trajectory tracking of a class of under-actuated system which translational dynamics is perturbed by an unknown constant disturbance that scales a bounded smooth state function. In [3] using back-stepping method, trajectory tracking and path following problem for a class of under-actuated vehicles with modeling parametric uncertainty were investigated, whereas just convergence of the tracking error signals to a small neighborhood of the origin was provided.

It is relevant to mention that the effect of disturbances and uncertainties has been coped with for a variety of nonlinear systems (just to cite a few, see [8,11,16,27,28,30]). In most of these works either a non-continuous control input is designed which cannot be extended directly to this class of under-actuated systems or only ultimate boundedness of the state trajectories is verified which is not the goal we are aiming at in this paper. Here, we further pursue a global asymptotic trajectory controller while a pair of time-varying disturbances perturbs the translational and rotational dynamics and there is no knowledge about the mass and inertia matrix of the vehicle.

A typical and intuitive way to deal with such systems is to exploit the cascade design whereby the required thrust direction is extracted. Afterward, the torque input is designed such that the desired thrust force is provided for the vehicle. The difficulty which arises in this method is the constraints immersed through the procedure of extracting the desired thrust force. All extraction algorithms suffer from singularity. Moreover, the required extracted thrust must be twice differentiable. These limitations along with the uncertainties and disturbances make the position control of this type of vehicles more challenging. Compared to other results reported in the literature, the prominent feature of our work is to consider the time-varying disturbances in the translational dynamics which makes us to put aside the most familiar and effective controller, i.e., sliding mode control. For example, in the work of [7,22], a back-stepping approach and extraction method are utilized for trajectory tracking for a thrust-propelled vehicle with constant disturbances. In both studies, to control the translational dynamics (which is the most complicated part of the thrust-propelled control), saturated controllers are designed based on position and velocity errors of the vehicle. Thereafter in the next step, these controllers were used in [7] through a back-stepping procedure and in [22] in the extraction algorithm calculations, both required the translational controller to be twice differentiable. However, the zero differentiation of the disturbance simplifies the design by incorporating smooth estimators in the translational controllers. The same argument holds for other researches as well [4,10,20]. In fact, although diverse methods were used in the studies (hybrid controller [10], singular perturbation theory [4], back-stepping approach [3,20]), they have one thing in common: All utilized saturated controllers in the translational control procedure in the absence of time-varying disturbance. Moreover, the translational virtual control should be also designed to be bounded a priori. To this effect, in our work the translational controller is designed in two steps in which a couple of variable structure controllers play the role of virtual control.

The main contribution of this paper is asymptotic tracking control for an under-actuated TPV, accommodating for time-varying translational and rotational disturbances. In addition, Our approach is robust against inertia matrix uncertainty. To the authors' best knowledge, this is the first time that the existence of time-varying disturbances is addressed for asymptotic tracking of this class of under-actuated vehicles. The difficulty lies in the fact that these systems are under-

actuated, and hence, using non-differentiable functions in control input is not allowed. On the other side, the vast majority of the controllers introduced so far to counteract the effect of the time-varying disturbances include the non-smooth control inputs such as sliding mode control. Having this constraint in mind, we propose a new variable structure control, which is twice differentiable, to deal with the translational disturbance and then can be used throughout the back-stepping procedure. As another contribution of the paper, the mass of the vehicle is assumed to be unknown which is required to be used in the extraction algorithm. Hence we estimate its value by implementing an adaptive estimator. However, the other problem which arises here is that our estimation should be always nonzero; otherwise, singularity would happen in the extraction algorithm. We also deal with this challenge by utilizing a smooth projection operator. It is to be noted that due to the under-actuated nature of these systems, in order that the attitude dynamics reaches the desired performance, the first and second time derivative of the virtual control input should be at hand as the components of the reference signal for rotational dynamics. We also solve this issue by implementing a robust attitude controller which not only cope with the effect of disturbances and uncertainties in inertial matrix but also it is designed in the presence of some unknown part of the reference signal. It is worth mentioning that the proposed approach also allows us to implement the controller in a conditions where the bounds for rotational disturbance and some part of the reference signal are unknown.

The rest of the paper is organized as follows. In the next section, preliminaries are given. The procedure design for the position and attitude control is explained in Sect. 3. The stability of the proposed control framework is provided in Sect. 4. An illustrative numerical simulation is given in Sect. 5, and the paper is finally concluded in Sect. 6.

2 Preliminaries

2.1 System model

The equations characterizing the motion of the TPV are given by

$$\begin{cases} \dot{p} = v, \\ \dot{v} = g\hat{z} - \frac{T}{m}R(Q)^T\hat{z} + b(t), \end{cases} \tag{1a}$$

$$\begin{cases} \dot{Q} = \frac{1}{2}\begin{pmatrix} \eta I_3 + q^\times \\ -q^T \end{pmatrix}\omega, \\ J\dot{\omega} = \Gamma - \omega^\times J\omega + d(t), \end{cases} \tag{1b}$$

where $\hat{z} = (0, 0, 1)^T$ and m is the total mass of the TPV and g is the gravitational acceleration. $p \in \mathbb{R}^3$ and $v \in \mathbb{R}^3$ are, respectively, the position and the linear velocity of the center of the mass of the TPV coordinated in the inertial frame. $b(t)$ and $d(t)$ are the translational and rotational time-varying disturbances, respectively. $J \in \mathbb{R}^{3\times 3}$ is the inertia matrix with respect to the body-fixed frame. The scalar T and vector $\Gamma \in \mathbb{R}^3$ are, respectively, the thrust and torque input for the vehicle. ω denotes the body-referenced angular velocity of the vehicle. The unit quaternion $Q = (q^T, \eta)^T$ is the attitude of the vehicle with respect to the inertial frame which composed of the vector part $q \in \mathbb{R}^3$ and the scalar part η and satisfies the constraint $q^Tq + \eta^2 = 1$ [12,23]. The inverse of unit quaternion Q is defined as $Q^{-1} = (-q^T, \eta)^T$ with the quaternion identity given by $Q = (0, 0, 0, 1)^T$. The unit quaternion multiplication is defined by $Q \odot Q_j = ((q^T\eta_j + q_j\eta + q^\times q_j)^T, \eta\eta_j - q^Tq_j)^T$ which is also a unit quaternion. The rotation matrix $R(Q)$ which brings the inertial frame into the body frame is obtained by $R(Q) = (\eta^2 - q^Tq)I_3 + 2qq^T - 2\eta q^\times$ where $\times$ is a skew symmetric matrix such that $x^\times = (0, -x_3, x_2; x_3, 0, -x_1; -x_2, x_1, 0)$, in which $x = [x_1, x_2, x_3]^T$.

