Charles University in Prague

Faculty of Social Sciences

Institute of Economic Studies

Operational Risk Scenario Analysis

Rigorous Thesis

Author: Mgr. Milan Rippel

Email: milanrippel@seznam.cz

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Thesis Leader: PhDr. Petr Teplý

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Statement

I hereby declare that except where reference is made to the work of others, the work described in this thesis is my own or was done in collaboration with my thesis supervisor.

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Abstract

Operational risk management and measurement has been paid an increasing attention in recent years – namely due to the Basel II requirements that were to be complied with by all international active financial institutions by January 2008 and also due to recent severe operational risk loss events. This rigorous thesis focuses on operational risk measurement techniques and on regulatory capital estimation methods. A data sample of operational losses provided by a Central European bank is analyzed using several approaches. Multiple statistical concepts for the Loss Distribution Approach are considered. One of the methods used for operational risk management is a scenario analysis. Under this method custom plausible loss events defined in a particular scenario are merged with the original data sample and their impact on capital estimates and on the financial institution as a whole is evaluated. Two main problems are assessed in this rigorous thesis – what is the most appropriate statistical method to measure and model operational loss data distribution and what is the impact of hypothetical plausible events on the financial institution. The g&h distribution was evaluated to be the most suitable one for operational risk modeling because its results are consistent even while using scenario analysis method. The method based on combination of historical loss events modeling and scenario analysis provides reasonable capital estimates for the financial institution and allows to measure impact of very extreme events and also to mitigate operational risk exposure.

Abstrakt

Zájem o problematiku řízení a meření operačního rizika se v posledních letech prudce zvyšuje – zejména kvůli požadavkům kapitálové přimeřenosti definovaných v Basel II, které musí k 1. lednu 2008 splňovat všechny mezinárodně aktivní finanční instituce a také kvůli závažným ztrátám v oblasti operačního rizika, které se staly v nedávné minulosti. Tato rigorózní práce se zaměřuje na techniky meření operačního rizika a metody odhadů kapitálové přiměřenosti. Soubor ztrát operačního rizika, který byl poskutnut středoevropskou bankou, je analyzován pomocí různých přístupů. Je posuzováno několik statistických konceptů používaných pro modelování rozdělení operačních ztrát. Jednou z metod řízení operačního rizika je metoda analýzy scénářů. V této metodě jsou definovány hypotetické ztrátové události a tyto události jsou přidány do souboru empirických událostí a následně je posuzován vliv výsledného souboru událostí na výpočet kapitálové přiměřenosti a na finanční instituci jako celek. Tato rigorózní práce se zejména věnuje následujícím dvěma problémům – jaká je nejpřijatelnější statistická metoda na měření a modelování rozdělení ztrát operačního rizika a jaký je vliv hypotetických událostí na finanční instituci. G&h distribuce byla vyhodnocena jako nejvhodnější pro modelování ztrát operačního rizika a výsledky kapitálových odhadů pomocí tohoto rozdělení jsou konzistentní i po aplikaci metody analýzy scénářů. Metoda založená na kombinaci empirických dat a analýzy scénářů tak poskytuje věrohodné odhady kapitálové přiměřenosti a dovoluje finanční instituci měřit vliv extrémních událostí a zavádět postupy zmírňující míru operačního rizika.

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Introduction

In this paper we focus on modeling and stress testing of economic and regulatory capital set aside to cover unexpected losses of a medium size Central European bank (BANK). There are two main questions this thesis is aimed to answer:

- What is the appropriate statistical method to model OR loss data distribution and measure reasonable capital estimates for the institution?
- What is the impact of extreme events defined in particular extreme case scenarios on the capital estimates and on the financial institution?

Firstly, the risk measurement statistical techniques must be evaluated and the most suitable ones used further for scenario analysis method in order to test whether those methods are consistent even if original data sample is enriched by adding a few extreme losses. The best method for capital estimate computation is then chosen and effects of scenarios to the financial institution are assessed.

Several statistical distributions are used to model loss severity distribution and compute capital estimates. It is expected that the best results will be provided by a distribution that can reasonable model body as well as the heavy right tail of the data sample. On the other hand, techniques that focus just on the tail of the distribution might not provide consistent results if the tail is contaminated by loss events defined during scenario analysis. The distribution that is expected to be the most suitable for modeling the operational risk data is the g&h distribution used by Dutta, Perry (2007). So the null and alternative hypothesis can be stated as:

H_0: The g&h distribution provides consistent capital estimates for scenario analysis method

H_1: Extreme Value Theory provides consistent capital estimates for scenario analysis method

Once this hypothesis is assessed the effects of extreme events on the financial institution can be evaluated. It might be assumed that the bank will not be able to cover the worst case joint scenario losses, because the loss amounts will be too high to be covered by the bank capital. On the other hand, the bank should be able to cover average joint scenario losses.

This rigorous thesis is organized as follows: The first chapter provides an overview of operational risk concepts, Basel II measurement and risk management techniques. The second chapter focuses on a detailed overview of the Loss Distribution Approach (LDA). The third chapter analyzes the data sample of BANK and proposes distributions that can best model the data sample. These distributions are then used for capital estimates computation. The fourth chapter provides a theoretical overview of stress testing and scenario analysis methodology. In the fifth chapter the loss events defined in particular scenarios are merged with original data sample and new capital estimates are computed. Finally, the last part of this paper makes conclusion of the findings and results and proposes ideas for future research.

Chapter 1 - Operational Risk & Basel II

This chapter provides a basic overview of literature about operational risk and scenario analysis. Afterwards the concepts of operational risk (OR), economic and regulatory capital and application of Basel II regulatory rules are being discussed.

1.1 Literature overview

Taken into consideration the scarcity and confidentiality of OR loss data, there are only few papers that explore specifics of OR data and are able to measure OR exposure with the accuracy and precision comparable with other sources of risk. The most comprehensive ones are Chernobai (2007), Dutta, Perry (2007), Embrechts (2006), de Fountnouvelle (2006) , Mignolla, Ugoccioni (2006) and Degen (2006). The scenario analysis method theory is just very briefly mentioned in papers from Cihak (2004), Rosengren (2006) or Arai (2006).

1.2 Classification of risks

Banks and other financial institutions are facing multiple risks that affect their financial results. A financial institution is required to manage these risks and allocate necessary amount of capital that is sufficient to cover potential losses caused by the occurrence of loss events in order to ensure sustainability of its business activities. A risk is thus defined as a measure to capture the potential of suffering a loss. Risks can be classified into several categories. The Bank for International Settlements (BIS)[1] classifies financial risk sources as shown in figure 1.1.

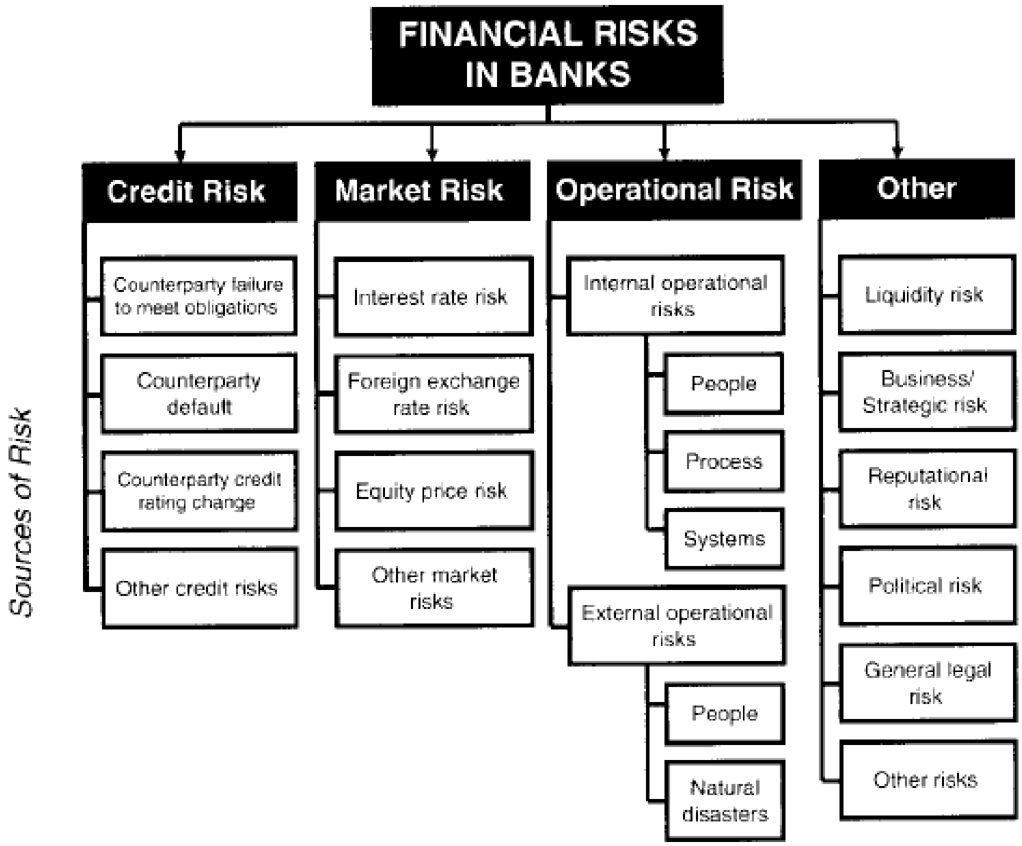
Until very recently risk management was focused just on credit risk - the risk that a counterparty will fail to meet its obligations – and market risk – the risk of losses due to changes in market conditions. The other sources of risks were not treated as important as would be relevant. Due to serious changes in financial market conditions the bank risk profiles significantly altered in the last years. The main drivers of these changes were “globalization and deregulation, accelerated technological innovation and revolutionary advances in the information network, an increase in the score of financial services and products”[2] and an increasing amount of mergers and acquisitions. Because of those changes banks

[1] BCBS (2006)

[2] Chernobai (2007)

became more exposed to losses that occur during the bank operations and the operational risk management has been given appropriate attention of both banks and financial market regulators.

Figure 1.1 - Topology of financial risks in banks



Source: Chernobai (2007)

Even though these events are of rather infrequent occurrence, they often cause large damage to bank operations and so financial institutions must consider such events for the risk management. “As a result, a bank will create provisions for expected losses and set aside capital for unexpected losses”[3].

1.3 Operational risk

The concept of operational risk (OR) is relatively new. OR has been defined as a separate source of risk just few years ago. But it has been given an increasing attention by both, banks and regulators, in

[3] Teply, Chalupka (2008)

recent years and OR has become a "hot topic for both practitioners and academics"[4]. Even though the OR measurement techniques are still under development and there is still much work left to do in the OR management field. This applies to the relevant terminology as well.

The most common definition of OR is given in Basel II as "the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk."[5] However, other definitions exist as well. A very general definition says that OR is a consequence of doing business. OR thus bundles relatively broad area of risks which differs it from market and credit risk. The common idea is that operational risk encompasses those risks, not covered under credit and market risk, that have a measurable financial impact. Table 1.1 categorizes OR by its main drivers.

Table 1.1: Main factors of operational risk

People	Systems	Processes	External Events
• Fraud, collusion and other criminal activities • Violation of internal or external rules • Management errors • Loss of important employees • Security violations	• IT problems • Unauthorized access • Unavailability of data • Communication failures • Utility outages	• Execution, registration, settlement errors *(transaction risk)* • Model and methodology errors (*model risk)* • Accounting errors • Compliance issues • Inadequate attribution of responsibilities	• Criminal activities • Political and military events • Supplier failures

Source: Based on Teply, Chalupka (2008)

There are some specifics of OR in comparison to market and credit risks that in general make OR more difficult to manage. "The main differences are the fact that operational risk is not taken on a voluntary basis but is a natural activity performed by a financial institution"[6] and a noticeable lack of hedging instruments. The main differences are summarized in Table 1.2.

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[4] Teply (2007)
[5] BCBS (2006)
[6] Teply, Chalupka (2008)

Table 1.2: Risk types comparison

Market an Credit Risks	Operational risk
• Consciously and willingly faced • Speculative risk, both losses and profits • Positive risk-return relationship • Easy to identify and understand • Easy to measure and quantify • Availability of hedging instruments • Comparatively easy to price and transfer	• Unavoidable • Pure risk, implies only losses • Non consistent risk-return relationship • Difficult to identify and understand • Difficult to measure and quantify • Lack of effective hedging instruments • Difficult to price and transfer

Source: Based on Teply, Chalupka (2008)

There are some widely known and severe magnitude of OR events that happened in recent years – the most publicly known examples of OR would be those caused by fraud, natural disaster or unauthorized trading – one very recent OR event from the Czech Republic is the theft of USD 31M in the G4S Cash Services. The other example would be a failure of internet banking of Ceska Sporitelna in 12/2007, or a loss of USD 12M suffered by BANK due to improper rounding in interbank transactions.

The mostly know foreign OR events starts with a large loss in the amount of USD 7,500M caused to Société Générale by unauthorized derivatives trading by Jerome Kerviel. The first "top class" OR event is attributed to Nick Leeson who caused 1,000M loss to the Barings bank by unauthorized trading. Another category of events is connected with terrorist acts or natural disasters – like losses caused by 9/11 or hurricane Katrina. Each of those events exceeds loss amount of USD 1,000M.

It is clear that those events are the most severe but very infrequent ones. They represent high risk and in some cases can be destructive for a financial institution. There are other loss events that are more common but cause much smaller loss to a bank – like an input error caused by an employee, a credit card fraud or a failure of a supplier.

1.4 Basel II

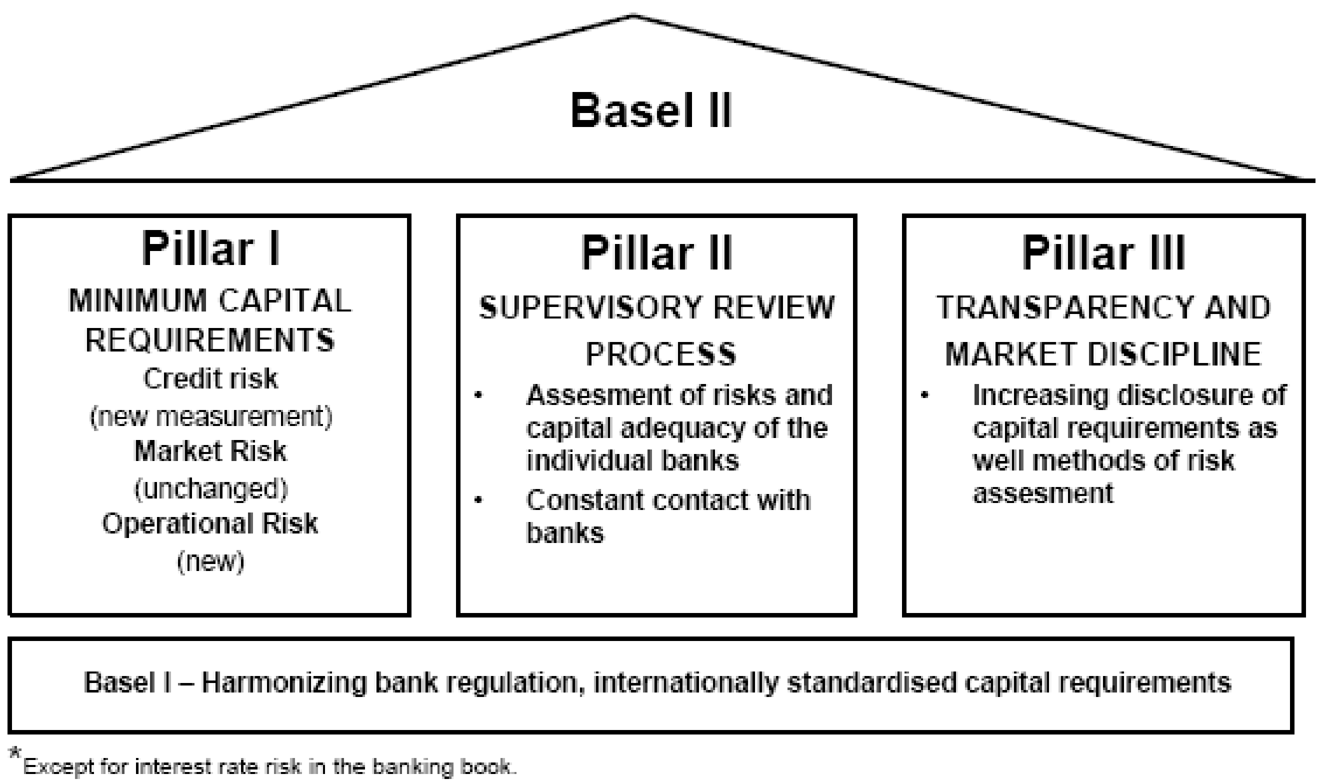
Since the number of OR events exceeding USD 100m loss is higher than 100 since 1980[7], it is clear that financial institutions must manage the OR and regulators are obliged to monitor how financial

[7] Fontnouvelle (2003)

institutions do it. Because of that, the Basel II Capital Accord document finalized in June 2006 specifically includes OR as one of the risks to be considered for the minimum capital requirements. The regulatory capital is designed to reflect the exposure of each bank to operational risk.

