Stimulus Control for Making Math Verbal

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ABSTRACT

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In three experiments, I first examined the correlation between the presence of transformation of stimulus function (TSF) across computation and the presence of TSF across saying and writing for spelling words, and then tested the effects of the establishment of TSF across saying and writing on the establishment of TSF across math operants. Eight middle school students with learning disabilities participated in experiments I and II. All participants demonstrated reader/writer and math skills such as textual responding and using counting strategies to solve one-step word problems. Four of the eight participants also demonstrated TSF across saying and writing for spelling. The dependent variables of Experiment I were the accuracy and fluency of solving word problems after receiving fluency training on math facts, as well as the number of counting strategies used when solving word problems. Results showed that all participants with TSF across saying and writing for spelling demonstrated significant increases in both their accuracy and fluency when responding to word problems (i.e., ES = 1) whereas participants who did not demonstrate TSF across saying and writing for spelling demonstrated minimal gain from accuracy and fluency training of math facts (i.e., mean ES = 0.3). Experiment II tested the effects of fluency and accuracy training of word problems on the accurate and fluent responding to math facts and other math operants. Results showed that accuracy and fluency training had large effects on all participants (i.e., ES = 1). Participants who did not demonstrate TSF also demonstrated larger improvement (i.e., ES > 0.67) compared to Experiment I. The results of Experiments I and II demonstrated an association between TSF across math operants and TSF across saying and writing for spelling. Experiment III further tested for a functional relation by

examining the effects of the establishment of TSF across saying and writing for spelling on the establishment of TSF across math operants with three of the participants who did not demonstrate TSF across saying and writing for spelling in the first two experiments. Upon establishment of TSF across saying and writing for spelling words, all three participants demonstrated TSF across math operants (i.e., increased accuracy and fluency of word problems, extinction of counting strategies). The results of the three experiments suggest the importance of teaching math as a verbal behavior, more specifically, as a speaker-as-own-listener behavior instead of as visual match-to-sample repertoires. Future replication of the procedure is needed to extend the external validity of the current experiments.

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Dedication

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Chapter I

Introduction

Former President Barack Obama addressed science, technology engineering, and math (STEM) as a critical way to understand, explore, engage with, and change the world (Obama, 2015). Acquiring proficient mathematical skills is one of the key predictors of one's academic achievement, even more predictive than verbal skills (Delaney & Devereux, 2020). However, only a fraction of American high school graduates demonstrated satisfactory mastery of math concepts upon graduation (USDOE, 2010). Foundational math skills are also vital for students to become independent and functional members of society (NMAP, 2008; NAEP, 2015). Phillips (2007) reported that more than half of Americans have difficulties calculating interest for loans or tips at restaurants. Schoenfeld and Stipek (2012) reported that gaps between performance levels among children emerged as early as kindergarten and that the gap only increases.

To address the performance gap in mathematics, the National Council of Teachers of Mathematics (NCTM, 2010) identified problem-solving with math, that is, applying learned math skills to solve real-life problems, as the focus of all mathematics teaching practices. Word problems, as the most fundamental form of problem solving, provide students with an opportunity to connect math concepts with real-life situations, apply learned concepts, and thus actively engage in their learning (Cai & Lester, 2010). However, solving word problems is a multi-step process that requires students to read and comprehend the problem, reason, translate words to mathematical expressions, and correctly solve the arithmetic that prevented a lot of students from excelling in math (Neef et al., 2003).

To target this multi-step process, early research focused on teaching schematic tools (i.e., visual prompts) to help students translate words to visual prompts and then translate visual prompts to mathematical expressions (Griffin & Jitendra, 2009; Hegarty & Kozhevnikove, 1999; Krawec, 2014; Marshall, 1995). It was not until recent decades that math education practice shifted the focus to the development of mathematical literacy. The Organisation for Economic Co-operation and Development (OECD) defined mathematical literacy as the "capacity to formulate, employ and interpret mathematics in *a variety of contexts*" (OECD, 2018, p. 67). This means that someone who is literate in mathematics applies previously acquired math skills to solve math problems presented in different contexts. From a behavioral science perspective, when an individual emits untaught behavior within a novel context as a result of previously learned stimulus control, transformation of stimulus function occurred (Barron et al., 2019; Dymond & Rehfeldt, 2000). Thus, the struggle to apply learned math concepts in different contexts is the struggle to demonstrate transformation of stimulus function.

Transformation of stimulus function across different stimuli and response topographies has been examined in previous research (Eby et al., 2010; Greer et al., 2005; Lamarre & Holland, 1985; Ross & Greer, 2003; Stafford et al., 1988; Tsiouri & Greer, 2003). They found that different types of operants are initially independent. That is, teaching one operant does not result in the acquisition of a different operant with the same response topography or function. For example, Eby et al. (2010) taught a set of four novel spelling words in written topography and tested the participants' responses when asked to spell the same words vocally. They found that, prior to the intervention, the participants did not emit 100% correct responses to the untaught topography. Greer et al. (2007) observed similar results in terms of the transformation of establishing operation across mand and tact. Students who acquired a vocal response to mand for a specific item did not emit the same word as a tact before receiving additional interventions to join those two responses.

However, the transformation of stimulus function across computation skills and problem-solving skills has not been closely examined or tested. The current experiments examined the transformation of stimulus function across computation skills (i.e., math facts) and problem-solving skills (i.e., word problems) and its association with the transformation of stimulus function across saying and writing for spelling words (i.e., Experiments I and II). A functional relation between the establishment of TSF across math operants and TSF across saying and writing for spelling was then concluded when the establishment of TSF across saying and writing for spelling resulted in the establishment of TSF across math operants (i.e., Experiment III).

The current experiments are significant and crucial to the development of both curriculum design and research on mathematical literacy and problem solving. The results of Experiments I and II informed better teaching practice, specifically, that for students who did not demonstrate TSF across saying and writing, math fact fluency training had low effects on their accuracy and fluency of math problem solving. However, when we conducted fluency training with word problems, all participants benefitted. The results of Experiment III suggested that by establishing TSF across saying and writing, students' reliance on visual prompts shifted to speaker-as-own-listener behavior which led to an increase in their computation and problem-solving fluency, suggesting that teachers should also teach mathematical skills as speaker behavior instead of listener behavior only.

Keywords: fluency, math facts, transformation of stimulus function, word problems.

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CHAPTER II

STUDY I MANUSCRIPT

Abstract

Existing literature shows mixed findings on the effectiveness of computation skills training on accurate responding to other math operants involving computation. In 2 experiments, we tested the effects of accuracy and fluency training of math facts on accurate and fluent responding to word problems and vice versa. The participants of the study were 8 middle school students with various learning disabilities aged from 11-14 years enrolled in a multi-grade special education classroom. All participants performed below grade level on numbers- and operations- related math tasks. Experiment I used a multiple probe design across dyads to test the effects of training of math facts to accuracy and fluency criteria on participants' accuracy and rate of responding to word problems employing the same math facts targeted in fluency training. Experiment II systematically replicated Experiment I to test the effects of training of word problems with accuracy and fluency criteria on the participants' accuracy and rate of responding to math facts and other math problem-solving employing the same number families targeted in the word problems. Results showed increases in accuracy and fluency for 4 of the 8 participants. The one consistent difference was that participants demonstrating effects also demonstrated transformation of stimulus function (TSF) across saying and writing whereas only 1 participant who did not demonstrate TSF showed weak transformation. This raised the possibility that transformation of stimulus function across saying and writing might be related to the transfer of accuracy and fluency from computation skills to word problem-solving.

Keywords: fluency, math facts, transformation of stimulus function, word problems.

The Effects of Fluency Training on Fluent Responding Across Math Skills

A recent Pew Research Center analysis showed that science, technology, engineering and math (STEM) related jobs grew 79% since 1990, three times faster than overall job growth (National Mathematics Advisory Panel, 2008). An average STEM worker also earns 26% more than an average non-STEM worker. Acquiring proficient mathematical skills, a significant predictor of children's academic achievement outcomes, is not only important for students who are interested in STEM-related professions, but also vital for students to become independent and functional members of society (Duncan et al., 2007; NAEP, 2015; NMAP, 2008). However, according to the United States Department of Education (USDOE), only 16% of American high school graduates are proficient in math. The United States ranked 33rd in math and 17th in science among 77 countries according to the most recent Programme for International Student Assessment (PISA), scoring below average among fifteen-year-olds who participated in the assessment from other countries (PISA, 2018).

The National Council of Teachers of Mathematics (NCTM, 2010) identified problem-solving with math, that is, applying learned math skills to solve real-life problems, as the focus of all mathematics teaching practices. However, Bae (2015) reported that problem-solving skills, or solving word problems as the most elementary form of problem-solving, has prevented many children from excelling in math. Nesher et al. (2003) defined *solving word problems* as "the ability to deduce new information given the information presented in words and numerals," which is a multi-step, "cognitive process" employing multiple strategies and frameworks (Hegarty et al., 1995; Mayer, 1999; Montague & van Garderen, 2003).

Although different factors play into the difficulties of learning math, students who struggled with math do share some general characteristics (Goldman, 1989; Mercer, 1997;

Rivera, 1997). However, the discussion of those characteristics often resorts to mentalistic, cognitive constructs such as working memory or other memory-related deficits (Kroesgergen & Van Luit, 2003). Such attribution of the difficulty in acquiring problem-solving skills to mentalistic, cognitive constructs has led to a large body of research focusing on using direct instructions and interventions to establish automaticity for computation or to teach students to solve basic word problems to release individuals' working memory to target problem-solving (Beirne-Smith, 1991; Case et al., 1992; Cassel & Reid, 1996; Cook et al., 2019; Fuchs et al., 2014; Ginsburt-Block & Fantuzzo, 1998).

Carnine (1997) argued that simply mastering computations and operations is not sufficient to accurately respond to word problems as students also need to know when and how to apply acquired skills to new situations. Fuchs et al. (2008) further pointed out that the major gap to be filled is linguistic information presented in word problems that required students to construct a problem model with the information presented. Most research targeting word problem-solving published over the past five years utilized strategies targeting the translation of words to mathematical expressions. However, these studies showed that after the participants mastered "schema-based" strategies, strategies that focused on developing a plan to solve a problem or an algorithm, they still did not emit 100% accurate responses to word problems (Browder et al., 2018; Chadli et al., 2017; Driver & Powell, 2017).

From a science of behavior perspective, Sidman (2008), building on the epistemology of behavior as a science (Skinner, 1954), referred to the source of memory, or the behavior of remembering, as stimulus control topographies established in the past. In addition, Delaney and Austin (1998) defined working memory as "stimulus control by perceptually unavailable stimuli over relatively short time periods" (p. 82). That is, the root of deficits in memory or working memory lies in insufficient stimulus control. However, little research has been done that focuses on the establishment of stimulus control for math operants and the transformation of such stimulus control between accuracy/fluency in calculation skills and the accuracy/fluency for word problems.

In fact, unlike reading comprehension skills, advanced math skills such as problem-solving have rarely been examined, tested, or taught from a behavior analytic perspective, although new findings portend new applications (Ross, et al., 2020). Most research in applied behavior analysis targeted teaching or increasing the fluency of specific tool or component skills when conducting math-related research (Browder, et al., 2008). Strategies and curricula rooted in principles of behavior such as Precision Teaching (Chiesa et al., 2000; Stromgren, et al., 2014), Direct Instruction (Al-Makahleh, 2011; Din, 1998; Firdaus, 2017; Kinder, 1991), peer tutoring (Mayfield, et al., 2007), and peer editing (Weber, 2016) have been utilized to teach math skills. Recently, more studies utilized an equivalence-based or relational frame approach to teach or bridge equivalent or related math concepts such as fractions and decimals (Verdun et al., 2019), algebraic and trigonometric functions (Ninness, et al., 2006), and size/area (Belisle, et al., 2019). However, most of those studies focused on teaching or connecting discrete math skills, leaving a gap in bridging the behavior chain of computation and problem-solving.

The few studies that addressed the relation between computation skills and problem-solving skills yielded mixed findings. McTiernan et al. (2016) tested the effects of Morningside Math Fact drills on the correct responses to math facts and application problems. However, the results showed that the participants only demonstrated increases in accurate responding to math computation but not more complex, application-based math operants. Singer-Dudek and Greer (2005) found that accurate and fluent responding to math facts resulted in better performance on composite skills during follow-up assessments but did not decrease the number of instructional trials required for students to master problem-solving objectives. With the limited number of participants involved in each study, it is hard to conclude any shared characteristics among participants that resulted in accurate responding to composite skills after receiving instructions on computation skills.

When solving word problems, students are typically presented with the same stimuli for computation skills and problem-solving skills once the students translate a word problem into a number sentence. For example, when given the word problem "Emily has 2 apples and she got 3 more from mom, how many apples does Emily have now?", a student will likely write down "2+3= " if the student correctly translated the word problem into a number sentence. However, those two skills are often taught as a behavior chain. Students are taught numbers and other computation skills before word problems or other problem-solving skills come into the scope and sequence of math curricula. Thus, some students would require additional instruction (e.g., multiple exemplar instruction across numerosity and application) to acquire the transformation of stimulus function for the initially separate responses to numerosity and application.

By Transformation of Stimulus Function (TSF), we mean that a stimulus that initially evokes or reinforces one or more responses (i.e., demonstrates convergent control) comes to control one or more additional stimuli forming a new relation and overarching operant (i.e., divergent control) (Greer, 2020; Morgan et al., 2020; Pohl et al. 2018). For example, when the number sentence 2+3=__ that evoked a response of 5 comes to also control a response of 5 when presented as a step in the chain of solving a word problem, TSF occurred. Moreover, when the natural reinforcement of problem-solving (e.g., finding the answer of a math problem in a real-life context by using manipulatives) also serves as a reinforcer for finding the answer for a mathematical expression, TSF occurred. Frequently, after students are trained to respond to math facts fluently, students still use fingers or other counting strategies when the same math facts were targeted as part of a word problem.

Past research focused on TSF between different response topographies such as mands and tacts or spelling words in vocal or written form (Eby et al., 2010; Greer et al., 2005; Ross & Greer, 2003; Lamarre & Holland, 1985; Stafford et al., 1988; Tsiouri & Greer, 2003). For example, Singer-Dudek et al. (2017) observed an initial functional independence of mands and tacts. That is, after the participants learned to emit an item name as a mand operant, the participants did not emit the item name as a tact operant. This required teachers to teach the same response as a mand and a tact separately. To join those initially separate responses, researchers utilized multiple exemplar instruction (MEI) to induce TSF. In Singer-Dudek et al. (2017), the experimenters alternated instructional trials between mand and tact opportunities. Once the participants emitted 90% accurate responses as both tacts and mands, the experimenters conducted post-intervention probes during which they taught new responses as tacts or mands and probed for the untaught response topography. They found that MEI successfully established transformation of establishing operations across mands and tacts. When the participants acquired a new response as a tact, they emitted those responses as a mand with no additional instructions. Eby et al. (2010) and Greer et al. (2005) observed similar results for TSF across spelling words in vocal or written form that after mastering the MEI, when taught to spell words vocally, students did not require extra instructions on spelling the same set of words in written form. Thus, teaching TSF across different response topographies and functions is essential to the optimal allocation of instructional time and resources.

The current study sought to test for the source of TSF across computation and problem-solving skills by examining the difference in the effects of accuracy and fluency training of computation skills (i.e., math facts) or problem-solving skills (i.e., solve word problems) on the accurate and fluent responding to the untrained skill between participants who demonstrated TSF across saying and writing for spelling and those who did not. With two experiments, we tested for the TSF across math operants by answering the following research questions: (1) Will the participants accurately and fluently respond to word problems when taught to accurately and fluently respond to math facts? (2) Will the participants accurately and fluently respond to math facts when taught to accurately and fluently respond to word problems? (3) Do accuracy and fluency training affect the behavior of the participants who demonstrated TSF across saying and writing for spelling differently comparing to the participants who did not demonstrate TSF across saying and writing?

EXPERIMENT I

Methods

Participants

The participants of the study were eight middle school students. All participants attended a public middle school in a school district located in a suburb outside a major metropolitan city. All participants were enrolled in a self-contained classroom utilizing the Comprehensive Application of Behavior Analysis to Schooling (CABAS®) model, where teachers based all instruction on scientific procedures and continuously measured student responses and performance (Greer, 1994; Greer, 2001; www.cabasschools.org; www.scienceofteaching.org).

All participants attended the same self-contained classroom for English language arts, mathematics, science and social studies. One participant was female, and seven participants were male. All eight participants had Individualized Education Plans (IEP). One head teacher and up to three paraprofessionals were in the classroom during all probe and intervention sessions. One participant, Gary, received one-to-one supervision from paraprofessionals as required by his IEP.

All participants demonstrated some relevant reader/writer repertoires and the prerequisite verbal behavior developmental cusps according to the verbal behavior development theory (See *Table 1*) (Greer & Ross, 2008). For example, the participants textually responded to words at grade levels ranging from 1st to 5th grade, demonstrated read-do, transcribed, and spelled words at their respective grade levels. Prior to the experiment, we assessed the presence or absence of TSF across saying and writing for all participants with the procedure outlined in Eby et al. (2010), during which the participants received instructions for novel spelling words with written or spoken responses. Once the participants mastered those novel words in one topography, the experimenters conducted a spelling probe in the untaught topography. The participants demonstrated TSF across saying and writing for spelling if they emitted 80% or more correct responses towards the spelling probe with untaught topography. Participants Gavin, Lucas, Kevin, and Evan demonstrated TSF for spelling across saying and writing whereas participants Jeff, Gary, John, and Sally did not. The experimenters selected those participants because they all performed two or more grade levels below their enrolled grade-level for reading and math according to *iReady Diagnostics Assessments* conducted at the beginning of the school year. We started teaching the prerequisites of the experiment after the assessment. The experiment started two weeks after the assessment. All participants received math facts fluency training for addition and subtraction facts using *Morningside Math Fluency* (Johnson, 2008). Prior to the intervention, all participants demonstrated mastery of using counting strategies (e.g., finger counting, drawing tallies, number line) to solve one-step multiplication/division word problems. We taught this to the participants as part of their IEP goals.

Settings and Materials

We conducted all probe and intervention sessions in the participants' classroom. The participants sat in a group of four with their own rectangular desk facing one another. For participants who required one-on-one or shared paraprofessional assistance, the paraprofessional sat across the table from the participant or one meter behind the participant.

The experimenter distributed word problem worksheets each with ten different word problems for probe sessions or fluency worksheets each with 100 math-facts problems targeting three multiplication number families. Students used a pencil to respond to worksheets and a timer to record how long it took them to complete a worksheet (See *Appendices B and C*). We placed additional blank paper on the desk for all participants regardless of their previous use of counting strategies. During intervention and probe sessions, the participants sat with an experimenter at one of the tables in one corner of the classroom, away from other students. We instructed the participants to keep their hands on the table so that they were visible to the experimenters at all time to facilitate observation of participants' use of counting strategies.

Dependent Variables

Accuracy

The dependent variables were the number and rate of correct and incorrect responses emitted to word problems, and the number of counting strategies used. Each word problem probe consisted of 10 word-problems targeting 10 different number sentences produced by the target number families. Each set of word problems contained exactly 314 words. A correct response consisted of two components: (a) correct selection of operation (i.e., multiplication vs. division) and (b) correct computation (e.g., 3 x 3 = 9). When the participant emitted an incorrect response, we coded incorrect selection of operation and incorrect computation differently to locate the true error within the behavior chain.

Fluency

During probe sessions, participants recorded the start and end time for each probe session. An Experimenter checked for accuracy and calculated rate of correct/incorrect responding using the formula: rate of correct responding=number of correct responses/duration in seconds*60, and rate of incorrect responding=number of incorrect responses/duration in seconds*60. In addition, experimenters only introduced intervention if the participants' rate of correct responding was descending or if the participants' rate of incorrect responding was stable or ascending. Otherwise, we conducted additional probe sessions until reaching a steady state of responding.

Counting Strategies

A third dependent variable was the participants' use of counting strategies. We defined counting strategies as visual prompts that helped participants visualize and count to solve the word problems. The use of visual prompts is referred to as schema-based strategies in most math education research. A schema is a plan for problem-solving developed by Marshall in 1995. With a schematic approach, students use a schema, mostly a graphic representation (e.g., tallies, number line, finger counting, etc.), to demonstrate number-object relations presented in words to graphically *demonstrate* the underlying structure of the problem (Powell, 2011). We recorded a maximum of 1 counting strategy per problem if the participants used multiple counting strategies for the same problem.

Independent Variables

The independent variables of the study were (a) the establishment of the accurate and fluent responding to math facts containing number families: (1) 3, 3, 9, (2) 3, 6, 18, and (3) 2, 9, 18 and (b) TSF across saying and writing for spelling words.

Accuracy and Fluency Criteria

We taught the target math facts to accuracy and then to fluency. We presented the participants with one of the three versions of the math fluency worksheets until the participants demonstrated accuracy by emitting 100% correct responses in one intervention session to one of the three versions of the worksheets. After three additional word-problem probe sessions, we further taught math facts using the same fluency worksheets until the participants reached their individualized fluency goal with 100% accuracy (See Table 1). During training, we implemented the CABAS® decision protocol (Greer, 2003; Greer et al., 2002). The CABAS® decision protocol informed decisions to continue (i.e., when observing three consecutive or five overall ascending data paths) or stop (i.e., when observing three consecutive or five overall descending or no trend data paths). We made a decision to stop the intervention and implement tactics when the participants emitted descending correct responses over four consecutive sessions or six overall sessions. We implemented the tactic for the participants to complete half a sheet of the math facts to criterion and re-introduced the full sheet. The tactics used to establish accuracy and fluency were not essential to the study since the independent variable was the establishment of accurate and fluent responding to math facts.