2.2 Objective

Design controllers thrust input $T(t)$ and torque input $\Gamma(t)$ for the system given in (1) such that the position of the vehicle asymptotically tracks the desired spatial path $p_d(t)$ from any initial conditions on position, linear velocity, attitude and angular velocity in the presence of time-varying disturbances and uncertainties in the mass and inertia matrix, in other words $p(t) \rightarrow p_d(t)$. To fulfill our goal, we assume that the position, velocity, attitude and angular velocity of the vehicle are available to be used in feedback.

Assumption 1 We assume that the mass of the vehicle has a known lower bound m_L and a known upper bound m_U. Hence without loss of generality, it is assumed that $1/m = \theta_0 + \bar{\theta}$, where θ_0 is $(\frac{1}{m_L} + \frac{1}{m_U})/2$ which is known, and θ is an unknown constant which has the property that $|\theta| \leq (\frac{1}{m_L} - \frac{1}{m_U})/2$.

Assumption 2 It is assumed that there exist known constants u_{m1} and U_m such that $\sup_{t>0} ||b(t)|| < u_{m1}$ and $\sup_{t>0} ||\dot{v}_d(t)|| + u_{m1} < U_m < g$ where v_d is derivative of the desired path. It is also assumed that $\ddot{v}_d$ and $v_d^{(3)}$ are bounded.

Assumption 3 The inertia matrix J is bounded with some unknown constant $||J|| < \bar{J}$.

Assumption 4 The torque disturbance $d(t)$ is assumed to be bounded with some unknown upper bound $\sup_{t>0} ||d|| = \bar{J}\bar{d}$.

3 Control design strategy

Let us define the error variables $\tilde{p} = p(t) - p_d(t)$ and $\tilde{v} = v(t) - v_d(t)$. Taking $\theta = 1/m$ for notational convenience, we add and subtract the terms $\hat{\theta}TR(Q)^T\hat{z}$ and $\hat{\theta}TR(Q_d)^T\hat{z}$ to the second equation in (1a) which leads to the following translational error dynamics by

$$\dot{\tilde{p}} = \tilde{v}, \tag{2}$$
$$\dot{\tilde{v}} = -\dot{v}_d(t) + F + \tilde{F} - \tilde{\theta}TR(Q)^T\hat{z} + b(t), \tag{3}$$

with

$$F = g\hat{z} - \hat{\theta}TR(Q_d)^T\hat{z}, \tag{4}$$
$$\tilde{F} = \hat{\theta}T\left(R(Q_d)^T - R(Q)^T\right)\hat{z}, \tag{5}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ and $\hat{\theta}$ is the estimation of the parameter θ, F is the intermediate controller for the translational dynamics and $\tilde{F}$ is the under-actuation error. Note that from Assumption 1, θ can be decoupled as a known part and an unknown part. Therefore, we take $\hat{\theta} = \theta_0 + \hat{\bar{\theta}}$ where $\hat{\bar{\theta}}$ is the estimation of the unknown part and consequently we have $\tilde{\theta} = \tilde{\bar{\theta}} = \bar{\theta} - \hat{\bar{\theta}}$.

As we can see from (2) to (3), we have the intermediate controller F as an input for a fully actuated system. When F is designed, the thrust controller T and the desired attitude can be obtained from the extraction algorithm described in "Appendix A." The error of the under-actuation is remained to be dealt with in designing the torque controller Γ. As we will see throughout the paper, by making F prior bounded, the under-actuation is also prior bounded which can be viewed as a bounded perturbation that will be vanished through a suitable design of the rotational controller. The block diagram of such a procedure is shown in Fig. 1.

Before going through the rest of the paper, we state the following lemma which will be invoked later.

Lemma 1 *Consider the adaptation law*

$$\dot{\hat{\theta}} = \gamma_\theta \, proj\left(\Upsilon, \hat{\theta}\right)$$
$$= \gamma_\theta\left(\Upsilon - \frac{\varpi_1 \varpi_2}{2(\varepsilon^2 + 2\varepsilon B)^{n+1}B^2}\hat{\theta}\right), \quad \gamma_\theta > 0 \tag{6}$$

with

$$\varpi_1 = \begin{cases} (\hat{\theta}^T\hat{\theta} - B^2)^2 & \hat{\theta}^T\hat{\theta} > B^2 \\ 0 & otherwise \end{cases}, \tag{7}$$
$$\varpi_2 = \hat{\theta}^T\Upsilon + ((\hat{\theta}^T\Upsilon)^2 + \bar{\delta}^2)^{\frac{1}{2}}, \tag{8}$$

where ε and $\bar{\delta}$ are arbitrary positive constants, $\hat{\theta}$ is the estimation of θ, $\tilde{\theta} = \theta - \hat{\theta}$, $B > 0$ is the bound on the estimation and $\Upsilon(t)$ is a known, continuously differentiable variable. Then the following properties hold

p1) $||\hat{\theta}|| \leq B + \varepsilon, \quad \forall t > 0,$
p2) $\tilde{\theta}^T proj\left(\Upsilon, \hat{\theta}\right) \geq \tilde{\theta}^T\Upsilon,$
p3) $||proj\left(\Upsilon, \hat{\theta}\right)|| \leq ||\Upsilon||[1 + ((B+\varepsilon)/B)]^2 + ((B+\varepsilon)/(2B^2))\bar{\delta},$
p4) $proj\left(\Upsilon, \hat{\theta}\right) \in \mathcal{C}^n.$

The proof is given in [9].

3.1 Designing the intermediate input F

As it is explained in "Appendix A," the feasibility of extraction of the thrust T and desired attitude Q_d requires that $F \neq g\hat{z}$. To satisfy this condition, we put the limitation on F as $|F_i| < g$ for $i = 1, \ldots 3$, where $|F_i|$ denotes absolute value of the ith component of vector F. To that goal, we introduce the following structure for F by

$$F(t) = f(u), \tag{9}$$
$$\dot{u} = h(u)^{-1}w, \tag{10}$$

where $f(u) = U_m \tanh\left(\frac{u}{U_m}\right)$ is a saturation function by the saturation level U_m in which $\tanh(x) = (\tanh(x_1), \tanh(x_2), \tanh(x_3))^T$ and

$$h(u) = \text{diag}\left(\frac{\partial \tanh(u_1)}{\partial(u_1)}, \frac{\partial \tanh(u_2)}{\partial(u_2)}, \frac{\partial \tanh(u_2)}{\partial(u_2)}\right). \tag{11}$$