The aim of Basel II is to set the same rules of the game for all internationally active banks. The regulatory framework is divided into three pillars as shown on figure 1.2. Pillar 1 sets the rule for minimum capital requirements for all risks that banks are subject to. Pillar 2 describes the revision process done by regulators and finally Pillar 3 defines policies for market discipline and information disclosure. The Basel II framework includes identification, measurement, monitoring, reporting control and mitigation of operational risk. All internationally active banks were obliged to comply with Basel II by January 2007.[8]

Figure 1.2 – Basel II concept



Source: Teply, Chalupka (2007)

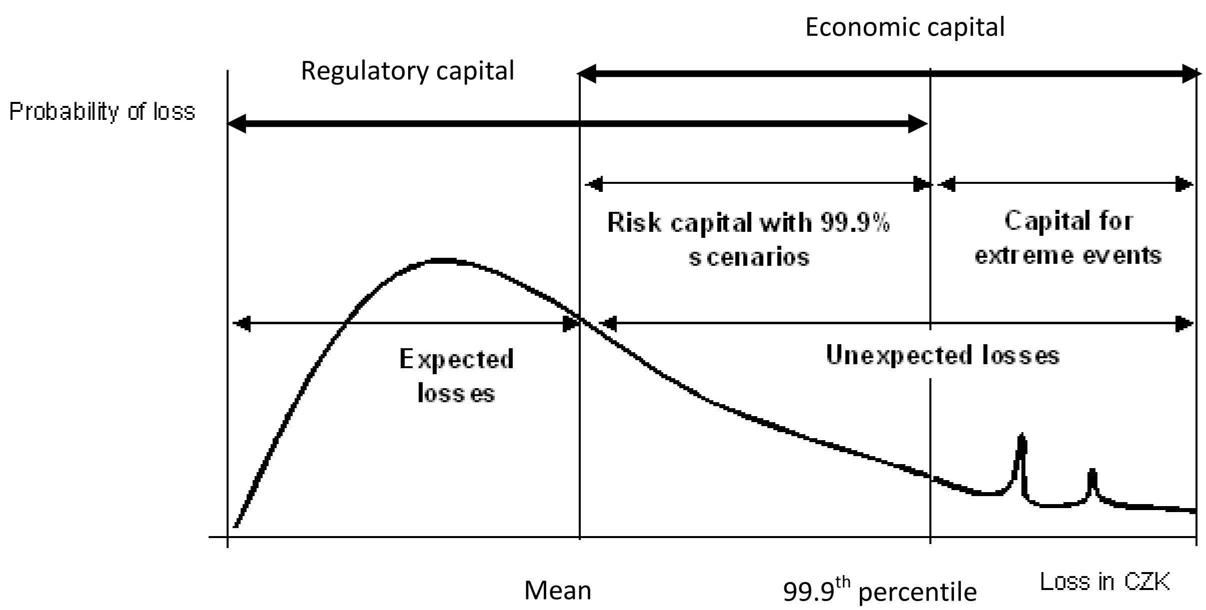
[8] There was a one year extension granted for this deadline, therefore the banks have to comply with Basel II by January 2008

1.5 Regulatory and economic capital

Regulatory capital is the amount of capital necessary to provide adequate coverage of banks' exposures to financial risks as defined in the capital adequacy rules set by the Basel II. "A one-year minimum regulatory capital is calculated as 8% of risk-weighted assets."[9] Empirical studies show that operational risk regulatory capital, in general, constitutes about 25% of overall capital adequacy requirements.

On the other hand, economic capital "is a buffer against future, unexpected losses brought about by credit, market, and operational risks inherent in the business of lending money"[10] or alternatively economic capital might be defined as the amount necessary to be in the financial business.

Figure 1.3: Classification of bank's requirements according to risk



Source: Based on Teply, Chalupka (2008) & BCBS(2006)

Further we will focus on modeling both regulatory and economic capital for OR because this concept is to be used for the Advanced Measurement Approach (AMA) as it should cover all unexpected losses – even the extreme events with the Value at Risk (VaR) higher than 99.9%. Regulatory capital covers expected losses and unexpected losses only to a certain confidence level and it does not consider

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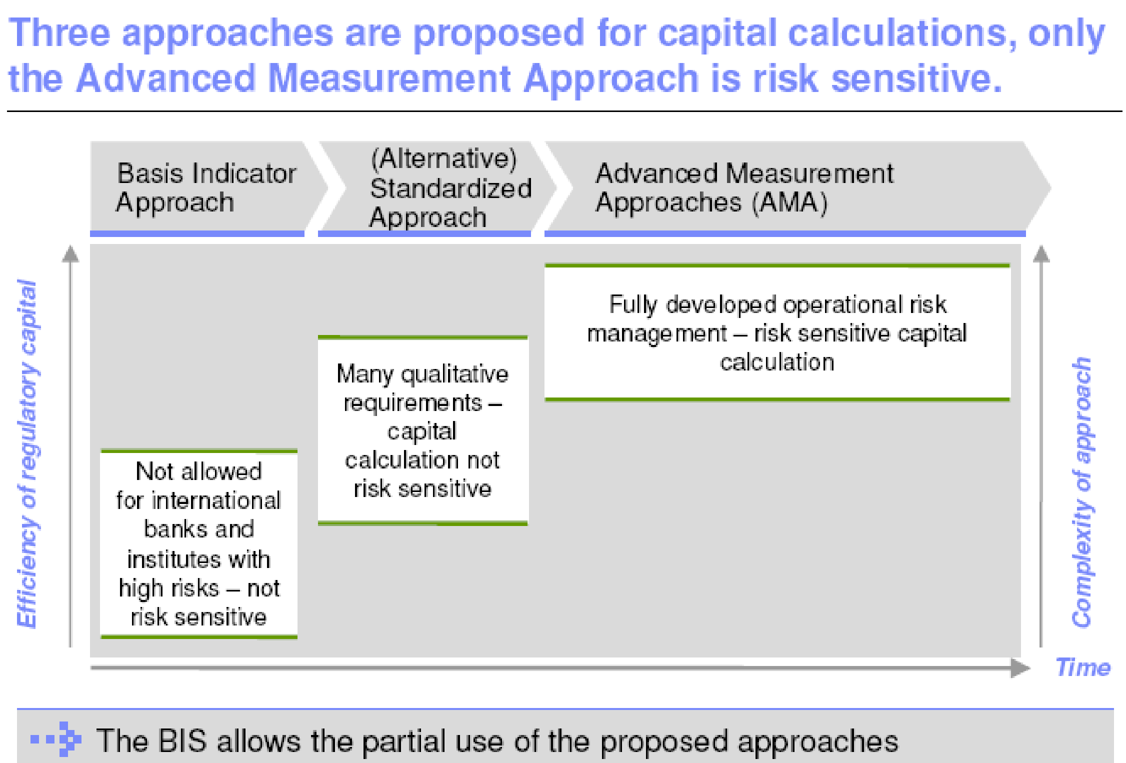
[9] Chernobai (2007)
[10] Mejstrik, Pecena, Teply (2007)

the extreme events[11] like economic capital does. The regulatory capital will be further defined as the $VaR_{0.999}$ measure and the economic capital as the $CVaR_{0.99}$ measure[12].

1.6 Basel II operational risk measurement techniques

Basel II sets three operational measurement methodologies for calculating operational risk capital charge "in a continuum of increasing sophistication and risk sensitivity"[13]. The first two approaches are top-down approaches, because the capital charge is allocated according to a fixed proportion of gross income. The third approach is a bottom-up approach, because the capital charge is estimated based on actual internal OR loss data.

Figure 1.4 – OR measurement approaches



Source: Napiontek (2004)

[11] Under AMA expected losses can be covered by provisions and can be excluded from regulatory capital charge
[12] For more info on VaR and CVaR measures see chapter 2
[13] BCBS (2006)

The motivation for banks to move from a less advanced to a more advanced technique is the increased risk sensitivity and in general lower expected capital requirement. Once a bank chooses to move to a more sophisticated approach there is no option to revert back. The relationship between the measurement techniques is shown on figure 1.4.

1.6.1 Basic Indicator Approach

The simplest approach is **The Basic Indicator Approach** (BIA) which estimates the capital charge as a fixed percentage of an average gross income[14] over the last three years. Currently the parameter α is set to 15%. Only those years, when the gross income was positive, are considered.

$$K_{BIA} = [\Sigma(GI_{1\ldots n} \times \alpha)]/n$$ [15]

This approach is fairly simple. It does not require too much calculation. On the other hand, the required capital level is quite high. This approach does not allow the differentiation among different bank activities, that are likely to have different operational risk exposure and sensitivity. Therefore BIA is not suitable for an international bank specific.

There are no specific criteria for use of the BIA. However, it is not very beneficial for a bank to choose this approach and it is expected that all financial institutions will move to a more sophisticated approaches. Internationally active banks are not allowed to use this method at all.

1.6.2 Standardized Approach

The second approach is **The Standardized Approach (SA)**. This technique improves BIA by dividing the bank activities into eight business lines. Each business line is assigned a different percentage, so called beta factor, as a measure of OR exposure. The regulatory capital charge is then computed as a sum of weighted averages of gross income per each of the business lines for the last three years. The particular business lines and beta factors are listed in Table 1.3. The formula for capital requirement under the Standardized Approach is:

$$K_{TSA}=\{\Sigma_{years\ 1\text{-}3} \max[\Sigma(GI_{1\text{-}8} \times \beta_{1\text{-}8}),0]\}/3$$ [16]

It is clear, that this approach might underpin the merits of OR better than BIA. Even though the mean value of beta factors is 15%, the business lines, that are most sensitive to low impact OR events,

[14] Gross income is net interest income plus net non-interest income
[15] BCBS (2006), GI denotes Gross Income
[16] BCBS (2006)

are assigned lower beta factor of 12% – it is namely the retail banking, where most of the OR events occur. On the other hand, the business lines more sensitive to extreme events are assigned a higher beta factor of 18%. Given the higher weights of the business lines with lower beta factors, the overall regulatory capital charge estimation is supposed to be lower than the one calculated using the BIA.

Table 1.3 – The standardized approach business line mapping

Business Lines	Beta Factors
Corporate finance	18%
Trading and sales	18%
Retail banking	12%
Commercial banking	15%
Payment and settlement	18%
Agency services	15%
Asset management	12%
Retail brokerage	12%

Source: BCBS (2006)

But even this approach lacks a risk sensitivity – the empirical loss data are not studied and just preset beta factors are used. Moreover, a perfect correlation is implied between the business lines and so the "results are likely to overestimate actual amount of capital required to capitalize operational risk[17]". Banks that want to use the SA are subject to many qualitative and quantitative requirements set by Basel II[18]. These arguments lead to the conclusion, that even the SA is not suitable for large and internationally active banks.

1.6.3 Advanced Measurement Approach

The most advanced approach for operational risk assessment is called **Advanced Measurement Approach** (AMA). "Under the AMA, the regulatory capital requirement will equal the risk measure generated by the bank's internal operational risk measurement system using the quantitative and qualitative criteria[19]" that are given in Basel II. Use of AMA is subject to a supervisory approval.

Under the AMA the OR data are divided into the seven event type classes. The classes are listed in Table 1.4. The particular AMA technique chosen by a bank should work with a matrix of seven event types and eight business lines.

[17] Chernobai (2007)
[18] Particular qualitative and quantitative requirements are listed in BCBS (2006). Comments on those requirements are beyond the focus of this paper
[19] BCBS (2006)

Table 1.4 – Event type classes

Event type class	Example
Internal fraud	Forgery, Insider trading
External fraud	Robbery, Hacking damage
Employment Practices, Workplace Safety	Discrimination, Termination issues
Clients, Products, Business Practices	Antitrust, Product flaws
Damage to Physical Assets	Natural disaster
Business Disruption, System Failure	Hardware or software failure
Execution, Delivery, Process management	Delivery failure, Model failure

Source: BCBS (2006)

Since the operational risk measurement techniques are still under development, Basel II does not fix any standard technique the for AMA, thus the banks are allowed to develop their own models. Basel II encourages the banks to further develop increasingly risk sensitive OR allocation techniques, that will correspondent with the empirical loss data for the particular bank. The AMA thus provides significant flexibility to banks – on the other hand, regulators are given better control than the AMA techniques used by a particular financial institution.

The criteria for adopting the AMA are more specific and strict than those for the SA. Except for organizational requirements for OR management Basel II requires the banks to adopt techniques that can "reasonably estimate unexpected losses based on the combined use of internal and external loss data, scenario analysis, back testing, Bayesian methods and bank-specific business environment and internal control factors"[20].

Basel II also mentions the use of both internal and external data, capturing of business environment and internal control factors of a bank as well as an allowance of partial risk mitigation using insurance[21]. Basel II differentiates between unexpected losses that are to be covered by the regulatory capital and the expected losses that are to be covered by provisions if it is allowed by national accounting standards.

In the BCBS(2006) document, three approaches were proposed for the AMA[22]. The *Internal Measurement Approach* constructs the matrix of business lines and event types. For each of the 56 cells the capital charge is determined as the product of exposure indicator, probability and loss amount of an event. This approach still assumes perfect correlation among the business lines/event types so the capital charge estimation is likely to be overestimated.

[20] BCBS (2006)
[21] Details can be found in BCBS (2006)
[22] BCBS (2001)

The *Scoreboard Approach* is "highly qualitative approach, under which the banks determine an initial level of operational risk capital at the business line level, and then modify these amounts over time on the basis of scoreboards"[23].

The final approach defined for the AMA is the *Loss Distribution Approach* (LDA). This approach makes use of the exact operational loss frequency and severity distributions analyzing the historic OR data. The operational capital charge is computed as the simple sum of the one-year value-at-risk measure for each business line/event type pair using a high confidence level (99.9%). Even though the LDA is highly risk sensitive and in general provides reasonable estimates of the capital charge, still the LDA tolerates the assumption of perfect correlation.

The approaches mentioned above complement each other – while the LDA is a backward looking quantitative technique, the scoreboard approach focuses on forward-looking qualitative indicators and so all these approaches should be combined together in order to successfully manage OR. In this rigorous thesis we will focus particularly on the LDA approach based on the available OR loss data provided by BANK. This approach will be then combined with scenario analysis method which is described in chapter 4.

1.7 Common OR management and measurement techniques

Even though Basel II is the main driver for implementing OR management and measurement techniques, other reasons to focus on OR exist as well. Even if a financial institution decides to use the BIA or the SA, it can benefit from deploying custom OR management techniques in order to manage economic capital and ensure, that the company will be able to survive some severe operational risk events. The appropriate OR management can help to improve economic results of a financial institution.

The other measurement methods not specifically mentioned in Basel II are also being used by financial institutions[24]. There are four main techniques used to measure OR. The basic features of those techniques are listed in Table 1.5.

The most theoretical measurement approach is the LDA. This method was already explained above and will be discussed in more details in the following chapter. Because of the fact, that the OR management is a relatively new concept, there are not enough historical OR events in internal loss database of a financial institution and thus statistical methods applied on a limited data sample may provide biased or inconsistent results. It is assumed that as the number of events in internal and external databases will grow, the LDA approach will become the prevalent one. Some other disadvantages of the

[23] Chernobai (2007)
[24] BANK internal instructions

LDA exist. The LDA is purely based on historical OR events that might not be the best predictor of the future and might reflect crucial changes in OR exposure of a financial institution with a several years gap. So even if the LDA is the most advanced, objective and theoretical method it is still useful to combine it with other approaches in order to control OR exposure of a financial institution.

Table 1.5: OR measurement techniques

LDA	RCSA	KRI	Scenario analysis
•Application of statistical methods on historical OR events •Quantitative methods	•Inherent and residual risk estimation •Risk mitigation techniques •Subjective qualitative methods	•Risk exposure measurement system •Objective qualitative method	•Based on hypothetical or historical scenario •Assess impact of extreme events •Quantitative method

Source: Author

The second method – the Risk Control Self Assessment (RCSA) - is a subjective qualitative method that constructs a map of inherent OR across all bank processes and departments. The inherent OR is the risk inherent to a process before adopting any control or precautionary mechanisms. Such a map provides a qualitative classification of OR exposure level. The responsible employees within a bank are sent a questionnaire and they are asked to classify severity of inherent OR category and also risk mitigation parameters and control precautionary mechanisms that would decrease overall risk exposure of a financial institution. Those questionnaires are then being aggregated and evaluated. They provide information about residual risk that a bank faces after implementation of the risk mitigation practices. If the residual risk level is high, then a bank should either further improve control mechanisms, decide to take the risk or outsource such a process. The RCSA method provides a useful information about quality of OR management in a bank. However, since the feedback given by a responsible person is subjective, the results might be biased – e.g. managers might have a temptation to overestimate risk exposure of their department just to be safe, should an extreme event happen. The main benefit of the RCSA method is that it can be applied to new banking products and so potential riskiness of those products can be assessed before releasing this product to public. This method is being widely used by financial institutions also because of the lack of sufficiently large OR event database. The RCSA method corresponds with the Scoreboard approach mentioned in the Basel II document.

