

## Three-Axes Rotation Algorithm for the Relaxed 3L-CVRP

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### ABSTRACT

*The purpose of this work is to present a developed three-axes rotation algorithm to improve the solving methodology for the relaxed 3L-CVRP (Three-Dimensional Capacitated Vehicle Routing Problem). Although there are reported works on solving approaches for the relaxed 3L-CVRP that consider product rotation to optimize load capacity, rotation on the three axes has not been thoroughly studied. In this aspect, the present work explicitly explores the three-axes rotation and its impact on load capacity optimization. In order to improve the relaxed 3L-CVRP problem, a two-phase solution was developed. The first phase consists of finding the solution for the CVRP problem, using a demand previously obtained with a heuristic developed to convert the 3L-CVRP demand into CVRP demand. The second phase is to obtain the loading of the vehicle using a heuristic developed to load the items using rules to obtain the rotation of the items. The proposed approach was able to improve the load assignment in 48.1% of well-known 3L-CVRP instances when compared to similar approaches on the relaxed 3L-CVRP. The outcomes of this research can be applied to transportation problems where package rotation on the z-axis is an option, and there are not fragile items to load in the vehicles.*

*Keywords: 3L-CVRP; constraint relaxation; three-axes rotation; load capacity optimization*

### INTRODUCTION

The Capacitated Vehicle Routing Problem (CVRP), proposed by (Dantzig & Ramser 1959), is a Non-deterministic Polynomial-time complexity class problem (NP-hard) in the transportation field. The CVRP consists of defining routes for every vehicle to minimize transportation costs or time with the restriction of capacity based on the weight or volume of the items. The problem has been addressed by different researchers using metaheuristic alternatives (Caballero-Morales, Martínez-Flores & Sánchez-Partida 2018; Hosseinabadi, Rostami, Kardgar, Mirkamali & Abraham 2017; Mazidi, Fakhrahmad & Sadreddini 2016) to obtain near-to-optimal solutions to large instances (>150 nodes or locations to be served by the vehicle) within a reasonable time. In contrast, exact algorithms can only provide solutions for instances with less than 137 nodes (Liu, Li, Luo & Chen 2013). Orrego (Orrego Cardozo, Ospina Toro, & Toro Ocampo, 2016) presents various metaheuristics used to solve CVRP. Recently, this type of problem evolved considering the transport of items of different sizes (dimensions), known as the Three-Dimensional Capacitated Vehicle Routing Problem (3L-CVRP) introduced by Gendreau (Gendreau, Iori, Laporte & Martello 2006). This problem is a combination of the CVRP and the Three-Dimensional Bin Packaging Problem (3D-BPP). The 3D-BPP has been solved

to optimality for instances with a maximum transportation load of 60 items (Martello, Pisinger, & Vigo, 2000).

Because the 3L-CVRP takes into account the dimensions of the items, not all items are suitable to be loaded within the vehicle. The loading task increases in complexity as more constraints are considered (i.e., LIFO, support, fragility). Because of the complexity of the transportation scenario, some or all constraints are relaxed. It leads to Relaxed 3L-CVRP.

The present work extends on the solving aspect of the Relaxed 3L-CVRP by proposing an algorithm to improve the loading task. It is performed by extending the three-axes rotation of the items and performing constraint relaxation on fragility, support area, and LIFO. Relaxation was performed as in the reported reviewed works, and it was found that three-axes rotation can improve load assignment and support area. Particularly, load assignment was improved in 48.1% of well-known 3L-CVRP instances when compared to similar approaches.

The advances of the present work are described as follows: in 3L-CVRP Section the technical background of the 3L-CVRP is presented; then in the next section recent works on the 3L-CVRP and Relaxed 3L-CVRP are presented and discussed; after the proposed algorithm is described; immediately the obtained results and the discussion of the

results are addressed, and finally the conclusions and future work are presented.

### 3L-CVRP

The 3L-CVRP is a combination of the CVRP and the three-dimensional bin packing problem or 3D-BPP. In this problem, packaging takes into account the geometry (i.e., dimensions) of the items, which causes that not all of the items can be packed. In mathematical terms and following the notation presented by (Fuellerer, Doerner, Hartl, & Iori, 2010) the 3L-CVRP is defined as a complete graph  $G=\{V, E\}$ , where  $V=\{0, 1, \dots, n\}$  is a set of  $n+1$  vertices and  $E$  the set of edges connecting each pair of vertices. The depot is vertex 0, and the vertices  $\{1, \dots, n\}$  are the  $n$  customers to be served. The edges are denoted by  $(i, j)$ , and they are associated with a routing cost  $C_{ij}$  ( $i, j = 0, \dots, n$ ). There is a set of  $v$  identical vehicles with a weight capacity  $D$ , and a loading space of dimensions composed by width ( $W$ ), length ( $L$ ), and height ( $H$ ). The vehicles have an opening on the rear for loading/unloading the items of  $W \times H$  dimensions.