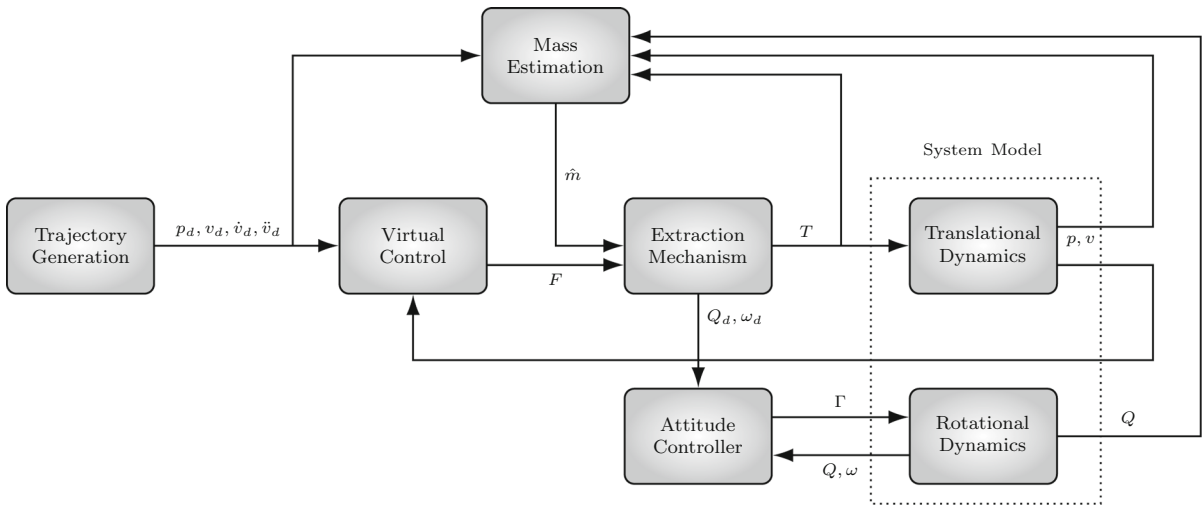Note that all entries of $h(u)$ are always greater than zero, and then, $h(u)$ is always invertible. We can

Fig. 1 Block diagram of hierarchical control structure

observe form (9) that the intermediate controller F is bounded by U_m. Hence taking $U_m < g$ ensures the feasibility of the extraction algorithm. Another reason for introducing the above structure for F is to have a completely known expression for $\dot{F}$. ω_d is required to be available for rotational control and it is a function of $\dot{F}$. It should also be pointed out that our approach does not need to a known $\dot{\omega}_d$ which implies $\ddot{F}$ is not required to be known.

It is worth mentioning that similar structures can be found in [30] and [27].

Let define the following variable error as

$$s_1 = \tilde{v} + k_1 \tilde{p}, \tag{12}$$

where $s_1 = (s_{11}, s_{12}, s_{13})^T$.

Consider the positive definite function V_{1T} as

$$V_{1T} = \sum_{i=1}^{3} \ln\left(\cosh\left(s_{1i}\right)\right) + \frac{1}{2\lambda_1} \kappa_1^2, \tag{13}$$

where $\lambda_1 > 0$ and κ_1 is the variable which is designed later. Differentiating (13) along (2), (3) gives

$$\dot{V}_{1T} = \tanh(s_1)^T \left(-\dot{v}_d + k_1 \tilde{v} + f(u)\right. \\ \left. + \tilde{F} - \tilde{\bar{\theta}} T R(Q)^T \hat{z} + b\right) + \frac{1}{\lambda_1} \kappa_1 \dot{\kappa}_1. \tag{14}$$

Consider the virtual control f_d and the adjustment law for κ_1 as

$$f_d = -k_2 \tanh(s_1) - \Phi_1 + \dot{v}_d - k_1 \tilde{v}, \tag{15}$$

$$\Phi_1 = \frac{u_{m1} \tanh(s_1)}{\sqrt{||\tanh(s_1)||^2 + (\kappa_1 \sigma_1)^2}}, \tag{16}$$

$$\dot{\kappa}_1 = -\lambda_1 \frac{u_{m1} ||\tanh(s_1)|| \sigma_1}{\sqrt{||\tanh(s_1)||^2 + (\kappa_1 \sigma_1)^2}}, \quad \kappa_1(0) > 0, \tag{17}$$

where $k_2 > 0$, $\lambda_1 > 0$ and $\sigma_1 > 0$ and $u_{m1} > 0$.

Note that after the system position converges to the desired trajectory, we should have $F = f_d$ and $s_1 \to 0$. Hence $f_d = -\Phi_1 - \dot{v}_d < u_{m1} + ||\dot{v}_d||$. On the other hand, each component of F should be smaller than g, so f_d should also satisfy this condition after the convergence of trajectories. Hence we should take U_m such that $u_{m1} + ||\dot{v}_d|| \leq U_m < g$.

After substituting (15)–(17) in (14) and upon use of

$$\frac{||\tanh(s_1)||^2}{\sqrt{||\tanh(s_1)||^2 + (\kappa \sigma_1)^2}} \\ = ||\tanh(s_1)|| \left(\frac{||\tanh(s_1)|| + \kappa_1 \sigma_1 - \kappa_1 \sigma_1}{\sqrt{||\tanh(s_1)||^2 + (\kappa_1 \sigma_1)^2}}\right) \\ \geq ||\tanh(s_1)|| \left(1 - \frac{\kappa_1 \sigma_1}{\sqrt{||\tanh(s_1)||^2 + (\kappa_1 \sigma_1)^2}}\right), \tag{18}$$

and exploiting $2ab < \delta a^2 + \frac{1}{\delta} b^2$ along with

$$\tilde{F} \leq 2\sqrt{2} \hat{\theta} T ||\tilde{q}|| = 2\sqrt{2} ||F - g\hat{z}|| ||\tilde{q}||, \tag{19}$$

$$0 < \frac{\partial \tanh(x)}{\partial x} \leq 1, \tag{20}$$

we get

$$\dot{V}_{1T} \leq -(k_2 - \delta_1 - \delta_2)||\tanh(s_1)||^2$$
$$-(u_{m1} - ||b||)||\tanh(s_1)||$$
$$+\frac{1}{4\delta_2}||s_2||^2 + \frac{2}{\delta_1}||F - g\hat{z}||^2||\tilde{q}||^2$$
$$+\tanh(s_1)^T\left(-\tilde{\bar{\theta}}TR(Q)^T\hat{z}\right), \tag{21}$$

where $s_2 = f(u) - f_d$ and δ_1 and δ_2 are arbitrary positive scalars. Now we define the positive definite function V_{2T} as

$$V_{2T} = s_2^T s_2 + \frac{1}{2\lambda_2}\kappa_2^2, \tag{22}$$

where $\lambda_2 > 0$ and κ_2 is defined later. The derivative of (22) is obtained by

$$\dot{V}_{2T} = s_2^T\left(\frac{\partial f}{\partial u}\dot{u} - \dot{f}_d\right) + \frac{1}{\lambda_2}\kappa_2\dot{\kappa}_2$$
$$= s_2^T\left(w + k_2h(s_1)\dot{s}_1 + \dot{\Phi}_1 - \ddot{v}_d + k_1\dot{s}_1 - k_1^2\tilde{v}\right)$$
$$+\frac{1}{\lambda_2}\kappa_2\dot{\kappa}_2, \tag{23}$$

where $h(.)$ is defined in (11) and $\dot{\Phi}_1$ is calculated by

$$\dot{\Phi}_1 = \varphi_1\dot{s}_1 + \varphi_2 \tag{24}$$

with

$$\varphi_1 = u_{m1}\left(\frac{h(s_1)}{\sqrt{||\tanh(s_1)||^2 + (\kappa_1\sigma_1)^2}} + \frac{h(s_1)\tanh(s_1)\tanh(s_1)^T}{\left(||\tanh(s_1)||^2 + (\kappa_1\sigma_1)^2\right)^{3/2}}\right), \tag{25}$$

$$\varphi_2 = \frac{u_{m1}2\kappa_1\dot{\kappa}_1\sigma_1\tanh(s_1)}{\left(||\tanh(s_1)||^2 + (\kappa_1\sigma_1)^2\right)^{3/2}}. \tag{26}$$

