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Master's Thesis

Comparison of Value-At-Risk Using Various Empirical Methods for the Portfolios of BRICT and G-7 Countries In The Long Run

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Master's Thesis

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Annotation

This master's thesis deals with Value-at-Risk for equity portfolios. The distribution of daily returns of equity returns is not perfectly normal. Therefore, the use of the Delta-Normal Value-at-Risk (VaR) method is misleading. Accuracy of estimation may turn out to be failure for portfolios to measure VaR time to time. Therefore, two further methods, Modified VaR and Filtered Historical Simulation, are used for VaR estimation. The former estimates using Cornish-Fisher (1937) expansion and then the latter estimates using autoregressive model for mean equation, EGARCH for volatility and Filtered Historical Simulation (FHS) for VaR estimation i.e. AR (1) - EGARCH (1,1) - FHS methods; and also the performance of both the VaR estimates with Delta-Normal VaR estimate are compared. Last but not the least the implementation of various methods are discussed and analyzed on the two passive historical index portfolios, which represent some of the most attractive financial markets in the world economy.

Keywords

Value-At-Risk, VaR, Gaussian VaR, Modified VaR, EGARCH, ARCH, Bootstrapping, Historical Simulation (HS), Filtered Historical Simulation (FHS)

JEL Classification: C15, C53, G10, G14, G15, G17

Authorship Declaration

I hereby declare and confirm that this thesis is entirely the result of my own work except where otherwise indicated.

In Prague, Özgür Gül

To Stephanie, my wife-to-be, for her endless source of encouragement and support until I had finished…

To Chloe, our daughter…

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Contents

Master's Thesis

THIS THESIS CONTAINS 94 NUMBERED PAGES

Preface

Almost everyone who lives in any part of the world not only heard what is going on about the recent subprime mortgage crisis but also seriously felt some tension in his or her daily life. This crisis affects everybody one way or another. Some people lost their houses because of the unpaid installments of the mortgage credit loans, some people lost their jobs because of defaulted payment contracts or simply lost their hope and prosperity because of being eager on their earnings on the financial markets. In fact, because of the nature of economics science, it is not too much clear yet whether this financial burden in the world is over or it just started inducing some other financial phenomena. All over the world, all economy newspapers, journals, magazines, news on television network and so on have been holding economic reviews, financial insights or talk shows with broad range of experts from public, business and academia. Main topic of the discussions is, of course, the turbulence of global financial market. Everybody talks over the risk on global financial crises knowingly or unknowingly. Furthermore, most of the market players in the developed and emerging countries rely on the pure dynamics of world economics, which obviously undermined economies around the globe. This vast majority of problems become apparent in assessing the risk in financial markets.

In last few decades, the growth rate of world economics and trading activity were tremendously increased, of course, together with the well-known bankruptcies and trading losses of well-known financial institutions. The loss of these multibillion capital-sized market companies (see Danielsson, 1998, Jorion, 2000, Mishkin, 2004) have forced financial regulators and supervisory committees to push to use quantitative techniques to measure possible losses that most likely occur. In fact, VaR has been recognized as one of the most popular of these techniques. What it made so popular actually is that it intuitively provides a straightforward answer to the following key question: "with a given confidence level (say 95 or 99.5 percent), what is the predicted financial loss over a given time horizon?" (Huang and Lin, 2004).

The finance, which is one of the sub-fields of economics, studies the management of money and other assets. Finance in portfolio theory has three main pillars like optimization over time, asset valuation, and risk estimation. When we take a closer look to the market risk estimation in finance, Value-at-Risk estimation method draws great attention on academic researches and studies. As of today, as far as author's knowledge and literature review, this approach has been employed neither earlier nor today's economic turbulence so that this study will contribute to debate by answering outstanding findings in the long-term perspective. Namely, it takes a closer look at the world leading stock markets especially during a subprime mortgage crisis a.k.a. the biggest economic depression of the 21st century when this crisis triggered the liquidity crisis due to increasing defaults in subprime mortgage credits. This liquidity crisis start drawing great attention after February 2007 for the first time.

The portfolio in finance has a very complex and multidimensional components in the actual risk management process. These components refer to different types of financial instruments and derivatives. Each portfolio product exposes to different risk factors. Notwithstanding anything to the contrary contained herein, this master thesis is mainly established on the analysis of two historical portfolio daily price index returns of Group of Seven (G-7) and BRICT nations'[1]. The importance of the academical and industrial interest in portfolio risk management and forecasting is induced by the regulation of banks whereby this risk is assessed by the certain risk measure methods, so called Value-at-Risk (VaR) which is also known as Capital-at-Risk or Money-at-Risk[2]. Furthermore, these BRICT and G-7 countries stock market whose indices were fluctuated with consequences of the economic crises are on the world's top 30 leading economies in term of both market capitalization of listed companies in their stock markets and GDP figures[3]. This was an outstanding advantage to take a closer look at the crisis impact on global scale. This is because by looking at GDP estimates of 2009

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[1] Group of Seven (G-7) is the seven leading industrial countries of the world: Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. BRICT is an abbreviation originates from BRIC countries that coined in a 2003 Goldman Sachs paper in which the authors predicted that the economies of the emerging markets of Brazil, Russia, India and China (BRICs) would overtake the world's wealthiest countries by 2050. Yet some of the researcher predicted that Turkey could be included into this group as well.

[2] See Riskmetrics Group (1996). Concepts and applications are Jorion (2000) and Simons (2000).

[3] See GDP (in USD) figures and market capitalization of listed companies (in USD) of BRICT and G-7 countries in 2008 which are converted from domestic currencies using single year official exchange rates. Data is retrieved from World development indicator of World Bank on May 9, 2010 and available at http://data.worldbank.org/indicator/NY.GDP.MKTP.CD and http://data.worldbank.org/indicator/CM.MKT.LCAP.CD.

and as well as 2010 most of these countries were about to enter economic recession where is a period of no or negative economic growth and high unemployment in it.

However, there is one significant downside, which may have discouraged potential researchers, is to deal with monthly data form where it requires substantial amount of transformation over subsample time series of the market indices and running some analyzes on this daily portfolio data, which does not clearly show evidence of a normal distribution on each subsamples. The advantage of this study is the results capture a significant aspect of risk in a single number on monthly basis. Therefore, it is easy to understand and it asks the fairly simple question: "How bad can things get throughout each subsample?"

Having said that the points mentioned above, there are other significant challenges. One of them is to work on some of the consequences that arise from the key assumptions of the methodology about the distribution of the portfolio returns, the parameter choice of VaR methods and the choice of the proper methodology. Moreover, analyzing a topic in which the potentially some of the interest groups have not reached a rigid decision is another challenge. Yet some of our findings will inevitably contradict with the commonly held opinions of portfolio risk managers, while some of them will further strengthen their opinions. The goal is not to pass any final verdict on any of these outstanding opinions of risk management. It is rather to contribute to the debate by providing sound perspective to them by analyzing empirical daily data in the long term period starting from 2002 to 2010 on the stock market indices of BRICT and G-7 countries.

Introduction

In today's world, the barriers of the capital movement did not disappear completely with the rapid growth in trading activity and advances in information technology. Yet investment is much easier than earlier. Both nationally and internationally financial markets are rapidly liberated. Besides, in the last few decades the financial instruments increased in volume and also in variety. Financial markets are grown and spread quickly and our giant global world is now becoming a tiny local financial market for the global market players. Although each market player, regardless of their size and magnitude of financial events, aims to reduce the risk by minimizing the error of risk models, large institutions started loosing more and more money due to the bad risk management. This cupidity had drawn academicians' attention so that risk measurement/management and controlling of risks had become a distinct sub-field of the theory of finance.

Long-Term Capital Management (LTCM) hedge fund, one of the large hedge funds in US, was collapsed in September 1998 and a consortium of banks bailed out the lost of billions of dollars of investors' money. John Meriwether, the founder of the LTCM hedge fund, drew our attention to clearly learn from this experience of extreme stock market crises in US and Europe due to Russian economic crisis; he declares that:

"With globalization increasing, you'll see more crises. Our whole focus is on the extremes now—what's the worst that can happen to you in any situation—because we never want to go through that again."[4]

After a great deal of losses and bankruptcies is observed in the last few decades, the risk has clearly become a critical issue for organizations, which often prefer not taking the risk in the first place. The most well known example of these is probably the collapse of Barings Bank, UK in 1995, that was caused by the Singapore based derivatives trader Nick Leeson, who took large positions in futures and options on Asian Stock Exchanges (Koupparis, 1995). Other worldwide well-known companies that have been seriously impacted by insufficient risk management techniques are the German

[4] See Embrechts (2008).

commodity and engineering conglomerate Metallgesellschaft in 1993 and Summito Corp. in 1996, that made lost over 1.8 billion USD through unauthorized copper trades (Danielsson, 1998). LTCM hedge fund which was a very large hedge fund caused financial crisis in US since LTCM hedge fund that was invested to Russian bonds that were defaulted by Russian government in 1998 become huge losses. In late 2001, Enron Corporation, a firm focusing on energy market trading and once the seventh largest corporation in the United States (Mishkin, 2004), filed for bankruptcy after recognizing a series of losses and debts that had previously been concealed in off-balance sheet deals with partnerships (Healy and Palepu, 2003).

Certainly, there is barely any circumstance where economic decisions are made with perfect certainty. The statistical uncertainty was traditionally associated with the financial risk on the final outcome (Bouchaud and Potters, 2001). Within increasing competition in financial markets, both the multinational companies and financial institutions become more fragile to financial risk. Subsequently they have taken much riskier positions on the market to cover underestimated loss. Thanks to this kind of gambling on financial markets, a framework of effective risk management came into picture as a necessity of measuring risk. The importance of risk measurement at this stage has been accelerated and is understood with the academic studies to possess risks, and to assess the risk measures.

A new approaches and new tools are invented to control the risk. Within the context of risk management process, these approaches vary from a broad perspective. More complex risk management techniques such as option pricing models, sensitivity analysis, simulation techniques are extensive used. The downside risk measure in finance (Value-at-Risk) method is used more and more in order to estimate market risk since late 1990s. The Value-at-Risk estimation methods are one of the complex risk measurement methods where we are going to explain in more details throughout this thesis.

There are various approaches used by VaR models to estimate the potential losses. Each estimation method, of course, has been evolved over time and we can not say any risk measurement technique which is a single ultimate technique or absolutely superior against other techniques. All techniques have the same intention, that is, in fact, nothing but measuring the size of possible future losses at a predetermined confidence level.

Models differ from each other in a way that they calculate the density function of future profits and losses of current positions and also the underlying assumptions that are based on.

From the evolutions of risk parameters, corresponding evaluations and estimations, a loss distribution is obtained from several risk measures (expected loss, value-at-risk, expected shortfall) which are chosen similar to the density function of the profits and losses of a credit portfolio. For large portfolios, the above calculation method is often simplified by mapping securities to homogeneous buckets with similar sensitivity to risk drivers.

In literature, of computing VaR estimation, there are three basic methods that are going to be explained in next chapter. These three methods draw the greatest attention to produce forecasts. They are as follows: variance-covariance method, historical simulation method and Monte Carlo Simulation Method (Baesens and van Gestel, 2009). Yet the objective of this thesis is to compare parametric and non-parametric VaR estimates for two historical portfolios using alternative methodologies to Delta-normal VaR.

However, the thesis forms by three major chapters. In the first chapter of this thesis, a rather unsophisticated review of VaR literature and our objectives are introduced. In the second part of the thesis, the general financial risk concept and market risk where the Value-at-Risk resides are explained. It continues how the risk concept in financial market industry is perceived. Financial risk has been detailed and market risk types are explained also here in this chapter.

Consequently, in the final chapter, measuring market risk with the Value-at-Risk (VaR) approach is covered with methods of risk measurement. The classification and high level details of various VaR methods are discussed in this section. Furthermore, the essential components of assessing the VaR estimate, which have to be kept in mind, are put in plain words. Furthermore, data and methodologies are discussed. This section will examine the VaR approaches used in this study to illuminate its central questions about how the risk estimate in the portfolio risk management is perceived, interpreted and used via the empirical methods in the long term. Brief descriptive analysis is given about the data we work on. The hypothetical portfolio selection process and the importance of portfolio management are vaguely discussed in this section. Furthermore, results of various VaR approaches that are used are both detailed and findings are shared with the readers. These approaches enables empirical analysis of VaR estimates on two historical portfolios for one-month holding period in the long-run to shed some light on the general perception about the accuracy of VaR estimates before, after and during the subprime mortgage crisis. Subsequently, further possible studies are discussed.

CHAPTER ONE

1 Literature

Literature on VaR calculation shows that there are numerous types of empirical methods available while calculating the passive portfolio VaR and each one of them has different assumption sets to measure VaR estimate. Of course, not all the methods give similar and singular outcomes; consequently, it draws our attention that research in this field is open to new contributions and further studies that can enhance the existing methods by measuring accuracy which can be supported with further robust diagnostic tests.

Bernstein (1998) has an eminently good textbook on the history of risk and probability with financial applications. His paper explains the history of uncertainty, evolution of risk management and the behaviors of financial market investors. Supplementary very helpful material on the risk management and its timeline in the 20th century is to be found in Field (2003).

There are various studies on the outstanding losses because of extremely risky speculative trading activities on financial derivative products. Dunbar (2000) and Lowenstein (2000) explained the LTCM hedge fond case very well. Jorion (2000) is particularly for the technical risk measurement issues involved. Boyle and Boyle (2001) enlighten the the Orange County[5] and Barings and LTCM cases very comprehensibly.

Crouhy, Galai and Mark (2001) give a very detailed overview of relevant useful issues and fundamentals of Risk Management. We also suggest a textbook emphasizing the use of VaR as a risk measure and containing several empirical examples in Jorion (2001), whose valuable teaching paper is a reference point on the same topic (Jorion, 2002).

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[5] See Jorion's Orange County Case Available at http://merage.uci.edu/~jorion/oc/case.html. Retrieved on May 19, 2010.

Zangari (1996) suggests using the Cornish-Fisher expansion[6] of normal distributions to capture extreme events much better than the "classical" normal distributions thanks to the much fatter tails.

In another study, the accuracy of several VaR estimation models on Dutch portfolios on interest rate is examined by Vlaar (2000). Consequently, he found that historical simulation (HS) model calculates satisfactory results only when a long run data is available.

The non-parametric statistical models include the family of HS models. Furthermore, the filtered historical simulation (FHS) is a special form of a generalized HS. All the positive properties of HS exist in FHS and it surmounts weaknesses of most of the historical simulation. This approach which is deliberately mixture of non-parametric and parametric statistical models is developed by Barone-Adesi, Burgoin and Giannopoulos (1998), Barone-Adesi, Giannopoulos and Vosper (1999, 2000) who proposed the FHS approach in a sound way and that allowed the introduction of GARCH models in VaR measures. They also compared the estimates with traditional historical simulation, which has also various shortcomings detailed.

For the first time, Bollerslev (1986) proposed the GARCH specification that puts together the serial dependence of volatility and includes the historical observations into the future volatility (Bollerslev et al. (1994)). Nelson (1991) put forward the EGARCH model, which allows taking into consideration the 'leverage effect' that the volatility of the stock returns increases more after bad news than after good news in the market and overcomes the standard GARCH model weakness regarding the assumptions of the positive and negative error terms have a symmetric effect on the volatility.

In the working paper of Cavenaile and Lejeune (2010) argue that confidence levels below 95.84% should never be used for the Modified Value-at-Risk to be consistent with investors' preferences for kurtosis. In addition, the use of higher confidence levels is restricted by the value of the skewness. Failure to respect these restrictions on confidence levels results in mistakenly assessing risk and potentially overweighting assets which exhibit undesirable properties in terms of higher moments.

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[6] We will elaborate the Cornish-Fisher (1937) expansion of the normal distribution throughout the discussion.

Baillie and Bollerslev (1992) construct ex-ante approximate prediction confidence intervals for GARCH(1,1) dynamic variance forecasts at multiple horizons but ignore estimation error. Furthermore, the estimation error issue is not explained in details in risk management textbooks such as for example Christoffersen (2003) and Jorion (2000).

CHAPTER TWO

2 Risk Concept and Market Risk

2.1 Risk Concept

2.1.1 Definition of Risk and Types of Financial Risks

The investment can be defined as the money or capital disposal, which generates profitable returns as like interest on principal value, fix or floating income, revenue or earnings in the future. As of today, the future can not be known; earnings of the investment can be hardly predicted without uncertainty. Any value prediction is assessed with risks of these profits and losses as investments are analyzed often involve high level uncertain and instability. Uncertainty and instability form a variety of risks in the financial markets. Each of these measures is calculated each day, every day for every market variable to which they are exposed to.

Risk, in general, is defined as to exposure the unexpected results. In terms of economic perspective, "economic risk consists in that actual positive conventional cash flows (income, inflows) turn out to be less than expected or actual negative conventional cash flows (expenditures, outflows) turn out to be larger than expected (in absolute terms)" (Galasyuk and Galasyuk, 2007). Yet then, we can have a deviation between actual results and the expected results can not always be a negative component of risk, the risk can be encountered as a positive including the direction of the deviation (Bolak, 2004). Although above sentences express some of the elements of risk, no single one-sentence definition is entirely satisfactory in all contexts (Embrechts et al., 2005).

"In a sense, the economics of risk is a difficult topic to cover; it involves understanding human decisions in the absence of perfect information" (Chavas, 2004). Maximizing the value of portfolio and providing positive balance on portfolio is some of the common goals to support any financial activities of any business organization that wants to survive on the market. This is not a binding statement and it is also applicable for organizations that rely on heuristic portfolio management models. Providing quick and accurate risk measure information is the responsibility of any (back office) business decision-support units in the organizations. Portfolio risk management is the main responsibility of the senior management team who is used to compose and decompose portfolios to achieve the common goal of any organization that wants to maximize profitability.

Often there are hundreds, or even thousands of market variables, which describe and form different aspects of the risk. In traditional sense, we can categorize market risk under two main pillars, that is, systematic risk and unsystematic risk.

Systematic Risk

Systematic risk can be also called "common risk" or "general market risk". Any The change in the transaction's value correlated with the behavior of the market such as inflation, production factor prices, interest rates, raw material costs affect the scales of economy negatively or any changes all over economy forms this type of the risk. There is a systematic relationship among the returns of a financial asset with the returns of an identical class of all financial assets. Systematic risk of a portfolio of financial assets is a combination of risk in the financial assets they contain.

Systematic risk can affect the return volatility of all financial asset prices in different proportions at the same time and in the same direction and does not reduce its risk in spite of changing the number of financial assets in portfolio or diversifying the risk of financial assets in portfolio. Therefore, systematic risk is the type of risk that can not be diversified.

Unsystematic Risk

The factors such as the management structure of enterprises, quality of management organizations, technical and technological developments, and consumer preferences can be considered unsystematic risk also known as an idiosyncratic risk or company specific risk. Unlike systematic risk, unsystematic risk is that any change in the transaction's value not correlated with the behavior of the market. It presents the characteristics of the company or the business of the underlying risk. Unsystematic risk affects specific industry or financial securities and this risk can be reduced by changing composition of portfolio.

However, in the RiskMetrics — Technical documentation, the risk is defined as "the degree of uncertainty of future net returns". Longerstaey and Spencer (1996) categorize this uncertainty in various forms. That is why most financial market players are subject to a variety of risks. Based on the source of the underlying uncertainty risk can be classified as follows:

- **Credit risk** estimates the potential loss as a result of the inability of a counterpart to pay back its obligations such as loans and bonds, due to the incapacitated payment ability of the borrower.

- **Operational risk** is the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. For instance, results from errors that can be made in unauthorized transactions or results in money laundering.

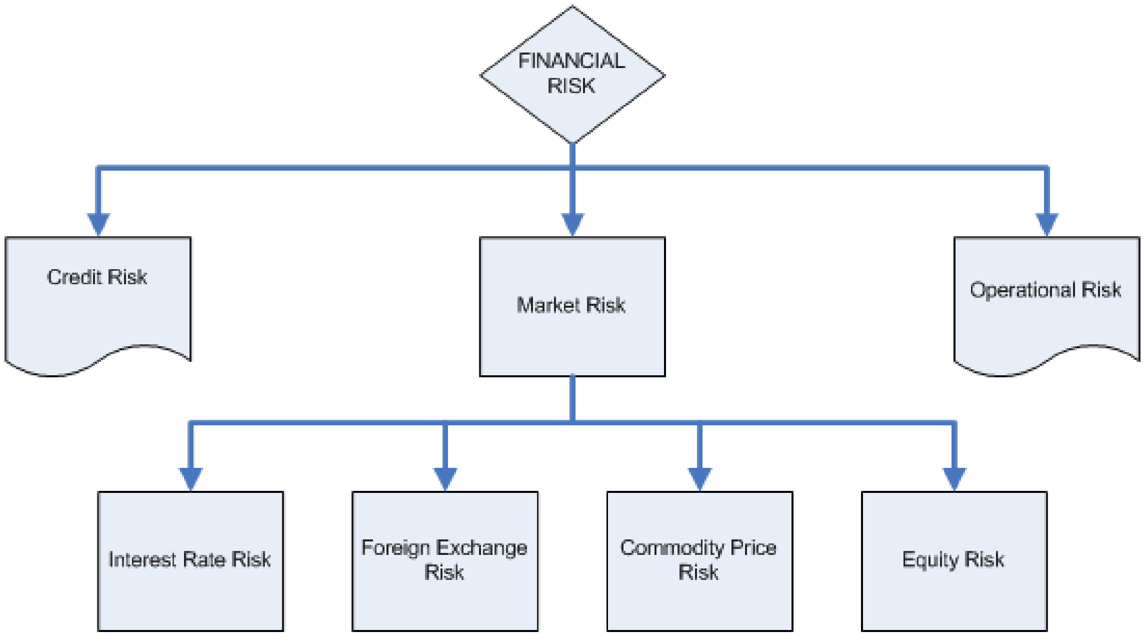
- **Market risk** is the risk that changes in market conditions such as stock and bond prices, interest rates, commodity prices, exchange rates and so on. It involves the uncertainty of future rate of returns resulting from changes. Since the last decade, measure of market risk has become identical with the Value-at-Risk (VaR) approach.

In trading activities, risk arises from both open (unhedged) positions and imperfect correlations between market positions that are intended to counterbalance each other (Crouhy et al., 2000). Besides, "the boundaries of these three risk categories are not always clearly defined, nor do they form an exhaustive list of the full range of possible risks affecting a financial institution" (Embrechts et al., 2005).

2.1.2 Types of Market Risk

In different contexts, market risk is given many different meanings since there is no canonical form of market risk types. For instance, in the case of a portfolio management, the measure of market risk is often relative to a benchmark index and hence the market risk is referred to as "risk of tracking error". Both positive and negative deviations of unexpected outcomes due to changes in financial variables imply that these movements should be viewed as sources of risk. Market risk often captures effect on portfolio value. Figure 2-1 illustrates the high level breakdown of four principal types of Market risk that is subcategory of a financial risk:

Figure 2-1 Structural Breakdown of Financial Risk



Source: Author

- **Interest rate risk** — the simplest form of interest rate risk is the any change in the interest rate value correlated with the behavior of the assets in the market. Changes in interest rates negatively affect asset prices and securities. On the other side, the components of the financial instrument like the maturity and the size and timing of cash flows, portfolio retention time of each asset affect the interest rate levels.

- **Foreign exchange risk —** risk refers to losses due to the foreign currency debt that is incurred a negative exchange rate. Namely, this type of risk is revealed in the value of foreign currency debt upon the appreciation or depreciation of national currency against foreign currency. As like all other market risks, open or imperfectly hedged positions cause foreign exchange risk. Exchange rates was not volatile between 1946 and 1973 thanks to the Bretton Woods system[7]

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[7] Bretton Wood system is the international monetary system of fixed exchange rates that was established in 1944 and lasted from 1946 to 1971 for more than quarter century. In a system of fixed exchange rates, each country's central bank intervenes in the national monetary market against any currency attacks from abroad to balance the exchange rate. (Bordo et al., 1993)

but have been volatile ever since the failure of this international monetary system of fixed exchange rates in 1970s (Dowd, 2002).

- **Equity price risk —** the price risk associated with the sensitivity of an instrument or portfolio value to a change in the value of stock prices (Jorion, 2005). "Specific" or "idiosyncratic" risk refers to this portion of an equity price volatility that is determined by characteristics of a firm such as its line of business, the quality of its management, or a breakdown in its production process. Therefore, equity markets have always been volatile, but sometimes extremely so.

- **Commodity price risk —** the price risk of commodities differs from interest rate and foreign exchange risk considerably. This is because the changes of supply in most commodities, which are traded in markets, can float price. Risks arise in both the spot market and from transactions that take place in the future (e.g., a final product delivery matures in one month's time) whilst spot market condition can be affected positively or negatively.

Accordingly, one of the simplest forms of risk measure is to calculate volatility by standard deviation of unexpected outcomes. There are two factors that can cause losses during combination of them. They are as follows: (i) volatility of underlying financial variable and (ii) exposure of underlying financial variable to this variable of interest like portfolio value, earnings, capital, particular cash flow and so on.

CHAPTER THREE

3 Measuring Market Risk: The VaR Approach

Effects of globalization rigorously observed in a variety of markets like exchange rates, interest rates, commodities and stock markets of the global economy. As a result of high capital mobility, risk management studies have gained incredible momentum to ensure stability of the investment in the second half of the 20^{th} century. Increasing interest and rapid development on the risk estimation models become somewhat a revolution in the risk management process. Around the globe, finance industry races to implement the new models to increase their accuracy on the risk estimation whose methodology is transformed as well as the underlying concept. More sophisticated risk management techniques such as option pricing models, sensitivity analysis, simulation techniques became widespread.

Hull (2005) underlines that Value-at-Risk (VaR) is an attempt to provide a single number summarizing the aggregate risk in a portfolio of financial assets for the senior management. VaR has become widely used by banks, securities firms, commodity merchants, energy merchants, and other trading organizations. Central bank regulators also use VaR while determining the capital requirement to bear with the market risks (Hull, 2006).

