# Paper <br> Optimization of Gyrotron Resonator's Dimensions 

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#### Abstract

This paper explains the procedure of determining the initial dimensions of a gyrotron resonator. In particular, the paper discusses which geometrical parameters impact the wave properties of the resonator. The solution is implemented using Matlab software and estimations are performed with the use of the SMath Studio spreadsheet.


Keywords-gyrotron design, cold cavity, Matlab code.

## 1. Introduction

Gyrotron-related theory and equations have been known since the 1960s [1]-[4]. Numerical computation methods and computer technologies have evolved significantly since that time. Simulation software, such as CST Microwave Studio, is capable of performing EM field calculations pertaining to any structure. However, due to fact that every computer simulation consists of input data and produces some output data, two questions are always valid: "what are the proper input data?" and "are the output results in agreement with the laws of physics?". In this paper, we will provide a Matlab numerical solution based on the cold cavity approach in order to obtain the initial dimensions of a gyrotron resonator. The proposed approach may be justified by the simplifications introduced to the equations, which, in turn, make computer-based implementation not excessively complicated. Further geometry optimizations may be performed using commercial, full-wave, microwave simulation software, but a starting point needs to be defined that is close to the optimal solution. The results obtained using CST Microwave Studio are much closer to actual experiment outcomes than those offered by simplified mathematical models, but are more computationally expensive. Therefore, the cold cavity approach and full-wave simulation are two complementary methods that are relied upon jointly.

## 2. Cold Cavity Formalism

In the presented approach, the cold cavity model is used to obtain the initial geometry of the resonator's microwave structure.

The formula is developed from the string equation (stationary Schrodinger equation):

$$
\left\{\begin{array}{l}
\frac{d^{2} f}{d z^{2}}+k_{z}^{2}(\omega, z) f=0  \tag{1}\\
k_{z}^{2}(\omega, z)=\left(\frac{k}{c}\right)^{2}-k_{\perp}^{2}(z), k_{\perp}=\frac{v_{m, k}}{R(z)}, k=\omega \sqrt{\left(1+\frac{i}{Q}\right)}
\end{array}\right.
$$

When substituting $k$ and $k_{\perp}$ with $k_{z}$ followed by substituting $k_{z}$ in Eq. (1), a differential formula for the cold cavity model is obtained:

$$
\begin{equation*}
\frac{d^{2} f}{d z^{2}}+\left[\left(\frac{\omega}{c}\right)^{2}\left(1+\frac{i}{Q_{d i f f}}\right)-\left(\frac{v_{m, k}}{R(z)}\right)^{2}\right] f=0 . \tag{2}
\end{equation*}
$$

This describes the RF field profile as a function of the $z$ dimension of the resonator. The longitudinal field profile $f$ is important for a few reasons. It is desired for the EM field to create a stationary wave that fulfills the boundary conditions and suffers from small diffraction losses, so that the overall microwave efficiency is high. There is also a second very important factor that requires knowledge of the EM field profile - electron cyclotron resonance. This is the mechanism for transferring energy from electrons to the EM field. It is desired that electron orbits and speeds of their travel in the $z$ direction be chosen in such a way that most electrons transfer as much energy as possible to the microwave field, and then leave the resonator. To optimize these two factors which have a direct impact on gyrotron efficiency, Eq. (2) is crucial.
The boundary conditions for the stationary state are expressed as:

$$
\left\{\begin{array}{l}
\left.\frac{d f}{d z}\right|_{z_{\text {in }}}-i k_{z}\left(z_{\text {in }}\right) f\left(z_{\text {in }}\right)=0  \tag{3}\\
\left.\frac{d f}{d z}\right|_{z_{\text {out }}}+i k_{z}\left(z_{\text {out }}\right) f\left(z_{\text {out }}\right)=0
\end{array}\right.
$$

