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Improved Graph Model for Interdependent Gas and Electricity Critical Infrastructures

Jesus Beyza and Hector F. Ruiz
PGIIE
Instituto Tecnologico de Morelia
Morelia, Mexico
jbbcia4@hotmail.com, hfruiz53@yahoo.com.mx

Jesus S. Artal-Sevil and Jose M. Yusta
Departamento de Ingenieria Electrica
Universidad de Zaragoza
Zaragoza, Spain
jsartal@unizar.es, jmyusta@unizar.es

***Abstract*—Interdependence between gas and electricity transmission networks is a subject of concern due to the expanding use of gas for electricity generation in combined-cycle power plants around the world. This paper proposes a novel and much more accurate representation of natural gas and electrical networks based on graph theory, which includes all the assets of both systems and their couplings and offers a more realistic topological model of the two coupled networks. The representation is proposed as a scale-free graph and is mathematically validated in test networks, finding that the representations maintain the same characteristics of traditional graphs, but with more topological detail of the infrastructures.**

***Index Terms*—Cascading failures, critical infrastructures, gas network, graph theory, power grid, vulnerability.**

I. INTRODUCTION

Electricity and natural gas networks are essential for the daily operation of the critical infrastructures of any country. The loss of functionality in either of these two systems would have devastating consequences for the economy and the quality of life of the population. In recent years, the growing development of natural gas transmission and distribution infrastructure, both for domestic and industrial supply, as well as for electricity generation in gas-fired power plants, has increased the likelihood that a problem in one infrastructure drastically affects the other.

On the one hand, natural gas infrastructure is composed of compression stations, pipelines, underground gas storage facilities, drilling platforms, and so on. Some of these facilities depend on a safe and reliable electricity supply for their operation. On the other hand, electrical infrastructure shows a high dependence on natural gas networks because an important share of the electricity production is obtained from natural gas combined-cycle power plants. Thus, to satisfy the optimum operation of the electrical system in terms of safety and reliability, natural gas networks must have enough capacity to facilitate the supply of gas to the combined-cycle power plants [1], [2]. Therefore, there is a growing mutual dependence between the two infrastructures.

The close relationship between both systems is increasing the potential risk of catastrophic events [3]. An example that illustrates this dependence took place on February 2011, in the southwest of the U.S, when extreme temperatures caused problems in the natural gas extraction due to freezing of storage wells, which caused significant pressure drops in the gas network and reduced the gas availability for combined cycle power plants. This event caused a power outage to 4.4 million customers and affected the gas compressors, which worsened the situation [3].

This article proposes a novel representation of both infrastructures for the first time, modelling the interaction between them with a greater level of topological detail than the proposals made so far and incorporating all the assets as nodes of a scale-free graph. This type of network is a graph in which several nodes are highly connected, which means that they have a certain number of edges (links) with other nodes [4], [5]. The importance of representing the electricity and gas infrastructures as scale-free networks is that this type of graph is very useful for studying the vulnerability of the network against failures, which allows us to characterise its basic properties, i.e., its resilience against random errors and deliberate attacks [6]. Also, these representations are useful for analysing the inherent structural characteristics of the two networks, since they use little model information, do not require specific modelling software, and need short computational time. The above allows multiple studies to be carried out according to the interests of researchers, as can be found in [7]–[10].

The rest of this paper is organised as follows: Section II reviews studies in the literature on the joint operation of gas and electricity infrastructures. Later, Section III presents a novel proposal for representing coupled gas and power networks. Finally, Section IV provides an example with larger test networks and Section V summarises the main conclusions of this paper.

II. ANALYSIS MODELS OF COUPLED ELECTRICITY AND GAS NETWORKS

In recent years, researchers have been interested in analysing the interdependencies in critical infrastructures, thus establishing a new area of study through the application of different analysis techniques for coupled networks [11]. Electricity and gas transmission systems can fail not only due to the complexity of their technical operation but also due to the interdependent relationships that bind them together. The interdependencies can contribute to the loss of joint operability of the coupled systems and also amplify the impact of small perturbations of one system on the other. To model the complex behaviour of electricity and gas systems, different strategies for representation of the assets of both networks have been developed, which have economic as well as technical perspectives [12].