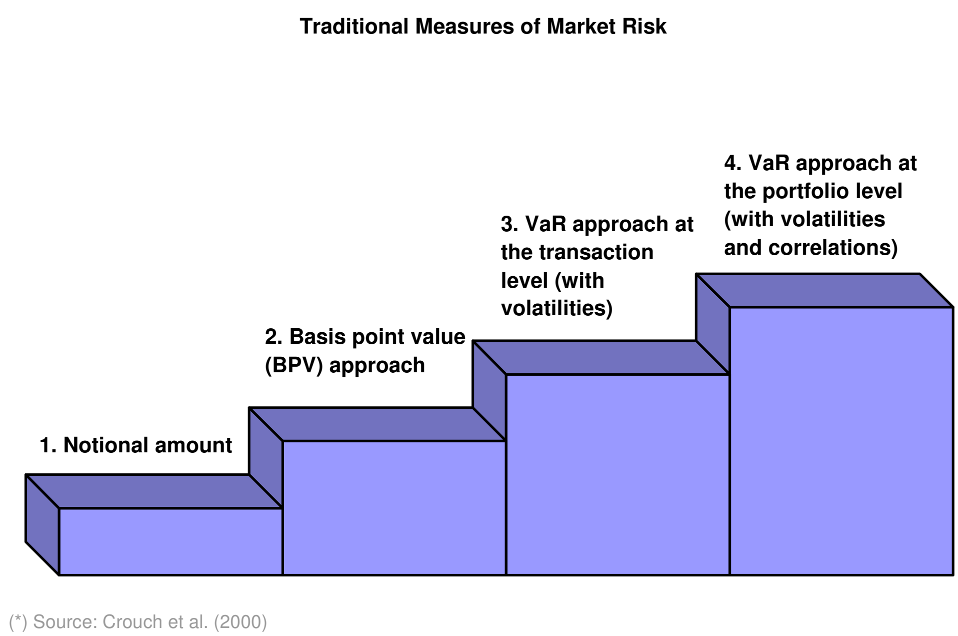
VaR method is used more and more as one of the common statistical methods to estimate the market risk as is. VaR estimate methods are the basis for sophisticated risk measurement in recent years and it is a commonly used risk measure method of the loss as well as profit on a designated portfolio. Yet we need to go back in the history of a theory of finance a bit and we have to have a look at how, why and when this method is brought up and is formed as a method in the theory of finance.

3.1 Measuring Risk: A Historical Perspective

The term "value-at-risk" (VaR) is coined before the new millennium, more precisely in the early 1990s. Yet VaR measure thrown out the consideration for the first time in the literature of the theory of finance is in the early 20th century. Holton states that this goes back in the history to prudential capital requirements of US securities firms, starting with an informal capital test to the New York Stock Exchange (NYSE) first applied to member firms around 1922 (Holton, 2003).

The measurement of risk has replaced with new improved approaches over time likewise other approaches in any science. It has changed from simple measures such as the face value or "notional" amount for an individual security, to more complex measures of price sensitivities such as the duration and convexity and Greek measures, to the most recent methods for measuring VaR estimates (see Figure 3-1). Put it simply, sophistication of methods increases from left hand side to right one. Each method has run at first to be applied to individual securities, and then to be modified to measure the risk of complex portfolios such as those that form with derivatives (Crouhy et al., 2000).

Figure 3-1 Traditional Measures of Market Risk



None of ad-hoc methods like notional amounts, sensitivity measures, and scenario measures that calculate risks was reasonable. These methods do not measure what actually matters, i.e., the downside risk for the total portfolio, while they provide some quick insight of risk. Hence, they give out to take in account differences in volatilities across risk factors, and also the probability of adverse moves in the risk factors (Jorion, 2005).

Holton (2003) asserts that portfolio theory influenced the regulatory and proprietary VaR measures directly or indirectly. Markowitz (1952) and Roy (1952) independently unveiled VaR measures to back up portfolio optimization theory. In 1952, processing power in computer technology was not sufficient to support any practical use of such methods, but the Markowitz risk-return diagram[8] is based on the idea that the natural equilibrium between risk and returns of a given investment portfolio. It represents the foundation of the "theory of portfolio selection" (see Markowitz 1952, 1959). Throughout the entire extent of an efficient portfolio frontier that correlates a portfolio's risk profile to expected returns, the portfolio manager could see natural fact that increasing returns refer bigger risks and also optimize the portfolio return for a given risk level. Markowitz and Sharpe had shown how diversification could easily reduce the risk. In the following decades, we saw a tremendous development in risk management methodology, including such ideas as the Sharpe ratio, the Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Model (APT) (Embrechts et al., 2005).

Even though VaR measures has been used since 1980's by few large financial institutions, it was not too much popular until 1993 when it became widely recognized by financial institutions especially banks and by the financial regulators. A linear VaR model was the main reason behind this extensive influence. The linear VaR is based on the variance-covariance of past portfolio returns, and introduced by JP Morgan, RiskMetrics (1993) (Barone-Adesi, Giannopoulos and Vosper, 2000).

In 1993, the G-30[9] released an influential report regarding the off-balance-sheet products, like derivatives for the first time. Meanwhile, the banking institutions clearly perceived the necessity for an appropriate risk management of these new financial products. For example, the famous Weatherstone '4.15' report which asked for a one-day, one sheet of paper summary of the JPMorgan bank's market risk to be delivered to the top senior manager (CEO) in the late afternoon (hence the "4.15"). RiskMetrics set an industry-wide standard for a market risk measure so that Value-at-Risk (VaR) was born (Phelan, 1997).

[8] Risk-return diagram shows rates of return and a measure of risk (a standard deviation) for a pre-specified time period on the horizontal axis and on the vertical axis respectively.
[9] Group of Thirty (G-30) is an influential private, nonprofit, international body consisting of senior administrators of the private, public sectors, and academia and established in the late 1970s to understand the dynamics of international economic and financial issues. (See http://www.group30.org/)

However, for market risk, the VaR measure catalyzed by the 1996 Amendment is still the most popular market risk measure at the moment of writing. In 1996, Basel Committee on Banking Supervision[10] published an amendment to determine an explicit capital cushion for market risk to which institutions are exposed. For large portfolios, the risk calculation is simplified by mapping securities to homogeneous buckets with similar sensitivity to risk drivers. Instead of evaluating each security, the impact of the risk-driver evolution is calculated on buckets.

3.2 Definition of Value-at-Risk

Market risk is primarily measured using VaR that is one of the most ordinary methods for assessing this type of risk. VaR is a statistical measure of market risk that is easy to interpret. VaR quantifies the total portfolio risk, taking into consideration the diversification and leverage of the portfolio.

VaR as a Downside Risk Measure

VaR is defined as the expected loss from adverse market movements over a target horizon such that there is a low, given probability that the actual loss will be larger (Jorion, 2007). Correspondingly, the RiskMetrics Technical Documentation (J.P.Morgan and Reuters, 1996) describes VaR as a measure of the maximum potential change in value of a portfolio of financial assets with a pre-specified probability over a target horizon. Furthermore, Linsmeier and Pearson (1996) underline VaR as such that is "a single, summary, statistical measure of possible portfolio losses".

Kevin Dowd defines that, "in its most literal sense, VaR refers to a particular amount of money, the maximum amount we are likely to lose over some period, at some specific confidence level" in his textbook "Beyond value at risk". Then again, the author, Cormac Butler, of the textbook "Mastering value at risk" defines with much broader terms. He defines that "Value at Risk is an attempt to identify what causes risk and what policies are effective at reducing risk". It is clearly meant to more complex financial instruments.

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[10] The Central-Bank Governors of the Group of Ten (G-10) founded the Basel Committee of Banking Supervision committee at the end of 1974. Much of the regulatory drive in the financial institutions originated from this committee. See Alexander and Baptista (2001) for further discussion on literature.

As a result of these definitions, the term of VaR can be summarized as VaR is a summary statistical measure of possible maximum portfolio losses due to the price changes of a portfolio. Furthermore, VaR aggregates all of the risky assets into a single number that is more than sufficient to report to stakeholders in the senior management, to regulators, or to put into activity closure documents such as periodical reports in the financial institutions. Losses greater than the VaR are suffered just with a given small probability which subject to the simplified assumptions used in its calculation. Therefore, the idea of VaR is straightforward to understand even if one does not have a competency on a statistical measure. Linsmeier and Pearson (1996) underline "it is simply a way to describe the magnitude of the likely losses on the portfolio".

3.2.1 Components of a Value-At-Risk Measure

In preparation of subsequent models, there are two important aspects of VaR estimates that need to be focused on. Put it differently, the VaR measure is subject to two arbitrarily chosen parameters—a holding (or horizon) period and a confidence level.

Holding Period

A holding (or horizon) period is the period of time over which profit or loss of the portfolio is measured. Since the holding period is a scalar variable, different VaR calculations could use different time. There is no typical "correct" time period while measuring VaR to answer how much the portfolio can lose in a time period. For instance, financial banks often measure a daily basis; then again, pension funds usually calculate a monthly VaR. Thus, the usual holding periods can be either one day or one week or two weeks or one month or even one-quarter year. Ideally, other things being equal, the holding period is suitable in any given market if the length of time ensures liquidation of positions in that market. However, other factors favor a short holding period:

- The assumption that the portfolio does not change over the holding period is more easily defended if we have a shorter holding period.

- A short-term holding period is preferable for model validation: reliable validation requires a large data set, and a large data set requires a short holding period. (Dowd, 2002)

VaR is obviously proportional to holding period and can be extended from a one-day holding period to T days by multiplying with the square root of T. This adjustment assumes daily returns are independent and identically distributed (i.i.d.) and position is constant during the full period of time (Jorion, 2005). When the distribution of returns are independent and identically distributed and also the moments of the distribution are known, any logical judgment made regarding potential portfolio losses will be accurate and unchanging over time (see Barone-Adesi and Giannopoulos, 2003). Therefore, any logical conversion of time period can be easily calculated for different holding periods. For instance; $\sigma_{daily} \cong \frac{\sigma_{annual}}{\sqrt{250}}$ or $\sigma_{monthly} \cong \frac{\sigma_{annual}}{\sqrt{12}}$ are approximation for the volatility estimates.

Generally speaking, many finance industry experts and academicians would agree that the value-at-risk of a passive portfolio is a maximum loss the portfolio itself may suffer within a certain holding period, during when the composition of the portfolio remains intact. The length of this holding period depends on the purpose but it is a quiet short-term, usually one day to a few weeks in the finance industry. Hence, the value-at-risk helps to quantify the maximum amount of portfolio value that can be lost in a short period of time (Huang and Lin, 2004).

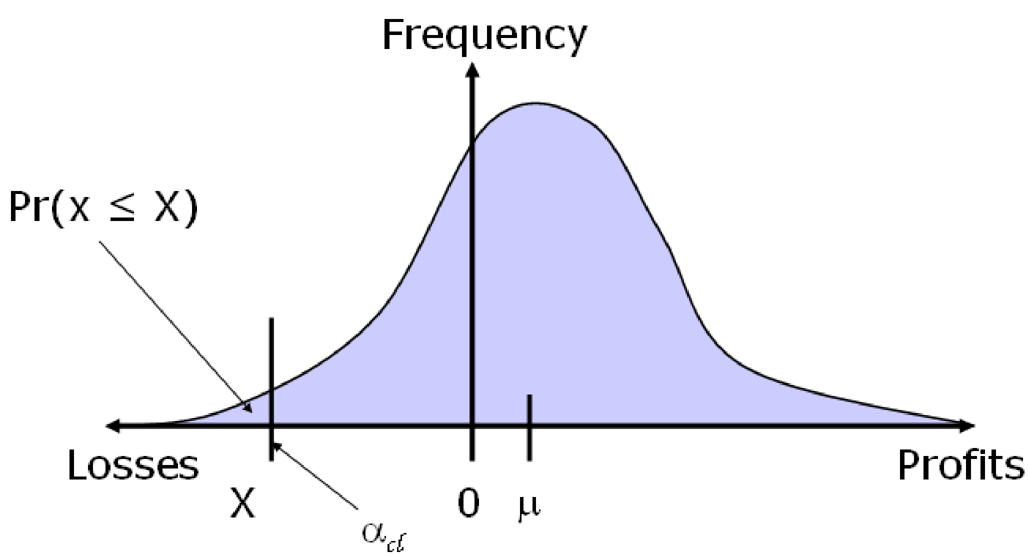
Confidence Level

The selection of confidence level specifies the probability that an outcome will not be worse than VaR estimate, and this value could be 90%, 95%, 99%, or any fraction between 0 and 1 (Dowd, 2002). The selection of confidence level also depends on the composition of portfolio and the liquidity of a market position.

More rationally speaking, the VaR is a single number such that captures a probability α_{cl} of a worse return performance over the T holding period. Both α_{cl} and T have to be determined beforehand by the portfolio risk manager. "The

VaR is thus simply a quantile of return distribution" (Christoffersen, 2009). At the below Figure 3-2 depicts that

Figure 3-2 Loss distribution and probabilities



(*) Source: Author

A random variable X is normally distributed with mean μ and variance σ^2 if the probability that X takes the value x, $f(x)$, obeys the following probability density function (pdf):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left((x-\mu)/\sigma\right)^2\right] \qquad (3\text{-}1)$$

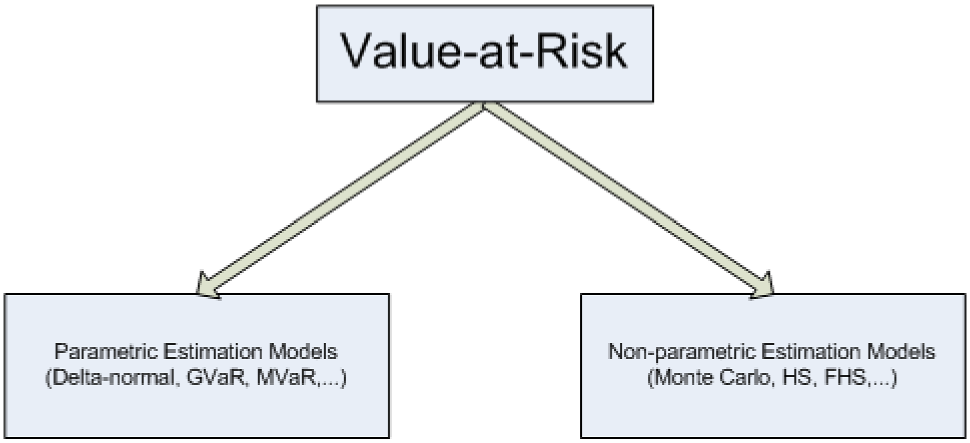
where X is defined over $-\infty < x < \infty$. Therefore, if we want to calculate the VaR estimate at the given confidence level, we can use below equation.

$$\Pr[x \le X] = \int_{-\infty}^{X} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx \qquad (3\text{-}2)$$

where x such that areas to their left represents a given probability α_{cl}. For normal distribution quantiles can be easily found from statistical tables

Having said that VaR is a category of market risk measures. In practice, VaR is generally estimated by means of conventional methods. In the theory of finance, we can classify VaR models into two main categories: parametric estimation (a.k.a. local valuation) and non-parametric estimation (a.k.a. full valuation).

Figure 3-3 Two categories of Value-at-Risk



(*) Source: Author

Parametric estimation means the empirical distribution of portfolio returns fits a parametric distribution with known parameters and non-parametric estimation means past realizations are used to generate historic simulation and assumes that their parametric and/or empirical distribution describe future outcomes (See Figure 3-3). Yet the latter has higher computational complexity than the former. Moreover, three popular VaR estimation models (like Variance-Covariance, Monte Carlo Simulation and Historical Simulation approaches) will be explained in this section together with their advantages and disadvantages. Afterwards, relatively more complex of two estimation models will be covered in the below following sections, "Implementation of Value-At-Risk Approach", "Implementation of Modified Value-At-Risk Approach", and "Implementation of Filtered Historical Simulation Approach".

3.3 Variance-Covariance Approach

The variance-covariance approach (a.k.a. the delta-normal method, the closed-form method, the parametric method, the analytical method or model-building approach) is the simplest VaR approach, which assumes that the portfolio can be adequately utilized as a linear combination of normally distributed risk factors. This parametric model imposes distributional assumption for the underlying distribution of portfolio returns. For instance, underlying distribution could be a normal, mixture of normal, Student's t, Generalized Error Distribution and so on. Therefore, the distribution is one of the core aspects of the method. In addition, the approach is also know as "linear" method since it follows the one common characteristic that all linear transformations are applicable to portfolios whose portfolio mapping[11] function is a linear polynomial. Such portfolios include not only portfolios of equities but also portfolios of commodities, or portfolios of futures.

Variance-Covariance VaR measures do not generally suit well to portfolios like financial instruments with embedded options, redeemable bonds[12], mortgage-backed securities (MBS) and many other structured notes.

The resulting profit and loss distribution is obtained assuming a normally distributed where the variance of returns is calculated based upon the variance and covariance matrix. The simplest method of variance-covariance method is referred to as the minimum data are needed. Apart from the given confidence level, weights of the assets in the portfolio, asset risk and their correlation of coefficients are required. The correlation between the assets and current risks of them are present in different web sites and the weight of the assets in the portfolio may be easily calculated (Butler, 1999).

Local valuation, in which the portfolio is valued once and the changes in value are described by a closed form solution. This is because both the expected return and standard deviation of returns are employed in this method. For instance, while calculating a daily VaR we calculate the standard deviation of daily returns in the past and assume it will be a plausible outcome for the future. Then again, using the expected daily return and standard deviation of a portfolio, we estimate the one day VaR at the given confidence level.

The assumption of normality may not be the most suitable to capture the risk of extreme market movements that are observed in market, but for sure it simplifies the computational burden (Baesens et al., 2009). A stylized fact[13] of empirical finance time series often shows that the distribution of the daily time series of returns has heavier-tail than one with the normal distribution. The assumption of the normal distribution is

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[11] The purpose of a mapping (redistribution) procedure is to characterize a portfolio's exposures to present value and duration.
[12] a.k.a callable bonds.
[13] The stylized facts of financial time series are a collection of observations and consequences obtained from these empirical observations that seem to suit to the most of daily series of returns of equities, indexes, exchange rates and commodity prices (EFM, 2005).

problematic just because many daily series of returns frequently exhibits the characteristic where there are occurrences at the tails, namely, far away from the mean than predicted by the normal distribution. More technically speaking, we are talking about the nature of the problem of the leptokurtosis of the daily returns. VaR will tend to underestimate the loss and its associated probability when a risk factor return distribution has "fat tails". Also, we have to remember that delta-normal VaR is calculated using the historical standard deviation, which may not be suitable if the composition of the portfolio changes, if the estimation period includes extreme events (like market crashes), or if market conditions have differed (Embrechts et al., 2005).

$$VaR = [\hat{R}_p - (\alpha_{cl})(\sigma_{daily})]V_p \tag{3-3}$$

where $\hat{R}_p$ is expected 1-day return on portfolio, V_p is value of the portfolio, α_{cl} is the degree of certainty with the desired level of significance and σ_{daily} is standard deviation of 1-day returns of the portfolio return which is quantified from historical returns variances and covariances for the constituent assets (the ones whose returns have an effect on the portfolio value). In fact, these changes are known as risk factors.

As an impact of the leptokurtic distribution problem, new VaR approaches have been proposed to use as estimators for the distribution of outcomes, but these are complex and have not been widely accepted by scholars yet.

We can list of series of advantages and disadvantages of this approach. They are as follows in the Table 3-1:

Table 3-1 Pros and Cons of the Variance-Covariance Approach

Pros	Cons
• The model is easy to implement • Calculations can be performed quickly • Conductive to analysis because risk factors, correlations, and volatilities are identified	• The need to assume a normal distribution • It requires the estimation of the volatilities of the risk factors as well as the correlations of their returns • The method results in a higher proportion of distributions with fat tails, either because of unidentified time variation in risk or unidentified risk factors/or correlations • Nonlinear relationships of options-

	like positions are not adequately described by the delta-normal method.

(*) Source: Schweser, 2008

3.4 Historical Simulation Method

Historical simulation (HS) (a.k.a. bootstrapping estimation[14]) method is used more and more in the financial risk management and is one of popular ways of estimating VaR. The HS method is based on the past returns. Put it differently, it takes into account historic data in a quite straight way as a seed to answer what might happen in the future. This is know as bootstrapping which is a method for estimating the distribution of an estimator by resampling past return data. HS is often much more accurate in finite samples than ordinary asymptotic approximations.

HS assumes that same parameters of the distribution of past and prediction returns has the same parameters and past and prediction moments of the density function of returns of a specific risk factor are equivalent to each other. Moreover, HS utilizes historical data to estimate future outcomes. Multipath simulation scenarios are generated with arbitrarily chosen historical returns coupled with each risk factor. Thanks to simulated values of a portfolio, the aggregate risk of all linear positions is generated. The procedure is repeated many times using only historical returns (Barone-Adesi and Giannopoulos, 2000).

The method uses one of two procedures: (1) a single-step procedure and long-term data; this procedure is used when the data has the same time scale as the time horizon of interest, or (2) a multi-step procedure and short-term data to create a longer term periods.

HS requires no statistical assumption except stationarity of the distribution of returns or especially their volatility. In addition to that, past returns are drawn with or without replacement. Thus, the historical simulation approach does not formulate any presumption on the density distribution of the returns. "The historical VaR is an

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[14] Bootstrap method is one of important methods in applied econometrics, because the familiar asymptotic normal and chi-square approximations can be very inaccurate. See Efron and Tibshirani (1993) for further bootstrap discussions.

extrapolative method that assumes the future is a faithful reproduction of the past and the present. Historical data are used to identify a hypothetical density function which is employed to calculate the current or future portfolio VaR" (Ajili, 2008).

In its simplest form, Barone-Adesi and Giannopoulos (2000) consider historical simulation as following. Given a data set of historical returns Θ, we draw an element e^*

$e^* = \{e_1^*, e_2^*, \ldots, e_T^*\}$ $\quad$ $e^* \in \Theta$ $\quad$ where $i = 1,\ 2, \ldots,\ T$ refers to past days to form a simulation price for asset Y:

$$Y_{T+1}^* = Y_T + Y_T^* e^* \qquad (3\text{-}4)$$

The process in (3-4) is repeated and the simulated price series Y^* is recursively updated up the last day of the VaR horizon. This sequence of simulated prices for day $T+1, T+2, \ldots, T+N$ forms a simulated pathway or scenario for the risk factor Y.

The ability to gather enough number of appropriate past data for analysis is the significant factor for the success of HS approach. If there are any gaps in the past data, if there are new unmapped risk factors in the model, if there are troubles fulfilling the historical record, these hassle decreases the effective number of observation and that also means, no doubt, accuracy of VaR suffers due to the poor empirical predictions.

In the Historical Simulation methodology, there are at least two techniques. First technique uses the present weights of assets, the correlations of assets and assets' volatilities to compute the volatility of portfolio. This technique approach is called as "ex-ante" technique. Second, "ex-post" technique computes the portfolio returns and then the portfolio volatility is estimated from it. This technique is also known as portfolio-normal (Bonollo, 2007). Therefore, a sample of historical portfolio returns with current portfolio weights is built as an initial step. The VaR is basically calculated as the unconditional quantile of the subsample history. The method therefore mostly takes no notice of the last 20 years of academic research on models of conditional asset return. Time variability is only captured through the rolling historical sample. Variation in time is merely captured through the rolling historical sample. Despite warnings of the nature of force-free model, (see Pritsker, 2001), the HS method is seen as a great benefit for many professionals. The well-known use of the technique of HS motivates us to concentrate on backtesting VAR calculated using this method (Christoffersen and Pelletier, 2004).

We can list of series of advantages and disadvantages of this approach. They are as follows in Table 3-2:

Table 3-2 Pros and Cons of the Historical Simulation Approach

Pros	Cons
• The model is easy to implement if enough data exists • Calculations can be performed quickly • Horizon is a positive choice based on the intervals of historical data used • It is exposed to model risk • It includes all correlations as embedded in market price changes	• There may be not enough historical data for all assets • Only one path of events is used (the actual history), which includes changes in correlations and volatilities that may have occurred only in that historical period • The past data is considered a representative of the future. Therefore the window may omit important data or may include not relevant data • Time variation of risk in the past may not represent variation in the future • The model may not recognize changes in volatility and correlations from structural changes, such as the introduction of the New Turkish Lira in January 2005 • It is slow to adapt to new volatilities and correlations as old data carries the same weight as more recent data • Small number of actual observations may lead to biased and insufficiently defined distribution tails • It can not be used to conduct sensitivity analyses

(*) Source: Schweser, 2008

3.5 Monte Carlo Approach

The method is identical to the historical simulation method. Instead of the historical time series, a series of pseudorandom numbers is generated via Monte Carlo simulation, which attempts to predict the maximum likely loss for a given a confidence interval over a prespecified holding period over the distribution of pricing pathways given arbitrarily generated data. It involves the creation of the distribution by taking samples from a normal (or Gaussian) distribution to simulate potential future outcomes. For every future scenario, a portfolio value can be generated and a corresponding VaR measure can be estimated (See Schweser, 2008, p. 198-200).

The advantage of the Monte Carlo simulation is that it is the most flexible method and does not assume linearity or normality and full valuation, in which the portfolio is fully valued for each scenario and can include variations in risk and correlations and can provide a nearly unlimited number of scenarios.

We can list of series of advantages and disadvantages of this approach. They are as follows in the **Error! Reference source not found.**:

Table 3-3 Pros and Cons of the Monte Carlo Simulation

Pros	Cons
• It is the most powerful model • It can accommodate any distribution of risk factors and account for both linear and nonlinear risks • It can include time variation in risk and correlations by aging positions over chosen horizons • Nearly unlimited numbers of scenarios can produce well-described distributions • It allows the user to perform sensitivity analyses and stress testing	• There is a lengthy computation time as number of valuations escalates quickly • It is expensive because of the intellectual and computing skills required. • It is subject to model risk of the stochastic processes chosen • It is subjected to sampling variation at lower numbers of simulations

(*) Source: Schweser, 2008

3.6 Usage of Value-at-Risk

Usually VaR has been a tool for measuring and managing short horizon risk (see (Pritsker, 2000) and (Kupiec, 1995)). However, for example financial institutions and corporations with long term liabilities, it is necessary to have a proper risk management methodology that control risk on longer horizons (Giannopoulos, 1995). Dowd (2002), author of an Introduction to Market Risk Measurement, notes that VaR information can be used in many ways. For instance, according to Dowd (2002),

- Top management can use the VaR estimate to set their goal of risk management objectives that determine the risk and position limits on the business activity.