Transformation of Stimulus Function

Another independent variable was the presence and absence of TSF across saying and writing for spelling. Results from a pilot study demonstrated an association between the presence of TSF across math facts and the presence of TSF across saying and writing for spelling (See *Appendix A*). Thus, we matched one participant who demonstrated TSF across saying and writing for spelling and one participant who did not demonstrate TSF across saying and writing for spelling in each dyad by their rate of responding to math facts to examine the difference between the effects of the intervention on participants in different experimental groups. To assess for TSF across saying and writing for spelling, we taught the participants to spell a set of five novel words in written form to mastery with a criterion of 90% accurate responding across two consecutive sessions. Each session consisted four presentations of the novel words in randomized order. Upon mastery of the novel words, we tested if the participants spelled those words vocally. If the participant emitted 80% correct vocal spelling responses, then the participant demonstrated TSF across saying and writing for spelling (Eby et al., 2010).

Interscorer and Interobserver Agreement

We conducted interscorer agreement (ISA) for the accuracy and fluency of all word problem probe sessions using permanent products with the formula ISA=number of agreement/(number of agreement+ number of disagreement)*100%. Experimenters obtained ISA for 100% of the probe sessions with 100% agreement. To conduct ISA for accuracy of word problems, two scorers independently scored the word problems and compared the scoring.

We also conducted interobserver agreement (IOA) on the use of counting strategies. The experimenters conducted IOA by having two observers simultaneously observe and independently collect data on the participants' use of counting strategies. We then compared data and calculated IOA with the formula: IOA=number of agreement/total numbers*100%. We gathered IOA for 49% of the probe sessions with 100% agreement.

We conducted interscorer agreement on students' responses to word problems and *Morningside fluency* (Johnson, 2008) worksheets using permanent products and the formula: ISA=number of agreement/(number of agreement+ number of disagreement)*100%. Experimenters obtained ISA for 100% of the probe sessions with 100% agreement. Obtaining ISA was especially crucial during fluency training as the participants did not receive another probe session until they mastered or fluently responded to fluency worksheets. Experimenters obtained 100% agreement for 100% of the accuracy and fluency training sessions.

Procedure and Data Collection

Prior to the intervention, the experimenters conducted a probe for writing and textual responding fluency to determine the fluency criterion. The experimenters instructed the participants to write numbers zero through nine repeatedly in a minute and reported the number as digits per minute. The experimenter then set fluency criterion at 75% of participants' writing rate. For example, students who wrote 60 digits per minute had a fluency criterion of 60*75%=45 digits per minute. The participants' target word problem fluency goal was then set at Goal=314/TR rate*60+600/Fluency Criterion. We used a random sequence generator to produce the sequence of word-problem worksheets that each participant followed.

During pre- and post-intervention probe sessions, the experimenter recorded the accuracy and fluency of participants' responses to each set of word problems. For accuracy, the experimenter marked a "+" for each correct response emitted by the participant, an "O" for each incorrect response with operation selection errors (e.g., student performed addition for a multiplication problem), or a "C" for each incorrect response with computation errors (e.g., student wrote 3x3=7). The experimenter then reported the number and the rate of correct/incorrect responses using the formula rate = number of questions (correct or incorrect)/duration (in seconds) on a stacked bar graph. We did not provide the participants with any feedback after the probe sessions.

During the intervention, we presented the participant with one of the three versions of fluency worksheets and instructed the participants to complete the sheet as accurately and as quickly as possible. When the participant finished the worksheet, the experimenter reinforced the emission of every correct response. If the participant emitted any incorrect responses, the experimenter presented a correction procedure during which the experimenter modeled the correct response and provided the participant with an independent opportunity to respond for all the incorrect responses at the end of each timed session. After the participant mastered a fluency sheet by emitting 100% correct responses to one worksheet, the experimenter conducted post-accuracy word-problem probes.

For fluency, participants used a timer or the experimenters observed the participants using a timer projected on a whiteboard to self-record duration of their word-problem solving. Before starting the timer, participants wrote “start” and “end” on their sheet and recorded the start time. When finished with the sheet, participants paused the timer and recorded the end time on their sheet. The experimenter used the formula: (minute end-minute start) *60+(second end-second start) to report duration in seconds and then used the formula number of problems/duration*60 to calculate the number of math facts responded to correctly or incorrectly per minute. The experimenter then displayed the data as line graphs. The experimenter also individualized participants’ fluency goal according to their rate of textual responding using the formula: target fluency=75% x writing rate. The experimenter then converted the target goal from seconds to minutes. If the participant met accuracy and fluency criterion during accuracy training, the experimenter only conducted one set of three post-intervention probes. After the participant emitted fluent responding to word problems by emitting 100% correct responses under the target duration, the experimenter conducted a set of three post-fluency probes. During fluency training, the participants only received reinforcement if they emitted 100% correct responses and completed the worksheet within the target time. The experimenter presented a correction procedure if the participants emitted an incorrect response.

Experimental Design

We utilized different methods of experimental control in response to different research questions. We used a multiple probe design across dyads of participants with simultaneous treatment to test for the effects of accuracy and fluency of math facts on the accuracy and fluency of responding to word problems (Horner & Baer, 1978). The multiple probe design provided a between-participants control for testing the intervention by showing that the behavior change occurred for multiple participants as a function of the intervention because behavior change only occurred when the experimenters introduced the intervention (Horner & Baer, 1978). Such a between-participants design rules out the possibility that behavior change occurred due to instructional history, maturation or other confounding variables outside of experimental settings. To do so, experimenters conducted initial probes for all participants and introduced intervention in a delayed manner (Horner & Baer, 1978; Johnston & Pennypacker, 2010).

Prior to accuracy and fluency training, the experimenter conducted an initial word problem probe for all participants. The first dyad of participants received at least two additional word-problem probes until we observed steady state responding. Then the experimenters taught three target number families to mastery to the first dyad of participants while withholding the intervention for all other participants. When the first two participants mastered the target math facts, the experimenters conducted three post-accuracy probes for word problem-solving for the first two participants as well as two additional pre-intervention word problem probes for the second dyad. Additional pre-intervention probes were conducted similarly until we observed steady state of responding for each of the dyads. The experimenter then taught math facts to fluency to the first dyad of participants and taught math facts to 100% accuracy to the second dyad of participants. When the second dyad of participants mastered the number facts, the experimenters conducted post-accuracy probes and two additional pre-intervention word problem probes for the third dyad of participants. The experimenter repeated the procedure until all participants completed all intervention and probe phases.

To compare the effects of accuracy and fluency training between participants who demonstrated TSF across saying and writing and those who did not, we matched participants based on their target fluency goal and their responses to the initial word problem probe. We assigned participants into dyads based on their past performance on computation fluency (i.e., fluency target) and the number of correct responses emitted towards the first word problem probe.

Results

Accuracy and Fluency of Word Problems

Among the participants who demonstrated TSF for spelling words across saying and writing, Gavin emitted a mean of 0 correct responses at a mean rate of 0 correct responses per minute across three pre-intervention probes. After emitting 100% accurate responses to math facts, Gavin emitted a mean of 6.7 correct responses at the mean rate of 1.42 correct responses per minute across three post-accuracy probes. During post-fluency word problem probes, Gavin emitted a mean of 10 correct responses at the rate of 3.96 correct responses per minute (See *Figure 1 and 2*). Lucas emitted 0 correct responses at a mean rate of 0 correct responses per minute during pre-intervention probes, a mean of 9.67 correct responses at a mean rate of 5.47 correct responses per minute during post-accuracy probes, and a mean of 10 correct responses at a mean rate of 16.09 correct responses per minute during post-fluency probes (See *Figure **1** and 2*). Evan emitted a mean of 3.33, 10, and 10 correct responses at a mean rate of 0.29, 4.16, and 12 correct responses per minute during pre-intervention, post-accuracy, and post-fluency conditions respectively (See *Figure 1 and 2*). Kevin emitted a mean of 0, 9.67, and 10 correct responses at a mean rate of 0, 3.68, and 6.16 correct responses per minute during pre-intervention, post-accuracy, and post-fluency conditions respectively (See *Figure 1 and 2*).

Among the participants who did not demonstrate TSF for spelling words across saying and writing, Jeff emitted a mean of 5.33 correct responses at a mean rate of 0.39 correct responses per minute during pre-intervention probes, a mean of 6.67 correct responses at a mean rate of 0.62 correct responses per minute during post-accuracy probes, and a mean of 8 correct responses at a mean rate of 0.83 correct responses per minute during post-fluency probes (See *Figure 1-2*). Gary emitted no correct responses after mastering math facts and performing math facts to fluency (See *Figure 1* and *Figure 2*). John emitted a mean of 0.67, 0, and 0 correct responses at a mean rate of 0.15, 0, and 0 correct responses per minute during pre-intervention, post-accuracy, and post-fluency conditions respectively (See *Figure 1-2*). Sally emitted a mean of 0, 1, and 0.33 correct responses at a mean rate of 0, 0.2, and 1.61 correct responses per minute during the pre-intervention, post-accuracy, and post-fluency conditions respectively (See *Figure **1-3***).

Counting Strategies

Among participants who demonstrated TSF across saying and writing for spelling, Gavin and Lucas did not use any counting strategies throughout the experiment. Evan used a mean of

5.67 counting strategies during the pre-intervention, 0.33 counting strategies during post-accuracy probes, and 0 counting strategies during post-fluency probes. Similarly, Kevin also demonstrated a decrease in the use of counting strategies by using 1.33, 0, and 0 counting strategies during each probe phase (See Figure 3).

Among the participants who did not demonstrate TSF across saying and writing for spelling, Gary did not use any counting strategies throughout the intervention. The use of counting strategies for Jeff, John, and Sally persisted. John used a mean of 10 counting strategies during pre-intervention probes, 9.67 counting strategies during post-accuracy probes, and 9 counting strategies during post-fluency probes. Jeff used a mean of 3, 4, and 2.33 counting strategies, and Sally used a mean of 10, 10, and 10 counting strategies during the pre-intervention, post-accuracy, and post-fluency probes, respectively.

Effect Size

We also calculated and reported effect size (ES) as Robust Improvement Rate Difference (IRD) (Altman, 1999, Parker et al., 2009). IRD is a non-overlap, single-case effect size that examines the degree of overlap between different experimental conditions. The experimenters reported three ES for each participant: (a) the ES between pre-intervention probes and post-accuracy probes, (b) the ES between post-accuracy probes and post-fluency probes, and (c) the ES between pre-intervention probes and post-accuracy probes. The effect sizes of the overall intervention for Gavin, Lucas, Evan, Kevin were 1.0, 1.0, 1.0, and 1.0 respectively, indicating that the accuracy and fluency training was very effective for those participants in terms of their accurate and fluent responding to word problems (Rakap, 2015) (See Table 2). The effect size of the overall intervention for Jeff was 1.0, whereas the overall effect sizes for John, Gavin, and Sally were 0, 0, and 0.3 respectively, falling under the ineffective category (See Table 2).

Accuracy and fluency training demonstrated larger effects for participants with TSF across saying and writing for spelling.

Discussion

We sought to test if accuracy and fluency training in math facts would affect participants' emission of correct and/or fluent responses to word problems. All participants who demonstrated TSF for spelling words across saying and writing (i.e., Gavin, Lucas, Evan, and Kevin) emitted a higher rate of correct responding after accuracy training compared to the rate of correct responses emitted during pre-intervention probes. They demonstrated 100% correct responses and further increased their rate of correct responding after demonstrating fluent responding to multiplication families (See *Figure 1*). The mean effect size for those four participants prior to and after accuracy and fluency training was 1.0 (See *Table 2*).

Participants who did not demonstrate TSF for spelling words across saying and writing, on the other hand, demonstrated limited gains from the intervention. Gary and John emitted no correct responses after mastering and fluently responding to math facts. Although Jeff and Sally emitted more correct responses overall after mastering math facts, the number of correct responses emitted remained unstable. Participants Jeff, John, and Sally, who emitted counting strategies during the pre-intervention probe sessions, also emitted a high number of counting strategies during post-accuracy and post-fluency probe sessions, suggesting a lack of TSF from math facts to word problems. The mean ES for participants who did not demonstrate TSF across saying and writing for spelling was 0.3 (See *Table 2*), demonstrating low to no effect of the intervention. Moreover, participants who did not demonstrate TSF across saying and writing for spelling showed a slight increase in their rate of incorrect responses to word problems. This suggests that, when a student does not demonstrate TSF across saying and writing, fluent training of math facts might negatively impact their performance for problem solving.

Interestingly, all participants who demonstrated TSF across saying and writing for spelling showed increases in the rate of correct responding to word problems after mastering math facts, whereas the participants who did not demonstrate TSF across saying and writing did not show significant gains. This suggests that the presence of TSF for spelling across saying and writing might be correlated to the presence of TSF across accuracy/fluency of math facts and accuracy/fluency of word problems. Replication of this experiment might yield more data to test for the correlation between the above mentioned two sets of TSF at a group level. Word-problems are not the only form of math problem-solving. Future studies can examine how the mastery of math facts affect students' performances to other math operants.

One limitation of the study was the lack of control over the number of sessions conducted each week and the duration between two experimental sessions. Participants received a mean of two sessions of probe or intervention per day during math classes. However, participants only have math classes every other day, resulting in a maximum of 3 consecutive days without receiving any interventions. This might have a negative effect on participants' mastery and maintenance of learned skills during the intervention. For future studies, experimenters can conduct one or two sessions in the morning every day to account for the impact of breaks and schedule changes.

Another limitation of the study was the fidelity of the intervention. Although the experimenter conducted interscorer and interobserver agreement, no tactics were present to ensure the correct implementation of the intervention. The experimenter might have worded the antecedent or consequence differently for different participants and this may have affected the fidelity of the intervention. For future studies, the experimenter should script out antecedents and consequences to minimize the variation of antecedents and consequences received by participants. The experimenter can also have an independent observer conduct fidelity checks using a checklist or the *Teacher Performance Rate and Accuracy (*TPRA) form to improve fidelity of instrumentation (Ingham & Greer, 1992).

Moreover, reactivity might be a limitation of the current study as the experimenters instructed the participants to keep their hands on the desk during all probe sessions. Knowing that the experimenters might be observing their hands might alter the frequency of the participants' use of finger-counting strategies. To address reactivity, future researchers can utilize recording devices angled to record participants potential use of finger-counting strategies above or under their desks or use desks with no cubbies so that the participants must leave their hands on the desk without receiving explicit instructions to do so.

Lastly, experimenters used vertical multiplication/division new facts sheets as fluency worksheets during all probe sessions whereas all participants wrote number sentences horizontally during the intervention. Future studies can use math fact worksheets that are presented in a horizontal manner and measure whether participants emitted more correct responses after accuracy/fluency training when presented with horizontal math fact worksheets.

With Experiment I, we found a functional relation between the mastery of math facts and the mastery of word problems for all participants who demonstrated TSF across saying and writing. However, only one participant who did not demonstrate TSF across saying and writing demonstrated minimal gain from the intervention. Previous researchers have argued that solving a word problem is a two-step process including the translation of word problem to number sentence and the computation of number sentence (Hegarty et al., 1995; Mayer, 1999; Montague

& van Garderen, 2003). Thus, one might argue that teaching only one component of the problem-solving process is not sufficient for students to master word problems. In Experiment II, we reversed the dependent and independent variables to test if mastery of problem-solving is sufficient for the mastery of math facts because computation is an essential step in the problem-solving process.

EXPERIMENT II

Methods

Methods of Experiment II were consistent with those of Experiment I in terms of participants, materials, settings, design, and procedure. However, we reversed the dependent variable and independent variables to examine the effect of accuracy and fluency training of word problems on the accurate and fluent responding to math facts. To address the limitation of having word problem being the only form of math problem-solving, we introduced additional novel problem-solving probes to test if the transformation of stimulus function occurred beyond math facts and word problems to other problem-solving math operants that involved both translation of a problem to number sentences (i.e., evaluating one-step algebraic expressions, finding area of a rectangle) and computation skills.

Dependent Variables

The dependent variables of the study were the numbers and rates of correct and incorrect responses emitted to math facts and the numbers and rates of correct/incorrect responses to two generalization probes (i.e., evaluating one-step algebraic expressions, finding area of a rectangle). Each math fact worksheet contained 100 math fact questions. We set participants' math fact fluency goal at 75% of their rate or writing numbers.

Accuracy and Fluency

During probe sessions, participants recorded the start and end time for each probe they completed. A teacher then checked for accuracy and calculated rate of correct/incorrect responding using the formula: rate of correct responding = number of correct digits/duration in seconds*60, and rate of incorrect responding = number of correct digits/duration in seconds*60. In addition, experimenters only started intervention if the participant demonstrated a steady state of responding, or if the participants' rate of correct responding was descending or if the participants' rate of incorrect responding was increasing. Otherwise, the experimenters conducted additional probe sessions until reaching a steady state of responding.

Novel Problem-Solving Probe

We also conducted two probes for the transformation of stimulus function across math operants with math operants that were not targeted in the intervention. The two novel operant probes conducted were evaluating algebraic expression (See *Appendix D*) and areas of rectangles (See *Appendix E*). We conducted probes two weeks after the participant completed the intervention. All participants had previously mastered evaluating one-step algebraic expressions by replacing a variable with an indicated number, and computing the expression using a calculator by emitting 90% correct responses across 20 opportunities. All participants also mastered finding the area of a rectangle given the length of two adjacent edges or finding the length of an edge given the area of a rectangle and the length of another edge using a calculator with 90% accuracy across 20 opportunities. The experimenter reported the number and rate of correct and incorrect digits performed by each participant towards the two new math operant probes as bar graphs along with the other probe sessions.

Counting Strategies

We measured the participants' use of counting strategies by observing the number of finger-counting or other visual counting strategies they used during each math fact probe session as described in Experiment I. To address the limitation of reactivity presented in the previous experiment, we seated the participants at a desk with no cubby so that the student had to leave their hands on the table without the experimenters instructing them to do so.

Independent Variables

Accuracy and Fluency

The independent variables of the study were the accurate and fluent responding to word problems containing number families: (1) 7, 8, 56, (2) 6, 9, 54, and (3) 8, 8, 64. We taught word problems to accuracy and then to fluency. Each word problem training phase contained ten word-problems each with exactly 314 words to account for participants' rate of textual responding. We conducted math fact fluency probes after accurate and after fluent responding. We used a random sequence generator for the sequence of presentation of word problem worksheets to account for potential sequencing effects. Upon completion of a word problem worksheet, the experimenter delivered consequences for each question. The experimenter delivered vocal approvals for correct responses (i.e., "You got it! Seven times eight is 56!") and a correction procedure for incorrect responses. The correction procedure consisted of a teacher model of a correct response and an opportunity for the student to independently vocally respond to the problem.

We implemented the CABAS® decision protocol during the intervention (Greer, Keohane, & Healy, 2002). We made a decision to stop the intervention and implement tactics when the participants emitted descending correct responses over four consecutive sessions or six overall sessions. We implemented the tactic for the participants to complete half a sheet of the math facts to criterion and re-introduced the full sheet.

TSF Across Saying and Writing

To examine the difference in responding between participants who demonstrated TSF across saying and writing and participants who did not, we matched the participants by their rate of responding to math facts prior to the experiments to enter the intervention as dyads. We kept the participants in the same dyad as Experiment I.

Interscorer and Interobserver Agreement

Experimenters conducted IOA for 67% of the probe sessions with 100% agreement. To address the limitation of fidelity in Experiment I, we used the Teacher Performance Rate and Accuracy form during intervention sessions (Ingham & Greer, 1992). An experimenter observed the implementation of the intervention and recorded if (a) the instructor delivered a correct antecedent, (b) the student emitted a correct response, and (c) the instructor delivered a contingent consequence. We collected IOA for 100% of the intervention sessions with 100% agreement. We conducted ISA for permanent products across all sessions with a 100% agreement.

Results

Accuracy and Fluency

Among the participants who demonstrated TSF for spelling words across saying and writing, Gavin emitted all correct responses at a mean rate of 23.48 correct responses per minute during pre-intervention probes, a mean rate of 33.51 correct responses per minute during post-accuracy probes, and a mean of 39.57 correct responses per minute during post-fluency probes (See *Figures 4 and 5*). For evaluating algebraic expression and finding area for rectangle generalization probes, Gavin emitted all correct responses at the rate of 50 and 50.70 correct responses per minute respectively (See *Figures 4 and 5*).

Lucas emitted a mean of 48 correct responses at a mean rate of 7.37 correct responses per minute during the pre-intervention probes, a mean of 100 correct responses at a mean rate of 20.07 correct responses per minute during the post-accuracy probes, and a mean of 100 correct responses at a mean rate of 28.54 correct responses per minute during post-fluency probes (See *Figures 4 and 5*). For evaluating algebraic expression and finding area for rectangle generalization probes, Lucas emitted all correct responses at the rate of 19.57 and 34.29 correct responses per minute respectively (See *Figures 4 and 5*).