On the one hand, economic models analyse how the perturbations propagate and how to implement effective preventive measures. These models measure the interdependencies through their economic relationships and are useful for macroeconomic analysis due to natural hazards, malicious attacks or accidental events. Some examples of this model have been focused on energy markets due to competition in the generation and commercialisation of electrical energy and the impact of combined-cycle plants and the electric network on the gas infrastructure [2], [13], [14].

On the other hand, technical models analyse the interaction by using load flows, where specific events on the coupled networks are studied, which requires the use of equations and physical parameters that describe the joint behaviours of both systems. In these studies, the electricity and gas systems are represented by a simple network that is composed of nodes and edges that associate some assets of the infrastructures [15], [16]. Other proposals represent the effects of the compressors, fuel types and limitations, generation capacity and start-up characteristics of the gas-fired power plants [17]. Most of the mentioned references use simulation and optimisation models because they study the behaviour of coupled networks under a condition, which involves the calculation of the nodal pressures and flows in the pipelines through complex solution methods.

More recently other methods propose using graph theory for the representation of the infrastructure through nodes and edges. This technique proves suitable for modelling the topological properties of large complex systems, but until now, it has been rarely used for studying the case of electricity and gas infrastructures. The surge of this technique is mainly due to the reduced use of technical parameters in the models. However, the representation of electrical and gas networks by graphs is performed in most cases in a very simple way because it excludes important components in the operations of both systems [18], [19]. Models should have a greater level of detail of the network assets and allow their scaling to real energy systems. Therefore, this work argues that the topological representations by the use of graph theory can be a very useful tool because they allow the analysis and visualisation of the physical behaviours of these two systems, with a minimum amount of information, short computational time, and without special hardware or software, which facilitates the analysis of multiple scenarios with great flexibility according to the analysts interests.

III. PROPOSAL FOR REPRESENTATION OF ELECTRICITY AND GAS INFRASTRUCTURES

This section proposes a novel way of representing natural gas and electricity infrastructures with greater topological detail, incorporating the assets of the systems as nodes in a scale-free graph [4], [5].

Mathematically, electricity and gas infrastructures can be represented as a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ that comprises a set of $\mathcal{V}$ nodes or vertices, which represent the infrastructure components, and a set of $\mathcal{E}$ edges or lines, which represent the interactions or relationships between the nodes or components [20]. The edges correspond to a set of i, j pairs such that $i, j \in \mathcal{E}$.

Most authors simplify the representation as a simple coupled network in which the electrical substations and some assets in the gas network correspond to the nodes of the graph and the electrical lines and gas pipelines correspond to the edges [18], [21]; however, this representation is incomplete because it does not consider all the assets of the coupled system. In our proposal, a more complete original model is developed for the first time, which considers all of the possible assets and can be applied to real energy infrastructures. The proposed models of both networks and their coupling are described below.

A. Electrical infrastructures

Electrical infrastructure is traditionally represented as a graph that is composed of nodes and edges, where the nodes represent connection points between two or more electrical components, and each edge represents a transmission line or an electrical transformer [21], [22]. In this type of network characterisation, the assets connected to the substations (generators, loads, and so on) are not considered in the scale-free graph but are implied in a single node. Fig. 1 (b) shows the traditional topological representation of the four bus network of Fig. 1 (a), where only the substations are considered to be nodes of the graph, and the electrical lines are the edges.

Fig. 1 (c) represents the novel proposal of this paper. In our topological model, the graph representation of the same network of Fig. 1 (b) is now composed of a scale-free graph formed by 14 nodes and 14 edges, instead of the four nodes and four edges. The scale-free graph considers the power lines as assets, which are represented as nodes of the graph and not as edges. The transformers are connected between buses in a substation, and the loads, as well as the capacitors and reactors, are also considered to be nodes of the graph.

B. Natural gas infrastructure

Natural gas infrastructure is also usually described as a graph that is composed of nodes and edges. The network interconnection points are represented by nodes in the graph, whereas the compressors and pipelines are represented by edges [18]. Fig. 2 (b) shows the basic representation of the 11-node gas network of Fig. 2 (a). This hydraulic representation includes the demands, pipelines, compressors, and so on, and its topological equivalent has only nodes and the edges that connect them. Gas supplies and demands are not considered.