Whenever they want the financial institution to increase their risk exposure relative to comparisons of benchmark value, they would increase the global VaR objective, and vice versa. Of course, there are circumstances where risk needs be considered on longer horizons for example financial institutions have budgeting and forecasting for up to one year or longer.

- Since VaR tells us the potential loss, we can use it to set the levels of capital cushion. It can be used to determine not only capital requirements at financial institution but also the level of individual investment decision. Put it simply, the riskier the activity, the greater the value at risk and the greater the demand for capital. Therefore, financial institutions decide on (internal) long run policy based on VaR estimates.

- VaR can be handy to inform and publicize the purposes and financial institutions. VaR is increasingly becoming benchmark information in their annual reports[15].

- We can use the VaR information to assess the risks of different investment opportunities before making decisions. VaR-based decision rules can guide investment, hedging strategies and trading decisions, and also undertaking any alternative options for the risk of the portfolio as a whole[16].

- VaR can help provide a more coherent and integrated approach to manage different risks, which also leads to greater transparency and better risk management strategy.

[15] Dowd (2002) also suggests seeing Dowd (2000b), Jorion (2001) or Moosa and Knight (2001) for more on the use of VaR for reporting and disclosure purposes.
[16] Dowd (2002) also suggests seeing Dowd (1999) for further information on VaR-based decision rules and suggests seeing Kuruc and Lee (1998) and Dowd (1999) for such portfolio-wide hedging strategies are explained in more detail.

4 Data and Methodology

First of all, there is hardly any circumstances where in any economic decision is taken into account the risk. This is understood by everyone without being openly stated. Of course, this tacit consent can not be made without any benchmark value to compare with the interest. In this section, this benchmark method would be Delta-normal VaR estimation. We intend to conduct this research after transforming the data. The descriptive statistics of the data before and after transforming the data will be elaborated as well. Particularly, we will clarify how historical portfolios and hypothetical portfolios of BRICT and G-7 countries and also subsamples are formed to run further methodological analysis on the data in the long-run. Afterwards, the Jarque-Bera normality test on the data is conducted whether daily returns are normally distributed or not.

Secondly, after how to prepare the subsample data will be illustrated in details, we will justify why we want to focus on two more methods that are much more complex than delta-normal VaR estimate that is a benchmark method in this thesis so that we will be able to compare the VaR estimates in the long run. In addition to that, we will plot the histogram diagrams of the periods with the high 'maximum loss' and compare them to the histogram diagrams of the periods with low 'maximum loss' and see how the market did change.

Last but not the least comparison of one another will be performed on VaR estimates for various confidence levels and findings will be shared in the following section, i.e., Conclusion and Further Studies. Throughout this section, the assumptions underpinning the research design will be made explicit and we assume that a reader has solid background in applied econometrics to get along with the concepts and details of the methodology.

Where does the Data Comes From?

Our data comes from the Reuters Wealth Manager[17] data dissemination website. We take into account daily stock index returns $r_1, \ldots, r_n$ and assume that these have been expressed in geometric form. Put it simply, the geometric return r_t is a first logarithmic differencing of a daily index series (Y_t).

$$r_t = \ln\left(Y_t / Y_{t-1}\right) \qquad (4\text{-}1)$$

where $t = 1,\ 2, \ldots,\ T$.

We decided to use the geometric returns data rather than the arithmetic return [18]data that does not ensure portfolio value is never negative if the returns will be small if we are dealing with the short horizon period even if the returns themselves are unbounded.

As per already mentioned in the preliminary of the thesis, we choose the below listed stock market indices (see Table 4-1) based on the GDP and market capitalization of listed companies in the stock market of BRICT and G-7 countries to analyze the VaR estimate among the world's 30 leading economies. Historical past data consists of 1531 observations[19] of daily closing index values of the below representative equity indices for the period of the trading dates between January 8, 2002 to February 26, 2010.

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[17] Historical stock exchange market index closing values had been retrieved from Reuters Wealth Manager Service prior to March 1, 2010. All data rights are reserved by http://www.reuters.com and data is available at http://www.gva.rapid.reuters.com/wealthmanager/login.aspx?culture=en-GB. We are certainly grateful to Institute of Economic Studies at Charles University in Prague for providing us the data.

[18] Data can also come in the form of arithmetic returns. The arithmetic return is defined as $r_t = (Y_t - Y_{t-1}) / Y_{t-1}$ where $t = 1,\ 2, \ldots,\ T$. This form is less meaningful economically. This is because a low realized return —or a high loss— implies that the asset value (Y_t) is negative, and a negative asset price seldom makes economic sense.

[19] A careful reader would easily catch that the number of observation is less than what it suppose to be because some records are filtered for making VaR prediction consistent if there is any missing observation of the closing index of any stock market where there is any local holiday or an off-day. Harri and Brorsen (2009, p. 2) argue that "most authors provide no justification for using overlapping data, but there must be some advantage to using it or it would not be so widely used". In fact, we will certainly do that too.

Table 4-1 List of the hypothetical portfolio which is composed with equal weights

Hypothetical Portfolio Composition	Country	Stock Exchange Market	Ticker Symbol
BRICT	Brazil	Bovespa	(.BVSP)
	Russia	RMX	(.RMX)
	India	Bombay SE	(.BSE500)
	China	Shangai SE	(.SSEC)
	Turkey	Istanbul SE	(.XUTUM)
G-7	Canada	TSX Composite	(.GSPTSE)
	France	CAC 40	(.FCHI)
	Germany	DAX	(.GDAXI)
	Italy	FTSE MIB	(.FTMIB)
	Japan	Nikkei 225	(.N225)
	UK	FTSE 100	(.FTSE)
	US	S&P 500	(.GSPC)

Figure 4-1 and Figure 4-2 illustrate the relative price movement of each index for BRICT and G-7 countries respectively. The initial level of each index has been normalized to demonstrate the comparison of relative performance.

Figure 4-1 Relative daily equity indices spanning the trading dates January 8, 2002 to February 26, 2010 for BRICT countries

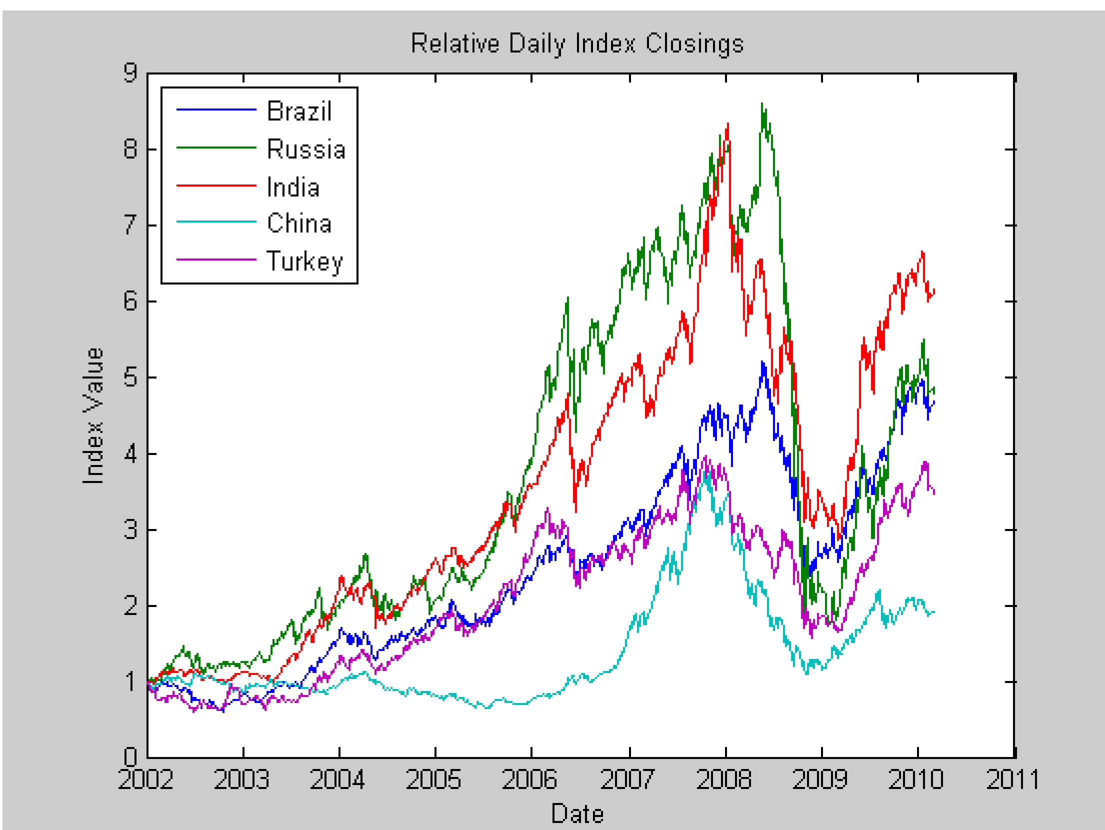
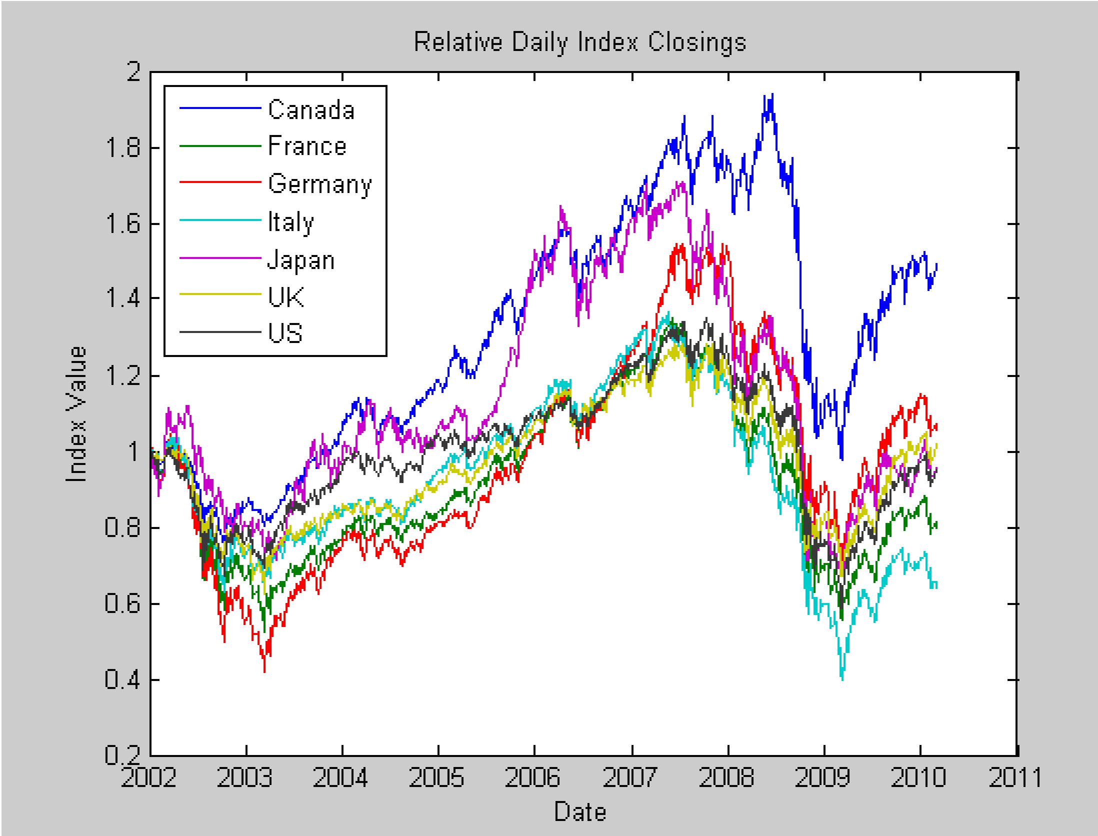


Figure 4-2 Relative daily equity indices spanning the trading dates January 8, 2002 to February 26, 2010 for G-7 countries



4.1 Data Description and Preliminary Analysis

Descriptive statistics on the price and return series, for the period January 8, 2002 to February 26, 2010, are shown Table 4-2 and Table 4-3 for BRICT and G-7 countries respectively. The results from descriptive statistic analysis of these two underlying data sets show that the distributions of all return series are characterized by negative skewness except for China and Turkey in BRICT and France and UK in G-7 countries. The coefficient of kurtosis on the daily returns is greater than that predicted by the normal (Gaussian) distribution.

Table 4-2 Descriptive statistics of the daily equity indices and log returns of whole sample (1531 observations) of BRICT countries

Country	Descriptive Statistics			Country	Descriptive Statistics		
		Equity Index	Log. Return			Equity Index	Log. Return
Brazil	Mean:	34866.8086	0.001	India	Mean:	3697.241	0.0012
	Std. Dev.:	18283.1654	0.0225		Std. Dev.:	1992.095	0.02
	Skewness:	0.3122	-0.3536		Skewness:	0.3056	-0.5315
	Kurtosis:	1.8605	8.9886		Kurtosis:	1.9466	13.1827
	Min:	8370.88	-0.1875		Min:	1021.3	-0.1684
	Max:	73516.8	0.1097		Max:	8778.98	0.1647

Country	Descriptive Statistics			Country	Descriptive Statistics		
		Equity Index	Log. Return			Equity Index	Log. Return
Russia	Mean:	1068.3808	0.001	China	Mean:	2182.572	0.0004
	Std. Dev.:	630.266	0.0295		Std. Dev.:	1131.713	0.0209
	Skewness:	0.5353	-0.8673		Skewness:	1.4034	0.0383
	Kurtosis:	1.8809	35.8561		Kurtosis:	4.2043	7.0682
	Min:	282.79	-0.3945		Min:	1011.499	-0.1276
	Max:	2478.87	0.2953		Max:	6092.057	0.1295
Turkey	Mean:	29039.1288	0.0008				
	Std. Dev.:	13796.2337	0.0233				
	Skewness:	0.0487	0.1286				
	Kurtosis:	1.7049	8.3364				
	Min:	8391.84	-0.1439				
	Max:	55720.33	0.1504				

Table 4-3 Descriptive statistics of the daily equity indices and log returns of whole sample (1531 observations) of G-7 countries

Country	Descriptive Statistics			Country	Descriptive Statistics		
		Equity Index	Log. Return			Equity Index	Log. Return
Canada	Mean:	10191.3214	0.0003	Italy	Mean:	30107.56	-0.0003
	Std. Dev.:	2481.7133	0.0137		Std. Dev.:	6970.361	0.0165
	Skewness:	0.1248	-1.4028		Skewness:	0.0673	-0.2301
	Kurtosis:	1.7796	23.9313		Kurtosis:	2.1574	12.1269
	Min:	5695.33	-0.17		Min:	12895	-0.1328
	Max:	15073.13	0.0871		Max:	44364	0.1372
France	Mean:	4176.0157	-0.0001	Japan	Mean:	12218.51	0.0000
	Std. Dev.:	911.4702	0.0176		Std. Dev.:	2935.497	0.0179
	Skewness:	0.3454	0.0125		Skewness:	0.4754	-0.5216
	Kurtosis:	2.0487	9.3636		Kurtosis:	2.0421	10.8942
	Min:	2403.04	-0.1148		Min:	7173.1	-0.1272
	Max:	6168.15	0.1129		Max:	18252.67	0.1181
Germany	Mean:	5096.1828	0.0000	UK	Mean:	5080.862	0.0000
	Std. Dev.:	1407.0255	0.0186		Std. Dev.:	836.8845	0.0151
	Skewness:	0.3139	-0.3596		Skewness:	0.1679	0.0345
	Kurtosis:	2.2409	8.5165		Kurtosis:	1.8713	10.7155
	Min:	2202.96	-0.1183		Min:	3287	-0.1033
	Max:	8092.77	0.1159		Max:	6732.4	0.1087
US	Mean:	1158.7023	0.0000				
	Std. Dev.:	196.0142	0.0148				
	Skewness:	0.0353	-0.7514				
	Kurtosis:	2.2066	13.0104				
	Min:	682.55	-0.1378				
	Max:	1565.15	0.094				

In here we will let the data explains itself so that there is not much to say about the stylized facts of daily index returns' variances which always exceed the mean and daily returns are not normally distributed (see Cont, 2001). Yet some of these aspects are meticulously explained for the hypothetical historical portfolio periods later on.

A time series of T hypothetical historical portfolio returns are computed using constant portfolio weights, and historical returns on N equity indices

$$\{r_t\}_{t=1}^{T} \equiv \left\{ \sum_{j=1}^{N} w_{T,j} r_{t,j} \right\}_{t=1}^{T} \tag{4-2}$$

where $r_{t,j}$ denotes the log returns on country equity index j from the market close on day $t-1$ to market close on day t, that is, (above equation on page 39) and where $w_{T,j}$ refers to weight of j equity index in the portfolio despite the fact that an equally weighted portfolio is assumed. The modeling of the properties of this univariate portfolio return set for the univariate risk model. Furthermore, the portfolio weights are fixed throughout the analysis horizon. Obviously, this approach prevents us to not deal with the correlations, quantify the past return variances and covariances and also other interdependencies between N equity indices. Disadvantage of this approach, though, is that it is restrictive on the portfolio weights. If the weight vectors are altered, then the estimated model should be measured again. Thus, "it does not directly allow for evaluating the effects of actively managing the risk of the portfolio by changing the portfolio weights" (Christoffersen, 2009). Moreover, dividend adjustments, margin payments, reinvestment income, storage costs, insurance, financing or changes in exchange rates are ignored in the evaluation. Even if it assumes the daily rebalancing process is self-financing and no transaction costs required to rebalance the portfolio are explicitly taken into account.

After hypothetical portfolio return series are formed, the sample minimum, maximum, standard deviation, skewness, kurtosis, Jarque-Bera and Kolmogorov-Smirnov statistics of the returns are parts of the descriptive (or summary) statistics that can be retrieved easily on the hypothetical return series, for the period January 8, 2002 to February 26, 2010, are shown Table 4-4 for BRICT and G-7 countries respectively. Kurtosis value is equal three for the normal distribution. Values are greater than three show that the distribution has 'heavy tails'. This clearly indicates the density of a distribution is in the center, is higher at the tails and the density in the areas in between is smaller than the density of a normal distribution. These are characteristic for financial data with daily frequency (Kirshgassner and Wolters, 2007).

Table 4-4 Descriptive statistics of the portfolio logarithmic returns of whole sample

Descriptive Statistics of the Logarithmic Returns		
	BRICT	**G-7**
Mean:	0.001	0.000
Std. Dev:	0.0158	0.0135
Skewness:	-0.4841	-0.6049
Kurtosis:	20.3329	13.3938
Min:	-0.1719	-0.129
Max:	0.1154	0.0872
JB test statistic:*	19212.13	6980.312
p-value:	0.001	0.001
KS test statistic:*	0.474438	0.478456
p-value:	0.000	0.000

(*) H_0 of normality tests can be rejected at 5% significance level

Mandelbrot (1963) noted that the time series of financial portfolio returns frequently exhibit the volatility clustering feature where probability of large price changes likely to cluster together, resulting in persistence of the amplitudes of subsequent price changes too. For instance, volatility clusters in 2009 on Figure 4-3 and Figure 4-4 are evidently distinguishable due to large price changes in the BRICT and G-7 hypothetical portfolios.

Figure 4-3 Hypothetical global equity index portfolio relative closings and returns spanning the trading dates January 8, 2002 to February 26, 2010 for G-7 countries

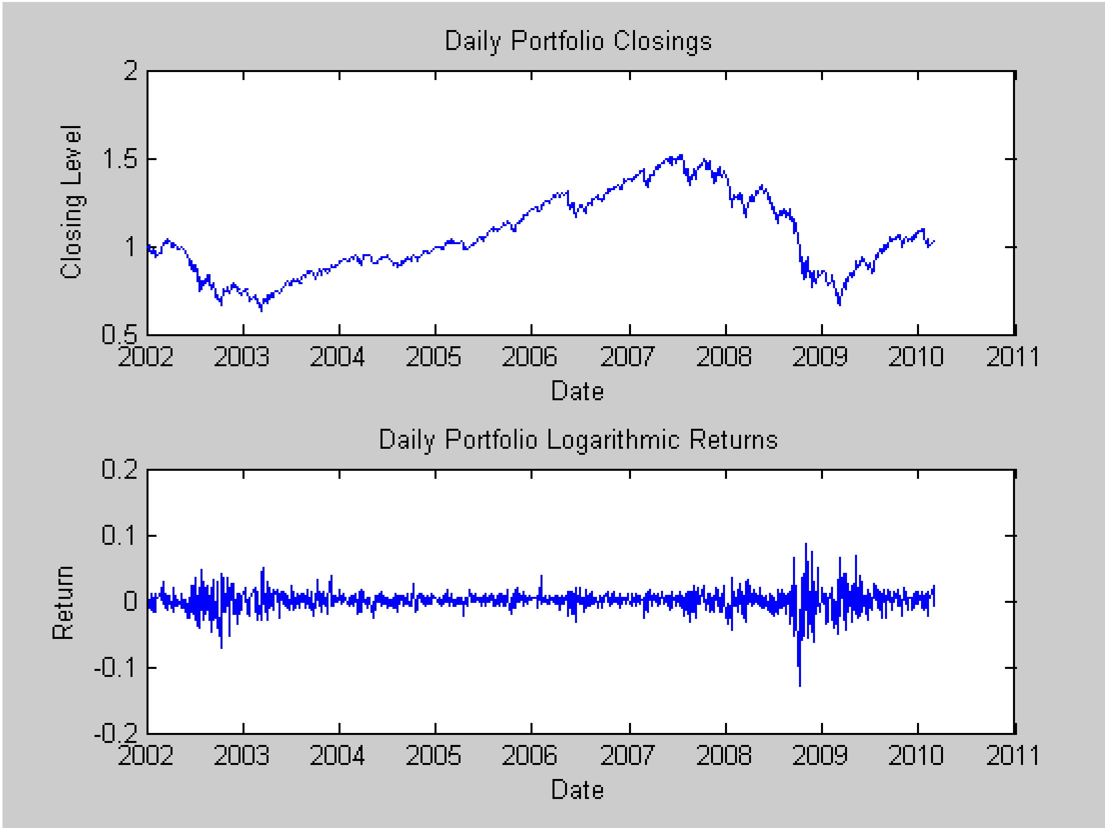
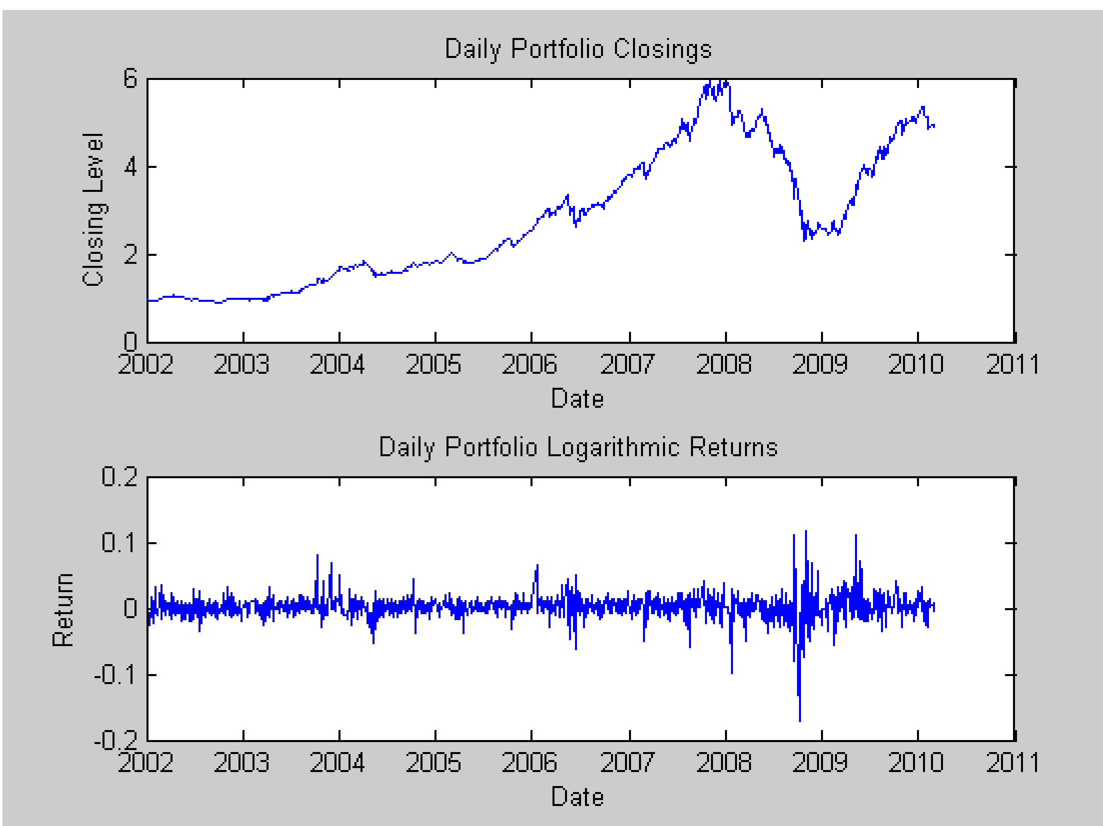


Figure 4-4 Hypothetical global equity index portfolio spanning the trading dates January 8, 2002 to February 26, 2010 for BRICT countries



A standard Jarque-Bera test is performed and the null hypothesis of normality assumption can be rejected in all subsamples except 1, 2*, 3*, 4, 5, 6* and 3, 4, 39, 48, 49* subsamples of BRICT and G-7 countries respectively for p-value greater than 0.05 (see Table 4-5 and see Table 0-1 for complete list of subsamples). We took eight years of data spanning the period 2002–2010 and formed daily logarithmic returns. For each subsample period, we calculated sample skewness and kurtosis and applied the Jarque–Bera test[20] to the univariate time series. The daily return data fail for majority of tests; especially, it is notable that there are some large values for the sample kurtosis. Of course, the reasons for this high number of rejections are due to days with extreme gains or losses, leading to excess skewness and kurtosis. As a result of that, the rejection rate decreases to about 100% for the subsamples in the one sample Kolmogorov-Smirnov test, which is more "tolerant" to outliers. Both tests indicate that the assumption of normality of the returns of the portfolio could lead to specification errors.