The first part $\left(z_{i n}\right)$ of Eq. (3) describes the input boundary condition for the left-hand side of Fig. 1. This boundary condition represents the total internal reflection of the EM wave. Moreover, electric field energy is not propagating in the $-z$ direction and, therefore, this boundary condition is that of an evanescence wave. The input boundary condition is used to define $\frac{d f}{d z}$ at the $z_{i n}$ - starting point for integration of the model Eq. (2). The second part ( $z_{\text {out }}$ ) describes


Fig. 1. Longitudinal profile of the resonator (along $z$ axis).
the minimum reflection boundary conditions that are required for the EM wave to leave the resonator. The output boundary condition may be expressed as Eq. (4). $|R|$ may be referred to as the reflection coefficient.

$$
\begin{equation*}
|R|=\left|\frac{\frac{d f}{d z}+i k_{z} f}{\frac{d f}{d z}-i k_{z} f}\right|_{z=z_{\text {out }}}=\text { minimum } . \tag{4}
\end{equation*}
$$

The solution is found by optimizing Eq. (2). To achieve this goal, minimalization of Eq. (4) in the two-dimensional space of $\omega$ and $Q$ is performed. In this model, the first harmonic and the single mode scenario are considered.
The presented formalism was chosen for consideration because of its simplicity which, in turn, leads to a shorter implementation time and makes it more computationally efficient. Moreover, higher accuracy is not required at this step of the gyrotron design process, due to the fact that a precise final geometry is obtained using commercial fullwave microwave simulation software.

## 3. The Solution

This chapter explains all the definitions and steps necessary to find the solution. Physical constant values are included in the source code in the "PhysicalConstants.m" file. Readers should pay particular attention to the units used, as the constants are shown in gigahertz, millimeters and nanoseconds. The source code for the sample gyrotron cavity design is attached to this paper and may be downloaded. The code is written using the Matlab package.
To solve the second order differential equation, the ode45 Matlab solver is used. In this model, the second order equation is simplified to the first order differential equation system, and therefore the ode 45 Matlab solver is used. It makes the problem much simpler and, more importantly, the Dormand Prince ode4 algorithm with fifth order verification is well documented in the literature [5].
In order to start the calculations, the working frequency needs to be assumed arbitrarily. In this paper, the gyrotron being designed will work with the frequency of approx. 24 GHz , which is common for technological applications. When designing a gyrotron, other restricting factors have to be considered, e.g. limitations due to the availability of components, such as the magnet, electron gun and fabricating technology. The most important factor that has to be taken into account at this stage is the magnet. The length
and value of the magnetic field will limit the maximum working frequency, resonator length and radius. In the calculations, it is assumed that magnetic field $B_{0}$ is constant in the parallel part $L_{2}$ of the cavity. It is desired to choose a magnet and resonator in order to fulfill this requirement as accurately as possible. The distance between the electron gun and the resonator is determined by the magnetic field profile, which is expressed as the $B_{0} / B_{c}$ compression ratio. The magnetic field at the interaction part of the resonator over the magnetic field at the electron gun cathode. All factors mentioned above must be kept in mind when determining the size of the resonator. It is an iterative process. Once a solution is found, it is required to reconsider all the limitations and to repeat the calculation steps, if necessary. In this paper, the gyrotron under consideration will work at the first harmonic. The purpose of this paper is to show how to understand the physical meaning and to solve cold cavity model equations in order for the chosen working mode to be arbitrary to $T E_{0,1}$. A higher order mode would require that mode competition analysis [6] need to be performed, but this task is not considered in this paper.