Evan emitted a mean of 99.33 correct responses at a mean rate of 10.88 correct responses per minute during pre-intervention probes, a mean of 100 correct responses at a mean rate of 17.58 correct responses per minute during post-accuracy probes, and a mean of 100 correct responses at a mean rate of 24.79 correct responses per minute during post-fluency probes (See *Figures 4 and 5*). For evaluating algebraic expression and finding area for rectangle generalization probes, Evan emitted all correct responses at the rate of 24.32 and 31.58 correct responses per minute respectively (See *Figures 4 and 5*). Evan did not use any counting strategies throughout the intervention.

Kevin emitted a mean of 87.33 correct responses at a mean rate of 10.88 correct responses per minute during pre-intervention probes, a mean of 100 correct responses at a mean rate of 21.14 correct responses per minute during post-accuracy probes, and a mean of 100 correct responses at a mean rate of 27.07 correct responses per minute during post-fluency probes (See *Figures 4 and 5*). For evaluating algebraic expression and finding area for rectangle generalization probes, Kevin emitted all correct responses at the rate of 21.42 and 31.58 correct responses per minute respectively (See *Figures 4 and 5).*

Among the participants who did not demonstrate TSF for spelling words across saying and writing, Jeff emitted a mean of 61 correct responses at a mean rate of 3.48 correct responses per minute during pre-intervention probes, a mean of 60 correct responses at a mean rate of 4.64 correct responses per minute during post-accuracy probes, and a mean of 90.33 correct responses at a mean rate of 16.15 correct responses per minute during post-fluency probes (See *Figures 4 and 5*).

Gary emitted all correct responses at a mean rate of 15.92, 15.23, and 20.28 correct responses per minute during pre-intervention, post-accuracy, and post-fluency conditions respectively (See *Figures 4 and 5*). For evaluating algebraic expression and finding area for rectangle generalization probes, Gary emitted all correct responses at the rate of 6.07 and 19.57 correct responses per minute respectively (See *Figures 4 and 5*).

John emitted a mean of 4 correct responses at a mean rate of 0.61 correct responses per minute during pre-intervention probes, a mean of 60.67 correct responses at a mean rate of 4.54 correct responses per minute during post-accuracy probes, and a mean of 70.67 correct responses at a mean rate of 5.55 correct responses per minute during post-fluency probes (See *Figures 4 and 5*).

Sally emitted a mean of 21.67 correct responses at a mean rate of 3.55 correct responses per minute during pre-intervention probes, a mean of 81 correct responses at a mean rate of 8.29 correct responses per minute during post-accuracy probes, and a mean of 95.33 correct responses at a mean rate of 10.78 correct responses per minute during post-fluency probes (See *Figures 4 and 5*).

Counting Strategies

Among participants who demonstrated TSF across saying and writing for spelling, Gavin, Lucas, and Kevin did not use any counting strategies. Evan used a mean of 1 counting strategy during pre-intervention probe sessions but did not use any counting strategies during subsequent probe conditions. Among participants who did not demonstrate TSF across saying and writing for spelling, Gary did not use any counting strategies. Jeff, John, and Sally used a mean of 3.48, 4.64, and 3, 0.61, 2.33, and 0.33, and 0.67, 0, and 0 counting strategies during the three probe conditions, respectively (See *Figure 6*).

Effect Size

The effect sizes of the overall intervention for Gavin, Lucas, Evan, and Kevin were 1, 1, 1, and 1, respectively, showing large effects (See Table 3). The effect size of the overall intervention for Jeff, Gary, John, and Sally were 1, 0.67, 1, and 1 respectively, also showing large effects (See Table 3). Although the intervention demonstrated larger effects for participants who demonstrated TSF across saying and writing for spelling, all eight participants demonstrated positive gains from the accuracy and fluency training of word problems.

Discussion

The findings of Experiment II were consistent with those of Experiment I. Participants who demonstrated TSF for spelling across saying and writing also demonstrated TSF across word problems and math facts by emitting accurate and fluent responses to math fact worksheets and generalization probes after mastering emitting accurate and fluent responses to math facts. However, in Experiment II, participants who did not demonstrate TSF across saying and writing also showed significant gain. Overall, the group mean effect size was 1 for those who demonstrated TSF and 0.92 for those who did not demonstrate TSF. However, participants who did not demonstrate TSF emitted either incorrect responses to math fact worksheets or emitted correct responses at a lower rate compared to the participants who demonstrated TSF who demonstrated a change in the level of rate of responding as well as 100% correct responses to math facts.

These findings have several implications for future practice. When students demonstrate TSF across saying and writing for spelling, teachers can focus on teaching word problems to accuracy and fluency to teach math facts in context instead of providing students with math facts training separately. When teaching word problems, teachers often focus on the translation of words to number sentences. However, findings of this study suggest that word problems can function as a motivating condition for students to learn and master novel math facts. All four participants who demonstrated TSF across saying and writing for spelling words immediately performed at fluency criterion level for math facts after meeting fluency criterion for word problems. The participants who did not demonstrate TSF across saying also demonstrated increases in their accuracy and rate of responses to math facts. This suggests that teachers can use fluency responding to word problems as a tactic or replacement for students' fluency training to math facts while obtaining the same effect. Fluency training to word problems might also function as a reinforcer sampling for students to contact the reinforcement and increase their motivation to meet fluency goal for math facts. Math fact fluency worksheets often contain many math fact problems. This can be discouraging at sight for some students. With the materials utilized in Experiment II, the participants only had to respond to ten word-problems to fluency criterion to demonstrate gains in their responses to math facts.

Although three of the four participants who did not demonstrate TSF emitted significantly more correct responses after receiving accuracy/fluency training for word problems, those participants still emitted some incorrect responses or emitted correct responses at a lower rate than those who demonstrated TSF. This suggests that although fluency responding might have transformed from word problems to fluency for participants who did not demonstrate TSF across saying and writing, accurate responding remained under separate stimulus control for those participants. That is, although the stimulus control for fluent responding has transformed as a function of fluency training of word problems, participants who did not demonstrate TSF across saying and writing for spelling demonstrated weaker stimulus control for accurate responding.

One of the limitations of Experiment II was the potential sequence effects of the maintenance and generalization probes. We administered those probes in the sequence of areas of rectangles first and evaluating algebraic expressions second. All participants emitted lower rates of correct responses during the area of rectangles probe. Several factors might contribute to this difference. First, the setup of the worksheet requires the participant to locate where to place the answer. For example, the participants had to write the area in the middle of the rectangle and write the length of an edge adjacent to a given edge. This process might have caused the participants to take additional time to respond. In addition, with the generalization probes functioning also as maintenance probes, the participants might have emitted lower rates of responding when first exposed to those math facts after two weeks. Future research can address this concern by counterbalancing the order of generalization probes across participants across groups.

Another limitation is the ceiling effect of participants who emitted 100% accurate responses during pre-intervention probes. Due to the ceiling effect, we cannot conclude if the stimulus control of accurate responding transformed as a function of accuracy and fluency training of word problems. Future studies can conduct multiple assessments to locate fact families that were not mastered by any of the participants.

General Discussion

The findings of the two experiments suggest that, when participants did not demonstrate TSF across math operants, accuracy and fluency in computation cannot be transformed or utilized for the accurate and fluent responding to problem solving. This is not consistent with the findings of McTiernan et al. (2016) where they found no significant difference in responses to application problems between control and intervention group, or Singer-Dudek and Greer (2005) where participants emitted criterion level correct responses to composite tasks after mastering math facts with fluency criterion. This might be due to the homogenous sampling of the two prior studies. Instead of grouping participants by their pre-intervention scores, we grouped the participants by their existing cusps which is a stronger indictor of what contingencies the participants contact in their environment that led to learning and their gains from fluency training.

The results also showed an association between the demonstration of TSF across computation and problem solving and TSF for spelling words across saying and writing. This raised the possibility that transformation of stimulus function across saying and writing might be responsible for the transformation of accuracy and fluency from computation skills to word problem-solving skills. Although those two types of TSF seem very different from each other, they do share some similarities. Both types of TSF address the transformation of stimulus function of different representations of the same stimulus. In TSF for spelling words, students were asked to spell the same words in written form or vocally. In TSF for math operants, students were asked to solve the same math sentence presented as a math fact, word problem, or diagram.

Researchers should collect more data for participants at all fluency levels to test for a statistically significant correlation between the two types TSF. Researchers can also test for the presence of TSF for spelling across saying and writing after the acquisition of TSF across math operants to examine if the onset of one type of TSF results in the demonstration of the other. Thus, further research needs to be conducted to test for the functional relation between the establishment of TSF for spelling across saying and writing and the generalization of computational accuracy and fluency for word problems or other types of problem solving.

The possibility that those two types of TSF are in fact the same remains a question given the strong association suggested by the results of Experiments I and II. TSF for spelling across saying and writing addresses the correspondence between the textual representation and the vocal representations of words given an audio input of phonemes that correspond to the letters. We can test if the correspondence between vocal and written from of math facts is also responsible for TSF across math operants by implementing MEI for spelling words across saying and writing. If the implementation of such procedure results in the presence of TSF across math facts, then the two types of TSF discussed above address the same type of correspondence and are thus in fact the same type of TSF. That is, TSF across math operants is another application of TSF across saying and writing. When we establish TSF across math operants, the stimulus control of those math operants shifts from various visual representations (e.g., counting strategies, printed number sentences) to be under verbal (i.e., speaker-as-own-listener) stimulus control. Marr (2015) argued that mathematics is indeed a verbal behavior, a behavior that one acquires through repeated exposure to verbal contingencies. Over the past decades, more studies demonstrated efficacy in utilizing behavioral intervention tactics to teach math as a verbal behavior by teaching the function of mathematics (Crosbie, 2018; Weber, 2016).

If the onset of TSF across saying and writing for spelling did not result in the onset of TSF across math operants, then those two types of TSF are functionally independent. In that case, we could implement MEI across math operants and then replicate the procedure outlined in Experiments I and II to test for the presence of TSF across math operants because establishing the correspondence between different responses involving the same stimulus seems to be the key to the establishment of TSF. Researchers used multiple exemplar instruction to rapidly alternate between different responses involving the same stimulus to establish correspondence between stimuli and responses. Speckman and Greer (2012) and Luke et al. (2011) taught the correspondence between autoclitic frames and their autoclitic function using MEI. Greer et al., (2015) and Gilic and Greer (2011) established bidirectional Naming using MEI. Previous researchers induced TSF for spelling across saying and writing using MEI (Eby et al., 2010; Greer et al., 2005). Future research should employ a similar approach to test the effects of MEI on the establishment of TSF across math operants.

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Table 1

Participants' Demographics and Related Skills

Dyad	Participant	Age	Gender	Grade	iReady Diagnostics Numbers and Operations Grade Level Equivalence	iReady Diagnostics Reading Grade Level Equivalence	Presence of TSF across saying and writing for spelling	Target Fluency Goal (digits per minute)	Rate of Textual Responding (wpm)
1	Gavin	13	M	7	5th Grade	2nd Grade	Yes	35	128
	Jeff	11	M	6	3rd Grade	3rd Grade	No	35	106
2	Lucas	14	M	8	4th Grade	2nd Grade	Yes	35	180
	Gary	13	M	7	4th Grade	2nd Grade	No	25	126
3	Evan	13	M	8	3rd Grade	2nd Grade	Yes	25	90
	John	11	M	6	2nd Grade	1st Grade	No	25	104
4	Kevin	14	M	8	1st Grade	1st Grade	Yes	25	55
	Sally	11	F	6	1st Grade	1st Grade	No	15	50

Table 2

Effect Size of Accuracy and Fluency Training of Math Facts

Participant	ES between Pre-intervention probes and post-accuracy probes	ES between Post-accuracy probes and post-fluency probes	ES between pre-intervention probes and post-fluency probes
Gavin	1	1	1
Lucas	1	1	1
Evan	1	1	1
Kevin	1	1	1
Jeff	0.67	0.67	1
Gary	0	0	0
John	0	0	0
Sally	0.67	0	0.33

Note. We reported effect size using robust Improvement Rate Difference (IRD). ES<= .5 indicates questionable or no effect. When .5< ES< .7, the intervention is effective. When ES>=.7, the intervention is very effective.

Table 3

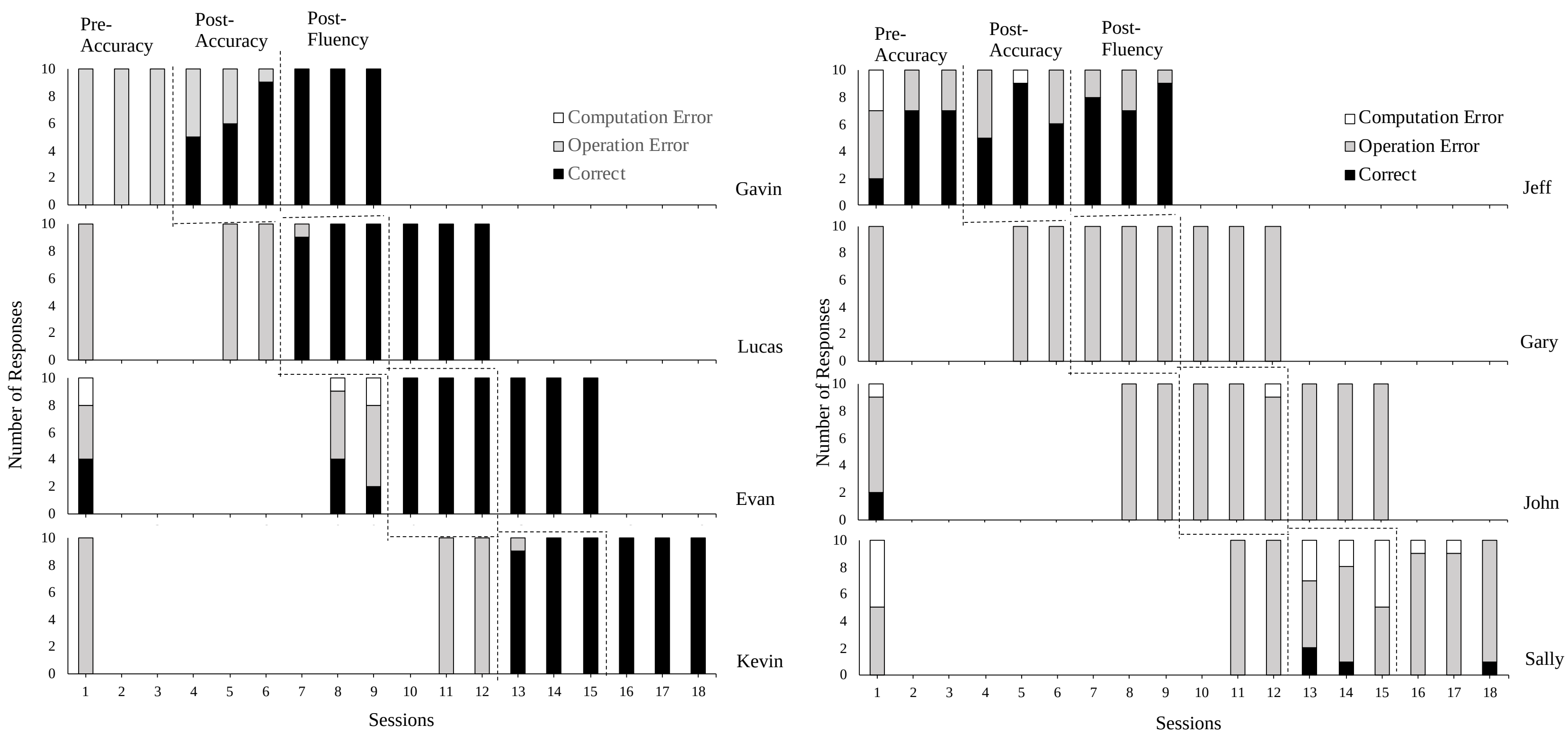
Effect Size of Accuracy and Fluency Training of Word Problems

Participant	ES between pre-intervention probes and Post-mastery probes	ES between post-mastery probes and post-fluency probes	ES between pre-intervention probes and post-fluency probes
Gavin	1	1	1
Lucas	1	1	1
Evan	1	1	1
Kevin	1	1	1
Jeff	0.67	1	1
Gary	0	1	0.67
John	1	0.33	1
Sally	1	0.67	1

Note. We reported effect size using robust Improvement Rate Difference (IRD). ES<= .5 indicates questionable or no effect. When .5< ES< .7, the intervention is effective. When ES>=.7, the intervention is very effective.

Figure 1

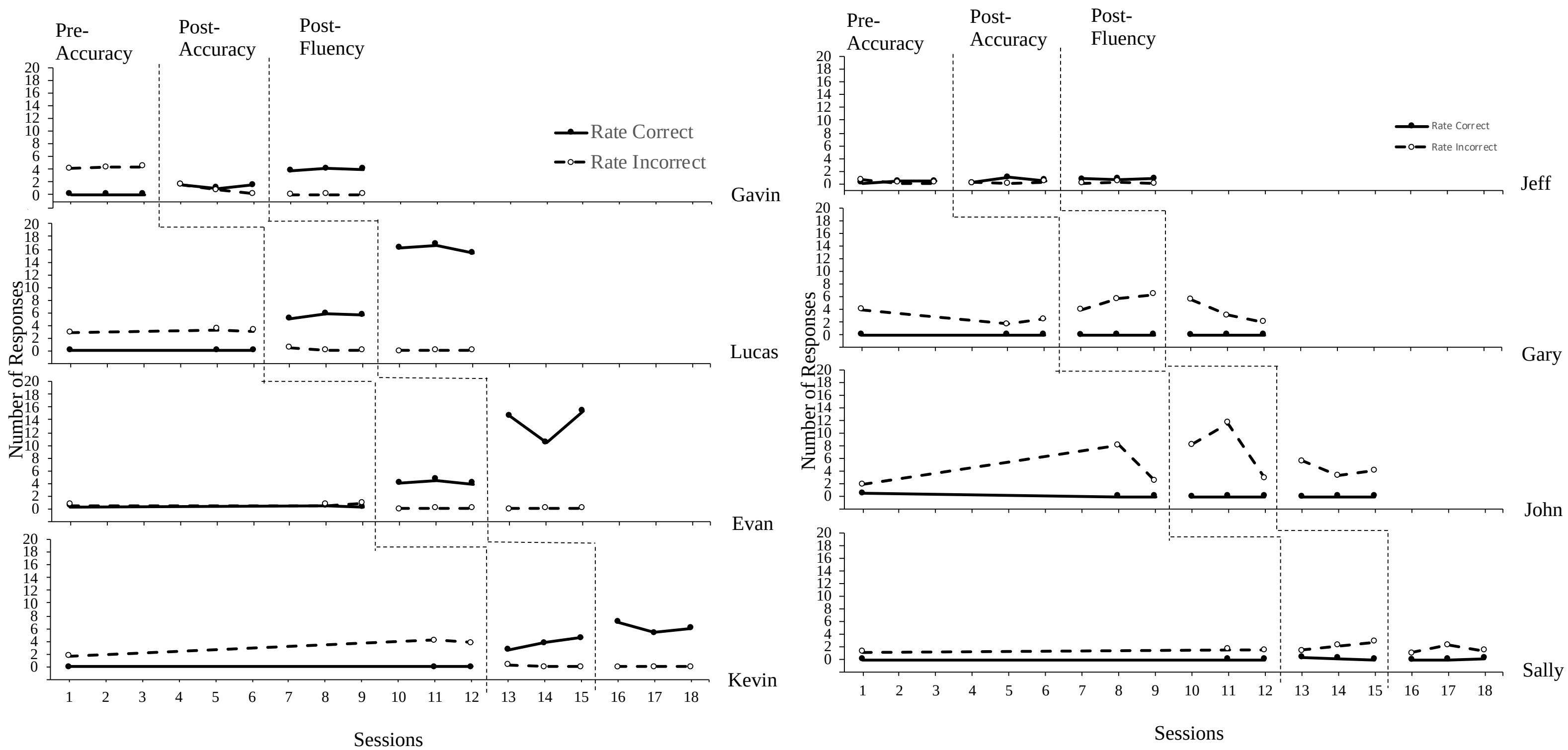
Responses to Word Problems



Note. The figure showed the number of correct and incorrect responses emitted to word problems by participants with or without TSF across saying and writing for spelling words on adjacent panels. The left panel showed responses emitted by participants who demonstrated TSF across saying and writing for spelling words and the right panel showed responses emitted by participants who did not demonstrate TSF across saying and writing for spelling words.