With this information, space (volume) required by the items requested by the customer,  $i$  can be represented as  $vol_i = \sum_{k=1}^{m_i} w_{ik} h_{ik} l_{ik}$  where  $i=1, \dots, n$ ,  $k=1, \dots, m_i$  where  $m_i$  is the number of items requested by customer  $i$ , and  $w_{ik}$ ,  $h_{ik}$ , and  $l_{ik}$  are the weight, height, and length of item  $k$  requested by customer  $i$ .

This type of problem has general constraints that the researchers consider when proposing new algorithms to solve this problem. These are classified into:

1. CVRP constraints
  - a. Each customer is served by only one vehicle and visited once.
  - b. The total weight of the items cannot exceed the vehicle's capacity.
  - c. The total volume of the items cannot exceed the vehicle's capacity.
  - d. Every route begins and ends in a single depot.
  - e. The demand of all customers must be satisfied.
2. 3L-CVRP loading constraints
  - a. Orientation. The loading must be orthogonal (only can be rotated  $90^\circ$  in the  $x$ - $y$  plane).
  - b. Fragility. Non-fragile items cannot be placed on top of fragile items; however, fragile items can be placed on top of fragile and non-fragile items.
  - c. Support area. When an item is placed on top of another item, its base must be supported by a minimum supporting area  $\alpha$ , which is frequently to 75.0%.
  - d. Sequential load (LIFO policy). When a customer  $i$  is visited, the items of the customer must be unloaded without moving the items of other customers.

Figure 1a) presents an example of a routing solution where node 0 represents the central depot, and three routes are considered to satisfy the demand of the customers. Thus, Route 1 serves customers 2, 4, 6 sequentially while Route 2 supplies customers 8, 1, 5, and finally, Route 3 supplies customers 3 and 7. To integrate the loading aspect of the 3L-CVRP, each item to be loaded into the vehicles is identified by two indexes:  $i$  for the customer and  $k$  for the item within the customer's demand (Fuellerer et al., 2010). Thus, item labeled as 52 identifies the second item that has to be delivered to customer 5. A detailed 3L-CVRP loading solution of Route 1 is presented in Figure 1b).

### APPROACHES FOR THE 3L-CVRP AND THE RELAXED 3L-CVRP

The 3L-CVRP reduces traveling costs or distances and avoids the assumption that the items of customers can be loaded into a vehicle without considering their dimensions (something which is considered by the CVRP). The 3L-CVRP considers the dimensions of the items, weight, and volume to determine a more accurate loading in the vehicle without exceeding its capacity. Thus, a solution for the 3L-CVRP consists of two aspects. First, a solution for the routing problem and second a solution for the loading problem. Table 1 presents an overview of works developed on the 3L-CVRP and their approaches for both aspects of its solution.

As presented, the 3L-CVRP has been solved with different approaches considering all constraints (orientation, fragility, support, LIFO), and some works have also provided solutions for the Relaxed (unconstrained) 3L-CVRP. However, in real situations, complying with all constraints is not practical, and increasing the number of constraints that affect the vehicle's capacity utilization.

An example of this situation is presented considering an instance reported by Lacomme (Lacomme et al. 2013). Table 2 presents information about the customers and demands (boxes), including load dimensions.

For this example, Figure 2 presents the 3L-CVRP loading solution, which complies with all constraints for the route 0-20-1-13-7-22-0. Following the standard placement rule, items are loaded from right-to-left (i.e., items of customer 22 are loaded first into the vehicle). As presented, this solution does not enable the items of customers 1 and 20 to be loaded. Thus, not all customers can be served, and the capacity is not fully used.

In contrast, Figure 3 presents the loading solution considering the relaxation of the constraints. As presented, the items of all customers in the route 0-20-1-13-7-22-0 are loaded, and capacity is highly used.

Because in practical situations, not all constraints must be set (Pedruzzi, Amorim Nunes, de Alvarenga Rosa & Passos Arpini 2016), relaxation is performed to provide the most suitable solution for the considered situation. For example, transportation of clothes or some tools have no special requirements regarding fragility and orientation.