The third commonly used method is the Key Risk Indicators (KRI) approach. The KRI are defined as quantifiable parameters that are able to indicate a change in the OR profile in a bank process or business line. An example would be a number of new employees, an amount of contractual penalty etc.

Once values of those parameters exceed predefined interval, then the OR exceeded acceptable limits and OR managers should therefore implement some corrective action. KRI method is used as a periodical measurement of OR exposure.

The last but not least method is the Scenario Analysis (SCA). This method can be classified as a stress testing method. A financial institution can obtain valuable results from analyzing scenarios that cover infrequent but severe risks that can have severe impact on bank operations. The other reason is to measure the exposition to plausible risks that has not happened so far and thus are not registered in the internal OR loss database. The theory of the SCA is of major importance in this rigorous thesis and will be further discussed in chapter 4. Application of the SCA, including creation of a custom scenario, is then the subject of chapter 5.

1.7.1 OR mitigation techniques

Once a financial institution determines the specifics of its OR exposure, its managers can take several actions to manage OR. There are five ways to manage OR – they are described in Table 1.6. The aim of a financial institution is to minimize the amount of residual OR. The procedure is to identify the level of inherent risk, implement risk mitigation techniques and then evaluate the level of residual risk. If some risk is not controllable by internal means, then the risk should be transferred either to insurance company,[25] to a 3rd party using outsourcing or such an activity should be limited.

Table 1.6: Risk mitigation techniques

Precautionary arrangements	Business continuity management	Transfer risk to insurance companies	Outsourcing	Taking the risk
• Aimed to avoid OR events • Control directives, standardization of processes, trainings etc. • Best practice for high frequency, low severity events	• Plans for getting back to normal situation, should an extreme OR event happen • Can mitigate impact of severe events • Requires periodical training of those plans	• Complex insurance plans for the case of extreme OR events • Usually used for low frequency, high severity events	• Transfer the activities that are not directly connected with bank activities to a third party	• If long term costs of OR management is higher than potential loss resulting from a specific OR event

Source: Author

[25] Basel II allows insurance coverage up to 20% to be considered for regulatory capital estimates

Chapter 2 - LDA methodology

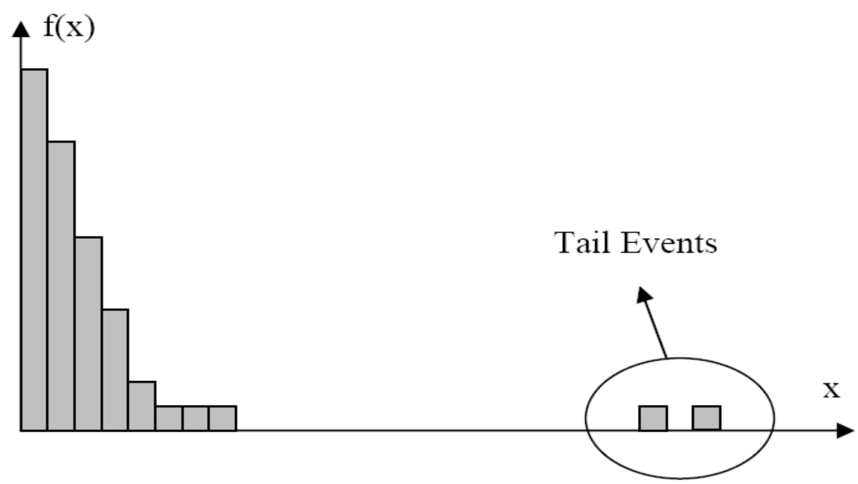
This chapter provides a comprehensive theoretical overview of the LDA methodology, describes the specifics of OR data and explains the statistical distributions and techniques that are used for OR loss severity and frequency distributions modeling and for regulatory and economic capital estimation.

2.1 Specifics of OR data

Empirical evidences prove that OR data have certain specifics, as mentioned above, which distinguish them from credit and market risks data and that causes techniques used for assessment of credit and market risks unsuitable for OR management. From this point of view, OR management has something in common with insurance mathematics and so some of the insurance methodology can be applied to OR assessment – e.g. Extreme Value Theory (EVT).

The OR data are specific by the fact that there exist events that cause very severe losses to a financial institution, but they are not so frequent. For example, there is a very low probability that Czech Republic would be affected by a thousand-year flood – but it did happen in 2002 and this event had negative consequences for all Czech banks. Example of distributions of OR loss severity data is shown on Figure 2.1. The x-axis denotes the loss amount and the y-axis shows the frequency of events for different loss amount levels.

Figure 2.1 – Example of OR severity distribution



Source: Chernobai (2007)

OR data suggest that there exists two kinds of events – the first category consists the losses of high frequency/low severity that are relatively unimportant for a bank and can often be prevented using risk mitigation techniques and covered by provisions. The second category consists of the low frequency/high severity events that are more important for a bank. “Banks must be particularly attentive to these losses as these cause the greatest financial consequences to the institutions.”[26]

If we consider statistical distribution of OR loss severity data the “existing empirical evidence suggest that the general pattern of operational loss data is characterized by high kurtosis, severe right-skewness and a very heavy right tail created by several outlying events.”[27] Distributions fitting such data are called leptokurtic. As will be shown later, the data sample provided by BANK exhibits the same characteristics.

Because of those specifics of OR loss data it is quite clear that the estimation based on elementary statistical distributions would not fit the data very well because the tail events would not be covered by e.g. normal distribution and the predicted loss estimations would be unacceptably low.

2.2 Models for OR measurement

There exist two fundamentally different approaches to develop models for OR :

- The top – down approach
- The bottom-up approach

The first one quantifies operational risk without attempting to identify the events or causes of losses while the second one quantifies operational risk on a microlevel being based on identified internal events. Both approaches should be combined.

The top-down approach group includes, among others, the *Risk indicator models* that rely on a number of OR exposure indicators to track operational risks and the *Scenario Analysis and Stress Testing models* that are “estimated based on the what-if scenarios generated with reference to expert opinion, external data, catastrophic events occurred in other banks, or imaginary high-magnitude events. Experts estimate the expected risk amounts and their associated probabilities of occurrence.”[28] The theory of stress testing will be covered in chapter 4.

The bottom-up approach group includes actuarial type models that will be further discussed in this chapter. Those models have two key components – frequency and loss severity distributions that model historical OR loss data sample. The capital charge is then computed as the value of $VaR_{0.99}$ measure of the one-year aggregate distribution loss. The actuarial models use different type of loss severity distribution and so they are classified to three groups:

- Empirical loss distribution

[26] Chernobai (2007)
[27] Chernobai (2007)
[28] Chernobai (2007)

- Parametric loss distribution
- Extreme value theory

We will focus on each of these categories later on.

Another important feature of OR is the scarcity of available historical data. As of now the banks usually do not have more than five years of loss data in their OR loss data internal databases and even within those five years the data collection methods underwent a significant development – the tail of the distribution cannot be modeled with a sufficient statistical fit, if only very few extreme events exist in such an internal database. So the limited data sample lacks sufficient explanatory power. There were some methods proposed to reduce this limitation:[29]

- Pooling internal and external data samples
- Supplementing actual losses with near-miss losses
- Using scenario analysis and stress tests

There exists a software, working with databases, that contains a sufficient amount of external losses, both empirical and imaginary. Because of the fact that those databases are not publicly available, we will not work with external data and so the overall capital charge might be underestimated.

Because of the scarcity of available historical data and because of confidentiality of those data, some researches even use simulated data which is quite uncommon technique for other risk areas.

Yet another topic should be mentioned – data arrival process. Both frequency and severity distributions of OR data are important and thus banks have to record both the date of occurrence and the amount of loss. There are likely to be observed a seasonal effects in data collection process because of the recording and accounting practices common in financial institutions.

2.3 Frequency distributions

The studies based on empirical data suggest that choice of frequency distribution is not as much important as an appropriate choice of loss severity distribution.[30] The banks should develop a solid mechanism for recording OR data. The most common frequency distributions are the Poisson distribution and the negative binomial distribution. The survey of studies done by Chernobai (2007) suggest that the Poisson distribution will be a reasonable solution for modeling OR data. We will use the Poisson distribution later on for modeling frequency distribution of the data sample provided by BANK.

[29] Chernobai (2007)
[30] De Fontnouvelle (2003)

2.3.1 Poisson distribution

The Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring during a fixed period of time under the condition that these events occur with a known average rate and independently of the time since the last event occurred. If the expected number of events during a unit time interval is λ, then the probability that there are exactly k occurrences is estimated using the following equation:

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!} \text{ , k = 0,1,...}$$

The Poisson process assumes constant mean and is therefore often called a homogeneous Poisson process. Both mean and variance of a Poisson random variable are equal to the intensity rate λ. The MLE parameter estimate of the intensity rate $\hat{\lambda}$ equals to the sample mean.

2.3.2 Negative binomial distribution

The negative binomial distribution is a generalized case of the Poisson distribution, in which the intensity rate λ is no longer constant but is assumed to follow a gamma distribution[31]. The probability mass function of negative binomial distribution is:

$$P(X = k) = \binom{n+k-1}{k} p^k (1-p)^n \text{ , } k = 0, 1, \dots \text{, where } p = \frac{\beta}{1+\beta} \text{ ,}$$

where X is a Poisson distributed random variable with intensity rate λ which is gamma distributed with parameters n and β.

2.4 Loss severity distributions

Loss severity distributions are divided into two main approaches:

- Nonparametric approach
- Parametric approach

The first approach simply uses the density of the data sample. This approach might be used, if it is assumed that the empirical data do not follow any conventional distribution or if the available data set is sufficiently comprehensive.[32] The latter assumption obviously does not hold in case of OR.

[31] See chapter 3.4.2.3 for details on gamma distribution

The second approach tries to find a conventional loss distribution that would best fit the empirical loss data. Such a distribution must be “right-skewed, possibly leptokurtic, and have support on positive values.”[33]

2.4.1 Empirical loss distribution

The empirical loss distribution approach makes two critical assumptions regarding the future loss data:

- Historical data are sufficiently comprehensive
- All past losses are equally likely to reappear in the future, and losses of other magnitudes cannot occur

Empirical distribution density function has the following form:

$$P(X = k) = \frac{number\ of\ losses < x}{total\ number\ of\ losses}$$

The empirical distribution is often used in goodness of fit tests – namely in Q-Q plots.[34]

2.4.2 Parametric loss distribution

Only those parametric distributions, which are able to underpin merits of OR data, should be used. Five key performance measures should be considered while choosing a particular severity distribution:[35]

- Good fit
- Realistic – consider whether realistic capital estimates are generated
- Well-specified – logically consistence with sample data
- Flexible
- Simple

There exist quite many parametric distributions used to model loss severity data. The most common ones are exponential, lognormal, Weibull, gamma and g&h distributions. Those distributions vary in the number of parameters starting from one (exponential) to four (g&h).

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[32] Rosenberg (2004)
[33] Chernobai (2007)
[34] See chapter 2.5 for more details on Q-Q plots
[35] Dutta, Perry (2007)

In the third chapter we will evaluate which distributions best fit the data provided by BANK and will follow up just with those distributions that satisfy the efficiency measures mentioned above. The maximum Likelihood (MLE) or Method of Moments (MoM) methods will be used for estimating parameters of a particular distributions. Based on the inverse distribution function, random variates will be generated using the Monte Carlo (MC) simulation and the results will be compared with the data sample in order to question statistical fit. The following section provides basic characteristics of the selected distributions.

2.4.2.1 Exponential distribution

The exponential distribution is the simplest distribution used to model loss severity data. It has just one scale parameter λ. This distribution has moderately heavy tail that exponentially decays. This does not correspond with the empirical data specifics very well.

The density function has the following form:

$$f(x) = \lambda e^{-\lambda x}, x > 0$$

The MLE estimator for scale parameter is a converse value of the sample mean $\hat{\lambda} = \frac{1}{\bar{x}}$. The random variates can be generated using the inverse transform method by $X = -\frac{1}{\lambda}\log U$,[36] where U is distributed uniformly on interval (0,1).

2.4.2.2 Lognormal distribution

The lognormal distribution consists of two parameters – location μ and scale σ. If the random variable X is lognormal distributed, then the random variable log(X) is normally distributed. This distribution has a fatter tail than the previous one. The density function has the following form:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}}, x > 0$$

The MLE estimators for the parameters are $\hat{\mu} = \frac{1}{n}\sum_{j=1}^{n} \log x_j$ for the location parameter and $\widehat{\sigma^2} = \frac{1}{n}\sum_{j=1}^{n}(\log x_j - \hat{\mu})^2$ for the scale parameter. The random variates can be generated using the inverse transform method by $X = e^{\phi^{-1}(U)\sigma + \mu}$, where ϕ is the standard normal distribution.

[36] Log denotes the natural logarithm

2.4.2.3 Gamma distribution

The gamma distribution is a generalization of an exponential distribution with a density function in the form of:

$$f(x) = x^{\alpha-1}\frac{e^{\frac{-x}{\beta}}}{\beta^{\alpha}\Gamma(\alpha)}, \qquad x, \alpha, \beta > 0,$$

where $\Gamma(\mathrm{a})$ denotes a complete gamma function characterized by $\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}dt$, α is the shape and β is the scale parameter. The MLE estimates for the parameters can only be evaluated numerically because no close form exists[37].

The random variates can be generated using $X = -\frac{1}{\beta}\log(\prod_{j=1}^{\alpha} U_j)$, where U_j are independent uniform random variables.[38]

2.4.2.4 Weibull distribution

The Weibull distribution is also based on the exponential distribution. The Weibull distribution has two parameters - α is the shape parameter and β is the scale parameter - that allow for better tail modeling. The density function has the following form:

$$f(x) = \frac{\alpha}{\beta}(\frac{x}{\beta})^{\alpha-1}e^{-(\frac{x}{\beta})^{\alpha}} , x, \alpha, \beta > 0$$

The MLE estimates can again be computed only numerically. To generate Weibull random variables one can use the following relation $X = \beta(-\log U)^{\frac{1}{\alpha}}$, where U is a uniform random variable from interval (0,1). For $\alpha < 1$ the distribution is very heavy tailed, which makes it suitable for reinsurance models.[39]

2.4.2.5 G&h distribution

The g&h distribution is the most advanced parametric distribution that will be used in this paper. It is "a strictly increasing transformation of the standard normal distribution Z defined by:

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[37] In this case statistical software R will be used

[38] This holds only if α is an integer (Chernobai 2007)

[39] Chernobai (2007)

$$X_{g,h}(Z) = A + \frac{B}{g}(e^{gZ} - 1)e^{\frac{1}{2}hZ^2},$$

where *A*, *B*, *g* and *h* ≥ 0 are the four parameters of the distribution."[40]

The parameters are estimated using the following algorithm. $\hat{A}$ is equal to median of the data sample $X_{0,5}$. The $\hat{g}$ parameter is defined as a median of $g_p = -\left(\frac{1}{Z_p}\right)\log(\frac{X_{1-p} - X_{0,5}}{X_{0,5} - X_p})$ where X_p is the p^{th} percentile of g-distribution and Z_p is the p^{th} of standard normal distribution. The other two parameters are determined using the OLS regression of *log (UHS)* on $Z_p^2/2$, where *UHS* is an upper half spread[41] defined as $UHS = \frac{g(X_{1-p} - X_{0,5})}{e^{-gZ_p} - 1}$. The $\hat{B}$ is estimated as the exponentiated value of the intercept of this regression and the $\hat{h}$ is estimated as the coefficient of that regression.

Random variates are then generated using the form for $X_{g,h}(Z)$. Please note, that some other distributions can be expressed using the g&h distributions for a specific value of its four parameters. Because of the specific tail behavior, the g&h distribution might fit the OR data considerably well.

2.4.3 Extreme value theory

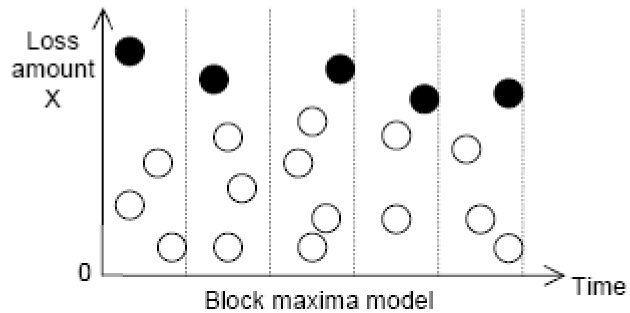
The EVT is a branch of statistics that is focused on the study of extreme phenomena – the rare events that are situated in a tail of a particular probability distribution. Based on the knowledge of OR data distribution, it is assumable that the EVT would be an ideal tool for OR capital charge estimation. There are several techniques for the EVT – each of them uses different method to pick up the low frequency/high severity loss events. They differ in the way how they set a threshold to cut loss data distribution into two parts – the body and the tail. Under the EVT, the body is being modeled using a different method (e.g. empirical sampling) and the tails are being modeled using specific EVT methods. The EVT relies on a sufficiently large data sample. This is not always the case for OR data, therefore the results can be biased.