Figure 2

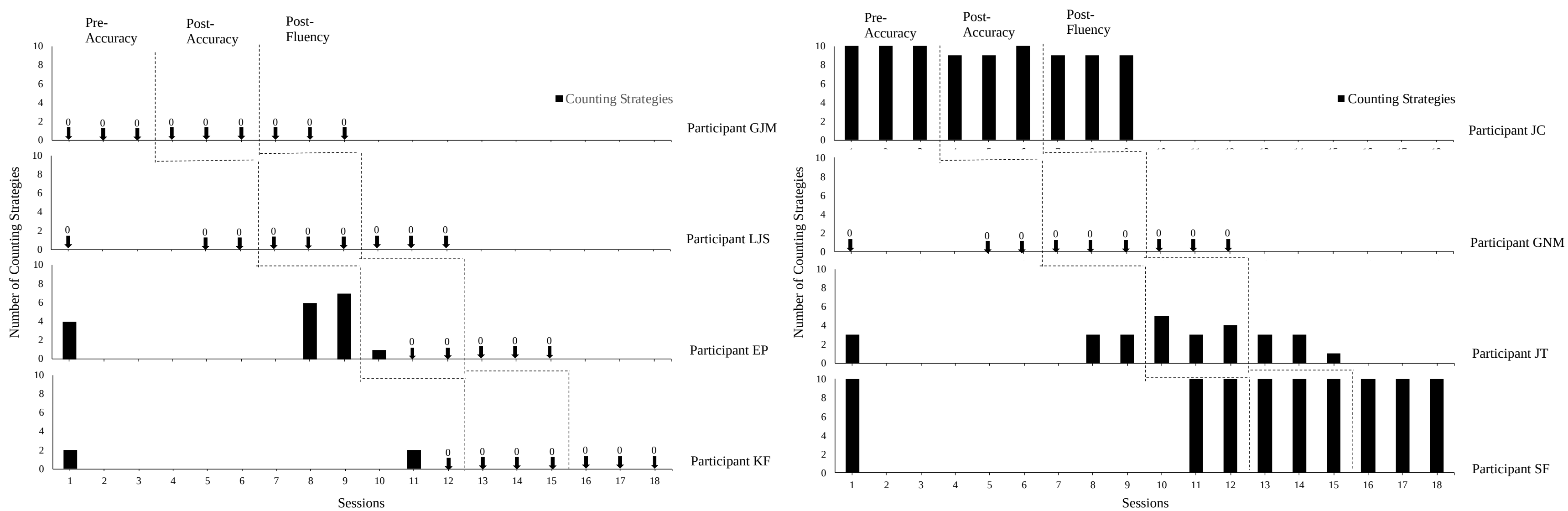
Rate of Responses to Word Problems



Note. The figure showed the rate of correct and incorrect responses for word problems emitted by participants with or without TSF across saying and writing for spelling words on adjacent panels. The left panel showed responses emitted by participants who demonstrated TSF across saying and writing for spelling words and the right panel showed responses emitted by participants who did not demonstrate TSF across saying and writing for spelling words.

Figure 3

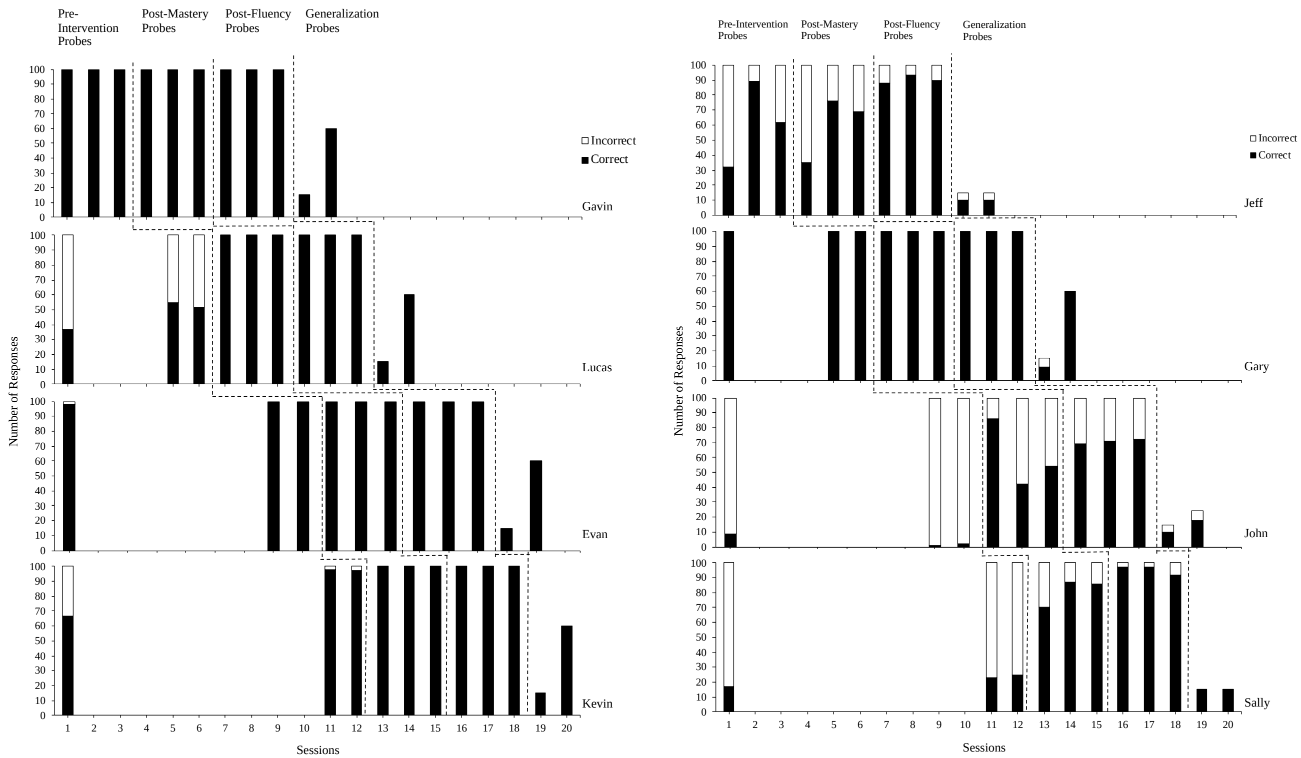
Number of Counting Strategies Used



Note. The figure showed the number of counting strategies used by participants with or without TSF across saying and writing for spelling words on adjacent panels during word problem probes. The left panel showed responses emitted by participants who demonstrated TSF across saying and writing for spelling words and the right panel showed responses emitted by participants who did not demonstrate TSF across saying and writing for spelling words.

Figure 4

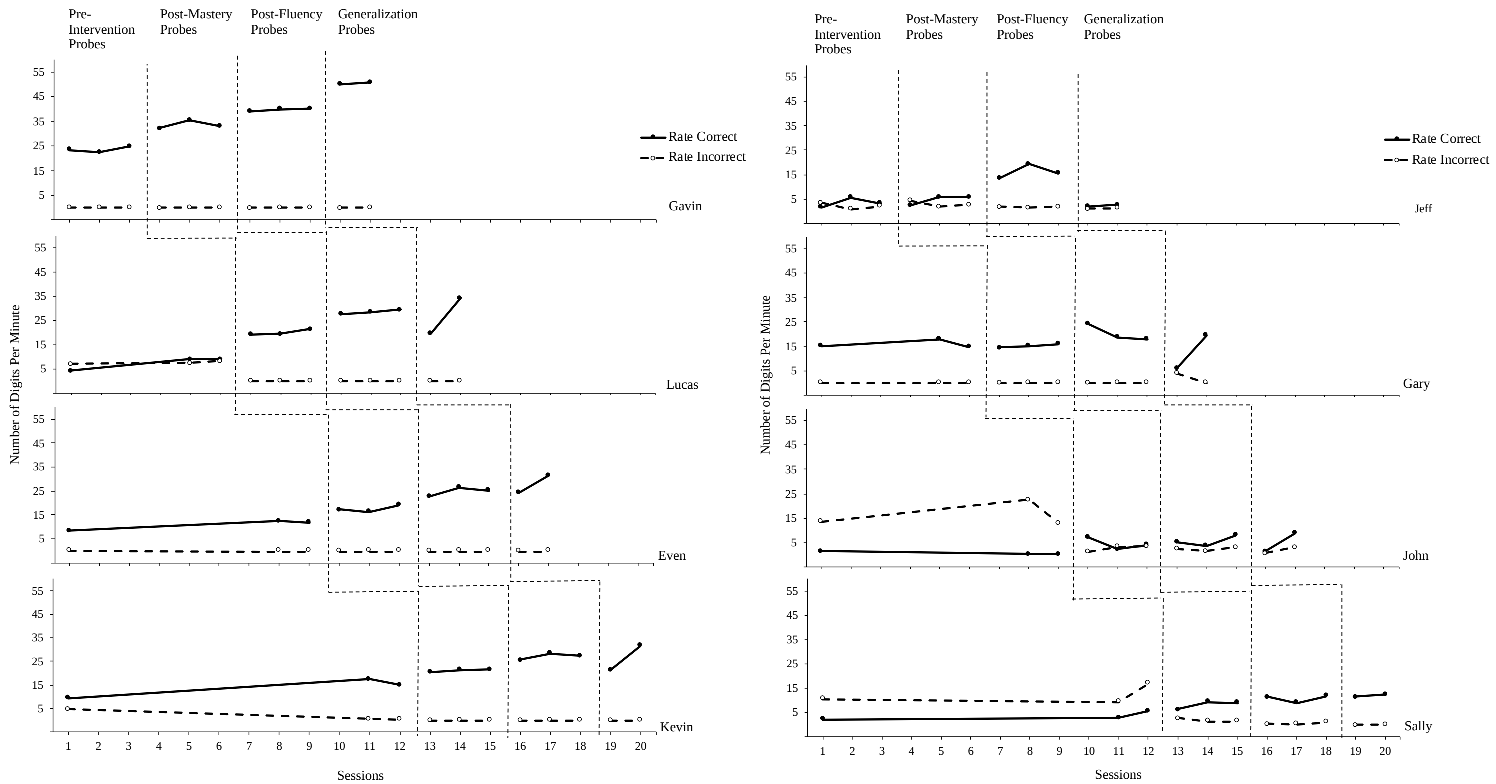
Responses to Math Facts



Note. The figure showed the number of correct and incorrect responses emitted to math facts by participants with or without TSF across saying and writing for spelling words on adjacent panels. The left panel showed responses emitted by participants who demonstrated TSF across saying and writing for spelling words and the right panel showed responses emitted by participants who did not demonstrate TSF across saying and writing for spelling words.

Figure 5

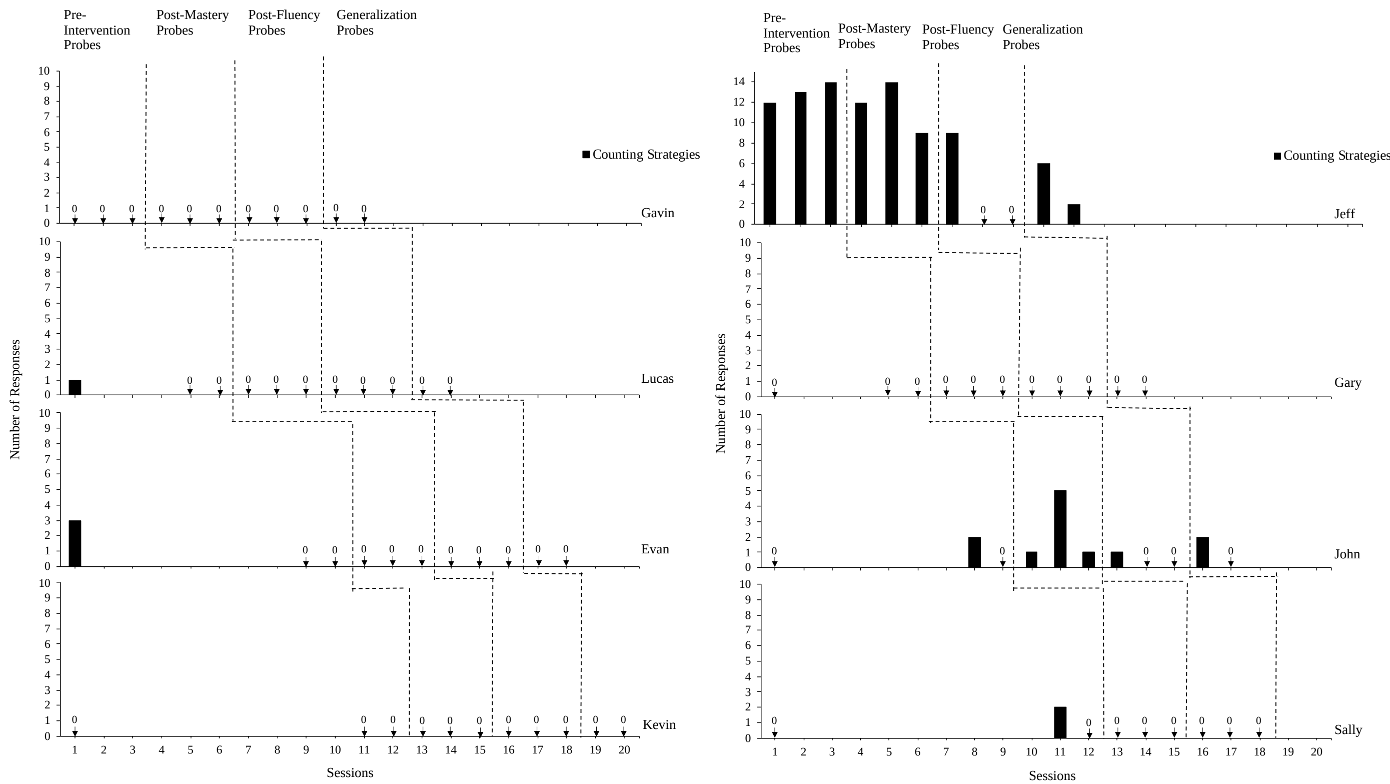
Rate of Responses to Math Facts



Note. The figure showed the rate of correct and incorrect responses emitted to math facts by participants with or without TSF across saying and writing for spelling words on adjacent panels. The left panel showed responses emitted by participants who demonstrated TSF across saying and writing for spelling words and the right panel showed responses emitted by participants who did not demonstrate TSF across saying and writing for spelling words.

Figure 6

Number of Counting Strategies Used



Note. The figure showed the number of counting strategies used by participants with or without TSF across saying and writing for spelling words on adjacent panels during word problem probes. The left panel showed responses emitted by participants who demonstrated TSF across saying and writing for spelling words and the right panel showed responses emitted by participants who did not demonstrate TSF across saying and writing for spelling words.

Appendix A

Pilot Study

Methods

Participants

The participants of the study were eight middle school students. All participants attended a public middle school in a school district located in a suburb outside a major metropolitan city. All participants enrolled in a self-contained classroom utilizing the Comprehensive Application of Behavior Analysis to Schooling (CABAS®) model, where teachers based all instruction on scientific procedures and continuously measured student responses and performances (Greer, 2010).

All participants attended the same self-contained classroom for English language arts, mathematics, science and social studies. Two participants were female, and six participants were male. All eight participants had Individualized Education Plans (IEP). One head teacher and 4 paraprofessionals were in the classroom during all probe and intervention sessions. Participant Kylie and Participant Gabe received one-to-one supervision from paraprofessionals as mandated by their IEPs. Participant Gray and Participant Collin shared assistance from a paraprofessional.

All participants performed on a reader/writer level of verbal behavior (See Table 4). This means that they demonstrated reader/writer skills such as transcription, dictation, and textually responding. Participants Dylan, Collin, Gray, and Kylie demonstrated transformation of stimulus function across saying and writing whereas participants Liam, Sam, Jo and Gabe did not. All participants performed below grade level for reading and math according to iReady Diagnostics Assessment conducted at the beginning of the school year. All participants received math facts fluency training for addition and subtraction facts through Morningside Math Fluency curriculum.

Settings and Materials

We conducted all probe and intervention sessions in participants' classroom. The participants sat in a group of four with their own rectangular desk facing one another. Participants were given the option to work at their own desk or at one of the two horseshoe tables located in the corner of the classroom. For participants who required one-on-one paraprofessional assistance, the paraprofessional seated across the table from the participant or three feet behind the participant.

Teachers distributed word problem worksheets for probe sessions and Morningside fluency math facts sheets for intervention sessions. Students used a pencil to respond to worksheets and a timer to record how long it took them to complete a worksheet. Teachers used a projector to project a timer on a whiteboard located in the front of the classroom for whole class fluency sessions. Teachers also used a data sheet to record the number of counting strategies the participants used during each session.

Experimental Design

We utilized a multiple probe design across participants to test for the effects of mastery and fluency in math facts on the accuracy and fluency in word problems (Horner & Baer, 1978). Prior to mastery and fluency training, teachers conducted a probe of one set of word problems. Then the experimenters trained the three target number families to mastery to two participants while withholding the intervention for all other participants. When the first two participants mastered the number facts, teacher conducted a post-mastery probe for those two participants as well as a second pre-intervention probe for two other participants. Teachers then taught math facts to fluency to the first dyad of participants and taught math facts to mastery to the second dyad of participants. When the second dyad of participants mastered the number facts, teachers conducted post-mastery probe and second pre-intervention probe for the third dyad of participants. Teachers repeated the procedure until all participants responded to math facts to fluency (See Figure 1).

In addition, for participants who received a second pre-intervention probe, experimenters started intervention if the participant emitted approximately the same number or fewer correct responses during the second probe. If the participant emitted more correct responses during the second pre-intervention probe, experimenters conducted a third probe in order to obtain steady state of responding.

The multiple probe design showed efficacy of the intervention by showing that the behavior change occurred for multiple participants as a function of the intervention because behavior change only occurred when the intervention was introduced (Horner & Baer, 1978). Such design ruled out the possibility that behavior change occurred because of maturation or other confounding variables outside of experiment settings (Horner & Baer, 1978).

Dependent Variables and Data Collection

Word Problems

Experimenters collected data on the number of correct responses participants emitted to a set of twenty word-problems. We created four sets of twenty word-problems with answers containing number families: (1) 6, 9 15, (2) 7, 8, 15, and (3) 8, 8, 16. Within each word problem set, each number sentence generated by the number families appeared twice (e.g., two of the word problems have the answer 6+9=15). Experimenters counterbalanced the number of words in each set of word problem to minimize the effect of reading fluency on the duration of word problem solving. Each set of word problem consisted of a mean of 551.5 words (ranging from 551 words to 552 words).

Experimenters recorded the accuracy and fluency of participants' responses to each set of word problems. For accuracy, experimenters marked a "+" for each correct response emitted by the student, a "1" for each incorrect response with operation errors (e.g., student performed addition for a subtraction problem), and a "2" for each incorrect response with computation errors (e.g., student wrote 6+9=23). Experimenters then reported data on a stacked bar graph with a maximum of twenty.

For fluency, participants used a timer or observed the big timer projected on the whiteboard to self-record the duration of their word-problem solving. Before starting the timer, participants wrote "start" and "end" on their sheet and recorded the start time. When finished with the sheet, participants paused the timer and recorded the end time on their sheet. We used the formula: (minute end-minute start) *60+(second end-second start) to report duration in seconds and then used the formula number of problems/duration*60 to calculate the number of problems responded to correctly or incorrectly per minute. We then reported data as bar graphs.

Counting strategies

Experimenters also collected data on counting strategies participants employed throughout all probe sessions. We defined a counting strategy as any use of visual or vocal prompt to add or subtract. Some of the counting strategies we observed during probe sessions were finger counting, usage of tally marks and/or vocally counting up or down. We recorded one tally mark for each word-problem for which the participants used any of the above listed counting strategies. We reported the number of counting strategies used during probe sessions as bar graphs.

Independent Variables

The independent variables of the study were the accuracy and fluency of Morningside Fluency worksheets. Teachers defined accuracy as 100% accurate responses to a Morningside Fluency worksheet with 100 questions. Teachers recorded duration for each worksheet during accuracy trainings, but it was not part of the criterion.

Prior to the intervention, teachers conducted a probe for writing fluency. We told the participants to write numbers zero through nine repeatedly in a minute and reported the number as digits per minute. Experimenters then set fluency criterion at 75% of participants' writing rate. For example, students who wrote 60 digits per minute had a fluency criterion of 60*75%=45 digits per minute. During fluency training, student recorded start and end time for each sheet they completed. An experimenter checked the sheet for accuracy and calculated rate of correct/incorrect responding using the formula: rate of correct responding=number of correct digits/duration in seconds*60, and rate of incorrect responding=number of correct digits/duration in seconds*60. During fluency training, teachers also utilized CABAS® decision protocol to make instructional decisions or implement performance tactics (Keohane & Greer, 2005). Experimenters made a decision to stop the current objective if the participant's responding demonstrated three consecutive descending data paths, five overall descending data paths, or three or five no trend data paths. We then implemented a tactic for the student to respond to half a sheet to fluency criterion and then reversed back to whole sheet.

Interobserver Agreement and Interscorer Agreement

Experimenters obtained Interobserver Agreement (IOA) for counting strategies by having two observers observing one participant simultaneously and recording data independently.

Experimenters then used the formula: IOA=number of agreements/(number of agreement+ number of disagreements)*100% to calculate interobserver agreement. Experimenters conducted IOA for 82% of the probe sessions with 100% agreement.

Experimenters also conducted interscorer agreement on students' responses to word problems and Morningside fluency worksheets using permanent product and the formula: ISA=number of agreements/(number of agreements+ number of disagreements)*100%. Experimenters obtained ISA for 71% of the probe sessions with 100% agreement. Obtaining ISA was especially crucial during fluency training as the participant did not receive another probe session until they mastered or fluently responded to fluency worksheets. Experimenters obtained 100% agreement for 100% of the mastery and fluency training sessions.