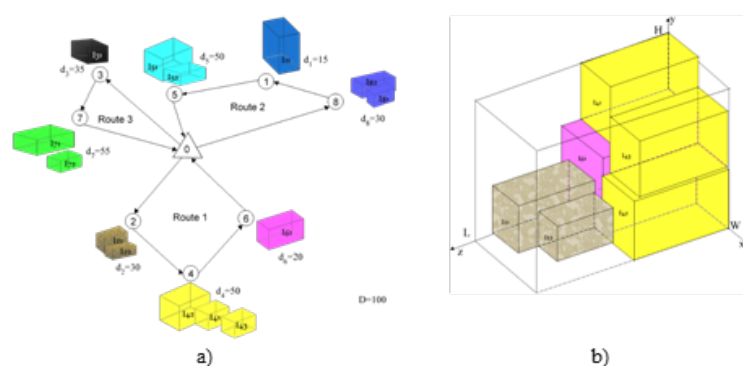


FIGURE 1. Example of a partial solution for a 3L-CVRP instance.

TABLE 1. Overview of solving approaches for the 3L-CVRP

| Work   | Type of problem           | Solving Approach  | Loading constraints                     | Visual Assessment |
|--|---------------------------|---|---|-------------------|
| Gendreau(Gendreau et al., 2006)                              | 3L-CVRP/Relaxed 3L-CVRP   | Tabu Search (TS) to solve the routing problem with two heuristics (bottom left algorithm, touching perimeter algorithm) to solve the loading problem. Additional optimization of routes by using 4-opt.   | Orientation, LIFO, Stability, Fragility | No                |
| Fuellerer(Fuellerer et al., 2010)                            | 3L-CVRP/Relaxed 3L-CVRP   | Ant Colony Optimization (ACO) to solve the routing problem and two heuristics (bottom-left-fill algorithm, touching perimeter algorithm) to solve the loading problem.  | Orientation, LIFO, Stability, Fragility | No                |
| Wang(Wang, Guo, Chen, Zhu, & Lim, 2010)                      | 3L-CVRP                   | TS to solve the routing problem with two heuristics (Deepest-Bottom-Left-Fill and Maximum Touching Area) for the loading problem.   | Orientation, LIFO, Stability, Fragility | No                |
| Borfeldt(Bortfeldt, 2012)                                    | 3L-CVRP/Relaxed 3L-CVRP   | TS to solve the routing problem and Tree Search Algorithm with Extreme Points to solve the loading problem.   | Orientation, LIFO, Stability, Fragility | No                |
| Junqueira(Junqueira, Oliveira, Carravilla, & Morabito, 2013) | 3L-CVRP                   | Integer Linear Programming (ILP)  | Orientation, LIFO, Stability, Fragility | Yes               |
| Zhu(Zhu, Qin, Lim, & Wang, 2012)                             | 3L-CVRP                   | TS to solve the routing problem and two heuristics (Deepest-Bottom-Left-Fill (DBLF), Maximum Touching Area (MTA)) to solve the loading problem.   | Orientation, LIFO, Stability, Fragility | No                |
| Ruan(Ruan, Zhang, Miao, & Shen, 2013)                        | 3L-CVRP/Relaxed 3L-CVRP   | Honey Bee Mating Optimization (HBMO) to solve the routing problem and six heuristics (Back_Left_Low, Left_Back_Low, Max_Touching_Area_W, Max_Touching_Area_No_Walls_W, Max_Touching_Area_L, Max_Touching_Area_No_Walls_L) to solve the loading problem.   | Orientation, LIFO, Stability, Fragility | No                |
| Lacomme(Lacomme, Toussaint, & Duhamel, 2013)                 | 3L-CVRP / Relaxed 3L-CVRP | Greedy Randomized Adaptive Search Procedure (GRASP) and Evolutionary Local Search (ELS) to solve the routing problem. The x-y axes are considered to place the items by only adding the height of the items in the same x-y coordinates; if the loading is feasible, then the items are pulled to the extremes, and the position in the z-axis is calculated. | None                                    | Yes               |
| Wei(Wei, Zhang, & Lim, 2014)                                 | 3L-FCVRP                  | Variable Neighborhood Search (VNS) to solve the routing problem and the Open Space-Based First Fit heuristic to solve the loading problem.  | Orientation, LIFO, Stability, Fragility | No                |

cont.

cont.

|   |                            |  |   |     |
|---|----------------------------|--|---|-----|
| Tao (Tao & Wang, 2015)  | 3L-CVRP/Relaxed<br>3L-CVRP | TS to solve the routing problem and two heuristics (least waste algorithm, touching perimeter algorithm) to solve the loading problem.     | Orientation,<br>LIFO, Stability,<br>Fragility | No  |
| Escobar(Escobar-Falc3n,<br>3lvarez-Mart3nez,<br>Granada-Echeverri,<br>Escobar, & Romero-<br>L3zaro, 2015) | 3L-CVRP/Relaxed<br>3L-CVRP | Branch-and-Cut (B&C) algorithm to solve the routing problem and GRASP algorithm to solve the loading problem.                              | Orientation,<br>LIFO, Stability,<br>Fragility | Yes |
| Mahvash (Mahvash,<br>Awasthi, & Chauhan,<br>2015)   | 3L-CVRP                    | Column-Generation(CG) algorithm combined with pricing problem to solve the routing problem and extreme points to solve the loading problem | Orientation,<br>LIFO, Stability,<br>Fragility | No  |