### 3.1. Cold Cavity Model Input Data

Calculations are performed for the initial frequency of 24 GHz . To estimate the input parameters for the cold cavity model "InputDataEstimation.sm", the SMath Studio spreadsheet is used. The following input data were estimated and assumed:

- freq $_{\text {init }}=24 \mathrm{GHz}$ desired design frequency;
- $m, k$ - chosen azimuthal and radial mode numbers. It is assumed that the harmonic number is equal to 1 . In this case $M=0, K=1$ is chosen;
- $R_{0}$ - resonator radius in the non-tapered part. In this region, electrons interact with the RF field. The radius has to be chosen in such a way that the cut-off frequency is slightly lower than the desired working frequency. The initial guess would be:

$$
R_{0 i n i t}=\frac{c \cdot v_{m, k}}{2 \pi f r e q_{i n i t}} .
$$

This value is rounded off reasonably. Here, it would be a tenth of a millimeter, which boils down to $R_{0}=$ $7,5 \approx R_{\text {0init }} \mathrm{mm} ;$

- $L=\left[L_{1}, L_{2}, L_{3}\right] \mathrm{mm}$ - resonator length vector, proposed initial value for:

$$
L_{2} \approx L_{2 i n i t}=6 \cdot \frac{c}{\text { freq}_{i n i t}}
$$

which is approximately six wavelengths. $L_{2}=75 \mathrm{~mm}$ in this case. $L_{1}$ and $L_{3}$ are supposed to be shorter than $L_{2}$. It is assumed that $L_{1}=L_{3}=30 \mathrm{~mm}$;

- $\phi=\left[\phi_{1} ; \phi_{2} ; \phi_{3}\right]$ deg - resonator's taper angle vector. $\phi_{2}=0$ (always) and $\phi_{1}$ and $\phi_{3}$ are typically in
the range $0-6$ degrees and are described in the literature as "slightly" tapered [7]. A sharp angle would cause a sudden impedance change, which has to be avoided. It is desired to extract maximum radiation from the resonator, and reflections at the output are not desired. It is assumed that $\phi_{1}=2.3$ and $\phi_{3}=3.0 \mathrm{deg}$;
- $z_{\text {step }}-z$ axis step -0.05 mm is a good initial choice for good accuracy of the calculation.


### 3.2. Solving the Cold Cavity Model Equation

The following theory is implemented in the "ColdCavityApproachFunction.m" Matlab script. Before it is possible to solve the differential equation, auxiliary variables need to be calculated. The resonator's geometry is described by vector $R(z)$. This parameter is the resonator's radius along the longitudinal direction of the gyrotron Fig. 1. The wall profile includes the tapered part and is calculated using a tangent function and input variables $L=\left[L_{1}\right.$, $\left.L_{2}, L_{3}\right] \mathrm{mm}$ and $\phi=\left[\phi_{1} ; \phi_{2} ; \phi_{3}\right]$ deg. The characteristic value is the $K$-th solution to the $M$-th order of the first kind Bessel function derivative $J_{m}^{\prime}\left(v_{m, k}\right)=0$. In other words, it is the $x$ axis value of the Bessel first kind $M$-th order function derivative at the location where it crosses the $x$ axis the $K$-th time. The characteristic value is presented in Fig. 2 together with the electric field profile that is calculated using the "RadialDistributionOfEMfieldBesselFunction.m" script. The electric field of mode TE has component $E_{z}=0$, while $E_{r} \neq 0$ and $E_{\theta} \neq 0$ [8]-[10]. Moreover, $E_{\theta}\left(R_{0}\right)=0$. This is implied by the physical fact that the electric field in the conductor and at its surface (components parallel to the surface) is equal to 0 . This is due to the fact that a perfect conductor has the same potential inside its entire volume and on its surface. The electric field is a property of space that expresses the potential difference between two points within that space. This phenomenon
implies that boundary conditions exist at the resonator's wall, which are applied to the general EM field equations and, therefore, the field profile in the $r, \theta$ plane may be calculated. For the TE mode, the following time-dependent formulations are obtained:

$$
\left\{\begin{array}{l}
E_{r}=-B_{0} m \omega R_{0}\left(\frac{1}{v_{m, k}}\right)^{2} J_{m}\left(v_{m, k} \frac{r}{R_{0}}\right) \cos \left(m \theta+k_{z} z-\omega t\right)  \tag{5}\\
E_{\theta}=B_{0} \omega R_{0} \frac{1}{v_{m, k}} J_{m}^{\prime}\left(v_{m, k} \frac{r}{R_{0}}\right) \sin \left(m \theta+k_{z} z-\omega t\right) \\
E_{z}=0
\end{array}\right.
$$

where: $B_{0}$ - magnetic field in the resonator, $\omega=2 E \pi$. freq E wave angular frequency, $k_{z}$ wave number in the $z$ direction, $t$ - time.