Results

Dylan emitted 19 correct responses and 1 incorrect response with the rate of 1.88 correct responses per minute and 0.10 incorrect responses per minute during pre-intervention probe. Dylan used counting strategies for five instances. After one session of mastery training, Dylan mastered the fluency set and received a post-mastery probe. Dylan emitted 20 correct responses at the rate of 7.45 correct responses per minute with no counting strategies. After meeting fluency criterion after seven fluency training sessions, Dylan responded to the post-fluency probe with 20 correct responses at the rate of 9.38 correct responses per minute with no counting strategies (See *Figures 7-9*).

Collin emitted 20 correct responses at the rate of 1.98 correct responses per minute during pre-intervention probe. Collin used counting strategies for two instances. After one session of mastery training, Collin mastered the fluency set and received a post-mastery probe. Collin emitted 20 correct responses at the rate of 3.85 correct responses per minute. After meeting fluency criterion after six fluency training sessions, Collin responded to the post-fluency probe with 20 correct responses at the rate of 6.25 correct responses per minute (See *Figures 7-9*).

Gabe emitted 17 correct responses and three incorrect responses at the rate of one correct response per minute and 0.18 incorrect responses per minute during the first pre-intervention probe with 0 counting strategies. During the second pre-intervention probe, Gabe emitted 16 correct responses and four incorrect responses at the rate of 0.97 correct responses per minute and 0.17 incorrect responses per minute with 0 counting strategies. After two sessions of mastery training, Gabe mastered the fluency set and received a post-mastery probe. Gabe emitted 19 correct responses and one incorrect response at the rate of one correct response per minute and 0.05 incorrect responses per minute with 0 counting strategies. After meeting fluency criterion after five fluency training sessions, Gabe responded to post-fluency probe with 16 correct responses and four incorrect responses at the rate of 2.16 correct responses per minute and 0.54 incorrect responses per minute with zero counting strategies (See *Figures 7-9*).

Gray emitted 11 correct responses and 9 incorrect responses at the rate of 2.63 correct responses per minute and 2.15 incorrect responses per minute during the first pre-intervention probe with 2 counting strategies. During the second pre-intervention probe, Gray emitted 11 correct responses and nine incorrect responses at the rate of 1.58 correct responses per minute and 1.05 incorrect responses per minute with 7 counting strategies. After one session of mastery training, Gray mastered the fluency set and received a post-mastery probe. Gray emitted 20 correct responses at the rate of 5.22 correct responses per minute with 0 counting strategies. After meeting fluency criterion after two fluency training sessions, Gray responded to the post-fluency probe with 20 correct responses at the rate of 12.40 correct responses per minute with zero counting strategies (See *Figures 7-9*).

Jo emitted 12 correct responses and eight incorrect responses at the rate of 0.64 correct responses per minute and 0.43 incorrect responses per minute during the first pre-intervention probe with 1 counting strategy. During the second pre-intervention probe, Jo emitted 11 correct responses and nine incorrect responses at the rate of 0.75 correct responses per minute and 0.61 incorrect responses per minute with 0 counting strategies. After two sessions of mastery training, Jo mastered the fluency set and received a post-mastery probe. Jo emitted 16 correct responses and four incorrect response at the rate of 1.27 correct responses per minute and 0.32 incorrect responses per minute with 0 counting strategies. After meeting fluency criterion after six fluency training sessions, Jo responded to the post-fluency probe with 15 correct responses and five incorrect responses at the rate of 3.41 correct responses per minute and 1.14 incorrect responses per minute with zero counting strategies (See *Figures 7-9*).

Liam emitted seven correct responses and 13 incorrect responses at the rate of 0.43 correct responses per minute and 0.80 incorrect responses per minute during the first pre-intervention probe with 1 counting strategy. During the second pre-intervention probe, Liam emitted six correct responses and 14 incorrect responses at the rate of 0.61 correct responses per minute and 1.42 incorrect responses per minute with 0 counting strategies. After two sessions of mastery training, Liam mastered the fluency set and received a post-mastery probe. Liam emitted eight correct responses and 12 incorrect response at the rate of 1.76 correct responses per minute and 2.65 incorrect responses per minute with 0 counting strategies. After meeting fluency criterion after three fluency training sessions, Liam responded to the post-fluency probe with 12 correct responses and eight incorrect responses at the rate of 2.18 correct responses per minute and 1.45 incorrect responses per minute with zero counting strategies (See *Figures 7-9*).

Sam emitted 16 correct responses and four incorrect responses at the rate of 1.83 correct responses per minute and 0.46 incorrect responses per minute during the first pre-intervention probe with eight counting strategies. During the second pre-intervention probe, Sam emitted 12 correct responses and eight incorrect responses at the rate of 1.42 correct responses per minute and 1.07 incorrect responses per minute with 11 counting strategies. After one session of mastery training, Sam mastered the fluency set and received a post-mastery probe. Sam emitted 12 correct responses and eight incorrect response at the rate of 1.6 correct responses per minute and 1.07 incorrect responses per minute with 8 counting strategies. After meeting fluency criterion after six fluency training sessions, Sam responded to the post-fluency probe with 12 correct responses and eight incorrect responses at the rate of 1.62 correct responses per minute and 1.08 incorrect responses per minute with nine counting strategies (See *Figures 7-9*).

Kylie emitted 12 correct responses and eight incorrect responses at the rate of 0.46 correct responses per minute and 0.31 incorrect responses per minute during the first pre-intervention probe with 18 counting strategy. During the second pre-intervention probe, Kylie emitted nine correct responses and 13 incorrect responses at the rate of 0.34 correct responses per minute and 0.41 incorrect responses per minute with 017 counting strategies. During the first mastery training session, Kylie spent more than 40 minutes on one worksheet at which point the experimenters stopped the intervention. Experimenters decided to discontinue the intervention with Participant Kylie (See *Figures 7-9*).

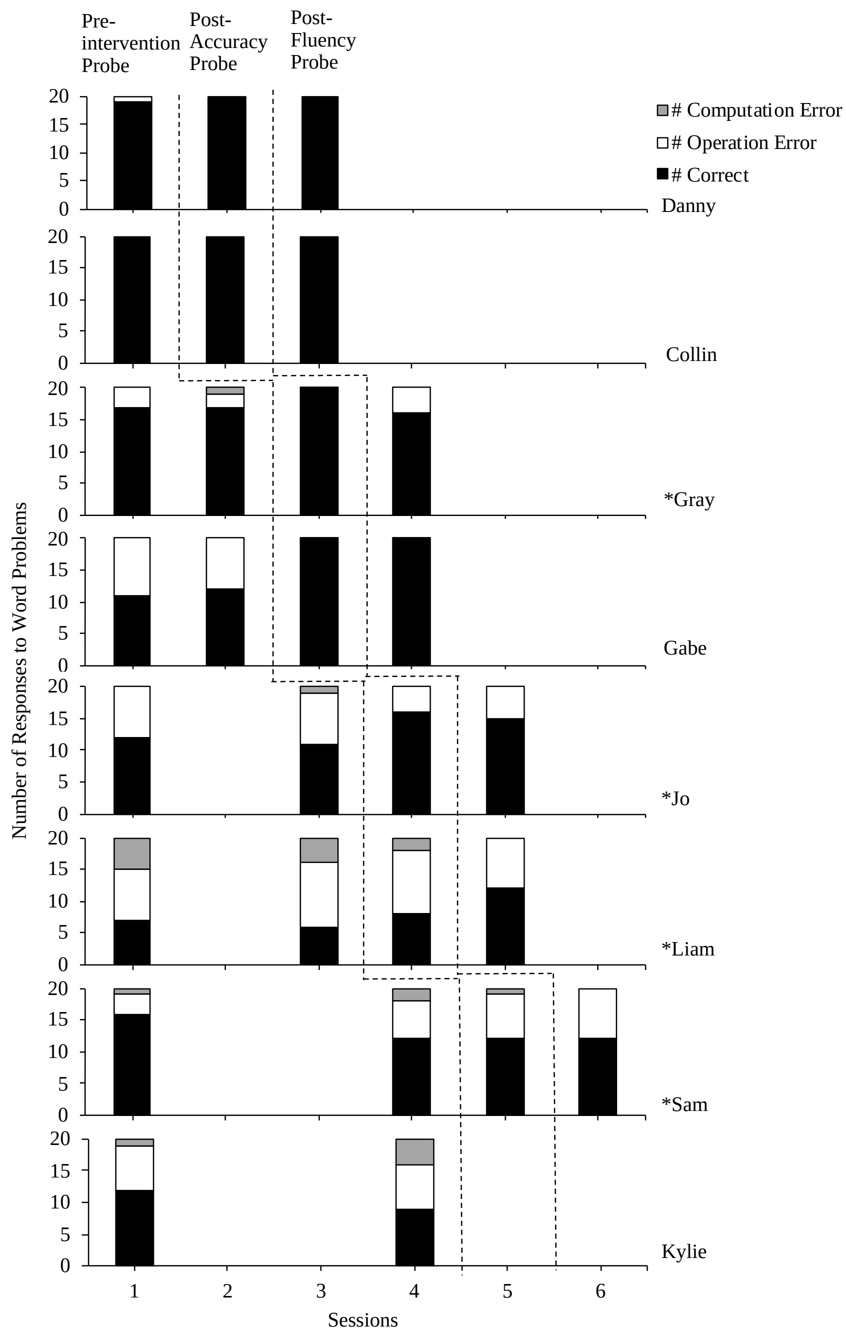
Table 4

Participants' Demographics and Related Cusps

Participant	Age	Gender	Grade	iReady Diagnostics Numbers and Operations Grade Level Equivalence	Presence of TSF across saying and writing
Danny	14	M	8	6th Grade	Yes
Collin	11	M	6	4th Grade	Yes
Gray	12	M	6	3rd Grade	No
Gabe	12	M	6	2nd Grade	Yes
Jo	13	F	8	3rd Grade	No
Liam	13	M	7	Kindergarten	No
Sam	13	M	8	1st Grade	No
Kylie	13	F	8	2nd Grade	Yes

Figure 7

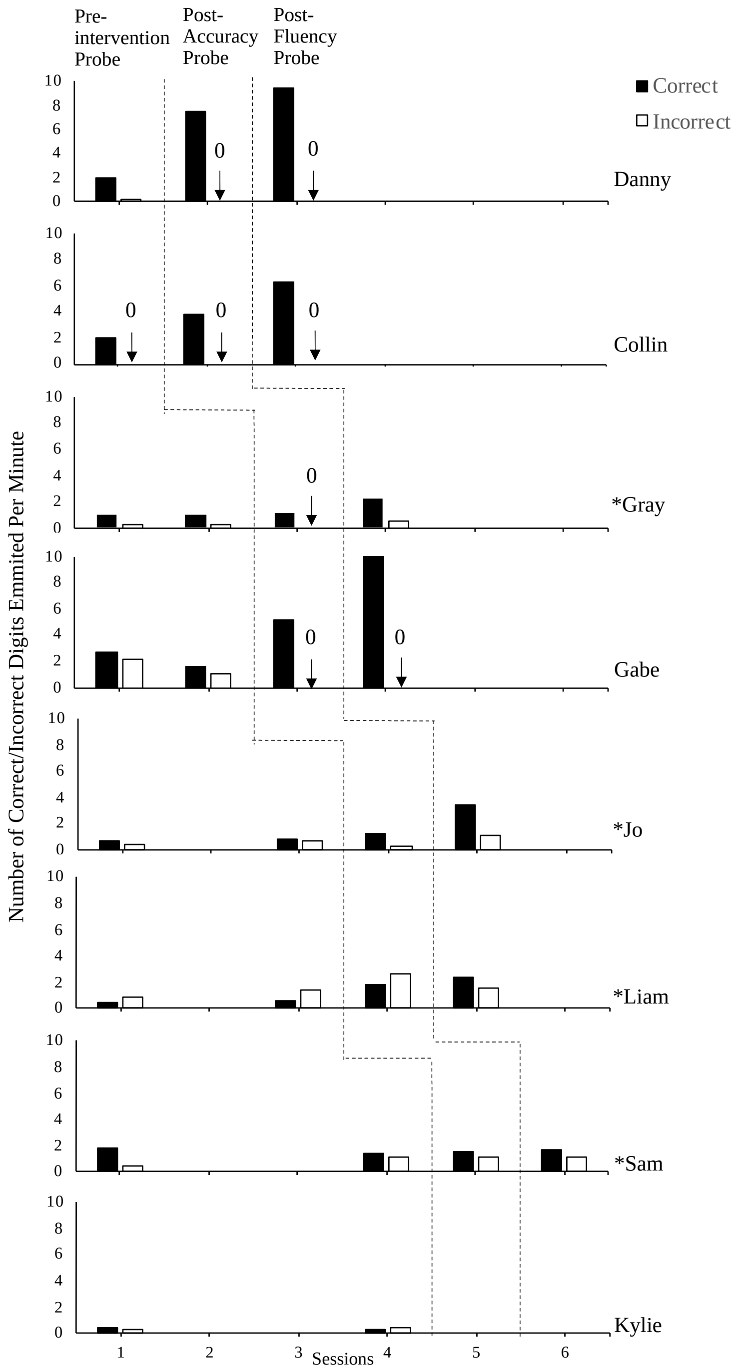
Number of Correct/Incorrect Responses to Word Problems



Note. Number of correct responses, incorrect operations, or incorrect computations emitted to 20 word-problems. Participants with a "*" next to their names did not demonstrate TSF across saying and writing.

Figure 8

Rate of Correct/Incorrect Responses to Word Problems

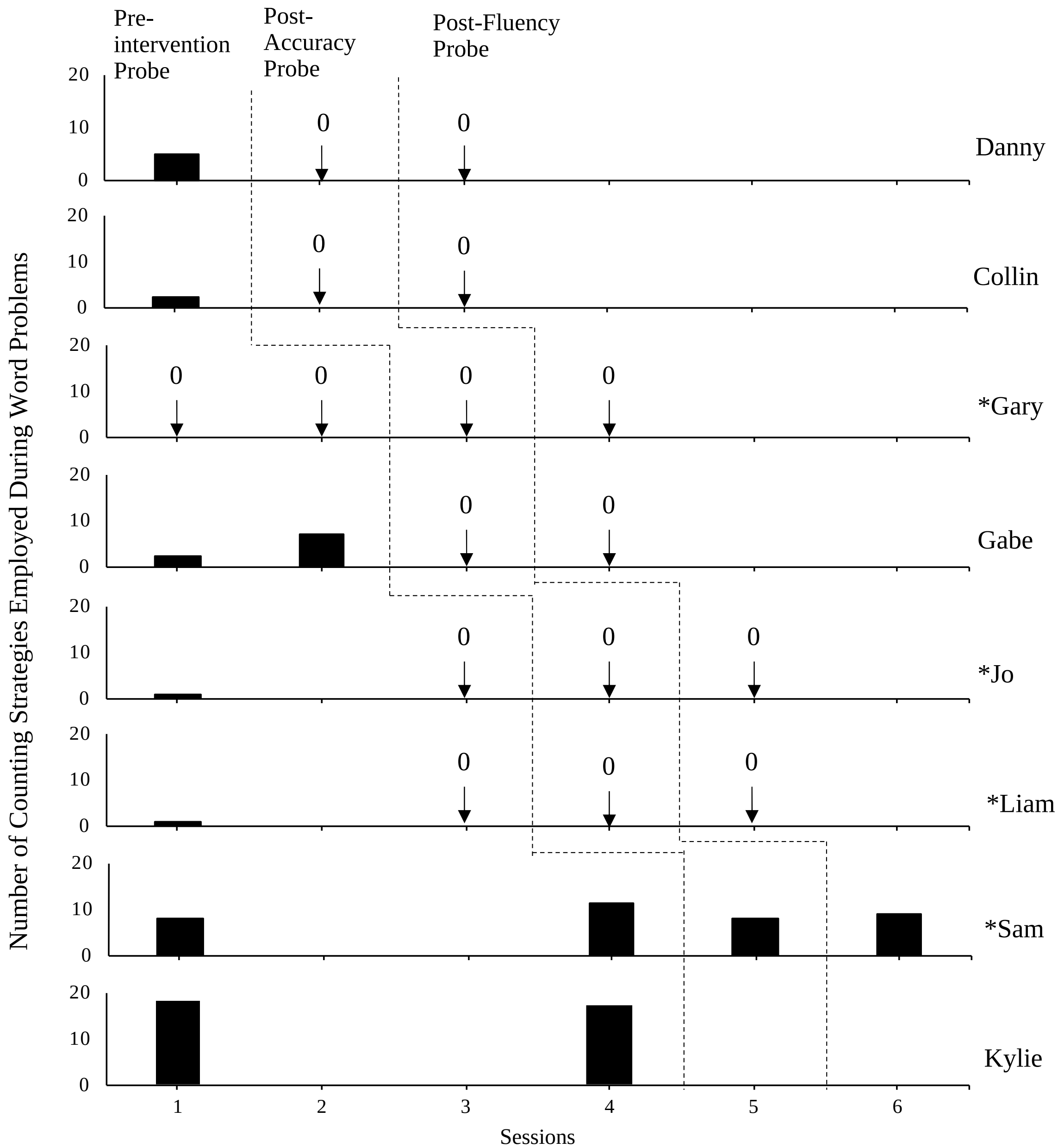


Note. Number of correct/incorrect responses emitted to 20 word-problems per minute.

Participants with a "*" next to their names did not demonstrate TSF across saying and writing.

Figure 9

Number of Counting Strategies Used



Note. Number of counting strategies used by participants during probe sessions. Participants with a "*" next to their names did not demonstrate TSF across saying and writing.

Appendix B

Sample Math Fact Worksheets

Name ______________

Morningside Math Facts Fluency
Multiply-Divide
2 9 18, 3 3 9, 3 4 12

New Facts 6
Fact Sheet

3 x 3	4)12	4)12	9 x 2	3)12	3)12	2 x 9	3)9	4)12	9 x 2	10
4 x 3	3 x 3	9 x 2	9 x 2	3)12	4 x 3	4)12	2 x 9	3 x 4	2 x 9	20
9)18	4)12	2)18	3)12	3)12	3)9	3 x 3	3 x 3	3 x 4	9 x 2	30
4)12	3 x 4	3)9	2 x 9	2 x 9	3 x 3	3 x 3	3)9	2 x 9	2)18	40
2)18	2 x 9	4 x 3	3 x 3	3)12	3)12	4 x 3	3 x 3	9 x 2	2)18	50
3)9	9 x 2	3 x 3	4 x 3	9)18	4 x 3	4 x 3	3 x 4	9)18	3)9	60
3 x 4	3)9	3)9	3 x 4	3)12	3 x 3	3 x 3	3 x 3	9)18	2 x 9	70
9)18	2 x 9	4 x 3	4 x 3	2)18	9 x 2	4)12	2)18	9)18	9)18	80
4)12	3 x 3	9 x 2	3)9	3 x 3	3 x 3	3)12	3 x 3	2)18	4)12	90
4 x 3	3)9	4 x 3	2 x 9	2 x 9	3 x 3	3 x 3	9)18	3 x 3	4 x 3	100

Appendix C

Sample Word Problem Worksheets

1

Adam has 18 minutes of free time at night before he has to go to bed. How many 2 minute long YouTube videos can Adam watch before he has to go to bed?

Stephen's Instagram page gains 9 followers every time he posts. If he posts 2 times a day, how many followers does he gain in a day?

Brandy is sorting her 9 crayons which she will place into small bags. Each bag can contain 3 crayons. How many crayon boxes does Brandy need?

Ty is having a video game night at his house and wants to make peanut-butter and jelly sandwiches for his friends. He has 4 friends coming over and wants to make 3 sandwiches for each friend. How many sandwiches should he make?

Courtney picked up all the paper clips that were on her desk. She was able to collect 12 paper clips. She wants to put 4 paper clips in a box. How many boxes will she need?

Patrick's company is building a new office for his client. Each floor of the new building is 3 meter/feet tall and the building has 4 floors. How tall is the building when it's done?

Quinn wants to buy her dogs some toys. If Quinn has 3 dogs and she buys 3 toys for each dog, how many toys did she buy?

Elyse is replanting her garden now that winter has ended. She is planting her favorite vegetable, corn. If Elyse plants 12 corn seeds in rows of 3, how many rows of corn seeds will there be?

Rosa went on a new diet that her doctor recommended, where she eats 2 apples a day. How many apples will she eat over 9 days?

Damien brought 18 pizzas to his friend's graduation party. If there were 9 guests at the party, how many pieces of pizza did each guest get?