2.4.3.1 Block maxima method

The block maxima method (BMM) divides data into independent blocks of the same size.[42] The data selection model is shown on figure 2.2. The block maxima model would be useful, if the extreme events were equally distributed over the whole time interval. This is not usually the case.

[40] Dutta, Perry (2007)
[41] This part of spread is relevant for operational risk data because they are right skewed
[42] E.g. a one month period

Figure 2.2 - Block maxima model



Source: Teply, Chalupka (2007)

"For very large extreme loss observation *x,* the limiting distribution of such normalized maxima is the Generalized extreme value (GEV)."[43] The probability density distribution function of GEV distribution has a form of:[44]

$$f(x;\, \mu, \sigma, \xi) = \frac{1}{\sigma}[1 + \xi\left(\frac{x-\mu}{\sigma}\right)]^{-1/\xi - 1} e^{-[1+\xi\left(\frac{x-\mu}{\sigma}\right)]^{-1/\xi}} \text{ for } 1 + \xi\left(\frac{x-\mu}{\sigma}\right) > 0,$$

where x refers to block maxima observations, $\mu \in R$ is the location parameter, $\sigma > 0$ is the scale parameter and ξ is the shape parameter. The GEV is supported under these conditions:

$$x > \mu - \frac{\sigma}{\xi} \text{ if } \xi > 0$$

$$x < \mu - \frac{\sigma}{\xi} \text{ if } \xi < 0$$

$$x \in R \text{ if } \xi = 0$$

The GEV distribution can be divided into three cases based on the value of the shape parameter.[45] The most important case called the Fréchet or the type II extreme value (EV) distribution is for $\xi > 0$. The tail of the Fréchet distribution is slowly varying and thus suitable for modeling high severity OR data. The other two cases (the Gumbel or the type I EV distribution for $\xi = 0$ and the Weibull or the type III EV distribution for $\xi < 0$) are of a less importance for OR data modeling because they do not fit the tail as well as in the Fréchet case.

Teply, Chalupka (2008) further details parameter estimation methods for the GEV distribution using the probability-weighted moments (PWM). A GEV random variate can be simulated using the

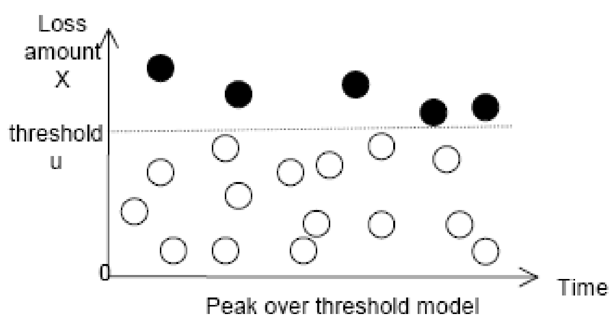
[43] Chernobai (2007)
[44] For more details see Embrechts (2005), Teply, Chalupka (2008), Chernobai (2007)
[45] Teply, Chalupka (2008)

inverse transform method $X = \mu - \sigma(1 - \log U)^{-\xi})/\xi$, where U is distributed uniformly on (0,1) interval.[46]

2.4.3.2 Peak over threshold method

The peak over threshold method (POTM) logic is shown on figure 2.3. This method uses all observations that exceed certain high threshold level. As argued by Embrechts (2005), these models are more frequently used in practice for OR exposure measurement.

Figure 2.3 – Peak over threshold model



Source: Teply, Chalupka (2007)

The limiting distribution for the POTM is the generalized Pareto distribution (GPD) with the probability density function in the form of:

$$f(x;\xi,\mu,\sigma) = \frac{1}{\sigma}(1 + \frac{\xi(x-\mu)}{\sigma})^{(-\frac{1}{\xi}-1)} ,$$

where *x* refers to the data exceeding the threshold, $\mu \in R$ is the location parameter,[47] $\sigma > 0$ is the scale parameter and ξ is the shape parameter. GPD is supported under these conditions:

$$x \geq \mu \text{ if } \xi \geq 0$$

$$\mu \leq x \leq \mu - \frac{\sigma}{\xi} \text{ if } \xi < 0$$

Similarly to the GEV, also the GPD has special cases based on the value of the shape parameter. The most important case from OR modeling point of view is when $\xi > 0$.[48] In this case the GPD has very heavy tails.

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[46] This form holds when $\xi \neq 0$

[47] The location parameter is usually assumed to be 0 which reduces number of parameters to two

The GPD parameters can be again estimated by using either the MLE or the PWM methods – for more details see Teply, Chalupka (2008). A GDP random variate can be simulated by using the inverse transform method in the form of $X = \mu - \sigma\,(1 - U^{-\xi})/\xi$.[49]

A critical task for designing the GPD distribution is to set an appropriate threshold level. This level should be set to be sufficiently high to fit extreme events. But on the other hand, the filtered data sample should not be limited too much in order to provide reasonable statistical evidence. Several approaches to solve this optimization task exist. The most commonly used one relies on "the visual observation of the mean excess plot,"[50] which is defined as the mean of all differences between the values of the data exceeding threshold level u and u. In case of the GPD the empirical mean excess function can be formalized into the following equation:

$$e_n(v) = \frac{\sum_{j=1}^{n}\left(x_j - v\right)I(v < x_j)}{\sum_{j=1}^{n} I(v < x_j)} = \frac{\beta}{1-\xi} + \frac{\xi}{1-\xi}u$$

where v is the value above threshold level u. "Threshold values against mean excess values provide the mean excess plot. If the data supports a GPD model, then this plot should become increasingly linear for higher values of v "[51]. A general practice is then to choose such u for which the mean excess plot is roughly linear.

Several other approaches for choosing the threshold exist – the most simple one is just to define the right tail as five or ten percent of the largest observations.

2.4.3.3 Estimation of the shape parameter ξ for GEV and GPD

Because of a low number of extreme observations that can be used for the POTM or the BMM, the MLE for the shape parameter ξ may by biased.[52] Nonparametric estimation methods can be used as a workaround for this problem: The Hill estimator and the Pickands estimator. But both of these methods require large number of extreme events observations.[53] Even though it might be beneficial to use those estimates together with the PWM method because they all concentrate on the tail events, while MLE method treats all the observation with an equal weight.

The formula for the Hill estimator is given as:

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[48] The GPD in this case is a reparameterized Pareto distribution (Chernobai 2007)

[49] In the case when $\xi \neq 0$
[50] Chernobai (2007)
[51] Based on Teply, Chalupka (2008)
[52] Chernobai (2007)
[53] The Pickands estimator requires even larger number of observations, therefore we will further focus only on the Hill estimator

$$\hat{\xi}^{H} = \frac{1}{k}\sum_{j=1}^{k} \log X_j - \log H_k$$

The optimal value of $\hat{\xi}^{H}$ is then computed using visual observation of the Hill plot – the optimality condition is when the curve of the Hill estimator values becomes flat.

2.5 Goodness of fit tests

The fit of distributions chosen should be tested by a set of goodness of fit tests (GOFT) in order to avoid model risk – risk of choosing bad distribution for the LDA approach. "An underestimated VaR would jeopardize the long-term ability of a bank to maintain a sufficient amount of capital reserves to protect against catastrophic operational losses, while a severely overestimated VaR would limit the amount of funds available for investment."[54] In order to determine optimality of the chosen model, a combination of GOFTs must be used.

There are two ways how to assess the GOFT – either by using in-sample GOFTs or backtesting. Backtesting is the opposite approach to stress testing which questions validity of a chosen model. Since backtesting is beyond the scope of this rigorous thesis, we will focus on in-sample GOFTs further on. They are divided into two classes – visual tests and formal tests.

2.5.1 Visual GOFTs

Visual GOFTs compare empirical and hypothesized distributions by plotting them to a chart and comparing their characteristics. One of the tests is the mean excess plot.

The most commonly used visual test is Quantile-Quantile (QQ) plot which plots empirical data sample quantiles against the quantiles of the distribution that is being tested for fit. If such a distribution fits the data well then the QQ-plot would follow a 45^0 line. The QQ plot is especially important in case of small sample sizes. "The reason is that as the sample size shrinks, formal statistical tests become more likely to fail to reject the fit of a distribution."[55]

[54] Chernobai (2007)
[55] Dutta, Perry (2007)

2.5.2 Formal GOFTs

Formal GOFTs test whether the data sample follows a hypothesized distribution. The null and the alternative hypothesis are stated as:[56]

H_0 : The data sample follows the specified distribution

H_1 : The data sample does not follow the specified distribution

Because of the OR the data specifics, the tests that are based on empirical distribution function[57] are adequate measures for testing the GOF of particular distribution for OR loss severity modeling. The common chi-square based tests rely on large sample size. The output of these tests might be biased because of the limited sample size of the data provided by BANK.

"Empirical distribution function-based tests directly compare the empirical distribution function with the fitted distribution function."[58] The tests belonging to this group are the Kolmogorov-Smirnov test, the Kuiper test and the Anderson-Darling test.

2.5.2.1Kolmogorov-Smirnov test

The Kolmogorov-Smirnov (KS) test is a supremum based test. The test statistic is based on the largest vertical difference between the empirical cumulative distribution function $F_n(x)$ and the cumulative distribution function (CDF) of the fitted distribution $F(x)$. Mathematically the KS statistic is calculated as:

$$KS = \sup_x |F_n(x) - F(x)|$$

The computing formula for KS statistic is expressed as follows:[59]

$$KS = \max\left\{\sup_j \left(\frac{j}{n} - z_{(j)}\right), \sup_j \left(z_{(j)} - \frac{j-1}{n}\right)\right\}$$

where $z_{(j)} = F\left(x_{(j)}\right), j = 1,2,\dots,n;\ x_{(1)} < x_{(2)} < \cdots < x_{(n)}$. The test output is then compared with critical values of the Kolmogorov distribution.

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[56] Chernobai (2007)

[57] An empirical distribution function is a cumulative distribution function that concentrates probability $1/_n$ at each n observations in a sample

[58] Chernobai (2007)

[59] For more details on definition of the computing formula see Chernobai (2007)

The KS statistic has some important drawbacks.[60] It is only usable for continuous distributions. It is more sensitive to deviations in the center rather than to deviations in the tails. And if the fitted distribution contains shape parameter then the critical value of the KS needs to be simulated.

2.5.2.2 Kuiper test

The Kuiper test is a modification of the KS test. It sums the largest differences between the EDF and the fitted CDF and conversely. The test statistic is calculated as:

$$V = (\sup_x \{F_n(x) - F(x)\} + \sup_x \{F(x) - F_n(x)\}$$

The computing formula is:

$$V = \left(\sup_j \left(\frac{j}{n} - z_{(j)}\right) + \sup_j \left(z_{(j)} - \frac{j-1}{n}\right)\right)$$

2.5.2.3 Anderson-Darling test

The Anderson-Darling (AD) test puts more weight on the tails of the distributions and thus it might be more suitable for the purpose of OR data modeling where one is supposed to deal with heavy tails. The quadratic AD test statistics is:

$$AD^2 = n\int_{-\infty}^{\infty} \frac{(F_n(x) - F(x))^2}{F(x)(1-F(x))} dF(x)$$

The computing formula for quadratic form of the AD test is expressed as follows:[61]

$$AD^2 = -n + \frac{1}{n}\sum_{j=1}^{n}(1-2j)\log(z_{(j)}) - \frac{1}{n}\sum_{j=1}^{n}\big(1+2(n-j)\big)\log(1-z_{(j)})$$

Different critical values must be used for each null distribution of the AD test which makes it more sensitive to the null distribution than the KS or the Kuiper tests .

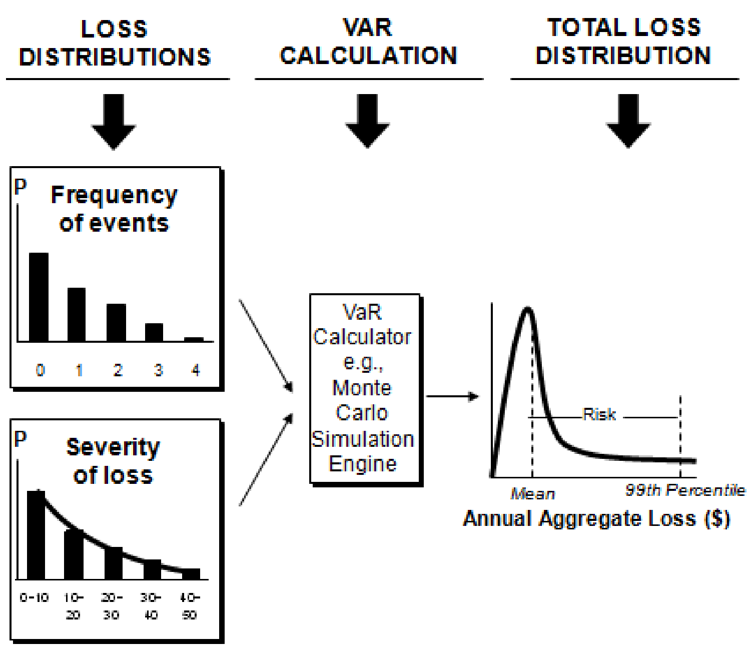
[60] Dutta, Perry (2007)
[61] The quadratic form will be used below

2.6 Aggregate loss distribution and capital charge estimates

Once the frequency and severity loss distributions are evaluated, an aggregated risk exposure of the bank should be estimated. Both types of distributions are to be aggregated to a single model which estimates the total loss over a one-year period. The measure used for the estimation of required capital charge is the Value-at- risk (VaR). "In the context of operational risk, VaR is the total one-year amount of capital that would be sufficient to cover all unexpected losses with a high level of confidence."[62]

2.6.1 Aggregate loss distribution

Figure 2.4: Aggregation of operational loss and frequency distributions



Source: Samad-Khan (2006)

According to Chernobai (2007), simple actuarial type models are used to aggregate the severity and frequency distributions into one single model. This method relies on the following assumptions:

[62] Chernobai (2007).

- Loss amounts are independent from each other and are identically distributed positive random variables
- Conditional on the given total number of loss events *n*, the distribution of the loss amounts is independent from *n*
- The distribution of the total number of loss events does not depend on the loss amounts

The aggregation process is shown on figure 2.4. Mathematical derivation of the aggregate loss distribution function is further discussed in Chernobai (2007).

Due to the fact that the cumulative distribution function is not linear in X nor in N, analytic expressions for the compound distribution function do not exist and thus the function must be evaluated numerically. The most common technique relies on numerical approximation of the compound distribution function using the Monte Carlo simulations of loss scenarios. The algorithm is as follows:[63]

i. Simulate a large number of Poisson random variates and obtain a sequence $n_1, n_2, \dots n_{MC}$ representing scenarios of the total number of loss events in a one-year period.
ii. For each of such scenarios n_k simulate n_k number of loss amounts using a specified loss severity distribution
iii. For each of such scenarios n_k sum the loss amounts obtained in the previous step in order to obtain cumulative one-year losses
iv. Sort the sequence obtained in the last step to obtain the desired aggregate loss distribution

The number of simulated observations differ. A minimum number of simulations that is considered to be statistically representative is 10,000. Due to the very high VaR confidence levels, 50,000 simulations will be used for the more advanced distributions and for scenario analysis in this rigorous thesis.

2.6.2 VaR

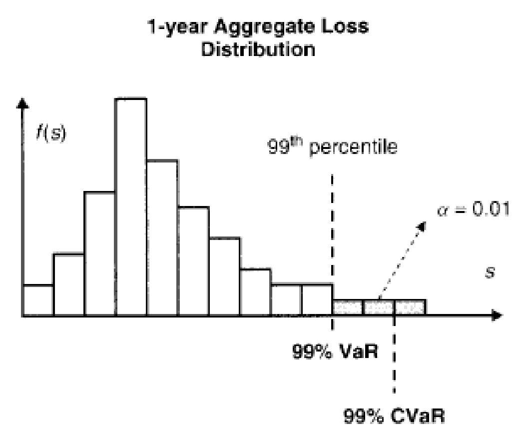
"VaR determines the worst possible loss that may occur with a given confidence level and for a given timeframe."[64] In case of OR VaR is also a measure for capital adequacy requirement defined by Basel II using a high confidence level, such as 99.9% and a one-year period. Thus under the LDA the VaR can be estimated as a corresponding percentile of the aggregated loss distribution.