Table 4-5 Jarque-Bera test statistics of normality where the null hypothesis can be rejected for subsamples of the portfolio logarithmic returns

Subsample Period	BRICT for %5 level		G-7 for %5 level	
	JB test statistic	p-value	JB test statistic	p-value
2	3.825573	0.110892*	14.702106	0.005845
3	3.933567	0.105027*	10.51584	0.013298
6	2.761934	0.199293*	47.666609	0.001
49	414.2035	0.001	5.014833	0.064407*

(*) Indicates where the null hypothesis can be rejected

However, rigorous assumption about the distributional properties is imposed by delta-normal VaR models. For instance, the density function of daily returns follows a normal distribution and has constant volatility of volatility estimate to produce current. The empirical studies on value changes and the distributional properties of them support these assumptions (see Kendall, 1953 and Mandelbrot, 1963). Furthermore, the frequency of the data these violates assumptions of normality. The hypothetical subsample data, which have daily frequency, tend to deviate from normality.

[20] Jarque-Bera (1987) test statistic is which is asymptotically distributed as a chi-squared random variable with two degrees of freedom, to test for the normality of r_t. One rejects H_0 of normality if the p-value of the JB statistic is less than the significance level. It can be applied on the time series itself and as well as on its differences.

In 1997 Alan Greenspan expressed his own concern for the financial returns show fat tail characteristic which means that the normal distribution is not a good estimator to predict extreme market events by stating "*…as you well know, the biggest problems we now have with the whole evolution of risk is the fat-tail problem, which is really creating large conceptual difficulties*" (see Danielsson, 1998-1999, p. 11.).

In fact, a quantile-quantile (Q-Q) plot is also used to see whether the daily observations of data follow normal standard distribution (see Figure 4-5 and Figure 4-6). Yet due to the fat tailed distribution, the distributions of both BRICT and G-7 countries are not normal, the blue colored '+' plot is not close to linear. It should be approximately linear if the data is normally distributed.

Figure 4-5 A quantile-quantile plot of the portfolio logarithmic returns for whole sample versus a normal distribution, for BRICT countries

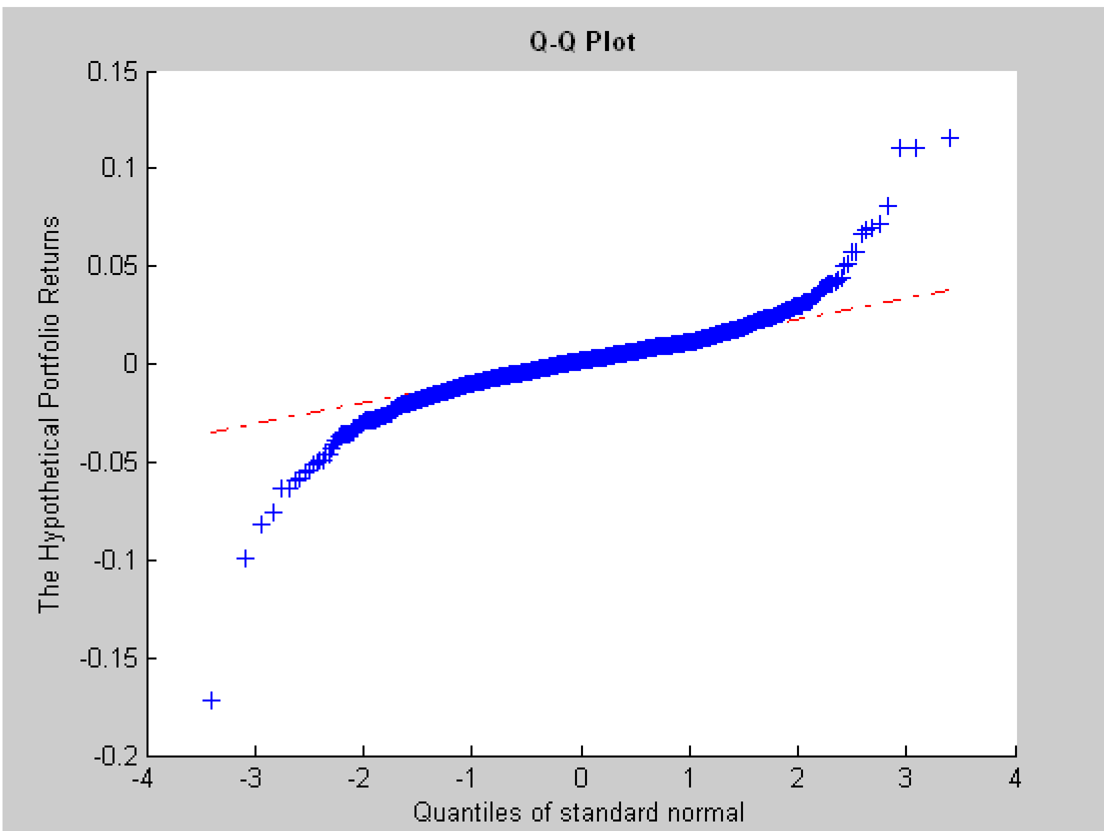
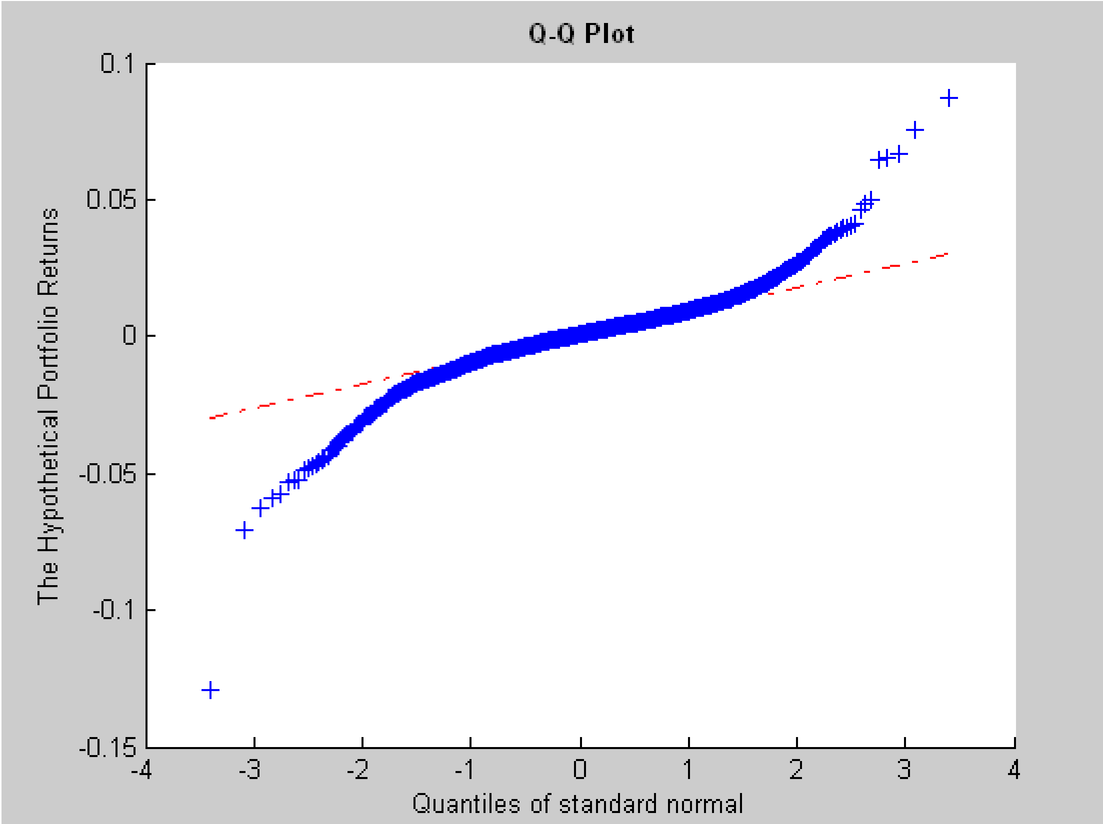


Figure 4-6 A quantile-quantile plot of the hypothetical portfolio returns for whole sample versus a normal distribution, for G-7 countries



In preparation for subsequent modeling of our methodology, first of all, we need to focus on the two important aspects of VaR estimates — the holding period and the confidence level — that are of interest in risk management and are often chosen arbitrarily based on the intention.

Our purpose is to demonstrate VaR estimates of rolling subsamples over 60 periods for comparison; we use three different confidence levels. These values are 90, 95 and 99%.

Therefore, in this study, we prefer the one month length (22 days) of the holding period in which the simulation of the hypothetical portfolio presents feasible and robust results. Time aggregation is possible if and only if it is independent and identically distributed.

As we mentioned earlier, the overlapping data of BRICT and G-7 countries whose VaR are predicted on one-month holding period consists of 1531 observations of daily returns on closing index values of the below representative equity indices for the period of the trading dates between January 8, 2002 to February 26, 2010.

These sequences of subsamples can be extracted with the following method

$$\{r_{i+h(p-1)}\}_{i=1}^{N} \quad for \quad p = 1,\ldots,P \quad and \quad P = floor\left(\frac{T-N}{h}\right) \qquad (4\text{-}3)$$

where T is the total sample size ($T = 1530$), N is the subsample size ($N = 200$) and h is one-month holding period ($h = 22$). Thus, we end up with P sequences of historical portfolio returns after (4-2) is applied to index returns.

However, not complete set of historical observations are used to estimate the VaR. Instead of doing this estimation one off, overlapping data is taken a sequential subsample from the full data whose size is 200[21] observations and data is forwarded one-month for next subsample. Namely, data is split into chunks of multiple periods by rolling one month. Afterwards, the VaR estimation of each subsample is analyzed for one-month holding period using both parametric and non-parametric methods (see Figure 4-7). Therefore, each underlying subsample has different volatility regime prevailing on the VaR estimate. Moreover, overlapping data are used not only to form a global portfolio but also to avoid the loss of any information. Furthermore, a global portfolio is subsequently used as part of the parametric/local valuation methods (such as a delta-normal model) and also the nonparametric/full valuation methods (such as a filtered historical simulation model) to use multipath innovations in it.

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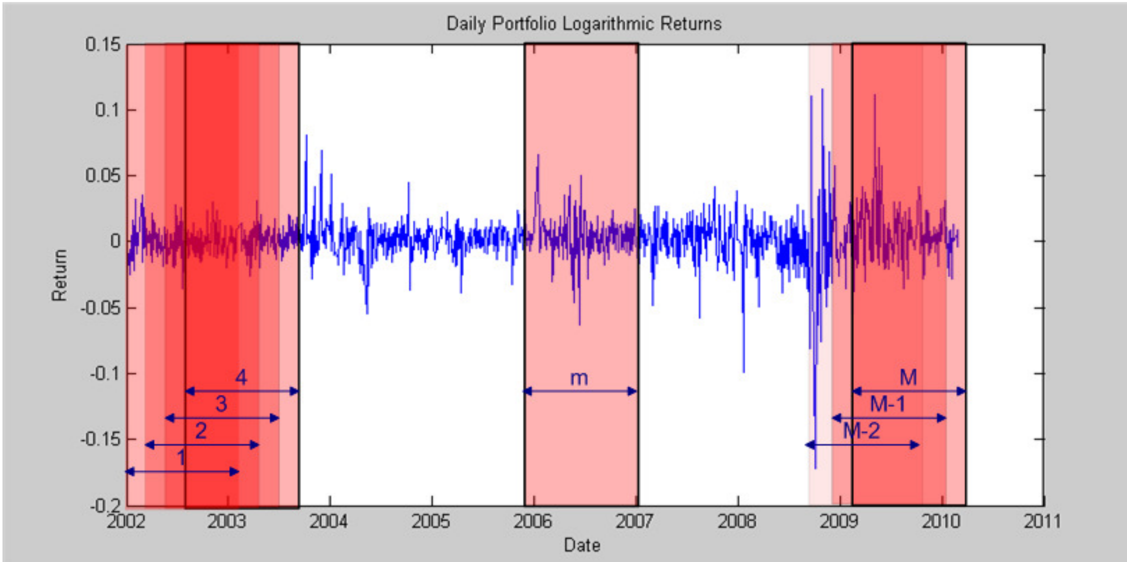
[21] The Market Risk Amendment to the Basel Capital Accord requires for 1%, 1-day VaR estimates for market risk using a 1-year historical return history (RiskMetrics — Technical Document, 1996). On the other side, because country holidays were not aligned, this one-year observation number is reduced to 200.

Figure 4-7 Illustration of rolling windows for subsamples



VaR estimates can be measured ex-post, i.e. when the true values are available. There are many different measures to do this and this is done on the line charts of the estimated values displaying trend over time throughout the thesis.

4.2 Implementation of Value-At-Risk Approach

Parametric VaR measurement is constrained by the strong assumptions about the distributional properties of returns. Instead of repeating what has been said, the details that have been covered already throughout the thesis previously, below Figure 4-8 and Figure 4-9 present the Delta normal VaR estimates of 60 periods across the time. These two will be used as a benchmark later on.

Figure 4-8 Delta-Normal Value-at-Risk for each period for BRICT countries

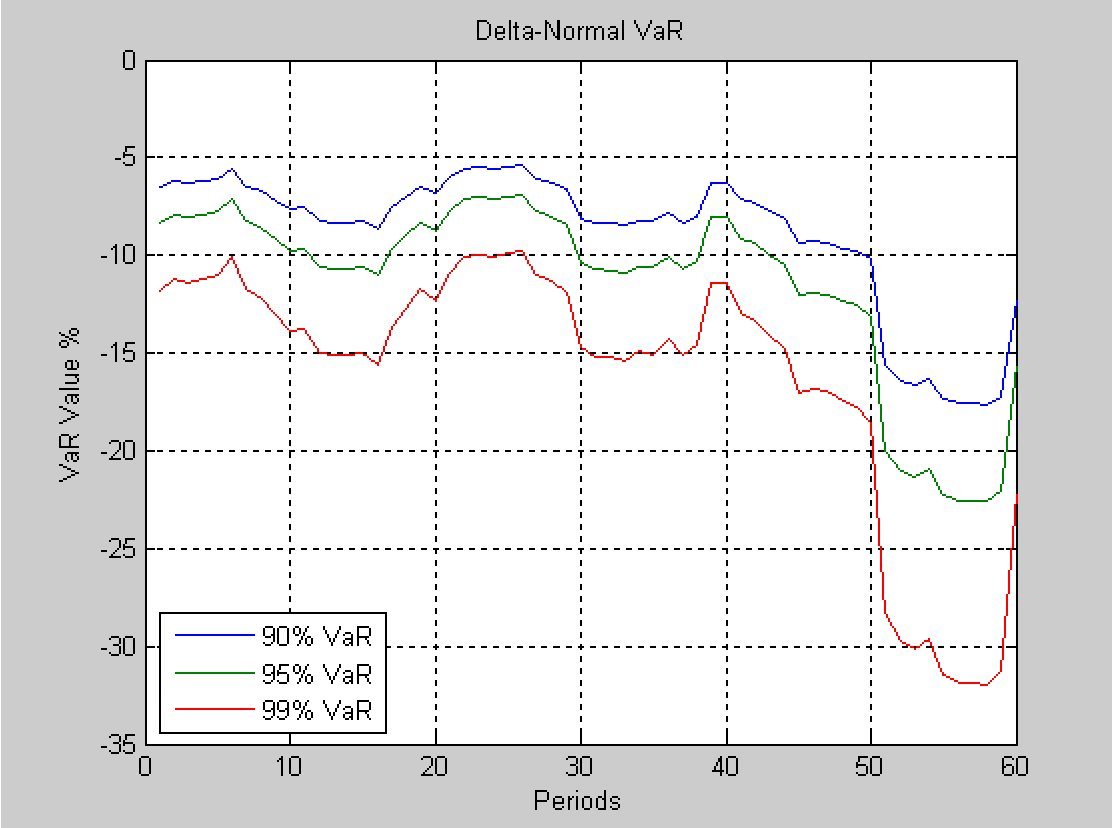
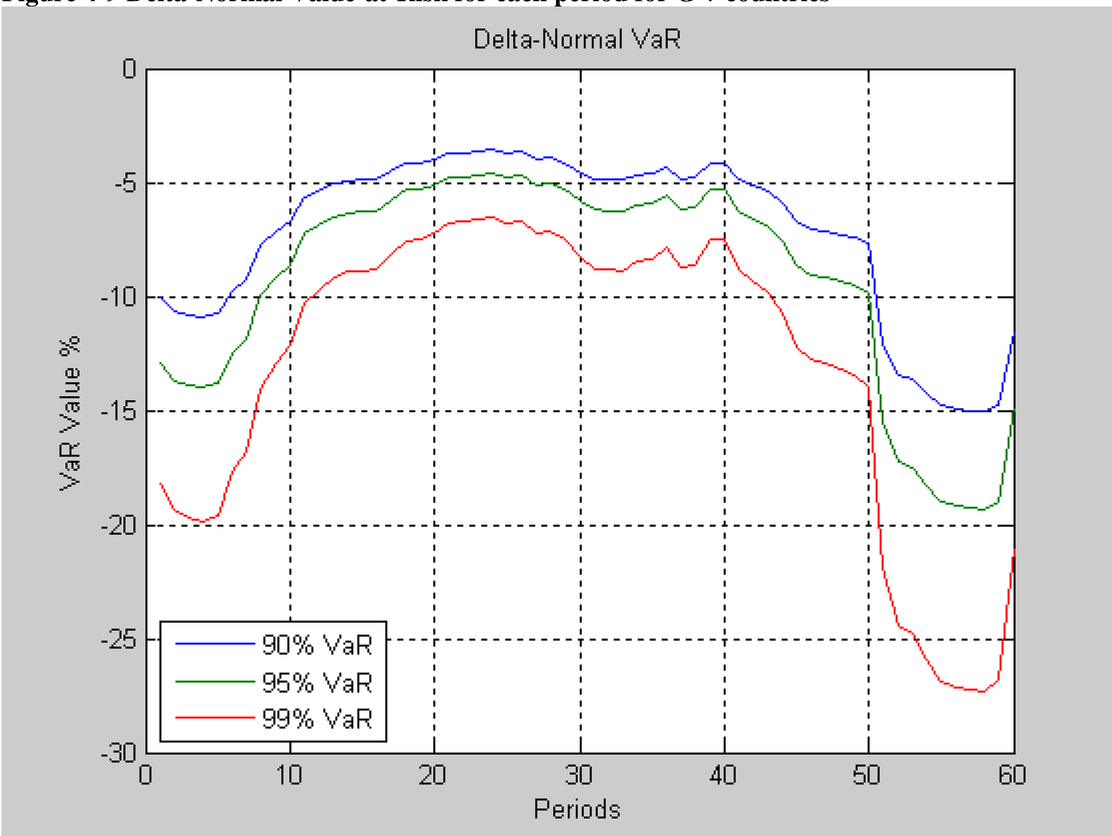


Figure 4-9 Delta-Normal Value-at-Risk for each period for G-7 countries



As per mentioned earlier in the previous chapter of the thesis (see Table 3-1) the normality assumption of overlapping data brings some undesirable statistical properties. One of these desirable motivations is also the violation of the assumption of the normal distribution; since VaR is totally depend on the standard deviation in returns. On the other side, Modified VaR allows certain flexibility to model our methodology while we deal with analysis and evaluation of VaR in the long-run. We focus on the Cornish-Fisher expansion under which the distribution of returns can be non-normal.

4.3 Implementation of Modified Value-At-Risk Approach

Zangari (1996) proposed to obtain reliable estimates of parametric VaR assuming the returns to be non-normal distributed to use the Cornish and Fisher (1937) expansion to rectify the problem due to skewness and heavy tails in the distribution of returns. This VaR estimator, so-called modified Value-at-Risk (MVaR), has become popular due to its high accuracy and computational efficiency. Campbell et al. (2001) and Favre and Galeano (2002) propose a modified VaR calculation that takes the higher moments (skewness, kurtosis and, the measurement of the 'heavy-tailedness' of the returns) of non-normal distributions into account by use of a Cornish and Fisher (1937) expansion, a better approximation of the shape of the true distribution.

$$VAR = -q_{\alpha}\sqrt{w'\Sigma w} \qquad (4\text{-}4)$$

where we present the equations for calculating VaR for a general portfolio of N assets with the portfolio weights, w, the negative value of a lower quantile (confidence level) of the portfolio returns, q_{α}, and covariance matrix, Σ. Here we present the rest of the parameters denoted by r the return on the hypothetical global index with mean μ. In the rest of the study, VaR has to be understood as the negative value of a lower quantile (confidence level) of the hypothetical global portfolio index returns, representing the random risk at hand too. This is because these returns can be non-normal, we also need the $\mathrm{N}\times\mathrm{N}^2$ co-skewness matrix of the returns:

$$\mathrm{M}_3 = \mathrm{E}\left[(r-\mu)(r-\mu)' \otimes (r-\mu)'\right] \qquad (4\text{-}5)$$

and the $\mathrm{N}\times\mathrm{N}^3$ co-kurtosis matrix:

$$\mathrm{M}_4 = \mathrm{E}\left[(r-\mu)(r-\mu)' \otimes (r-\mu)' \otimes (r-\mu)'\right] \tag{4-6}$$

where $\otimes$ stands for the Kronecker product (see e.g. Jondeau and Rockinger, 2006). In the portfolio theory, the moments could be computed using the past returns of the whole portfolio. Hence, the q^{th} centered portfolio moment $\mathrm{M}q = E\left[(r_p - w'\mu)^q\right]$. We have:

$$m_2 = w'\Sigma w \tag{4-7}$$

$$m_3 = w'M_3(w \otimes w) \tag{4-8}$$

$$m_4 = w'M_4(w \otimes w \otimes w) \tag{4-9}$$

(see Jondeau and Rockinger, 2006). The portfolio skewness s_p and excess kurtosis k_p are the given by:

$$s_p = m_3 / m_2^{3/2} \tag{4-10}$$

$$k_p = m_4 / m_2^2 - 3 \tag{4-11}$$

For the probability α such as 90%, 95% and 99%, portfolio VaR under the assumption of normally distributed returns is given by:

$$GVAR = -w'\mu - q_\alpha \sqrt{w'\Sigma w} \tag{4-12}$$

where q_α is the α-quantile of the standard distribution and also GVaR stands for Gaussian VaR.