Appendix D

Evaluating One-step Algebraic Expression Probe

$Find\ x \cdot 8\ when\ x = 7$

$Find\ u \div 8\ when\ u = 64$

$Find\ t \div 9\ when\ t = 54$

$Find\ y \div 7\ when\ y = 56$

$Find\ m \cdot 7\ when\ m = 8$

$Find\ n \times 6\ when\ n = 9$

$Find\ t \cdot 9\ when\ t = 6$

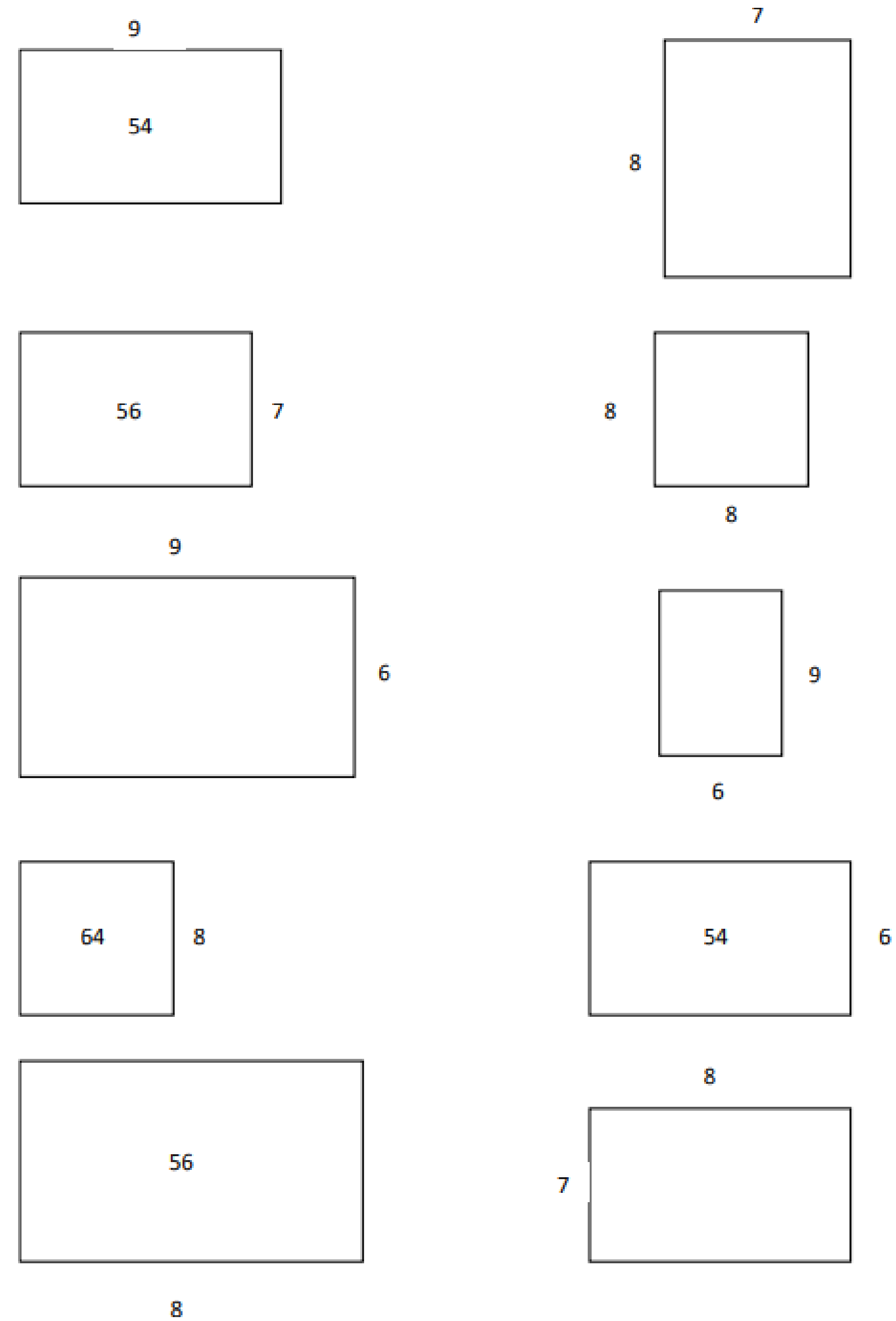
$Find\ t \div 6\ when\ t = 54$

$Find\ x \cdot 8\ when\ x = 8$

$Find\ t \div 8\ when\ t = 64$

Appendix E

Area/Side of Rectangle Probe



CHAPTER III

STUDY II MANUSCRIPT

Abstract

Research targeting math competency of children with or without disabilities has largely focused on theoretical, cognitive constructs such as developing schematic tools to aid children's needs in "processing" information presented both as math symbols and English language while overlooking the importance of teaching math as a language and verbal communicative behavior. In prior studies focused on math as verbal behavior, researchers reported an association between the demonstration of the transformation of stimulus function (TSF) across saying and writing for spelling and the demonstration of transformation of stimulus function between math problem solving and computation accuracy. This called for an experiment to test for possible functional relations between the two types of TSF. In the current study, I used a multiple probe design to test the effects of the induction of TSF across saying and writing for spelling using multiple exemplar instruction (MEI) on the presence of TSF across math facts and word problems. The participants of the study were three sixth grade students with individualized education plans (IEPs) who did not demonstrate TSF for spelling or math operants. Once they acquired TSF across saying and writing for spelling, all participants demonstrated TSF across math facts and word problems, suggesting that TSF for math operants is indeed a form of TSF across saying and writing. The results suggest that teachers should implement teaching tactics that allow students to respond to math operants both as a listener and as a speaker, and most importantly, as speaker-as-own-listener to teach math as verbal repertoire for students.

Keywords: fluency training, transformation of stimulus function, word problems.

Stimulus Control for Making Math Verbal

Albert Einstein (1935) once regarded mathematics as “the poetry of logical ideas.” Galileo Galilei more explicitly described mathematics as “the *language* in which God has written the universe” (Galilei, 1623). Although mathematicians have long regarded math as a language that communicates ideas, math educators in the past focused mainly on teaching the manipulation of numbers and symbols while making occasional connections between numbers, math symbols, and words by using words to bridge mathematical concepts (Kliman et al., 1996; Nesher et al., 1986; Wakefield, 2000). It was not until recent decades when math education researchers and organizations started to focus on incorporating mathematical discourse as a crucial component of everyday math education practice (Ryve, 2011).

In 1998, The National Council of Teachers of Mathematics (NCTM) acknowledged that math “can be thought of as a language that must be meaningful if students are to communicate mathematically and apply mathematics productively.” Numerous math education studies focusing on the effective teaching of mathematical discourse sprouted after the publication of the NCTM article. However, the education of mathematical discourse, or the use of mathematical language to communicate mathematical ideas, did not go beyond using terminologies or frameworks from other fields (Niss, 2007). Ryve (2011) pointed out that a majority of the articles reviewed (60 of 108) simply defined discourse as speech, or the use of mathematical terminologies.

However, discourse is much more than speech. Skinner (1957) pointed out that “speech” only emphasizes vocal behavior whereas “language” refers to the practices of a linguistic community, which he later referred to as “verbal behavior” and “verbal community.” Palmer (2008) further elaborated that “verbal behavior emerges in a community that maintains contingencies of reinforcement for behavior that reflects conventional but arbitrary relationships between stimuli and responses" (p. 299). That is, verbal behavior addresses an arbitrarily defined relation between an object and language as a function of the reinforcement between listener and speaker. Individuals function as listeners and speakers in the verbal community where speakers' behaviors are reinforced by listeners and listeners' behaviors are conditioned by speakers (Palmer, 2008). Skinner (1957) pointed out that people are not limited to one single verbal community. For example, one can speak two or more languages and thus participate in two different verbal communities. Just like anyone who participates in the English-speaking verbal community, mathematicians participate in the verbal community of mathematics.

When considering learning math as the participation in a new verbal community, the parallel between math and language acquisition becomes evident. Then why can't we learn and teach mathematics as verbal behavior? Numerous studies stemmed from Skinner's verbal behavior to teach children to listen (Goswami, 2014; Sterkin, 2012), to emit first instances of language (Pistoljevic et al., 2010, Tsiouri & Greer, 2003), to read (Helou-Care, 2008), to write (Broto & Greer, 2014; Helou et al., 2007; Reilly-Lawson et al., 2006), and to become independent learners and thinkers (https://www.cabasschools.org).

In a review of current literature, recent research also started to focus on using behavior analytic interventions to teach mathematics or teaching math as a verbal behavior through teaching the function of it. Those studies used procedures and tactics developed to teach reading/writing skills to successfully improve participants' performances in math. Weber (2016) and Crosbie (2018) used a peer-editing procedure that Pellegren (2015) used to target writing skills to increase the number of correct written math algorithms to solve math problems. Maurilus (2018) used procedures that were previously used to condition book stimuli to establish the reinforcement value for math and tested how that affected children's rate of acquisition of math operants. All those studies attempted to teach math skills as verbal behavior by building functions to math. Other behavior analytic strategies and curricula such as Precision Teaching (Chiesa et al., 2000; Stromgren, et al., 2014), Direct Instruction (Al-Makahleh, 2011; Din, 1998; Firdaus, 2017; Kinder, 1991), peer tutoring (Mayfield, et al., 2007), and peer editing (Weber, 2016) were studied and tested in attempts to teach math in a more systematic manner.

While those studies addressed crucial aspects missing in the current field of math education, most of them targeted specific math skills: math facts or math problem-solving, leaving a gap concerning making connections across those math skills. A few studies that tested the connection between math facts and problem solving yielded mixed findings (McTiernan et al., 2016; Singer-Dudek & Greer, 2005). From a behavioral perspective, the connection between different math operants is a form of transformation of stimulus function. For example, when given a word problem "John spent $2 and now has $3 left. How much did John have before?", a student who demonstrates solving one-step algebraic equations will approach the problem by stating x-2=3 and find the sum of two and three while those who did not demonstrate that verbal stimulus control will use trial and error or schematic tools. This is not always the case. Students often need additional instruction on every component of a novel type of problem even when the problems involve mastered skills (Kroesbergen & Van Luit, 2003). Those students who were not under the joint stimulus control emitted more errors and demonstrate greater needs in mastering novel math operants.

Another gap in existing research is that although researchers adopted procedures that were previously used to teach language arts-related skills to successfully teach math skills, none of them explicitly examined the association between the establishment of reading skills and the acquisition of mirroring math skills. A pilot study on this matter found a strong association between the presence of TSF for spelling across saying and writing and TSF across math facts and word problems (See *Appendix A*). After mastering novel math facts, all participants who demonstrated TSF for spelling across saying and writing also demonstrated mastery and fluent responding to word problems involving those math facts whereas the participants who did not demonstrate TSF for spelling across saying and writing did not demonstrate mastery or fluent responding. Participants without TSF across saying and writing for spelling also relied more on the use of visual prompts such as finger counting, tally marks, or other number-object prompts. However, the nature of the relation remains to be tested.

To fill in the gap in experimental testing on the relation between TSF and expertise in math word problems, the current experiment sought to test for a functional relation between the establishment of TSF across saying and writing for spelling and the establishment of TSF across math facts and word problems.

Methods

Participants

The participants of the study were three sixth grade students who participated in Experiments I and II. All participants attended a public middle school in a school district located in a suburb outside a major metropolitan city. All participants were enrolled in a self-contained classroom utilizing the Comprehensive Application of Behavior Analysis to Schooling (CABAS®, www.cabasschools.org, www.scienceof teaching.org) model for math, reading, social studies, and science. Teachers in the class based all instruction on scientific procedures and continuously measured student responses and performance (Greer, 1998; Greer, 2010; Singer-Dudek et al., 2010; Singer-Dudek, Keohane, & Matthews, in press; www.cabasschools.org). The students participated in general education settings for other subjects such as music, art, physical education, and technology.

All participants attended the same self-contained classroom for English language arts, mathematics, science, and social studies. All participants had Individualized Education Plans (IEP) addressing their specific academic, behavioral, and social needs. One head teacher and up to three paraprofessionals were in the classroom during all probe and intervention sessions. All participants demonstrated reader/writer skills such as textually responding at a rate between 60 to 100 correct words per minute for 3rd grade level texts and dictating 3rd grade level words using phonics.

None of the participants demonstrated TSF across saying and writing for spelling or for math operants prior to the study. This means that upon mastery of spelling a set of words in written form, the participants emitted less than 80% correct responses to spelling the same words vocally and that upon mastery of math facts, the participants emitted less than 80% correct responses to word problems targeting the same math fact families. They all performed below grade level for reading and math according to *iReady Diagnostics Assessments* conducted at the beginning of the school year (See Table 1). All participants received math facts fluency training for addition and subtraction facts through the *Morningside Math Fluency Curriculum* (Johnson, 2008). They also received fluency training for early multiplication and division math facts. However, participants did not receive accuracy or fluency training for the math facts targeted in the current study. Pre-experimental assessments demonstrated that the participants did not respond to the targeted math facts with accuracy or fluency. Prior to the intervention, all participants mastered solving one-step multiplication/division word problems with numbers lower than 10 using visual prompts (e.g., tallies, finger counting, etc.). That is, when given a one-step multiplication/division word problem, the participants used tally marks and circles to draw a visual representation to solve the word problem.

Settings and Materials

We conducted all probe and intervention sessions in the participants' classroom. During intervention and probe sessions, the participants sat with an experimenter at one of the tables in one corner of the classroom, isolated from other students. Those tables did not have cubbies, so the participants had to keep both hands on the tabletop. This was done in order for experimenters to track data on potential finger counting as visual prompts to solve word problems. Students completed experimental tasks using printed *Morningside Fluency* (Johnson, 2008) worksheets for math fact families (6, 7, 42), (6, 8, 48), and (7, 7, 47) or word problem worksheets, blank scrap paper, and pencil.

The math fact worksheets contained math facts covering the ten math sentences that are generated with the three fact families (i.e., 7x7=49, 49/7=7, 6x7=42, 7x6=42, 42/7=6, 42/6=7, 8x6=48, 6x8=48, 48/8=6, 48/6=8) presented in random order. The worksheets consisted of 10 rows of math fact questions with 10 questions in each row, totaling 100 math fact problems. The word problem worksheets used for this study contained ten different word problems the experimenters generated from the three target number families. Each number sentence generated by the target number families occurred exactly once in the worksheets. For example, with number family (8, 6, 48), we wrote four word -problems with the answer 8x6=48, 6x8=48, 48/8=6, and 48/6=8 each appearing exactly once. Each word problem worksheet contained exactly 314 words to control for students' rate of textual responding.

Dependent Variables

The dependent variable of the study was the presence of TSF across math operants (i.e., computation of math facts and word problem solving). The onset of TSF for math operants consisted of two components: accurate and fluent responding to word problems after demonstrating accurate and fluent responding to math facts, and extinction of counting strategies. We defined accurate responding to word problems as emitting 100% correct responses to word problems with the same fact families taught during math fact fluency training. We defined the fluent responding to word problems as emitting 100% accurate responses to word problems within a pre-determined duration of time. We individualized the target duration based on the participants' rate for textual responding and handwriting with the formula Target Duration (in seconds) =314/rate of textual responding x 60 seconds + 10 questions x 60 seconds/rate of handwriting.

The experimenter considered a participant to demonstrate TSF from math facts to word problems when, after trained to accurately and fluently respond to math fact families (3, 3, 9), (3, 6, 18), and (2, 9, 18) for pre-intervention probes and (6, 7, 42), (6, 8, 48), and (7, 7, 49) for post-intervention probes by emitting 100% accurate responses and meeting their individualized fluency goal, the participant responded to ten word problems involving the trained fact family with 100% accuracy within their target duration.

The experimenter considered a participant to demonstrate extinction of counting strategies when the participant who used counting strategies during pre-intervention probes ceased to use those strategies after being trained to accurately and fluently respond to math facts or word problems. The experimenters defined counting strategies as the use of fingers, drawing, tallies, or any other visual representations or visual prompts of the math problem. That is, the participants moved away from verbally mediated strategies to contingency-shaped behavior without directly contacting contingencies concerning the word problems.

Independent Variables

The independent variable of the study was the acquisition of TSF across saying and writing for spelling. We used the same assessment and treatment procedure outlined in Eby et al. (2010). To test for the absence or presence of TSF across saying and writing for spelling, we taught the written spelling of 5 novel words to mastery using learn units (Albers & Greer, 1991; Greer & Hogin-McDonough, 1999). We then tested for TSF for spelling by asking the participants to spell those mastered spelling words vocally. We did not consequate spelling responses during probe sessions. If the participant emitted 80% or more correct responses, we considered them as having demonstrated TSF across saying and writing for spelling. We conducted an initial probe for TSF across saying and writing for spelling for all participants prior to the assessment for TSF across math facts and word problems. We conducted another TSF for spelling probe prior to establishing TSF across saying and writing for spelling to show that participants of the current study did not demonstrate TSF for spelling as a function of daily school instruction over the course of the previous study.

If the participant did not emit 80% or more correct responses towards the vocal spelling probe, we implemented multiple exemplar instruction (MEI) to teach the transformation of stimulus function across saying and writing for spelling. During MEI, we taught the spelling for 5 novel words and alternated between vocal spelling responses and written spelling responses using learn unit instruction (Albers & Greer, 1991; Greer & Hogin-McDonough, 1999). We provided consequences after the participants completed each intervention task during intervention phases. We provided reinforcement (i.e., praise, points) to the participants for each correct response and a correction (i.e., teacher model of how to solve the problem and an independent opportunity for the participant to respond for up to three times) for each incorrect response (Albers & Greer, 1991; Greer & Hogin-McDonough, 1999).

We presented instruction for each word 4 times in each response topography, totaling to a 40-learn unit instructional session. With MEI, we alternated between vocal and written responses. We also delivered learn units for the words in a random order. The participants responded to criterion level when they emitted 90% or more correct responses across two consecutive sessions or 100% correct responses for one session. Upon mastery of the MEI instruction, we conducted another TSF across saying and writing for spelling probe where we taught written spelling of words to mastery and tested for vocal spelling. We also conducted an additional probe session during which we taught vocal spelling of a set of 5 novel words to mastery and conducted probes for the written spelling of those words.

Data Collection

Math Operants

The experimenter recorded the accuracy and fluency (i.e., number correct, number incorrect, rate correct, rate incorrect) of participants' responses to each set of word problems and math facts throughout the experiment. For word problems, the experimenter marked a "+" for each correct response emitted by the student, an "O" for each incorrect response with operation selection errors (e.g., student performed addition for a multiplication problem), and a "C" for each incorrect response with computation errors (e.g., student wrote 3x3=7). For math facts, the experimenter recorded a plus "+" for each correct response and a minus "-" for each incorrect response. The experimenter then visually represented the number of correct/incorrect responses as a stacked bar graph.

The experimenters also recorded the duration of each probe and intervention session to convert the number of correct/incorrect responses to the rate of correct/incorrect responses using the formula: rate of correct/incorrect response (per minute) =number of correct/incorrect responses/duration (in seconds) *60. During the intervention, we individualized participants' fluency goals based on their pre-experimental rates of number writing and rate of textual responding.

The experimenters recorded the number of uses of visual prompts using event recording. The experimenters recorded a tally mark for each problem for which the participant used counting strategies. If the participant used multiple counting strategies for one problem, we only recorded it as one occurrence. For example, if the student used finger counting for a word problem and then re-calculated the same word problem using tallies, we only counted it as 1 use of a counting strategy.

Spelling Words

We reported the number of correct responses emitted to spelling words. For vocal spelling, the experimenter delivered a vocal antecedent instruction "Spell the word _____" and marked a plus "+" for a correct vocal spelling response demonstrating one-to-one correspondence or a minus "-" for incorrect spelling responses. We marked a plus if the participant self-corrected their response by starting from the beginning of the word (i.e., emitted an intraverbal chain) before finishing spelling. For written spelling responses, the experimenter delivered a vocal antecedent instruction "Write the word _____" and marked a plus "+" for a correct written spelling response or a minus "-" for an incorrect response.

Experimental Design

The experiment consisted of three major components: (a) pre-experimental probes which included probes for TSF across saying and writing for spelling words and probes for TSF across math operants to identify participants of the study, (b) induction of TSF across saying and writing for spelling words with MEI, and (c) post-intervention probe for TSF across math facts (See *Figure 1*). We utilized a multiple probe design across participants by conducting initial probes for all participants at the same time, and then introduced the intervention to the first participant while withholding the intervention for subsequent participants (Horner & Baer, 1978). Participants received additional pre-intervention probes as the previous participant received post-intervention probes.

A multiple probe design is also embedded within pre- and post-intervention probes for TSF across math operants (See *Figure 2*). During the pre- and post-intervention probes, all participants received the first word problem probe at the same time, the first participant received two more probes and entered math fact training when the baseline data demonstrated steady state responding. After the first participant completed accuracy training for math facts and started receiving post-accuracy probes, the subsequent participant received two more pre-accuracy word problem probes. The multiple probe design showed that the behavior change occurred for multiple participants as a function of the intervention by providing between-participants control for testing, because behavior change only occurred when the experimenters introduced the intervention (Horner & Baer, 1978). Such designs ruled out the possibility that behavior change occurred because of other instruction outside of the experimental setting by conducting initial probes for all participants and introducing intervention in a delayed manner (Horner & Baer, 1978).

Interscorer and Interobserver Agreement

Experimenters conducted interobserver agreement (IOA) on the participants' use of counting strategies. The experimenters conducted IOA by having two observers simultaneously observe and independently collect data on the participants' use of counting strategies for an entire probe or intervention session. We then compared data and calculated IOA with the formula: IOA=number of agreements/total number of counting strategies observed*100%. We gathered IOA for 49% of the dependent variables with 100% agreement. Experimenters also conducted interscorer agreement on students' responses to word problems and Morningside fluency worksheets using permanent products and the formula: ISA=number of agreement/(number of agreements+ number of disagreements)*100%. Experimenters obtained ISA for 56% of all sessions with 100% agreement.