The electric field profile in the $r, \theta$ plane, for mode $T E_{0,1}$, is presented in Fig. 2. The TE modes in the cylindrical cavity are linearly polarized, but in the gyrotron resonator, polarization is spinning with time and space ( $z$ axis), while in a cylindrical waveguide mode, it is not rotating in time. Modes with $m \neq 0$ have two potential polarizations [10] which may be obtained from Eq. (5) by replacing sine with cosine, and cosine with sine, and they will appear as:

$$
\left\{\begin{array}{l}
E_{r}=B_{0} m \omega R_{0}\left(\frac{1}{v_{m, k}}\right)^{2} J_{m}\left(v_{m, k} \frac{r}{R_{0}}\right) \sin \left(m \theta+k_{z} z-\omega t\right)  \tag{6}\\
E_{\theta}=B_{0} \omega R_{0} \frac{1}{v_{m, k}} J_{m}^{\prime}\left(v_{m, k} \frac{r}{R_{0}}\right) \cos \left(m \theta+k_{z} z-\omega t\right) \\
E_{z}=0
\end{array}\right.
$$

The solution of the cold cavity task consists of two steps. It is first required to choose the sweep range of the optimization variables freq (E field frequency) and $Q_{\text {diff }}$ (cavity diffractive quality). The differential Eq. (2) has to be then solved iteratively for a different freq and $Q_{d i f f}$ pair to find the values that will result in the minimal Eq. (4) value. This becomes a typical optimization problem. To find the optimal solution, the frequency and quality search space


Bessel function of the first kind $M$-th order derivative

Fig. 2. Bessel function derivative and electric field in the resonator's cross-section for mode $T E_{01}$ at $z=0$ and $t=0$. (For color pictures see the digital version of the paper.)
must be defined. The frequency search range proposed in this paper [11] is:

$$
\left\{\begin{array}{l}
\text { freq }_{\min }=\frac{c}{2 \pi} \sqrt{\left(\frac{v_{m, k}}{R_{0}}\right)^{2}+\left(\frac{\pi}{L_{1}+L_{2}+L_{3}}\right)^{2}}  \tag{7}\\
\text { freq }_{\max }=\frac{c}{2 \pi} \sqrt{\left(\frac{v_{m, k}}{R_{0}}\right)^{2}+\left(\frac{\pi}{L_{2}}\right)^{2}}
\end{array}\right.
$$

where: $c$ - light velocity, $v_{m, k}$ - characteristic value for mode $T E_{m, k}$.
The proposed search range for the resonator's quality factor [12] is:

$$
\left\{\begin{array}{l}
Q_{d i f f_{\min }}=4 \pi\left(\frac{L_{2}}{\lambda}\right)^{2}  \tag{8}\\
Q_{d i f f_{\max }}=4 \pi\left(\frac{L_{1}+L_{2}+L_{3}}{\lambda}\right)^{2} \cdot 4
\end{array}\right.
$$

where: $\lambda=\frac{c}{0.5 \cdot\left(\text { freq }_{\text {min }}+\text { freq } q_{\text {max }}\right)}$.
The proposed range for frequency is found in the literature [11], but the range for the quality factor's maximum values are the author's ideas and are based on the literature and observation of the RF field profile function in the domain of the freq, $Q_{\text {diff }}$ sweep. The formulation for the $Q_{\text {min }}$ value found in the literature is correct [4], but the $Q_{\text {max }}$ value cannot be estimated from the maximum resonator length, because it appears to be too narrow and does not always include the final solution within the optimization range.
One way to determine whether the search range was properly chosen is to check if the best solution does not lie at the edge of the search space. If it does, the optimization variable range needs to be expanded in this direction.

