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Faculty of Social Sciences
Institute of Economic Studies

RIGOROUS THESIS

Information Complexity of Strategic Voting

Author: **Mgr. Ján Palguta**
Supervisor: **Prof. RNDr. Ing. Frantiġek Turnovec, CSc.**
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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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All remaining errors are my own.

Abstract

This thesis in political economy considers the concept of strategic optimisation of voting behaviour under imperfect information. Under strategic voting we understand an act of voting for other than voterὂs best preferred (order of) alternatives. Motivation for this thesis comes from the empirically witnessed fact that a substantial portion of the electorate votes for their second or third best preferred alternatives, seeing that their most preferred alternatives face in expectation low probabilities of voting success. At other instances, the voters vote strategically with the intentions of strengthening the coalitional partners to their best choices or to weaken the coalitional partners of the undesired parties. Despite to the evident individual rationality of the strategic voting, strategic voting is typically socially suboptimal. Strategic voting leads to social choices that do not reflect the truthful preferences of the public.

Via a series of computation-based simulations the thesis studies the relative vulnerability of the most common voting procedures to strategic manipulation. The thesis categorizes these voting procedures by their degree of susceptibility to voting manipulation. By standard econometric techniques it confirms that strategic voting is most threatening in small groups, typically in committees, boards of directors, or in other small collective decision-making bodies. The thesis then relaxes the assumption of complete information, which is central for the Gibbard-Satterthwaiteὂs impossibility theorem to predict strategic voting. We confirm that in small decision-making bodies even a small reduction in the amount of possessed information can severely threaten the agentὂs ability to strategically manipulate the vote. For some procedures such reduction in information precludes strategic voting. This finding applies both for an absolute and relative drop in the amount of possessed information. On the other hand, if a committee member knows exactly the voting patterns of her colleagues, the probability of strategic manipulation rises substantially.

JEL Classification: C72, D72, D81

Keywords: strategic voting, information, voting behaviour, distance in preferences, computation-based simulations

Author's e-mail: jan.palguta@cerge-ei.cz

Supervisor's e-mail: turnovec@fsv.cuni.cz

Abstrakt

Práce z oblasti politické ekonomie se zabývá schopností voliļe strategicky optimalizovat svoje voliļské chování v podmínkách nedokonalé informovanosti. Pod strategickým hlasováním práce rozumí odevzdání voliļských hlasŢ jiné alternativŊ, neģ voliļ upŚímnŊ preferuje. Motivací práce je empiricky pozorovatelný fakt, ģe nezanedbatelná ļást voliļŢ hlasuje za svou druhou anebo tŚetí nejlepġí alternativu, protoģe první nejlepġí alternativa nemusí mít vysokou nadŊji na vítŊzství. Jindy voliļi hlasují pro koaliļní partnery ke svým alternativám nebo v snaze oslabit koaliļní potenciál neģádoucích stran. Naproti tomu, ģe strategické hlasování je individuálnŊ racionální, spoleļensky je neģádoucí. Strategické hlasování totiģ ļasto vede k rozhodnutím, která neodráģejí skuteļné preference voliļŢ.

Práce pomocí poļítaļových simulací voliļských preferencí zkoumá odolnost nejpouģívanŊjġích volebních procedur vŢļi strategické manipulaci v hlasování. SeŚazuje tyto procedúry od nejmanipulovatelnŊjġí aģ po nejménŊ manipulovatelnou. Pomocí standardních ekonometrických metod potvrzuje, ģe strategické hlasování je nejkritiļtŊjġí pŚi malém poļtu hlasujících, typicky v komisích, pŚedstavenstvech anebo jiných malých kolektivních orgánech. Dále práce opouġtí pŚedpoklad dokonalé informovanosti, kritický pro Gibbard-SatterthwaithŢv teorém nemoģnosti k tomu, aby pŚedpovŊdel strategické hlasování. Práce ukazuje, ģe pŚi malém poļtu hlasujících i minimální redukce v mnoģství informací o zpŢsobu hlasování ostatních voliļŢ výraznŊ znesnadŔuje strategické hlasování. U nŊkterých volebních procedur redukce v míŚe informovanosti strategickému hlasování pŚedchází. Práce potvrzuje tento nález pro absolutní i relativní úbytek v míŚe informovanosti. Naopak, pokud ļlen komise ví, jak budou hlasovat jeho kolegové, pravdŊpodobnost manipulace hlasování je vyġġí.

Klasifikace JEL: C72, D72, D81

Klíļová slova: strategické hlasovanie, informácia, volebné správanie, vzdialenosŠ preferencií, poļítaļové simulácie

E-mail autora: jan.palguta@cerge-ei.cz

E-mail vedoucího práce: turnovec@fsv.cuni.cz

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Chapter 1

Introduction

Strategic voting disturbs the optimality of the social choice and collective decisions. At the local level strategic voting may lead to implementation of publicly inferior projects or to undertaking of collectively undesired investments. Due to strategic voting the power may be delegated to unwanted or controversial representatives. On the state level, if the democracies have legislatively set minimum quotas for parties to enter their parliaments, strategic voting may favour large established parties. In voting when committees or parliaments face more than two alternative movements, strategic voting may bias the decision-making towards maintaining the status quo. Strategic voting often leads to attainment of the second-best, third-best or even worse outcomes. Under strategic voting voters wilfully vote for alternatives, which they would otherwise abandon having faced different respective probabilities of their success.

Considerable amounts of money are given out in election campaigns by small parties in efforts to counter the effects of strategic voting. Despite of the efforts, at the moment of the elections many voters tend to strategically support their larger and established rivals. The money spent on campaigns gets evaluated as expended in vain. It may only startle an unbiased observer if pushing the small parties out from the political scene was not in winnersô best interests: the large parties often lose potential coalition partners. Strategic voting or its mismanagement may lead to considerable swings in the division of the ruling power and despite all pre-election expectations it may change such important things as is the direction of a country. A prototypical example might have been e.g. observed in 2010 on the case of Slovakia or on other examples.

There are several approaches of how to methodologically seize the concept of strategic voting. The rational voter model, a descendant from the rational choice theory, has been established and used as the workhorse in the field of political economy. Rational voter model does predict strategic voting. In this paradigm, the voters consistently with their utility maximisation and in their expectations of the election result reveal their preferred orders of alternatives not necessarily truthfully, but rather vote for their second or third-best (orders of) alternatives with intentions to attain individually more favourable social outcome. The central variable here is the voterôs information, which helps the agent to update her beliefs on the respective probabilities of social outcomes.

Regardless of the individual viewpoint, from the social perspective the strategic behaviour is typically undesired and suboptimal. The question arises to what extent the strategic voting can be deterred and by which electoral institutions or voting procedures it can be best precluded. The feasible candidates for variables, through which the potential institutional optimisation can be performed, are the size of the committees, the voting rule determining the social outcome or again the information, beliefs and expectations that the voters form of each othersô voting patterns.

This rigorous thesis develops and extends its predecessor, authorôs master thesis on the Informational complexity of strategic voting. As a requirement for the rigorous thesis, the master thesis was extended by its third chapter. This chapter forms a standalone research article aiming at the very same research questions as the original thesis. The article has been submitted and accepted to the óAUCO Czech Economic Reviewô journal, prior to what it has been awarded by the óHonourable Mention of the Czech Economic Society's award for economists younger than 25 years of ageô. The article is to be published in AUCO in the second half of year 2011.

Numerous spelling mistakes suggested by the referee in the report on the master thesis were corrected in the article. Their detailed list is too excessive to be included at this very place. Next, we justify the choice of the used aggregation rules by the argument that we focus on the most common voting rules. We also justify the use of the uniform distribution for modelling of votersô preferences by the neutrality of this distribution. Better justifications for the choice of the definition of strategy-proofness were provided, together with better positioning of the definition into the preceding literature. In the thesis we will use the concept of strategy-proofness positioned neither into the pure probabilistic framework, as it is standard in one strand of the literature, nor in pure non-probabilistic framework, where the strategy-proofness is approached by focusing on the best and/or the worst alternatives in the choice sets. Our definition will be on a midway between the probabilistic and non-probabilistic frameworks. The expected utility will not over the standalone alternatives, but over voting distances between the individualǿs preference ordering and final social voting outcome. We argue that already Bossert and Storcken (1992) use Kemenyǿs distance to evaluate the voting distances between preference orderings. Also Duddy, Perote-Peña and Piggins (2009) have lately investigated the problem of constructing a social welfare function that is non-manipulable in a context, where individuals attempt to manipulate a social ordering as opposed to a social choice. The authors have managed to prove the impossibility theorem in the framework of manipulation of social orderings, which justifies our choice of the definition of strategy-proofness.

Last, we acknowledge that some choices of the modelling techniques may still seem ex-post arbitrary in our approach; we refer e.g. to the choice of the weights used in the distance function or to the use of Euclidian metric rather than of the Kemenyǿs distance. Nevertheless, making such and similar modelling choices is a natural feature of any simulation study on preference modelling.

Chapter 2

Information complexity of strategic voting

2.1 Introduction

A modern social choice theory is dominated by two results. First is the famous Arrowôs impossibility theorem, which states that there exists no voting system for three of more alternatives, which would be universal, would not break independence of irrelevant alternatives assumption, weak Pareto efficiency or non-dictatorship and which still would produce transitive and consistent results. The other result is the Gibbard-Satterthwaite theorem, which states that there exists no voting system with three or more alternatives designed to select a single winner, which would as well be unrestricted in domain, would not be dictatorial, and which would not provide an agent, who has a full knowledge of other votersô preference profiles, with an incentive to strategically misrepresent her voting preference so as to swing the election outcome into her favour. The prediction of the theory is clear; no voting system will ever be able to satisfy all listed desirable conditions.

On the other hand, it is more than easy to comply with these negative results. Knowing that a perfect voting system does not exist, much effort is unexpectedly saved and instead of attempting for a construction of a faultless voting rule, efforts can be taken so as to analyse the sensitivity of the assumptions of the two stated theorems. Alternatively, the currently existing voting procedures can be gathered and we may inspect their susceptibility towards the undesirable predicted properties.

Our work reacts on the Gibbard-Satterthwaite theorem, which predicts susceptibility to strategic manipulation for all non-dictatorial and universal voting procedures. We react on the assumption made about the full knowledge possessed by the strategic voter about the individual preference profiles of all voters and we are going to subject this assumption to a sensitivity analysis.

For this purpose we propose in the first part of the second chapter a function, which evaluates a distance between any social preference order and strategic voterôs sincere preference order. Minimisation of the distance between the two orders will in our set-up prompt the strategic agent towards strategic manipulation. In the second half of the chapter we are going to computationally simulate 10 different voting procedures. Via a series of voting simulations we will evaluate the susceptibility to manipulation of the particular voting procedures. Thirdly, we will study the vulnerability of strategic voting to the variation in the amount of information that the individual strategic agent holds about other votersô voting preferences.

In our work we find that the susceptibility to strategic voting manipulation is a function of the number of voting participants, of the number of competing alternatives, of the currently used voting procedure and prominently of the amount of information that the individual voter holds. Once we strip the agent from the full knowledge of the collective preference profile, we confirm the vulnerability of strategic voting both to an absolute and relative reduction in the amount of owned information. A minimal reduction in her holding of information severely threatens her ability of strategic manipulation. The precision in selecting the correct best manipulating voting pattern is also decreasing in the relative amount of information withheld. Consistently, the agent more often ends up with payoffs worse than sincere voting would yield, when a relatively larger share of information is withheld from her. These and other results are step by step documented in our work.

2.2 Game representation of voting

This subchapter proposes a game representation of voting, through which we shall study the role of information on optimal individual voting strategies. We describe how the voter decides to cast one ballot rather than another under specific informational circumstances. For this purpose we need two components. Primarily we need to set-up a voting environment and secondly to specify an underlying individual decision process. These are our primary goals for this subchapter. The first component, the voting environment, accrues to a world of alternatives and a number of voters possessing individual voting preference and a degree of information about other votersô preferences. The second component corresponds to one of two alternative modes of voting behaviour, either to sincere voting or strategic voting, which occur under altering voting aggregation rules. We aim to introduce these components through the terminology and necessary assumptions of the game theory employing some notions from the rational voter model and contrasting our approach to the approach of the social choice theory.

Social choice theory analyses the extent to which individual preferences can be aggregated into social preference, or more directly into social decisions. This aggregation has to be compatible with the fulfilment of a variety of desirable conditions. We shall use in this subchapter some aggregation methods realised by different voting procedures. From rational voter models voters we will borrow assumptions on votersô sincere preferences or utility rankings, by which they rate the voting alternatives. We use this microeconomic approach to determine an objective for an individual voter to maximise. This objective will materialise in a distance function between the individual preference ordering and an aggregated social preference order. Game theory will merge both approaches and will permit strategic interactions between players. The resulting game representation of voting will constitute one of our main contributions in this study. Study of increased interaction between players, which we expected to positively correlate with deeper profoundness of playerọs information about each other is the other principal contribution and is studied in next subchapter.

The considered voting procedures correspond to the most common ones. We specify the majority rule and general majority rule, plurality voting, the approval voting procedure, Bordaọs count and numerous other procedures. In this subchapter we will draw closely on following literature: Mas-Colell, Whinston, Green (1995); Feldman, Serrano (2006); Turnovec (2001) and Nurmi (1987).

The subchapter is organised into three sections. First section introduces the voting environment; second section specifies the modes of voting behaviour. The third and last section specifies eleven different voting procedures, which we use throughout the whole thesis.

2.2.1 Voting environment

Let us first introduce **U as a universe of alternatives**, let it be finite, non-empty set of all possible alternatives, the elements of which are denoted a, b, cé Note, that 2^U, so-called power set of U, is a set of all subsets of U. **Let T = $2^U \setminus \emptyset$** stand for the set of all non-empty subsets of U. Let set **A, call it an opportunity set,** be such set that $A \in U$. Let it be an unstructured set of finite cardinality, $card(A) = m$. That means that A contains precisely *m* alternatives, where *m* is a positive finite integer. Use index j to represent a particular alternative from A.

What are the alternatives? The alternatives may be anything from allocations in an exchange economy, with or without externalities, to production plans or production and consumption patterns in the economies with production, or it may be levels of public goods expenditure or alternatively political candidates, etc. It may just be any alternative pool of choices subject to collective choice.

Assume a **set of individuals N = {1,2,é ,n}** to be a non-empty finite set, where all these individuals clearly understand what will happen to them, if option $a \in A$ is chosen rather than $b \in A$. Use i as an index representing particular voter.

Preference profiles

Let us assume that each individual $i \in N$ has a **binary preference relation** defined on A. Let $\mathbf{R_i}$ denote this *preference relation*. We assumed R_i to be a *weak* relation, aR_ib stands for ñindividual i regards a as at least as good as bò. The strict preference relation $\mathbf{P_i}$ and indifference $\mathbf{I_i}$ are defined from R_i in the usual way:

$$aI_ib \Leftrightarrow aR_ib \text{ and } bR_ia$$
$$aP_ib \Leftrightarrow aR_ib \text{ and not } bR_ia$$

For the rest of the study, each individual preference relation R_i is assumed to be *complete, reflexive, transitive* and *anti-symmetric*. These assumptions are necessary and sufficient for R_i to be characterised as a total preference ordering.

Definition: (Completeness) *For all* $a, b \in A$, *either* $aR_i b$ or $bR_i a$

Completeness of the individualôs preference relation implies full awareness of the results of binary comparisons between any two alternatives. It may not happen that an individual cannot tell, in what relation two alternatives stand against each other. If she cannot state a strict preference, she has to weakly prefer one alternative or she has to be able to say that she is indifferent between the alternatives.

Definition: (Reflexivity) *For all* $a \in A$, $aR_i a$

Note that assumption of reflexivity is abundant for total preference orderings, as reflexivity is a necessary condition for completeness. If completeness was not satisfied, but reflexivity, transitivity and anti-symmetry conditions were satisfied, then the individual preference relation would take form of a partial preference ordering.

Definition: (Transitivity) *For* all $a, b, c \in A$, $aR_i b$ and $bR_i c \Rightarrow aR_i c$

Transitivity implies that it is impossible to face the voter with such sequence of binary comparisons that would lead to a cycle in her preferences. The voter cannot rank choice C strictly above choice A, if she previously stated that A is at least as good as B, and B is at least as good as C. The assumption of transitivity relates strongly to the very concept of individual rationality and breaks it if it fails.

Definition: (Anti-symmetry) *For all* $a, b \in A$, $aR_i b$ *and* $bR_i a \Rightarrow a = b$.

Anti-symmetry guarantees that indifference between any two options a and b is precluded. In consequence, this assumption is the last necessary and sufficient condition for considered preference relations R_i to be identified as total preference orderings.

Definition: (Total preference ordering) Let A be a set of finite cardinality and let R_i be a weak binary preference relation defined on A. Then the ordered set $\Re_i = [a_1\ a_2$ é $a_m]$ of all elements of A such that $a_1R_ia_2R_ia_3$é $a_{m-1}R_ia_m$ is called a *total preference ordering* of a_j on A. Preference relation R_i defines a total preference ordering $\Re_i$ on A, if and only if the preference relation R_i is complete, transitive and anti-symmetric.

Let us introduce an equivalent notation for a preference order involving m alternatives. Let it take form $[a\ \mathrm{b\ c}] \Leftrightarrow \mathrm{aR_i} bR_i c$ in case of a strict preference, and $[a\ (\mathrm{b\ c})] \Leftrightarrow \mathrm{aR_i} bI_i c$ or $[(a\ \mathrm{b\ c})] \Leftrightarrow \mathrm{aI_i} bI_i c$ in cases of a weak preference. Let us further denote the elements in the individual preference order by [r_1 r_2 é r_m], where j denotes the j^{th} position of an element in the individual preference ordering.

Example 2.1 Total preference ordering Think three possibilities a, b, c and a weak binary preference relation R_i, which we assume to be complete, transitive and anti-symmetric. On this set of alternatives we may think of 6 different preference orderings $\Re_i$ = [r_1 r_2 r_3]. They are [a b c], [a c b], [b a c], [b c a], [c a b] and [c b a]. Ĭ

Gibbard (1973) calls ñchain orderingñ what we call a total preference ordering.

Example 2.2 Relaxation of anti-symmetry Think the same three possibilities a, b, c, but relax anti-symmetry of R_i, while maintaining completeness and transitivity. We can now think of 13 different preference rankings, which are however no longer necessarily preference orderings: [a b c], [a c b], [b a c], [c b a], [c a b], [c b a], [a (b c)], [b (a c)], [c (a b)], [(a b) c], [(a c) b], [(b c) a] and [(a b c)]. Ĭ

We materialize the collection of preferences of n voters in a collective preference profile R.

Definition: (Collective preference profile) A set of n total individual preference orderings on A with one and only one total preference ordering for each individual i from N is called **a collective preference profile R**, such that

$$R = (\Re_1, \Re_2, \ldots, \Re_n) \in \Re_A^n$$

where $\Re_A$ would be a set of all possible individual preference orderings on A and $\Re_A{}^n$ is an n-fold Cartesian product of $\Re_{A.}$

Preference aggregation

The theory of social choice discerns several approaches to preference aggregation, which we describe here for reference. In social choice we could aggregate the collective preference profile either into a ranking of alternatives, or we select a set of socially best alternatives or eventually we select a single alternative as socially superior. The structure of an aggregation outcome depends on the particular chosen technique of preference aggregation. If the aggregation translates the collective preference profile into a complete ranking of alternatives, we assume this ranking to be a social preference ordering.

Definition: (Social preference ordering) Let A be the opportunity set of finite cardinality and let R_i be a weak binary preference relation defined on A and R be the collective preference profile. The ordered set S = [a_1 a_2 é a_m] of all elements of A, which can be characterised by $a_1R_ia_2R_ia_3$é $a_{m-1}R_ia_m$, is called a *social preference ordering* of a_i on A, if S is a total preference ordering generated through a preference aggregation of the collective preference profile R. Let us denote the particular elements of a social order by [s_1 s_2 . . . s_m], where s_j represents the j^{th} position of an alternative in the social order.

In social choice the preferences aggregation of may be undertaken by the means of:

a) Social welfare functions,

b) Social choice functions,

c) Social choice correspondences.

Social Welfare Function (SWF) (in the Arrowian sense) looks for the same type of preference relation on the collective level as one is assuming on the individual level. It is a function f mapping the n-tuples of complete, transitive and anti-symmetric individual preference orderings into a complete, transitive and anti-symmetric social preference ordering. Formally we write:

$$f : \Re_1 \times \Re_2 \times \ldots \times \Re_n \to S$$

where $\Re_i \ (i = 1,...,n)$ are individual preference orderings of n individuals satisfying completeness, transitivity and anti-symmetry and S is the social preference ordering. This approach is perhaps best known in the social choice theory due to the famous contribution of Arrow (1963). Arrow in his work attempted to find a general method of determining the social preference S, given individual preference rankings $\Re_i$, so that the social preference would possess the same properties of completeness, transitivity and anti-symmetry as the individual preferences $\Re_i$. The impossibility result, involving a set of other desirable requirements on S is in the literature widely known as the *Arrowôs Impossibility Theorem.*

Social Choice Correspondence (SCC) is a practical application of reduction in requirements on the Arrowian SWFs, where for the purposes of preference aggregation we are interested only in finding such set of alternatives, which the society deems ñbestò. Formally, we construct a function F of the following sort:

$$F : U \times \Re_1 \times ... \times \Re_n -> T$$

where U is the universe of all alternatives and T is a set of all non-empty subsets of U. Given the alternatives, the function F translates the preference n-tuple into a set of socially best alternatives. This function by construction allows for ties between alternatives and in its range allows eventually for a case when all alternatives in U are chosen as ñbestò. This approach originates in Fishburn (1973) and Plott (1976).

Social Choice Function (SCF) is a special case of the social choice correspondence allowing the social choice to be single-valued only, i.e. eventual ties must be broken and a single winner pronounced. The approach is justifiable by practicality in those cases, when technical circumstances allow for an implementation of a single alternative only or for an appointment of a sole candidate. Given such circumstances, the counsel of social choice correspondence is of a limited use to the decision maker. Gärdenfors (1977) uses the label óresoluteô for this kind of social choice functions. Resolute SCFs provide a justification for the choice of one alternative before all other. In the social choice theory the Arrowian social welfare functions, social choice functions and social choice correspondences are typically realised by the means of **voting procedures.** It takes a specific voting procedure to aggregate votes into a voting outcome. Note the difference from ñvotingò, by which we understand just stating oneôs own individual preference.