TABLE 2. Dimensions of items (boxes) to be loaded into the vehicle

| Customer | Box | Length | Width | Height |
|----------|-----|--------|-------|--------|
| 20       | 1   | 36     | 10    | 10     |
| 20       | 2   | 29     | 10    | 8      |
| 20       | 3   | 29     | 8     | 9      |
| 1        | 1   | 34     | 10    | 13     |
| 13       | 1   | 14     | 9     | 11     |
| 13       | 2   | 24     | 7     | 7      |
| 13       | 3   | 15     | 10    | 8      |
| 7        | 1   | 15     | 11    | 17     |
| 7        | 2   | 22     | 6     | 12     |
| 22       | 1   | 18     | 13    | 11     |
| 22       | 2   | 22     | 8     | 11     |
| 22       | 3   | 19     | 12    | 17     |

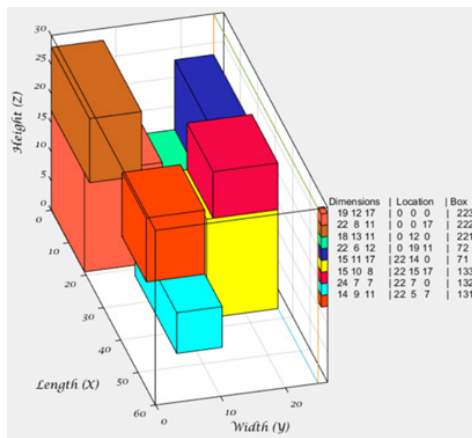


FIGURE 2. Example of a fully constrained 3L-CVRP loading solution

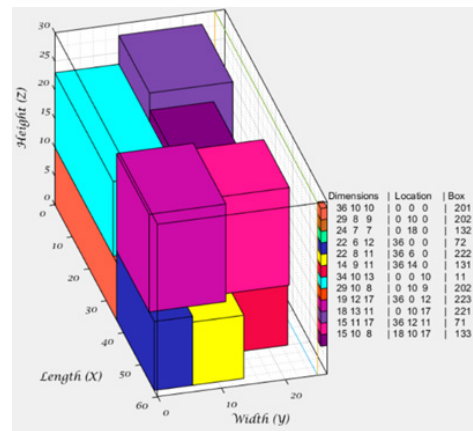


FIGURE 3. Example of relaxed (unconstrained) 3L-CVRP loading solution

Assessment of solving methods for the fully constrained 3L-CVRP (such as those reviewed in Table 1) have considered the Relaxed (unconstrained) 3L-CVRP as a reference to study the implications of relaxation. Thus, providing a reference solution for the Relaxed 3L-CVRP can lead methods to improve the fully constrained 3L-CVRP. Particularly, in emergencies where the full load is required without specific loading restrictions, the Relaxed 3L-CVRP can be of better application.

#### PROPOSED HEURISTIC APPROACH FOR LOADING

The methodology used to solve the 3L-CVRP is composed by three-phase:

Phase a) establish the instances to use. The necessary data on each instance are capacity and dimensions of vehicle, position in  $x$ - $y$  axes and demand for each customer, and finally, dimensions for each one of the items required for customers. In this phase the instances selected to use were the used in (Gendreau et al. 2006).

Phase b) a routing solution. This phase was obtained with the support of the VRP Solver V.1.3 tool developed by Larry Snyder. This VRP solver tool uses a randomized version of Clarke and Wright, swap, 2-opt, and or-opt to solve CVRP problems. In this phase a heuristic was developed to adapt the 3L-CVRP demand in CVRP demand.

Phase c) a loading solution. For the loading solution, a heuristic was developed based on the model of Lacomme (Lacomme et al. 2013) for the 3L-CVRP. However, the proposed heuristic differs from the model of Lacomme in that the positioning in the  $z$ -axis is performed while positioning in the  $x$  and  $y$ -axes. Figure 5 presents the structure of the loading heuristic.

As presented in Figure 5, the items are rotated based on three rules previous to their loading into the vehicle:

1. In the first iteration of the algorithm (rule = 1), the items are not rotated.
2. In the second iteration (rule = 2), only the items which do not exceed the dimensions of the vehicle are rotated in the  $x$ - $y$  axes.
3. In the third and final iteration (rule = 3), only the items which do not exceed the dimensions of the vehicle are randomly rotated in the  $x$ - $y$ - $z$  axes.

Then, the rotated and not rotated items (**O**) are sorted according to the following sorting schemes which are randomly selected: (a) descending order based on their volume, (b) descending order based on their bottom area, (c) descending order based on height, (d) ascending order based on height, and (e) sorted to fill the width (f) random sorting.