### 3.3. Solving the Equation System

To solve Eq. (2), the second order problem is rewritten as a first order equation system. To do this, new variables $y_{1}=f$ and $y_{2}=\frac{d f}{d z}$ are introduced. It is then possible to deduce that $\frac{d y_{2}}{d z}=\frac{d^{2} f}{d z^{2}}$, but from Eq. 2 it is known that $\frac{d^{2} f}{d z}=-k_{z}^{2}(\omega, z) f$, and, therefore, the computationally friendly first order form is:

$$
\left\{\begin{array}{l}
\frac{d y_{1}}{d z}=y_{2}  \tag{9}\\
\frac{d y_{2}}{d z}=-k_{z}^{2}(\omega, z) y_{1}
\end{array}\right.
$$

where $y_{1}$ and $y_{2}$ are two variables in the equation system. The first order differential equation system may be solved using the Runge-Kutta routine [7] implemented in the ode45 algorithm. From the Matlab manual, it is known that to run a solver, the following representation of the problem must be presented as: $[z, f]=\operatorname{ode} 45(\operatorname{odefun}(z, f)$, $\left.z_{\text {span }}, f_{0}\right)$, where:

- odefun - defines the function that returns the first derivative $\frac{d f}{d z}$. For the equation system, the function receives $f$ as a vector and returns a vector
that contains one derivative for each equation in the system;
- $z_{\text {span }}$ - the vector that defines the $z$ values for the algorithm;
- $f_{0}$ - the initial value for function $f$. It is a vector containing the starting values.

In the ode 45 solver manual, there is a sample solution to the problem that is very similar to this case. The odefun for the Matlab solver is implemented in the "OdeEqColdCavityField" function as part of the "EqColdCavityField.m" script. Technical realization of the second-to-first order differential equation reduction requires function values $y_{1}$ and $y_{2}$ to be passed into the odefun at each iteration. It might be confusing, but $y_{1}=f$ and $y_{2}=\frac{d f}{d z}$. The return values are the first derivatives of $y_{1}$ and $y_{2}$ calculated inside the ode function using the following formulation:

$$
\left\{\begin{array}{l}
\frac{d f}{d z}=\frac{d^{2} y}{d z^{2}}  \tag{10}\\
\frac{d^{2} f}{d z^{2}}=-k_{z}^{2}(\omega, z) \frac{d y}{d z}
\end{array}\right.
$$

It might be confusing, but in the case under consideration, the return values of the odefun are in fact $\frac{y_{1}}{d z}=\frac{d f}{d z}$ and $\frac{y_{2}}{d z}=\frac{d^{2} f}{d z^{2}}$.
To solve the differential equation system, the initial conditions must be supplied. In this model, the following equations define the initial conditions [13] for the ode45 solver:

$$
\left\{\begin{array}{l}
f_{\text {init }}=\mathrm{e}^{i k_{z}}  \tag{11}\\
\frac{d f_{\text {init }}}{d z}=\mathrm{e}^{i k_{z}} \cdot f_{\text {init }}
\end{array}\right.
$$