The potential loophole in our approach consists in the fact that many voting procedures allow declaring voting outcomes that involve *ties* between alternatives. The generated social ranking would not then satisfy the sufficient conditions for a preference ordering, because it would break the assumption of anti-symmetry. Let us consider in such cases a randomisation device, which assigns equal probability to all potential social orderings that could occur if the tie was broken randomly. Let us introduce it by the means of a *lottery*.

Definition: (Lottery) Let there be K possible social orderings that could occur after random breaking of all ties involved in a given voting outcome. A simple lottery L is then a list $L = (p_1, ..., p_K)$ with $p_k \geq 0$ for all k and $\sum_k p_k = 1$, where p_k is interpreted as the probability of social ordering k occurring. In our study we work with $p_1 = p_2 = ... = p_k = k^{-1}$.

Example 2.3 Breaking ties Think of a voting procedure that has assigned scores 1, 4, 1 to alternatives a, b, c respectively. Allowing for ties, a social ranking would take form [b (a c)]. Let us therefore consider a randomisation device that gives equal probability 1/2 to both possible social orderings [b a c] and [b c a] and such decides about the final realised social ordering. Ĭ

Example 2.4 Voting outcomes ï majority voting

Let us illustrate the outcome, which would occur if all players stood at the same time at the riverbank without any informational advantages. Let us employ a Social choice function realised by majority voting (see section 2.5.1 for details on majority voting). Each player may state just his best choice and the alternative with most votes wins.
Think of three voters choosing a single option from alternatives a, b, c.

Table 2.1 ï Voting outcomes, majority voting

	Votes of 2 and 3								
	(a, a)	(a, b)	(a, c)	(b, a)	(b, b)	(b, c)	(c, a)	(c, b)	(c, c)
Vote of 1	**Outcome**								
a	[a]	[a]	[a]	[a]	[b]	[?]*	[a]	[?]*	[c]
b	[a]	[b]	[?]*	[b]	[b]	[b]	[?]*	[b]	[c]
c	[a]	[?]*	[c]	[?]*	[b]	[c]	[c]	[c]	[c]

* The question mark indicates where the social decision is to be taken randomly. All competing options are assigned an equal probability of 1/3 in the voting outcome determination. Ĭ

Let us look how the players evaluate particular voting outcomes.

Utilities associated with the voting outcomes

Utility associated with a selected alternative

The assumptions on properties of individual preference relations allow us to describe these preference relations by the means of *utility functions*. By **utility function $u(a_i)$**, which represents individual preference relation R_i, we map elements from the opportunity set A into real numerical values. Hence we rank the elements in A in accordance with the individual preference relation.

Definition: A function u: AŸ $\Re$ (set of real numbers) is a *utility function representing preference relation* R_i, if

$$\text{for all a, b} \in \text{A}, \; aR_ib \Leftrightarrow u(a) \geq u(b).$$

The utilities are the payoffs that individual i receives, if particular element from A is selected by the selection mechanism. The value that the individual forgoes is equal to the value of the second best alternative.

Utility derived from a social preference ordering

A bit more challenging concept emerges, when we want to cardinalise the individual utility derived not from a particular alternative but from a complete social preference ordering. We need to construct such payoff function that would reflect both original individualôs preference and the aggregated social preference order. Payoffs should at the same time reflect to what degree these two orders agree or how close is the generated voting outcome from the original voterôs preference. Last, the agreement between the two orders should influence the payoff function with higher significance at beginning of an order than as at its end. For these reasons we compare the two orderings by the means of distance function. We consider a distance function, which resembles the mathematical Euclidian distance function. Minimization of distance between the individual ordering and the social preference ordering then corresponds to maximization of utility of a particular voter.

Definition: (Distance function) Let r_j and s_j constitute two systems of non-negative weights attached to all alternatives $a_j \in A$ of an individual and social preference order, respectively. The two systems of weights are intertwined in the following manner: if particular alternative a_j is located at the j^{th} position in the individual preference order $\Re_i$, then it bears individual weight r_j. An equal weight s_x will be attached to such position in the social order S, at which alternative a_j was placed by the voting aggregation rule. The distance function D_{iS} between i^{th} individual preference ordering $\Re_i$ = [r_1 r_2 é r_m] and social preference ordering S = [s_1 s_2 é s_m] is subsequently given by:

$$D_{iS} = \sqrt{\sum_{j=1}^{m} (r_j - s_j)^2} .$$

Distance function D_{iS} effectively maps the two systems of weights attached to elements of the individual and social orderings into one real number. The quantified distance then enters the utility function as its main argument, where utility is monotonically decreasing in distance:

$$\frac{\partial u_i(D_{iS})}{\partial D_{iS}} \leq 0.$$

The distance function hence effectively turns any utility function into a disutility function from distance.

Example 2.5 Distance function Consider an individual ordering $\Re_i$ = [r_1 r_2 r_3] = [b a c] and a social ordering S resulting from a vote between n individuals S = [s_1 s_2 s_3] = [c a b]. Assume that individual i attaches following weights to his ordering: [b a c] $\rightarrow$ [3 1 0]. By definition of the distance function, the individual evaluates the aggregated social order S = [c a b] by weights [0 1 3]. The distance between the social preference order S and the individual preference order is then given by $D_{iS} = \sqrt{3^2 + 0^2 + (-3)^2} = 3\sqrt{2}$.Ĭ

The weights attached to alternatives in the individual order are expected to be marginally diminishing or at least marginally non-increasing in j. Nonetheless, fulfilment of this assumption is not crucial here. Attachment of weights to particular positions of an individual preference order allow for personalisation of any distance function.

Example 2.6 Weights of a distance function Imagine an individual, who cares only about the first winning option and disregards the order of allocation of all other options. Here one would attach weight $r_j = 1$ for j=1 and $r_j = 0$ for all j≠1. Contrast with an individual, which cares for a complete social order of alternatives. Her weights are non-negative in the whole social order. Ĭ

Any specific imposition (personalisation) of individual weights is necessarily arbitrary. We nevertheless need to proceed in this direction if we want to analyse the responsiveness of individuals to an amount of information about other votersô preferences. The weights that we attach to alternatives shall be unified since now on for the rest of our study. We assume them to correspond to scores, by which an individual would evaluate alternatives during **Bordaôs voting**, i.e. the winning option scores (m-1) points and then the weights attaches to other options are consecutively falling by 1, with the last option scoring 0 points. (See section 2.5.6 for details on Bordaôs voting.)

Example 2.7 Weights of a distance functions II Think a player, who ranks five competing options a, b, c, d, e in an individual order $\Re_i$ = [r_1 r_2 r_3 r_4 r_5] = [a c e b d]. Let a specified voting procedure aggregate the options into a social order S = [s_1 s_2 s_3 s_4 s_5] = [a b d e c]. Vector of voter 1 Borda scores assigned to her individual ordering [r_1 r_2 r_3 r_4 r_5] = [a c e b d] reads [4 3 2 1 0]. Voterôs 1 evaluation of a social order [s_1 s_2 s_3 s_4 s_5] = [a b d e c] is then evaluated as [4 1 0 2 3]. Distance function translates the difference between the two orderings into a real number $D_{iS} = \sqrt{0^2 + 2^2 + 2^2 + (-1)^2 + (-3)^2} = 3\sqrt{2}$.Ĭ

Utility from voting outcomes involving tie(s)

Voting outcomes involving tie(s) between alternatives are inconclusive outcomes, which lead to uncertainty in voting strategic considerations. The uncertainty consists in random breaking of tie(s) and corresponding varying disutility from distance between potential social orderings and individual voting preference order. We resolve the issue of uncertainty by the translating the argument of a distance function into a form of average distance.

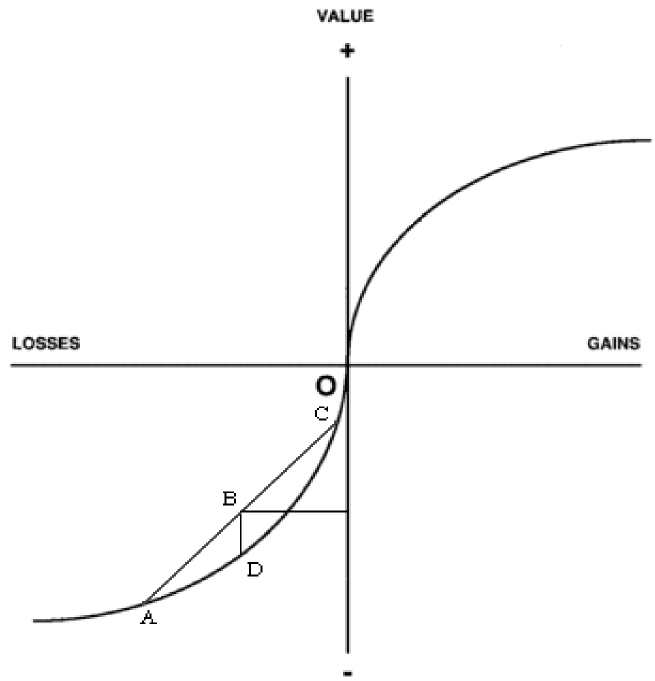
Definition (Average distance) Let $L = (p_1,..., p_K)$ be a lottery, which assigns equal probabilities to all potential social orders k that may occur after a random breaking of tie(s) involved in a voting outcome. Let D_{iSk} represent k potential distances of social ordering S_k from individual ordering $\Re_i$ for all k =(1,é , K). Then

$$\overline{D_i} = p_1 D_{iS1} + é + p_k D_{iSK}$$

is the average distance between the individual preference ordering $\Re_i$ and all potential social preference orders. $\overline{D_i}$ further enters the distance function as its main argument.

Average distance is effectively an average distance that occurs between k potential social preference orders and the individual preference order $\Re_i$. We chose to approach the uncertainty issue by calculation of average distances rather than by calculation of expected disutility from k potential social orderings for one reason. Disutility from an average distance corresponds to the maximal disutility that individual may face. Expected disutility is smaller in absolute terms. Consider Figure 2.1 for graphical illustration.

Figure 2.1 ï Theoretical utility function



Source: own

In Figure 2.1, points A and C mark on the vertical axis distinct disutilities from two different social orderings, where point A represents larger distance of the social order away from the individual preference order on horizontal axis. Point B then corresponds to an expected disutility derived from these two distances. Point D corresponds to the disutility of an average distance. Clearly D marks the upper bound (maximum in absolute terms) of disutility that may be inflicted upon the individual. This of course relies on the convexity of the utility function on its negative domain. Taking disutility corresponding to D as the reference disutility corresponding to voting outcomes involving tie(s) then allows for comparison and utility maximization with respect to voting outcomes, which result directly in unique social orderings.

Example 2.8 Normal form representation of a voting game Think three alternatives a, b, c and think three players 1, 2, 3. Let each of them vote by stating their most preferred alternative. Let the social order be aggregated by plurality voting, i.e. the order shall correspond to an order of alternatives with most votes. Table 2.2 captures all possible social rankings that may occur under all potential strategies of these three non-cooperating players.

Table 2.2 ï Social rankings in plurality voting

Vote of 1	Votes of 2 and 3 (respectively)								
	(a,a)	(a,b)	(a,c)	(b,a)	(b,b)	(b,c)	(c,a)	(c,b)	(c,c)
	Outcome								
a	[a,(b,c)]	[a,b,c]	[a,c,b]	[a,b,c]	[b,a,c]	[(a,b,c)]	[a,c,b]	[(a,b,c)]	[c,a,b]
b	[a,b,c]	[b,a,c]	[(a,b,c)]	[b,a,c]	[b,(a,c)]	[b,c,a]	[(a,c,b)]	[b,c,a]	[c,b,a]
c	[a,c,b]	[(a,b,c)]	[c,a,b]	[(a,b,c)]	[b,c,a]	[c,b,a]	[c,a,b]	[c,b,a]	[c,(a,b)]

Then for each voter, may he have whatever individual preferences we may construct a payoff matrix by the means of a distance function. Construction of payoff matrices is the last step needed in composition of a normal form of a non-cooperative voting game.

Let the individual 1 preference ordering be [a b c] Ÿ [2 1 0]. Let Table 2.3 document the distances of the generated social orders from this individualôs voting preference.

Table 2.3 ï Voting distances associated with social preference orderings, plurality voting

Vote of 1	Votes of 2 and 3 (respectively)								
	(a,a)	(a,b)	(a,c)	(b,a)	(b,b)	(b,c)	(c,a)	(c,b)	(c,c)
	Distance from [a b c]								
a	0.707	0*	1.414*	0*	1.414*	1.759*	1.414*	1.759*	2.449*
b	0*	1.414	1.759	1.414	1.932	2.449	1.759	2.449	2.828
c	1.414	1.759	2.449	1.759	2.449	2.828	2.449	2.828	2.639

* marks the best response of player 1 with preference [a b c] to respective pairs of strategies of players 2 and 3

Ĭ

Individual information

We have already established that each individual voter possesses certain information about the alternatives from the opportunity set A, in the sense that he fully understands what will happen if a particular alternative or a particular social ordering is selected rather than another and what (dis)utility she will consequently derive from this selection. In addition to this information we discern a further informational aspect of voterôs knowledge. This aspect concerns the information that the voter commands about **other individualsô preferences**. The information about other votersô preferences differs in its amount and profoundness.

We will distinguish three levels of profoundness of this information. The voter can command information on:

a) only her own preference ordering,

b) preference orderings of some subset of voters (eventually set of all voters),

c) truncated preference orderings of some subset of voters,

The minimum level of information is described by full information only about oneǿs own preference ordering. The agent knows then nothing further about other votersô preferences, which puts the voter into a relatively extreme position with no strategic considerations, as we will learn later. The position is extreme in the sense that it is rather common to possess at least some degree of knowledge of other votersô preference. The other extreme is when the voter knows the entire collective preference profile, i.e. the complete preference orderings of all n individuals. Such voting situation offers a wonderful background for individual strategic voting considerations. What we mean by strategic considerations we will explain in the subchapter 2.3.

Majority of voting situations then move between these two informational extremes of no and with full information of other voterǿs preference. The voter can usually identify orderings of some of her counterparts or only their truncated orderings; say when she knows e.g. first best choice of other voter or of a group of voters.

Example 2.9 Knowledge of the best option of player 2 Consider example 2.8 and the generated payoff matrix displayed in Table 2.3 for voter 1 with preference [a b c]. If voter 1 knew that the best option of player 2 was e.g. c, she would not need to consider all columns of other playersô strategies. Her payoff matrix would reduce to a matrix captured by Table 2.4.

Table 2.4 ï Reduction in the knowledge of individual player

Vote of 1	Votes of 2 and 3		
	(c,a)	(c,b)	(c,c)
	Distance from [a b c]		
A	1.414*	1.759*	2.449*
B	1.759	2.449	2.828
C	2.449	2.828	2.639

* marks the best response of player 1 with preference [a b c] to respective pairs of strategies of players 2 and 3, while knowing that playerǿs 2 best option is option c.

Nevertheless, an assumption that all other voters cast their votes in accordance to their true preference is crucial here. Ĭ

Information could be incorporated by further means. If voter i knew rankings of all other voters and assumed their honest voting, then her payoff matrix would reduce to one column of the full matrix. As we have seen, if a voter knows ranking of some of other voters and assumes their honest voting, then her payoff matrix reduces to a sub-matrix of the full matrix. If voter assumes other voters to act strategically, we can expect them to coordinate on some of the arising Nash equilibria.

Having specified votersô preferences, means of preference aggregation, mechanisms of utility determination and means of consideration of informational aspects, we have completed the necessary specifications on the voting environment, in which voters play the strategic games and optimise their voting patterns.

2.2.2 Voting behaviour

Voting as a collective vs. individual decision problem

Voting is an individual act of registering a choice between alternatives, which groups use to come at collective decisions. In economics voting is used to judge among Pareto optimal alternatives and emerges to substitute market mechanisms, where necessary. Voting is used in democratic societies to resolve conflicting situations, while letting all members of a society to participate in the collective decision-making by expressing their personal attitudes. Voting generally aims to come at those solutions, which best fit the collective opinion.

Much of the social choice theory looks at voting from the collective perspective and it studies the compatibility of fulfilment of various desirable properties of different voting procedures. Most prominently step up two impossibility theorems: Arrowŝ impossibility theorem (Arrow, 1963) and further interpreted Gibbard- Satterthwaithe theorem (Gibbard, 1973, Satterhwaite, 1975), unified by e.g. Reny (2000).

Approach of these and numerous following authors focuses on the properties of social choice functions and correspondences realised by different voting procedures and on theoretical consequences of their use. These authors forgo the individual view of voters involved in voting. We provide just a few from a long list of examples dealing with manipulability of SCFs and SCCs without intentions of actual reviewing them: Bandyopadhyay (1983), Barbera (1977), Feldman (1979), Gärdenfors (1979), Pattanaik (1973), and more recently Barbera, Dutta, Sen (2001), or Rodriguez-Alvarez (2007). Here we specify, what we shall understand in our considerations under voting viewed as a collective act.

Definition: (Voting as a collective act) Given a set of alternatives A, and a set of voters N, and given their exogenously specified individual preferences, which are assumed to be orderings, **the group is required** to choose an alternative on the basis of stating and aggregating all individual preferences or alternatively to produce a ranking of alternatives from the most to the least preferred (Turnovec, 2001).

Contrary, numerous empirical studies focus on econometric detection and measurement of strategic voting in multiparty systems or in systems with numerous candidates, such are Alvarez, Nagler (2000), Blais et al. (2001), Blais, Bodet (2007), Fisher (2001a, 2001b), Schmitt (2001) and numerous other. Other studies rely on simulations of votersô preferences, e.g. Laslier (2009), economic experiments on strategic voting show up occasionally.

Last branch of economics related to voting provides microeconomic rationales underlying either voting turnout or voting patterns including strategic voting. Much of the literature in this branch descends from **rational voter model** stemming from rational choice theory. Rational voter models are well described by Myerson, Weber (1993), Feldman, Serrano (2006); or Edlin, Gelman, Kaplan (2007).

Definition: (Individual voting problem) Given a set of alternatives A, and a set of voters N, given information that voters have about other votersó preferences, **each voter is required** by the act of voting to state her preference, which after all voters have done so, on the grounds of a settled mechanism will be aggregated into a winning alternative or into a ranking of alternatives.

We have to nevertheless assume that people do vote, by which we avoid the consequences of the theory of rational ignorance (Downs, 1957; Aldrich, 1997). Given the probabilities of voting outcomes and the utilities associated with them, we assume that there are either no costs of voting or that voting benefits derived from the expected influence on voting outcome outweigh the voting costs. The voters view it gainful to vote rather than not to vote.

The voter in her individual voting problem tries to determine her optimal individual decision. For this purpose the voter takes into regard primarily her own utility. The voter does not care for what is best for the group unless this aspect directly enters her utility function. Aldrich (1997) incorporates social welfare into individualōs utility to explain voting turnout.

Sincere and strategic voting

As it was theoretically proposed and empirically shown, voters are capable of voting strategically, which means ñnot in accordanceò to their honest preference orderings. The reason is that voter typically has not only individual preference, but also perceptions of chances of winning of particular alternatives. These perceptions are influenced by the information that the voter has about other voterōs preferences.

Example 2.10 Strategic voting Imagine a vote between four alternatives a, b, c, d among 5 voters in Bordaōs voting procedure. (See section 2.5.6 on Bordaōs voting procedure for detailed description.) Each alternative earns points for its relative position in voterōs ranking. If preferences were revealed sincerely, i.e. as they are stated in Table 2.5, alternative b would win the vote with 12 points.

Table 2.5 ï Sincere preference orderings, Borda voting

	Preference orderings of voters				
Points	1	2	3	4	5
3	a	a	a	b	b
2	b	b	b	c	c
1	c	c	c	a	a
0	d	d	d	d	d

Scores: a: 11 = 3x3 + 2x1 c: 7 = 2x2 + 3x1

b: 12 = 2x3 + 3x2 d: 0 = 5x0

Nevertheless, if we allow e.g. first voter to react *strategically*, and we let her know the entire preference profile of all other players, her best move would involve moving option b in his ranking to the 4th place, while maintaining ñaò at the 1st rank in her stated rank. That would reduce the total score of b to 10 and cause option ñaò to become a winner. Winning of alternative ñaò would make the 1st player better off than she would be under sincere voting. Ĭ

Minimisation of distance between individual preference and a generated social order could have been observed also in earlier examples. The proposed minimisation would involve also misrepresentation of individual preferences, where necessary.

Two kinds of voterõs rationality and voter behaviour therefore emerge:

a) **sincere rationality / sincere voting,** i.e. voter states her ordering (or her bets choice) independently from the information about other voters,
b) **strategic rationality / strategic voting,** i.e. voter states her ranking (or her best choice), while taking into her account information about other votersô preferences

Sincere voting occurs either by assumption, i.e. if we simply assume that the voters do not vote strategically, or it occurs via the optimality of such decision in the strategic rationality framework, e.g. there may be no way how to strategically influence the voting results, or it occurs through to a lack of information on other votersô preferences, which would allow for a strategic vote.

Strategic voting (tactical voting, sophisticated voting) in contrast occurs thanks to the possession of information on other voterõs preferences. This information allows for the creation of expectations on other voterõs voting pattern as well as for the creation of expectations on the probabilities of winnings of particular alternatives.

Strategic voting is standardly analysed through the rational choice framework, mostly because it is the only theory that would predict strategic voting. Fisher (2001a) poses three criteria to distinguish a strategic vote in constituency elections:

1. Voters are assumed to be **short-term instrumental rational**, i.e. voter wants to influence, who wins the election.
2. Vote is **different from the voterôs sincere ordering**.
3. Vote is **consistent with utility maximization**, given the **expectations of voting results** and the **utilities associated with the alternatives.**

These criteria in our view capture the essence of strategic voting, although Fisher (2001a) uses these criteria to find a theoretical voting rule for single member simple plurality electoral systems. This voting rule nevertheless fits in line with previous literature: McKelvey and Ordershook (1972), Cox (1997) or Myatt (2000).

Instrumentality of voterôs motivations implies that voterôs utility is affected only by the voting outcome. Other issues, like the margin of the victory or the order of alternatives or any other aspects of voting do not enter votersô considerations unless they can be translated into the utility associated with who won. If voters derive utility simply from the act of voting, they are clearly not instrumental. Short-term aspect restricts the voterôs interest always only on an actual voting situation.

The difference between the actual vote and sincere ordering is the most intuitive and consistent criterion on distinguishing a strategic vote. Nevertheless, it triggers a few questions. For example, if a voter with instrumental motivations persuades herself that it is the óbestô to vote for the most probable winners, than the difference wades away. That would mean that the voter has adjusted not only her voting pattern but also her preference according to the information she has. The voter might do so for many reasons, e.g. many voters cling to backing a winner. In such situations it is essential to make the voter reveal if she would vote in the same pattern if she did not perceive the probabilities on probable winnings. If she would vote differently with and without perceptions and information on probable winnings, we face a strategic voter. Luckily, the concepts of preference and the perceptions of probabilities of winning are separable in this way.