After the items are sorted, the arrays  $L_x$ ,  $L_y$ , and  $L_z$  are initialized ( $\{0,0,0\}$ ). These arrays contain the

advancing coordinates ( $pos_x$ ,  $pos_y$ ,  $pos_z$ ) on the axis  $x$ ,  $y$ , and  $z$ , respectively. Also, the variables  $count$ ,  $ok$  and  $i$  are initialized ( $count=0$ ,  $ok=true$ ,  $i=1$ ).

While there are items in **O** and  $ok$  is true, the items will be loaded into the vehicle, and the variable  $count$  will be increased by one until a limit established will be reached. The first item loaded into the vehicle is placed in coordinates  $(0,0,0)$ ; the following items are loaded in the available space within the vehicle. The available space is performed sequentially through the  $x$ - $y$ - $z$  axes as follows (always the space with lower values on  $pos_x$ ,  $pos_y$ ,  $pos_z$  are first considered):

1. Available space is searched on the  $x$ -axis. If space is available, the item is loaded and removed from **O**, and  $L_x$ ,  $L_y$ , and  $L_z$  are updated. Otherwise, available space is searched on the  $y$ -axis.
2. Available space is searched on the  $y$ -axis. If space is available, the item is loaded and removed from **O**, and  $L_x$ ,  $L_y$ , and  $L_z$  are updated. Otherwise, available space is searched on the  $z$ -axis.
3. Available space is searched on the  $z$ -axis. If space is available, the item is loaded and removed from **O**, and  $L_x$ ,  $L_y$ , and  $L_z$  are updated. Otherwise, the item is moved to the end of the list **O** to be loaded into the final available spaces.

If the variable  $count$  has reached the limit established, the variable  $ok$  is changed from real to false to end the loop. Heuristic developed to adapt 3L-CVRP demand data into standard CVRP demand data to use with VRP Solver. Figure 4 presents the heuristic for the adaptation of demand for the solver tool.

The initial routing is performed without change the characteristics of vehicles or instances. The result obtained with VRP Solver permits the second step, which is to proceed with the loading of items in vehicles. If the loading is not possible, the heuristic shown in Figure 4 is followed.

As presented in Figure 5, the items are rotated based on three rules previous to their loading into the vehicle:

1. In the first iteration of the algorithm (rule = 1), the items are not rotated.
2. In the second iteration (rule = 2), only the items which do not exceed the dimensions of the vehicle are rotated in the  $x$ - $y$  axes.
3. In the third and final iteration (rule = 3), only the items which do not exceed the dimensions of the vehicle are randomly rotated in the  $x$ - $y$ - $z$  axes.

Then, the rotated and not rotated items (**O**) are sorted according to the following sorting schemes which are randomly selected: (a) descending order based on their volume, (b) descending order based on their bottom area, (c) descending order based on height, (d) ascending order based on height, and (e) sorted to fill the width (f) random sorting.

**Algorithm.** 3L-CVRP demand into CVRP demand

1. **Procedure** demand
2. **Input parameters**
3. Instance (demand weight, capacity, dimensions of items, quantity of items)
4.  $ok=true$
5. **Output parameters**
6. Instance with fixed parameters
7. **begin**
8. **while**  $ok$
9. load the items in vehicles
10. **if** items can be loaded in vehicles
11.  $ok=false$
12. **else**
13. decrease the capacity of the vehicle
14. **if** items can be loaded in vehicles
15.  $ok=false$
16. **else**
17. **for**  $i=1$  to 3 **do**
18. modify the weight of the demand of customers according to the quantity and dimensions of items they required
19. **endfor**
20. **endif**
21. **endif**
22. **endwhile**