Results

In addition to reporting the dependent variables, we also calculated and reported effect size (ES) as Robust Improvement Rate Difference (IRD) (Altman, 1999, Parker et al., 2009). We reported three ES for each participant: (a) the ES for accuracy training, (b) the ES for fluency training, and (c) the ES for overall intervention.

Jeff emitted a mean of 5.33, 6.67, and 8 correct responses with a mean of 0.39, 0.62, and 0.83 correct responses per minute to word problems with the use of a mean of 10, 9.67, and 9 visual prompting strategies during pre-intervention probes, post-mastery of math facts probes, and post-fluency training of math facts probes respectively (See *Figures 1-3*). The overall effect size of mastery and fluency training of math facts on word problems was IRD=1.00 (See *Table 2*). After the induction of TSF across saying and writing for spelling, Jeff emitted a mean of 5.67, 10, and 10 correct responses with a mean of 0.67, 1.02, and 1.17 correct responses per minute to word problems with the use of a mean of 10, 0, and 0 visual prompting strategies during pre-intervention probes, post-mastery of math facts probes, and post-fluency training of math facts probes respectively (See *Figures 1-3*). The overall effect size of accuracy and fluency training of math facts on word problems was 1.00, demonstrating a significant increase in the effects of accuracy and fluency training of math facts on the accuracy and fluency of word problems after Jeff acquired TSF across saying and writing for spelling.

Prior to the establishment of TSF of spelling across saying and writing, John emitted a mean of 0.67, 0, and 0 correct responses with a mean of 0.15, 0, and 0 correct responses per minute to word problems with the use of a mean of 3, 4, and 2.33 visual prompting strategies during pre-intervention probes, post-mastery of math facts probes, and post-fluency training of math facts probes respectively (See *Figures 1-3*). The overall effect size of mastery and fluency training of math facts on word problems was 0, which demonstrated no effects (See *Table 2*). After the induction of TSF across saying and writing for spelling, John emitted a mean of 2, 10, and 10 correct responses with a mean of 0.41, 15.92, and 29.09 correct responses per minute to word problems with the use of a mean of 8.67, 0, and 0 visual prompting strategies during pre-intervention probes, post-mastery of math facts probes, and post-fluency training of math facts probes respectively (See *Figures 1-3*). The overall effect size of mastery and fluency training of math facts on word problems was 1.00, falling under the high effect size category.

Sally emitted a mean of 0, 1, and 0.33 correct responses with a mean of 0, 0.20, and 1.61 correct responses per minute to word problems with the use of a mean of 10, 10, and 10 visual prompting strategies during pre-intervention probes, post-mastery of math facts probes, and post-fluency training of math facts probes respectively before TSF across saying and writing for spelling is established (See *Figures 1-3*). The overall effect size of mastery and fluency training of math facts on word problems was 0.33, showing no effects (See *Table 2*). After the induction of TSF across saying and writing for spelling, Sally emitted a mean of 0.67, 9.67, and 10 correct responses with a mean of 0.08, 4.04, and 4.30 correct responses per minute to word problems with the use of a mean of 8.33, 0, and 0 visual prompting strategies during pre-intervention probes, post-mastery of math facts probes, and post-fluency training of math facts probes respectively (See *Figures 1-3*). The overall effect size of mastery and fluency training of math facts on word problems was 0.67, showing large effects.

Discussion

The purpose of the current study was to build on previous findings regarding the association between TSF across saying and writing for spelling and TSF across math operants to establish a functional relation (Sun & Greer, 2019). We sought to test if the establishment of TSF across saying and writing for spelling would result in the presence of TSF across math operants and whether TSF across math operants is just an application of TSF across saying and writing beyond spelling.

With the results of the current study, we found a functional relation between the induction of TSF across saying and writing and TSF across math operants for all three participants. After acquiring TSF across saying and writing for spelling, all three participants demonstrated significant increases on the number of correct responses and the rate of correct responses emitted to word problems or math facts after being trained to respond to the other math operant. The experimenters also observed a shift from visual prompts (e.g., finger counting) to vocal prompts (e.g., talking aloud to themselves) after the participants acquired TSF across saying and writing for spelling. Participants started to use vocal prompts by repeating the numbers they wrote down to solve the word problem instead of using finger-counting or other visual prompt strategies, suggesting that the two types of TSF we were looking at might actually be two applications of the same type of TSF, TSF across saying and writing.

The implications of the findings of the current study are important. For decades, educators and researchers have been focused on teaching counting strategies that are visual prompt tools to help students "self-regulate" and problem solve (Griffin & Jitendra, 2009; Krawec, 2014; Powell, 2011; Zhang & Xin, 2012). The use of such visual prompts, or verbal stimuli, as behavior analysts would call them, however, not only stayed mostly at a visual level, but also was not faded. For math to function as a verbal behavior for students learning math, for students to be verbal in math, they must join listener and speaker responses to become speaker-as-own-listeners of math (Greer et al., 2005; Lodhi & Greer, 1989). Barnes-Holmes et al. (2000) argued that verbal behavior involves some degree of arbitrary application of relations. One is not verbal, or the operant emitted is not verbal, if it is acquired through explicit contingencies (Barnes-Holmes et al., 2000). Applying this definition to the participants' behavior prior to the induction of TSF across saying and writing for spelling, they acquired math facts or word problems as a function of direct contingencies presented as learn units without demonstrating derived relational responding when the learned operants were presented in a different form (i.e., word problem or math facts). Thus, they were not verbal in math.

This is comparable to the findings in how we teach students to be verbal in reading and writing. When a child is verbal in English, after learning the relation between the printed word "dog" and the animal dog, and the printed word "jump" and the action of jumping, they will read the sentence "the dog is jumping" and "see" the image of a dog jumping without additional training or a physical image present (Mercorella, 2017). Similarly, when a child is verbal in math, after learning the relation between number and quantity (i.e., number five and five objects), and signs and operations (i.e., addition sign and the operation of adding), they will read the number sentence 5+3 = and “see” five and three objects put together without having to make tallies or count on their fingers without receiving explicit trainings.

Although extensive existing research in math education established the importance of incorporating speaker behaviors or what some researchers call classroom discourse in math classes, math is rarely taught as a verbal behavior (Pimm, 1987; Lampert, 1990; Alexander, 2006). Morgan et al. (2014) reviewed a collection of articles on language use in mathematics education research and concluded that the focus of language use has been on the acquisition of mathematical ways of speaking or writing, rather than the application and use of mathematical verbal behavior. That is, the inquiry of math discourse in the classroom is different from what we would consider the verbal behavior of math because math discourse focused on the *form* of using mathematical terminologies rather than the *function* of communicating mathematical ideas.

But how are the students supposed to be verbal in math, to become speakers-as-own listeners in math, if the teachers do not present the materials in a way that allows them to do so? The use of MEI and multiple exemplar training (MET) has long been shown effective in establishing relational frames for participants to emit derived relational responding among different response topographies or stimuli. In the current study, participants demonstrated the joining of saying and writing responses for both spelling stimuli and math stimuli after receiving MEI for written and vocal spelling words. This implies that teachers should rotate between speaking and writing responses for students who do not have TSF across saying and writing in repertoire so that they will join those two initially separate responses under the same control of mathematical verbal stimuli and become speakers-as-own-listeners of math. TSF across different response topographies and different relations within math needs also to be further investigated.

One limitation of the current study was the limited number of participants and the homogeneity of the participants. All three participants were sixth grade students attending the same self-contained classroom. This limited the external validity of the study. Future studies should replicate the procedure with participants across different ages and levels of verbal behavior to further test the external validity of the study.

Another limitation of the study was the lack of measurement on the long-term impact of TSF across saying and writing on students' academic performances. If our hypothesis that being verbal in math or being speaker-as-own-listeners in math allows students to join separate math responses under the control of one or a class of stimuli is true, then, as previous research on TSF or other developmental cusps and capabilities have demonstrated, students would demonstrate accelerated learning (Greer & Ross, 2008). The experimenters anecdotally reported the accelerated acquisition of new math facts and word problems after the acquisition of TSF across saying and writing for spelling. However, future studies should measure and report students' rate of acquisition across various math objectives to further solidify our proposition.

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Table 1

Participants' Demographics and Related Cusps

Participant	Age	Gender	Grade	iReady Diagnostics Numbers and Operations Grade Level Equivalence	Presence of TSF across saying and writing	Target Fluency Goal (digits per minute)	Rate of Textual Responding (wpm)
JC	11	M	6	3rd Grade	No	35	106
JT	11	M	6	2nd Grade	No	35	104
SF	11	F	6	1st Grade	No	35	68

Table 2

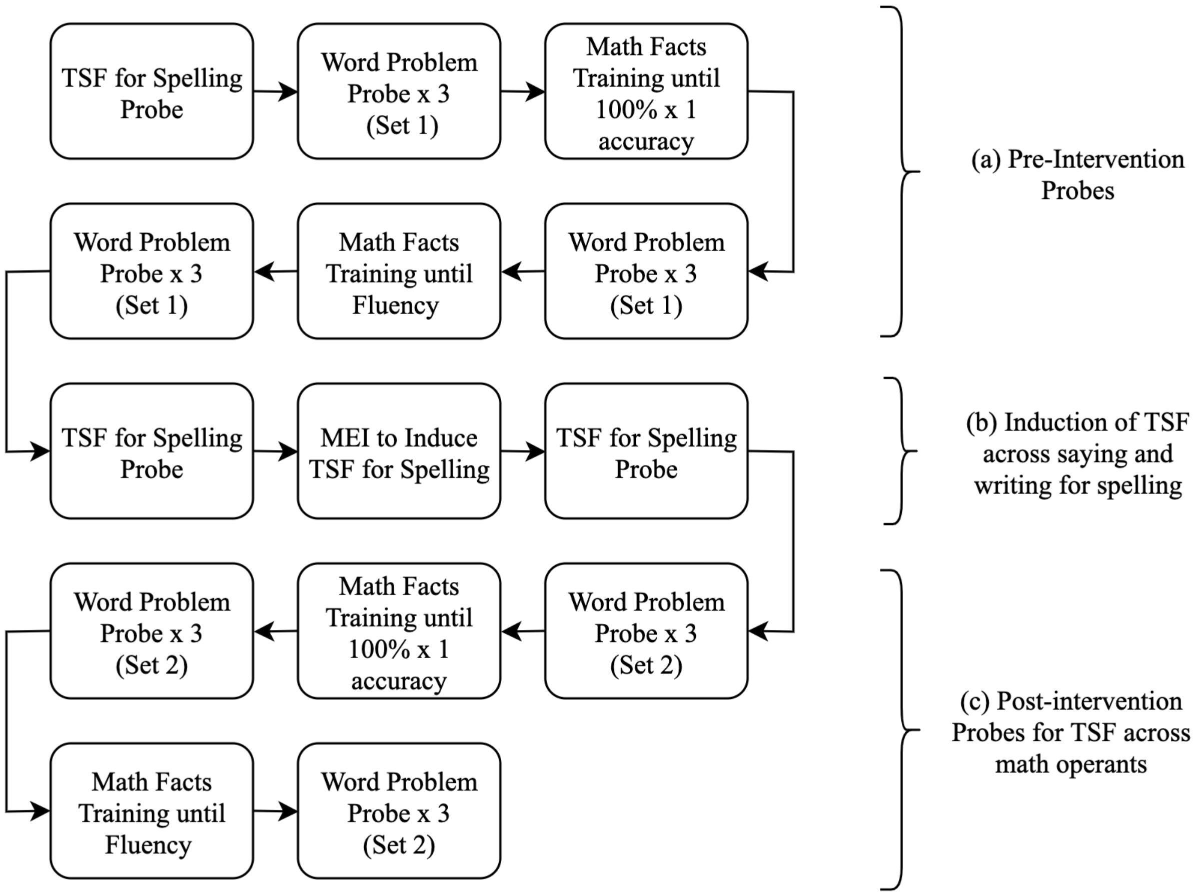
Effect Sizes Across Participants and Probe Conditions

		Prior to the induction of TSF across saying and writing for spelling			After the induction of TSF across saying and writing for spelling		
	Participants	Jeff	John	Sally	Jeff	John	Sally
Train math facts, test word problems	ES between pre-intervention to post-mastery	0.67	0	0.67	1	1	1
	ES between post-mastery to post-fluency	1	0	0	1	1	1
	ES between pre-intervention post-fluency	1	0	0.33	1	1	0.67

Note. We reported effect size using robust Improvement Rate Difference (IRD). ES<= .5 indicates questionable or no effect. When .5< ES< .7, the intervention is effective. When ES>=.7, the intervention is very effective.

Figure 1

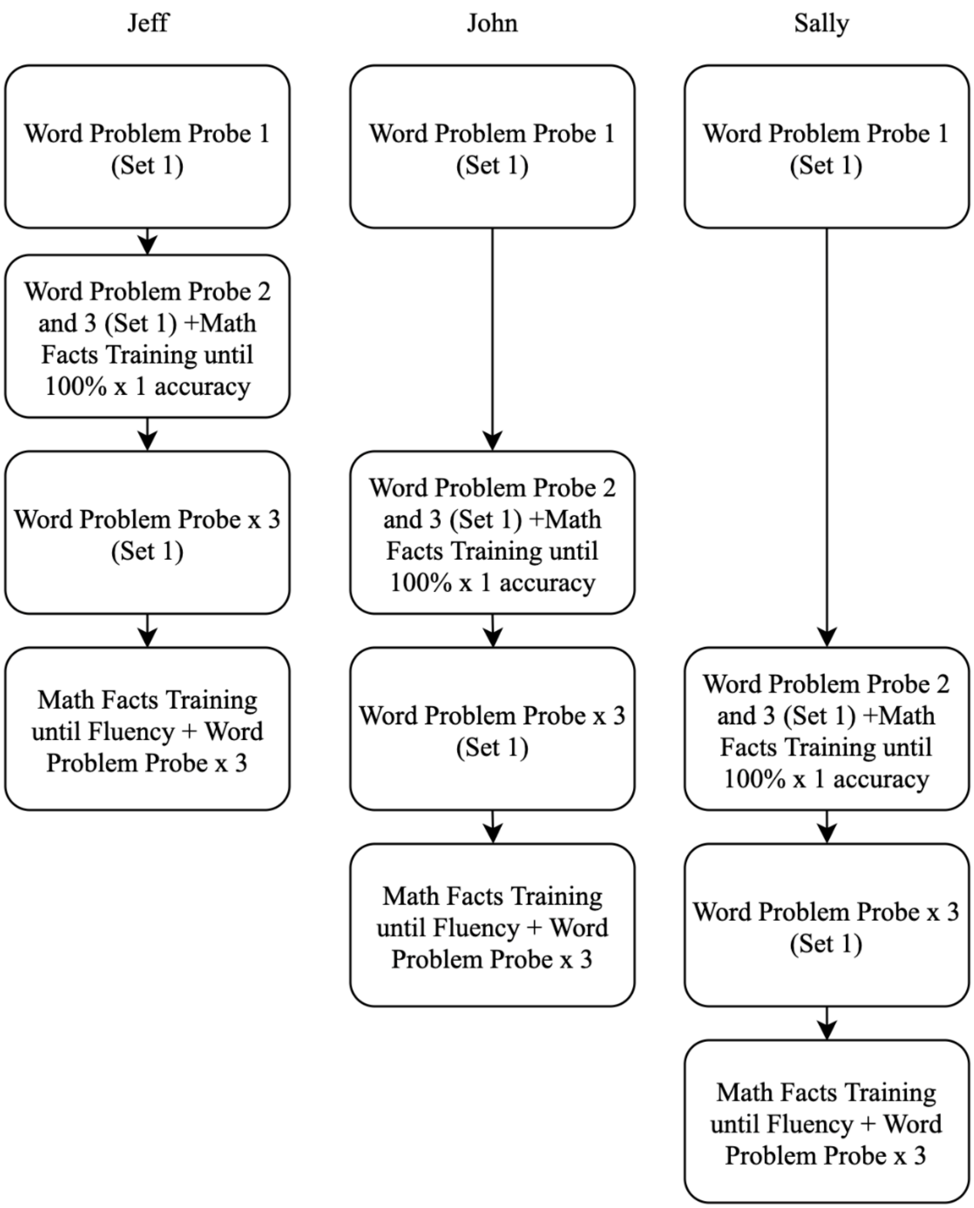
Experiment Procedure



Note. The figure demonstrates the experimental procedure for each participant. The experiment consisted of three parts: (a) initial TSF probes for both spelling and math operants, (b) the induction of TSF of spelling, and (c) the assessment for TSF across math facts with novel math facts.

Figure 2

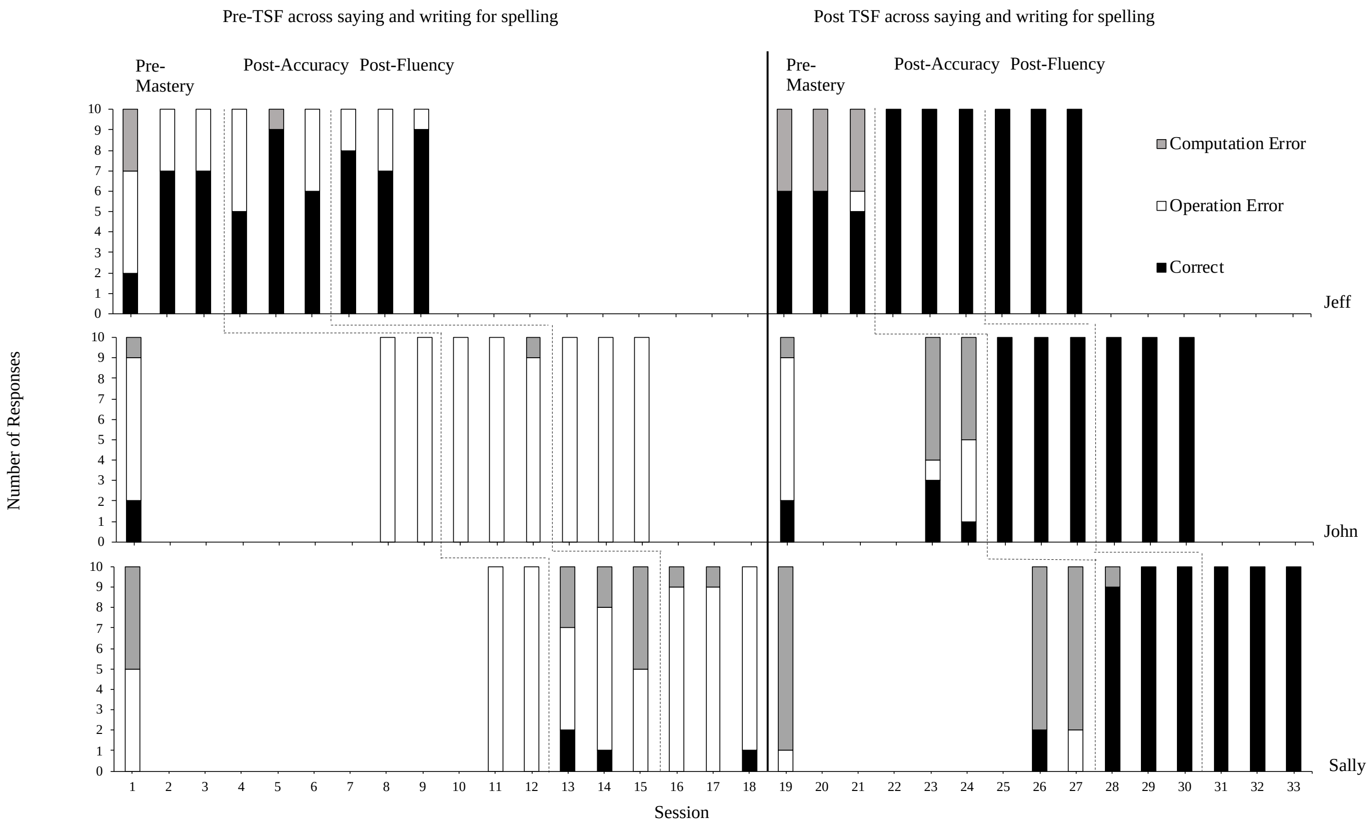
Experiment Procedure Within Pre- and Post-intervention TSF across Math Operants Probes



Note. The figure demonstrated the embedded multiple probe design utilized during the pre- and post-intervention TSF across math operants probes.