Consistent with utility maximization

Since we work in a deterministic framework, where we solve the only uncertainty that enters our considerations by calculation of average distances, consistency with utility maximisation means simply voting for such pattern that minimises the distance between an individual preference ordering and the aggregated social preference ordering. Many authors propose voting rules, which specify when a voter would vote strategically in probabilistic frameworks, where probability measures over alternatives are given (Barbera et al., 2001; Myatt, 2000, 2002; Rodriguez-Alvarez, 2007). A voter should vote strategically here to maximise her expected utility.

Types of strategic voting

Burying

An example of burying an alternative was provided in example 2.10. Individual 1 having a preference ordering [a b c d] knew that a social preference ordering may be changed from [b a c d] to [a b c d] by placing option b to the bottom of her stated preference order. Individual has such minimised the distance between the individual preference ordering and the social preference ordering to 0.

The basic principle of burying is to let a strong alternative competing with our preferred alternative earn low scores in the preference aggregation. Burying can swing much with voting procedures, which allocate scores to all alternatives or base their aggregation on mutual pair-wise comparisons between alternatives.

Bullet voting

Bullet voting is another simple method of how to make strong competing alternatives earn lower points. In voting procedure such as in approval voting a voter does not assign scores to all alternatives that she would otherwise honestly approve, but only to an alternative that she prefers to strengthen in the vote.

Compromising

Compromising is the most common form of strategic voting, where a voter perceives that her best preferred alternative has little chances of winning and thus decides to support alternatives, which are lower in her rank but compete with alternatives that are even lower. Many voters perceive this voting pattern as useful voting. In multiparty environments this approach often leads to so-called Duvergerian equilibria (Duverger, 1972), where only two strongest alternatives in the plurality rule elections get votes and third alternatives are devastated by strategic voting. (Niou, 2001)

Sincere rationality induces no choice problem and therefore is of a lesser interest to us in this work. Strategic rationality, which spurs strategic interactions, leads to revealing more aspects of voterés behaviour. The strategic component of the voterés considerations incites the analysis of arisen voting situations by the means of game-theoretic tools as was indicated earlier.

2.2.3 Voting procedures

Various voting procedures can lead to various voting results and do influence the strategic considerations of voters; therefore it is always beneficial to specify the voting procedure in each voting situation exogenously and before the preferences are determined. There are a vast number of different voting procedures suggested by the theory and used in the practice. What binds all the procedures in this work together, are the intuitively plausible democratic properties of all listed procedures. Our procedures respect the axioms of **anonymity** (no-oneés preferences are favoured because of who she is), **universal admissibility** (any preference profile is admissible) and **neutrality** (no alternative is favoured due to aspects different from votersô preferences).

Basic majority rule

If we apply the basic majority rule to any situation involving a choice between two alternatives x and y, then **x wins if it gets more votes than y and they tie, if they obtain same numbers of votes**. Majority principle of few giving way to the many conveys a natural alternative to dictatorship in cases, when unanimity cannot get reached. Majority rule is a trivial voting procedure saying nothing about the cases with more than two alternatives and its basic principle remains undisputed in the democratic societies.

Simple plurality voting

Plurality voting is the simplest extension to majority voting and simplest scoring rule as well. It involves broader range of competing alternatives than simple majority rule, i.e. three or more. Each voter needs to decide, to which single alternative to assign a score of 1, while assigning 0 to all other alternatives. Plurality winner is such an alternative that collects the highest number of votes. Other common names of this procedure are first past the post or winner-takes-all. There is no need for absolute majority in this procedure. Furthermore observe that only the first rank of each voter matters, thus implicitly a large mass of information on votersô preferences is lost in this procedure.

Condorcetôs voting procedure

A simple extension of the basic majority rule to a choice involving more than two alternatives while satisfying many desirable properties is embodied in the Condorcetôs voting procedure. A winning alternative is chosen by this procedure if and only if it is not defeated by a strict majority by any other alternative in a pair-wise vote. Such alternative is then called a **Condorcetôs winner**. May (1952) shows in his recognised Mayôs theorem that a SCF satisfies the axioms of anonymity, neutrality, and positive responsiveness under the condition of universal admissibility if and only if it is the general majority rule choice. The problem with this voting procedure is that it is not applicable just on any voting situation; particular cases may emerge when CVP selects no winner. Example 2.11 illustrates a simple famous case when CVP cannot select a winner.

Example 2.11 Condorcetôs paradox

Table 2.6 ï Condorcetôs paradox - preferences

Voter no.	1.	2.	3.
1st best choice	a	b	c
2nd choice	c	a	b
3rd choice	b	c	a

Table 2.7 ï Table of pair-wise comparisons

No. of wins	a	b	c
a	-	1	2
b	2	-	1
c	1	2	-

We see from the Table 2.7 that none from alternatives wins all pair-wise comparisons; a strict majority defeats each alternative at least once. Thus no alternative can be selected as a Condorcetôs winner. Ĭ

Approval voting

If we comply with approval voting, we allow each individual to vote for as many options as he desires, i.e. he may assign a score of 1 to as many alternatives as he wishes and assign 0 to all the others. Approving one alternative does not prevent from approving any other alternatives. The winning alternative is the one, which gathers the most votes. The underlying motivation attempts to foster truer revelation of votersô preferences, just because for example in contrary to plurality voting, the voters are not tempted to vote for other than their most preferred alternative, given that its probability to win is small, or for other instrumental reasons. In approval voting there is no cost of voting for an alternative that faces low probability of winning.

Plurality voting with runoff

Just like under the standard plurality voting only first-place ranks enter the count. The modification is that if no absolute majority is reached in the first round, a second round of elections takes place. The second round involves a vote only between two alternatives with the highest scores obtained in the first round. The purpose of the first round, so-called runoff is to eliminate the least preferred options. The method is widely used for single member constituencies or for presidential elections.

Bordaôs voting procedure

Bordaôs voting is sometimes called as well weighted voting. It is a scoring rule, where the scores are assigned in the following simple manner: given m alternatives, each voterôs first stated alternative obtains (m-1) points, the second stated alternative obtains (m-2) points, the third one gets (m-3) points, and so forth, down to a minimum of 0 points for the worst alternative. The scores are added up across the individuals and the option with the highest score becomes the **Bordaôs winner,** (see example 2.10). One great advantage of Bordaôs count is that this voting procedure never fails to select a winning alternative.

Blackôs voting procedure

Blackôs procedure (Black, 1958) is not demanding on description. It simply chooses the Condorcetôs winner if one exists. Otherwise it chooses the Bordaôs winner, which as we have suggested always exists.

Hare's voting procedure - single transferable vote system

In Hare's voting procedure voters are asked to reveal their full rankings concerning the alternatives. If some alternative is ranked first by more than 50% of voters, it wins the election. If none such alternative exists, the alternative with fewest first ranks is eliminated from the count and the rest of alternatives is being pushed upwards in the preference lists of the voters. Subsequently, we again determine if any alternative ranks first by more than 50% of the voters. If so, it becomes a winner. If not, another round of eliminations proceeds. Eventually, after a number of rounds of eliminations one alternative must become **Hare's winner** or a tie is established in the final round. Thomas Hare first proposed this voting procedure in year 1861.

Example 2.12 Hare's STV Let us consider 4 competing alternatives and 5 voters, whose complete preference orderings are captured in Table 2.8.

Table 2.8 – Sincere preferences, Hare's STV

	1.	2.	3.	4.	5.
1st best choice	d	b	b	a	a
2nd choice	b	a	a	b	c
3rd choice	a	c	d	c	b
4th choice	c	d	c	d	d

Table 2.9 – Shift of vote in the Hare's STV

	1.	2.	3.	4.	5.
1st best choice	d	b	b	a	a
2nd choice	b	a	a	b	b
3rd choice	a	d	d	d	d

Since any player has not placed option c at the first rank, option c is eliminated from the count and the preferences are shifted upwards as shown in Table 2.9. Since no alternative was ranked by a majority of voters on the first rank, we proceed in eliminations. We eliminate option d from the rank, since fewest voters have placed it at the first rank, as Table 2.9 shows. By eliminating option d, option b gains a majority of first ranks and wins the Hare's count. The final ordering is [b a d c]. ∎

Coombs' technique

Coombs suggested a slight modification to Hare's voting procedure and that was to eliminate during the rounds of elimination such an alternative that is ranked last by the largest number of voters. The qualification criterion for victory stayed the absolute majority of the first ranks in voters' stated preference profiles.

Example 2.13 Coomb's technique Take the preference profile displayed in Table 2.8 from the previous example. Since in Coomb's technique we eliminate the alternative with largest number of last ranks, we eliminate option d instead of c, as we would do in Hareôs voting procedure. Table 2.10 captures the preference profile after the first round of eliminations.

Table 2.10 ï Shift in preferences, Coombôs technique

	1.	2.	3.	4.	5.
1st best choice	b	b	b	a	a
2nd choice	a	a	a	b	c
3rd choice	c	c	c	c	b

Now already after the first round of elimination, option b has scored a majority of first ranks and thus became the Coombôs winner. In next round option c would be eliminated, since most voters have ranked it at the last place. The final social ordering becomes [b a c d]. We can see how a simple change of a voting procedure may lead to two different voting results. Ĭ

Max-min voting technique

Max-min voting procedure requires from all voters to form their strict individual preference orderings to allow pair-wise comparisons between alternatives. The procedure first finds a number of individual wins of each alternative over every other alternative summing the votes across all voters. In other words we construct a binary comparison matrix across all alternatives. The voting procedure then finds the lowest number of these pair-wise wins related to every alternative, which equals to finding a lowest number in a row of the binary comparison matrix. Finally, the procedure ranks the alternatives according to the retrieved minima. Consider example 2.14.

Example 2.14 Max-min voting procedure with 4 voters and 4 alternatives

Consider a preference profile of 4 voters displayed in Table 2.11 and construct a table of pair-wise wins. Those are displayed in Table 2.12.

Table 2.11 ï Honest preferences of 4 voters

	1.	2.	3.	4.
1st best choice	a	c	c	c
2nd choice	d	a	a	a
3rd choice	b	d	d	b
4th choice	c	b	b	d

Table 2.12 ï Pair-wise comparisons between alternatives

	a	b	c	d	Min	Order
a	-	4	1	4	1	2.
b	0	-	1	1	0	3.
c	3	3	-	3	3	1.
d	0	3	1	-	0	3.

e.g. first row reads: 4 voters prefer a to b, 1 voter prefers a to c, and 4 voters prefer a to d.

Find the minima from the numbers of pair-wise wins. In our example those are summarised in column ñMinò in Table 2.12. Rank the alternatives according to the number of minima. The voting procedure in this way looks for the highest minimum of individual wins in pair-wise votes between the alternatives, or equivalently this function chooses those alternatives, whose worst showing against other alternatives is as good as possible. (Turnovec, 2001) Ĭ

Copelandôs voting procedure

The last proposed procedure as well requires revelation of total preference orderings from all voters. The procedure attributes a number of wins and a number of losses to each alternative. The alternative wins over other alternative, if it gains a majority of votes in a pair-wise vote. The alternative looses if it gains less votes in a pair-wise vote than the competing alternative. The social ordering consists of an ordered list of differences between a sum of wins and sum of losses of each alternative. Copelandôs procedure obviously selects the Condorcetôs winner if it exists, because the Condorcetôs winner essentially collects all wins in pair-wise comparisons and suffers no losses.

Example 2.15 Copelandôs procedure Consider a case with 7 voters and three competing alternatives a, b, c. The preference orderings of the 7 voters correspond to those displayed in Table 2.13.

Table 2.13 ï Sincere preferences, Copelandôs procedure

Voterôs preferences	**1.**	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**
1st best choice	a	a	b	a	d	c	d
2nd choice	c	c	a	d	b	a	c
3rd choice	b	d	d	c	a	b	b
4th choice	d	b	c	b	c	d	a

Table 2.14 captures how a win or a loss is determined.

Table 2.14 ï Counting the wins and losses in Copelandôs procedure

Comparison	**a>b**	**a>c**	**a>d**	**b>c**	**b>d**	**c>d**
No. of Votes	4	5	5	2	3	3
Comparison	**b>a**	**c>a**	**d>a**	**c>b**	**d>b**	**d>c**
No. of Votes	3	2	2	5	4	4
Wins	**a**	**a**	**a**	**c**	**d**	**d**
Losses	**b**	**c**	**d**	**b**	**b**	**c**

Table 2.15 counts the numbers of wins and losses, determines the relevant difference out of which the final social ordering is formed.

Table 2.15 ï Final social ordering in Copelandôs procedure

Alternative	**No. of Wins**	**No. of Losses**	**Difference**	**Social order**
a	3	0	3	1.
b	0	3	-3	4.
c	1	2	-1	3.
d	2	1	1	2.

Example 2.16 Copelandôs procedure selecting no winner: Consider Condorcetôs paradox in example 2.11. Here alternative a wins over b (thus it counts 1 win), but looses with c (it counts 1 loss): No. of Wins minus No. of Losses = 1-1 = 0. Alternative b wins once over c, but looses with a; final score is again 0. Alternative c wins over a, but looses with b, 1-1 = 0. Copelandôs procedure has therefore selected no winner. Ĭ

2.3 Voting experiments: Measuring sensitivity of strategic voting to information

The third subchapter is devoted to computation-based simulations of voting. We use computation-based simulations to randomly generate a collective preference profile of a set of voters. All but one of these voters will cast their votes sincerely in order to come at a collective decision, which the residual voter will attempt to manipulate through her strategic vote. We target to estimate the change in the success rate of strategic voterös manipulations, given that her information about other votersô preferences would shrink. The information that the agent possesses will shrink because we will assume away some voters were able to vote rationally. In consequence they will not be able to form their complete preference orderings. They will instead vote randomly under a specified probability distribution. In closing part of the chapter we will analyse the informational issues while allowing numerous strategic voters to interact. Moreover they will not share the information about what they individually aim to accomplish.

2.3.1 Methodology

The random generation of a collective preference profile is commonly in literature called **a culture**. An overview of different preference generating cultures has been provided in the exposition of Laslier (2009). From among different specified Rousseauist, distributive or spatial cultures, the ó*impartial culture*ô seems to be the most adequate for our simulations.

The impartial culture attributes to each individual sincere voter a preference ordering from among m! strict total preference orderings, where m is the number of competing alternatives. This agrees consistently with our assumptions on the individual preference relation. The other essential characteristic is that the culture chooses the individual preference orderings **uniformly and independently**. The culture hence treats all the alternatives symmetrically and learning something about preference orderings of some voters yields no information about the rest of voters or alternatives. We obtain a **uniform probability distribution** over the set of individual preference profiles.

Due to the symmetrical treatment of alternatives, we may fix the preference profile of the single strategic voter by attributing to her an alphabetical ordering of alternatives [a b c dé m]. The uniformity and independence property of the preference generating process allows us to choose this approach without the loss of generality.

Knowledge of the full collective preference profile

The role of a fully informed strategic voter is straightforward. She calculates all possible distances that could occur between her individual preference ordering and the aggregated social preference orderings and she selects such voting pattern so as to minimise this distance. We have specified the means of calculating the voting distance earlier.

First the voter evaluates her preference ordering by her individual weights. We have contended these weights to correspond to Borda scores of m alternatives. She adds up all the votes of other voters accordingly to a selected voting procedure. She determines, which social preference orderings could occur given her vote. She evaluates all these potential social orderings by social weights consistently with an earlier described manner: if particular alternative a_j bears weight r_j in the individual preference order, an equal weight s_x will be attached to such position in the social order, at which alternative a_j was placed by the aggregation rule. As a next step, the voter calculates the respective distances between her individual ordering and all potential social preference orderings, which could emerge given her vote. Finally she chooses such voting pattern, which minimises the relevant distance argument and she votes accordingly.

The voter may do as described for various reasons. First her choice finalises the aggregation of the social preference ordering, and second she is the sole strategic voter and hence she faces no uncertainty about voting patterns of other voters.

We shall ask a particular question, namely: how successful is the voter in her strategic manipulation? That decomposes into how many times **did the voter have** and how many times **did she use** the opportunity to strategically manipulate the voting result so as to come at a social ordering, which is **closer** to her preference than an ordering resultant from her eventual sincere voting? The success rate may be calculated either as a number of cases when the strategic voter succeeded to lower the relevant distance relatively to distance occurring after sincere voting or we may calculate the success rate as a number of cases when the voter succeeded to manipulate the voting result so as to make it copy her own individual preference order and thus made the relevant distance equal zero. In our simulations we shall evaluate the former statistic.

Under full information, the number of opportunities that the strategic voter **had** to manipulate the voting result fully matches the number of opportunities that the voter **used.** The difference between the two statistics emerges under voterõs restrained information.

We simulate the preference profile of (n-1) voters and m competing alternatives using 100 000 independent draws for all voting procedures specified in previous chapter except for the majority voting procedure. Majority voting, as it was earlier specified, is clearly non-manipulable, since it involves a choice between only two alternatives and hence yields a trivial result. All simulation codes are provided on a CD carrier attached to the thesis, whose contents are described in Appendix 2.D.

Information about full rankings of a subset of voters

The manipulating ability of a limitedly informed voter may be hampered by a lack of knowledge about voting patterns of a subset of the electorate. This may happen, for instance, when some part of the electorate does not meet all sufficient conditions for their preference rankings to be classified as preference orderings. Alternative interpretation says that a part of the electorate may from various reasons behave **non-rationally** in their decision-making. May it be due to their bounded rationality, inadequate cognitive abilities, indifference, laziness, should they be constrained by time pressure, lack of appropriate incentives, or by any other feasible constraint, due to some of these reasons some voters may not able to construct complete, transitive, reflexive and anti-symmetric preference orderings from alternatives that are offered to them. From now on we will refer to such voters as to **ónon-rational votersô**.

The individual strategic voter can nevertheless determine some partial scores that the alternatives have gathered from sincere voters, about which she has information and which do behave rationally. Nevertheless the voter has to think about all possible voting patterns of the residual non-rational voters. We will assume the strategic voter to know the distribution, in which the non-rational voters do vote. Particularly we will assume their voting patterns to be distributed **uniformly and independently.**

Now, we may think of some simple heuristic rules that the strategic voter could use given her limited information. For instance, we may think her to attempt to manipulate the partially aggregated social ordering as if it was the fully aggregated social ordering. Then we could calculate the number of successful manipulations over the number of all manipulations that were possible if the voter had known the complete collective preference profile. Nevertheless such heuristic rule could often lead into situations, where the strategic voter would end up with even worse payoffs than she would receive under sincere voting.

Another heuristic option is to make the strategic voter calculate all possible social orderings, into which the partially aggregated social ordering could lead and make the voter vote according to a min-max principle. That means to make her select such voting pattern, which would lead to such potential social orderings, from among which the furthest one from the individual order is the closest one across different voting patterns. The voter would minimise the maximum distance.

Example 3.1 Min-max heuristic rule Think an abstract case, in which the manipulating agent can either vote honestly H or has 3 different strategies of manipulation of the voting result M1-M3. Given information that she has about a subgroup of voters and given 3 different possible combinations C1-C3 of voting patterns of the residual voters, she can calculate the potential distances between hers and the possible final social orderings. We depict them in Table 2.16.

Table 2.16 ï Max-min heuristic rule for decision making, reduced information

Vote of 1	Votes of residual voters		
	C1	C2	C3
	Distance from [a b c]		
H	2	1	3
M1	4	1	0
M2	3	2	2
M3	5	0	1

The voter discerns that if the real combination of voting patterns of residual voters was the one corresponding to C3, her best choice of manipulation would be M1. Similarly, if the real combination of voting patterns of the residual voters corresponded to C2, her best response would be to vote accordingly to M3. Nonetheless, given her lack of knowledge she heuristically chooses one of strategies H or M2, given that the worst payoff that she could end up with is distance 3, whereas using M1 she could end up with distance 4 and using M3 she could end up even worse off with distance 5.Ĭ

There are many other heuristic rules that the voter may stick to. For instance she can simply opt for a pattern that could bring her the highest utility, given a lucky chance. She would always select such voting pattern, which could lead her to the minimal voting distance under one combination of other votersô strategies, but she would not take into regard other potential larger distances associated with the same pattern but different strategies of other voters.

Alternatively, the voter could stick to a minimalistic approach to strategic voting. She would opt for strategic voting only in cases, where the payoffs from her insincere voting strategy would never be dominated by payoffs accruing to her honest voting. That means that under any possible combination of other votersô preferences the payoffs from insincere voting need to be always higher or equal under strategic voting than under sincere voting. Otherwise the voter votes sincerely.

Nevertheless to be consistent with our previous calculations, we make the voter decide for a concrete voting pattern according to a minimalisation of a weighted distance between her individual preference ordering and all plausibly aggregated social orderings associated with that voting pattern. The weights would be the probabilities of a particular combination of voting patterns to take place. Since now it is not a random chance, but concealed or non-rational preferences that determine the voting result, the attached weights need not to be uniform.

Example 3.2 Voting under limited knowledge Let us think a voting situation with 6 voters, out of which voter 1 is a strategic voter, other 3 voters vote sincerely, and last 2 voters vote non-rationally. Let us think 3 competing alternatives a, b, c, aggregated using a simple plurality rule. Let us assume that the strategic voter possesses information about complete rankings of all rational players 2, 3, and 4, and naturally the strategic voter has no information with regard to the preference of voters 5 and 6. For simplicity, let us assume that the votes of players 2, 3, 4 cancel each other out, for instance voter 2 votes a, voter 3 votes b, voter 4 votes c. Possible pairs of votes of the two residual voters and the corresponding social distances are captured in 17.

Table 2.17 ï Decision making on the basis of weighted distances, reduced information

	Possible combinations of votes of 5 and 6						
	(a,a)	(a,b)	(a,c)	(b,b)	(b,c)	(c,c)	
Probabilities	1/9	2/9	2/9	1/9	2/9	1/9	
Vote of 1	**Distance from [a b c]**						**Weighted distance**
A	0.707	0	1.414	1.414	1.759	2.449	1.21
B	0	1.414	1.759	1.932	2.449	2.828	1.78
C	1.414	1.759	2.449	2.449	2.828	2.639	2.29

There are 6 different combinations of residual votersô voting patterns, which obviously differ in probabilities to occur. Combinations (a,a), (b,b) and (c,c) are more rare under the uniform distribution of preferences with probability 1/9 to occur each; combinations (a,b), (a,c) and (b,c) are more probable, each with associated probability 2/9. In such situation, voter 1 should naturally vote sincerely, what would be prescribed to him by the weighted distance associated with sincere voting. Ĭ

To wrap up, the strategic voter under a lack of knowledge about the preference orderings of some subset of other voters undergoes through this mental exercise. She calculates the partially aggregated social ordering from the information she already possesses, she lists all possible combinations of other votersô voting patterns and determines their respective probabilities, she determines the weighted distances corresponding to all of her voting patterns, and finally she votes accordingly.