Fig. 2 (c) represents our topological proposal for the gas network as a scale-free graph. In this topological model, the same network is composed of 32 nodes and 35 edges. The scale-free graph now considers the pipelines as nodes of the system and not as edges of the graph. Also, the demands,

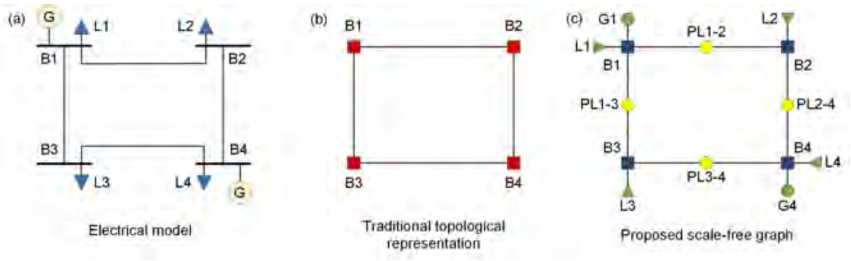

Fig. 1. Electrical network traditional graph versus proposed electrical network graph.

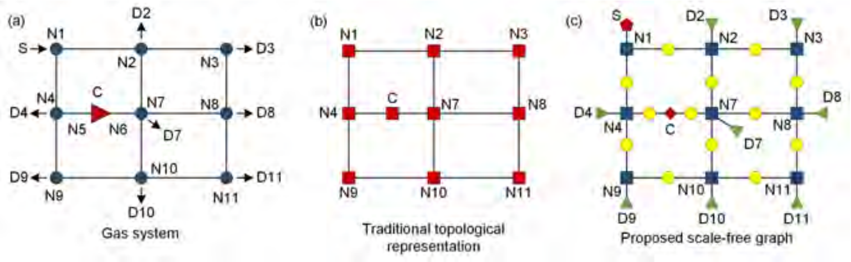

Fig. 2. Gas network traditional graph versus proposed gas network graph.

supplies and compressors are now considered to be nodes of the graph.

The gas system represented with the traditional model in Fig. 2 (b) is straightforward because it does not consider all of the assets of the real gas infrastructures. However, the model proposed here, in the Fig. 2 (c), allows all these assets to be taken into account.

C. Coupling between infrastructures

The coupling between both systems is conducted through the interactions between certain facilities of the two networks. On the one hand, the combined-cycle plants that use natural gas to generate electricity. On the other hand, through the electrical energy supply for the operation of the gas network compressors.

In Fig. 3 our novel coupling proposal is represented, where the scale-free graphs of Figs. 1 (c) and 2 (c) are considered. The natural gas combined-cycle plants implied in nodes *G1* and *G4* of the electrical network are fed from nodes *N2* and *N8* of the gas network, respectively. Likewise, compressor *C* operates through an external power supply provided by a substation *B2*. The resulting network is composed of 49 nodes and 55 edges, instead of 14 nodes and 19 edges of the traditional representation.

Our proposal considers couplings as nodes, where *G1-N2* and *G4-N8* couplings represent pipelines that transport gas to generators and *B2-C* coupling represents power lines that transport electricity to compressors. This novel proposal for a scale-free graph offers a more realistic topological model of both coupled networks.

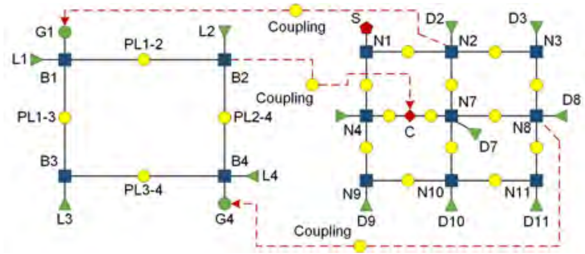

Fig. 3. Electricity and natural gas coupled scale-free networks.

D. Validation of the proposed representations as scale-free networks

Once the proposed topological networks have been shown, it is necessary to validate these representations as scale-free graphs. Firstly, in Fig. 4 the nodal degree distribution for the coupled network of Fig. 3 is presented, i.e., the probability that a randomly chosen node has exactly k connections. The nodal degree distributions for each of the individual graphs are also represented (Figs. 1 (c) and 2 (c)).