[63] Chernobai (2007)
[64] Chernobai (2007)

Many empirical studies show that in case of OR only few rare events account for the major part of the VaR.[65] Because of that even while using a high confidence level such as 99.9%, the VaR measures would not be able to account for extreme loses. And so the VaR can be used for estimation of required capital charge but not for estimation of required economic capital.[66] Other pitfalls of the VaR are discussed in Chernobai (2007) – the most important one is that the VaR fails a subadditivity property[67] and so a sum of VaR across different business lined can overestimate the required capital charge.

Because of those facts, alternative risk measures, which are able to account even for extreme events, were designed. The most common one is the Conditional Value at Risk (CVaR) – also known as the Expected Tail Loss or Expected Shortfall. The concept of the VaR and the CVaR is shown on Figure 2.5. "CVaR determines the amount of money one is expected to lose if an event in the right tail of the distribution beyond VaR takes place."[68] In case of OR modeling CVaR is the corresponding percentile of a right tail aggregate loss distribution, where right tail is defined as a 1 - confidence level used for the VaR.

Figure 2.5: Concept of VaR and CVaR



Source: Chernobai (2007)

The CVaR thus better captures tail events and can provide better measure of economic capital. While using EVT methods, the VaR measure can be used for the body and the CVaR for the tail of the aggregate loss distribution. On the other hand in presence of very extreme events the results obtained by the CVaR might be unrealistically high and thus not feasible for OR management. The median tail loss method can be used instead – this method relies on computing median of tail observations that lie beyond the VaR focus.

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65 Ebnother, Vanini, McNeil, Antolinez-Fehr (2001)
66 See chapter 1.4 for more details on economic capital
67 Subadditivity property: $\rho(X_1 - X_2) \leq \rho(X_1) + \rho(X_2)$
68 Chernobai (2007)

Chapter 3 - Empirical data sample analysis

This chapter applies theoretical approaches discussed in the previous chapter to the OR data sample provided by BANK and evaluates the results of regulatory and economic capital estimates as well as appropriateness of a particular statistical distribution for model OR data.

3.1 Data classification and empirical distribution

The data sample provided by BANK consists of 657 loss events registered between 26th March 2003 and 17th April 2007[69]. Several corrections and assumptions were made about the data sample.

3.1.1 Assumptions

The following assumptions about the data sample were made:

- Observations are registered according to loss registration and not OR event occurrence date
- The base currency (EUR) is used
- Exchange rate and inflation impacts are not considered, nominal values in EUR are used
- The data sample is truncated from below, but the threshold is set to a very low value,[70] so we do not use corrections for left truncation bias
- In order to have complete four years sample (4/2003-4/2007), two loss events registered in the year 2001 were excluded from the analysis
- Also seventeen other loss events were excluded from the data sample because they have no registered loss amount
- The impact of insurance is not considered – neither from the time or magnitude points of view – because only the actual loss amount is important for a financial institution
- The business line and event type classification defined by BANK is used – this classification differs from the one set by Basel II in some aspects (four business lines instead of eight)
- Only internal loss data are used and thus estimates provided by using the LDA might be underestimated because no external loss data were employed[71]

[69] Some events were excluded from the original sample – see assumptions section for more details
[70] The threshold level is set to app. EUR 300, which is significantly less than the threshold typically used by foreign banks – according to several papers (Dutta, Perry (2007), Chernobai (2007), de Fountnouvelle (2003), the typical threshold level is set to $10,000 and the left truncation correction methods must be used
[71] It was not technically possible to be provided with an access to some external data database

- While the SA approach uses 15% of gross income as a regulatory capital charge it might be assumed that by using the LDA approach the reasonable interval for capital charge estimates is 5-15%

3.1.2 Data sample statistics

Table 3.1: Data sample statistics – whole sample

Mean	Median	Std. deviation	Skewness	Kurtosis
41,738	3,114	280,538	14	225

Source: BANK data sample

The common statistics for the whole sample (Table 3.1) show a significant difference between the mean and the median and a very high standard deviation which signals a heavy right tail. The same information is given by the skewness measure – its positive value signals that right tail contains more data than in case of normal distribution. The high value of the kurtosis measure signals that the high standard deviation is caused by infrequent extreme observations.

Table 3.2: Data sample statistics – lower 90% of the losses

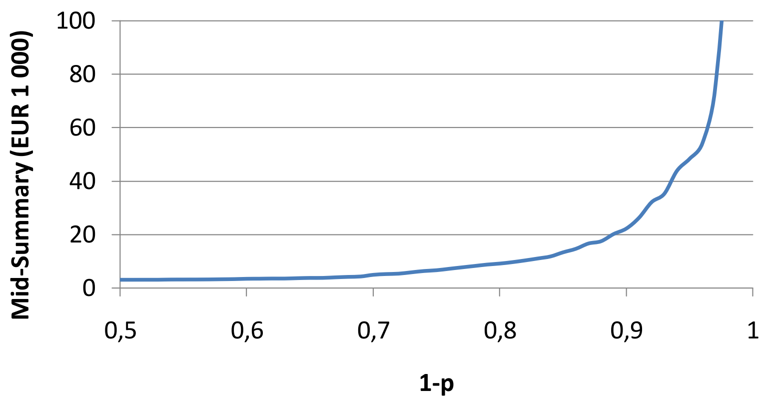
Mean	Median	Std. deviation	Skewness	Kurtosis
6,242	2,143	8,913	2	4

Source: BANK data sample

If 10% of the largest loss events are not considered, then the data sample statistics change significantly and exhibit relative similarities with normal distribution. The skewness and the kurtosis measures decrease to a low value, which means that there are not many extreme observations anymore and that the distribution is a relatively symmetric one. Those 10% of the largest losses account for 97% of variance in the data sample and cause an upward bias in the vital statistics. Chernobai (2007) suggests to use robust statistics methods for the LDA, where extreme observations are excluded from the analysis and body of the distribution is modeled using classical statistical methods. This approach is however beyond the focus of this paper.

The empirical data distribution exhibits a heavy right tail. This fact can be assessed by using one of the explanatory data analysis methods suggested by Dutta, Perry (2007). They suggest to measure skewness of the data sample by using a mid-summary approach. If the data are symmetric, then the mid-summary, defined as $mid_p = \frac{1}{2}(X_p + X_{1-p})$, equals to the median of the data for all percentiles *p.*[72] This "a plot of mid_p vs. *1-p* is useful in determining whether there is a systematic skewness of the data."[73]

Figure 3.1: Plot of mid-summaries to percentiles



Source: Author based on BANK data sample

The mid-summaries curve is plotted on Figure 3.1. It significantly diverges from the median for percentiles higher than 90%. This proves that the data sample exhibits severe right skewness. So those distributions which are able to sufficiently model such a right tail must be used for loss severity distribution modeling in order to achieve appropriate goodness of fit.

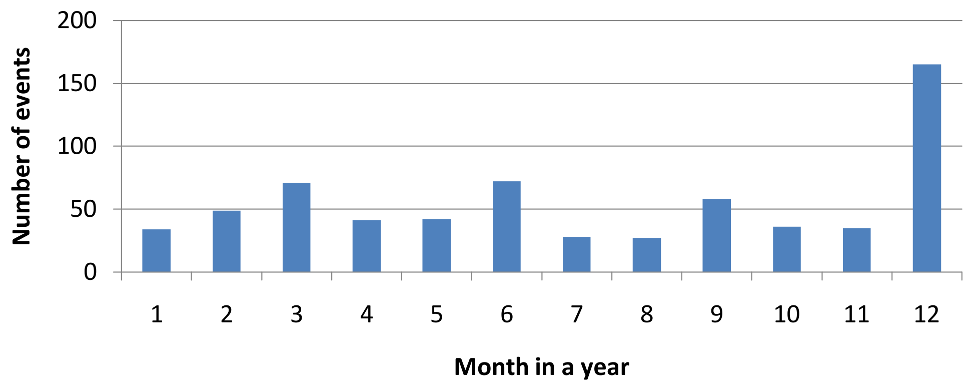
3.1.3 Empirical loss frequency

As mentioned above, the data sample consists of full four years of observations from 4/2003 to 4/2007.

The observations from a month in a year are summed over the four-year period as shown on Figure 3.2. The observations are not distributed evenly over the months in a year because of the data collection mechanism. This issue can be easily solved by modeling each month separately using a Poisson distribution, so the overall results should not be impacted by this inequality.

[72] X_p and X_{1-p} denotes *p-th* and *(1-p)-th* percentiles of the data sample
[73] Dutta, Perry (2007)

Figure 3.2: Distribution of loss observations over months in a year (number/month)



Source: BANK data sample

3.1.4 Business lines & event types classification

Even though the biggest attention is devoted to the data analysis and the capital estimation process on the institutional level, it should be also mentioned that differences between the empirical loss observations exist on both business line and event type levels, both in the number of events and the average loss in a particular category.

3.1.5 Economic results of BANK

According to the implementary regulations set by ČNB in the ammendment to EU Directive 2006/48/EC, the gross income does not include operating expenses, provisions and also outsourcing services.

The result (in EUR millions) is then used as a benchmark to compare capital adequacy ratios estimated using loss severity distributions listed in chapter 2.[74]

[74] Frequency distribution is modeled using the Poisson process

3.2 LDA results on a company level

The procedure described in chapter 2.6.1 was used to aggregate the loss frequency and the loss severity distributions. The Monte Carlo simulation method is used for the parameter estimation as well as for the aggregation function. In those cases, where a generally lower fit is expected, 10,000 trials are simulated. For distributions where a better fit is expected, 50,000 trials are simulated in order to achieve higher statistical relevance while working with such a high confidence level of the VaR and the CVaR measures. Some of the estimated parameters were computed by using the R-software[75] or the EasyFitXL Excel plug-in[76] or by using the direct computation method in MS Excel. The MLE or the MOM method is used. Three VaR measures are provided – 99% VaR, 99.9% VaR, which is used as a required capital charge estimate defined by Basel II, and also 99% CVaR measure which is used as an economic capital estimate. The capital estimates are then related to gross income of BANK. The loss distributions that do not fit the data sample very well will not be further used during the stress testing.

The fit of the distributions to the sample data is evaluated by using the QQ plot, the KS and the AD tests.[77] The test values are compared with the critical values for the KS and the AD tests which are listed in Table 3.3. If the test statistics are higher than the critical value, then the null hypothesis that the particular distribution is able to model the OR data sample cannot be rejected.

Table 3.3: Critical values for KS and AD GOFT for the sample[78]

Test	0.1	0.05	0.01
KS	0.0477	0.0529	0.0635
AD	1.9286	2.5018	3.9074

Source: EasyFitXL

3.2.1 Loss frequency distributions

Since the loss events are not distributed equally over months in a year, each month is being modeled separately by the Poisson distribution. The MLE estimate of the intensity rate $\hat{\lambda}$ is set as an average number of events per particular month divided by number of years. In case of BANK data

[75] http://www.r-project.org/
[76] http://www.mathwave.com/products/easyfit.html
[77] Except for the g&h distribution, where just QQ plot is used
[78] Note, that the critical values depend on the distribution used and also on the number of observations in a sample, so the numbers listed apply just to the specific case of the full BANK data sample

sample, the value of the estimated parameter corresponds to the one fourth of the number of events per month in a year on Figure 3.2.

3.2.2 Loss severity empirical sampling

The Empirical sampling method (ESM) relies on the VaR estimation based on a generation of large number of loss severity events that are randomly drawn from an empirical data sample using the MC method. Since the algorithm for choosing a particular item from a data sample tends to put the equal probability to both high and low severity events it might be rightly assumed that the $VaR_{0.999}$ measure would be quite low for this method. And thus a bank would not be able to use this method by its own in order to compute reasonable regulatory capital estimates. On the other hand, this method is very useful to modeling the body of the empirical data sample, while the tail is being modeled using a different, EVT type model – more details about this approach is given below. Because of the fact that the ESM does not use any statistical distribution, the GOFTs cannot be done.

Two ESMs were used – the first one considers the whole data sample, while the second one considers just those observations with the loss amount higher than EUR 1,000. The reason for the second method was already mentioned above – most of the banks use higher loss amount thresholds than BANK does and so the very low severity events might decrease regulatory capital estimate which is intended to focus especially on high impact events.

Table 3.4: ESM estimates

ESM type	Whole sample	EUR 1,000+ sample
Cap. estimate/Gross income	**2.31%**	**2.92%**

Source: Author

The capital estimates provided by both ESMs comply with the assumptions stated above – the overall result is quite low and thus the ESM itself might not be considered as a credible AMA method that would be approved by a regulator, because it underestimates required capital charge.

3.2.3 Loss severity parametric distributions

The distributions discussed in chapter 2.4 are used to fit the empirical loss severity data. The test statistics, parameter estimates and the economic and regulatory capital estimates are commented and evaluated from both economical and statistical points of view.

3.2.3.1 Exponential distribution

The exponential distribution exhibits a very poor fit to the data sample that has a heavy right tail. Both the KS and the AD null hypothesis are rejected on any conventional confidence level. The QQ plot suggests the very same conclusion – empirical quantiles are of a significantly larger magnitude than estimated ones, what result in a downward slope of the QQ plot curve.

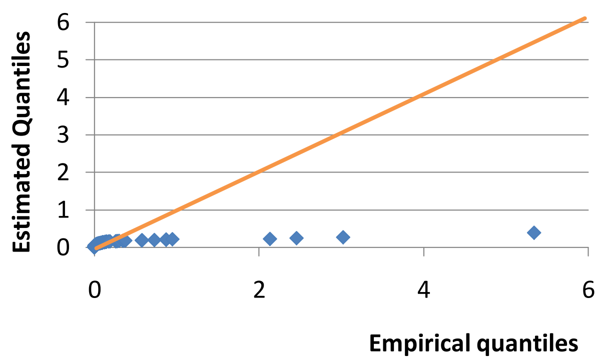
Table 3.5 Exponential distribution statistics

$\hat{\lambda}$	2.3959E-5
KS	0.52544
AD^2	572.13
Cap. estimate/Gross income	**0.95%**

Source: Author

There is not a significant difference between the $VaR_{0.99}$ and the $CVaR_{0.99}$ measures, thus the right tail is not very long under the exponential distribution. The capital estimate obtained by exponential distribution gives very low results, which means that the exponential distribution is not a suitable distribution for the stress testing purposes.

Figure 3.3: QQ plot for exponential distribution[79]



Source: Author

[79]The quantiles are measured in millions of Euros

3.2.3.2 Lognormal distribution

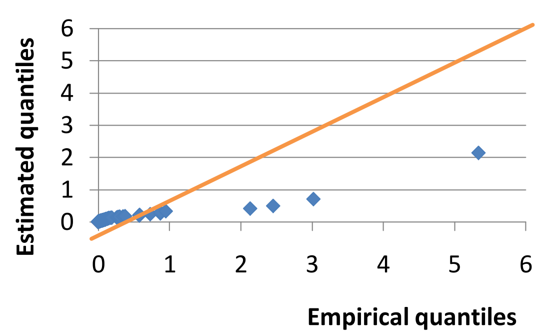
The lognormal distribution provides better fit than the exponential distribution. The KS test can still be rejected on any conventional confidence level up to the 99.99% but the test statistics value is significantly lower than in case of the exponential distribution and thus the tail is fitted little better, however still not sufficiently.

Table 3.6: Lognormal distribution statistics

$\hat{\boldsymbol{\mu}}$	8.2246
$\hat{\sigma}$	1.8208
KS	0.0872
AD^2	9.2307
Cap. estimate/Gross income	**1.49%**

Source: Author

Figure 3.4: QQ plot for lognormal distribution



Source: Author

The QQ plot for the lognormal distribution suggests a moderate fit to the empirical distribution. The difference between the $VaR_{0.99}$ and the $CVaR_{0.99}$ measures is significant, so that the right tail of the estimated distribution is moderately long. But the estimated regulatory capital value is just 1.5% of the gross income which is unrealistically low. Because of these reasons, the lognormal distribution does not seem to be suitable for the modeling OR data and thus it will not be used for stress testing purposes.

3.2.3.3 Gamma distribution

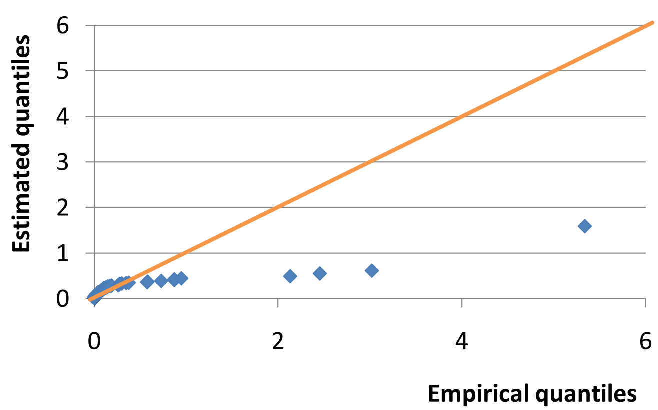
The gamma distribution provides results that are similar to the Weibull and to the exponential distribution, including the CVaR and the VaR measures. All GOFTs can be rejected on any conventional significance level which suggests that the gamma distribution is not fitting the empirical data very well neither.