The questions we ask under limited information look in principle for identical answers as the question raised earlier under full information. Given a number of voters about which the strategic voter possesses information, how many times **did the agent use the opportunity** to manipulate the voting result into her advantage? How many times **was she successful** in her manipulation, in the sense that the resulting social ordering was closer to her individual ordering than would be a social ordering from sincere voting? How many times did she manipulate with **adverse consequences**, in the sense that the resulting social ordering was further than would be a social ordering resultant from sincere voting? How do these answers change, given that the strategic agent **knows of fewer** other votersô profiles? At what fraction of voters about which the strategic voter has information **does the strategic agent lose the ability** to manipulate the voting result? We contrast all these figures to figures obtained from agentõs full information to obtain relative measures of ñsuccessful manipulationò. We again answer our questions for all 10 manipulable voting procedures as earlier.

Information about uniformly truncated rankings of all voters

The other manner of introducing incompleteness of knowledge of the whole collective preference profile is to **truncate** the orderings, about which the strategic voter has information. The option of limiting the strategic voterõs knowledge through truncation of other votersô known preferences makes available a multitude of combinations of how the orderings could be truncated and for which subsets of voters these truncations would apply. It is almost impossible to effectively approach all of these different combinations. Therefore we choose to truncate the known orderings **symmetrically** across all non-rational voters.

We truncate the orderings from their end. The strategic voter hence will not know the precise order of the two, three or more last alternatives from all non-rational votersô preference orderings. These non-rational voters still vote sincerely to the extent, to which they are able to form their preference orderings. For instance, their inability to compare last two alternatives does not prevent them to state which alternative is the best one for them, if the total number of alternatives is three or more. The assumption of truncations from the end is reasonable in the sense that the non-fully-rational voters feasibly do not care for the lower end of the social preference order and they tend to rather care for the winning or for a number of first few winning alternatives.

The strategic voter proceeds in her decision-making as previously. She aggregates all information she has got into a partial social ordering. She determines all combinations of residual voting patterns that could complement the partial social ordering and their respective probabilities. Given her own voting pattern she calculates the weighted distances that could occur, and she votes accordingly to minimise the voting distance.

We study this sort of limitation of strategic voterӧs knowledge for all voting procedures, which take into regard full preference orderings of sincere voters. Those are namely Condorcetӧs voting procedure, Bordaӧs count, Blackӧs procedure, Hareӧs STV, Coombsô procedure, max-min procedure and Copelandӧs voting procedure.

Voting procedures, which do not take full preference orderings into account, are manipulable to at a constant rate, with no respect to how many first ranks the strategic agent knows. The information about the full orderings of the sincere voters is hence abundant for the strategic voter.

Cases with numerous strategic voters

The last method of how to introduce incomplete knowledge of the collective preference profile by an individual voter is to relax the assumption of a single strategic voter. Whereas previously the strategic agent has lost the information about particular voters, because we had assumed away their rationality and hence the ability to form preference orderings, now the strategic voter looses the information about other voters, because some of them newly became sophisticated voters in the sense that they no longer vote only according to their sincere preference orderings. They are themselves capable of strategic utility maximization through strategic voting behaviour and all assumptions as on the first strategic voter apply likewise on them. Not to stray away from our subject of study, we do not allow these numerous strategic voters to vote in coalitions under complementary strategies. That would open too many new questions regarding to the possibilities of manipulation, which are beyond the scope of our study.

The strategic voters vote in our simulations individually and simultaneously. They vote individually, because their preferences may differ and moreover we do not allow them to communicate them through. Even if we allowed the strategic voters to communicate their preferences between each other, they would most probably do so through a cheap talk, which none would believe. They vote simultaneously, because we see no reason as of why some of voters should be endowed with an advantage of playing second or later. We treat all the strategic voters symmetrically.

Regarding informational endowment of the strategic voters, these possess all complete information about sincere votersô orderings, given that we assume all sincere voters to be fully rational and hence capable of forming complete, transitive and anti-symmetric preference orderings. We place no non-rational voters into this setting and we rather limit the information of strategic voters by having more of them. Increase in numbers of the strategic voters, while maintaining constant top number of all voters limits the individual information of a strategic voter, because as we said, they are assumed not to know about each otherôs preference orderings. They just know that the preference orderings of other strategic voters are generated independently from a uniform probability distribution.

We could have let the strategic voters know of each other preferences by assumption. Nonetheless, such knowledge would lead our strategic voters to search for their mutual best responses to their strategies and for the corresponding Nash equilibria. This would again circumvent the strategic voting issues under informational constraints.

The principal question stays: how many times is the strategic voter **successful** in her voting manipulations, given that she is limited in her knowledge by lack of knowledge of preferences of other strategic voters? How does the success rate of manipulation **change** if we **add more strategic voters**? How many times does the strategic voter choose a strategy that leads to **adverse results**? We contrast our results from this setting with the results from under the cases of incomplete information due to presence of non-rational voters. These counterfactual results provide a benchmark of successful manipulations under limited knowledge.

The strategic voter decides identically as earlier: to her the other strategic voters are just voters, about which she has no information. The strategic voter aggregates the partial social orderings from the information she already possesses. She lists the same possible combinations of other voterόs voting patterns and attributes them the same probabilities as under the presence of non-rational voters. She determines the same weighted distances corresponding to all of her voting patterns and she votes identically as she would vote if other strategic voters were non-rational voters, because likewise she has no information about them. All strategic voters act symmetrically in this voting exercise.

What is different is the voting result and corresponding social ordering that emerges, since other strategic voters do not vote randomly as non-rational voters would do. Against these altered results we evaluate the success rate of each strategic voterόs voting manipulation.

2.3.2 Results

In the following graphs and figures we present (when possible) the probabilities of a successful strategic voting manipulation, which we have obtained by computation-based simulations of individual votersô preferences for ten different voting aggregation rules. We present the probabilities of strategic manipulation for varying informational degrees; we do so for different numbers of interacting players and for differdent numbers of competing alternatives. We naturally start by the benchmark model, which assumes full information of a sole strategic voter.

Results ï full knowledge of the collective preference profile

Table 2.18 provides the complete tabulated overview of the opportunities for strategic manipulation of a sole strategic voter under full information. As we have already suggested, the number of opportunities for manipulation under full information mirrors the number of actual successful manipulations. Fully informed strategic voter moreover cannot end up with worse payoff by voting strategically than by voting sincerely. Table 2.19 and Table 2.20 present summary statistics on the probability of manipulation under full information by number of players and number of competing alternatives. Figure 2.2 and Figure 2.3 graphically outline the evolution of room for strategic manipulation for all considered voting procedures, related to the number of players. Figure 2.4 shows the histogram of probability of manipulation under full information, for all voting procedures, which are subject to strategic manipulation.

Table 2.18 ï Optimal number of voting manipulations, full information

Full information		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	M = 3	*	0.111	0.136	0.159	0.150
	M = 4	*	0.376	0.265	0.376	0.378
Condorcet's voting	M = 3	0	0	0	0	0
	M = 4	0.207	0.150	0.124	0.107	0.082
Approval Voting	M = 3	*	0.332	0.432	0.160	0.289
	M = 4	*	0.684	0.704	0.592	0.562
Plurality w\ runoff	M = 3	*	0.222	0.136	0.122	0.107
	M = 4	*	0.406	0.345	0.486	0.474
Borda' s Count	M = 3	0.333	0.196	0.232	0.234	0.219
	M = 4	0.794	0.578	0.598	0.585	0.536
Black' s Procedure	M = 3	0.332	0	0.023	0.014	0.016
	M = 4	0.625	0.259	0.316	0.267	0.225
Hare' s STV	M = 3	0	0.111	0.137	0.174	0.118
	M = 4	0.249	0.189	0.272	0.314	0.305
Coombs' Procedure	M = 3	0.166	0.111	0.114	0.079	0.107
	M = 4	0.247	0.275	0.254	0.228	0.251
Max - min Procedure	M = 3	0.166	0.361	0.354	0.325	0.282
	M = 4	0.542	0.541	0.576	0.582	0.562
Copelandôs Procedure	M = 3	0	0	0	0	0
	M = 4	0.167	0.147	0.128	0.109	0.087

* For plurality, Condorcetôs and Approval voting procedures, the results are trivial for n=2

Table 2.19 - Summary statistics for probability of manipulation, full information, m=3

Full information, m=3	n = 2	n = 3	n = 5	n = 7	n = 11
Average	0.142	0.144	0.156	0.127	0.129
Min	0	0	0	0	0
Max	0.333	0.361	0.432	0.325	0.289
Variance	0.019	0.016	0.019	0.010	0.010

Table 2.20 ï Summary statistics for probability of manipulation, full information, m=4

Full information, m=4	n = 2	n = 3	n = 5	n = 7	n = 11
Average	0.404	0.361	0.358	0.365	0.346
Min	0.167	0.147	0.124	0.107	0.082
Max	0.794	0.684	0.704	0.592	0.562
Variance	0.052	0.032	0.036	0.032	0.031

We observe three apparent and yet anticipated results: 1. strategic manipulation opportunity levels vary substantially across the used voting procedures, 2. strategic manipulation opportunity levels for four competing alternatives surpass those of three alternatives in every simulated procedure for all considered numbers of voters, 3. the number of sincere voters does not affect the manipulation opportunities, if we allow for wider confidence intervals, the opportunity for strategic manipulation is shown to be marginally diminishing in the number of voters. Let us analyse these points separately.

1. Levels of strategic voting vary substantially across the used voting procedures

Consider Figure 2.2 and Figure 2.3 in this regard. In both figures we can upon a careful look discern a distinct arrangement of layers.

Figure 2.2 ï Probabilities of manipulation, full information, m=3

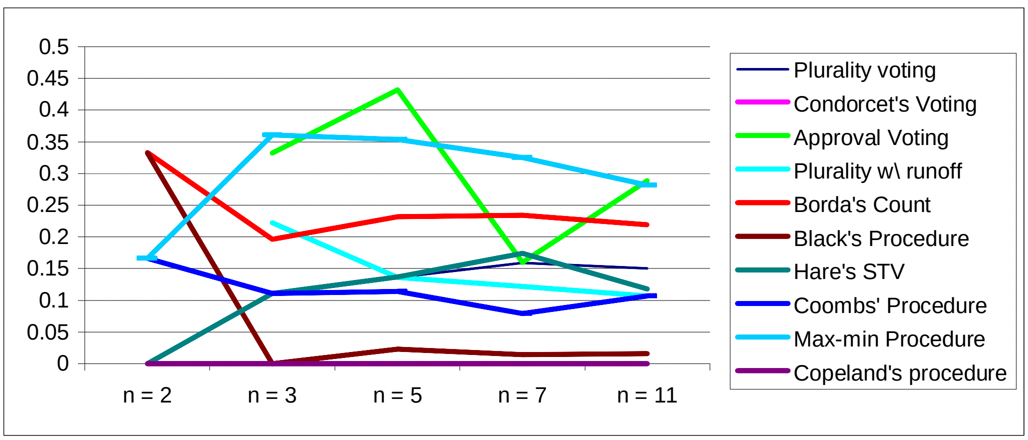
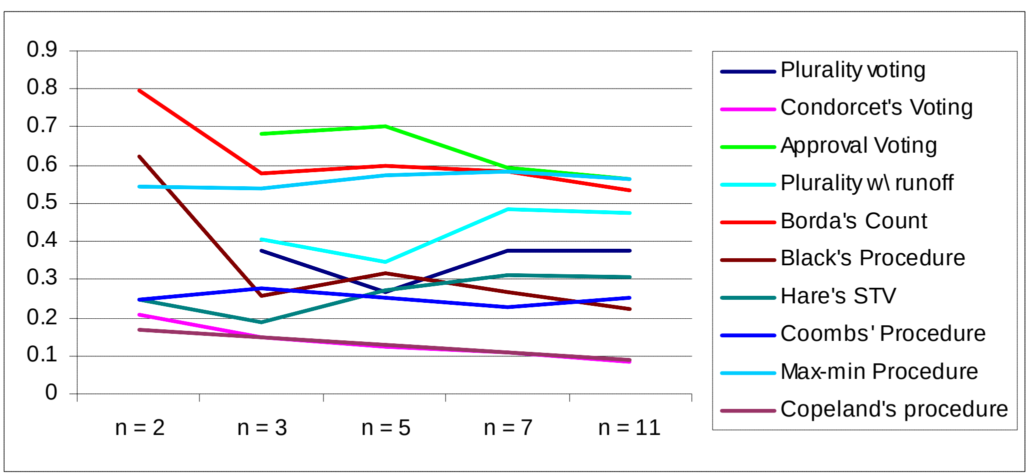


Figure 2.3 ï Probabilities of manipulation, full information, m=4



We see that the lowest probability of manipulation can be attributed to Copelandøs, Condorcetøs and Blackøs voting procedures. This comes at no surprise, as these procedures are exactly the Condorcet- consistent procedures, in other words they always select a Condorcetøs winner if it exists. The only outlier from the relevant group of low probabilities is the case of 2 voters in Blackøs procedure. Here the strategic voter misrepresents his preferences so as to force the procedure to select the winner on the basis of the Bordaøs count; since the number of voters is minimal, the opportunity for strategic voting appears frequently. The second layer of manipulability of voting procedures involves three elimination procedures: Coombsô and Hareøs procedures and Plurality with runoff voting procedure. Although these procedures are not Condorcet- consistent, the probability of manipulation is only slightly higher than in the former group. The reason is the difficult process of consecutive rounds of eliminations, where it is not only necessary for the strategic voter to find a situation where her vote is pivotal, but moreover she has to find such pattern of misrepresentation of her preferences, which does not harm her in later rounds of eliminations through the transferability of points in the preference aggregation. The last most manipulable layer groups together the remaining procedures. Those are Approval voting, Max-min voting and Bordaøs count.

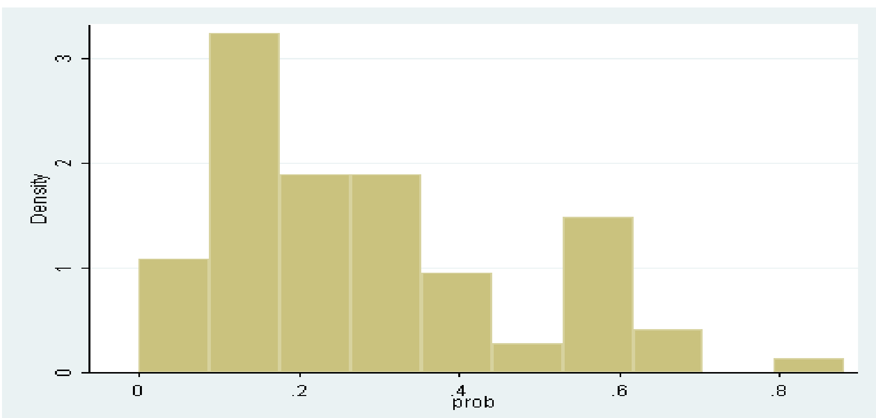
Approval voting could have been expected to be one of the most manipulable procedures, since in our design with only strict preference orderings, all sincere voters vote by one point, whereas the strategic voter selects from among 7 different strategies in case of 3 competing alternatives: [1 0 0], [0 1 0], [0 0 1], [1 1 0], [1 0 1], [0 1 1] and [1 1 1] or from 15 strategies in case of 4 competing alternatives: [1 0 0 0], [0 1 0 0], [0 0 1 0], [0 0 0 1], [1 1 0 0], [1 0 1 0], [1 0 0 1], [0 1 1 0], [0 1 0 1], [0 0 1 1], [0 1 1 1], [1 0 1 1], [1 1 0 1], [1 1 1 0], [1 1 1 1]. Approval voting effectively creates on average identically partially aggregated social orders as Plurality voting does, but approval voting offers more manipulating strategies to the strategic voter. Necessarily the probability of manipulation is always higher in Approval voting than in Plurality voting. Objections to the assumptions on the individual preference orderings come natural at this place. The Approval voting procedure is moreover the only procedure that effectively allows non-voting to the strategic player. In such case the strategic voter allocates 1 point to all competing alternatives, which cancel each other out, leaving the score unchanged.

Beside approval voting, Bordaôs count and Max-min voting procedures are characterised with the highest susceptibility to individual manipulation. The common feature of these three procedures is that they allow the strategic voter to allocate wide ranges of scores to individual alternatives. This is natural for Bordaôs voting procedure, where the strategic voter can by misrepresentation of her preferences make a particular alternative score by 2 points less or more in case of 3 alternatives or by 3 points less or more in case of 4 alternatives. This property of the Bordaôs voting procedure gives to the strategic voter power to swing with voting scores more flexibly.

In case of the max-min procedure, the voter effectively swings with the votes by manipulation of the structure of the binary comparison matrix. Since the procedure orders the minima of the particular rows of the aggregated binary comparison matrices, the use of the impartial preference generating culture contributes to arising of many voting ties between these minima. That does in turn facilitate strategic manipulation.

Simple plurality voting rule has not been so far mentioned in our comments. An analogical argument related to the use of impartial preference generating culture, which fosters voting ties, applies here. Had we been using other than the impartial preference generating culture, fewer ties would occur in the preference aggregation and consequently the strategic voter would face fewer opportunities for gainful misrepresentation of her preferences.

Figure 2.4 ï Histogram of probabilities of strategic manipulation, full information



The considerably higher susceptibility to strategic manipulation of the three afore-mentioned voting procedures (Borda, Max-min, Approval) manifests itself visibly on the histogram of the probabilities of manipulation. The three procedures contribute to the second peak in the probability distribution, just below the 60% mark in Figure 2.4. The Table 2.21 confirms the different levels of voting manipulation under varying voting aggregation rules. We use simple ordinary least squares (OLS) regression to explain the variability in the susceptibility to strategic manipulation. The susceptibility is captured in the explained variable Prob. The reader should understand that it is the entries of Table 2.18, which correspond to our observations examined by the regression. Regarding the explanatory variables n_i captures the number of players, while $m4_i$ is a dummy signifying that we choose from 4 voting alternatives rather than from 3 alternatives. The rest of the explanatory variables Plurality to Copeland are dummy variables corresponding to the 10 different voting aggregation rules. They are included in the (10x10) vector $proced_i$, to which correspond 10 coefficients contained in the (10x1) vector ŭ. The formal model can be expressed as follows:

$Prob_i$ = б n_i + ɔ $m4_i$ + ŭô$proced_i$ + Ǘ,

index i does not stand here for the individual voter, but for a particular observation of the susceptibility to strategic manipulation.

Regression table 2.21 ï Probability of manipulation, full information

Source	SS	df	MS
Model	9.06521324	12	.755434436
Residual	.482688736	72	.00670401
Total	9.54790197	84	.1136655

Number of obs	84
F(12, 72)	112.68
Prob > F	0.0000
R-squared	0.9494
Adj R-squared	0.9410
Root MSE	.08188

Prob	Coef.	Std. Err.	t	P>|t|		[95% Conf.	Interval]
n	-.005	.002	-1.99	0.051	*	-.011	.000
m4	.250	.019	13.14	0.000	***	.212	.288
Plurality	.155	.035	4.36	0.000	***	.084	.226
Condor	-.084	.044	-1.91	0.060	*	-.172	.003
Approval	.381	.035	10.68	0.000	***	.309	.452
Runoff	.198	.035	5.58	0.000	***	.127	.270
Borda	.337	.031	10.58	0.000	***	.273	.400
Black	.114	.031	3.59	0.001	***	.050	.177
Hare	.093	.031	2.93	0.004	***	.029	.157
Coombs	.089	.031	2.82	0.006	***	.026	.153
Max-min	.335	.031	10.53	0.000	***	.272	.399
Copeland	-.090	.044	-2.05	0.044	**	-.179	-.002

*** significant at 1%, ** significant at 5%, * significant at 10%

The column of coefficients allows us to rank the particular procedures according to their susceptibility to manipulation. The previously described pattern applies: the Condorcet-consistent procedures are least manipulable, the elimination procedures follow, while the scoring rules like Approval, Max-min or Bordaǿs count are most manipulable. Use of the Copelandõs or Condorcetõs voting procedures relatively lowers the susceptibility to manipulation in a given voting situation.

2. Levels of susceptibility to strategic manipulation for 4 competing alternatives surpass those of 3 voting alternatives in every simulated voting procedure for all considered numbers of voters

This result is apparent from the both the regression captured in Table 2.21 and from the relative comparison of levels of susceptibility to manipulation across Figures 2.2 and 2.3. The regression table 2.21 suggests that the susceptibility to strategic manipulation grows by one quarter, if we let 4 alternatives compete. In other words, if we use 4 competing alternatives instead of 3, there is a 25% higher chance that the strategic voter comes to a situation where it is beneficial for her to strategically manipulate her voting preference. Nonetheless, we have to be very cautious not to generalise this result with respect to the higher numbers of competing alternatives. The pattern does not have to be increasing in the number of alternatives in the least. A sound expectation for this pattern would be to be non-linear and rather depend on the difference (n-m) if not on a ratio of the number of voters and number of competing alternatives (n/m).

We have moreover seen that the Copelandõs and Condorcetõs procedures were not manipulable at all in our simulations under 3 alternatives, while they were manipulable by a positive probability under 4 alternatives. From the summary statistics in Table 2.19 and Table 2.20 we observe a jump in all minimal, average and maximal levels of susceptibility, when we compare the cases with 3 alternatives and 4 alternatives. The maximal levels jump roughly by 30%.

Table 2.22 ï Two-sample t test with unequal variances

```
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
 m=4 (0) |      47    .3724255    .0278792    .1911299    .3163077    .4285434
 m=3 (1) |      37    .1772162    .0183765    .1117801    .1399469    .2144855
---------+--------------------------------------------------------------------
combined |      84    .2864405     .020462     .187537    .2457424    .3271385
---------+--------------------------------------------------------------------
    diff |            .1952093    .0333908                .1287095    .2617091
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t =   5.8462
Ho: diff = 0                     Satterthwaite's degrees of freedom =  76.2608

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 1.0000         Pr(|T| > |t|) = 0.0000          Pr(T > t) = 0.0000
```

This difference in means can be also confirmed by the two-sample t test with unequal variances performed in Table 2.22. The two compared groups coincide to two classes of simulated probabilities, group 0 corresponds to the simulations with 4 competing alternatives and group 1 with 3 competing alternatives. The t test rejects the H_0 hypothesis of equal means, while accepting the H_a hypotheses of unequal means or a positive difference between the means.