Substituting (24)–(25) in (27) leads to

$$\dot{V}_{2T} = s_2^T\left(w + (k_2h(s_1) + \varphi_1 + k_1I_3)\dot{s}_1\right.$$
$$\left.+\varphi_2 - \ddot{v}_d - k_1^2\tilde{v}\right) + \frac{1}{\lambda_2}\kappa_2\dot{\kappa}_2, \tag{27}$$

which can be rewritten in the form

$$\dot{V}_{2T} \leq s_2^T\left[w - k_2(k_2h(s_1) + \varphi_1 + k_1I_3)\tanh(s_1)\right.$$
$$+(k_2h(s_1)+\varphi_1+k_1I_3)s_2 + (k_2h(s_1) + \varphi_1 + k_1I_3)\tilde{F}$$
$$-(k_2h(s_1)+\varphi_1+k_1I_3)\left(\tilde{\bar{\theta}}TR(Q)^T\hat{z} + ||b||\right)$$
$$\left.-(k_2h(s_1)+\varphi_1+k_1I_3)\Phi_1 + \varphi_2 - \ddot{v}_d - k_1^2\tilde{v}\right]$$
$$+\frac{1}{\lambda_2}\kappa_2\dot{\kappa}_2. \tag{28}$$

It is noted that from (25) to (26), φ_1 and φ_2 are bounded if κ_1 is always greater than zero which will be proved

later in the paper. Therefore, let us assume $||\varphi_1||_2 \leq \bar{\varphi}_1$, $||\varphi_2||_2 \leq \bar{\varphi}_2$ in which $\bar{\varphi}_1$ and $\bar{\varphi}_2$ are unknown constants and $||.||_2$ denotes the euclidean norm. Taking w as

$$w = -k_3s_2 - \Phi_2 + \ddot{v}_d + k_1^2\tilde{v}, \tag{29}$$

with Φ_2 and adjustment law for κ_2 given by

$$\Phi_2 = \frac{u_{m2}s_2}{\sqrt{||s_2||^2 + (\kappa_2\sigma_2)^2}}, \tag{30}$$

$$\dot{\kappa}_2 = -\lambda_2 u_{m2}||s_2||\frac{\sigma_2}{\sqrt{||s_2||^2 + (\kappa_2\sigma_2)^2}}, \quad \kappa_2(0) > 0, \tag{31}$$

where λ_2 and σ_2 are strictly positive and u_{m2} is to be determined later, and using the following inequality

$$\frac{||s_2||^2}{\sqrt{||s_2||^2 + (\kappa_2\sigma_2)^2}} \geq ||s_2||\left(1 - \frac{\kappa_2\sigma_2}{\sqrt{||s_2||^2 + (\kappa_2\sigma_2)^2}}\right), \tag{32}$$

we can have

$$\dot{V}_{2T} \leq -k_3||s_2||^2 + \sqrt{3}k_2(k_2 + \bar{\varphi}_1 + k_1)||\tanh(s_2)||$$
$$+2\sqrt{2}(k_2 + \bar{\varphi}_1 + k_1)||F - g\hat{z}||||\tilde{q}||$$
$$+(k_2 + \bar{\varphi}_1 + k_1)||\tanh(s_2)||||s_2||$$
$$+(k_2 + \bar{\varphi}_1 + k_1)||b||||\tanh(s_2)|| - u_{m2}||s_2||$$
$$+(k_2h(s_1) + \varphi_1 + k_1)u_{m1} - (k_2h(s_1)$$
$$+\varphi_1 + k_1)\tilde{\bar{\theta}}TR(Q)^T\hat{z}. \tag{33}$$

Putting (14) and (33) together, we can write

$$\dot{V}_{1T} + \dot{V}_{2T} \leq -(k_2 - \delta_1 - \delta_2)||\tanh(s_1)||^2$$
$$-(u_{m1} - ||b||)||\tanh(s_1)|| - k_3||s_2||^2$$
$$+\mu_1||s_2|| + \frac{2}{\delta_1}||F - g\hat{z}||^2||\tilde{q}||^2$$
$$-u_{m2}||s_2|| - k_2h(s_1)\tilde{\bar{\theta}}TR(Q)^T\hat{z}, \tag{34}$$

where μ_1 is an unknown constant such that

$$(k_2 + \bar{\varphi}_1 + k_1)\left(\sqrt{3}k_2 + \sqrt{3}(1 + u_{m1})+\right.$$
$$\left.||b|| + \sqrt{3} + \bar{\varphi}_2 + 2\sqrt{12}g + |\tilde{\bar{\theta}}|T\right) \leq \mu_1, \tag{35}$$

and $||F - g\hat{z}||||\tilde{q}|| \leq \sqrt{6}g$ was used. It should be noticed, in view of (63), the boundedness of the last term in the left-hand side of (35) can be ensured if $\bar{\theta}$ and T are bounded. As we show later, it will ensure a priori boundedness of $\bar{\theta}$ by using the projection operator. The input thrust T is also a priori bounded since it comprises of a priori bounded terms $\hat{\theta}$ and F [see (63)].

Now we introduce the following positive definite function

$$V_T = V_{1T} + V_{2T} + \frac{1}{2\lambda_{\bar{\theta}}}\tilde{\bar{\theta}}^2 + \frac{1}{2\lambda_{\mu_1}}\tilde{\mu}_1^2, \qquad (36)$$

where $\lambda_{\bar{\theta}}, \lambda_{\mu_1} > 0, \tilde{\bar{\theta}} = \bar{\theta} - \hat{\bar{\theta}}$ and $\tilde{\mu}_1 = \mu_1 - \hat{\mu}_1, \mu_1$ is defined in (35) and V_{1T} and V_{2T} are given, respectively, in (13) and (22). Taking $u_{m2} = \hat{\mu}_1 + \epsilon$ with $\epsilon > 0$ and

$$\dot{\hat{\mu}}_1 = \lambda_{\mu_1}||s_2||, \qquad (37)$$

with adaptation law

$$\dot{\hat{\bar{\theta}}} = \lambda_\theta \, proj(\Upsilon, \hat{\bar{\theta}}) \quad \Upsilon = -T \tanh(s_1)^T R(Q)^T \hat{z}. \qquad (38)$$

Viewing (34) and using property (2) of the projection operator given in Lemma 1, the derivative of (36) is obtained by

$$\begin{aligned} \dot{V}_T &\leq -(k_2 - \delta_1 - \delta_2)||\tanh(s_1)||^2 \\ &\quad -(u_{m1} - ||b||)||\tanh(s_1)|| \\ &\quad -(k_3 - \delta_2)||s_2||^2 - \epsilon||s_2|| + \mu_2||\tilde{q}||^2, \end{aligned} \qquad (39)$$

where

$$\frac{2}{\delta_1}||F - g\hat{z}||^2 \leq \frac{12g^2}{\delta_1} = \mu_2 \qquad (40)$$

was used. Note that all terms in (39) are non-positive except the last one. Since the positive definite function in (36) is to be involved as a part of the Lyapunov function for the whole system, this term is handled later by the rotational dynamics design.