Figure 3

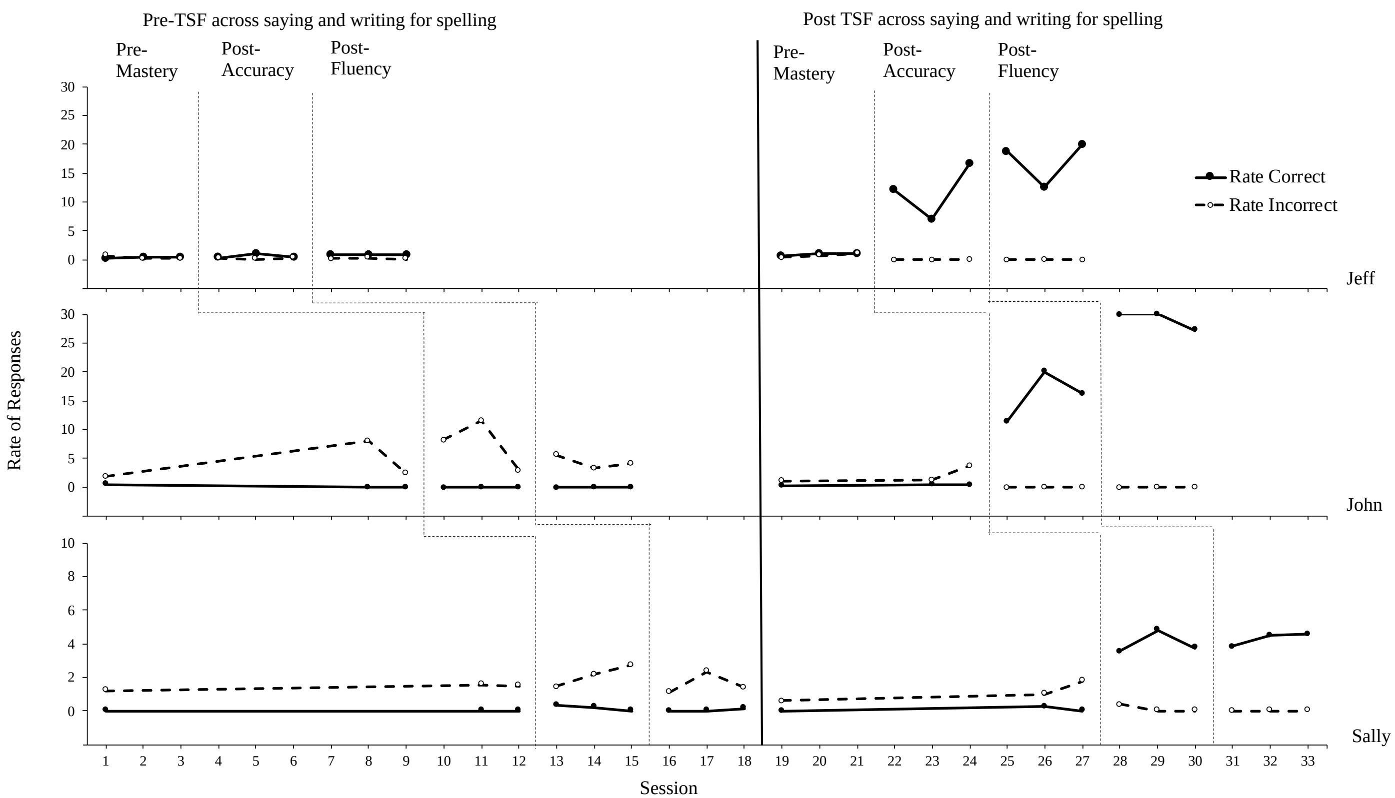
Number of Correct/Incorrect Responses Emitted to Word Problems



Note. The figure demonstrates the number of correct responses, incorrect responses with operation selection error, incorrect responses with computation error emitted by each participant to ten word-problems before and after receiving instructions on math facts. The left panel of the figure were reported in the previous study.

Figure 4

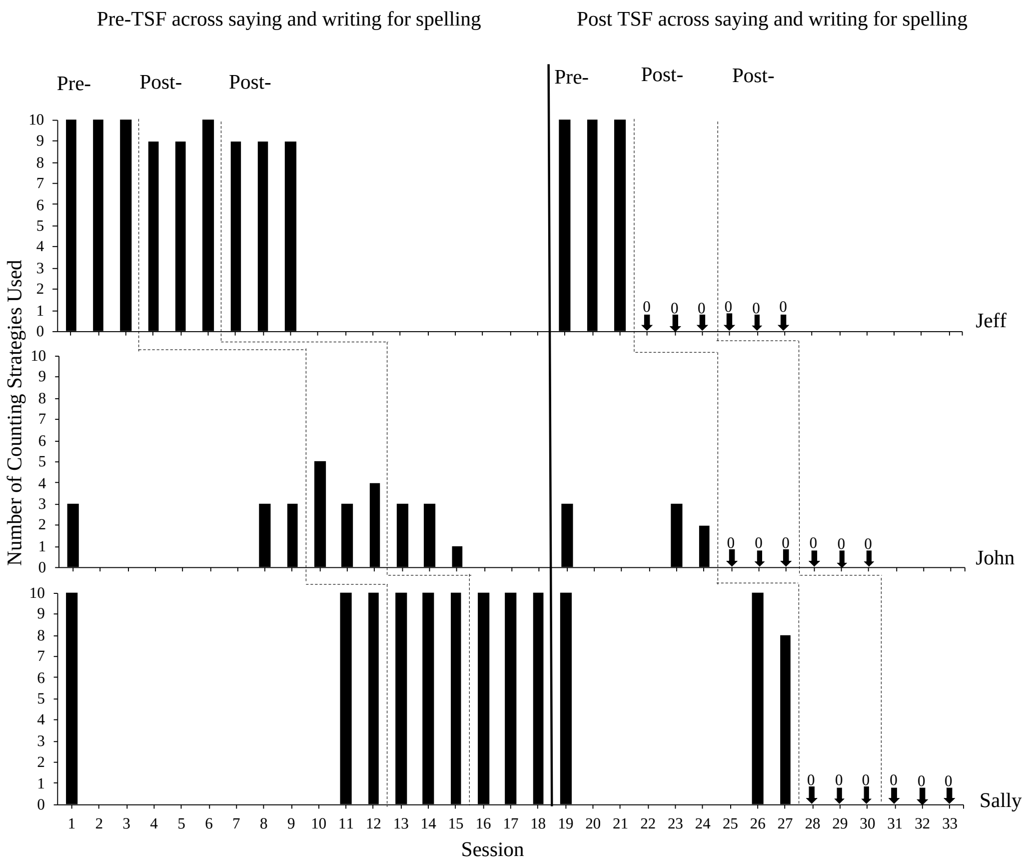
Rate of Correct/Incorrect Responses Emitted to Word Problems



Note. The figure demonstrates the rate of correct or incorrect responses emitted by each participant to ten word-problems measured as number of correct/incorrect responses per minute before and after receiving instructions on math facts. Data on the left panel of the figure were reported in the previous study.

Figure 5

Number of Counting Strategies Used for Word Problems



Note. The figure demonstrates the number of finger-counting, tallies, or other counting strategies used by each participant when solving ten word-problems before and after receiving instructions on math facts. Data on the left panel of the figure were reported in the previous study.

Appendix A.

Sample Math Fact Fluency Worksheet

____
Name

Morningside Math Facts Fluency
Multiply-Divide
6 7 42, 6 8 48, 7 7 49

New Facts 14
Fact Sheet

6)42	7 x 6	7)42	6)42	6)42	6)48	6 x 8	7)49	8)48	7 x 7	10
8)48	7 x 7	7)42	7)49	6 x 8	6 x 8	7)49	7)42	6 x 8	7)49	20
7 x 7	6)48	7 x 7	8)48	7)49	6)48	6 x 7	6)48	8 x 6	6)48	30
6)42	6)42	7)49	8)48	7 x 7	7)49	7 x 6	7)49	7 x 6	7)49	40
7)42	6 x 8	7)49	7 x 7	6 x 8	7 x 7	7 x 7	7)49	7)49	8 x 6	50
8 x 6	7 x 7	7)49	6 x 7	6)42	6 x 7	6 x 8	7)49	7)49	7)49	60
6 x 8	7)49	6 x 7	7)49	7)49	7)42	6 x 7	7 x 7	6 x 8	7 x 7	70
7 x 6	6)42	7 x 7	7)49	7)49	8)48	8 x 6	7)49	6 x 8	8)48	80
7 x 7	8 x 6	7 x 7	7)49	8 x 6	6)48	8)48	6 x 8	7)42	7)49	90
7 x 7	6 x 7	7)42	7 x 7	6 x 8	7)49	8 x 6	7 x 6	7)49	7)42	100

Appendix B.

Sample Word Problem Worksheet

1	Name	Date
	Cynthia is training for New York Marathon. She started off running 7 miles a day. How many miles did Cynthia run during the first 6 days?	Chidi likes to read books. He builds a bookshelf in his library to store all of his books. Chidi has 48 books and can fit 6 books on each shelf on the bookshelf. How many shelves does he need on his bookshelf to fit all of his books?
	Sally baked 42 cookies. Her class at school has 7 students, including Sally. If she gives the same number of cookies to each student, how many cookies will each student get?	John has 6 hens. Each hen lays 7 eggs per week. How many eggs can John collect every week?
	Eleanor ordered 6 pizzas for a Super Bowl party she is hosting at her apartment. Each pizza is sliced into 8 slices. How many slices of pizza does Eleanor have for her guests?	Charlie is planting peas in his garden for Thanksgiving dinner. He plans 7 rows of pea plants. Each row has 7 plants in it. How many pea plants can he plant?
	Nathan plays a video game for 42 minutes. One round lasts 6 minutes. How many rounds does he play?	Jianyu meditates for 48 minutes each day. When he meditates, he recites a phrase over and over again. He takes 8 minutes to say the whole phrase. How many times can he repeat the phrase when he meditates?
	For his holiday shopping, Charlie needs to buy 49 presents. If he buys 7 presents each week, how many weeks will it take for him to buy all he presents he needs?	There are 8 checkout lanes at the ACME nearby Sydney's apartment. When Sydney is ready to check out, there are 6 customers at each checkout lane. How many people are waiting to check out before Sydney?

CHAPTER IV

GENERAL DISCUSSION

One area of research in mathematics teaching that has been growing exponentially over the past decades is the investigation of teaching mathematical discourse (Ryve, 2011). Researchers found that building function into mathematical discourse, that is, teaching math as a verbal behavior, not only improves mathematical problem solving but also conditions mathematics as a reinforcer (Weber, 2016). From a verbal behavior developmental perspective, an individual is only fully verbal when they emit speaker-as-own-listener behaviors, that is, when individual functions as a speaker and as their own listener within one's skin (Greer et al., 2016). From a relational frame perspective, Barnes-Holmes et al. (2000) argued that verbal behavior involves some degree of arbitrary application of relations. One is not verbal, or the operant emitted is not verbal, if it is acquired through explicit contingencies (Barnes-Holmes et al., 2000). Thus, to teach students the verbal behavior of math, for students to be verbal in math, their listener and speaker responses must join to become speaker-as-own-listeners of math (Greer et al., 2005; Lodhi & Greer, 1989).

Major Findings

Across the first two experiments, we first tested the effects of fluency training of multiplication/division math facts on students' accurate and fluent responding to word problems. We found that all participants who demonstrated TSF across saying and writing for spelling words also demonstrated the transformation of fluent responding of math facts to word problems while participants who did not demonstrate TSF across saying and writing for spelling did not demonstrate such transformation. Participants who did not demonstrate TSF across saying and writing for spelling also persisted in the use of counting strategies (i.e., visual representations) throughout the experiment. This is consistent with the results and patterns observed of the pilot study conducted with addition/subtraction math facts.

We then tested the effects of fluency training of word problems, instead of fluency training on math facts, on the accurate and fluent responding to math facts. All participants demonstrated significant gain after they were taught to respond to word problems fluently. One difference between the intervention from Experiment I and that of Experiment II was the participation of verbal stimuli of English words presented in word problems in Experiment II. Combining this observation with the between-group pattern that only participants with TSF across saying and writing for spelling words benefitted from fluency training of word problems, we speculated that the use of verbal stimuli plays a crucial role in the transformation of stimulus function of computation skills (i.e., math facts) and problem-solving skills (e.g., word problems).

Experiment III then tested whether the onset of TSF across saying and writing for spelling led to the onset of TSF between computation skills and problem-solving skills. We found that upon acquisition of TSF across saying and writing for spelling words, all three participants demonstrated increase in their levels of responding to word problems after receiving fluency training for math facts. Following the demonstration of TSF across saying and writing for spelling words, all three participants emitted 100% accurate responses to word problems at an accelerated rate after receiving fluency training on math facts. Based on the findings across all three experiments, I suggest the presence of TSF across saying and writing, is a determining factor for the TSF from math fact fluency to word problems.

Verbal Stimulus Control

In educational settings, teachers utilize various approaches to build fluent responding to math facts, hoping to make that step of the problem-solving process automatic for students

(Boaler, 2015). When a student responds correctly to all steps of a problem except for the math fact, teachers, parents, and/or the student often contribute the source of the problem to not knowing/remembering the math facts. Sidman (2008) and Delaney and Austin (1998) referred to the behavior of remembering as stimulus control topographies established in the past that are perceptually unavailable. That is, when an individual emits a response while a discriminative stimulus is not readily available, they "remembered" the response. However, math facts rarely show up in the form that they were taught beyond students' initial encounter with math facts. A student will have to solve 2+3 that shows up in so many different forms and scenarios, as numbers, as words, as visual representations, to find the sum of money, distance, angles, lengths of edges, counts of various objects, etc. The one constant in all those scenarios is that they can all be translated to the verbal stimulus "two plus three."

Prior to the establishment of TSF across saying and writing, math fact 3 x 3 only functions as a textual stimulus to students who did not demonstrate TSF across saying and writing. Thus, the process of solving math facts is a process of visual match-to-sample. The participants solved their first encounter of 3 x 3 with counting strategies and the rest were all visual matching. They referred to the first instance that they solved and matched the answer. However, when 3 x 3 showed up in a different form such as in a word problem, there was no identical exemplar ready for matching. In Experiment I, some participants who did not demonstrate TSF across saying and writing emitted incorrect computation responses to word problems even after correctly translating the word problems to number sentences. Thus, the participants without TSF used counting strategies again even after mastering the math facts. The participants who demonstrated TSF across saying and writing, on the other hand, were under the verbal stimulus control of "3 x 3" printed on the page. Thus, despite the form of the math problem, the responses were joined under the stimulus control of the verbal stimulus for the participants who demonstrated TSF across saying and writing.

The use of verbal stimuli might also supply an explanation for the functional relation between TSF across saying and writing for spelling and TSF across math operants. Prior to the induction of TSF across saying and writing for spelling, participants were visually matching their written spelling of a word to a printed word (e.g., “apple” printed on the teacher’s whiteboard and “apple” that they are writing down on their whiteboard). When asked to spell the word vocally when the textual stimulus is no longer available as an exemplar, the participants who did not demonstrate joint control of verbal and textual stimuli did not vocally spell the words correctly. To induce TSF across saying and writing for spelling, we established the mutually entailed relation among printed words, written words, and spoken words through rapid rotation across different operant and topographies (i.e., writing and vocally spelling) during MEI. By establishing this mutually entailed relation for spelling words, we also established a similar relation among spoken numbers, printed numbers, and written numbers to be under the joint stimulus control of spoken numbers. Thus, when the participants translated a word problem to a number sentence, they did not need textual exemplars or additional counting strategies for them to solve the number sentence.

Cross-Modal Relations

Existing literature also suggests that individuals acquire and demonstrates cross-modal relations such as the vocal-auditory-visual-relation that was present in these studies faster than responses with one modality such as the visual-visual relation for participants who did not demonstrate TSF across saying and writing (Arntzen, 2004; Belanich & Fields, 1999; Dye et al., 2010). By demonstrating or acquiring TSF across saying and writing, the participants demonstrated the joined stimulus control across different response modalities, thus equivalent stimulus classes emerged at a faster rate without requiring additional instructions.

Skinner (1957) defined "problem-solving" as the process where "the speaker generates stimuli to *supplement* other behavior already in his repertoire" (p. 442). For participants who engages in visual-visual relations during problem-solving, those supplementary behaviors for problem-solving were the use of counting strategies to re-produce a textual exemplar so that they can visually match their answer to the textual exemplar. For participants who engages in auditory-visual relations, the process can be as simple as saying the number sentence, overtly or covertly, to supplement the behavior of solving a math fact problem, which was already in repertoire as a result of math fact fluency training. It is apparent that although engaging in both visual-visual relations and auditory-visual relations could eventually produce the same answer, the auditory-visual relation is more efficient and allows for a wider range of applications (e.g., spelling, math).

Educational Implications

The results of the current studies are educationally significant as they revised the interpretation of previous findings on the relation between fluency training and problem-solving by providing a new perspective. Instead of examining what prerequisite skills allow students to perform new, more complex skills, we should first look at the essential stimulus control that makes the target behavior possible. More specifically, the results of the experiments showed that when verbal stimulus control is present, the participants applied acquired math facts to solve other types of math problems involving the same types of math facts. Therefore, this suggests that it is important for students to acquire verbal stimulus control at early stages of math education. This allows teachers to optimize the allocation of instructional time to target the introduction of new math concepts instead of re-teaching acquired math concepts every time when they are presented in new forms. If the students do not demonstrate TSF, teachers and practitioners should also present instructions that rotate between written and spoken responses as much as possible during instruction to provide students with an MEI experience so that different math responses can fall under the same verbal stimulus control.

In addition, the results of Experiment II showed that whether the participants demonstrated TSF across saying and writing for spelling or not, they all benefitted from fluency training of word problems. Instead of allocating instructional time for individuals to practice math facts, fluency training for word problems, whatever math operants students are currently learning, or a mixture all learned math operants can be beneficial for students in terms of the fluent responding to math facts and the maintenance of learned operants. This allows students to acquire new math facts through learning the function of them. “Why am I learning this?” or “When am I going to use this?” are probably the two most asked questions in a math class (Schwartz, 2006). The lack of function (i.e., embedded reinforcement for solving real world application) of mathematical concepts has prevented students from acquiring and loving math (De Corte et al., 2000; Meyer et al., 2001). By building function to fluency training, students can learn 2+3 as finding the sum of two measures through various mathematical problems instead of learning 2+3=5 as a math fact.

Limitations and Future Research

One major limitation of the study concerns the limited number of participants. Four students who demonstrated TSF across saying and writing and four participants who did not demonstrate TSF across saying and writing participated in the first two experiments. Although the between-group comparison showed differences in their levels of responding, we cannot conclude anything with statistical significance. Homogeneous sampling of the study also limited the external validity of the study. Future research should involve participants in different educational settings and age groups to replicate the procedure to improve the external validity of the current study.

In addition, although we conducted some generalization probes, we did not examine the long-term impact on the students' rate of acquisition for math operants. We observed an increase in the participants' rate of acquisition of math facts after the induction of TSF across saying and writing. However, more data over a longer period across various math operants need to be collected to demonstrate any long-term effects. Future research should collect pre-TSF and post-TSF data across various math objectives counterbalanced across participants and calculate the rate of acquisition to demonstrate long-term effects of the induction of TSF across saying and writing and the participants' acquisition of other math operants.

In another limitation, we did not examine how the onset of other cross-modal capabilities might have affected the participants' response to word problems. As discussed above, individuals acquire cross-modal relations faster than relations of the same modality. Other than TSF, Bidirectional Naming (BiN) is another cross-modal capability that accelerates one's language acquisition (Kobari-Wright & Miguel, 2014; Lee et al., 2015; Miguel et al., 2008). MEI was also an effective strategy to induce BiN (Fiorile & Greer, 2007; Gilic & Greer, 2011). The onset of BiN might then be a mediating variable in our investigation of TSF for math operants. The utilization of MEI might have accidentally induced the incidental learning of object-name relations, or BiN, for the participants and is eventually responsible for the change in their responding. Future research should then test for other cross-modal cusps or capabilities and examine whether they also demonstrate correlation or functional relation with TSF across math operants.

Furthermore, for the current study, we matched the participants based on their rate of responding to math facts prior to the study. The dyads stayed the same during Experiment 2. Such matching criterion did not account for the discrepancy between participants' initial responding to word problems or math facts. Future studies can explore matching participants based on their rate of responding to initial probes or various other matching criteria.

In the current study, we found that the induction of TSF across saying and writing for spelling resulted in the presence of TSF across math operants for participants like these. We chose to induce TSF across saying and writing first based on our speculation of the important role verbal stimuli played in the process of TSF across math operants. However, the induction of TSF across saying and writing might be a sufficient but not necessary condition for TSF across math operants (Kleinert, 2018; Lo, 2016; Miguel et al., 2008; Morgan et al., 2020). For us to conclude that TSF across saying and writing for spelling is functionally equivalent to TSF across math operants, future research needs to induce TSF across math operants for participants and examine if that results in the induction of TSF across saying and writing for spelling as well.

Conclusion

The results of these experiments add to the existing, on-going discussion on the discrepancy between students' acquisition of math facts and their performance on mathematical problem-solving. Experiment I demonstrated a correlation and between the presence of TSF across saying and writing for spelling and the TSF from math facts to word problems. However, the results of Experiment II showed that when teaching word problems to fluency, whether the participants demonstrated TSF or not, they demonstrated significant increases in their responding to math facts. One difference between the intervention of the first two experiments was the participation of verbal stimulus. This opens the possibility that whether the students apply acquired math facts to other types of math problem solving depends on whether verbal stimuli participated in the process of acquiring math operants. Experiment III confirmed the hypothesis by demonstrating a functional relation between the establishment of TSF across saying and writing for spelling and the establishment of TSF across math operants. Therefore, this research has implications on the organization of the instructional sequence and delivery of instructional materials. Instead of teaching math facts as standalone concept, we should teach it as part of problem-solving procedure ensuring that the motivating conditions are acquired along with the function (i.e., reinforcement). Vocal responses should also be involved in the teaching process to facilitate the joining of listener and speaker responses in math, to make math verbal.

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