3. The number of sincere voters does not affect the manipulation opportunities under full information, if we permit just one individual voter to vote strategically
We have simulated the voting processes for 5 different numbers of voters, that is for n = {2, 3, 5, 7 and 11}. We have chosen these particular values since we focus on voting manipulation in small groups or committees, where the informational assumption that a particular voter might know all or majority of other votersô preference profiles is feasible. We have simulated the voting procedures for odd numbers of voters (and for 2 voters), so that we would preclude already high number of voting ties.

Table 2.21 shows a very slight and marginal negative trend of susceptibility to voting manipulation in the number of voters. We cannot reject the H_o hypothesis of no impact of this variable at 5% confidence level, and we have to allow for wider confidence intervals to be able to reject the H_0. On the other hand if we do so, the 90% confidence interval includes 0 as a feasible regression coefficient. The logic of our expectations for the coefficient to be negative is nevertheless straightforward: the more voters are involved in a voting situation, the lesser relative weight of one vote should become, in the sense that the strategic voter becomes less often pivotal.

Judging the different voting procedures separately in this regard would be dangerous from the statistical point of view, since twice we would consider only 5 observations and never more than 10 observations. We might not even postpone this question until our dataset grows by the observations from the reduced informational settings. There could be found explanations on why the susceptibility to strategic manipulation would grow in the number of voters under reduced informational settings.

Still, if we decided for evaluation of the n coefficient for different voting procedures separately, we could only conclude from Table 2.18 that it is only in Condorcetôs, Copelandôs and Plurality voting with runoff voting procedures with 3 alternatives that the susceptibility to strategic voting decreases monotonically in the number of eligible participants. Such conclusions are rather weak.

To conclude our inference about the statistical properties of the susceptibility to voting manipulation under full information, we present in Figure 2.5 the scatter plot of the actual values of the susceptibility to manipulation versus the values predicted by the linear regression. The grey area represents the 95% confidence interval of the linear prediction.

Figure 2.5 ï Probability of manipulation vs. fitted values, full information

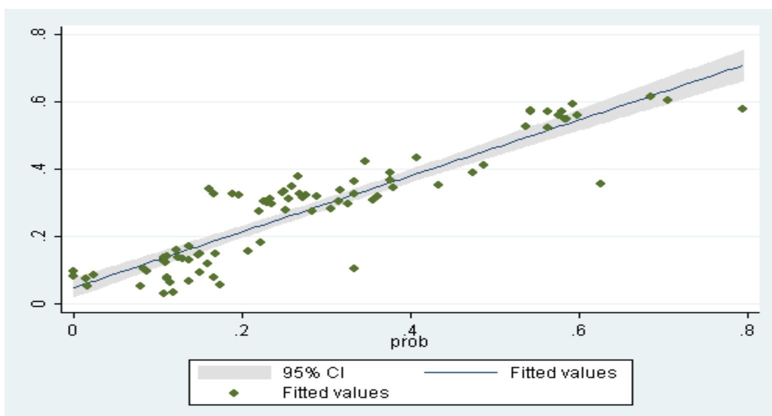


Figure 2.5 discloses slight differences in the variance between the group with smaller susceptibility to manipulation and group with higher susceptibility. This observation is not uncommon, since as we saw in Figure 2.4 and even previously that there are much more observations on lower susceptibilities to strategic manipulation, which tends to be associated with larger variance.

Table 2.23 ï Shapiro-Wilk test for the normality of residuals

Shapiro-Wilk W test for normal data

Variable	Obs	W	V	z	Prob>z
resid	84	0.86043	9.972	5.053	0.00000

In Table 2.23 we perform a Shapiro-Wilk test for the normality of residuals, as a statistical test about the assumptions for OLS. We fail to find enough supportive evidence to confirm the normality, which would break one of the OLS assumptions. Nonetheless we perceive the result of this test only as a supportive statistic.

Results - information about full rankings of a subset of voters

The results of our simulations in settings where the strategic voter does not know the full collective preference profile provide a wider spectrum of aspects to analyse. In this section the reduction of information consists in letting the strategic voter know about the complete preference profile except for a preference ordering of one sincere voter. Our simulations in these settings have yielded results of 4 kinds.

First, we have simulated a probability that the strategic voter **attempts for strategic manipulation**. Since the strategic voter does not have the full information, she is coerced to decide on the basis of the weighted distances (see methodological part 3.2.2 or example 3.2) instead of actual voting distances. The number of attempts for strategic voting may therefore be both higher and lower than the actual number of cases when the strategic voter would strategically manipulate her voting pattern had she known the full collective preference profile. The fact that the voter does not know whether to manipulate or not, propagates the residual three kinds of results.

We provide the simulated probabilities of occurrence of cases, when the strategic voter decides to manipulate the result and she acquires the same voting distance as she would acquire had she known the full collective preference profile. We present this probability under the title of ô**Maintained best manipulation**ö. This statistic does <u>not</u> include the cases when it was optimal for the strategic voter to vote sincerely and she correctly chose such voting pattern. Only those cases are included, when the voter under reduced information achieved the lowest voting distance, which would have been possible in a given voting situation, and which is moreover strictly lower than the distance associated with sincere voting. As a consequence the ôMaintained best manipulationö can be only equal or lower than the number of successful manipulations displayed previously when full informational settings were considered in Table 2.18.

Thirdly, an alternative measure of successful voting manipulation under reduced informational settings was produced in our simulations. We speak of a probability that the strategic voter under reduced information on the grounds of a weighted distance chooses such voting pattern, which yields not necessarily the best voting distance, but nonetheless **better** distance **than sincere voting** would yield. We provide the tabulated results in Appendix 2.A. In many cases the statistics on ôMaintained best manipulationö and ôBetter than sincereö do not differ or they differ only marginally. The reader should understand that the measure of ôBetter than sincereö could only be equal or higher than the statistics of ôMaintained best manipulationö.

Last, in the reduced informational settings we provide a statistic on the number of cases when the attempt for voting manipulation has lead to even **worse** voting distance **than sincere** voting would lead to. Even this statistic can be considered as an alternative measure of successful voting manipulation. The residual number of cases, i.e. (100 000 simulations ï ôWorse than sincereö) captures the number of cases when the strategic voter decided either correctly to manipulate or incorrectly but the voting distance was not worse than if she had voted sincerely, or thirdly the cases when the voter correctly decided not to manipulate are included. In other words, the complement to the measure of ôWorse than sincereö captures the number of cases when the strategic voter did not bring about worse voting result than sincere voting would do.

Table 2.24 displays the summary statistics on the listed four measures of individual manipulation success. Table 2.25 then measures the correlations between these statistics and the statistic on the probability of manipulation under full information, which is included in variable ôProbö. All observations on manipulability for n=2 were dropped together with the observations for non-manipulable Condorcetôs and Copelandôs procedures both for m=3.

Table 2.24 ï Summary statistics for measures of individual manipulation success

Variable	Obs.	Mean	Std. Dev.	Min	Max
Attempts	72	.261	.220	0	.749
Maint. Best	72	.114	.099	0	.359
Better	72	.129	.119	0	.496
Worse	72	.044	.057	0	.336

Table 2.25 ï Correlation table for measures of individual manipulation success

	Prob	Attempts	Worse	Better	Maint. best
Prob	1.0000				
Attempts	0.7899	1.0000			
Worse	0.4820	0.6781	1.0000		
Better	0.8219	0.9348	0.5989	1.0000	
Maint. Best	0.7501	0.8903	0.6259	0.9627	1.0000

We shall address the 4 statistics in the following order: first we will comment on the probability of maintaining the best voting manipulation; where we will among other merge the datasets under reduced information and full information and we will comment on the present patterns; second we will continue with the analysis on the attempts for manipulation and last we conclude with the results on the óWorse than sincereô statistics. We do not intend to comment on the statistic of óBetter than sincereô, whereas here the results are consistent with those of óMaintained best manipulation.ô

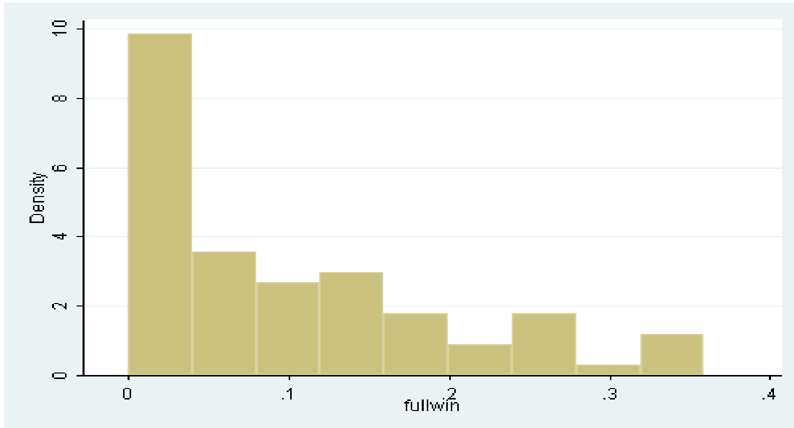
Maintained best manipulation

Table 2.26 shows the tabulated results on the óMaintained best strategic manipulationô in the reduced informational settings. Figure 2.6 shows the histogram of probabilities constructed from Table 2.26. Figure 2.7 and Figure 2.8 show the evolution of the probability of maintaining the best voting manipulation by the number of voters and by the used voting procedure. Regression table 2.27 aims to explain the voting patterns under reduced information by an OLS regression.

Table 2.26 ï Probability that a voting manipulation hits the individually best outcome

Maintained best manipulation		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.062	0.106	0.099
	m = 4	*	0.251	0.117	0.156	0.175
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.012	0.021	0.025
Approval Voting	m = 3	*	0.331	0.359	0.160	0.242
	m = 4	*	0.253	0.264	0.352	0.357
Plurality w\ runoff	m = 3	*	0	0.061	0.067	0.072
	m = 4	*	0.171	0.127	0.222	0.235
Borda' s Count	m = 3	0	0	0.099	0.131	0.148
	m = 4	0	0.029	0.124	0.156	0.185
Black' s Procedure	m = 3	0	0	0	0.001	0.004
	m = 4	0	0.068	0.100	0.085	0.071
Hare' s STV	m = 3	0	0	0.062	0.060	0.072
	m = 4	0	0.126	0.100	0.187	0.205
Coombs' Procedure	m = 3	0	0	0.020	0.036	0.048
	m = 4	0	0.046	0.072	0.098	0.126
Max ï min Procedure	m = 3	0	0	0.114	0.150	0.172
	m = 4	0	0.132	0.241	0.278	0.304
Copelandôs Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.014	0.025	0.031

Figure 2.6 ï Histogram of maintained best manipulation, reduced information



The results can be summarised in the following 5 points: 1. the levels of susceptibility to manipulation vary less significantly under reduced information than under full information; 2. this is associated with a rapid drop in the susceptibility in all considered voting procedures; 3. the order of manipulability of individual voting procedures remains nonetheless unchanged; 4. the susceptibility to strategic manipulation grows in the number of voters under reduced information; 5. the levels of manipulability are again higher in cases with more competing options. We approach the first four findings separately.

1. The lower variability in the susceptibility across individual voting procedures is well observable from both Figures 2.7 and 2.8, and could be also documented on a lower dispersion in the coefficients from Regression table 2.27. Numerous coefficients are moreover found not being significantly different from zero.

Figure 2.7 - Probabilities of manipulation, reduced information, m=3

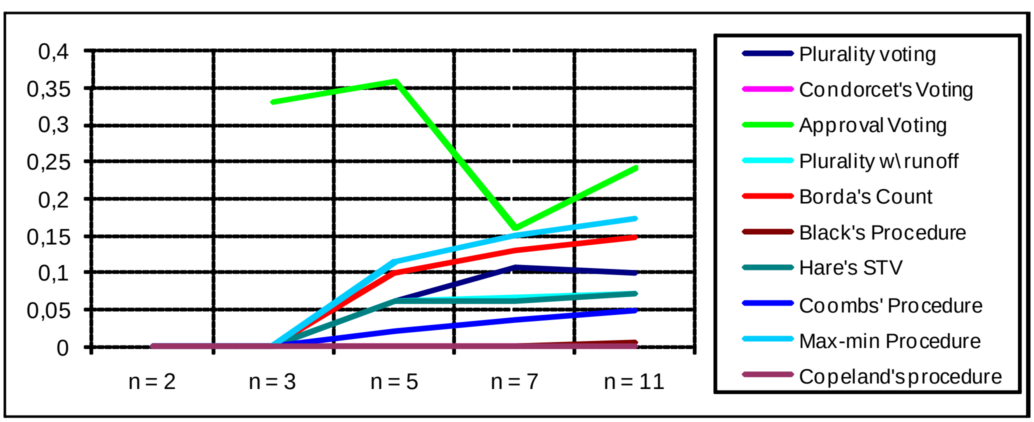
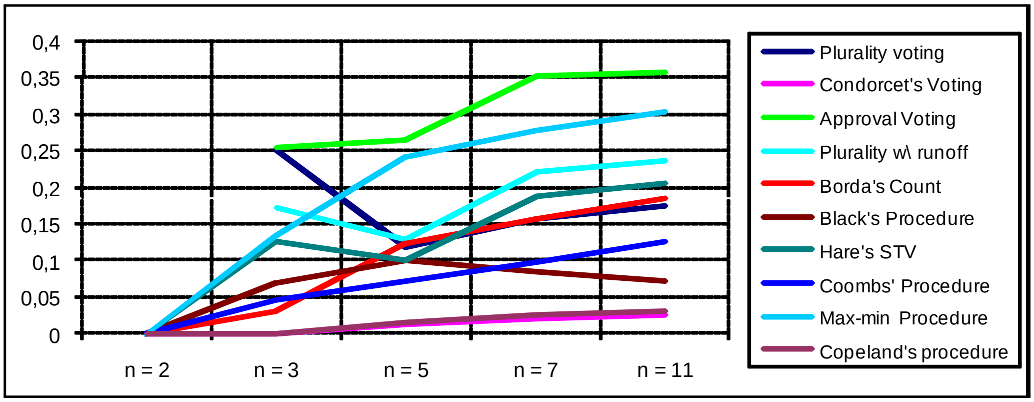


Figure 2.8 - Probabilities of manipulation, reduced information, m=4



Noteworthy, having 3 alternatives and 3 players all voting procedures except for Approval voting became immune to manipulation. Having 4 alternatives and 3 players the levels of manipulability remained significantly positive.

Regression table 2.27 - Probability of manipulation, reduced information

Source	SS	df	MS
Model	1.51674706	12	.126395588
Residual	.126827921	60	.002113799
Total	1.64357498	72	.02282743

Number of obs	=	72
F(12, 60)	=	59.80
Prob > F	=	0.0000
R-squared	=	0.9228
Adj R-squared	=	0.9074
Root MSE	=	.04598

Maint. Best	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
n	.007	.001	4.22	0.000***	.004	.011
m4	.085	.011	7.44	0.000***	.062	.108
Plurality	.027	.020	1.32	0.190	-.014	.069
Condorcet	-.121	.028	-4.28	0.000***	-.177	-.064
Approval	.196	.020	9.39	0.000***	.154	.238
Runoff	.026	.020	1.26	0.213	-.015	.068
Borda	.016	.020	0.76	0.448	-.025	.057
Black	-.051	.020	-2.48	0.016**	-.093	-.009
Hare	.008	.020	0.41	0.686	-.033	.050
Coombs	-.037	.020	-1.78	0.081*	-.079	.004
Max-min	.080	.020	3.86	0.000***	.038	.122
Copeland	-.118	.028	-4.18	0.000***	-.174	-.061

*** significant at 1%, ** significant at 5%, * significant at 10%

2. The rapid drop in the susceptibility to voting manipulation is best observable from the regression Table 2.28. Here the formal model resembles the previous models, apart from the facts that here the explained variable $Prob_i$ includes both the simulated probabilities from full and reduced informational settings, and that a dummy variable (Reduced Info) controls for this difference among the explanatory variables.

The formal model and the results captured in Table 2.28 follow:

$$Prob_i = б\ n_i + ɔ\ m4_i + ŭô\ proced_i + ẚ(\text{Reduced Info})_i + Ų_i,$$

Regression table 2.28 - Probability of manipulation, merged informational groups

Source	SS	df	MS
Model	9.87427496	13	.759559613
Residual	1.31720199	151	.008723192
Total	11.191477	164	.068240713

Number of obs	164
F(13, 151)	87.07
Prob > F	0.0000
R-squared	0.8823
Adj R-squared	0.8722
Root MSE	.0934

Prob	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
n	.003	.002	1.33	0.186	-.001	.007
m4	.162	.015	10.56	0.000***	.131	.192
Reduced Info	-.185	.014	-12.70	0.000***	-.214	-.156
Plurality	.173	.029	5.87	0.000***	.115	.232
Condorcet	-.021	.039	-0.53	0.593	-.099	.057
Approval	.371	.029	12.53	0.000***	.312	.429
Runoff	.194	.029	6.58	0.000***	.136	.253
Borda	.253	.026	9.50	0.000***	.200	.306
Black	.114	.026	4.31	0.000***	.062	.167
Hare	.128	.026	4.82	0.000***	.075	.181
Coombs	.108	.026	4.07	0.000***	.055	.161
Max-min	.278	.026	10.45	0.000***	.225	.331
Copeland	-.000	.039	-0.01	0.991	-.079	.078

*** significant at 1%, ** significant at 5%, * significant at 10%

The regression suggests the reduction in the knowledge of the strategic voter about the preference profile of one sincere voter reduces the probability of maintaining the best voting manipulation by 18%. That is however only a partial result since we need to take into regard also the coefficients on particular voting procedures and on the increase in the number of alternatives that have subsided. Lower amount of information depresses all of these coefficients simultaneously.

The drop in the susceptibility can be also observed in the shift to the left of the probability distribution of the susceptibility to manipulation, displayed in Figure 2.6.

Speaking in absolute terms, the susceptibility does not exceed 35% under both 3 or 4 alternatives. Compare with Table 2.18 and associated Figures 2.2 and 2.3. Under 3 alternatives, the level of 35% is only approached by the Approval voting procedure, where we have pointed out on the substantial asymmetry in the voting rights of the strategic and sincere voters. Omitting Approval voting, the level of susceptibility would not overcome 18% under 3 alternatives.

3. The order of manipulability of individual voting procedures stayed unchanged

We again find the Condorcet-consistent procedures to be least manipulable, followed by the elimination-based procedures and placing the Approval, Bordaös and Max-min voting procedures at the highest ranks in the manipulability of the voting procedures.

We may also order the procedures according to the **vulnerability of strategic voting to the reduction in information** about the other voterös profiles. We construct a ratio of óMaintained best manipulationô over óProbability of successful manipulation under full informationô, which gives us the percentage value of the óMaintained best manipulationsô from the possible manipulations instead from the total number of simulations. The higher is the percentage of maintained best manipulations, the less vulnerable is the voting procedure to the reduction in information. We present the ratios in Table 2.29.

Table 2.29 ï Vulnerability of voting procedures to reduction in information

Variable*	Obs	Mean	Std. Dev.	Min	Max
Plurality	8	.471	.21	0	.66
Condorcet	4	.149	.13	0	.30
Approval	8	.705	.25	.36	1
Runoff	8	.426	.19	0	.67
Borda	8	.316	.23	0	.67
Black	7	.219	.12	0	.31
Hare	8	.463	.22	0	.67
Coombs	8	.307	.18	0	.50
Max-min	8	.384	.19	0	.60
Copeland	4	.173	.15	0	.35

* (Maintained best manipulation / probability of successful manipulation under full information)

We can see that the order of manipulability of individual voting procedures stays unchanged exactly because of the extent of vulnerability of the voting procedures to the amount of information. Those procedures, which are least manipulable are in the largest extent further harmed by the incompleteness of the information and those which are more susceptible to strategic manipulation do not suffer that much. Most vulnerable are the Condorcetös and Copeland voting procedures, followed by Blackös procedure, Coombsô procedure, surprisingly Bordaös procedure, Max-min procedure, with Runoff, Plurality and Hareös procedures being least manipulable. Approval voting procedure stands as on outlier highly above all listed voting procedures.

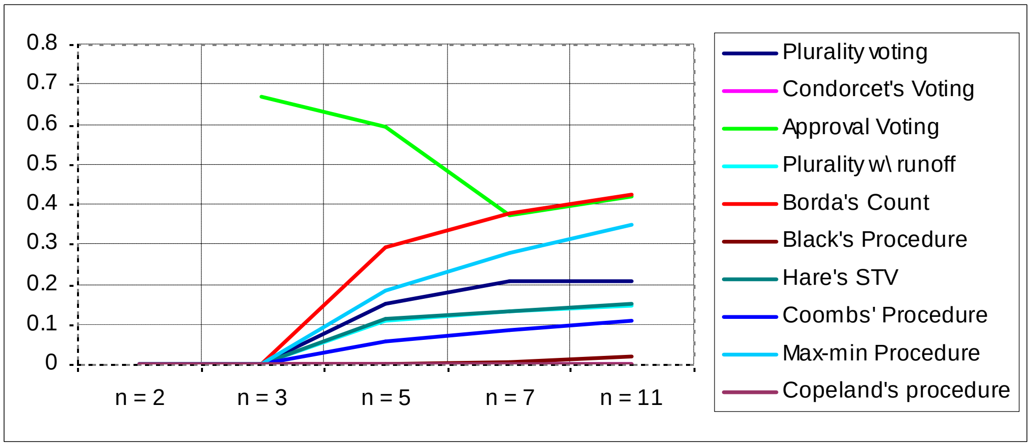
4. Under reduced information the susceptibility to manipulation grows in the number of voters

We infer this finding from the Regression table 2.27 from the β coefficient associated with the n_i variable. The increasing pattern is easily discerned also from the Figures 2.7 and 2.8. We do not have to look for some demanding explanations; the reason for the increasing manipulation in the number of voters can be attributed to the relatively lower share of withheld information from the strategic voters at higher numbers of voters. Knowing less of 1 sincere voter's profile when there are 11 voters is less important for the individual ability of strategic manipulation than knowing less of 1 sincere voter's profile when there are just 3 voters. Hereby we confirm the vulnerability of strategic manipulation not only to an absolute reduction in the individual information, but also to a relative reduction.