Remark 1 The projection operator in (38) is used to keep our estimation within a priori bounded set by property (1) of the projection operator given in Lemma 1. Since from Assumption 1 we know that $|\bar{\theta}| \leq \left(\frac{1}{m_L} - \frac{1}{m_U}\right)/2$, we can choose the parameter B of the projection operator as $B = \left(\frac{1}{m_L} - \frac{1}{m_U}\right)/2 - \varepsilon$ where B and ε are defined in Lemma 1. By this selection, one can guarantee that the estimations $\hat{\bar{\theta}}, \hat{\theta}$ are always bounded a priori and $\hat{\theta}$ never touches zero. This avoids the possible singularity in extraction algorithm (63).

3.2 Attitude control

3.2.1 Attitude error dynamics and kinematics

The attitude error dynamics is obtained by

$$\dot{\tilde{q}} = \frac{1}{2}\left(\tilde{\eta}I_3 + \tilde{q}^\times\right)\tilde{\omega}, \quad \dot{\tilde{\eta}} = -\frac{1}{2}\tilde{q}^T\tilde{\omega}, \qquad (41)$$

$$\begin{aligned} J\dot{\omega} &= -\omega^\times J\omega + J\left(\tilde{\omega}^\times R(\tilde{Q})\omega_d - R(\tilde{Q})\dot{\omega}_d\right) \\ &\quad + \Gamma + d(t), \end{aligned} \qquad (42)$$

in which $\tilde{Q} = (\tilde{q}^T, \tilde{\eta})^T$ is the discrepancy between the vehicle's attitude and the desired one and obtained by $\tilde{Q} = Q_d^{-1} \odot Q$ and $\tilde{\omega}$ is the angular velocity error obtained by $\tilde{\omega} = \omega - R(\tilde{Q})\omega_d$.

3.2.2 Designing torque input

We make the following coordinate transformation

$$\Omega = \tilde{\omega} + c_1\tilde{q}. \qquad (43)$$

In order to design the torque input, we introduce the torque input Γ as

$$\Gamma = -c_2\Omega - k_q\tilde{q} - \nu, \qquad (44)$$

with

$$\nu = \begin{cases} \frac{\Omega}{||\Omega||}\left(\hat{\vartheta}_1 + \hat{\vartheta}_2||\tilde{\omega}||\right) & ||\Omega|| \neq 0 \\ 0 & ||\Omega|| = 0, \end{cases} \qquad (45)$$

where $\hat{\vartheta}_1$ and $\hat{\vartheta}_2$ is obtained by the following adaptive laws

$$\dot{\hat{\vartheta}}_1 = \lambda_{\vartheta_1}||\Omega||, \qquad (46)$$

$$\dot{\hat{\vartheta}}_2 = \lambda_{\vartheta_2}||\tilde{\omega}||||\Omega||, \qquad (47)$$

and $c_1, c_2, \lambda_{\vartheta_1}, \lambda_{\vartheta_2}, k_q$ are strictly positive and ω_d and $\dot{\omega}_d$ are obtained from F designed in (4) and its first and second time derivatives by the extraction algorithm given in "Appendix A."

Now based on the boundedness of ω_d and $\dot{\omega}_d$ which is discussed in "Appendix B" and Assumption 4, we make a reasonable assumption that there exist unknown constants ϑ_1 and ϑ_2 such that

$$\bar{J}\left(c_1||\omega_d|| + ||\omega_d||^2 + \bar{d} + ||\dot{\omega}_d||\right) \leq \vartheta_1, \qquad (48)$$

$$\bar{J}\left(c_1 + 2||\omega_d|| + \frac{c_1}{2}\right) \leq \vartheta_2, \qquad (49)$$

which is used in the sequel.

Define the positive definite Lyapunov function as

$$V_R = \Omega^T J \Omega + 2k_q (1 - \eta) + \frac{1}{2\lambda_{\vartheta_1}} \tilde{\vartheta}_1^2 + \frac{1}{2\lambda_{\vartheta_2}} \tilde{\vartheta}_2^2. \tag{50}$$

where $\tilde{\vartheta}_1$, $\tilde{\vartheta}_2$ are error estimation defined as $\tilde{\vartheta}_1 = \vartheta_1 - \hat{\vartheta}_1$, $\tilde{\vartheta}_2 = \vartheta_2 - \hat{\vartheta}_2$.

Calculating the derivative of (50) leads to

$$\dot{V}_R = -\Omega^T \left(-\Omega - c_1 \tilde{q} + R(\tilde{Q})\omega_d \right)^{\times}$$
$$J \left(\tilde{\omega} + R(\tilde{Q})\omega_d \right)$$
$$+ \Omega^T J \left(\tilde{\omega}^{\times} R(\tilde{Q})\omega_d - R(\tilde{Q})\dot{\omega}_d \right) + \Omega^T \Gamma$$
$$+ \frac{c_1}{2} \Omega^T J \left(\tilde{q}^{\times} + \tilde{\eta} \right) \tilde{\omega} - c_1 k_q ||\tilde{q}||^2 + \Omega^T d(t)$$
$$- \tilde{\vartheta}_1 ||\Omega|| - \tilde{\vartheta}_2 ||\Omega|| ||\tilde{\omega}||,$$

which follows with

$$\dot{V}_R \leq ||\Omega|| ||J|| \left((c_1 + ||\omega_d||) (||\tilde{\omega}|| + ||\omega_d||) \right.$$
$$+ ||\omega_d|| ||\tilde{\omega}|| + ||\dot{\omega}_d|| + \frac{c_1}{2} ||\tilde{\omega}|| \right)$$
$$+ ||\Omega|| \bar{J} \bar{d} - c_1 k_q ||\tilde{q}||^2 + \Omega^T \Gamma - \tilde{\vartheta}_1 ||\Omega||$$
$$- \tilde{\vartheta}_2 ||\Omega|| ||\tilde{\omega}||, \tag{51}$$

where the property of operator $^{\times}$, $\Omega^{\times} \Omega = 0_{3 \times 1}$, was used. Substituting (48), (49) in the above inequality, one can get

$$\dot{V}_R \leq -c_2 ||\Omega||^2 - c_1 k_q ||\tilde{q}||^2. \tag{52}$$

The Lyapunov function introduced here along with the one in the previous section is used to analyze the stability of the overall system explained in the next section.