Table 3.7: Gamma distribution statistics

$\hat{\alpha}$	0.2879
$\hat{\beta}$	144,980
KS	0.2162
AD^2	66.1629
Cap. estimate/Gross income	**1.12%**

Source: Author

Figure 3.5: QQ plot for gamma distribution



Source: Author

The QQ plot for gamma distributions shows just the same – the empirical quantiles are much higher than the estimated ones. The capital estimate is very low, meaning that the gamma distribution would underestimate required capital charge and so it is not suitable to model the BANK data sample.

3.2.3.4 Weibull distribution

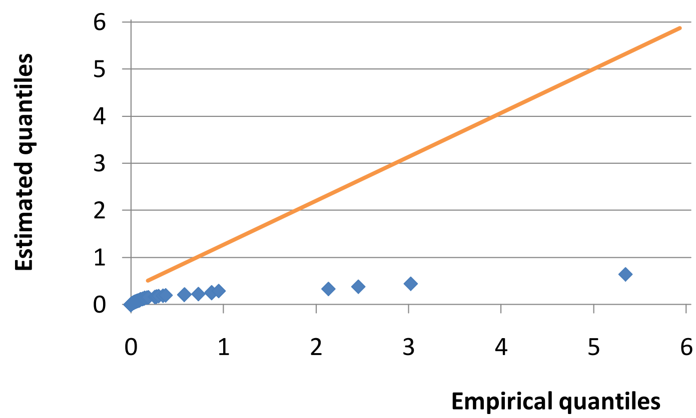
The Weibull distribution provides poor fit to the empirical data. Even though the KS and the AD^2 statistics get better results than in case of exponential distribution, still the null hypothesis can be rejected on any significance level – so the Weibull distribution does not match the empirical distribution. The VaR and the CVaR measures are very low and the overall capital estimate is the lowest one among all of the loss severity distributions used for the analysis.

Table 3.8: Weibull distribution statistics

$\hat{\alpha}$	0.4682
$\hat{\beta}$	9,839
KS	0.1784
AD^2	23.0713
Cap. estimate/Gross income	**0.69%**

Source: Author

Figure 3.6: QQ plot for Weibull distribution



Source: Author

The QQ plot for the Weibull distribution exhibits poor fit to the empirical distribution and the capital estimate is very low, which signals that the Weibull distribution is not suitable to model the right tail with a sufficient statistical significance.

3.2.3.5 G&h distribution

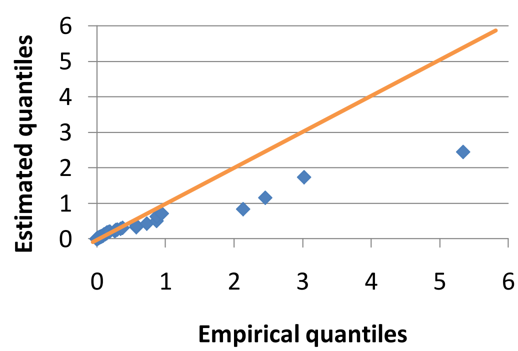
Table 3.9: G&h distribution statistics

$\hat{A}$	3,113
$\hat{B}$	6,197
$\hat{g}$	2.0747
$\hat{h}$	0.0545
Cap. estimate/Gross income	**4.43%**

Source: Author

Due to the fact that the g&h CDF is unknown,[80] the KS and the AD GOFTs were not computed. The statistical fit is measured just visually by using the QQ plot. The g&h distribution provides the most reasonable results in terms of capital estimate among all parametric distributions. Even though the capital charge is still underestimated (4.4%) as is clear from the QQ plot, the fit to the empirical data is much better than for all other parametric distributions. The value of the $VaR_{0.99}$ measure is more than five times lower than the value of $CVaR_{0.99}$ measure, supposed the g&h is able to model the very heavy right tail and can fit even the extreme loss events. The economic capital estimate which the bank is required to hold in order to stay in business, might be significantly higher than the capital charge required by Basel II, which corresponds with our assumptions.

Figure 3.7: QQ plot for g&h distribution



Source: Author

[80] Headrick (2008)

Because of these features the g&h distribution can be considered as sufficiently suitable for modeling the OR data and will be used further in this paper. The fact that the capital estimate is lower than the expected one can be explained by lack of extreme observations in the limited data sample. If there were more loss events in the data sample – e.g. external ones – the g&h distribution would then provide even better result. Therefore so this distribution might be evaluated as a very suitable one to model the OR data.

3.2.4 Loss severity EVT approach

The two EVT approaches described in chapter 2.4 were used to model the right tail of the data sample – the BMM and the POTM. The body was modeled using the ESM explained above – because the extreme events were not considered for the ESM, this method sounds reasonable for the EVT approach. The following section contains the parameter estimates and the test statistics of the distribution used for the tail modeling and also for the overall capital estimates. The critical values of the KS and the AD tests change as the number of observations in the sample changes and so p-values of those tests are provided. The EasyFitXL software was used for the parameter estimation and the MC method. In order to achieve better statistical relevance in a situation with limited empirical sample, 50,000 trials were used for the MC. The loss aggregation algorithm described in section 2.6.1 was slightly modified, because the random events were generated by two methods – the ESM and the EVT. The total one-year loss is then a sum of the loss events generated by the ESM and the loss events generated by the EVT.

It should be noted, that the KS and the AD test null hypothesis significantly differs from the one for parametric distributions because only the tail of the data is now modeled and not the whole data sample. Thus better statistical fit can be expected because of the lower number of observations and relatively lower variance, skewness and kurtosis measures. So the higher the threshold for the POTM or the longer the period for BMM method, the lower number of observations are used for particular EVT method – which has an advantage of generally better statistical fit but, on the other hand, it might bring some bias. The conclusion is that GOFT results are not comparable with each other because the null hypothesis differs. The parameter estimates depend very heavily on the number of observations in the tail of the data. Large discrepancies exists if the original loss database is merged with the loss events defined in a scenario. This issue will be discussed in more details in chapter 5.

3.2.4.1 EVT – Block maxima method

Two approaches for the BMM were used – they differ in the length of period used for the block maxima. The first one uses a month period, thus the total number of observations shrinks to 48. But still large discrepancies exist among these observations, which might lead to the overestimation of the capital charge. And so the second approach considers just the maximum observations per quarter. The number of observations further reduces to 16 but the differences between those observations are not as significant as in the previous case. Furthermore increasing the period would impose a bias on the parameter estimates, because the number of observations would be too low.

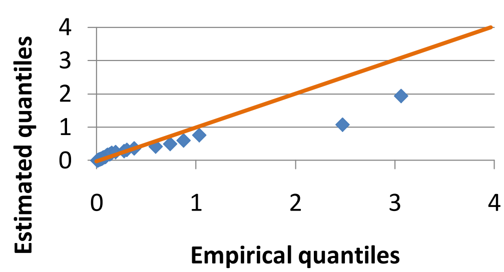
In both cases the GEV distribution was used to fit the extreme events. The shape parameter $\hat{\xi}$ estimates were positive in both cases, which corresponds with the assumptions that the OR data will fit the Fréchet type GEV distribution. Because of the relatively limited sample, advanced methods for the shape parameter estimation might be employed – namely the Hill's estimator discussed above.

Table 3.10: Block maxima – Max per month GEV statistics[81]

$\hat{\mu}$	47,202
$\hat{\sigma}$	78,284
$\hat{\xi}$	0.7746
KS	0.1288 (0.37)
AD^2	1.1055 (0.25)
Cap. estimate/Gross income	**14.95%**

Source: Author

Figure 3.8: QQ plot for BMM – Max per month



Source: Author

The Max each month method provides much better fit to the tail of the data than any of the parametric distributions discussed so far. Neither the KS nor the AD GOFTs can be rejected even on 80%

[81] The numbers in brackets denote the p-value of particular test. The same applies to all tables with the EVT type statistics

significance level. And so those GOFTs together with the QQ plot[82] prove, that the GEV distribution fits the empirical distribution of the tail quite well – but it might not fit the body of the data, so the test statistics are not comparable with the parametric distributions approach.

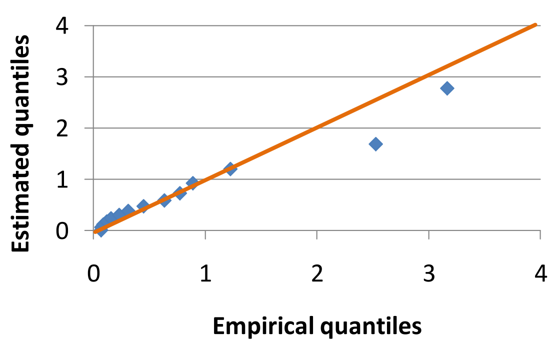
The data generated by the Max per month BMM exhibit a very heavy right tail – as can be seen from the tremendous difference between the $VaR_{0.99}$ and the $CVaR_{0.99}$ measures. The capital estimate provided by this method is quite high and almost equals the capital charge level used by the SA approach. So even though the method is suitable from the statistical point of view, it will not be used further in this paper, because it overestimates the capital charge[83].

Table 3.11: Block maxima – Max per quarter GEV statistics

$\hat{\mu}$	235,394
$\hat{\sigma}$	376,549
$\hat{\xi}$	0.5775
KS	0.1674 (0.7)
AD^2	0.5663 (0.6)
Cap. estimate/Gross income	**11.53%**

Source: Author

Figure 3.9: QQ plot for BMM – Max per quarter



Source: Author

The second BMM approach, which uses the maximum per year quarters, provide even better fit to the data because both the KS and the AD tests have the p-value higher than 50%. The QQ plot

[82] Note that the 100th percentile for estimated distribution is not plotted to the figure due to its extreme value
[83] These results comply with the conclusions of Dutta, Perry (2007)

suggests the same and thus the tail of the BANK data sample is modeled almost perfectly. The results can, however, be impacted by the low number of observations in the data sample (only 16).

The random data generated by the Max quarter GEV method exhibit the very heavy right tail as well, but the differences between the $VaR_{0.99}$ and the $CVaR_{0.99}$ are not that big as in the previous case. Also the capital estimate measure provides significantly better result than in the previous case which favors this method over the previous one. Therefore the BMM – Max quarter will be used further on during the stress testing procedures.

3.2.4.2 EVT – Peak over threshold method

Three approaches for the POTM were used. The first two set the threshold to 5% and 10%, respectively, of the observations with the highest loss amount from the data sample. The third one is more advanced. Under this approach the optimal value of the threshold is estimated by using the mean excess plot.

All approaches work with a different number of the highest observations – namely with 32, 65 and 61 observations, respectively. It might be assumed that the results provided by the second and the third approach will be very similar, while the results provided by the first approach will differ. In all cases the tail of the data sample was modeled by the GPD distribution and the body of the data sample was modeled by the ESM. The EasyFitXL was used for the parameter estimation and the random number generation. Other, more advanced, approaches could be used for the parameter estimation. For more details see Teply, Chalupka (2008). In all three cases the estimate of shape parameter $\hat{\xi}$ was positive, therefore the GPD distribution exhibits very heavy tails.

Table 3.:12 POTM – max 5% GPD statistics

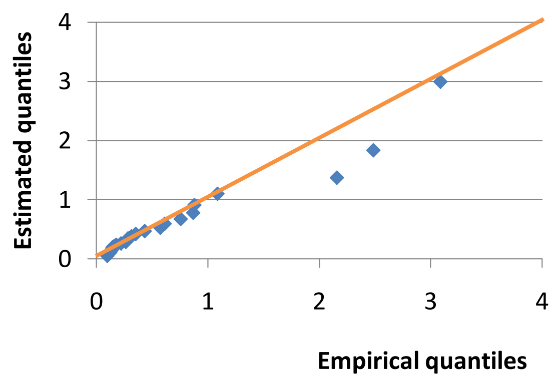
$\hat{\mu}$	45,785
$\hat{\sigma}$	254,863
$\hat{\xi}$	0.5919
KS	0.1844 (0.20)
AD^2	1.0667 (0.25)
Cap. estimate/Gross income	**9.32%**

Source: Author

The first method works with the 32 highest observations from the tail and 625 observations from the body of the data sample. There should be enough observations for the GPD in order to provide statistically relevant estimates and, on the other hand, not too many observations to impose a bias because of including unreasonable high amount of observations from the body of the data sample. None of the statistical tests can be rejected at conventional significance levels and so the GPD is able to model

5% of the highest observations very well. The QQ plot shows the same result - the estimated and empirical quantiles are very similar.

Figure 3.10: QQ plot for POTM – max 5%



Source: Author

The $VaR_{0.99}$ and the $CVaR_{0.99}$ measures again significantly differ from each other, this approach is thus able to model heavy tails. The capital estimate computed by this method is very reasonable and in accordance with the assumptions stated above. Even though the length of the tail was set using a up-down method, the results are very promising. This method will be used for scenario analysis.

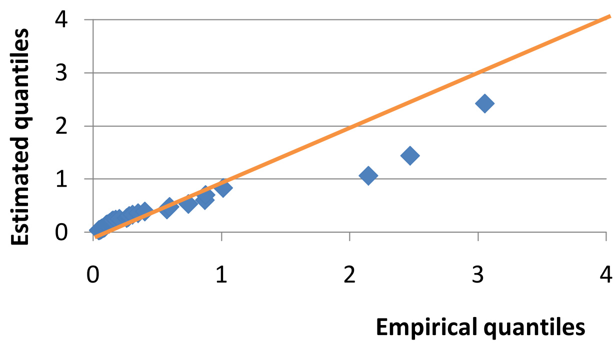
Table 3.13: POTM – max 10% GPD statistics

$\hat{\boldsymbol{\mu}}$	34,018
$\hat{\sigma}$	90,159
$\hat{\xi}$	0.72804
KS	0.1264 (0.23)
AD^2	1.7131 (0.17)
Cap. estimate/Gross income	**16.12%**

Source: Author

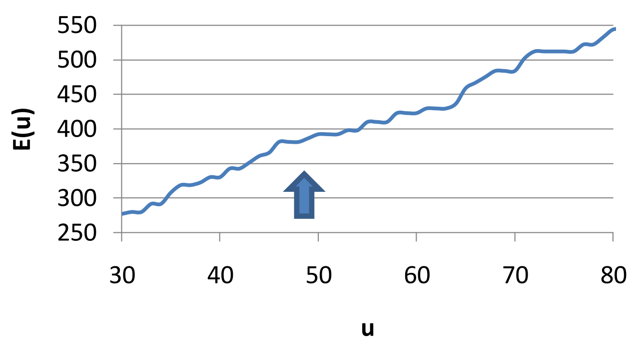
The second POTM works with the 65 highest observations from the data sample. This number is probably too high, because it contains also observations with a lower loss than EUR 50,000. Due to this fact, the results might be worse than in the previous case and the GPD might not fit the empirical tail distribution that well. This assumption is confirmed by both the KS and the AD tests which have a lower p-value and also by the visual inspection of the QQ plot for which the estimated quantiles do not correspond with the empirical ones that well. The capital estimate computed by this method is approximatelly the same as the one computed by the BMM – Max quarter method. Accroding to the assumptions stated above the capital estimate of 16% seems to be too high – especially since only internal loss data were considered.

Figure 3.11: QQ plot for POTM – max 10%



Source: Author

Figure 3.12: Mean excess plot for POTM



Source: Author

Table 3.14: POTM – threshold GPD statistics

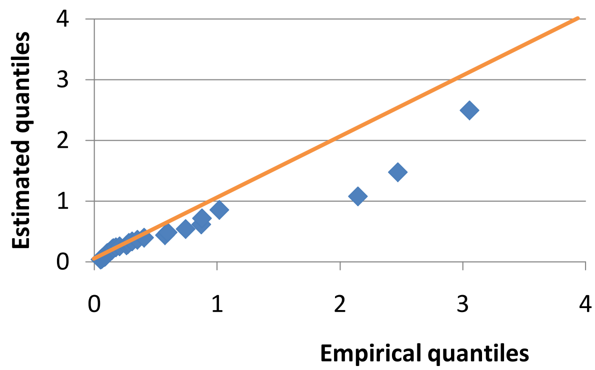
$\hat{\mu}$	36,948
$\hat{\sigma}$	97,041
$\hat{\xi}$	0.7223
KS	0.1335 (0.21)
AD^2	1.8575 (0.16)
Cap. estimate/Gross income	**13.58%**

Source: Author

The last POTM method sets the threshold level using a bottom-up approach by a visual inspection of the mean excess plot. In case of the BANK data sample the mean excess plot, defined as a relation between the threshold level *u* and mean excess of observations above the threshold level *E(u)*, becomes flat for the threshold of EUR 49,000 – as shown on Figure 3.12 where the blue arrow points to the optimal threshold level. The number of observations above the threshold level is 61, which is about the same as for the previous POTM – Max 10% method, thus one can expect approximately the same results. The parameter estimates are very similar to the previous case – the GOFTs p-values are even lower and the QQ plot suggest a lower fit between the empirical and the GPD distribution.