Figure 4-10 Gaussian Value-at-Risk for BRICT countries

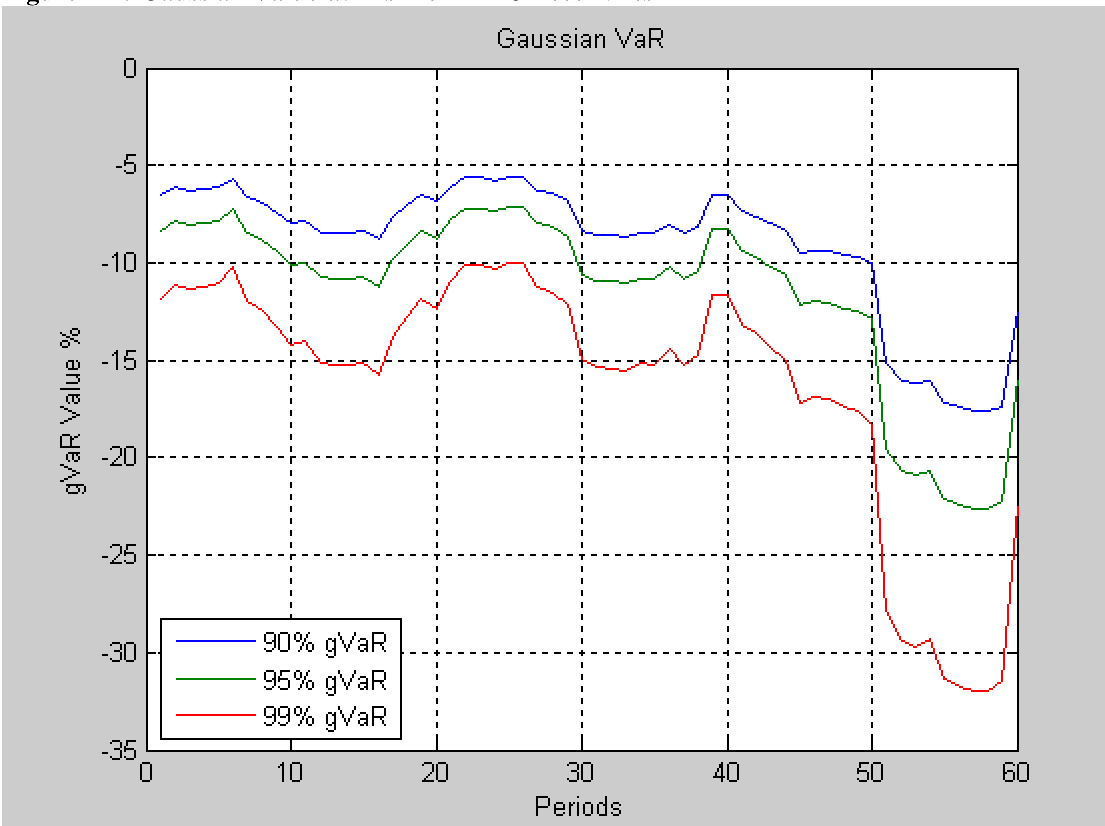
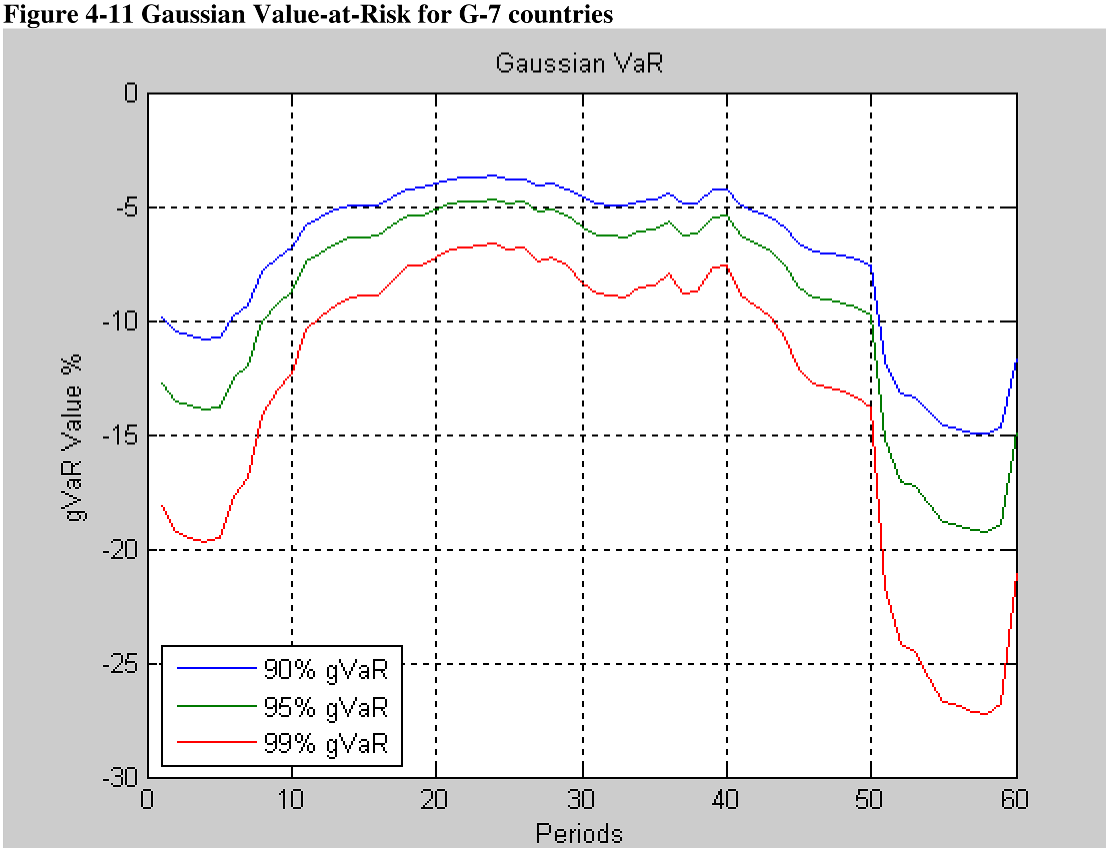


Figure 4-11 Gaussian Value-at-Risk for G-7 countries



Having said that the returns of many financial assets are heavy-tailed and skewed, we can get a better estimate of VaR by accounting for the non-normality. Zangari (1996) provides Modified VaR (MVaR) calculation that takes the higher moments of non-normal distributions (skewness, kurtosis) into consideration through the use of Cornish and Fisher (1937) expansion. MVaR is defined as follows:

$$MVAR = -w'\mu - q_{\alpha}^{cf}\sqrt{w'\Sigma w} \tag{4-13}$$

where q_{α}^{cf} is the α-Cornish&Fisher quantile. Using the below formulas, this quantile can be easily calculated as:

$$q_{\alpha}^{cf} = q_{\alpha} + \frac{\left(q_{\alpha}^{2}-1\right)s_{p}}{6} + \frac{\left(q_{\alpha}^{3}-3q_{\alpha}\right)k_{p}}{24} - \frac{\left(2q_{\alpha}^{3}-5q_{\alpha}\right)s_{p}^{2}}{36} \tag{4-14}$$

where s_p is the skewness and k_p is the kurtosis of the return series.

Modified VaR gives a larger loss estimate than delta normal VaR when returns are negatively skewed or highly heavy-tailed. On the contrary, you can expect it gives a smaller loss estimate if returns are leptokurtotic or positively skewed. Not surprisingly, Modified VaR becomes Gaussian VaR provided that skewness and excess kurtosis are zero. The Cornish-Fisher expansion covers much of the non-normality in returns that could be overrun by more computationally intensive techniques such as filtered historical simulation (which is going to be covered later on) or direct assumption of an ideal distribution.

For estimation of portfolio VaR especially, MVaR is much better than GVaR. This is because of the fact that MVaR takes into accounting for asymmetry and fat-tails in the marginal distribution of the returns of components, as well as accounting for non-linear dependence between the component returns. MVaR utilizes not only correlations, but also high-order cross-moments such as co-skewness and co-kurtosis, to aggregate risk of the portfolio (Boudt and Peterson, 2007).

In the Figure 4-12 and Figure 4-13, the modified VaR model is used to estimate for various confidence levels over each subsample of 200 data observations.

Figure 4-12 Modified Value-at-Risk for G-7 countries

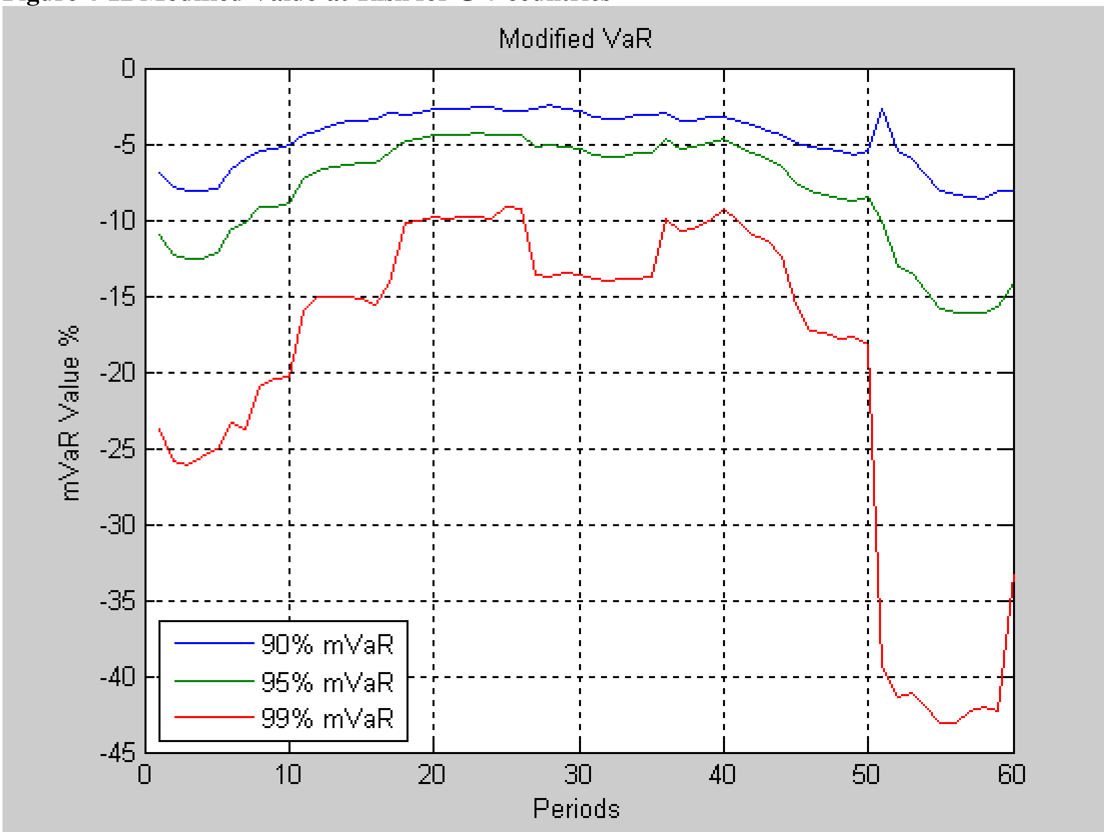
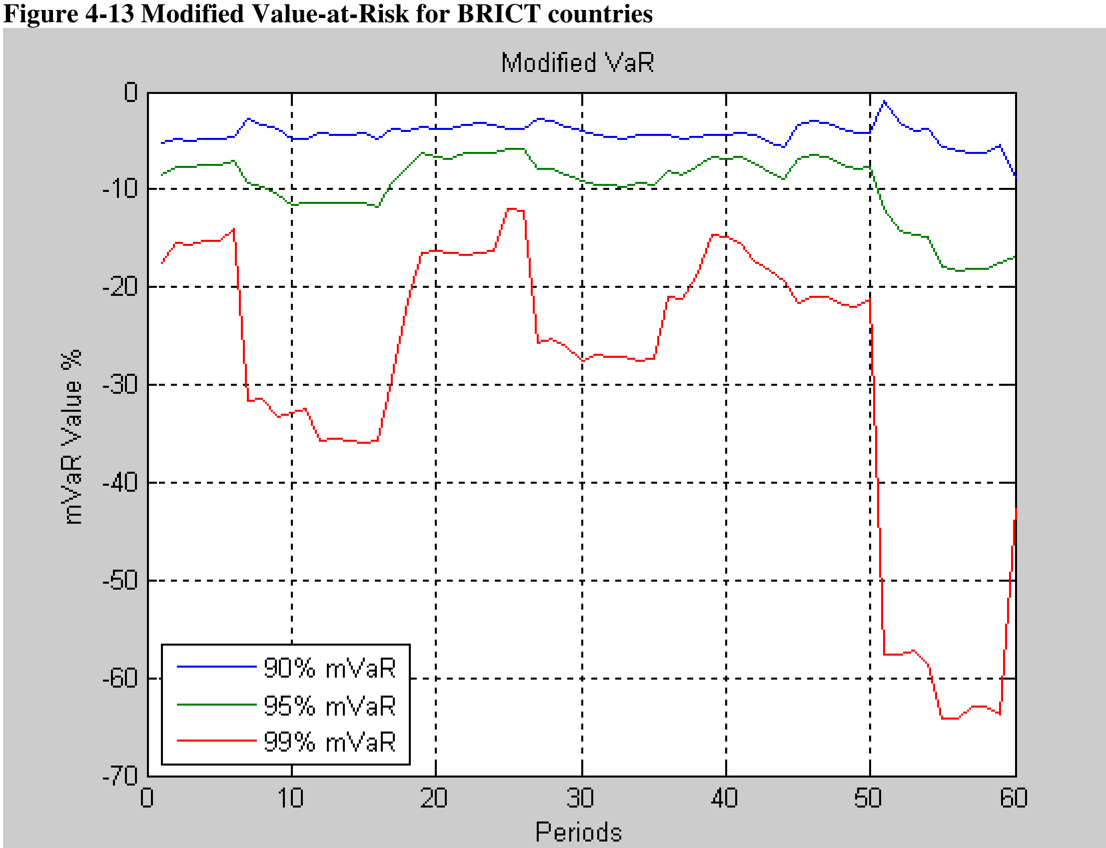


Figure 4-13 Modified Value-at-Risk for BRICT countries



Although the simplest manner to carry on is to suppose the distribution of returns has the theoretical normal distribution, we know from the empirical studies on the stylized facts of the daily frequency can not be described by a theoretical distribution. Instead of employing alternative distribution, we rely on an enhancement of the standard VaR model on the subsamples, which ignores the fact that volatility regimes are changing each time by using modified VaR. Furthermore; the autocorrelation in the dynamics of risk measurement is not taken into account across each subsample as parametric VAR measurement described before. The application of conditional heteroscedastic models to estimate the volatility of the return distribution can consider autocorrelation and / or conditional heteroscedasticity.

4.4 Implementation of Filtered Historical Simulation Approach

Filtered historical simulation (FHS) is derived from a generalized historical simulation. It surpasses most of the weaknesses of historical simulation thanks to its positive features. This approach is introduced by Barone-Adesi, Bourgoin and Giannopoulos (1998) and Barone- Adesi, Giannopoulos and Vosper (1999).

In this thesis, we will use of one of the non-parametric models, i.e. FHS, on an underlying daily data. The above mentioned limitations of HS at Table 3-2, a filtered set of results and improved response to changes in market volatility can be bypass with the use of FHS. Yet it is better to go over HS in order to grasp the FHS technique.

. In the "Handbook of Financial Time Series" textbook edited by Anderson et al., Christoffersen (2009) explains that there are two steps to compute VaR in FHS method. First, a series of hypothetical historical portfolio returns are constructed with weights of the today's portfolio and the past performance of returns. Second, the quantile of the historic returns of the hypothetical portfolio is calculated. Some of the researchers focus on the HS approach because of its "model-free" nature. Nonetheless, it is completely not "assumption-free". HS fundamentally assumes the distribution of returns is independent and identically distributed. In fact, it is unfortunately not the case for the most of the empirical daily data. Besides, Christoffersen (2009) asserts that the HS method has become known as the standard for estimating VaR.

Apart from stationarity assumption of the portfolio returns, HS does not require any statistical assumption in particular to the volatility. In HS method, we consider a series of daily historical returns of hypothetical portfolio returns Θ we choose e^* $e^* = \{e_1^*, e_2^*, ..., e_T^*\}$ $e_i^* \in \Theta$ where $i = 1, 2, ..., T$ refers to past days to form a simulated portfolio index.

The HS technique basically presumes that the distribution of forecasted portfolio returns, e_{i+1}^*, is very close to the empirical distribution of the actual T observations, that is, $\{e_{i+1-j}^*\}_{j=1}^T$. Put it differently, the distribution of e_{i+1}^* is captured by the histogram of $\{e_{i+1-j}^*\}_{j=1}^T$. Consequently, we simply sort the returns in $\{e_{i+1-j}^*\}_{j=1}^T$ in ascending order and choose the VaR to be the negative value of a lower quantile of the observations is smaller than the quantile.

However, in the simplest form of HS, subsamples of the data are analyzed repeatedly rather than analyzing subsets of the data repeatedly. Each subsample return is a randomly drawn replacement from the full data set. The size of observations in the subsample is chosen depending on empirical data. In addition, bootstrap method does not need formulae so that any limiting parametric assumptions can be simply avoided. There are more sophisticated bootstrap methods that can be assessed for estimating errors as well as estimating confidence intervals (Efron and Tibshirani, 1993).

As we mention in the Table 3-2, as well as Pritsker (2000) enlightens, the accuracy of HS model to forecast future losses may be made it less strong and less secure if the distribution of any risk factor does not change over the time. The Filtered Historical Simulation model tries to bring together the most desirable features of the HS model with the parametric estimation model. The latter attempts to capture conditional heteroscedasticity but assumes a normal distribution while the former does not assume a specific distribution but does not capture conditional heteroscedasticity. More specifically, The FHS method forecasts volatility by means of a parametric volatility model and utilizes the standardized returns' the quantile to measure the VaR estimate. It relies on a model based approach for the volatility, typically using a GARCH type model, while remaining model free in terms of the distribution. In particular, this method has the notable advantage of being able to simulate extreme losses even if they are not available in the sample past returns used for the simulation, thus taking tail risk into consideration more accurately. Therefore, "FHS requires a shorter historical record than HS to simulate the tails of the distribution of price changes. This is because this process is essentially an extrapolation, its validity must be carefully tested" (see Barone-Adesi et al., 2000).

In FHS approach, the stationarity assumption is eased; historical returns are first standardized by volatility estimated on that particular day (that's why the name of filtered), $z_t = \frac{\varepsilon_t}{\sqrt{\hat{h}_t}}$. This filtering procedure gives in return for historical simulation suitable i.i.d. past returns. Before filtered returns are used as innovations, which are scaled (standardized) by the current forecast of conditional volatility, they reproduce simulated market conditions:

$$Y^*_{T+n} = Y^*_{T+n-1} + Y^*_{T+n-1} z \sqrt{h_{T+n}} \qquad (4\text{-}15)$$

where $z^* = \{z^*_1, z^*_2, ..., z^*_T\}$ $z^*_i \in \Theta$ where $i = 1, 2, \dots, T$ refers to historical observations and $\left\{\sqrt{h_T}\right\}_{t=1}^{T}$ is the simulated conditional variance of the process and is estimated recursively by the time-series model, such as the one illustrated in equation (Barone-Adesi and Giannopoulos, 2000).

The FHS method can be expedited by explaining step-by-step to get i.i.d. standardized returns as innovations to calculate VaR. The steps —derived from (Bakshia and Panayotovb, 2009) — in applying the FHS are as follows:

- fit a AR(1) model to the conditional mean of returns and get the residuals;
- filter these residuals with contemporaneous volatility estimates obtained, e.g., in an EGARCH(1,1) model, thus removing serial correlation and volatility clustering;
- draw random samples from the standardized residuals to be used as an input in a bootstrap procedure, and
- use them recursively as innovations in a conditional variance equation to simulate forecast values of returns

Engle (1982) proposed the method of Autoregressive Conditional Heteroscedasticity (ARCH), which drew a significant attention and recognized as a significantly powerful tool in the quantitative finance and method is used on stock returns especially. ARCH models allow the conditional variances to change over time with respect to the past error terms. Subsequently, Bollerslev (1986) improved the ARCH model and introduced the General Autoregressive Conditional Heteroscedasticity (GARCH). Nonetheless, there are drawbacks in all models considered so far. This drawback of pure GARCH model is equivalent impact is observed for positive or negative impact on the conditional variance since the sign effects of shocks get lost because of squaring. In contrast, the volatility response is different to positive and negative shocks, i.e. if the volatility response occurs due to good or bad information. In fact, the 'leverage effect' causes to larger volatility due to negative shocks as unlike to positive ones. Therefore, one of extension models of the symmetric GARCH (1,1) model, i.e. EGARCH (1,1), with Student's t distribution data generation process presented is capable to take care of such asymmetric effects.

NELSON (1991) proposed an Exponential GARCH model (EGARCH). His proposed solution takes control of asymmetries and also guarantees that the conditional variance is always positive. By calibrating the EGARCH (1,1) model to the past information formed via residual returns of data, Residual returns are filtered to apply a first order autoregressive model to the conditional mean of the portfolio returns

$$r_t = \delta + \theta r_{t-1} + \varepsilon_t \tag{4-16}$$

where $\varepsilon_t \sim N(0, h_t)$, $z_t = \dfrac{\varepsilon_t}{\sqrt{h_t}}$ and an asymmetric exponential GARCH (EGARCH) model to the conditional variance

$$\ln(h_t) = \kappa + \alpha \ln[h_{t-1}] + \phi \left(|z_{t-1}| - \mathrm{E}[|z_{t-1}|]\right) + \psi z_{t-1} \tag{4-17}$$

Here, the standardized residuals $z_t = \dfrac{\varepsilon_t}{\sqrt{\sigma_t}}$ are used as an input in a bootstrap procedure for estimating VaR and a series of i.i.d. random variables with Student's distribution to compensate for heavy tails that is one of the stylized facts of daily returns. The absolute value of the standardized residuals produces the ARCH effect. The standardized residuals capture the asymmetry as well. An ARCH effect of ϕ for positive residuals is true for $\phi \neq 0$. If there is a 'leverage effect'[22], ϕ will be negative which means the increase in volatility following a previous fall in portfolio returns. Particularly, the outstanding benefit of this model is to be able to simulate extreme losses even if they are not present in the sample historical returns used for the simulation, consequently tail risk is taken into account more accurately (Kirchgässner et al., 2007).

Having said about FHS method, we can keep on talking about the analysis of the bootstrapped FHS method to produce a series of independent and identically distributed observations, which fit a first order autoregressive model to the conditional mean of the returns, i.e., above equation (4-16). In addition to that, the term "filtered" comes from the set of shocks, z_t , which are returns filtered by the EGARCH model. Thus, "the thrust of FHS method is to obtain an i.i.d. series of standardized returns to be used as an input in a bootstrap procedure for estimating VaR" (Bakshia and Panayotovb, 2009).

As per mentioned, initially the changes that have been observed in the hypothetical portfolio returns are analyzed thorough a pre-specified period of time; in our case, it is 200 observations for each subsample. The current analyzed portfolio is then simulated for 100,000 multipaths for the number of pre-specified holding period in the historical sample. Hence, forecast model creates the distribution of the forecast returns in the simulation process can be derived from which the VaR of the distribution of portfolio returns. Volatility is forecasted in a sample period and standardized returns are obtained. The standardized returns are bootstrapped and multiplied with each random drawing by the most recent volatility forecast. Each daily forecast return of the portfolio is considered as data in the histogram of portfolio values is finally constructed so as to measure the VaR that captures the given confidence level of the distribution on the left side. In other words, all of the past historical returns in terms of the lowest to the highest are literally ranked, and at the predetermined confidence level, what the lowest return is

[22] Leverage Effect appears firstly in Black (1976), who noted that:
"a drop in the value of the firm will cause a negative return on its stock, and will usually increase the leverage of the stock. [...] That rise in the debt-equity ratio will surely mean a rise in the volatility of the stock". The existence of a "leverage effect" is empirically shown that there is increase in volatility after smaller changes.

computed thanks to the FHS method. For example, if we had 100,000 simulations to approximate the true but unknown distributions of returns on the VaR estimates, 1000 simulations on the tail observations will be taken into account when computing a VaR for 99% confidence level. Afterwards, for all periods values are plot in a figure along with other confidence levels.

Figure 4-14 Value-at-risk values of the simulated one-month hypothetical portfolio returns for BRICT countries

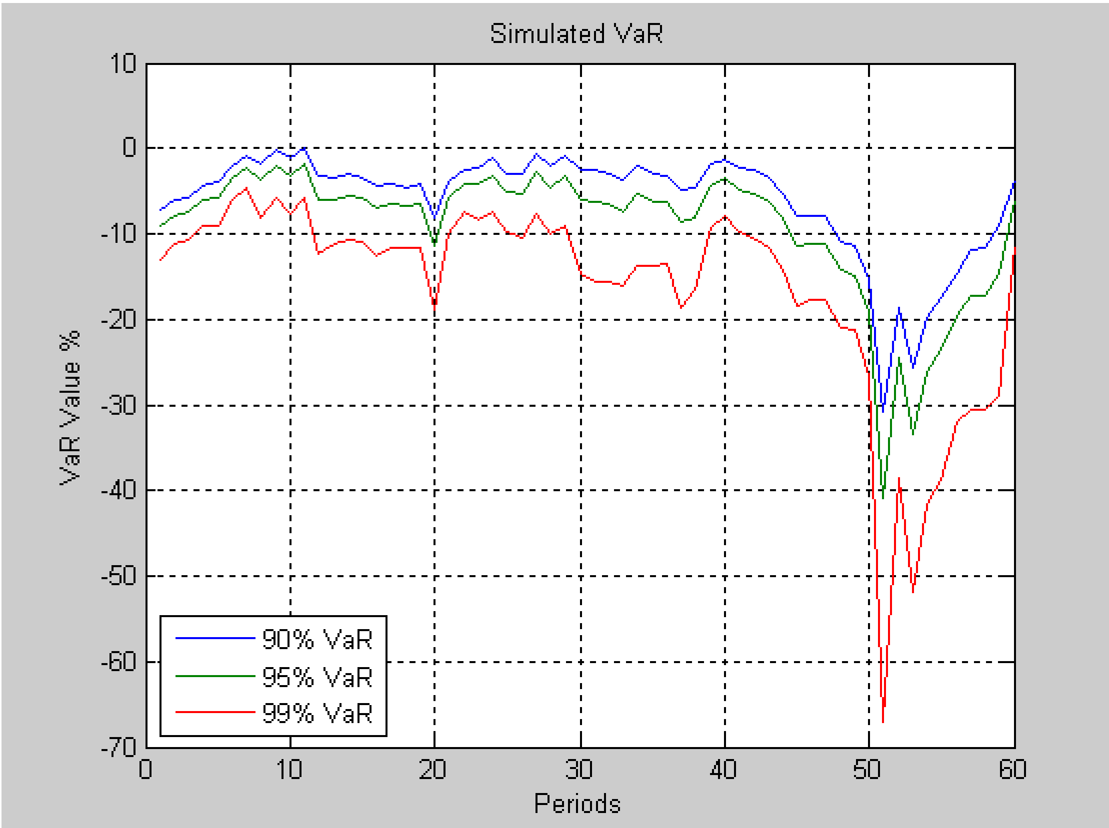
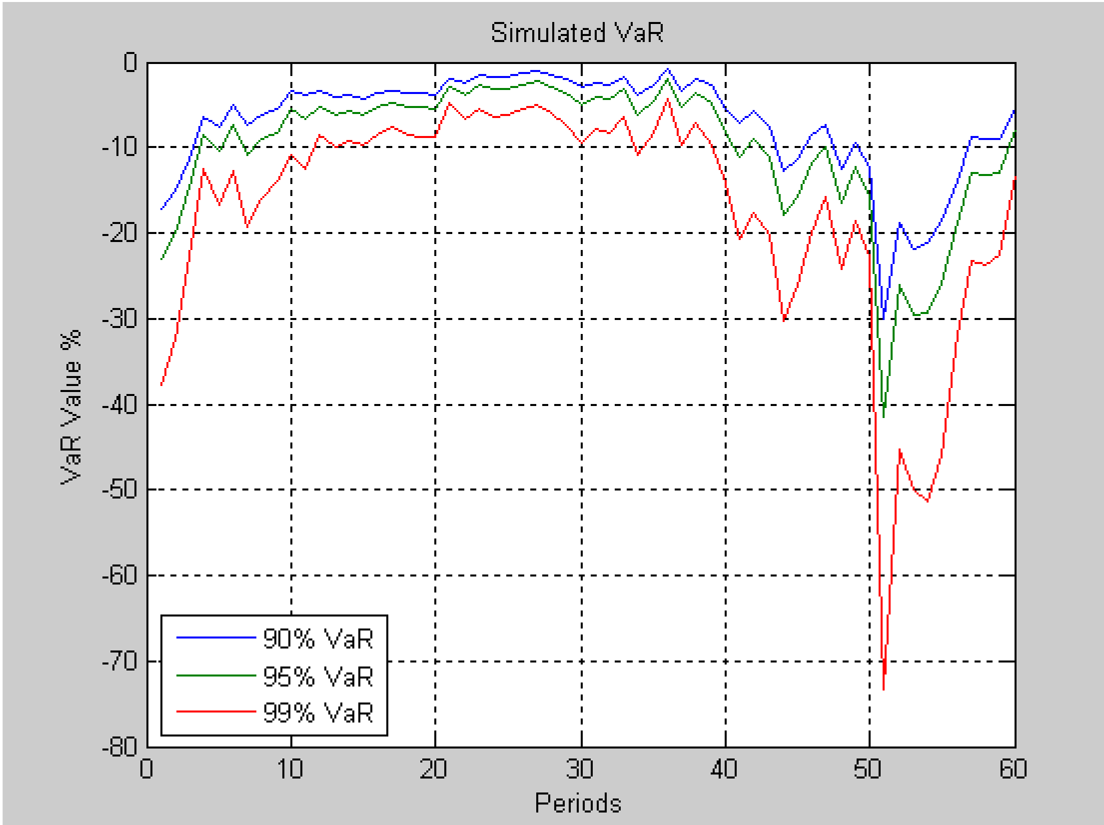


Figure 4-15 Value-at-risk values of the simulated one-month hypothetical portfolio returns for G-7 countries



The aim of these analyses is to examine the VaR estimates for the subsamples with the ARCH family of volatility models for BRICT and G-7 countries respectively. In the figures, an overview of unconditional and conditional volatility models is provided for three different confidence levels. As we highlighted before, the unconditional volatility model is based on constant volatilities whereas the conditional volatility model uses historical subsample's information to simulate a series of i.i.d. observations, which fit a first order autoregressive model to the conditional mean of the portfolio returns. "Unconditional models are based on rigorous assumptions about the distributional properties of returns while the conditional models are less rigorous and treat unconditional models as a special case. In order to simplify the VaR calculations unconditional models make strong assumptions about the distributional properties of financial time series. However, the convenience of these assumptions is offset by the overwhelming evidence found in the empirical distribution of returns, e.g. fat tails and volatility clusters. VaR calculations based on assumptions that do not hold, underpredict uncommonly large (but possible) losses" (Giannopoulos, 2000).