FIGURE 4. Adaptation heuristic for 3L-CVRP demand into CVRP demand

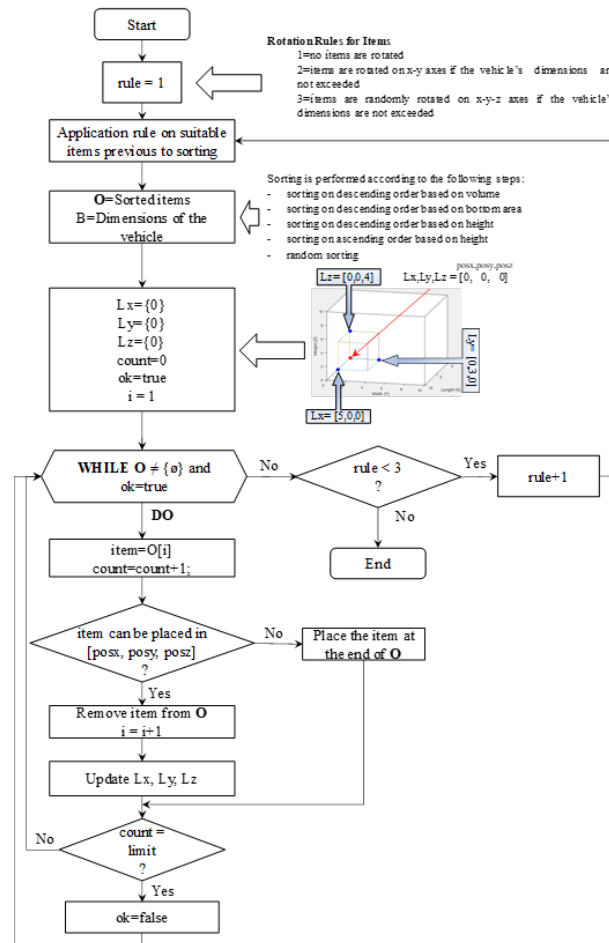


FIGURE 5. Loading heuristic for Relaxed 3L-CVRP.



After the items are sorted, the arrays  $L_x$ ,  $L_y$ , and  $L_z$  are initialized ( $\{0,0,0\}$ ). These arrays contain the advancing coordinates ( $pos_x$ ,  $pos_y$ ,  $pos_z$ ) on the axes  $x$ ,  $y$  and  $z$ , respectively. Also, the variables  $count$ ,  $ok$  and  $i$  are initialized ( $count=0$ ,  $ok=true$ ,  $i=1$ ).

While there are items in  $\mathbf{O}$  and  $ok$  is true, the items will be loaded into the vehicle, and the variable  $count$  will be increased by one until a limit established will be reached. The first item loaded into the vehicle is placed in coordinates  $(0,0,0)$ ; the subsequent items are loaded in the available space within the vehicle. The available space is performed sequentially through the  $x$ - $y$ - $z$  axes as follows (always the space with lower values on  $pos_x$ ,  $pos_y$ ,  $pos_z$  are first considered):

1. Available space is searched on the  $x$ -axis. If space is available, the item is loaded and removed from  $\mathbf{O}$ , and  $L_x$ ,  $L_y$ , and  $L_z$  are updated. Otherwise, available space is searched on the  $y$ -axis.

2. Available space is searched on the  $y$ -axis. If space is available, the item is loaded and removed from  $\mathbf{O}$ , and  $L_x$ ,  $L_y$ , and  $L_z$  are updated. Otherwise, available space is searched on the  $z$ -axis.
3. Available space is searched on the  $z$ -axis. If space is available, the item is loaded and removed from  $\mathbf{O}$ , and  $L_x$ ,  $L_y$ , and  $L_z$  are updated. Otherwise, the item is moved to the end of the list  $\mathbf{O}$  to be loaded into the final available spaces.

If the variable  $count$  has reached the limit established, the variable  $ok$  is changed from true to false to end the loop

## RESULTS & DISCUSSION

Implementation of the algorithm described in Figure 5 was performed in MATLAB R2015a in a DELL laptop with Intel Core i-7 CPU 8750H at 2.20 GHz with 16 GB RAM. Tests

TABLE 3. Comparison of performance of the proposed algorithm for the Relaxed 3L-CVRP.