Note that in Fig. 4, the probability that a node of the coupled scale-free graph has a connection degree $k=2$ is $P(k)>0.40$, which means that more than 40% of the nodes have only two connections with the other nodes of the network. However, close to 12% of the nodes have a connection degree of $k=3$ (compression stations fed by an electrical supply, or substations in the electrical network where three transmission lines converge). The value $k=2$ corresponds to the representation in the graph of the pipelines or electrical transmission lines. The value $k=1$ refers to the nodes in the graph that have only one link, a situation that usually corresponds to generators, load, or gas supplies.

In the case of individual gas and electricity networks (Figs.

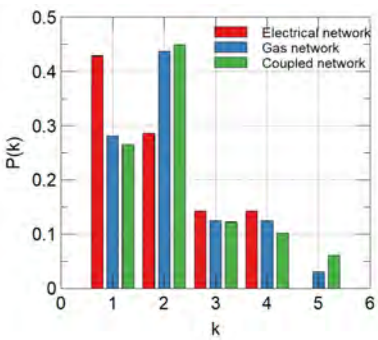

Fig. 4. Nodal degree distribution for the electricity and gas coupled network.

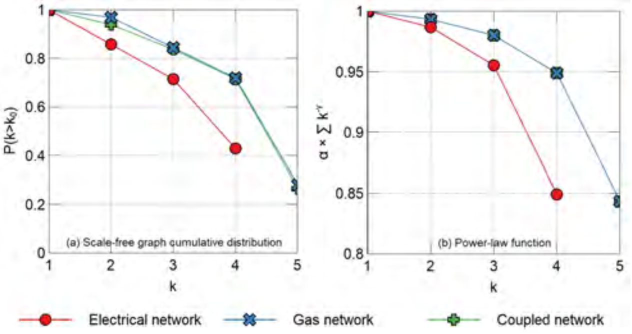

Fig. 5. Graphic representation of the nodal degree cumulative probability function.

TABLE I
PEARSON'S CORRELATION COEFFICIENTS BETWEEN CUMULATIVE PROBABILITY FUNCTIONS.

Network	**Electrical**	**Natural gas**	**Coupled**
Pearson's correlation	0.9722	0.9944	0.9954

1 (c) and 2 (c)), for nodal degree *k=1*, the electrical network presents a distribution of nodal degree *P(k)>0.40*, versus *P(k)>0.28* for the gas network, which means that the former network has more assets with only one edge for the same number of nodes. In addition, the proportion of nodes that have only one edge in the electrical network is greater than for the other nodal degrees because the network is not as meshed. Meanwhile, for *k=2* the natural gas network presents a *P(k)>0.40*, which indicates that it has a higher proportion of assets with two edges, versus *P(k)>0.25* for the electrical network. However, the gas network has nodes with connection degrees of up to *k=5*, in comparison with *k=4* of the electrical network, and thus, in both cases, such nodes are essential in the robustness of their respective networks.

To demonstrate the suggested approach, a comparison between the results obtained by an cumulative distribution and an analytical equivalent of power-law, as specified in the graph theory, is presented below [4], [23].

The cumulative distribution comprises the sum of the probabilities associated with the nodal degree's distribution of Fig. 4. The calculation of the cumulative probability is performed for a range $k_i \in$ [1, k_{max}]. Conceptually, it is the probability that a randomly chosen node has more than k_0 edges.

$$P_{cum}(k_i) = P(k_i \geq k_0) = \sum_{i=1}^{k_0} P(k_i) \tag{1}$$

The power-law is obtained analytically, where the probability value is proportional to $k^{-\gamma}$ with $\gamma = 3$, taking into account that the result $P_{cum}(k_i) \in$ [0,1] should be normalised. Equation (2) represents the cumulative probability of the nodal degree distribution as a power-law, where the coefficient α allows normalising the expression in the range $P_{cum}(k_i) \in$ [0,1], with $k_i \in$ [1,k_{max}], $k_0 \in$ [1,k_{max}].

$$P_{cum}(k_i) = P(k_i \geq k_0) = \underbrace{\sum_{i=1}^{k_{max}} k_i^{-3}}_{\alpha} \cdot \sum_{i=1}^{k_o} \frac{1}{k_i^3} = \alpha \cdot \sum_{i=1}^{k_0} \frac{1}{k_i^3} \tag{2}$$

Fig. 5 contains the comparative results of the nodal degree cumulative distribution (1) and the power-law function (2) for Figs. 1 (c), 2 (c) and 3. In both results, it can be observed that $P(k \geq 1) = 1$, which means that all nodes in the graphs have more than one connection. In addition, the probability of having nodes with over two connections $k \geq 2$ is high in all cases, with a probability close to 1, and so on with the other nodal degrees $\forall$ $k_i \in$ [1, 5].