4.5 Comparisons and Findings

To measure VAR estimates it is very essential to project the distribution of probable returns. We can accomplish this many possible ways to do so and now we have focused on just few of these methods. The parametric method is formed after specifying the distribution of returns on the holding period and then using one of the delta-normal and MVaR valuation models together with the presumed standard normal distributions to measure the VAR estimates. Of course, historical data is utilized to estimate parameter values required for answering our vital questions regarding the probability.

The modified VaR method responds strongly for the 99 % confidence level and estimate is rising in magnitude compare to delta-normal VaR estimates. In the Figure 4-16 and Figure 4-17 that VaR estimates at 99% confidence level are outperformed by the FHS forecasts for both BRICT and G-7 countries. Interestingly, magnitude values are exaggerated for again past 10 months from April 2010.

Figure 4-16 Delta-Normal Value-at-Risk versus Modified Value-at-Risk for BRICT countries

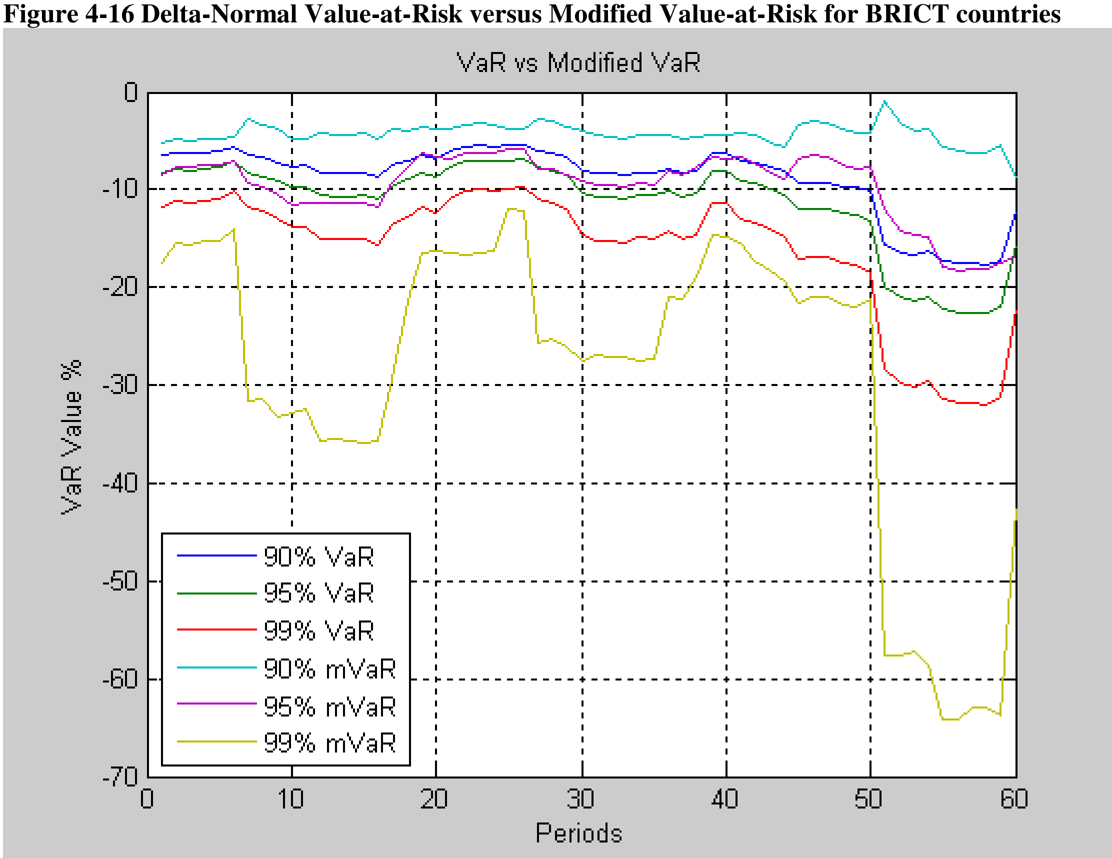
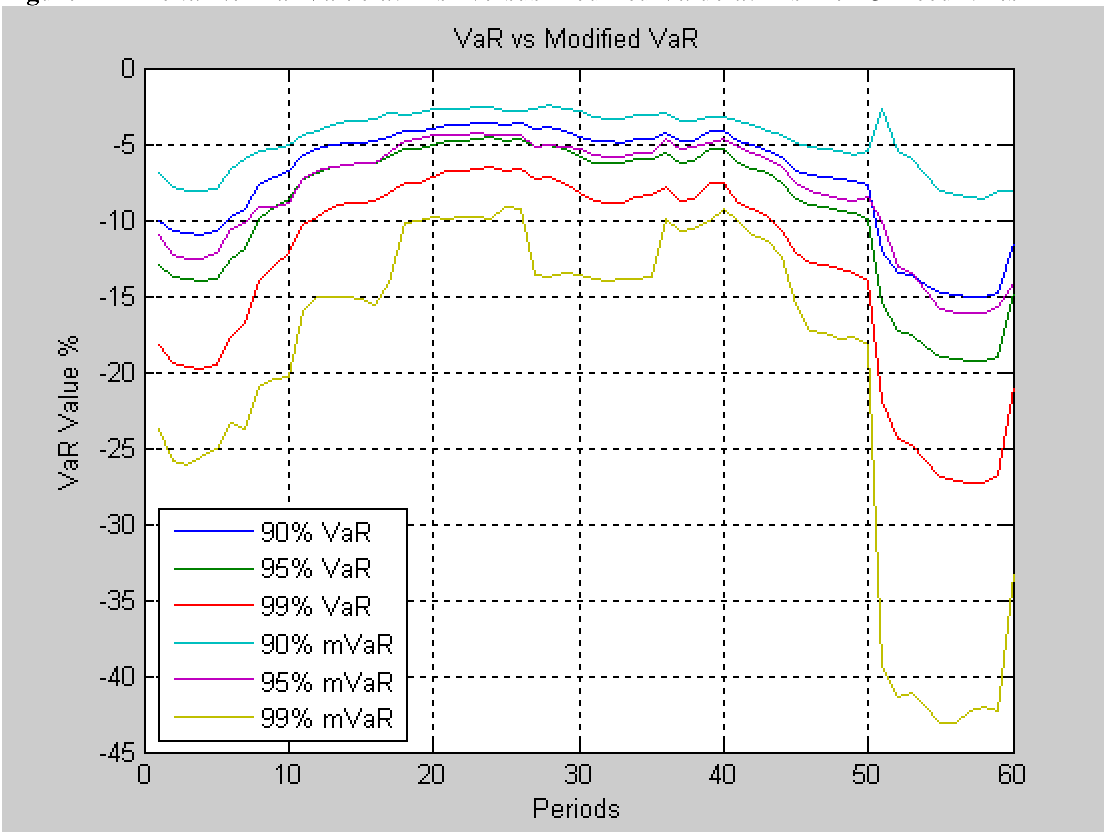


Figure 4-17 Delta-Normal Value-at-Risk versus Modified Value-at-Risk for G-7 countries



In the Figure 4-18 and Figure 4-19, the filtered historical simulation model is estimated over each subsample of 200 data observations. Their forecasting ability is compared to delta-normal VaR estimates. It could be noticed VaR estimates at 99% confidence level are outperformed by the filtered historical simulation forecasts for both BRICT and G-7 countries. This is also in parallel with the modified VaR results.

Figure 4-18 Delta-Normal value-at-risk versus the filtered historical simulation value-at-risk for BRICT countries

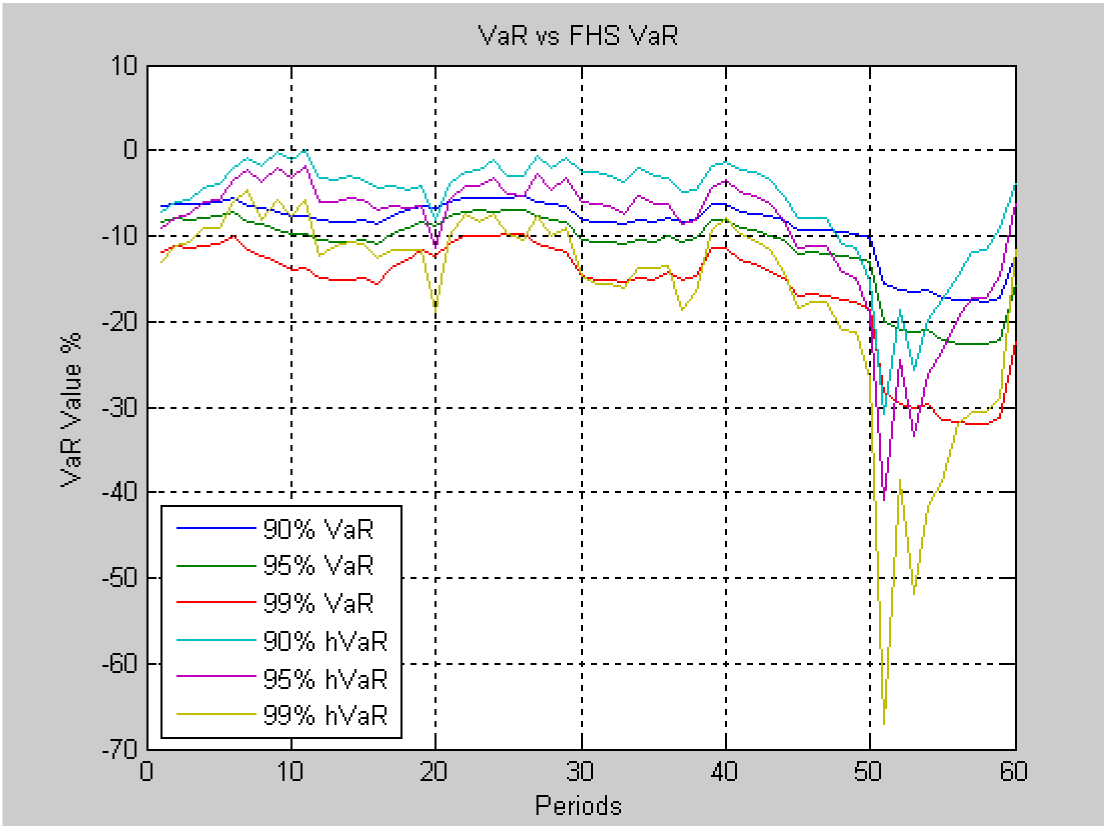
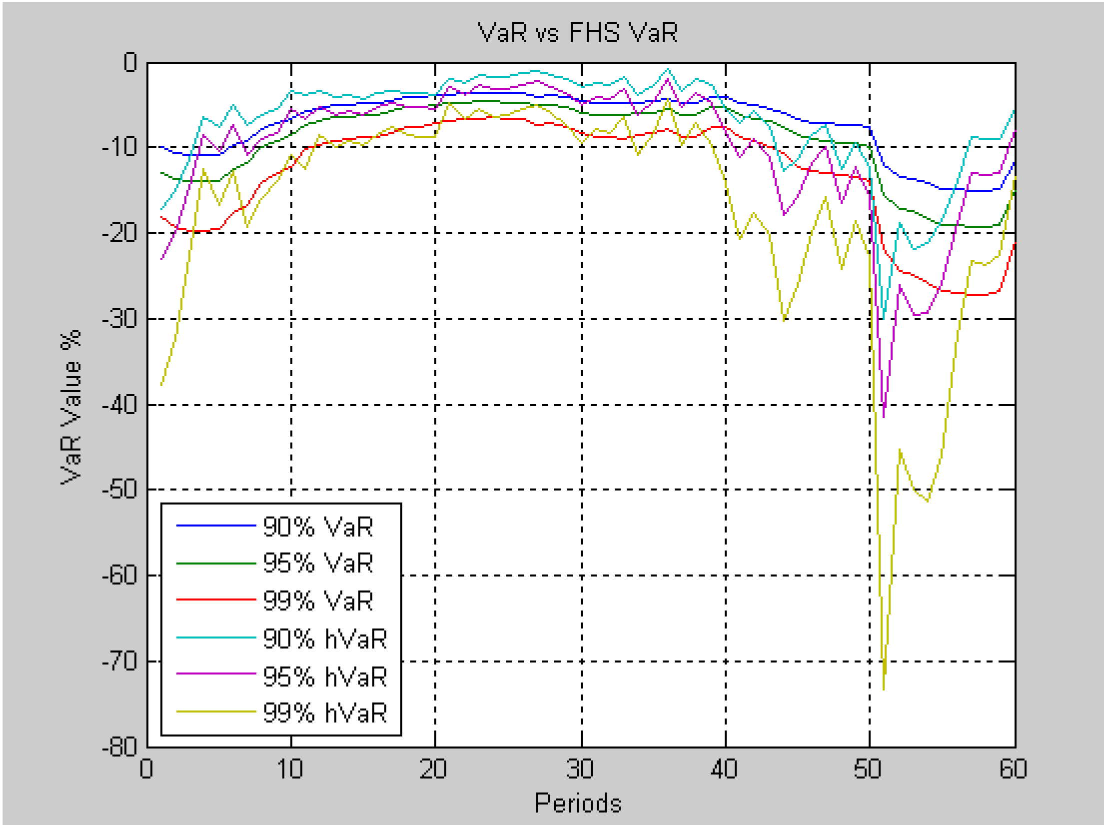


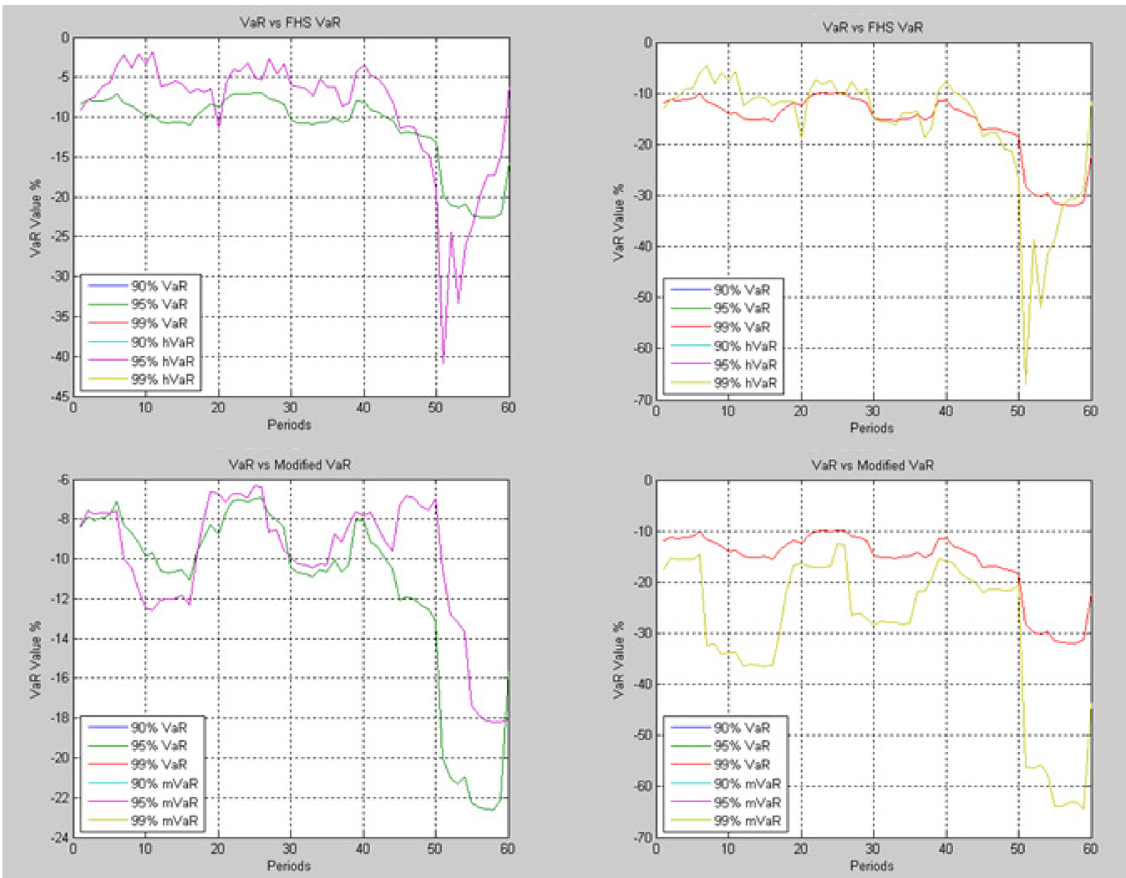
Figure 4-19 Delta-Normal value-at-risk versus the filtered historical simulation value-at-risk for G-7 countries



One of the most important challenges of this empirical thesis, of course, is modeling the conditional volatility of the portfolio index. Univariate process forecast are generated by EGARCH(1,1) process to estimate the portfolio index outcomes. The delta normal VaR approach is unable to be accurate regarding the estimate due the pitfalls of the delta normal approach. On the other hand, the simulation methods suffer from the well known difficult of computing because of a huge number of parameters in the conditional covariance and/or conditional correlation matrices. Yet computational burden is not what we focus on this thesis.

Once VaR and its confidence interval have been calculated, accuracy of the VaR model should be checked by inquiring the normality assumptions are justifiable or not. Having simulated the returns, these simulation values for all three confidence levels evidently show that risk gets higher and higher after second half of the 2008 and this estimate is peaked at the end of the first half of 2009. VaR estimates at 95% and 99% confidence levels are transposed in the Figure 4-20 and Figure 4-21 for BRICT and G-7 countries. Furthermore, the Figure 4-22 and Figure 4-23 illustrate the maximum simulated gains and losses of the returns for each subsample over the one-month holding period. This maximum simulated gains and losses show that the portfolio index can be interpreted that market risk of BRICT is much higher than the G-7's one. These results are, in fact, in parallel with the Lehman Brothers[23] collapse in September 2008. Maximum losses are increasing after September 2008 for both BRICT and G-7 countries.

Figure 4-20 Comparison of value-at-risk estimates at 95% and 99% confidence levels for BRICT countries



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[23] Lehman Brothers Holding Inc. was one of the oldest and largest investment banks in USA and collapsed from toxic financial instruments on mortgage-backed securities (MBSs) and collateralized debt obligations (CDOs) of real estates holdings. In September 2008, it went bankruptcy and became the first largest bank to bankrupt since the start of the subprime mortgage crisis. (Telegraph, September 15, 2008 Available at http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/2963415/Credit-Crunch-timeline-From-Northern-Rock-to-Lehman-Brothers.html and retrieved on May 10, 2010).

Figure 4-21 Comparison of value-at-risk estimates at 95% and 99% confidence levels for G-7 countries

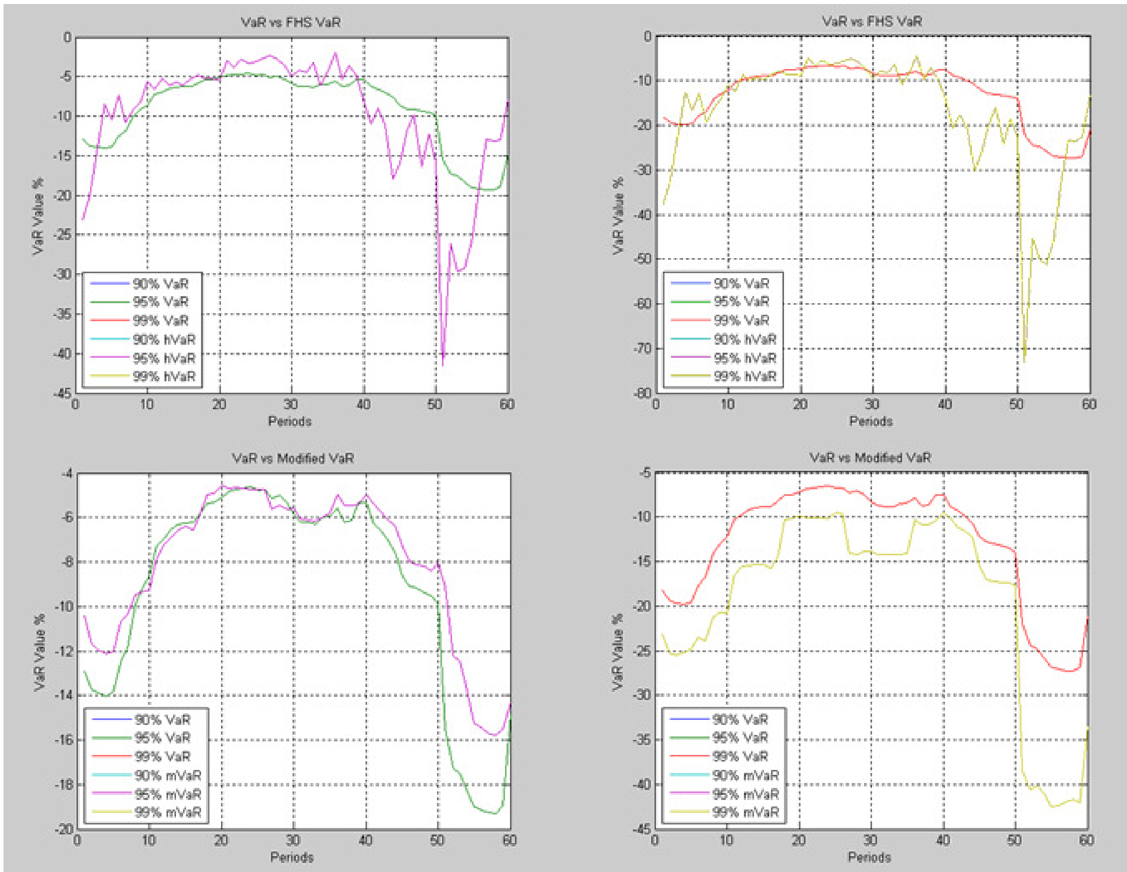


Figure 4-22 Maximum simulated gains and losses of the one-month hypothetical portfolio returns for BRICT countries

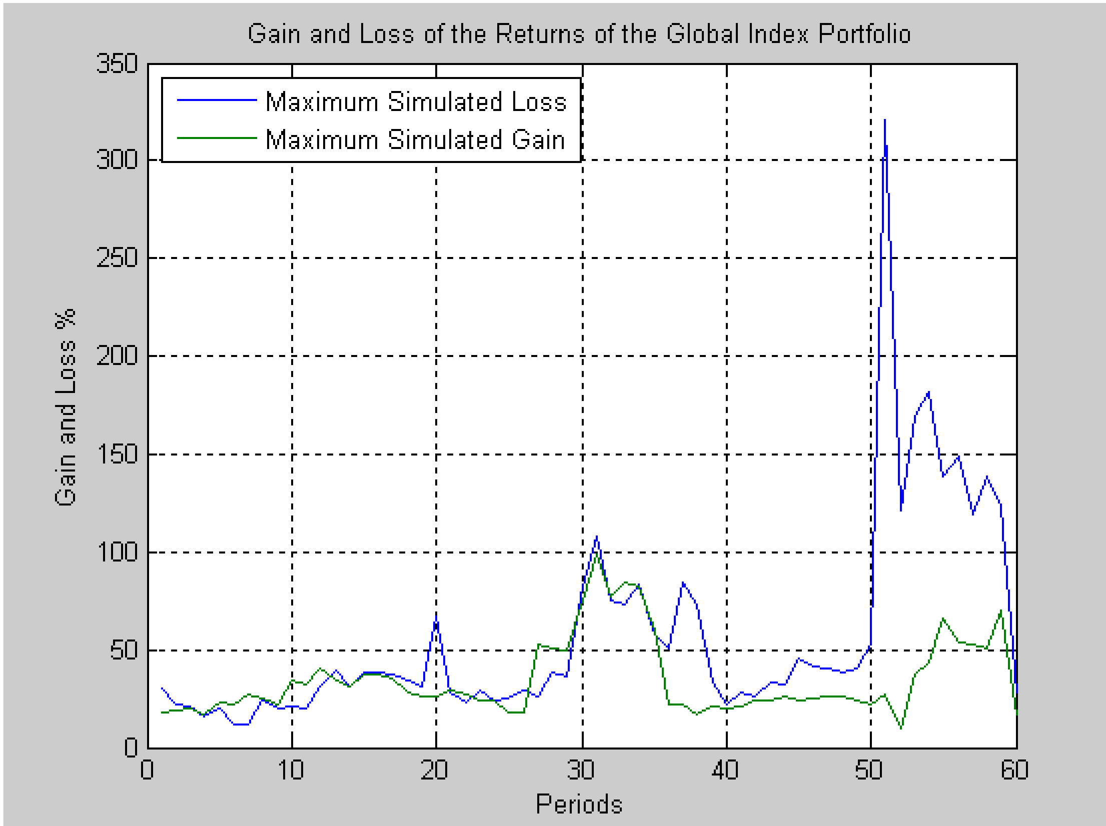
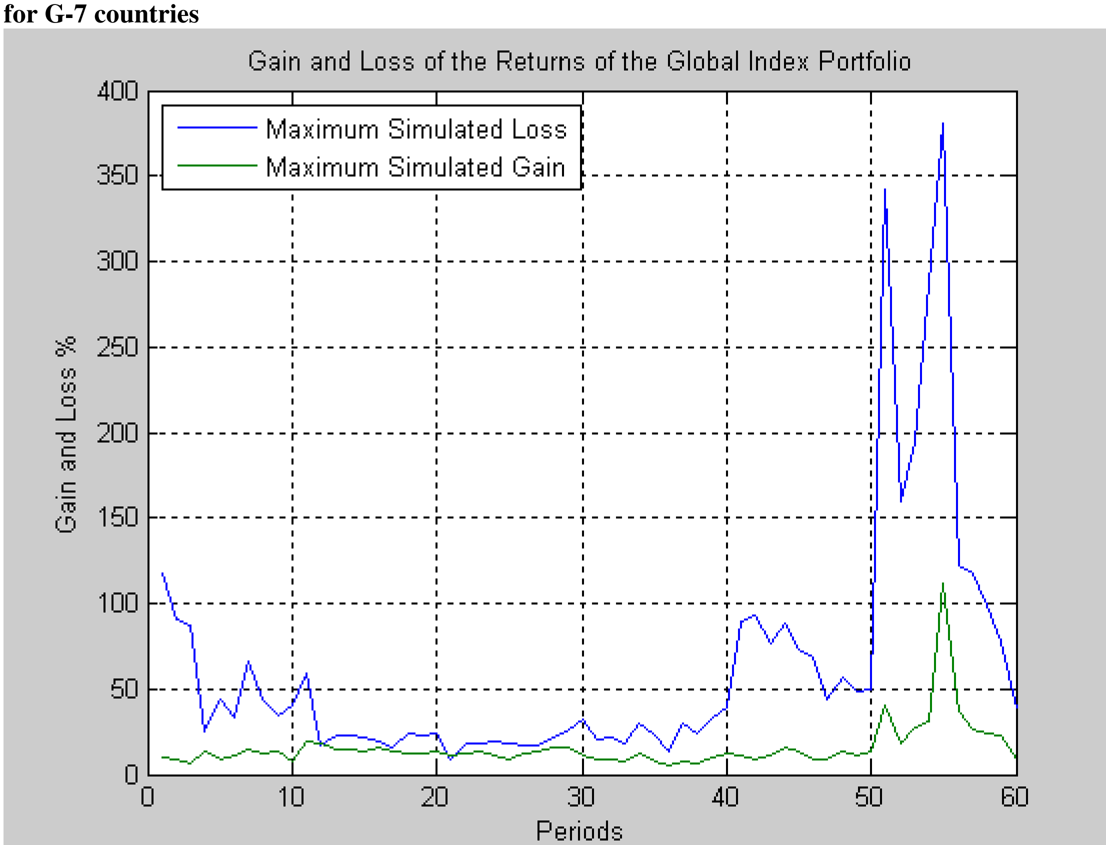


Figure 4-23 Maximum simulated gains and losses of the one-month hypothetical portfolio returns for G-7 countries



Below figures of the histogram of the period with high and low 'maximum loss' show the associated cumulative probability distribution utilizes a plot of the probability that the actual VaR estimate will not be greater than each of a set of possible values and we compare them to see how did the market change.