Attempts for voting manipulation

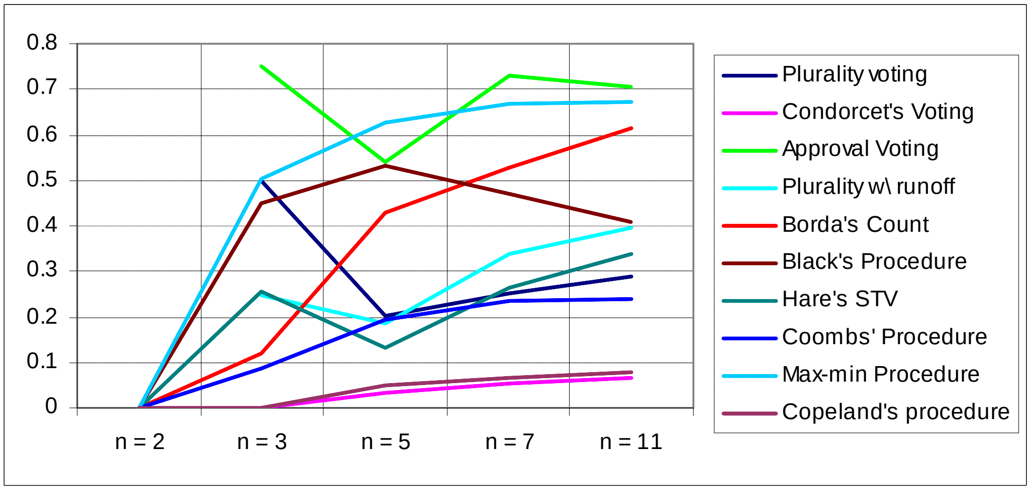
The table of "Attempts for voting manipulation" as explained at the beginning of this subchapter is provided in Appendix 2.B and is graphically outlined in Figures 2.9 and 2.10. These figures study the number of attempts by the number of players and by the used voting procedure. The Regression table 2.30 attempts to explain the number of attempts for strategic manipulation by an OLS regression.

Figure 2.9 – Attempts for manipulation, reduced information, m=3



Noteworthy, the reader must not draw direct inference from Figures 2.9 and 2.10, since these figures ignore the correlation between the number of attempts and the actual probability of strategic manipulation. It is natural that the weighted distances bid the strategic voter to attempt for strategic manipulation more often in those procedures, which are more susceptible to strategic manipulation. The most susceptible procedures are those that are most often attempted to be manipulated. On the other hand, regressing the number of attempts on the probability of voting manipulation induces endogeneity issues, since both variables are caused by third factors, such as by the number of voters, by the relative amount of withheld information, etc.

Figure 2.10 ï Attempts for manipulation, reduced information, m=4



The formal model used for drawing inference about the number of attempts for strategic manipulation hence puts on the left side of the regression the ratio of the number of attempts over the probability of strategic manipulation under full information. This ratio is captured in the variable (Rel. Attempts). The formal model and the results follow:

$$(\text{Rel. Atempts})_i = \frac{Attempts_i}{\Pr ob_i} = \text{б}\, n_i + \text{ɔ}\, m4_i + \text{ŭô}\, proced_i + \text{Ǚ}_i,$$

Regression table 2.30 ï Number of attempts for manipulation, reduced information

Source	SS	df	MS
Model	505.076542	12	42.0897118
Residual	12.9608789	51	.254134881
Total	518.037421	63	8.22281621

Number of obs	= 63
F(12, 51)	= 165.62
Prob > F	= 0.0000
R-squared	= 0.9750
Adj R-squared	= 0.9691
Root MSE	= .50412

Rel.Attempts	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
n	-.058	.022	-2.58	0.013**	-.104	-.013
m4	-.008	.139	-0.06	0.949	-.288	.270
Plurality	2.33	.270	8.65	0.000***	1.794	2.878
Condorcet	3.04	.379	8.02	0.000***	2.280	3.803
Approval	2.48	.250	9.95	0.000***	1.985	2.989
Runoff	2.11	.270	7.84	0.000***	1.575	2.660
Borda	3.69	.270	13.69	0.000***	3.155	4.239
Black	6.26	.291	21.51	0.000***	5.679	6.849
Hare	2.20	.270	8.17	0.000***	1.664	2.749
Coombs	2.75	.270	10.19	0.000***	2.209	3.294
Max-min	2.77	.270	10.28	0.000***	2.233	3.318
Copeland	2.94	.361	8.13	0.000***	2.214	3.665

From the Regression table 2.30 we can say that there are only few voting procedures where the relative number of attempts significantly differs from other procedures. In other words, the strategic agent attempts relatively for strategic manipulation in majority of procedures to a comparable extent. Majority of coefficients accruing to individual voting procedures fall into the 95% confidence intervals of the coefficients of other voting procedures. Only Blackös and Bordaös procedures differ from the other procedures in this respect, and their relative number of attempts for manipulation is higher. Nevertheless, as we will see only in the case of Blackös procedure this increased number of attempts leads eventually also to an increased number of adverse outcome of strategic manipulation.

As a positive result we view also the independence of the number of attempts on the used number of competing alternatives. The strategic agent opts for the relative number of attempts irrespective of the number of alternatives, which makes her decision making consistent.

Thirdly, we decrease in the relative number of attempts in the number of voters can be interpreted as a getting more exact in attempting for manipulation, which we perceive just as well positively.

Overall, we can see that the number of attempts exceeds the number of cases when voting manipulation was optimal by twofold or even more. Luckily for the strategic voter, in cases when she attempts for a voting manipulation and she does not succeed she brings about either a result that is equally good as sincere voting would yield or is even better than sincere voting although it might not be the best manipulating option. The cases when these eventualities did not occur are described in the following last section.

Adverse consequences of attempting for strategic manipulation

The tabulated results for the number of voting outcomes, which are óWorse than sincereô voting would deliver are provided in Appendix 2.C. The graphical outline of these results sorted by the number of voters and by the used voting procedure is provided in Figures 2.11 and 2.12. Regression table 2.31 explains the results through an OLS regression.

Figure 2.11 ï Manipulation into worse than sincere outcome, reduc. info, m=3

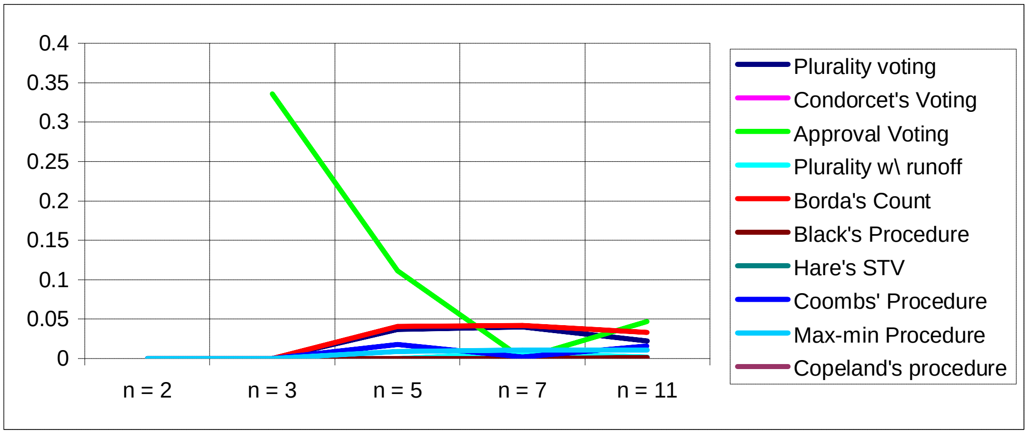
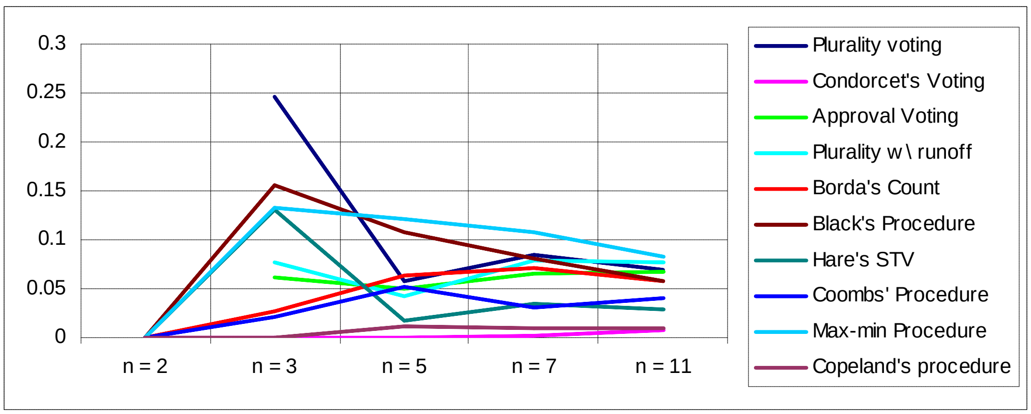


Figure 2.12 - Manipulation into worse than sincere outcome, reduc. info, m=4



Absolutely speaking, the voting outcome is worse than sincere voting would yield in 5% of simulated voting situations when we are selecting from 3 alternatives or the outcome is worse in 15% of situations when we are selecting from 4 alternatives. This percentage appears as a relatively small price to be paid for attempting for voting manipulation, given how many times the strategic agent succeeded in misrepresentation of her preferences. Moreover, since the strategic agent decides on a basis of a weighted distance, she might have come in the end to a worse result than sincere voting would yield, nonetheless the associated voting distance is most probably not that much different from the distance associated with sincere voting.

Speaking of relative figures, we relate the number of ôWorse than sincere outcomesô to the number of actual cases when voting manipulation was optimal. We capture the ratio of these two variables in a variable (Rel.Worse)$_i$. We do so for the potential endogeneity problems between the two variables, just as previously between the number of attempts and the number of actual opportunities for manipulation. The regression equation uses identical explanatory variables. The formal model follows:

$$(\text{Rel. Worse})_i = \frac{Worse_i}{\Pr ob_i} = \beta\, n_i + \gamma\, m4_i + \sum \delta\, proced_i + \varepsilon_i,$$

Regression table 2.31 - Manipulation into a worse than sincere outcome, reduced information

Source	SS	df	MS
Model	1.92406925	12	.160339104
Residual	.462447355	51	.009067595
Total	2.3865166	63	.037881216

Number of obs	63
F(12, 51)	17.68
Prob > F	0.0000
R-squared	0.8062
Adj R-squared	0.7606
Root MSE	.09522

Rel. worse	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
n	-.017	.004	-4.14	0.000***	-.026	-.009
m4	.084	.026	3.22	0.002***	.031	.137
Plurality	.348	.051	6.84	0.000***	.246	.451
Condorcet	.106	.071	1.48	0.145	-.037	.249
Approval	.219	.047	4.64	0.000***	.124	.314
Runoff	.224	.051	4.40	0.000***	.122	.327
Borda	.210	.051	4.12	0.000***	.107	.312
Black	.226	.055	4.12	0.000***	.116	.337
Hare	.197	.051	3.88	0.000***	.095	.300
Coombs	.261	.051	5.12	0.000***	.158	.363
Max-min	.200	.051	3.92	0.000***	.097	.302
Copeland	.189	.068	2.77	0.008***	.052	.326

We see that the voting procedures are not statistically distinguishable between each other in the regard of how many "Relative worse than sincere" outcomes they deliver. In other words the strategic agent selects on average the unsatisfactory voting pattern in a similar extent across all voting procedures.

We observe that the number of relatively worse outcomes is diminishing in the number of voters. A careful reader has noticed, that to an increased number of voters we have previously attributed an increasing exactness of attempting for strategic manipulation. Now we discover, that the increase in exactness extends also on the ability of attempting for such voting patterns, which do not harm the individual strategic voter relatively to her sincere voting. This increase may originate in the lowest relative share of withheld information at higher numbers of voters.

The selection of unsatisfactory voting patterns is higher when selecting from 4 competing alternatives. We nevertheless find no motivation for this result.

2.4 Concluding remarks

Strategic voting is not only an act predicted by the economic theoretical models but also empirically manifested and widely observed pattern of the voting behavior. The sophisticated voters, who out of their short-term instrumental motivations want to best influence the election result, misrepresent in the elections their individual voting preferences in the expectation of manipulating the aggregated social preference order into an order, which would reflect their own sincere wishes as closely as possible. On the other hand, the strategic voters in their effort for best influencing the outcome stumble upon different impediments of the voting situation.

This study has taken the effort to computationally simulate 10 different voting procedures for small numbers of voters and small numbers of competing alternatives so as to study the vulnerability of these procedures to strategic voting. This was followed by a study of vulnerability of strategic voting to the variation in the amount of information that the individual agents possessed.

The susceptibility to strategic voting manipulation was found to be a subtly diminishing function of the number of election participants and an increasing function of the number of voting alternatives. All procedures could be characterised by their own specific extent to which they were susceptible to voting manipulation. The procedure-specific extent of manipulation was in turn dependent on the amount of information that the procedure typically requires from a participating agent to disclose, in combination with the strictness of the voting procedure, which is the amount of points that the procedure allows the agent to manipulate with. Least susceptible voting procedures were the Condorcet-consistent procedures: Blackôs, Copelandôs and Condorcetôs procedure itself. The second group of relatively more susceptible voting procedures involved three elimination procedures: Coombsô, Hareôs and Plurality with runoff voting procedures. As the most manipulable procedures were found the plurality voting procedure, approval voting procedure, max-min voting procedure and Bordaôs count.

If the strategic agent has had a full access to the information about other votersô voting patterns, the opportunity for a strategic manipulation has occurred in up to 80%

of cases, although the average moved around 15% for 3 competing alternatives and 40% for 4 competing alternatives. Once we have stripped the agent from the full knowledge of the collective preference profile, we have confirmed the vulnerability of strategic voting to both an absolute and relative reduction in the amount of information. Having withhold information from the strategic agent about just one sincerely voting agent has reduced the number of cases, when the strategic agent was able to correctly choose the best manipulating voting pattern, by approx. 15-30 %. The precision of selection of the best manipulating voting pattern was decreasing in the relative amount of information withheld from the strategic agent. Consistently, the agent has more often ended up with worse payoff than sincere voting would yield, when a relatively larger share of information was withheld from her.

There is much work left undone in this field, which is mostly related to the alternative specifications of the preference generating cultures or to the means of withholding information from the strategic voter, not speaking of the cases with numerous strategic voters. Having formed the theoretical predictions, the future research may aim at the design of economic experiments simulating the voting environment suitable for strategic voting under varying informational settings. Nonetheless, these issues are beyond the scope of our work.

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Appendix 2.A Alternative measure of manipulation

Table 2.32 ï Probability of manipulation to achieve individually better outcome than sincere voting would

Better than sincere		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.062	0.106	0.099
	m = 4	*	0.251	0.117	0.156	0.175
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.012	0.021	0.026
Approval Voting	m = 3	*	0.331	0.359	0.160	0.242
	m = 4	*	0.496	0.315	0.439	0.407
Plurality w\ runoff	m = 3	*	0	0.061	0.067	0.072
	m = 4	*	0.171	0.130	0.239	0.247
Borda' s Count	m = 3	0	0	0.099	0.131	0.148
	m = 4	0	0.040	0.175	0.211	0.238
Black' s Procedure	m = 3	0	0	0	0.001	0.004
	m = 4	0	0.091	0.144	0.121	0.099
Hare' s STV	m = 3	0	0	0.062	0.060	0.072
	m = 4	0	0.126	0.103	0.187	0.208
Coombs' Procedure	m = 3	0	0	0.020	0.036	0.048
	m = 4	0	0.046	0.073	0.105	0.135
Max - min Procedure	m = 3	0	0	0.114	0.150	0.172
	m = 4	0	0.235	0.314	0.337	0.353
Copelandôs Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.015	0.025	0.032

Appendix 2.B Number of attempts for strategic manipulation, reduced information

Table 2.33 - Number of attempts for voting manipulation, reduced information

Attempts for manipulation		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.149	0.209	0.205
	m = 4	*	0.498	0.203	0.251	0.290
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.032	0.052	0.065
Approval Voting	m = 3	*	0.667	0.592	0.374	0.419
	m = 4	*	0.749	0.542	0.729	0.707
Plurality w\ runoff	m = 3	*	0	0.109	0.132	0.146
	m = 4	*	0.248	0.185	0.338	0.397
Borda' s Count	m = 3	0	0	0.291	0.375	0.422
	m = 4	0	0.120	0.429	0.527	0.616
Black' s Procedure	m = 3	0	0	0	0.007	0.019
	m = 4	0	0.449	0.530	0.470	0.410
Hare' s STV	m = 3	0	0	0.111	0.134	0.151
	m = 4	0	0.257	0.130	0.265	0.338
Coombs' Procedure	m = 3	0	0	0.057	0.083	0.110
	m = 4	0	0.088	0.194	0.234	0.239
Max - min Procedure	m = 3	0	0	0.184	0.277	0.347
	m = 4	0	0.505	0.627	0.666	0.671
Copelandŝ Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.049	0.066	0.080

Appendix 2.C Cases with worse outcomes than sincere voting would yield

Table 2.34 - Manipulation to achieve individually worse outcome than sincere voting

Worse than sincere		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.037	0.040	0.022
	m = 4	*	0.247	0.058	0.085	0.069
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.0006	0.001	0.008
Approval Voting	m = 3	*	0.336	0.111	0	0.047
	m = 4	*	0.061	0.050	0.066	0.067
Plurality w\ runoff	m = 3	*	0	0	0.007	0.002
	m = 4	*	0.077	0.043	0.079	0.076
Borda' s Count	m = 3	0	0	0.041	0.042	0.033
	m = 4	0	0.027	0.063	0.072	0.058
Black' s Procedure	m = 3	0	0	0	0	0.001
	m = 4	0	0.156	0.108	0.080	0.057
Hare' s STV	m = 3	0	0	0	0	0
	m = 4	0	0.130	0.017	0.034	0.028
Coombs' Procedure	m = 3	0	0	0.018	0.002	0.016
	m = 4	0	0.021	0.052	0.030	0.041
Max - min Procedure	m = 3	0	0	0.009	0.011	0.011
	m = 4	0	0.133	0.122	0.108	0.082
Copelandõs Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.011	0.010	0.009

Appendix 2.D CD carrier with Matlab simulation codes, Stata data files, Stata code

The CD carries beside a copy of the rigorous thesis also two folders, in which we include the Matlab code for our voting simulations. The first folder includes the simulation code for all considered voting procedures under full informational settings. The name of a particular Matlab .m files indicates the used voting procedure, the used number of alternatives and currently used number of voters as follows:
Name: simulation_Procedure_Voters_x_Alternatives.m

The second folder includes the simulation codes for the environment of incomplete information. The generic name of a particular Matlab .m file follows:
Name: incomplete_Procedure_Voters_x_Alternatives.m

Apart form the simulation codes we include on the CD carrier also manually gathered results from the voting simulations contained in three Stata data files. They correspond to the two voting environments of full or incomplete information. To each of the three Stata data files corresponds a Stata code, which performs the regressions and inference presented in the thesis.

Chapter 3

Voting experiments: Measuring vulnerability of voting procedures to manipulation

3.1 Introduction

Strategic voting (tactical voting, manipulation) is not only predicted by the economic theoretical models (Myerson, Weber, 1993); (Feldman, Serrano, 2006); (Edlin, Gelman, Kaplan, 2007); but is also empirically manifested and widely observed pattern of the voting behavior, (e.g. Alvarez, Nagler (2000), Blais et al. (2001), Schmitt (2001) and multitude of other authors). The sophisticated voters, who out of their short-term instrumental motivations want to best influence the election result, misrepresent in the elections their individual voting preferences (Fisher, 2001a, 2001b). They do so in the expectation of manipulating the aggregated social order into such an order, which would reflect their sincere wishes as closely as possible. Nevertheless, the strategic voters in their effort for best influencing the voting outcome stumble upon different impediments of the voting situation, one of which may be an incomplete knowledge of other votersô preference profiles.

In the modern social choice theory two impossibility theorems step up most prominently: the famous Arrowös impossibility theorem (Arrow, 1963), and further interpreted Gibbard-Satterthwaithe impossibility theorem (Gibbard, 1973, Satterhwaithe, 1975), unified among others by Reny (2000). Gibbard-Satterthwaiteös theorem states that there exists no voting system with three or more alternatives designed to select a single winner, which would be universal, non-dictatorial, and which would not provide an agent, who has a full knowledge of the collective preference profile, with an incentive to strategically misrepresent her voting preference so as to swing the election outcome into her favour. The impossibility result is dismal in the sense that all feasible voting procedures are vulnerable to manipulation and hence it is impossible to achieve socially desirable results through voting.

In this paper I relax the assumption of the complete information, which is central for the Gibbard-Satterthwaiteєs theorem. Via a series of computation-based simulations of voting I aim to assess the role of strategic voterєs knowledge on her ability to successfully strategically select her individually optimal voting pattern. To do so I first estimate, which procedures are most-to-least vulnerable to strategic voting. Successively I ask how the probability to strategically manipulate behaves under constrained information. This is important both for checking of the robustness of the ranking of most-to least manipulable procedures and for the insights into restrained informational setting per se. By standard econometric analysis I analyse the systematic patterns of the probability to manipulate with regard to the number of voters and number of offered alternatives. That leads us to predictions, which compositions of committees and public boards would in general most discourage their members from strategic voting considerations. Last I look at the relationship between the intensity of information, which voting procedures require from voters to disclose and voterєs ability to successfully manipulate them in the settings of constrained information.

My main findings are that strategic voting is vulnerable both to an absolute and relative reduction in the amount of possessed information. That is a positive result with regard to the dismal predictions of Gibbard-Satterthwaitheєs theorem. A minimal reduction in the agentєs holding of information severely threatens her ability to strategically manipulate. I show that the vulnerability of voting procedures is a diminishing function of the number of participants, and an increasing function of the number of competing alternatives. Consistently, I show that strategic manipulation is most vulnerable to reduction in information especially in the least information intensive procedures. The strategic agent attempts less often for manipulation, when more information is withheld from her and in line she more often ends up with worse payoffs than sincere voting would yield, when more information is concealed.

3.2 Methodology

I use computation-based simulations to randomly generate a collective preference profile of a set of voters. All but one of these voters cast their votes sincerely in order to come at a collective decision, which the last voter attempts to manipulate through her strategic vote. I evaluate susceptibility to manipulation of ten most common voting procedures, many of which are used also in practical daily life. I target to estimate the change in the success rate of strategic voterӳ manipulations, given that her information about other votersô preferences shrinks. The information that the agent possesses shrinks because I assume away her full knowledge of other voters` preference profiles. The strategic voter instead expects other voters to vote according to some previously specified probability distribution.