To confirm the great similarity observed between the curves of Figs. 5 (a) and (b), presented in Table I is the calculation of the Pearson's correlation coefficient between both series of values; this coefficient is obtained by the ratio between the covariance of two variables and the product of their standard deviations.

In all cases, given that the coefficient is close to one, it can be inferred that the two curves are highly correlated. Therefore, the argument made in this work for considering the electricity and gas networks as a scale-free graph is correct because the power lines, transformers, compressors, gas demands, and supplies, among others, are also nodes in the network topologies.

IV. NODAL DEGREE DISTRIBUTION IN THE IEEE TEST NETWORK AND NATURAL GAS NETWORK

To analyse the representations proposed in this work, larger test networks are chosen, and a comparison is performed with the traditional models proposed in the literature. The electrical network is represented by the IEEE-30 bus test system and the gas network by a 22-node gas test network [24], [25].

In Fig. 6, the traditional graphs are shown compared with the representation proposed in our work. The IEEE-30 bus test network is composed of 30 buses, 41 transmission lines, 6 generators, 21 loads and 2 capacitors. The traditional representation only takes into account the 30 buses and the 41 transmission lines. In contrast, our topological model considers 100 nodes and 111 edges, which constitute all of the network assets. The natural gas network is composed of 22 nodes, 35 pipelines, 18 loads, 3 compressors and 1 supply. The

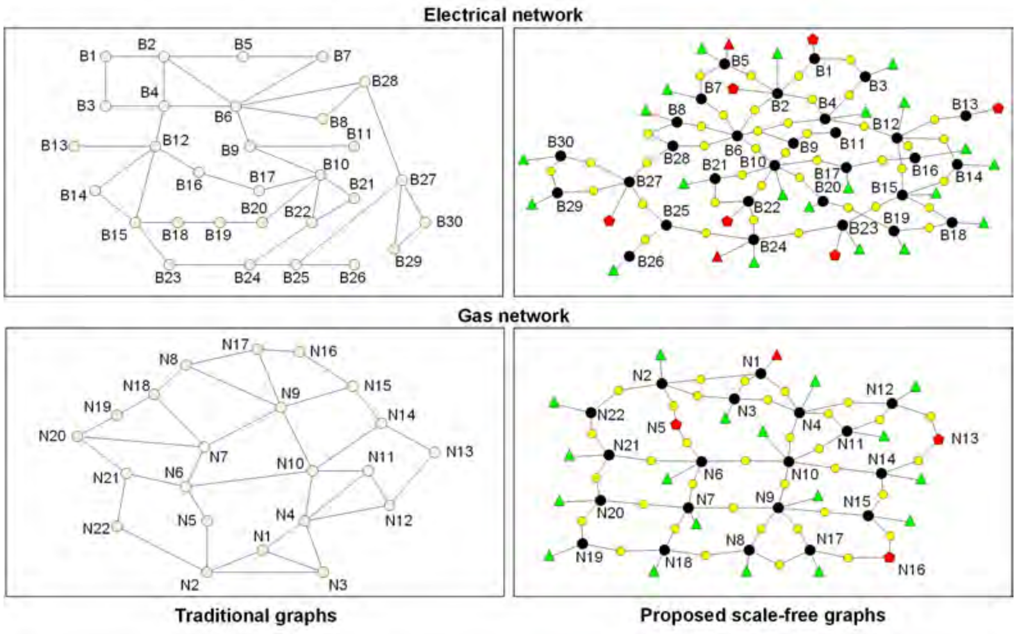

Fig. 6. Comparative analysis of traditional graphs vs proposed scale-free graphs.

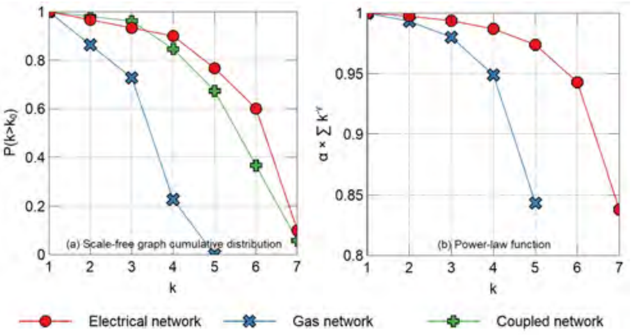

Fig. 7. Graphic representation of the nodal degree cumulative distribution and power-law function in the traditional graphs.