Even though the VaR measure approach assumes normality, the changes in daily returns exhibit positive kurtosis. This is absolutely predictable for the daily returns and the findings of Cont (2001) regarding the empirical properties of daily returns are in parallel with this. This also implies that extreme events in the returns are more expected than a normal distribution would forecast. Figure 4-25 and Figure 4-27 for BRICT countries and Figure 4-29 and Figure 4-31 for G-7 countries compare empirical distributions exhibiting positive kurtosis with normal distributions[24]. Both distributions have the different descriptive statistical properties like mean, variance and so on. Nevertheless, the positive-kurtosis distribution is spikier and has heavier tails for the high 'maximum loss' periods of BRICT and G-7 countries. It is noticeable what happens while we can still observe some of the stylized facts of the daily returns within the high and low maximum losses of the data generation process. Probability mass of the returns is accumulated at the positive central part and added to the tails for the high 'maximum loss' of both BRICT and G-7 countries (See Figure 4-24 and Figure 4-28). On the other side, probability density is more concentrated from tails of the probability distribution that are off the tails. The effect of kurtosis is therefore to increase the probability of very small moves in the returns of low 'maximum losses' of BRICT and G-7 countries while decreasing the probability of extreme returns.

[24] The normal distribution in the figures is scaled by using mean and variance of the simulated portfolio returns. In addition, bin width is selected as 0.02.

Figure 4-24 Plot of the cumulative probability distribution of the period with the high 'maximum loss' for BRICT countries

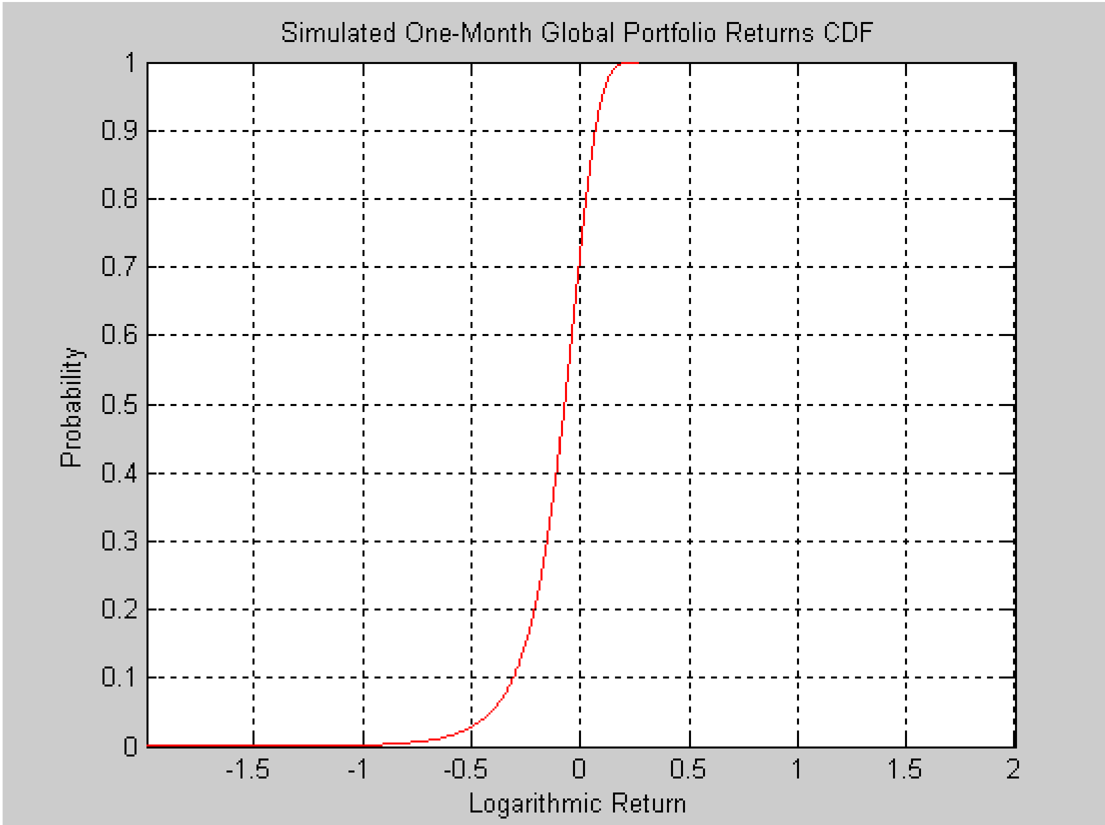


Figure 4-25 Histogram of the period with the high 'maximum loss' for BRICT countries

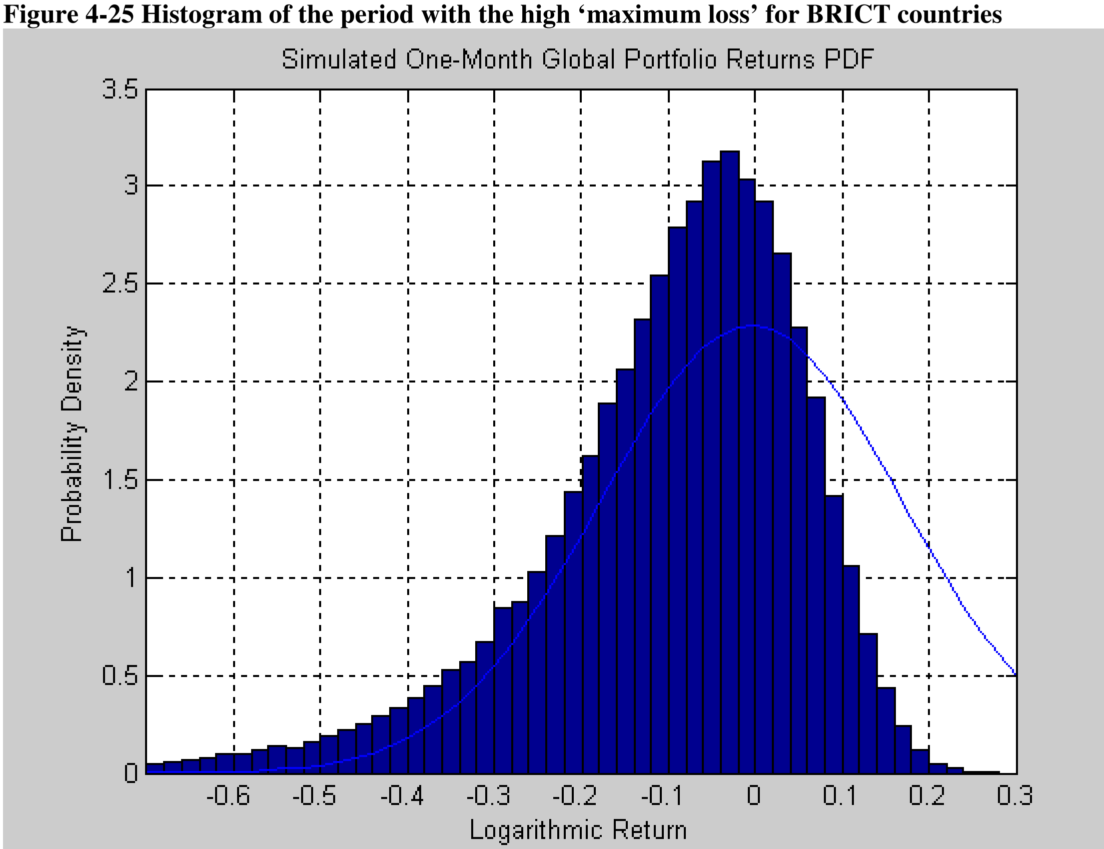


Figure 4-26 Plot of the cumulative probability distribution of the period with the low 'maximum loss' for BRICT countries

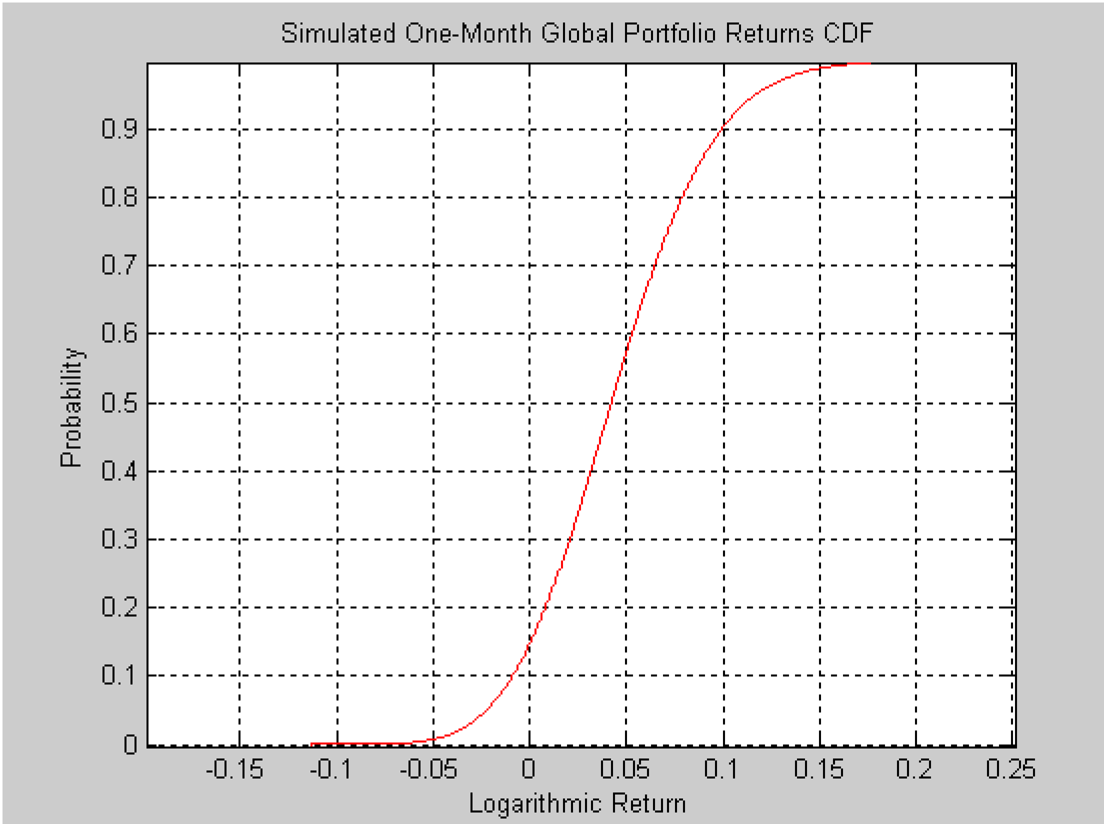


Figure 4-27 Histogram of the period with the low 'maximum loss' for BRICT countries

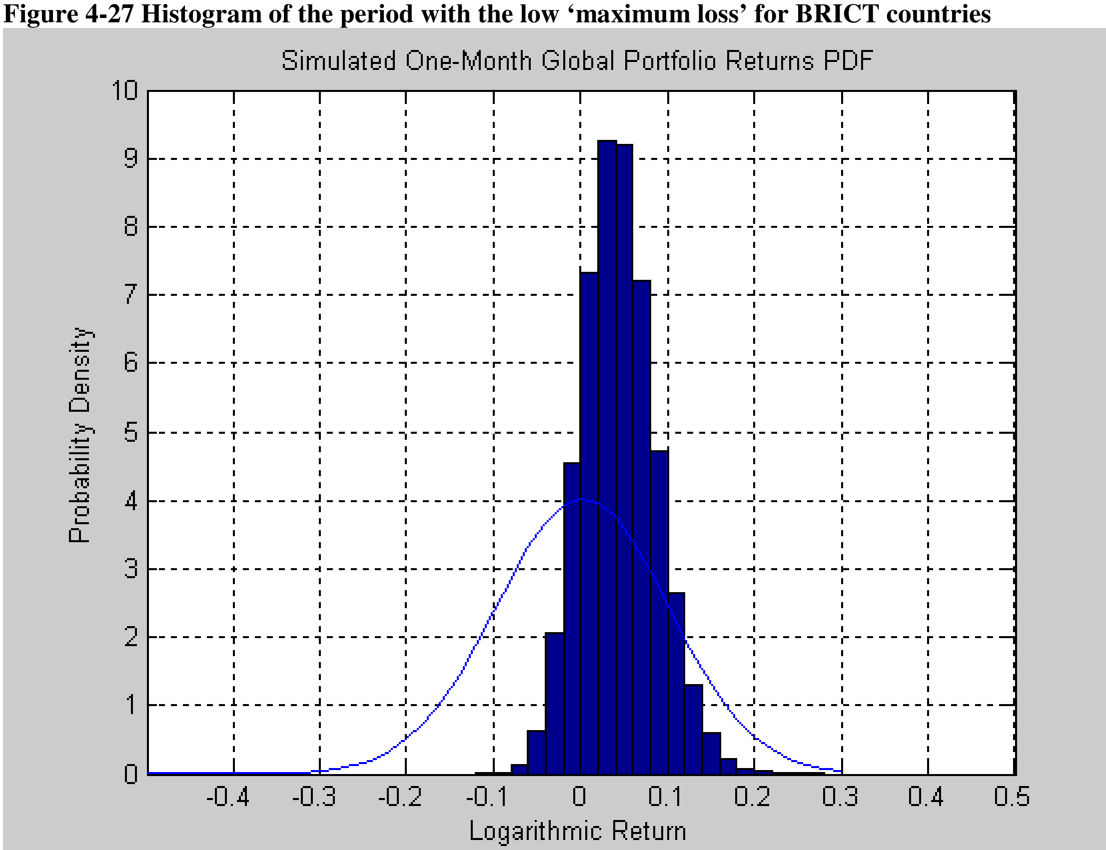


Figure 4-28 Plot of the cumulative probability distribution of the period with the low 'maximum loss' for G-7 countries

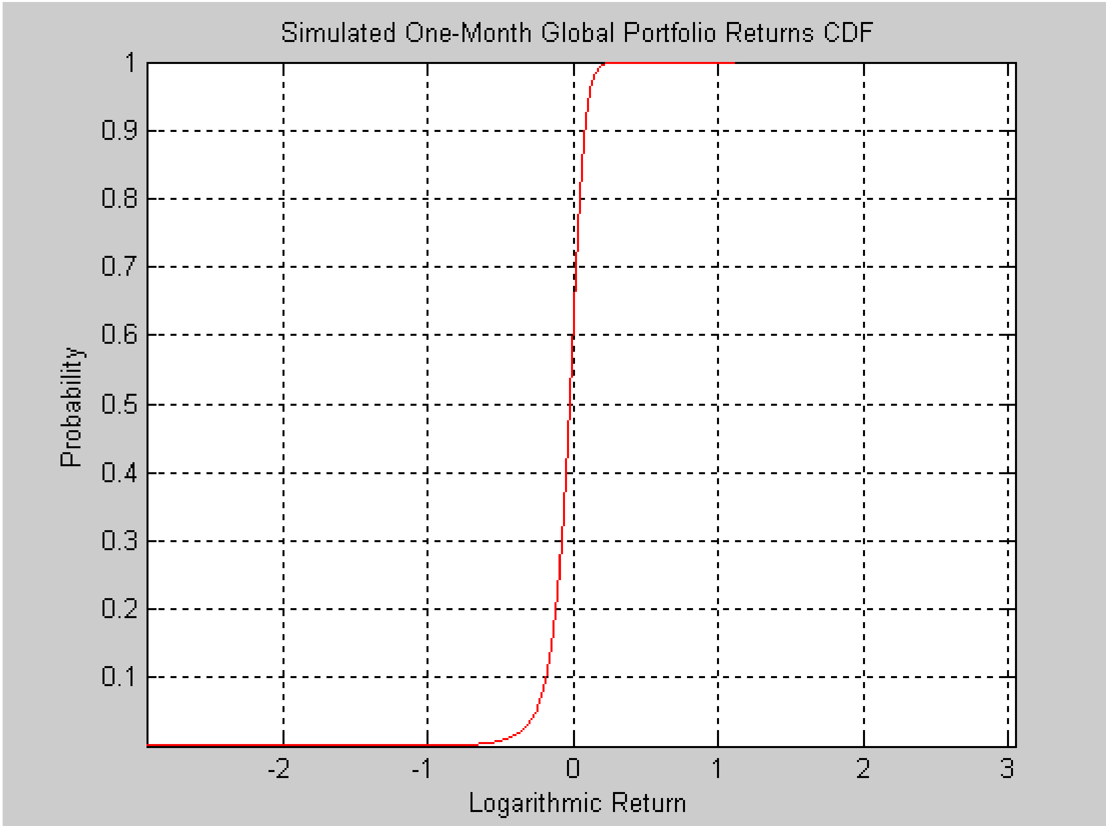


Figure 4-29 Histogram of the period with the high 'maximum loss' for G-7 countries

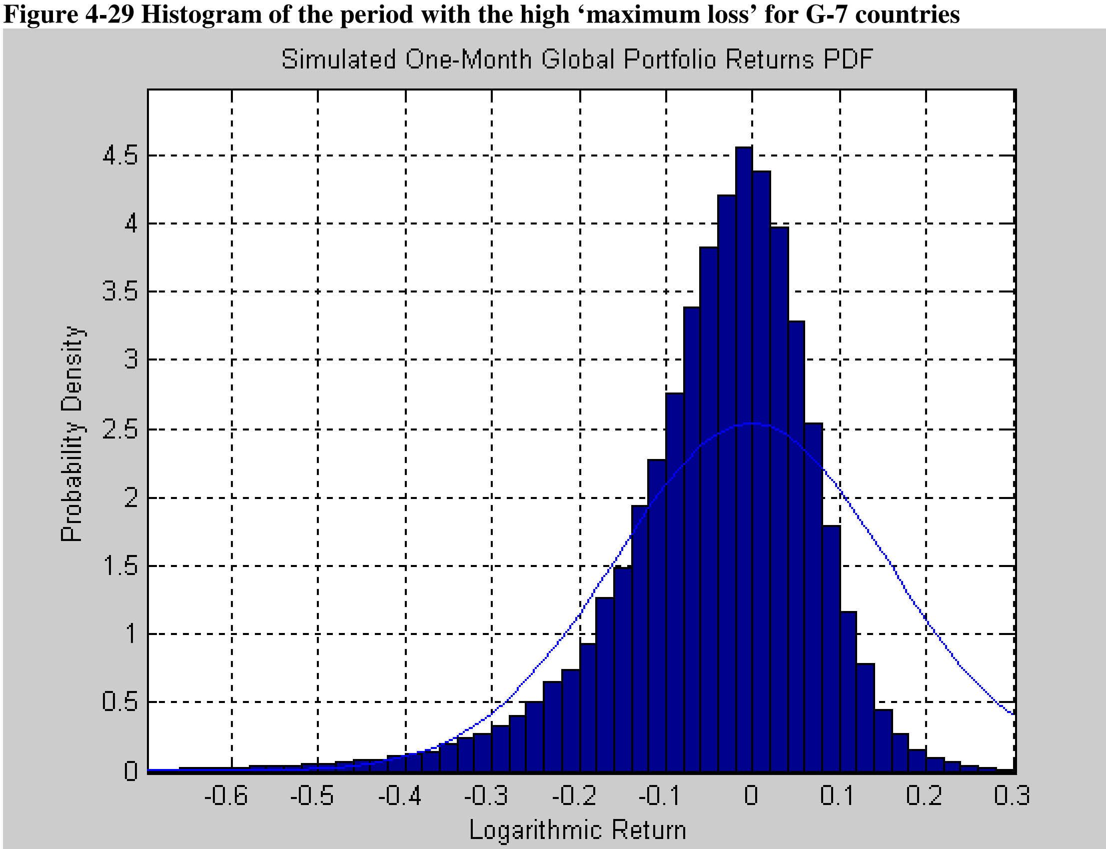


Figure 4-30 Plot of the cumulative probability distribution of the period with the low 'maximum loss' for G-7 countries

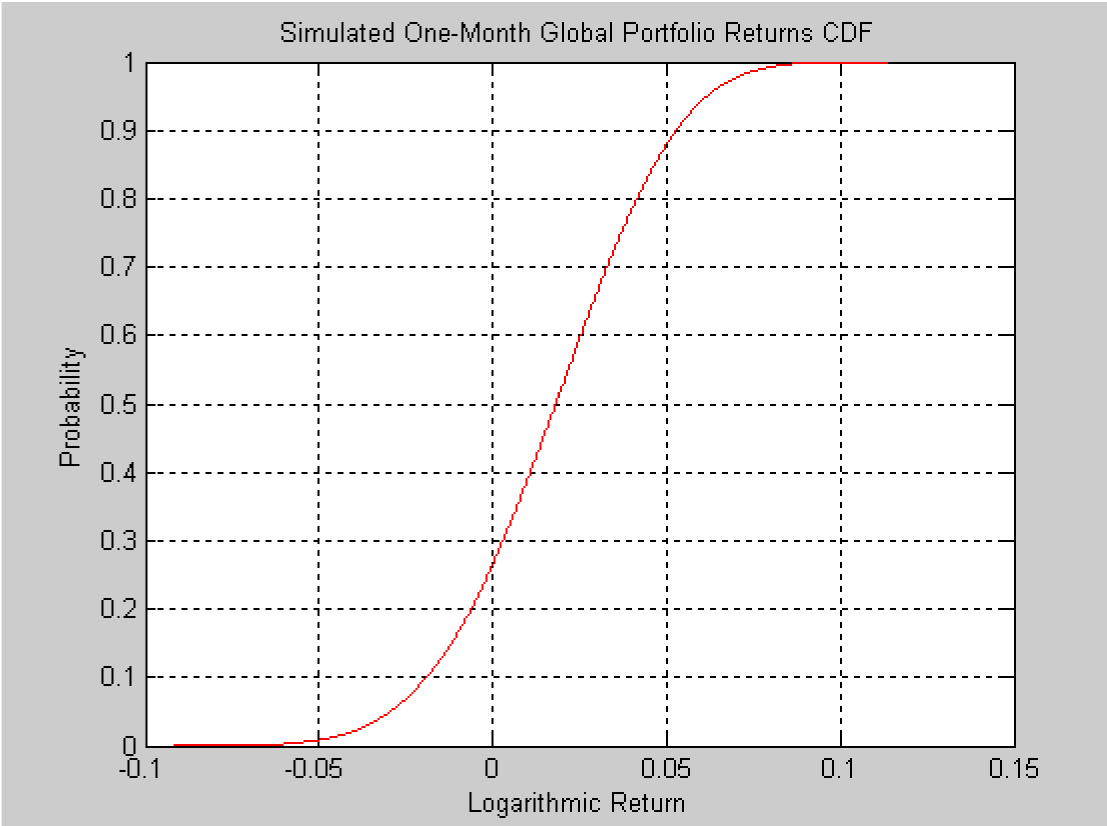
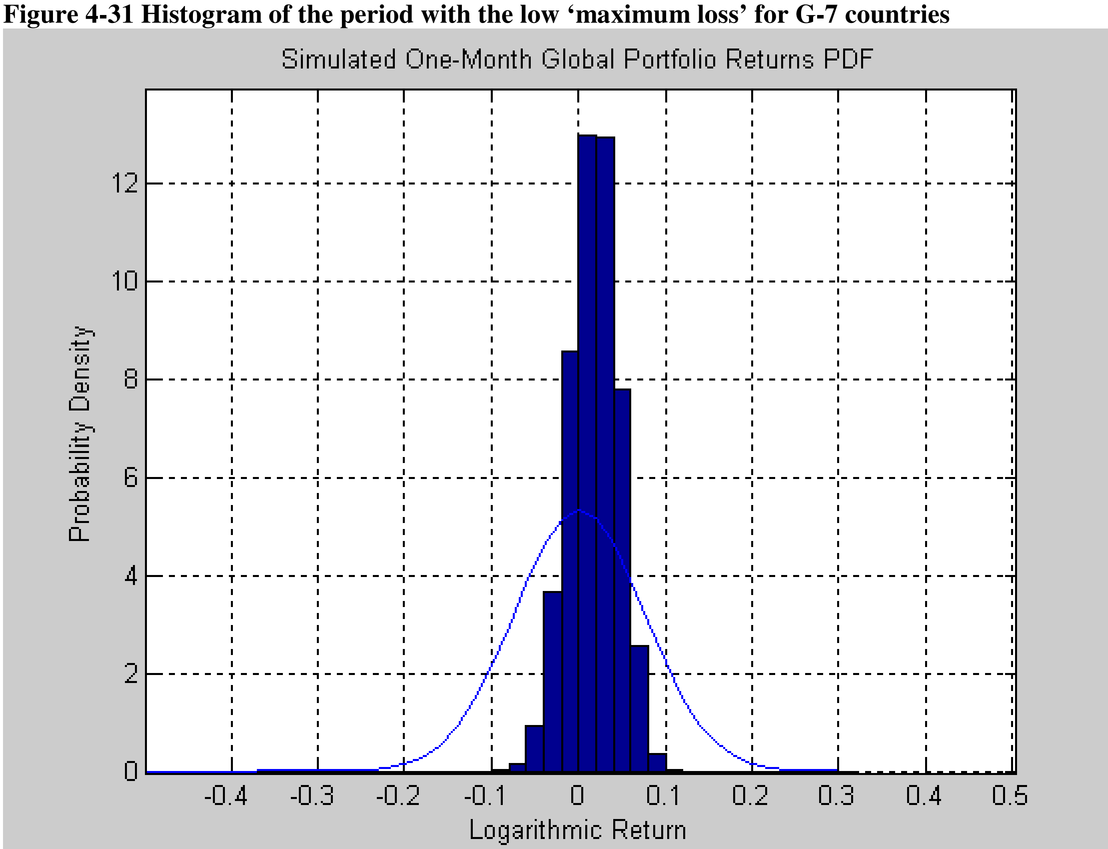


Figure 4-31 Histogram of the period with the low 'maximum loss' for G-7 countries



5 Conclusion and Further Studies

Risk assessment, which is very subjective to terms and conditions, is entailed to any economic activity. If we would like to implement the VaR calculation as an objective measurement to any of these risk assessment activities, we must ask ourselves what VaR estimate can really measure. In each case, some mechanism must be found that will support the VaR measure to help human judgment—without replacing it by default. For portfolio risk managers, the answer is value at the risk limits. Rhetorically, the reliability remains fairly open question for other possible applications.