3.2.1 Preference generating cultures

The random generation of a collective preference profile is in literature commonly called **a culture**. An overview of different preference generating cultures has been provided in the exposition of Laslier (2009). From among different specified Rousseauist, distributive or spatial cultures, I use the ó*impartial culture*ô for our simulations. Impartial culture is most suitable for general evaluation of voting systems, where we do not want to account for any specific prior information about the alternatives or for any assumption on the shape of distribution of voting preferences.

The impartial culture attributes to each individual sincere voter a preference ordering from among m! strict total preference orderings, where m is the number of competing alternatives. Letӳ assume n individual voters with complete, transitive and anti-symmetric preference relations R_i on the set of alternatives A, where i is an index for an individual voter and separate alternatives are denoted by [a b c ... m]. The assumptions on voterӳ preference relations are equivalent for any R_i to be characterised as a total preference ordering.

An essential characteristic of the impartial preference-generation culture is that the preference orderings attributed to individuals are **distributed according to uniform distribution** over m! logical strict preference orderings. The culture treats all the alternatives symmetrically and learning something about preference orderings of some voters yields no information about the preferences of the rest of voters.

Due to the symmetrical treatment of alternatives, I may fix the preference profile of the single strategic voter by attributing to her an alphabetical ordering of alternatives [a b c dé m]. The uniformity and i.i.d. properties of the preference generating culture allow us to proceed this way without the loss of generality.

3.2.2 Strategy-proofness and distance function specifications

There are a numerous approaches to specifying strategy-proofness. Umezawa (2009) provides an overview of the literature, which either makes explicit references to expected utilities, where probability measures over alternatives are given (e.g. Feldman, 1980; Barbera et al., 2001; Rodriguez-Alvarez, 2007; and numerous other.) The other approach defines strategy-proofness in a non-probabilistic framework, where individuals evaluate the sets of alternatives based on their preference orders over alternatives by focusing on the best and/or the worst alternative in the sets (see, e.g. Bandyopadhyay, 1983; Barbera, 1977; Pattanaik, 1973).

A midway between these two approaches is a specification of strategy-proofness based on probabilistically measured expected utilities, which are however not over specific alternatives but over voting distances between individualôs preference ordering and ordered voting outcomes. Bossert, Storcken (1992) use Kemenyôs distance to evaluate the voting distances between preference orderings. Duddy, Perote-Peña and Piggins (2009) investigate the problem of constructing a social welfare function that is non-manipulable in a context, where individuals attempt to manipulate a social ordering as opposed to a social choice. It is this middle way that I take in this paper, with an important distinction of using Euclidian rather than Kemenyôs distance to construct the evaluation of particular social orderings.

For the purpose of cardinalization of individualôs utility we need to use a function that reflects both original individualôs preference and the aggregated social preference order. Utilities reflect to what degree these two orders agree or how close is the generated voting outcome from the original voterôs preference. For these reasons I compare the two orderings by the means of a distance function. I consider a distance function, which resembles the mathematical Euclidian distance. Minimization of a distance between individualôs preference ordering and a social preference ordering then corresponds to maximization of utility of the strategic voter.

Definition: (Distance function) Let r_j and s_j constitute two systems of non-negative weights attached to all alternatives of an individual and social preference order, respectively. The two systems of weights are intertwined in the following manner: if particular alternative is located at the j^{th} position in the individual preference order $\Re_i$, then it bears individual weight r_j. An equal weight s_x will be attached to such position x in the social order S, at which the alternative was placed by the voting aggregation rule. The distance function D_{iS} between i^{th} individual preference ordering $\Re_i$ = [r_1 r_2 é r_m] and social preference ordering S = [s_1 s_2 é s_m] is subsequently given by (3.1):

$$D_{iS} = \sqrt{\sum_{j=1}^{m}(r_j - s_j)^2} \tag{3.1}$$

Distance function D_{iS} maps the systems of weights into one real number.

The weights that I attach to alternatives shall be unified since now on for the rest of our study. I assume them to correspond to scores, by which an individual would evaluate alternatives during **Bordaôs voting.** Individualôs first and best alternative is associated with a weight of (m-1) points. The weights attached to consecutive options are consecutively falling by 1 point, with the last option being weighted by 0.

In case the voting outcome involves tie(s) between alternatives, I resolve the issue by calculating an average distance between all potential social outcomes and the individual preference ordering.

Definition: (Average distance) Let $L = (p_1,..., p_K)$ be a lottery, which assigns equal probabilities to all potential social orders k that may occur after a random breaking of tie(s) involved in a voting outcome. Let D_{iSk} represent k-th potential distance of social ordering S_k from individual ordering $\Re_i$ for all k = (1,é , K). Then

$$\overline{D_i} = p_1 D_{iS_1} + é + p_K D_{iS_K} \tag{3.2}$$

(3.2) is the average distance between the individual preference ordering $\Re_i$ and all potential social preference orders.

3.2.3 Voting procedures

I compare the susceptibility to voting manipulation for a list of 10 voting procedures. For details on these procedures see e.g. Nurmi (1987).

1. **Simple plurality voting**

Each voter needs to decide, to which single alternative to assign a score of 1, while assigning 0 to all other alternatives. Plurality winner is such an alternative that collects the highest number of votes.

2. **Condorcetô s voting procedure**

In standard understanding, a winning alternative is chosen by this procedure if and only if it is not defeated by a strict majority by any other alternative in a pair-wise vote. In this paper I order the alternatives according to the number of wins of an alternative over other alternatives in a pair-wise vote.

3. **Plurality voting with runoff**

Plurality voting with runoff involves two rounds. The first round proceeds just like simple plurality voting. The second round involves a vote between two alternatives with the highest scores obtained in the first round. The purpose of the first round, so-called runoff is to eliminate the least preferred options.

4. **Bordaô s voting procedure**

Given m alternatives each voterô s first ranked alternative obtains (m-1) points, second ranked alternative obtains (m-2) points, the third one gets (m-3) points, and so forth, down to a minimum of 0 points for the last alternative. The scores are added up and the option with the highest score becomes the **Bordaô s winner.**

5. **Approval voting**

Individual voter may assign a score of 1 to as many alternatives as she wishes and assign 0 to all other. The winning alternative is the one, which gathers the most votes.

6. **Blackô s voting procedure**

Blackô s procedure simply chooses the Condorcetô s winner if one exists. Otherwise it chooses the winner and ranks the alternatives according to Bordaô s voting.

7. Hare’s single transferable vote system

If some alternative in Hare’s voting procedure is ranked first by more than 50% of voters, it wins the election. If none such alternative exists, the alternative with fewest first ranks is eliminated from the count and the rest of alternatives is being pushed upwards in the preference lists of the voters. We again determine if any alternative ranks first by more than 50% of the voters. If so, it becomes a winner. If not, another round of eliminations proceeds. Eventually, after a number of rounds of eliminations one alternative must become **Hare’s winner** or a tie is established in the final round.

8. Coombs’ technique

Coombs suggested a slight modification to Hare’s voting procedure and that was to eliminate during the rounds of elimination such an alternative that is ranked last by the largest number of voters. The qualification criterion for victory stayed the absolute majority of the first ranks in voters’ preference profiles.

9. Max-min voting technique

Max-min procedure counts how many voters rank an alternative above each of other alternatives. For every alternative the procedure finds the lowest of these numbers. The procedure then ranks the alternatives according to the retrieved minima.

10. Copeland’s voting procedure

The procedure attributes a number of wins and a number of losses to each alternative. The alternative wins over other alternative, if it gains a majority of votes in a pair-wise vote. Otherwise it loses. The social ordering consists of an ordered list of differences between a sum of wins and sum of losses of each alternative. Copeland’s procedure obviously selects the Condorcet’s winner if it exists.

3.2.4 Voter’s behaviour given the level of information

Knowledge of the full collective preference profile

The role of a fully informed strategic voter is straightforward. The agent calculates all possible distances that could occur between her individual preference ordering and the aggregated social preference orderings and then selects such voting pattern so as to minimise the voting distance.

The role is straightforward, because voterӳ choice finalises the aggregation of the social preference ordering, and because the agent is the sole strategic voter, which means she faces no uncertainty about voting patterns of other voters.

To evaluate how successful is the voter in her endeavours, we need to decompose the question into two parts. How many times **did the voter have** and how many times **did she use** the opportunity to strategically manipulate the voting result? The success rate may be calculated either as a number of cases when the strategic voter succeeded to lower the relevant distance relatively to distance associated with her sincere voting or we may calculate the success rate as a number of cases when the voter succeeded to manipulate the voting result so as to make it copy her own individual preference ordering. I shall evaluate the former statistic. Under full information, the number of opportunities that the strategic voter **had** to manipulate and the number of opportunities that the voter actually **used** do fully match**.** The difference between the two statistics emerges under voterӳ restrained information.

I simulate the preference profile of (n-1) voters and m competing alternatives using 100 000 independent draws for all listed voting procedures. I simulated the voting processes for n = {2, 3, 5, 7 and 11}. I use these values since I focus on voting manipulation in small groups or committees, where the informational assumption that a particular voter might know all or majority of other votersԠpreference profiles is feasible.

Information about full rankings of a subset of voters

The manipulating ability of a voter may be hampered by a lack of knowledge about voting patterns of a subset of the voters. This may happen, for instance, when some part of the electorate does not meet all sufficient conditions for their preference rankings to be classified as preference orderings. Alternative interpretation says that a part of the electorate may from various reasons behave **non-rationally** in their decision-making. May it be due to their bounded rationality, inadequate cognitive abilities, indifference, laziness, should they be constrained by time pressure, lack of appropriate incentives, or by any other feasible constraint. I shall avoid such explanations and will simply assume away strategic voterӳ full knowledge of the collective preference profile. The reduction of information consists in letting the strategic voter know about the full collective preference profile except for a preference ordering of one sincere voter.

The strategic voter can nevertheless determine some partial scores that the alternatives have gathered from voters, about which she has information. We will assume that the strategic voter knows that the voting patterns of all voters are i.i.d. from the uniform distribution.

Now, we may think of some simple heuristic rules that the strategic voter could use given her limited information. For instance, we may think her to attempt to manipulate the partially aggregated social ordering as if it were the fully aggregated social ordering. Nevertheless such heuristic rule could often lead into situations, where the strategic voter would end up with even worse payoffs than she would receive under sincere voting.

Another heuristic option is to make the strategic voter calculate all possible social orderings, into which the partially aggregated social ordering could lead and make her vote according to a min-max principle. That means to make her select such pattern, which would lead to potential social orderings, from among which the furthest one from the individual order is the closest one across different voting patterns.

Alternatively, the voter could stick to a minimalistic approach to strategic voting. She would opt for strategic voting only in cases, where the payoffs from her insincere voting strategy would never be dominated by payoffs accruing to her sincere voting.

Nevertheless to be consistent with previous specifications, we make the voter decide for a concrete voting pattern according to a minimisation of a weighted distance between her individual preference ordering and all plausibly aggregated social orderings associated with that voting pattern. The weights would be the probabilities of a particular combination of voting patterns to take place.

The questions I ask under limited information look for identical answers as the question raised under full information. Given a number of voters about which the strategic voter owns information, how many times **did the agent use the opportunity** to manipulate the voting result? How many times **was she successful** in her manipulation, in the sense that the resulting social ordering was closer to her own preference ordering than a social ordering associated with sincere voting would be? How many times did the agent manipulate with **adverse consequences**, in the sense that the resulting social ordering was further from her own preference ordering than would be a social ordering associated with sincere voting? How do these answers change, if the strategic agent **knows of fewer** votersô profiles? I contrast all these figures to figures obtained from cases of agentŏs full information to obtain relative measures of ñsuccessful manipulationò. I again answer these questions for all 10 specified voting procedures.

3.3 Results

3.3.1 Full knowledge of the collective preference profile

Appendix 3.B in Table 3.6 provides the complete tabulated overview of the opportunities for strategic manipulation of a sole strategic voter under full information. As I have already suggested, the number of opportunities for manipulation under full information mirrors the number of actual successful manipulations. Fully informed strategic voter cannot end up with worse payoff by voting strategically than by voting sincerely. Table 3.1 and Table 3.2 present the summary statistics on the probability of manipulation under full information by number of players and number of competing alternatives. Figure 3.1 and Figure 3.2 graphically outline the evolution of room for strategic manipulation for all considered voting procedures.

We observe 3 results:

1. strategic manipulation opportunity levels vary substantially across the used voting procedures,

2. strategic manipulation opportunity levels for 4 alternatives surpass those of 3 alternatives in every simulated procedure for all numbers of voters,

3. the number of sincere voters does not affect the manipulation opportunities, if we allow for wider confidence intervals, the opportunity for strategic manipulation is diminishing in the number of voters.

1. Levels of strategic voting vary substantially across the used voting procedures

Consider Figure 3.1 and Figure 3.2 in this regard. In both figures we can discern a distinct arrangement of layers.

Table 3.35 - Summary statistics for probability of manipulation, full information, m=3

Full information, m=3	n = 2	n = 3	n = 5	n = 7	n = 11
Average	0.142	0.144	0.156	0.127	0.129
Min	0	0	0	0	0
Max	0.333	0.361	0.432	0.325	0.289
Variance	0.019	0.016	0.019	0.010	0.010

Table 3.36 ï Summary statistics for probability of manipulation, full information, m=4

Full information, m=4	n = 2	n = 3	n = 5	n = 7	n = 11
Average	0.404	0.361	0.358	0.365	0.346
Min	0.167	0.147	0.124	0.107	0.082
Max	0.794	0.684	0.704	0.592	0.562
Variance	0.052	0.032	0.036	0.032	0.031

The lowest probability of manipulation can be attributed to Copelandôs, Condorcetôs and Blackôs voting procedures. This comes at no surprise, as these procedures are exactly the Condorcet- consistent procedures, in other words they always select the Condorcetôs winner if it exists. The second layer of manipulability of voting procedures involves three elimination procedures: Coombsô, Hareôs and Plurality with runoff voting procedures. Although these procedures are not Condorcet- consistent, the probability of manipulation is only slightly higher than in the preceding group. The reason is the difficult process of consecutive rounds of eliminations, where it is not only necessary for the strategic voter to find a situation where her vote is pivotal, moreover she has to find voting pattern, which does not harm her in later rounds of eliminations.

Figure 3.13 ï Probabilities of manipulation, full information, three alternatives

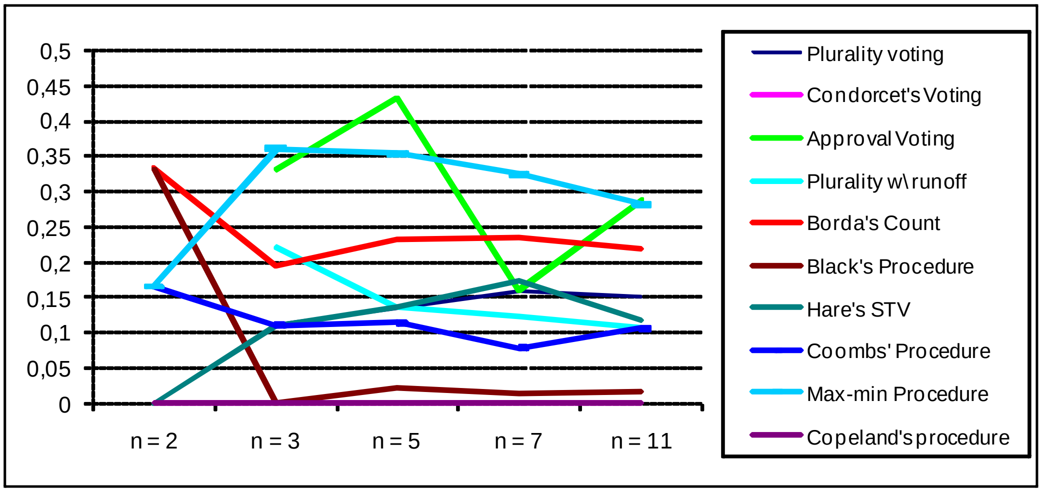
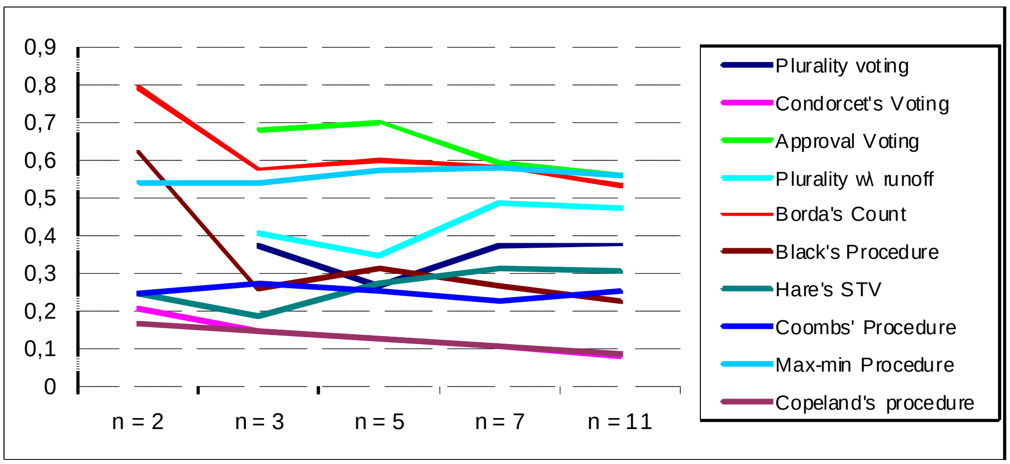


Figure 3.14 ï Probabilities of manipulation, full information, four alternatives



The last most manipulable layer groups together the remaining procedures: approval voting, max-min voting and Bordaôs count. The common feature of these three procedures is that they allow the strategic voter to allocate wide ranges of scores to individual alternatives. This property gives to the strategic voter power to swing with scores more flexibly.

Noteworthy, some level of susceptibility to manipulation needs to be attributed also to the use of the impartial preference generating culture. Had I been using some other culture, where fewer ties would occur during the preference aggregation, the strategic voter would face fewer opportunities for gainful manipulation. That applies most apparently for procedures, where the range of points that determine the social ordering is the narrowest, e.g. max-min procedure.

In the first column of Appendix 3.A I use simple ordinary least squares (OLS) regression to explain the variability in the susceptibility to strategic manipulation. The susceptibility is captured in the explained variable ñ$Prob_i$ò. Regarding the explanatory variables n_i captures the number of players, while $m4_i$ is a dummy signifying that we choose from 4 voting alternatives rather than from 3 alternatives. The rest of the variables Plurality to Copeland are dummy variables corresponding to the 10 voting aggregation rules. They are included in the (10x1) vector ñ$proced_i$ò, to which correspond 10 coefficients contained in the (10x1) vector ŭ. The formal model can be expressed by (3.3):

$$\text{Prob}_i = \beta\, \text{n}_i + \gamma\, \text{m4}_i + \delta'\, \text{proced}_i + \varepsilon_i \ . \qquad (3.3)$$

Index i does not stand here for an individual voter, but for a particular observation of the susceptibility to strategic manipulation.

The regression coefficients allow us to rank particular procedures according to their susceptibility to manipulation consistently with previous discussion.

2. Susceptibility to manipulation in cases with 4 alternatives surpasses that of cases with 3 alternatives in every procedure for all considered numbers of voters

A coefficient on number of alternatives in the first regression reads that if we are choosing from among 4 alternatives rather than from 3, there is a 25% higher chance that the strategic voter comes to a situation where it is beneficial for her to manipulate her vote. Nonetheless, it is rather sound not to generalise this result with respect to the higher numbers of competing alternatives. The pattern does not have to be increasing in the number of alternatives in the least. A sound expectation for this pattern would be to be non-linear and rather depend on the difference (n-m) if not on a ratio of the number of voters and number of competing alternatives (n/m).

3. The number of sincere voters does not affect the manipulation opportunities under full information

From the same regression we can observe a very slight decline of susceptibility to voting manipulation in the number of voters. We cannot reject the H_0 hypothesis of no impact of this variable at 5% confidence level, and we have to allow for wider confidence intervals to be able to reject the H_0. The logic of our expectations for the coefficient to be negative is nevertheless straightforward: the more voters are involved in a voting situation, the lesser relative weight of one vote should become, in the sense that the strategic voter becomes less often pivotal.

If we evaluate the ñnò coefficient for different voting procedures separately, we could conclude that it is only in Condorcetõs, Copelandõs and Plurality voting with runoff voting procedures with 3 alternatives that the susceptibility to manipulation decreases monotonically in the number of voters.

3.3.2 Incomplete information about rankings of a subset of voters

First, I have simulated a probability that the strategic voter **attempts for strategic manipulation**. The strategic voter does not have the full information, so she is coerced to decide on the basis of the weighted distances whether to attempt for a manipulation or not. The number of attempts may therefore be both higher and lower than the number of cases when the strategic voting was actually optimal. The uncertainty of the voter whether to manipulate or not propagates the other three kinds of results.

I provide the number of cases, when the strategic voter decides to manipulate and then acquires the same voting distance as she would acquire had she had the full information. This is captured in the variable of ó**Maintained best manipulation**ô. The variable does <u>not</u> include the cases when it was optimal for the strategic voter to vote sincerely and she correctly chose to do so. As a consequence the variable óMaintained best manipulationô can be only equal or lower than the number of successful manipulations in the settings with full information.

Thirdly, an alternative measure of successful manipulation was produced. It is a probability that the strategic voter on the grounds of a weighted distance chose such voting pattern, which yielded not necessarily the best voting distance, but nonetheless **better** distance **than sincere voting** would yield. Appendix 3.B Table 3.7 provides the results.

Last, I provide a variable of the number of cases, when the attempt for voting manipulation has lead to a **worse** voting distance **than sincere** voting would lead to. Even this variable can be considered as an alternative measure of successful manipulation. The residual number of cases, i.e. (100 000 simulations – "Worse than sincere") captures the number of cases when the strategic voter either correctly decided to manipulate or decided incorrectly but the voting distance was not worse than if she had voted sincerely, or the voter decided correctly not to manipulate.

Table 3.3 displays the summary statistics on the listed four measures of individual manipulation success. Table 3.4 measures correlations between these variables and the probability of manipulation under full information. All observations on manipulability for n=2 were dropped together with the observations of non-manipulable Condorcet's and Copeland's procedures for m=3.

Table 3.37 – Summary statistics for measures of individual manipulation success

	Obs.	Mean	Std. Dev.	Min	Max
Attempts	72	.261	.220	0	.749
Maintained	72	.114	.099	0	.359
Better	72	.129	.119	0	.496
Worse	72	.044	.057	0	.336

Table 3.38 – Correlation table for measures of individual manipulation success

	Prob	Attempts	Worse	Better	Maintained
Prob	1				
Attempts	0.7899	1			
Worse	0.4820	0.6781	1		
Better	0.8219	0.9348	0.5989	1	
Maintained	0.7501	0.8903	0.6259	0.9627	1

I do not intend to comment on the statistic of "Better than sincere", whereas here the results are tightly correlated with those of "Maintained best manipulation."