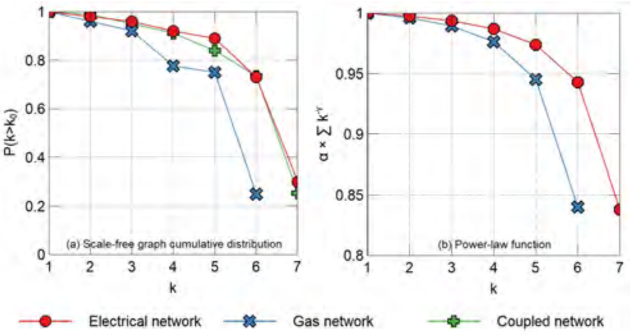

Fig. 8. Graphic representation of the nodal degree cumulative distribution and power-law function in the proposed scale-free graphs.

traditional topological representation considers one graph of 22 nodes and 35 edges, whereas the novel representation as a scale-free network proposed in this paper takes into account all of the assets, which form a graph of 76 nodes and 89 edges.

Now, consider that the electrical generators coupled to substations B22, B23 and B27 of the electrical network are natural gas combined-cycle plants that are fed from nodes N3, N2 and N12 of the gas network, respectively, and that compressors N5 and N13 operate through an external electrical supply provided by substations B24 and B29 of the electrical network, respectively. This coupling constitutes five new nodes that represent the interactions between these two networks, i.e., three nodes that represent the dependence of the electricity network on the gas network, and two nodes that indicate the dependence of the gas network on the electricity network.

Following the validation procedure proposed in Section III-D, Figs. 7 and 8 contain the comparative results of the nodal degree cumulative distribution and the power-law function for the traditional and proposed graphs in Fig. 6. The results show that the probability with k= 1 is equal to 1 in both figures, so all nodes have at least one edge. Similarly, for k=2, the nodes have a degree distribution close to 1. For the other nodal degrees k, the probability of degree distribution decreases.

On the other hand, Table II shows Pearson's correlations between Figs. 7 (a) and (b), and 8 (a) and (b). Note that the correlation coefficients tend to +1 in both cases, so it is possible to claim that the proposed scale-free graph still keeps the same characteristics as the traditional graph, but with more detail of the infrastructure. Likewise, the improvement percentages of 0.5%, 9.1%, and 5.5% in the correlation demonstrate that the proposed representations are better in terms of scale-free graph properties.

The importance of representing these infrastructures as a scale-free graph is because this network is more similar to the

TABLE II
PEARSON'S CORRELATION COEFFICIENTS BETWEEN THE CUMULATIVE PROBABILITY AND POWER-LAW FUNCTIONS FOR FIGS. 7 AND 8.

Network	**Electrical**	**Natural gas**	**Coupled**
Traditional graph	0.9937	0.9054	0.9457
Proposed graph	0.9990	0.9879	0.9981
Improvement percentage	0.5%	9.1%	5.5%

reality of the systems. Therefore, this proposal may facilitate vulnerability and resilience studies, and comparisons between topologies for robustness analysis, among other works. The studies using the proposed scale-free graphs are advantageous since they require few technical information on the systems and low computational cost, which also allows their application to real energy infrastructures by using only the network topology. The proposed approach already supports the additional research of the authors. Some other works have been developed based on the representation developed in this article [7]–[10].

V. CONCLUSIONS

This article has proposed a topological representation closer to the reality of interdependent electricity and gas infrastructures, emphasizing that power lines, transformers, electrical and gas loads, pipelines, among others, should make up nodes in the coupled network. Likewise, the proposed graph has considered as nodes the links that form the coupling between the two networks, i. e. the pipelines that transport the gas to the generators and the electrical lines that carry the power supply to the compressors. The representations have been mathematically validated by the nodal degree cumulative distribution and power-law function, finding that the proposed networks are better than traditional graphs, not only because they have more detail of the assets but also because they have better validation as scale-free graphs.

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