VaR has become one of the key subject matters while quantifying portfolio market risk. It provides information not only that can be easily understandable for portfolio managers but also that is especially practical for risk management while measuring the capital requirements that their inspectors have set up. In addition, VaR estimate can be utilized as an instrument to evaluate the performance of portfolio executives.

Whatever approach is used to obtain VaR—the variance-covariance, historical simulation, or Monte Carlo simulation approach—the outcome of the method is basically an algebraic estimation. The "accurate" VaR can not be estimated with sureness due to the nature of the dynamics of economics and also the core assumptions that are vulnerable to any third party dynamics of economics.

Crouchy et al. (2000) notes that the measurement errors of the VaR estimation depends on the accuracy of the VaR calculation parameters like the mean, the variance, and/or the quantiles of the distribution of the portfolio return. Clearly, any estimate is only informative and useful for portfolio risk manager or analyst unless it is irrationally wrong within confidence intervals.

All-in-all, a delta-normal VaR approach is more accurate over shorter holding periods than longer holding periods and also the filtered simulation approach does not bring out any additional value to estimation for shorter holding periods. On the other side, the filtered historical simulation method totally depends on not only the number of scenarios to mimic the market returns in scenarios but also parameters of the simulation where data can be generated by true estimate process on assumed parametric distribution on standardized residuals. In applied econometrics, the filtered historical simulation method is significant, because the asymptotic student's t distribution of returns can be very accurate even though the distribution has higher density on the tails and lower peak at the center compare to the empirical data unless a degree of freedom is too large. Thus, historical simulation approach itself allows easing on the normality assumption of daily returns. When this happens, the difference between the probability of true and path simulations of a confidence interval can be very large, and inference can be highly misleading for the risk management reports. Yet it does not often response the market events very well as expected or it may exacerbate the estimation results. We can say that neither the portfolio of BRICT nor the portfolio of G-7 countries is impacted from the subprime mortgage crisis by looking at the comparison results in the Figure 4-18 and Figure 4-19. Filtering historical returns through an appropriate filter leads to time-consistent estimates of risk. The implementation of filtered historical simulation through EGARCH(1,1) filters by using bootstrapped standardized residuals as the i.i.d. input noise process. However, if this empirical study shows the impact of the crisis as a whole time, then we had to be cautious about the last 10 months before April 2010. While looking for answer to the inquiry of "How bad can things get throughout each subsample?" under perfectly "normal" market conditions for more precise calculations of market risk, "normal" market condition conceals the limitations of the simplest calculations of market risk. Filtered historical simulation VaR is providing evidence to be an exceedingly robust way of evaluating the risk of hypothetical global index portfolio over a prespecified holding period, such as one-month period, and predefined confidence level. In fact, the FHS method allows us to calculate the risk as a single number that we can rely on by imposing input noise process on autocorrelation and heteroscedasticity observed in the historical time series.

Schachter (1998) noted that VaR does not measure "event" (e.g., market crash) risk. This is what we disagree due to our findings that VaR estimates for both two global portfolios that are measured against the risk associated with equity positions are incorporated with all plausible affiliations on this credit crunch crisis and aftermath of it. Filtered historical simulation VaR supplies us a common risk benchmark, and this benchmark enables for portfolio risk managers to keep eye on the risk of the portfolios risks in new outlooks that were not feasible before. However, we agree with Schachter on portfolio should be tested more rigorously to add on VaR estimates. VaR does not promptly detect liquidity differentiations amongst financial instruments but VaR does not measure model risks with good grace.

5.1 Limitations of VaR as a Risk Measure

A very significant concern in financial risk management, and one that makes it predominantly appealing as an outstanding topic in the financial statistics and probability as well, is the indigence to concentrate on extreme results, rather than the expected results that are at the center of lots of traditional tools. Alan Greenspan at the first "Risk Measurement and Systemic Risk" conference in Washington (DC), 1995:

"From the point of view of the risk manager, inappropriate use of the normal distribution can lead to an understatement of risk, which must be balanced against the significant advantage of simplification. From the central bank's corner, the consequences are even more serious because we often need to concentrate on the left tail of the distribution in formulating lender-of-last-resort policies. Improving the characterization of the distribution of extreme values is of paramount importance."[25]

Until certain extents, any VaR measure supplies an ordinary reliable criterion of risk across different positions for institutions to manage the portfolio risks. Theoretically, portfolio directors are supposed to not focus on the whole distribution of gains and losses over the target horizon. In fact, this distribution is assessed to one figure, i.e. the maximum loss at a given confidence level at either 90%, 95%, 99% or 99.9%. Jorion (2005) asserts that VaR is only one of the measures that financial portfolio directors take into consideration. Then again, in technical terms, the probability distribution of future gains and losses has to be assessed assuming an unchanged position. He suggests it ought to be validated by stress-testing, which discovers possible losses under extreme circumstances, which are associated with much higher confidence level.

On the contrary, VaR has its shortcomings as a 'risk measure' as well. Some of these shortcomings are quite obvious that VaR estimations can tend to model risk arising from inadequate presumptions on which models are based or implementation risk arising from the technique in which model is implemented. However, these are common problems for any risk evaluation methods, and such problems are not exclusive to VaR approach.

[25] See Hartmann et al. (2005).

Like Jorion(2005) suggests, running complementary tests are obviously correct way to test our hypothesis when we think of verifying the model that is derived from the observation of empirical data analysis and we are supposed to take into consideration risk management is not a pure science which we can test our hypothesis again and again.

What could be done more on the study?

Even though one of the most regular methods for assessing the risk is VaR, it is often calculated via delta normal and/or simulation techniques. However, VaR analysis is of limited use to describe extreme losses that exceed a certain threshold. Extreme value theory (EVT) is gaining popularity as a complementary to VaR analysis by focusing exclusively on tail events (Dowd, 2002).

There are various methods utilized by VaR measures to calculate most likely losses. Having said that, of course, each estimation method has been evolved over time and we can not put a final verdict any risk measurement technique which is a single ultimate technique or absolutely superior against other techniques. Furthermore, any econometric model has to come across with two typical types of errors, viz Type (I) Reject a correct model and Type (II) Accept an incorrect model. Type I and II errors have to be verified and validated by questioning the stability of our model. Hence, VaR model is validated by backtesting and this test is complemented by stress testing. Very high level we will introduce these two testing procedures.

Backtesting is a procedure where model based VaR is continually implemented and compared with the actual performance of the portfolio over time. This process of monitoring is fulfilled by evaluating a subjected model to calculate the accuracy of the existing approach. Put it simply, backtesting involves systematically comparing the associated ex-ante forecasted VaR measures with subsequent returns of actual measures. For example, reviews of various approaches can be found in Hurlin and Tokpavi (2006). The test method presented here is not ultimate and many other tests do exist. Depending on the type of forecast (quantile or density), different approaches are used.

For instance, calculate one-month holding period with 95% confidence level VAR estimates for a frozen weighted portfolio each month for some rolling period of time (i.e., 200 Days) with non-benchmark models. For backtesting purposes: use again lower confidence level (i.e., 95%) to ensure that estimates "over" actually occur during holding periods and then compare the VaR estimates on the succeeding trading month with the previous month's losses[26]. Finally, count the number of times the loss goes beyond the VAR estimates.

Assume that y_t denotes the realization of the forecasted variable, and $VaR_{t|t-1}(\alpha_{cl})$ the associated ex-ante forecasted value-at-risk for a confidence level α, predicted with both MVaR and FHS methods on the information available at time $t-1$ against delta-normal VaR on the information available at time t. Backtesting procedures for α-quantile forecasting models (e.g. value-at-risk models) based on the exception indicators can be written as follows:

$$f(\alpha_{cl}) = \begin{cases} 1 & if \quad y_t \leq VaR_{t|t-1}(\alpha_{cl}) \\ 0 & else \end{cases}$$

Given the history of this indicator function for $t = 1, 2, \ldots, T$, the accuracy of the forecasting model is determined. The random variable $f(\alpha_{cl})$ is said to follow a Bernoulli distribution whose expected value is α_{cl}.

Stress Testing essentially defines and reviews validity of exceptional but plausible events in the macro scenarios and associated parameters so that we could determine possible changes in the spot rate of a portfolio that could arise due to non-normal movement in one or more market parameters. Thus, stress testing concentrates on the occasional but large scale events that happen in the left tail of the histogram and consider current scenarios that are of relevance to risk profiles. These are precisely the events that traditional VaR can not accommodate. For instance, we can pay attention to risks in emerging markets considering political and economic developments in these countries or liquidity crisis in developed countries (Lee, 2003) despite the global effect of the subprime mortgage crisis.

In VaR models stress test can be run by inputting the stressed values of the risk factors and also recalculating the portfolio value using the new data.

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[26] The focus of this method is on losses but it can also apply to gains.

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List of Annexes

Figures

Tables

Annex 1: Jarque-Bera tests of normality for all subsamples of the hypothetical portfolio log returns

Table 0-1 Jarque-Bera tests of normality for all subsamples of the hypothetical portfolio log returns

Subsample Period	BRICT for %5 level		G-7 for %5 level	
	JB test statistic	p-value	JB test statistic	p-value
1	10.39779	0.013648	29.809994	0.001
2	3.825573	0.110892*	14.702106	0.005845
3	3.933567	0.105027*	10.51584	0.013298
4	5.744428	0.048825	10.467594	0.01344
5	7.773181	0.026058	16.573587	0.004272
6	2.761934	0.199293*	47.666609	0.001
7	1685.61	0.001	68.461479	0.001
8	1284.036	0.001	40.17268	0.001
9	1675.226	0.001	35.170141	0.001
10	1103.016	0.001	56.690882	0.001
11	1166.55	0.001	26.038505	0.001195
12	780.3541	0.001	29.404908	0.001
13	723.9198	0.001	52.760461	0.001
14	735.8	0.001	72.073096	0.001
15	820.6298	0.001	85.157884	0.001
16	610.0243	0.001	106.348917	0.001
17	423.0937	0.001	107.155179	0.001
18	183.2597	0.001	24.007467	0.001511
19	210.4839	0.001	24.434898	0.001437
20	153.0694	0.001	43.978067	0.001
21	90.51063	0.001	31.294065	0.001
22	147.0134	0.001	34.261905	0.001
23	163.9335	0.001	43.430443	0.001
24	145.9899	0.001	38.578811	0.001
25	44.72876	0.001	18.845951	0.003012
26	56.51943	0.001	21.862865	0.001976
27	653.9888	0.001	171.248001	0.001
28	529.1315	0.001	218.276192	0.001
29	453.4072	0.001	128.910606	0.001
30	320.2407	0.001	122.928432	0.001
31	240.2221	0.001	68.173897	0.001
32	229.3361	0.001	63.926291	0.001
33	200.4315	0.001	58.283322	0.001
34	266.7388	0.001	87.760967	0.001
35	244.6853	0.001	93.530202	0.001
36	181.8643	0.001	47.821155	0.001
37	147.5258	0.001	24.17756	0.001481
38	179.7475	0.001	29.786308	0.001
39	73.37938	0.001	8.445721	0.021794
40	61.36815	0.001	16.802962	0.004117
41	182.1326	0.001	32.225542	0.001

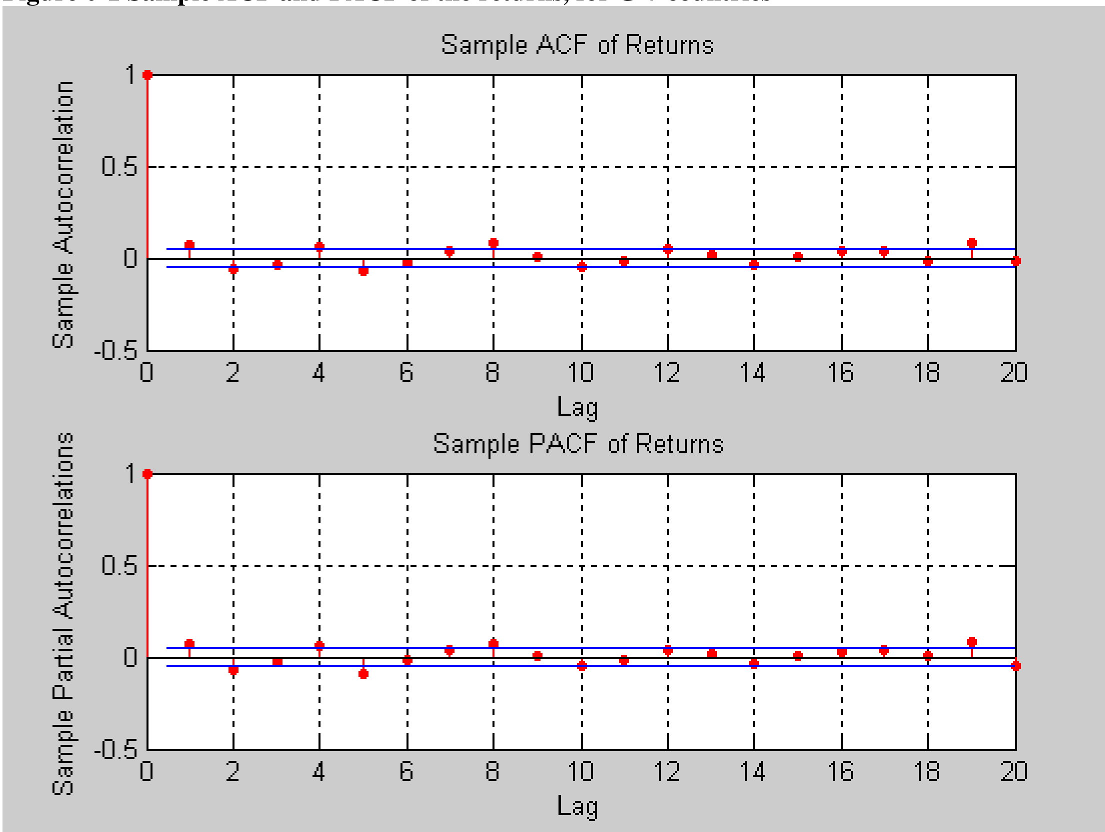
Subsample Period	BRICT for %5 level		G-7 for %5 level	
	JB test statistic	p-value	JB test statistic	p-value
42	163.1377	0.001	28.398285	0.001
43	81.90966	0.001	14.22252	0.006366
44	52.5663	0.001	14.15875	0.006439
45	680.88	0.001	17.363553	0.003768
46	750.3877	0.001	16.406733	0.004389
47	694.4338	0.001	12.726297	0.00842
48	487.4281	0.001	7.032203	0.032185
49	414.2035	0.001	5.014833	0.064407*
50	427.6462	0.001	23.955938	0.001521
51	1334.337	0.001	811.380057	0.001
52	878.105	0.001	374.70344	0.001
53	760.9832	0.001	315.87448	0.001
54	802.7233	0.001	222.836293	0.001
55	601.1293	0.001	165.903638	0.001
56	542.2731	0.001	148.035714	0.001
57	540.9041	0.001	142.922821	0.001
58	534.7242	0.001	140.922987	0.001
59	643.0896	0.001	173.090534	0.001
60	196.9754	0.001	52.691109	0.001

(*) Indicates where the null hypothesis can be rejected at 5% significance level

Annex 2: Sample ACF and PACF of the returns, for G-7 countries

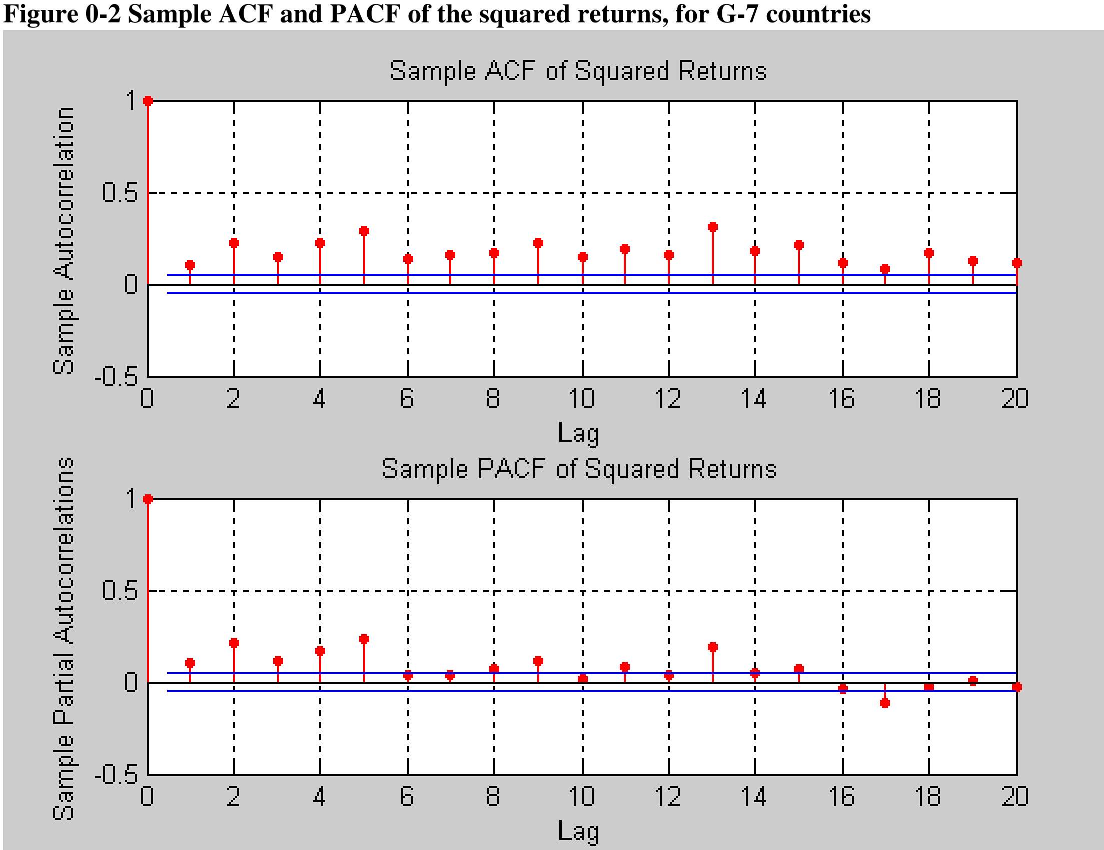
The lag at which the PACF goes below the upper and lower threshold boundaries is the indicated as a number of AR terms. If the lags in a PACF of the stationary series exhibits a sharp cutoff and/or the lags of sample autocorrelation are positive, then we consider adding an autoregressive term to the volatility estimate model. That is why in general we used AR(1) term throughout the analysis. However, there are more rigorous diagnostic tests such as the information criterion and so on but we did not apply those for any periods.

Figure 0-1 Sample ACF and PACF of the returns, for G-7 countries



Annex 3: Sample ACF and PACF of the squared returns, for G-7 countries

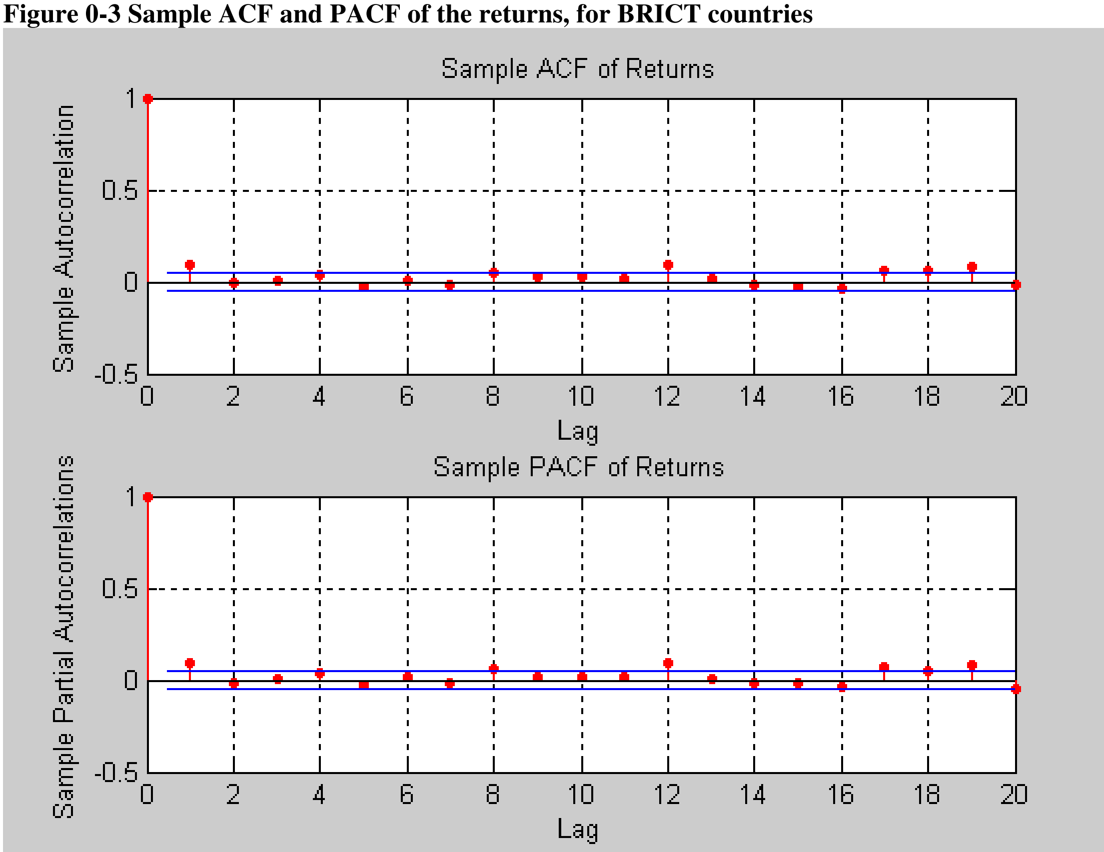
Figure 0-2 Sample ACF and PACF of the squared returns, for G-7 countries



Annex 4: Sample ACF and PACF of the returns, for BRICT countries

The lag at which the PACF goes below the upper and lower threshold boundaries is the indicated as a number of AR terms. If the lags in a PACF of the stationary series exhibits a sharp cutoff and/or the lags of sample autocorrelation are positive, then we consider adding an autoregressive term to the volatility estimate model. That is why in general we used AR(1) term throughout the analysis. However, there are more rigorous diagnostic tests such as the information criterion but we did not apply those for any periods.

Figure 0-3 Sample ACF and PACF of the returns, for BRICT countries



Annex 5: Sample ACF and PACF of the squared returns, for BRICT countries

Figure 0-4 Sample ACF and PACF of the squared returns, for BRICT countries