This method again provided quite high capital estimate measure, given the fact that the original historical loss database was not merged with the external loss data. This method thus also provides overestimated capital charge and therefore the method is not very suitable to model the OR data.

Figure 3.13: QQ plot for POTM - threshold



Source: Author

3.2.5 Comparison of the results for regulatory capital charge

The frequency distribution was not evaluated from the statistical point of view and it was not compared with an alternative distribution neither. It has been suggested by several researchers that frequency distribution is not of such an importance as severity distribution and that the Poisson distribution should be suitable for modeling the OR loss frequency.

The ESMs provide quite low capital estimates, because the data sampling algorithm puts the same weight on the body and on the tail of the data sample. And thus ESM is not suitable to model the whole data sample – on the other hand, it is a suitable distribution in combination with an EVT method.

Table 3.15: ESM results

Method	*Capital estimate*
ESS – Whole sample	2.31%
ESM – 1,000+ sample	2.92%

Source: Author

As concluded above, all parametric loss severity except the g&h distribution provided a poor fit to the OR data and therefore were not considered suitable for the estimation of the regulatory and the economic capital required to cover potential losses. The Table 3.16 lists the statistical fit and capital estimates as the percentage of the gross income for the parametric distributions used above. The conclusion is that parametric distribution underestimate economic and regulatory capital. The only suitable parametric distribution is the g&h distribution. It can be assumed, that the g&h distribution would be suitable even for the stress testing measures, where the tail of the data is enriched by extreme events defined in a particular scenario. For more details see chapter 5.

The results might differ, if the robust statistic methods or different threshold levels are used for the parametric distributions. However, this issue is beyond the focus of this rigorous thesis.

Table 3.16: Comparison of parametric distribution used for LDA

Distribution	*Statistical fit*	*Capital estimate*
Weibull	Very poor	0.69%
Exponential	Very poor	0.95%
Gamma	Poor	1.12%
Lognormal	Poor	1.49%
G&h	Moderate	4.43%

Source: Author

At the first sight, the better results were achieved by modeling the body and the tail of the data sample using different distributions – i.e. by using the EVT approach. Since the extreme events were separated from the body of the data sample, the ESM can better fit the body, because the skewness, kurtosis and variance measures are much lower. So the ESM MC method can provide reasonable estimates based on the body of the data sample. On the other hand, the tail can be modeled by using EVT methods that can better capture effects of the extreme events. The crucial task for the EVT is to split the body and the tail of the data sample. Five different methods for this task were used. The best results were achieved by using the BMM – max per quarter and the POTM – max 5% methods. These methods will be used further on for stress testing purposes.

It should be noted that even if the number of MC trials was set to 50,000 the simulation results and the statistical measures such as the $CVaR_{0.99}$ might be biased –it might thus be beneficial to increase the number of MC trials even to 100,000 or more. There are also several methods for the EVT GEV and GPD parameters estimation. Due to the low number of tail events, the PWM method for parameter estimation might provide biased results and so more advanced methods can be used – e.g. the Hill's or the Pickand's estimator. Also the VaR measures were computed based on the aggregated one-year losses distribution and not by using a computational formula, which might result in slightly different estimates. But even though these objections exist, it can be assumed that the EVT results are statistically relevant and that they can be used as estimates of the capital charge under Basel II even though the overall results are often overestimated. The limited data sample makes the GEV and the GPD parameter estimation method very sensitive to each new observation that is added to the data sample.

Table 3.17: EVT method statistics

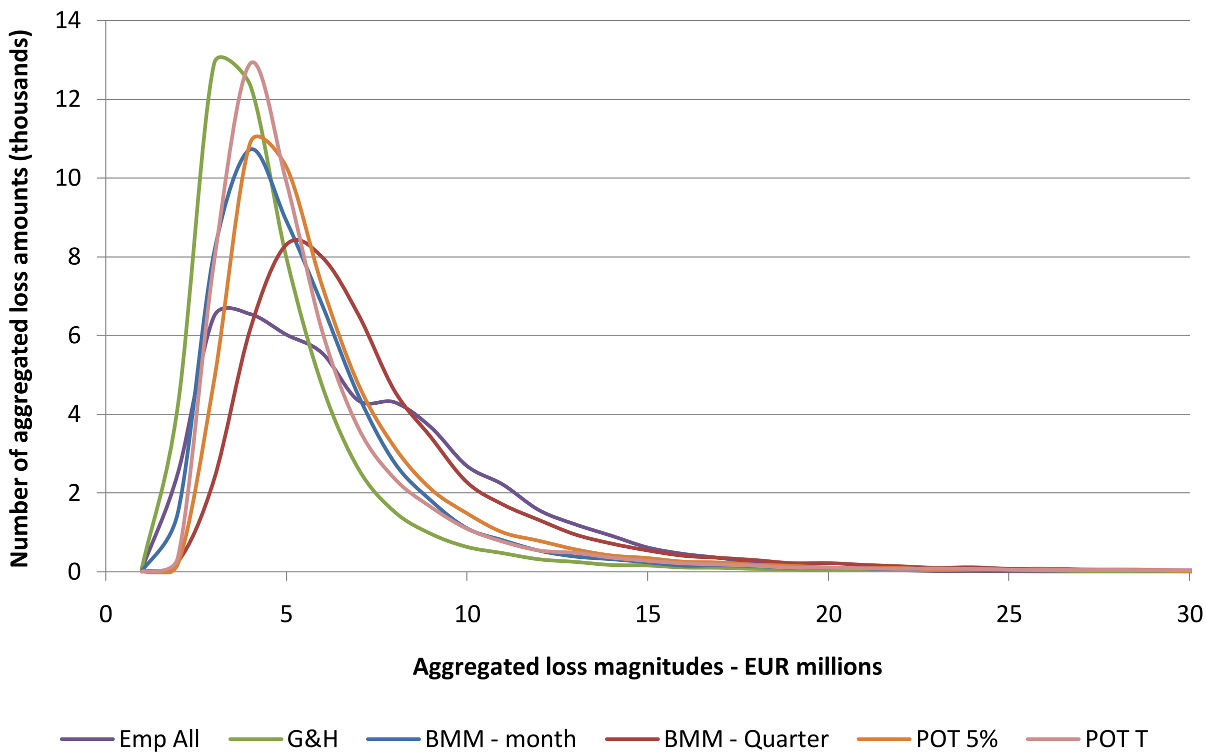
Method	*Distr. Body*	*Distr. Tail*	*Stat. fit*	*Cap. estimate*
BMM – max per month	Empirical	GEV	Good	14.95%
BMM – max per quarter	Empirical	GEV	Very good	11.53%
POTM – max 5%	Empirical	GPD	Very good	9.32%
POTM – max 10%	Empirical	GPD	Good	13.58%
POTM – threshold	Empirical	GPD	Good	16.12%

Source: Author

The figure 3.14 provides a comparison of the methods used for modeling the OR data[84] by plotting the one-year aggregated loss amounts on x-axis (in EUR millions) and the number of aggregated loss amount trials (out of 50,000 trials), in particular loss intervals on the y-axis. Note that the x-axis is trimmed and just the first 30 items are shown. For some of the distributions the x-axis range up to EUR 4,000M, which signals extremely heavy tails! But for the main part of the one-year loss distribution, the distribution provided by the ESM is best copied by the BMM – Max quarter approach. The g&h and the POTM – max threshold methods have higher concentration of aggregated one-year losses in the lower loss magnitudes intervals. But all of the approaches used on figure 3.14 provide a reasonable fit to the empirical one-year loss amounts. The EVT methods have much longer tails than the ESM or the parametric distribution approaches. This is also signaled by on the $CVaR_{0.99}$ measure.

[84] Only those methods that provide reasonable fit to the data were used

Figure 3.14: Aggregated loss magnitudes distribution



Source: Author

The conclusion for the LDA approach on the institution level is that only the g&h, the BMM – Max quarter and the POTM – Max 5% methods seem to be suitable for modeling the OR data for Basel II purposes and thus these methods will be used for the stress testing purposes in chapter 5.

3.2.6 Comparison of results for economic capital estimates

As already explained above, even such a high VaR confidence level as 99.9%, that is used to estimate the regulatory capital charge required by Basel II might not be sufficient enough to cover very extreme events. The economic capital that would cover such extreme events can thus exceed the regulatory capital. We used an alternative method which estimates the economic capital as the $CVaR_{0.99}$

measure.[85] The measure value is very high for most of the distributions, especially for the EVT methods. Thus some other parameter than the gross income of BANK should be used as a proxy, because the bank might not be able to cover these extreme losses just from the gross income – it might be needed to employ the total bank equity or even some portion of assets. The Table 3.18 provides the comparison of the CVaR measures for particular distributions as a proportion of the gross income and the total equity of BANK.

Table 3.18: Comparison of the economic capital measures

Distribution	$CVaR_{0.99}$/Gross Income	$CVaR_{0.99}$/Total Equity
Empirical	2.85%	1.51%
G&H	12.66%	6.71%
BMM – Month	91.62%	48.58%
BMM – Quarter	75.06%	39.80%
POT – 5%	35.81%	18.89%
POT - T	43.72%	23.18%

Source: Author

As it is clear from the Table 3.18, the ESM provides just a very low economic capital estimate. Even if a higher CVaR confidence level, such as $CVaR_{0,999}$, is used, the economic capital estimates given by the ESM are quite low. The other methods, especially the EVT methods, exhibit very high values of $CVaR_{0.99}$ measures. In case of the BMM – Max month method the measure value almost equals the gross income. But even by considering these very extreme events, BANK is able to survive and cover the losses from the gross income. Just if the $CVaR_{0.999}$ measure is employed, the value of this measure would be too high to be covered from the total equity. The probability that this situation happens is as low as $0.01 * 0.001 = 0.00001$. Such scenario might be e.g. a civil war or a loss as big as the one imposed by recent unauthorized trading case that happened in Société Générale (SG) in 2008[86]. In such case other bank funds would have to be used – such as new share issuance or a bond capital.

The conclusion is that while employing the very high significance levels for EVT methods, the economic capital is being overestimated. But even despite of the overestimation, it was shown that BANK would be able to survive those very severe OR events. Because of the high sensitivity of the EVT methods, it can be concluded that the g&h method provides more reasonable estimates than any EVT method used.

[85] It should be noted that estimates of the $CVaR_{0,99}$ measure might not be relevant enough, because the number of the MC simulations was just 50,000 and so the $CVaR_{0,99}$ measure employed only 500 simulations, which means that the result can be biased

[86] See chapter 5 for more details on this event.

3.3 LDA results per business lines & event types

Because of the fact that the loss events differ per business line – there are business lines with low frequency but high severity events such as BANK commercial and, on the other hand, there are business lines with high frequency/low severity events such as BANK trading - the data distribution for each business line can be modeled separately in order to better fit the empirical data. However, this task is beyond the focus of this rigorous thesis and might be considered as a suggestion for the future research.

Chapter 4 - Stress testing and scenario analysis

Because of the fact that the LDA approach is a historical one – the capital charge is estimated based on historical loss events - alternative methods for the OR management were developed, as discussed in chapter 1.6. One of those methods is the scenario analysis or, generally, the stress testing. This method is supposed to examine whether a financial institution would be able to undergo exceptional risk losses. Stress testing can be defined as "the examination of the potential effects on a bank's financial condition of a set of specified changes in risk factors, corresponding to exceptional but plausible events."[87] An alternative definition is given by Chernobai (2007): "Stress tests are intended to explore the effect of low probability events that lie outside the predictive capacity of any statistical model on VaR" or the one used by the BIS Committee on the Global Financial System, where stress testing is defined as "a generic term describing various techniques used by financial firms to gauge their potential vulnerability to exceptional, extreme or simply unexpected but plausible events."[88]

The stress testing should be used as a complementary approach to the VaR based LDA approach in order to ensure that a bank would be able to cover the losses even if a bank faces more severe risk events – such as the worst-case scenario. Stress testing should be used as a complementary method to the Scoreboard and the KRI approaches, where impact of a particular risk factor can be stress tested. Illová (2005) emphasizes two reasons for using stress testing as a complement to the VaR approach – the first is that VaR is based only on historical data which might not serve as the best predictor of the future and the second reason is that VaR relies on the normal distribution of risk factors.[89] Thus stress testing should be used in order to ensure that a financial institution would be able to handle such risk events that did not happen in the past and to ensure that the regulatory capital estimate is robust enough to outlast these extreme events. Stress tests are used primarily for understanding the risk profile of a firm. However, they are being used for capital allocation as well. "Whenever the stress tests reveal some weakness, management must take steps to manage the identified risks. One solution could be to set aside enough capital to absorb potential large losses. Too often, however, this amount will be cripplingly large, reducing the return on capital."[90] The goal is to ensure that the institution can survive such events, while keeping return on capital on a reasonably level.

The concept of stress testing might seem straightforward in comparison to a complex statistical LDA method. Just few theoretical or empirical papers devoted purely to stress testing theory exist – probably due to the fact, that the tests are usually based on empirical and often confidential data and so

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[87] Illová (2005)
[88] BCFGS (2000)
[89] This reason does not usually apply to OR modeling
[90] Jorion (2007)

they are not available to the public. The field of stress testing in the area of OR are still being developed, so there is a high flexibility of choosing specific methods that would best fit the financial institution. On the other hand, stress testing methods are not comparable with each other. Neither the applications of the same stress tests to different financial institutions are comparable with each other, because the results are always bound to the specific risk profile of a financial institution. The stress testing methods are thus subjective. Adopting bad assumptions or using irrelevant scenarios will lead to irrelevant losses.

Even though Basel II specifically refers to the stress testing methods for credit and market risks as one of the seven conditions financial institutions are required to satisfy in order to use internal models, the stress testing methods for the OR are mentioned just merely in paragraph No. 675 which states that "a bank must use scenario analysis of expert opinion in conjunction with external data to evaluate its exposure to high severity events."[91] The Incorporation of scenario analysis is also required by regulators.

As for the other methods, the stress testing was originally applied for market and credit risks and it was modified to fit the OR management later on. The models for credit and market risk stress testing are discussed in Illová (2005). There are none widely accepted theoretical models for the OR stress testing – however some literature exists on scenario analysis. An interesting application is provided in paper by Kuhn, Neu (2004) - they apply methods used for collective phenomena and phase transition on modeling the OR. They use stress tests in order to asses metastability of networks of interacting processes with different dependencies and failure rates. However, this advanced application of stress testing is beyond the focus of this rigorous thesis.

Since the stress tests often define events with a very low probability of occurrence[92], the results become difficult to interpret and it is not clear which actions should be taken by the management in order to mitigate the risks. Quite often the results of stress tests appear unacceptably large and they are just ignored and dismissed as irrelevant. As Jorion (2007) states, a financial institution is not supposed to handle all the possible states of the world like a widespread nuclear war. The central banks are supposed to support financial institutions in case of systematic crisis. Other actions besides increasing economic capital were proposed – such as insurance for the extreme events, business restructuralization in order to achieve better diversification and lower exposure to the risks in question or developing a special plan for corrective actions once a scenario starts happening. Such actions should ensure that the institution would be able to survive, should the scenario happen while keeping a reasonable level of economic capital. However, "a general way" to interpret results of stress tests does not exist, because the results are highly subjective and they depend on the choice of the test methods and the scenarios. This differs stress testing from the LDA approach, which provides more objective estimates of the economic capital.

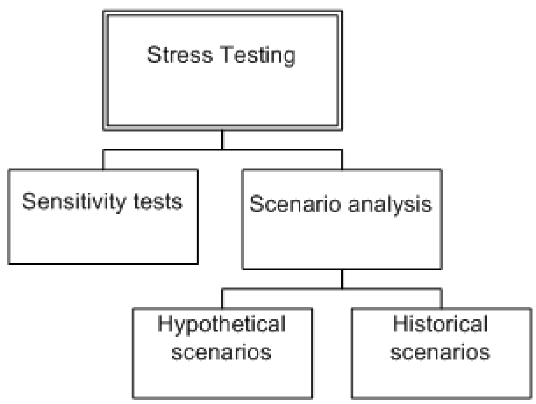
[91] BCBS (2006), differences between stress testing and scenario analysis will be discussed in the following chapter
[92] Or in the case of hypothetical scenarios this probability is defined very merely

The alternative approach to scenario analysis is a robust statistics method discussed in Chernobai (2007). While scenario analysis method adds new extreme observations to the data sample, the robust methods exclude the most severe losses and focus on modeling the data sample that is trimmed from the right side.