4 Stability analysis

Theorem 1 *Consider the TPV vehicle with the model given in (1) and the intermediate control input F given in (4), the extraction algorithm given in "Appendix A" and Assumptions 2–4. By the intermediate control given in (9)–(10) and (29), the adaptation law given in (38) for estimating $\bar{\theta}$ and the torque input Γ given in (44) with (45)–(47) and gains and parameters satisfying $k_2 > \delta_1 + \delta_2$, $k_3 > \delta_2$, $u_{m1} > ||b||$, $u_{m1} + ||\dot{v}_d|| \leq U_m < g$ and $c_1 k_q > \mu_2$, the vehicle position asymptotically converges to the desired spatial path $p_d(t)$ from any arbitrary initial conditions.*

Proof Since the intermediate controller F is chosen such that $|F_i| < g$, extraction of the thrust and desired

attitude described in (63), (64) is always possible. Let introduce the following Lyapunov function for the complete system

$$V = V_T + V_R, \tag{53}$$

with V_T and V_R, respectively, given in (36) and (50). The derivative of (53) is obtained by

$$\dot{V} \leq -(k_2 - \delta_1) || \tanh(s_1)||^2$$
$$- (u_{m1} - ||b||) || \tanh(s_1)||$$
$$- (k_3 - \delta_2) ||s_2||^2 - \epsilon ||s_2||$$
$$- c_2 ||\Omega||^2 - \left(c_1 k_q - \mu_2 \right) ||\tilde{q}||^2, \tag{54}$$

which is negative semi-definite, if the gains are chosen as stated in Theorem 1. In consequence, boundedness of $s_1, s_2, \tilde{\bar{\theta}}, \kappa_1, \kappa_2, \tilde{\mu}_1, \Omega, \tilde{\vartheta}_1, \tilde{\vartheta}_2$ is concluded. Invoking Barbalat's lemma [24], the convergence of s_1, s_2, and Ω to zero is concluded which follows with $\tilde{p} \to 0$, $\tilde{v} \to 0$, $\tilde{q} \to 0$. □

As it can be observed from Eqs. (16)–(30), approaching κ_1 and κ_2 to zero can cause singularities. In the rest, we derive conditions to avoid such singularities. From (54), we can say

$$\int_0^{\infty} || \tanh(s_1)|| d\tau \leq \frac{1}{(u_{m1} - ||b||)} (V(0) - V(\infty)), \tag{55}$$

$$\int_0^{\infty} ||s_2|| d\tau \leq \frac{1}{\epsilon} (V(0) - V(\infty)). \tag{56}$$

As $(V(0) - V(\infty))$ is bounded, there exist γ_1 and γ_2 such that

$$\int_0^{\infty} || \tanh(s_1)|| d\tau \leq \gamma_1, \tag{57}$$

$$\int_0^{\infty} ||s_2|| d\tau \leq \gamma_2. \tag{58}$$

Regarding (17) and (31), we can have

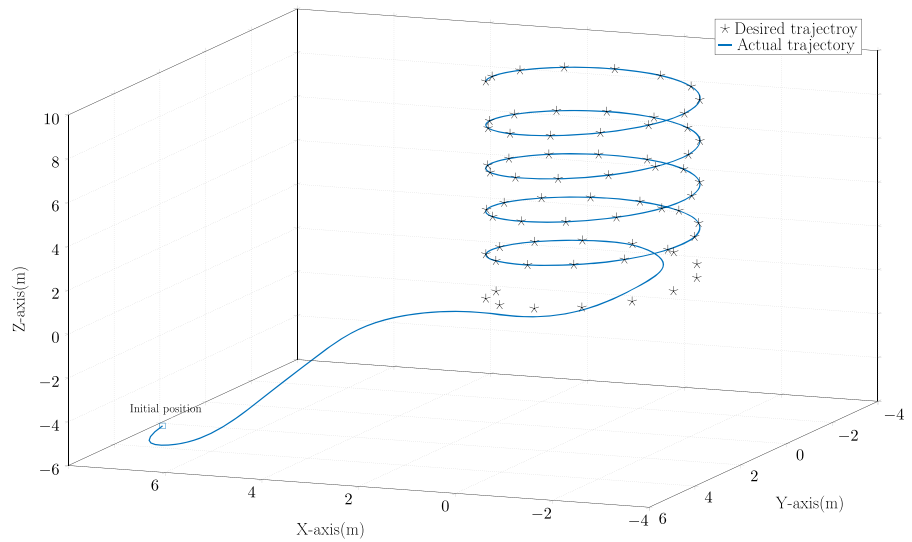$$\kappa_1 \dot{\kappa}_1 \geq -\lambda_1 u_{m1} || \tanh(s_1)||, \tag{59}$$
$$\kappa_2 \dot{\kappa}_2 \geq -\lambda_2 u_{m2} ||s_2||, \tag{60}$$

integrating each side of the above inequalities gives

$$\kappa_1^2(t) \geq \kappa_1^2(0) - 2\lambda_1 u_{m1} \int_0^t || \tanh(s_1)|| d\tau \geq \kappa_1^2(0)$$
$$- 2\lambda_1 u_{m1} \gamma_1.$$
$$\kappa_2^2(t) \geq \kappa_2^2(0) - 2\lambda_2 u_{m2} \int_0^t ||s_2|| d\tau \geq \kappa_2^2(0)$$
$$- 2\lambda_2 \epsilon \gamma_2^2,$$

Fig. 2 3D plot of the TPV vehicle trajectory in comparison with the desired trajectory

where we have used the fact that $m_2 \geq \epsilon + k_{\mu 1}\gamma_2$, which can be inferred from (37) and (58). Hence

$$\kappa_1(t) \geq \sqrt{\kappa_1^2(0) - 2u_{m1}\lambda_1\gamma_1}, \tag{61}$$

$$\kappa_2(t) \geq \sqrt{\kappa_2^2(0) - 2u_{m2}\lambda_2\epsilon\gamma_2^2}. \tag{62}$$

Therefore, κ_1 and κ_2 are always greater than zero and can be kept far from zero by the bounds, respectively, given in (61) and (62) which are determined by proper selection of initial values $\kappa_1(0)$ and $\kappa_2(0)$ along with the gains λ_1 and λ_2. Although finding these proper values seems not to be straightforward, it is observed that by choosing λ_1 and λ_2 small enough, κ_1 and κ_2 never cross zero.

Remark 1 In order to tackle the existence of uncertainty in the mass, we implement the adaptive law (38) to estimate $\theta = 1/m$. However, in our approach, a prefect estimation is not needed , i.e., convergence of estimation error $\tilde{\theta}$ to zero is not mandatory. In fact, $\hat{\theta}$ converges to the boundary of its actual value θ and the small imperfection of the estimation is compensated for along with the disturbance $b(t)$. Furthermore, if we have a rather precise knowledge about the value of θ, we can simplify the proposed controller by omitting the adaptive estimation. In this case, the conjecture of θ is used instead of the estimation $\hat{\theta}$.