| Instance  | (Gendreau et al., 2006) | (Fuellerer et al., 2010) | (Bortfeldt, 2012) | (Lacomme et al., 2013) | Proposed algorithm |
|-----------|-------------------------|--------------------------|-------------------|------------------------|--------------------|
| 1         | 297.65                  | 297.65                   | 297.65            | 297.65                 | 297.37             |
| 2         | 334.96                  | 334.96                   | 334.96            | 335.67                 | 299.64             |
| 3         | 362.27                  | 362.27                   | 381.36            | 362.27                 | 371.36             |
| 4         | 430.89                  | 430.89                   | 430.89            | 430.89                 | 372.98             |
| 5         | 395.64                  | 406.50                   | 397.16            | 379.43                 | 425.62             |
| 6         | 495.85                  | 495.85                   | 498.07            | 495.85                 | 452.95             |
| 7         | 742.23                  | 732.52                   | 741.80            | 725.43                 | 951.08             |
| 8         | 735.14                  | 735.14                   | 735.14            | 735.14                 | 726.67             |
| 9         | 630.13                  | 630.13                   | 631.82            | 630.13                 | 621.95             |
| 10        | 717.90                  | 711.45                   | 739.94            | 687.57                 | 920.23             |
| 11        | 718.24                  | 718.25                   | 723.44            | 718.24                 | 719.31             |
| 12        | 614.60                  | 612.63                   | 623.10            | 610.05                 | 584.34             |
| 13        | 2316.56                 | 2391.77                  | 2348.48           | 2306.04                | 2734.09            |
| 14        | 1276.60                 | 1222.17                  | 1234.54           | 1186.96                | 1458.01            |
| 15        | 1196.55                 | 1182.86                  | 1202.34           | 1161.20                | 1757.66            |
| 16        | 698.61                  | 698.61                   | 704.47            | 698.61                 | 680.38             |
| 17        | 906.42                  | 862.18                   | 928.93            | 861.80                 | 859.32             |
| 18        | 1124.33                 | 1112.18                  | 1108.37           | 1084.26                | 1214.00            |
| 19        | 680.29                  | 671.60                   | 678.59            | 670.44                 | 679.85             |
| 20        | 529.00                  | 515.39                   | 520.55            | 510.95                 | 527.65             |
| 21        | 1004.40                 | 951.87                   | 964.66            | 943.05                 | 918.14             |
| 22        | 1068.96                 | 1030.12                  | 1041.92           | 1029.87                | 1010.62            |
| 23        | 1012.51                 | 971.05                   | 995.22            | 987.06                 | 1041.14            |
| 24        | 1063.61                 | 1057.39                  | 1053.41           | 1056.33                | 1039.31            |
| 25        | 1371.32                 | 1207.97                  | 1238.83           | 1232.73                | 1276.87            |
| 26        | 1557.12                 | 1453.39                  | 1444.58           | 1415.15                | 1376.72            |
| 27        | 1378.52                 | 1333.16                  | 1342.23           | 1317.38                | 1356.29            |
| Avg. cost | 876.31                  | 856.67                   | 864.53            | 847.04                 | 913.83             |

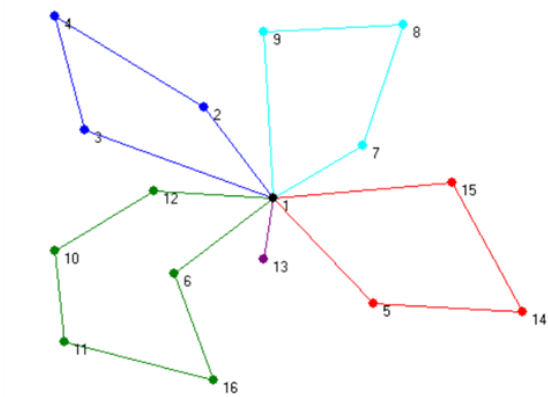
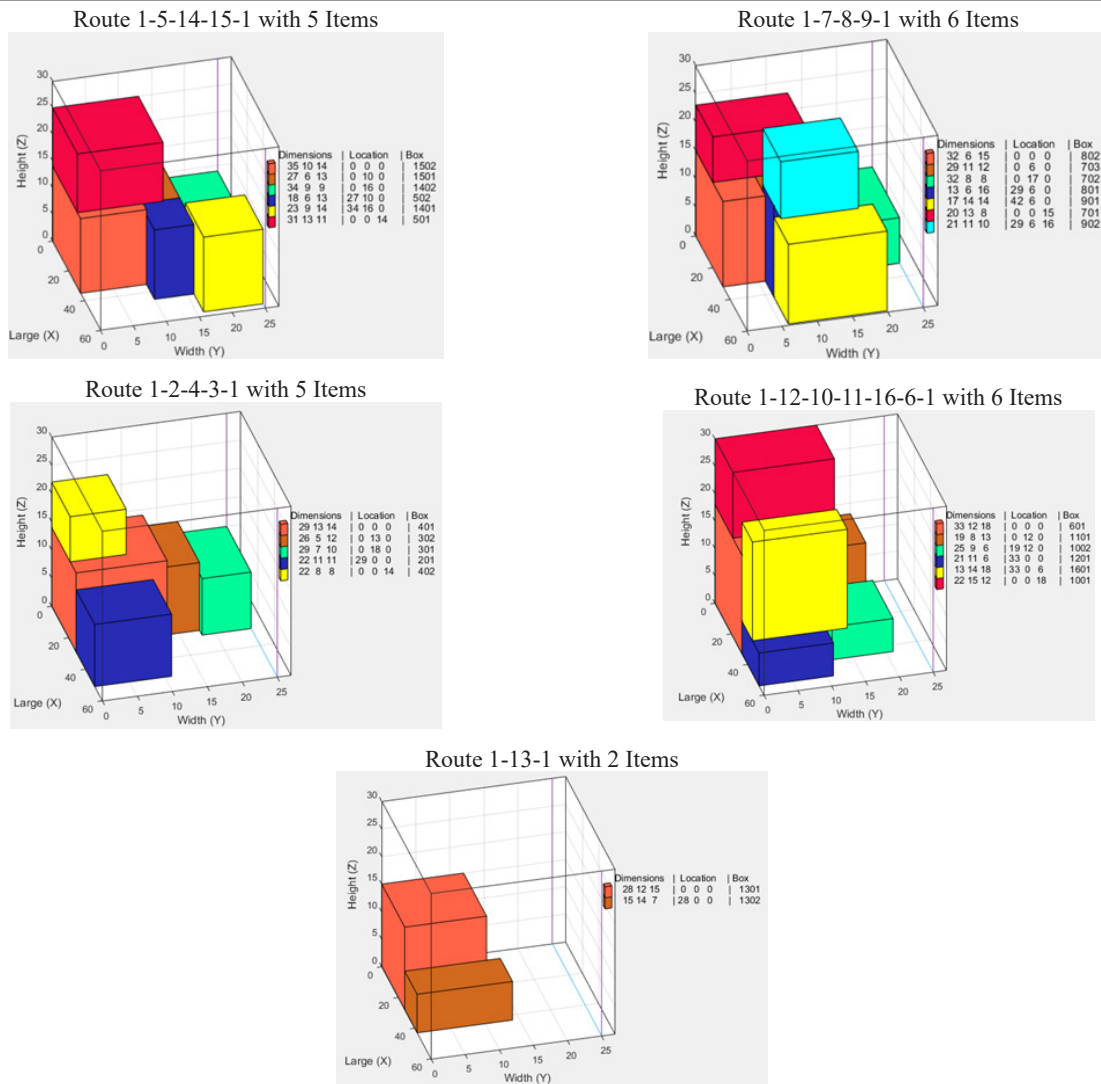