Maintained best manipulation

The results, provided in Appendix 3.B Table 3.8 can be summarised in 5 points:

1. the levels of susceptibility to manipulation of various procedures differ less significantly under reduced information than under full information;

2. we observe a rapid drop in the susceptibility to manipulation of all considered procedures under limited information, i.e. we confirm the severe vulnerability of strategic voting to an absolute reduction in information owned;
3. the order of the most susceptible to the least susceptible voting procedure remains unchanged when compared to the order of procedures under full information;
4. the susceptibility to strategic manipulation grows in the number of voters under reduced information, i.e. we confirm the vulnerability of strategic voting to a relative reduction in possessed information;
5. the levels of manipulability are higher for cases with more alternatives.

1. The lower variability in the susceptibility across different voting procedures is well observable from both Figures 3.3 and 3.4, and could be also documented on a lower dispersion in the coefficients on Procedures in OLS regression in the second column of Appendix 3.A. Numerous coefficients are found not being significantly different from zero.

Figure 3.15 - Probabilities of manipulation, reduced information, three alternatives

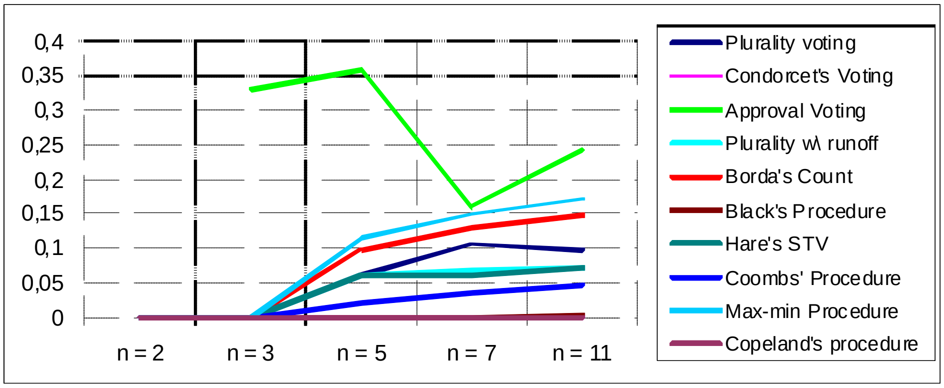
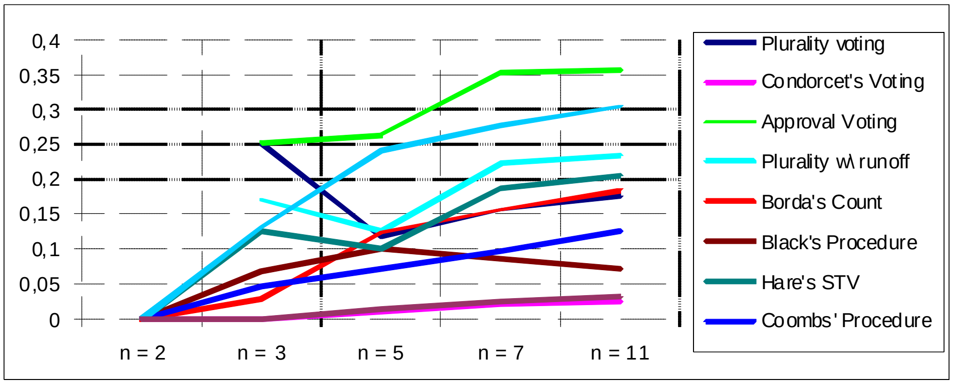


Figure 3.16 - Probabilities of manipulation, reduced information, four alternatives



Noteworthy, having 3 alternatives and 3 voters all voting procedures except for approval voting became immune to strategic manipulation. Having 4 alternatives and 3 voters the levels of manipulability remained significantly positive.

2. The rapid drop in the susceptibility to voting manipulation is best observable from the regression in third column in Appendix 3.A. Here the formal model resembles the previous model, apart from the facts that the explained variable Prob_i includes both probabilities from full and reduced informational settings, and an additional explanatory dummy variable $(\text{Reduced Info})_i$ controls for this difference.
A formal model follows in (3.4):

$$\Pr ob_i = \beta\, \mathrm{n}_i + \gamma\, \mathrm{m4}_i + \delta\,'\, \mathrm{proced}_i + \lambda\, (\text{Reduced Info})_i + \varepsilon_i, \qquad (3.4)$$

The regression results suggest that the reduction in the knowledge of strategic voter about a preference profile of one sincere voter reduces the probability of maintaining the best voting manipulation by 18%. This is moreover only a partial reduction as we need to take into regard also a decline in coefficients of particular voting procedures. Lower amount of information depresses all these coefficients simultaneously.

Speaking in absolute terms, under limited information none of the voting procedures is susceptible to manipulation in more than 35% of cases. Under 3 alternatives, the level of 35% is only approached by the approval voting procedure. Disregarding approval voting, the level of susceptibility would not overcome 18%.

3. The order of manipulability of individual voting procedures stayed unchanged
We again find the Condorcet-consistent procedures to be least manipulable, followed by the elimination-based procedures, placing the approval, Bordaôs and max-min procedures at the highest ranks in the order of the most to the least manipulable voting procedures.

Next, I order the procedures according to their **vulnerability to information reduction**, i.e. according to their **information intensity**. I construct a ratio of óMaintained best manipulationô over óProbability of successful manipulationô from under full information. The lower is the ratio, the more vulnerable to information reduction the procedure is. I present the ratios in Table 3.5.

Table 3.39 ï Vulnerability of voting procedures to reduction in information

Procedure (a)	Obs.	Mean	Min	Max
Plurality	8	.471	0	.66
Condorcet	4	.149	0	0.30
Approval	8	.705	0.36	1
Runoff	8	.426	0	0.67
Borda	8	.316	0	0.67
Black	7	.219	0	0.31
Hare's STV	8	.463	0	0.67
Coombs	8	.307	0	0.50
Max-min	8	.384	0	0.60
Copeland	4	.173	0	0.35

(a) Maintained best manipulation / probability of successful manipulation under full information

The least information intensive procedures are the approval, plurality, Hareõs and plurality with runoff procedures. The most vulnerable are Condorcetõs, Copelandõs and Blackõs procedures.

We observe that the order of manipulability of voting procedures is co-determined by the information intensity of the procedures. The least manipulable procedures are in the largest extent vulnerable to the reduction in the amount of information and the most susceptible procedures to manipulation do not suffer from information reduction that much.

4. Under reduced information the susceptibility to manipulation grows in the number of voters

The reason for the increasing manipulation in the number of voters can be attributed to the relatively lower share of withheld information from the strategic voters at higher numbers of voters. Knowing less of 1 sincere voterõs profile when there are 11 voters is less important for the agentõs ability of strategic manipulation than knowing less of 1 sincere voterõs profile when there are just 3 voters. Hereby I confirm the vulnerability of strategic manipulation not only to an absolute reduction in the individual information, but also to a relative reduction.

5. The levels of manipulability are higher for cases with more alternatives.

The discussion is analogous to the one in the section of full information

Attempts for voting manipulation

Numbers of "Attempts for voting manipulation" are provided in Appendix 3.B Table 3.9. Appendix 3.A in its fourth column explains the number of attempts for manipulation by an OLS regression.

The reader must not draw direct inference from absolute figures in Appendix 3.B, since these figures ignore the correlation between the number of attempts and the actual probability of manipulation. It is natural to expect that the weighted distances bid the strategic voter to attempt for strategic manipulation more often in those procedures, which are more susceptible to manipulation. On the other hand, regressing the number of attempts on the probability of voting manipulation would induce endogeneity issues, since both variables are caused by third factors, such as by the number of voters, by the relative amount of withheld information, etc.

The formal model used hence puts on the left side of the regression the ratio of the "number of attempts" over the "probability of strategic manipulation under full information". This ratio is captured in the variable $(\text{Rel. Attempts})_i$.
The formal model (3.5) follows:

$$(\text{Rel. Atempts})_i = \frac{Attempts_i}{\Pr ob_i} = \beta\, \mathrm{n}_i + \gamma\, \mathrm{m4}_i + \delta'\, \mathrm{proced}_i + \varepsilon_i \ , \qquad (3.5)$$

From the regression we can say that there are only few voting procedures where the relative number of attempts significantly differs from other procedures. In other words, the strategic agent attempts relatively for strategic manipulation in majority of procedures to a comparable extent. Majority of coefficients accruing to individual voting procedures fall into the 95% confidence intervals of the coefficients of other voting procedures. Only Black's and Borda's procedures differ from the other procedures in this respect, and their relative number of attempts for manipulation is higher. Nevertheless, we will see that in the case of Black's procedure this increased number of attempts leads eventually to an increased number of adverse outcomes.

As a positive result I view the independence of the number of attempts on the number of alternatives. The agent opts for attempts for manipulation irrespectively of the number of alternatives, which makes her decision making consistent.

Thirdly, a decrease in the relative number of attempts in the number of voters can be interpreted as getting more exact in attempting for manipulation, which I perceive just as well positively.

Overall, we can see that the number of attempts exceeds the number of cases when voting manipulation was optimal by twofold or even more. Luckily for the strategic voter, in cases when she attempts for a voting manipulation and she does not succeed she brings about either a result that is equally good as sincere voting or is better than sincere voting although it is the best manipulating option.

Adverse outcomes of attempting for manipulation

The results for outcomes, which are óWorse than sincereô voting would deliver are provided in Appendix 3.B Table 3.10. Appendix 3.A in its fifth column explains the results by an OLS regression.

Absolutely speaking, the voting outcome is worse than sincere voting would yield on average in 5% of simulated situations when we are voting over 3 alternatives or in 15% of situations when we are voting over 4 alternatives. This percentage appears as a relatively small price to be paid for attempting for manipulation, given how many times the strategic agent succeeded in misrepresentation of her preferences. Moreover, since the strategic agent decides on a basis of a weighted distance, this distance is most probably not that much worse than the distance associated with sincere voting.

Speaking of relative figures, I relate the number of óWorse than sincere outcomesô to the number of actual cases when voting manipulation was optimal. I capture the ratio of these two variables in a variable $(\text{Rel. Worse})_{\text{i}}$. The formal model (3.6) follows:

$$(\text{Rel. Worse})_{\text{i}} = \frac{Worse_i}{\text{Pr}\,ob_i} = \beta\,\text{n}_{\text{i}} + \gamma\,\text{m4}_{\text{i}} + \delta'\,\text{proced}_{\text{i}} + \varepsilon_{\text{i}}\,, \qquad (3.6)$$

We can see that the voting procedures are not statistically distinguishable between each other in the regard of how many óRelative worse than sincereô outcomes they deliver. The strategic agent selects on average the unsatisfactory voting pattern in a similar extent across all voting procedures.

The number of relatively worse outcomes is diminishing in the number of voters. A careful reader has noticed that to an increased number of voters we have previously attributed an increasing exactness of attempting for strategic manipulation. Now we discover that the increase in exactness extends also on the ability of attempting for such voting patterns, which do not harm the individual strategic voter relatively to her sincere voting. This increase may originate in the lowest relative share of withheld information at higher numbers of voters.

3.4 Concluding remarks

This study has computationally simulated 10 most common voting procedures for small numbers of voters and small numbers of competing alternatives so as to study the susceptibility of these procedures to strategic voting. This was followed by a study of vulnerability of strategic voting to the variation in the amount of information that an individual strategic agent possessed.

The paper points out that even if the theoretical prediction is that all feasible voting rules are vulnerable to voting manipulation, the practical circumstances, when the voters know little or more (but not everything) about other votersô voting patterns, the fears of the result being diverged far from socially optimal result are unjustified.

In my paper I have shown that the susceptibility to strategic voting manipulation is a diminishing function of the number of election participants and an increasing function of the number of voting alternatives. All procedures could be characterised by their own specific extent to which they were susceptible to manipulation. The procedure-specific extent of manipulation was in turn dependent on the amount of information that the procedure typically requires from a participating agent to disclose, in combination with the strictness of the voting procedure, which is the amount of points that the procedure allows the agent to manipulate with. Least susceptible voting procedures were the Condorcet-consistent procedures: Blackøs, Copelandøs and Condorcetøs procedure itself. The second group of relatively more susceptible voting procedures involved three elimination procedures: Coombsø, Hareøs and Plurality with runoff voting procedures. Most manipulable procedures were the plurality voting procedure, approval voting procedure, max-min voting and Bordaøs count.

If the strategic agent had a full access to information about other votersô voting patterns, the opportunity for a strategic manipulation has occurred in up to 80% of cases for some procedures, although the average percentage of opportunities moved around 15% for 3 alternatives and 40% for 4 alternatives. Once I have stripped the agent from the full knowledge of the collective preference profile, I have confirmed the vulnerability of strategic voting to both an absolute and relative reduction in the amount of information. Having withheld information from the strategic agent about just one sincerely voting agent has reduced the number of cases, when the strategic agent was able to correctly choose the best manipulating voting pattern, by approx. 15-30 %. I found that strategic voting was most vulnerable to the reduction in information in the least information intensive procedures. That is strategic voting was least vulnerable to a reduction in possessed information in approval, plurality, Hareós and plurality with runoff procedures and most vulnerable in Condorcetós, Copelandós and Blackós procedures. The precision of selection of the best manipulating voting pattern was decreasing in the relative amount of information withheld from the strategic agent. Consistently, the agent has more often ended up with worse payoff than sincere voting would yield, when a relatively larger share of information was withheld from her.

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Appendix 3.A Regression results

Probability of successful manipulation

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Prob full info	Prob reduced info	Prob merged groups	Attempts reduced info	Worse outcome reduced info
n	-0.00567*	0.00773***	0.00310	-0.0589**	-0.0179***
	[0.00285]	[0.00183]	[0.00234]	[0.0229]	[0.00432]
m=4	0.250***	0.0855***	0.162***	-0.00896	0.0846***
	[0.0190]	[0.0115]	[0.0154]	[0.139]	[0.0263]
Reduced info			-0.186***		
			[0.0146]		
Plurality	0.156***	0.0278	0.174***	2.336***	0.349***
	[0.0357]	[0.0210]	[0.0296]	[0.270]	[0.0510]
Condoret	-0.0845*	-0.121***	-0.0212	3.042***	0.106
	[0.0443]	[0.0283]	[0.0396]	[0.379]	[0.0716]
Approval	0.381***	0.197***	0.371***	2.487***	0.219***
	[0.0357]	[0.0210]	[0.0296]	[0.250]	[0.0472]
Runoff	0.199***	0.0264	0.195***	2.118***	0.225***
	[0.0357]	[0.0210]	[0.0296]	[0.270]	[0.0510]
Borda	0.337***	0.0160	0.253***	3.697***	0.210***
	[0.0319]	[0.0210]	[0.0267]	[0.270]	[0.0510]
Black	0.114***	-0.0519**	0.115***	6.264***	0.227***
	[0.0319]	[0.0210]	[0.0267]	[0.291]	[0.0550]
Hare STV	0.0935***	0.00850	0.129***	2.207***	0.198***
	[0.0319]	[0.0210]	[0.0267]	[0.270]	[0.0510]
Coombs	0.0898***	-0.0372*	0.108***	2.752***	0.261***
	[0.0319]	[0.0210]	[0.0267]	[0.270]	[0.0510]
Max-min	0.336***	0.0809***	0.279***	2.776***	0.200***
	[0.0319]	[0.0210]	[0.0267]	[0.270]	[0.0510]
Copeland	-0.0909**	-0.118***	-0.000472	2.940***	0.189***
	[0.0443]	[0.0283]	[0.0399]	[0.361]	[0.0683]
Observations	84	72	164	63	63

Standard errors in brackets

*** p<0.01, ** p<0.05, * p<0.1

Appendix 3.B Simulation tables*

Table 3.40 ï Optimal number of voting manipulations, full info

Probability of manipulation		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0.111	0.136	0.159	0.150
	m= 4	*	0.376	0.265	0.376	0.378
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0.207	0.150	0.124	0.107	0.082
Approval Voting	m = 3	*	0.332	0.432	0.160	0.289
	m = 4	*	0.684	0.704	0.592	0.562
Plurality w\ runoff	m = 3	*	0.222	0.136	0.122	0.107
	m = 4	*	0.406	0.345	0.486	0.474
Borda' s Count	m = 3	0.333	0.196	0.232	0.234	0.219
	m = 4	0.794	0.578	0.598	0.585	0.536
Black' s Procedure	m = 3	0.332	0	0.023	0.014	0.016
	m = 4	0.625	0.259	0.316	0.267	0.225
Hare' s STV	m = 3	0	0.111	0.137	0.174	0.118
	m = 4	0.249	0.189	0.272	0.314	0.305
Coombs' Procedure	m = 3	0.166	0.111	0.114	0.079	0.107
	m = 4	0.247	0.275	0.254	0.228	0.251
Max - min Procedure	m = 3	0.166	0.361	0.354	0.325	0.282
	m = 4	0.542	0.541	0.576	0.582	0.562
Copeland’s Procedure	m = 3	0	0	0	0	0
	m = 4	0.167	0.147	0.128	0.109	0.087

* For plurality, Condorcetôs and Approval voting procedures, the results are trivial for n=2

Table 3.41 ï Alternative measure of manipulation, reduced info

Better than sincere		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.062	0.106	0.099
	m = 4	*	0.251	0.117	0.156	0.175
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.012	0.021	0.026
Approval Voting	m = 3	*	0.331	0.359	0.160	0.242
	m = 4	*	0.496	0.315	0.439	0.407
Plurality w\ runoff	m = 3	*	0	0.061	0.067	0.072
	m = 4	*	0.171	0.130	0.239	0.247
Borda' s Count	m = 3	0	0	0.099	0.131	0.148
	m = 4	0	0.040	0.175	0.211	0.238
Black' s Procedure	m = 3	0	0	0	0.001	0.004
	m = 4	0	0.091	0.144	0.121	0.099
Hare' s STV	m = 3	0	0	0.062	0.060	0.072
	m = 4	0	0.126	0.103	0.187	0.208
Coombs' Procedure	m = 3	0	0	0.020	0.036	0.048
	m = 4	0	0.046	0.073	0.105	0.135
Max - min Procedure	m = 3	0	0	0.114	0.150	0.172
	m = 4	0	0.235	0.314	0.337	0.353
Copeland’s Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.015	0.025	0.032

* For plurality, Condorcetôs and Approval voting procedures, the results are trivial for n=2

Table 3.42 ï Probability to maintain optimal outcome, reduced info

Maintained		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.062	0.106	0.099
	m = 4	*	0.251	0.117	0.156	0.175
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.012	0.021	0.025
Approval Voting	m = 3	*	0.331	0.359	0.160	0.242
	m = 4	*	0.253	0.264	0.352	0.357
Plurality w\ runoff	m = 3	*	0	0.061	0.067	0.072
	m = 4	*	0.171	0.127	0.222	0.235
Borda' s Count	m = 3	0	0	0.099	0.131	0.148
	m = 4	0	0.029	0.124	0.156	0.185
Black' s Procedure	m = 3	0	0	0	0.001	0.004
	m = 4	0	0.068	0.100	0.085	0.071
Hare' s STV	m = 3	0	0	0.062	0.060	0.072
	m = 4	0	0.126	0.100	0.187	0.205
Coombs' Procedure	m = 3	0	0	0.020	0.036	0.048
	m = 4	0	0.046	0.072	0.098	0.126
Max – min Procedure	m = 3	0	0	0.114	0.150	0.172
	m = 4	0	0.132	0.241	0.278	0.304
Copeland’s Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.014	0.025	0.031

* For plurality, Condorcetôs and Approval voting procedures, the results are trivial for n=2

Table 3.43 ï Number of attempts for manipulation, reduced info

Attempts for manipulation		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality Voting	m = 3	*	0	0.149	0.209	0.205
	m = 4	*	0.498	0.203	0.251	0.290
Condorcet's voting	m = 3	0	0	0	0	0
	m = 4	0	0	0.032	0.052	0.065
Approval Voting	m = 3	*	0.667	0.592	0.374	0.419
	m = 4	*	0.749	0.542	0.729	0.707
Plurality w\ runoff	m = 3	*	0	0.109	0.132	0.146
	m = 4	*	0.248	0.185	0.338	0.397
Borda' s Count	m = 3	0	0	0.291	0.375	0.422
	m = 4	0	0.120	0.429	0.527	0.616
Black' s Procedure	m = 3	0	0	0	0.007	0.019
	m = 4	0	0.449	0.530	0.470	0.410
Hare' s STV	m = 3	0	0	0.111	0.134	0.151
	m = 4	0	0.257	0.130	0.265	0.338
Coombs' Procedure	m = 3	0	0	0.057	0.083	0.110
	m = 4	0	0.088	0.194	0.234	0.239
Max - min Procedure	m = 3	0	0	0.184	0.277	0.347
	m = 4	0	0.505	0.627	0.666	0.671
Copeland’s Procedure	m = 3	0	0	0	0	0
	m = 4	0	0	0.049	0.066	0.080

* For plurality, Condorcetôs and Approval voting procedures, the results are trivial for n=2

Table 3.44 ï Worse outcomes than sincere voting would yield, reduced info

Worse than sincere		n = 2	n = 3	n = 5	n = 7	n = 11
Plurality	m = 3	*	0	0.037	0.040	0.022
Voting	m = 4	*	0.247	0.058	0.085	0.069
Condorcet's	m = 3	0	0	0	0	0
voting	m = 4	0	0	0.0006	0.001	0.008
Approval	m = 3	*	0.336	0.111	0	0.047
Voting	m = 4	*	0.061	0.050	0.066	0.067
Plurality	m = 3	*	0	0	0.007	0.002
w\ runoff	m = 4	*	0.077	0.043	0.079	0.076
Borda' s	m = 3	0	0	0.041	0.042	0.033
Count	m = 4	0	0.027	0.063	0.072	0.058
Black' s	m = 3	0	0	0	0	0.001
Procedure	m = 4	0	0.156	0.108	0.080	0.057
Hare' s STV	m = 3	0	0	0	0	0
	m = 4	0	0.130	0.017	0.034	0.028
Coombs'	m = 3	0	0	0.018	0.002	0.016
Procedure	m = 4	0	0.021	0.052	0.030	0.041
Max - min	m = 3	0	0	0.009	0.011	0.011
Procedure	m = 4	0	0.133	0.122	0.108	0.082
Copeland’s	m = 3	0	0	0	0	0
Procedure	m = 4	0	0	0.011	0.010	0.009

* For plurality, Condorcetõs and Approval voting procedures, the results are trivial for n=2