5 Simulation results

To test the validity of the proposed controller, we consider a TPV with $m = 0.1\,\text{kg}$, $g = 9.8\,\text{m/s}^2$,
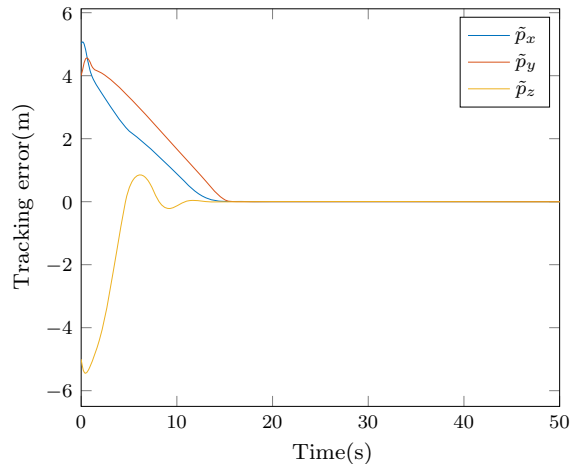

Fig. 3 Tracking error $\tilde{p} = p - p_d = (\tilde{p}_x, \tilde{p}_y, \tilde{p}_z)^T$

$J = \text{diag}(0.25, 0.15, 0.3)\text{kg m}^2$. The desired path to be tracked is $(2\sin(t/\pi), 2\cos(t/\pi), 0.1t)$ and translational disturbance and torque disturbance are chosen, respectively, as

$$b(t) = (0.2\sin(0.2t), 0.1\cos(0.1t), 0.15\cos(0.01t))^T \\ (\text{m/s}^2),$$

$$d(t) = (0.1\sin(0.5t), 0.1\cos(t), -0.15\cos(0.25t))^T \\ (\text{N.m}).$$

The initial position and velocity are selected randomly in $[-10, 10]$ and $[-2, 2]$, respectively. $\omega(0) = (0.1, 0.2, .03)\,rad/s$ and $Q(0)$ is chosen in random. Other initial values are selected as $\hat{\bar{\theta}}(0) = -1, \theta_0 = 10,$ $\kappa_1(0) = 1, \kappa_2(0) = 2, \hat{\mu}_1(0) = 5, \hat{\mu}_2(0) = 5,$
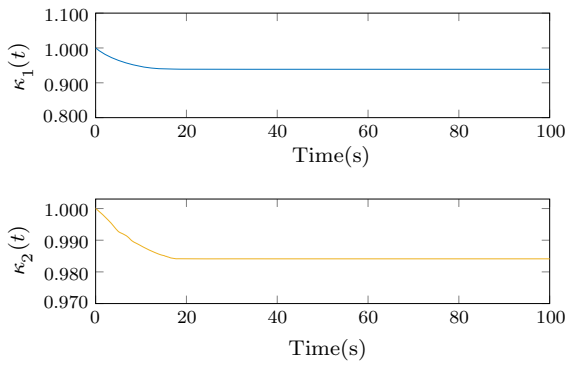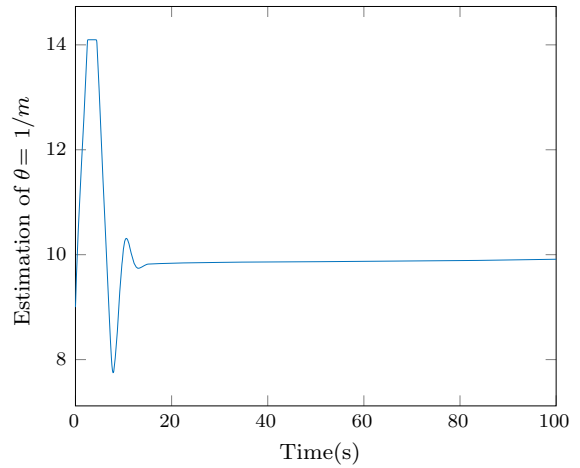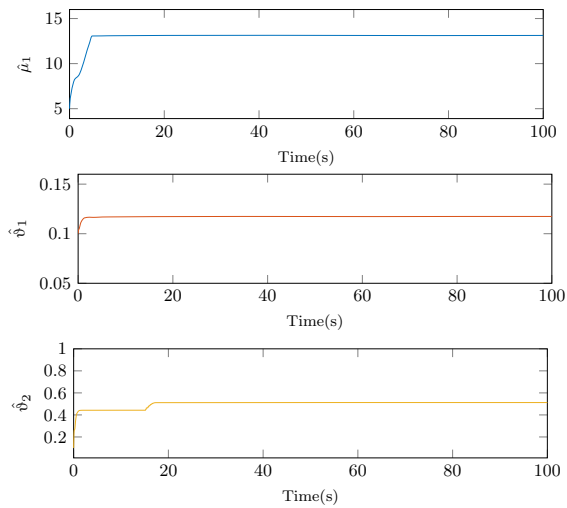
Fig. 4 Response of $\kappa_i(t)$