### 3.4. Solving the Optimization Task

It is necessary to consider if the optimized function is monotonous. If it has one minimum, then the Matlab "patternsearch" function can be used. The pattern search algorithm group is capable of finding the local minimum, but it does not guarantee that a global solution will be found, as the algorithm requires a starting point [14]. Success in finding the global minimum depends on the assumed starting point, unless the function has one minimum. In the considered case, there is only one minimum (Fig. 3), so it is possible to simply start searching from the middle of the assumed range of $\omega$ and $Q$, as there is only one minimal solution within the entire search range.
For each pair of the $\omega$ and $Q$, the RF field profile along the $z$ axis is found using Eq. (2). The boundary conditions on the left- and right-hand side of the resonator are then checked - Eq. (4). The best solution will have a minimal value of $|R|$ for $\omega_{\text {optimal }}$ and $Q_{\text {optimal }}$. This is done using the "EqColdCavityField.m" script. In this way, the optimal solution for the chosen $T E$ mode and fixed dimension is found.
To find the best geometry for the resonator, a parameter sweep is proposed. The 7th parameters have an influence


Fig. 3. $|R|=f(\omega, Q)$ boundary condition minimized function.
on the RF field, which are $L=\left[L_{1}, L_{2}, L_{3}\right], \phi=\left[\phi_{1}\right.$; $\phi_{2} ; \phi_{3}$ ] and $R_{0}$. Two main factors influence the solution $R_{0}$ and $L_{2} . R_{0}$ is fixed and estimated from the cutoff frequency dependency for a certain mode (see "SMath studio" sheet). Therefore, $L_{2}$ is a geometrical dimension that is the best candidate to be optimized. For each value of $L_{2}$, it is required to repeat the procedure of $\omega$ and $Q$ optimization. All values of $|R|$ for the different dimension have to be collected and compared in order to find the optimal $L_{2}$ value. This step is implemented in the "RunFindBasicSolution.m" script. Because the cold cavity model does not consider the electron current flow (and other factors), it is important to know that the final solution might be slightly different from the one chosen to be the best. Therefore, it is actually advised to consider 10 or 20 of the best solutions found by this method and to investigate them using a full-wave software package, or a more accurate but more time-consuming model.
Intermediate variables are not described here due to the fact that formulations may be found in the code. Moreover, they are self-explanatory.


Fig. 4. Electric field intensity longitudinal profile $f(z)$.

## 4. Results

As a result of the calculations, the following output variables are obtained:

- $f(z)-\mathrm{RF}$ field profile along the $z$ axis - Fig. 4,
- $f_{\text {optimal }}=\frac{\omega_{\text {optimal }}}{2 \pi}-$ optimal value of frequency,
- $Q_{\text {optimal }}$ - optimal value for the diffractive quality factor.
for certain resonator: $L=\left[L_{1}, L_{2}, L_{3}\right] \mathrm{mm}, \phi=\left[\phi_{1} ; \phi_{2}\right.$; $\phi_{3}$ ] deg and $R_{0} \mathrm{~mm}$.
The optimal value is the lowest $|R|$ value, which is equivalent to the minimal reflection at the output tapper.


## 5. Conclusion

In this work, the first step of the gyrotron resonator design process is presented. Due to the fact that the device has so many variables and factors that need to be considered, it is not obvious how to start the calculations. This work presents which variables have to be assumed and what initial values or value ranges may be relied upon to commence the design process. It is worth noting that, if a frequency below cutoff is chosen, the mode will not propagate, which will cause the code to crash, as $k_{z}$ will be imaginary. It is also worth keeping in mind that choosing a low value of $m$ and $k$ will cause the Brillouin angle to be low, and therefore microwave power will have to be extracted in the direction parallel to the $z$ axis. If perpendicular power output is desired, it is advised to use a high order mode so that the launcher component in the form of an oversize waveguide will have a reasonable length. It is possible to use GPU (graphic card processor) computing for this problem. The reason for this is that it may be turned into a parallel algorithm, due to fact that every solution is independent form the other. The main problem, causing implementation to be time consuming is, the requirement to implement the ode 45 solver in the GPU. This could be the next step in the development of this code. However, the presented model is solved efficiently using CPU, and this was the main reason for choosing this approach.

## Supplementary Materials

Supplementary materials: source code and animations may be downloaded from: http://sci.vberry.net.

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