FIGURE 6. Benchmark relaxed routing solution for 3L-CVRP instance 2

TABLE 4. Benchmark relaxed loading solution for 3L-CVRP instance 2.





were performed with the 3L-CVRP instances reported by (Gendreau et al. 2006).

The heuristic proposed is based on the relaxation of all the constraints, including the rotation in z-axis, this option was not considered by other investors.

The proposed heuristic was probed with 27 instances obtaining better results in 48.1% (13/27) of the instances.

The instances were divided in three groups with nine elements in each one of them, to analyze the results, the first group has 15-25 customers and 26-50 items; the second group has 29-44 customers and 58-94 items; the last group has 50-100 customers and 99-198 items.

The results show that in the first group, the result of six instances are improved, in the second group the results of three instances are improved and in the last group the results of four instances are improved. The better results are obtained with less than 26 customers and less than 51 items, for this reason we can conclude that the algorithm works better with little instances.

The results also were analyzed to know how much is improved in each group, obtaining that for the first group the average improvement was 5.23%; for the second group the average improvement was 2.14%; and for the last group the average improvement was 2.18%. The group with less customers and less items obtained the better results.

Table 3 presents the comparison of the performance of the proposed algorithm for the Relaxed 3L-CVRP with the algorithms reported by (Gendreau et al. 2006; Fuellerer et al. 2010; Bortfeldt 2012; Lacomme et al. 2013).

The details of the solution, for instance two, are described in Figure 6 and Table 4.

#### CONCLUSION AND FUTURE WORK

In this work, an algorithm to solve the 3L-CVRP when the relaxation of constraints is needed and appropriate to enhance load capacity in transportation was developed. In practice, not all constraints are used, and their consideration depends on the type of items to be distributed.

For example, although the LIFO constraint allows items to be unloaded without additional movements, it is one of the constraints that can lead to significant empty spaces in the loading space. As more constraints are used, the utilization of the vehicle's capacity is decreased. Thus, emptier vehicles are obtained, requiring additional vehicles to serve a distribution network.

On the other hand, the support constraint can be relaxed when no fragile items are transported, especially when the items are loaded first over the  $x$ - $y$  axes and finally on the  $z$ -axis because it improves the support of the items.

In an emergency or urgent request of items, it is necessary to evaluate what is more important, to avoid having extra movements (i.e., comply with LIFO constraints) or to transport the highest quantity of items with full utilization of the vehicle's cargo.

Although there are reported works on solving approaches for the relaxed 3L-CVRP that consider product rotation to optimize load capacity, rotation on the three axes has not been thoroughly studied. In this aspect, the present work explicitly explored the three-axes rotation and its impact on load capacity optimization.

This approach led to obtain better solutions than those reported in reviewed works for the Relaxed 3L-CVRP in 48.1% of well-known instances. However, more research is needed due to the complexity of 3L-CVRP. Thus, future research will be focused on the following actions:

1. Improve the algorithm to achieve improvements on the fully constrained 3L-CVRP. The use of Tabu-Search to improve diversification of the sorting options can be considered.
2. Adapt the algorithm for the 3L-CVRP with more than one depot.
3. Extend on the constraints for the 3L-CVRP.

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#### DECLARATION OF COMPETING INTEREST

None.

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