Fig. 5 Parameter estimations
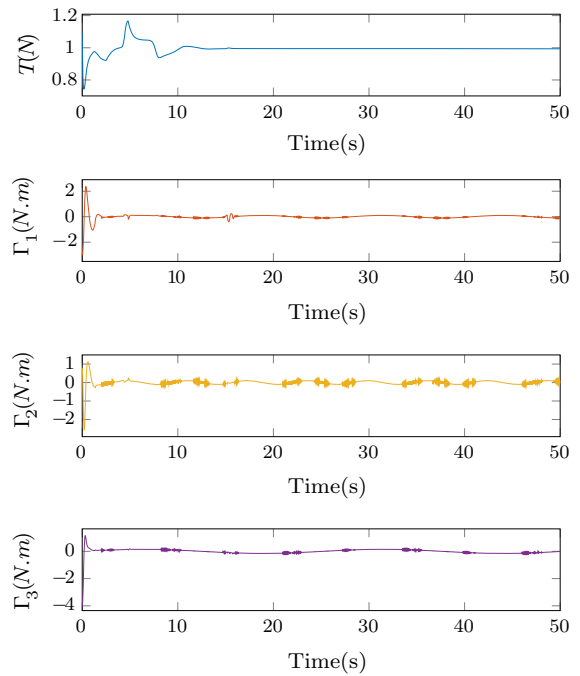

Fig. 6 Norm of attitude error

Fig. 7 Estimation of $\theta = 1/m$

Fig. 8 Thrust input and toque input
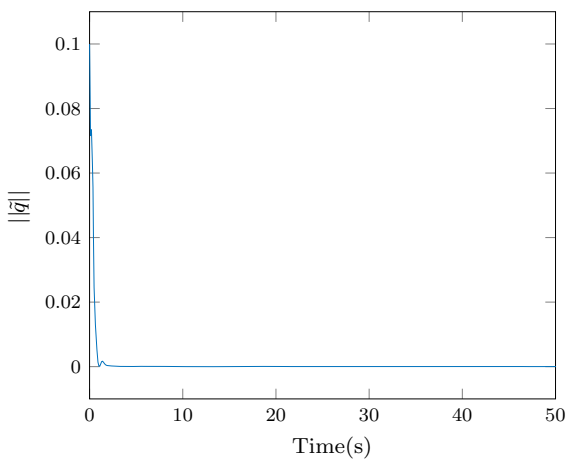
$\hat{\vartheta}_1(0) = 0.1$, $\hat{\vartheta}_2(0) = 0.1$. The gains are chosen to be $u_M = 9.5$, $\lambda_1 = 0.1$, $\lambda_2 = 0.01$, $\sigma_1 = \sigma_2 = 0.01$, $\lambda_\theta = 2$, $k_1 = 4$, $k_2 = 1$, $k_3 = 8$, $c_1 = 1$, $c_2 = 1$, $k_q = 5$, $\lambda_{\mu_1} = 1$, $\epsilon = 5$, $\lambda_{\vartheta_1} = 0.01$ and $\lambda_{\vartheta_2} = 0.1$. The parameters of the projection operator are chosen as $n = 1$, $\bar{\delta} = 1$, $\varepsilon = 0.1$, $B = 4$.

The simulation is performed over [0, 100 s] and the results are shown in Figs. 2, 3, 4, 5, 6, 7, and 8. In Fig. 2, the three-dimensional plot of the vehicle trajectory ver-

sus the desired trajectory is depicted. As we can see, the vehicle starts from random initial conditions and then after a while it tracks the desired path which is shown by star marker. The discrepancy between system trajectory and the desired trajectory is shown in Fig. 3. Figure 4 shows the evolution of $\kappa_1(t)$ and $\kappa_2(t)$. In Fig. 3, we can observe more exactly that it takes about 16 seconds that the position of the vehicle converges to the desired position and we can see from Fig. 4 that system becomes stable after almost 20 seconds. The estimations of μ_1, ϑ_1 and ϑ_2 are shown in Fig. 5, the norm of attitude error is depicted in Fig. 6 and the estimation of $\theta = 1/m$ is depicted in Fig. 7. Finally the thrust input and toque input are shown in Fig. 8.

6 Conclusion

Robust trajectory tracking control for a thrust-propelled vehicle in the presence of uncertainties in the mass and inertia matrix as well as a pair of disturbances was investigated. A hierarchical two-stage controller was developed in such a way that the vehicle tracks a predefined spatial path. With the aid of back-stepping technique, variable structure approach, adaptive control approaches and sliding mode control, the adverse effect of disturbances and uncertainties was compensated for. Stability of the whole closed loop system was also proved by Lyapunov function technique, and finally, the efficiency of the proposed controller was tested by a numerical simulation.

Appendix

A Extraction algorithm

Here we introduce the extraction algorithm for obtaining Q_d and T form the intermediate control $F = (F_1, F_2, F_3)^T$ given in (4).

$$T = \frac{1}{\hat{\theta}} \|F - g\hat{z}\|, \tag{63}$$

$$\eta_d = \sqrt{\frac{1}{2} + \frac{g - F_3}{2\|F - g\hat{z}\|}}, \quad q_d = \frac{1}{2\|F - g\hat{z}\|\eta_d} \begin{pmatrix} F_2 \\ -F_1 \\ 0 \end{pmatrix}. \tag{64}$$

As it is clear from (64), this extraction is well defined if

$$F \neq g\hat{z}. \tag{65}$$

The desired angular velocity ω_d and its derivative $\dot{\omega}_d$ can also be obtained by the following expressions

$$\omega_d = \Xi(F)\dot{F}, \tag{66}$$

$$\dot{\omega}_d = \dot{\Xi}(F, \dot{F})\dot{F} + \Xi(F)\ddot{F}, \tag{67}$$

with

$$\Xi(F) = \frac{1}{\ell_1^2 \ell_2} \begin{pmatrix} -F_1 F_2 & -F^{2^2} + \ell_1 \ell_2 & -F_2 \ell_2 \\ F^{1^2} - \ell_1 \ell_2 & F_1 F_2 & -F_1 \ell_2 \\ F_2 \ell_1 & -F_1 \ell_1 & 0 \end{pmatrix}, \tag{68}$$

where $\ell_1 = \|F - g\hat{z}\|$, $\ell_2 = \ell_1 + (g - \mu_3)$ and $\dot{\Xi}(F, \dot{F})$ is the time derivative of $\Xi(F)$ and the subscript i is omitted for notational simplicity. The proof can be found in [2].

B Analysis of boundedness of ω_d and $\dot{\omega}_d$

From (66) to (67), boundedness of ω_d and $\dot{\omega}_d$ can be guaranteed if F, $\dot{F}$, $\ddot{F}$ are bounded. Regarding the structure of F defined in (9)–(10), it is obvious that F is bounded and $\dot{F}$ and $\ddot{F}$ are bounded if, respectively, w and its derivative are bounded. From (29), we have

$$w = -k_3 s_2 - \Phi_2 + \ddot{v}_d + k_1^2 \tilde{v},$$

and boundedness of w can be easily concluded by Assumption 2 and boundedness of s_2 and $\tilde{v}$ which are provided by the discussion in Sect. 4. The derivative of w is obtained by

$$\dot{w} = -k_3 \dot{s}_2 - \dot{\Phi}_2 + v_d^{(3)} + k_1^2 \dot{\tilde{v}}.$$

Viewing (3) and Assumption 2, the last two terms in the above equation are bounded based on boundedness of s_1, F, $\dot{F}$, $\tilde{\bar{\theta}}$ which is concluded from the discussion in Sect. 4. Based on (9)–(10) and (15), we have

$$\dot{s}_2 = \dot{f}(u) - \dot{f}_d = w + k_2 \tanh(s_1) + \Phi_1 - \dot{v}_d + k_1 \tilde{v},$$

which is also bounded. It now just remains to prove that $\dot{\Phi}_2$ is bounded. From (30), we can obtain

$$\dot{\Phi}_2 = u_{m2} \frac{\dot{s}_2^T \left(\|s_2\|^2 + (\kappa_2 \sigma_2)^2\right) - s_2^T \left(s_2^T \dot{s}_2 + \kappa_2 \dot{\kappa}_2 \sigma_2\right)}{\left(\|s_2\|^2 + (\kappa_2 \sigma_2)^2\right)^{3/2}}, \tag{69}$$

which is also bounded since $\dot{s}_2$, s_2, κ_2 are bounded, $\dot{\kappa}_2$ is bounded from (31), and the fact that κ_2 is kept away from zero by suitable selection of the gain λ_2 and the initial value $\kappa_2(0)$ as explained in Sect. 4.

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