4.1 Stress testing methods

Two stress testing methods are mentioned in the literature – they differ in the number of risk factors used. The classification of the stress test methods is shown on figure 4.1:

Figure 4.1: Stress testing methods



Source: Author

4.1.1 Sensitivity tests

Sensitivity tests are single-factor tests that analyze impact of a single risk factor on the risk exposure of a bank. These tests are quite easy to perform but are not very useful to assess overall risk robustness of the institution. They are more or less a statistical concept that modifies some parameter of the LDA model. Sensitivity tests are not very often used in case of OR and they are also not being mentioned in the literature on the OR management. They are more useful in case of market or credit risk, where theoretical models are more developed. In case of the OR it is very difficult to isolate the impact of a particular risk factor. The only LDA factors are the frequency and the severity of the OR events, while for the market risk one can study impact of changes in volatility, correlation, equity index or yield curves.

4.1.2 Scenario analysis

"Scenario analysis consists of evaluating the portfolio under various extreme but probable states of world."[93] The purpose of defining a scenario is to answer the question: "How a bank would suffer under such scenario?" The advantage of this method over the LDA is that linking the loss to a specific event is more intuitive for the OR managers.

The scenarios can be divided into two groups based on the type of event they define. The first group uses historical events like 9/11 terrorist attack or unauthorized trading that happened in SG in 2007. Risk managers study a potential impact of those events on the financial institution. The second group, which is more widely used in practice, uses hypothetical scenarios. The scenarios are based on some plausible risk events that have not happened yet, but a non-zero probability of their occurrence exists. A scenario can also be based on an analysis of a new product a bank is going to implement.

Risk managers should build a scenario based on the risk factors that have a direct impact on the financial institution – such as an electricity failure or internet banking security breach. And so the scenario construction is usually a top-down method, when at first the stress event is defined and then the impact of this event is being discussed and evaluated. The bottom-up approach is also possible – under this approach "each department sets its own scenario through comprehensive analysis of operational errors and accidents, internal control condition, business environment, etc."[94] In both cases OR managers are required to estimate the frequency and severity of a potential event.

A typical scenario consists of the description of a complex state of the world that would impose an extreme risk event on a financial institution, including probabilities and frequencies of occurrence of the particular state of the world, business activities impacted by the event and maximum internal and external loss amounts generated by occurrence of such event, possible mitigation techniques including insurance against such an extreme event. Even though such a scenario claims to be realistic, it is not possible to comprise all possible risk factors and features – however, risk managers are trying to define the scenarios, so that they correspond to the reality as much as possible. Such scenarios, that are likely to severely impact the financial institution, should be used. It is clear that "the generation of relevant scenarios is a time-consuming process that requires quantitative skills as well as good economic understanding of the factors"[95] financial institution are exposed to. However, some adverse motivations for risk managers may exist – either to underreport the risk exposure in order to produce "nice results" or to use a too large number of scenarios just to be sure any likely state of the world is covered. Internal control methods should be used in order to achieve the optimal stress testing method application.

[93] Jorion (2007)
[94] Arai (2006)
[95] Jorion (2007)

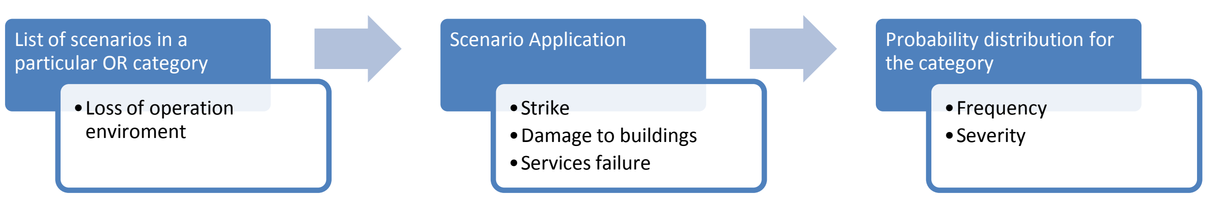
Rosengren (2006) describes a usual way how a scenario is created. Based on his experience from the US environment a scenario is defined on a workshop that "brings together managers to have a structured scenario discussion."[96] External consultants are often asked to cross-check results of a scenario. It is crucial to question, which type of data is used and also which scenario types are used. It is also very important for the management to ensure that the risk managers are sufficiently motivated to accurately evaluate frequency and severity of the losses. All the scenarios should be back-tested and validated by other internal or external experts in order to minimize the subjectivity of a scenario.

If a financial institution is able to implement appropriate scenario analysis policy, then this method provides a comprehensive overview of the impact of plausible events. It provides creditable quantitative basis, where the results can be further aggregated with the LDA methods on a company or business line levels and impact of such a scenario on the economic capital and the regulatory capital charge can be estimated. Concrete scenarios, together with its integration process with the method based on historical loss data, will be described and analyzed in the following chapter.

4.1.2.1 Scenario analysis implementation in BANK

BANK combines all four main approaches for the OR management – including the scenario analysis. Each scenario requires special ad hoc analysis by experts from the departments the scenario is connected with. Those experts evaluate all possible consequences of such a scenario from the financial losses and frequency point of view. The experts are obliged to take into account all available empirical data in order to evaluate, what is the proportion and magnitude of internal and external losses. They also must consider external (legal, business) and internal (internal controls) environment specifics to determine, what is the frequency of such a scenario and how to control potential losses. The aim of using scenarios is, as explained above, "to get an overview about low frequency events that might have severe impact on BANK." BANK is using a set of scenarios that will be used for stress testing.

Figure 4.2: Scenario analysis process

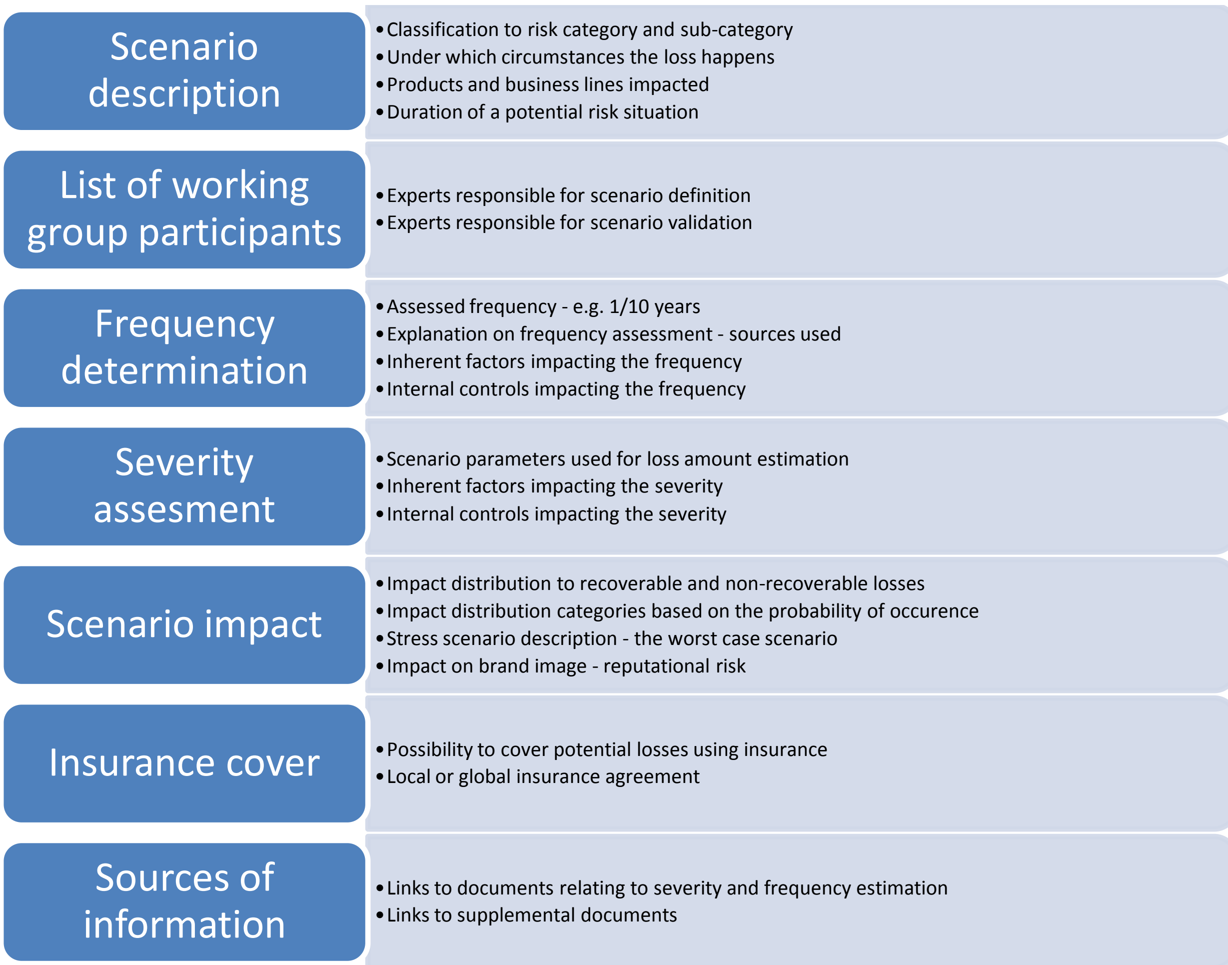


Source: BANK

[96] Rosengren (2006)

There exists a unified form used for a scenario definition. The process of scenario definition is shown on figure 4.3.

Figure 5.3: Scenario creation method – BANK form



Source: BANK

Chapter 5 - Applied scenario analysis

This chapter describes scenarios used for the stress testing of the LDA results. Several simple scenarios based on historical events were used. The impact of the scenarios developed by BANK will be evaluated as well as the custom scenarios will be defined and their impact will be analyzed.

5.1 Sensitivity test application

Since the model for OR considers just the severity and frequency factors, it is not very beneficial to perform a sensitivity analysis based on changes in one of these parameters. It might be assumed that a change in a parameter would cause a corresponding change in the regulatory capital estimate. It is of a higher importance to stress test the VaR measures by employing particular scenarios.

5.2 Scenario analysis application

The scenario analysis method was used to examine the impact of plausible events on the regulatory capital and the economic capital estimates and also on the business continuity plan of BANK. Two main approaches were used to aggregate losses generated by the scenarios with the database of historical events. The first one uses a set of the worst-case losses defined by a particular scenario and aggregates these losses to the historical loss data sample. The second approach calculates an average loss given by probability distribution of the loss amounts defined by a particular scenario and aggregates those average losses to the historical loss data sample. In both cases the statistical distributions mentioned above, the g&h, the POT – Max 5% and the BMM – Max quarter, were used for the severity distribution of the aggregated loss sample. The Poisson distribution was used for the loss frequency. Both distribution were then aggregated and the economic and regulatory capital estimates were computed by using the VaR and the CVaR measures.

In case of the g&h loss severity distribution, the aggregation method of losses generated by the scenarios with the historical data sample is straightforward, because the additional losses are simply added to the database. However, in the of the EVT approaches, where the body and the tail of the distribution are being modeled by using a different statistical distribution, the aggregation algorithm is more complicated, because all of the losses generated by the scenarios belong to the tail of the aggregated database distribution and thus it directly impacts the EVT methods. The most complicated case is the BMM, for which an additional algorithm had to be used in order to randomly distribute the additional losses over the whole four-year period.

Multiple scenarios are combined together. It should be noted, that the probability, that the worst-case joint scenario combination would occur to BANK during the observed four-year period, is very low. Further details are provided below.

The alternative approach to combine the forward-looking scenario analysis method and the backward-looking LDA method was implemented in several papers. Rosengren (2006) suggests using the scenario analysis as a separate method for the regulatory capital estimation. Mignola, Ugoccioni (2007) suggest to use more advanced method, where “the risk measures from historical losses and scenario analysis are finally compound using Bayesian methods.”[97] Guidici (2003) further details the process of using the Bayesian model that allows “integration between qualitative and quantitative data”[98] by constructing a Bayesian network. However, this method is beyond the focus of this paper. As mentioned above, a less sophisticated aggregation method was used, where the LDA approach is applied to a database consisting of both the historical and scenario generated loss events.

In section 5.3 scenarios are combined into several packages, denoted by test IDs. Both the worst-case and the average losses are considered. We merge those losses with the original loss database and then estimate the VaR and the CVaR regulatory and economic capital estimates using the aggregation method described in chapter 2.6. The tests differ by the number of scenarios they use – at first all scenarios defined by BANK as well as the custom scenarios are considered.[99] Then the number of scenarios considered is gradually decreased. Separate tests are run for the custom scenarios and for more frequent BANK scenarios.

5.2.1 Scenarios defined by BANK

The losses generated by the BANK scenarios were merged with the historical loss events from the years 2003-2007 using the method explained above. The scenarios will be denoted by IDs further on. The worst-case losses together with the probability that such worst-case loss happens as well as average losses, which have been computed as a probability weighted average of the loss estimates defined by the particular scenario, are considered for the stress testing purposes. In most cases the average loss is many times lower than the worst-case loss, which means, that even if an event defined in a scenario happens, usually the loss severity is sustainable. The average loss amounts for all of the scenarios are

[97] Mignola, Ugoccioni (2007)
[98] Guidici (2003)
[99] “Custom” denotes a scenario defined for the purpose of this rigorous thesis

comparable to the other tail losses from the original historical data sample, thus these eight losses just enrich the original tail of the data. On the other hand, the magnitudes of the worst case losses are apparently higher than the magnitude of the highest historical losses and so the right tail of such merged sample is much heavier than for the case of the historical data sample.

A financial institution should evaluate, whether it would be able to survive even the most extreme cases of the scenario it assesses or not. The probability that all the worst-case events defined by the joint scenario combination occur during the observed period limits to zero. But if this happens, then it can be rightly expected that the impact on a financial institution would be very severe. In some cases a financial institution might even default, because it would not be able to cover those extreme losses.

5.2.2 Custom scenarios

The following sections list custom scenarios defined by the author. Three different historical scenarios were defined – the first one is based on an unauthorized trading, the second one is based on an external fraud and the third one is based on process management failure loss even types. All of those scenarios are based on concrete historical events – the loss amounts are rescaled to fit the size of BANK.

On the other hand, the custom hypothetical scenario ID12 is more complex and copies the format of scenarios defined by BANK. The frequency and the loss severity distribution of the scenario is estimated based on the stated assumptions and empirical data.

5.2.2.1 Historical scenarios

Table 5.1: Historical scenarios list – loss amounts in EUR 1,000s

ID	*Scenario name*	*Estimated loss*
9	**Unauthorized trading – Kerviel**	112,000
10	**Process management failure – software loss**	7,300
11	**External fraud – Prochazka**	21,180

Source: Author

The historical scenarios are based on three operational risk events that happened in the recent years. Table 5.1 lists these historical scenarios including the loss amount occurred. Since the historical events will not reoccur in the future, we have not estimated the frequency of those events, nor the probability distribution of the loss. The estimated losses are quite high and thus they will be treated as the worst-case losses. The historical scenarios will not be used for tests based on average losses.

The first historical scenario ID9 is based on a recent unauthorized trading of Jerome Kerviel in SG.[100] The trader was concluding hidden deals on security trading, hoping to reverse losses from the past trading. At the end of his actions the loss amounted to EUR 5,000M. This event was the most severe OR loss event ever happened – the loss amount was four times higher than the loss caused by Nick Leeson to the Barings bank in 1995. The loss amount was rescaled to fit the BANK size.

The second historical scenario ID10 is based on a recent process management failure – software loss event that happened directly to BANK. The interbank transaction fees were rounded to a slightly lower value (1/100 of 1 CZK). Given the huge number of transactions and the four years duration of this incorrect system settings, the total loss to BANK amounted to CZK 200M which is about EUR 7.3M.

Finally, the last historical scenario ID11 is based on a recent external fraud – robbery event.[101] Frantisek Prochazka, an employee of a security agency, stole cash in the amount of CZK 564M. More than half of these money belonged to CSOB, a competitor of BANK. This event was the biggest robbery event ever happened in the Czech Republic. The loss amount in EUR is 21.18M.

5.2.2.2 Complex hypothetical scenario

A scenario of BANK employee strike that would hit all the regions is considered. This type of scenario was chosen because of the fact, that historical evidence of similar events exists. Such scenario belongs to the Employment Practices and Workplace safety Basel II event category.

The frequency of the scenario assessment was estimated to 1 per 40 years based on the following facts: according to the historical data there were several bank employee strikes in recent years - two of them in India, one in Canada TD Trust bank, one in Greece national bank.[102] The duration of the strike ranged from 1 day to 1 week. It is assumed that the frequency of strikes would be quite low in the region of Central Europe. The strikes in Central Europe are more likely to happen in the public sector (schools, railways) or in manufacturing firms (car industry). Usually the duration of such strike is limited only to several hours.

[100] http://en.wikipedia.org/wiki/J%C3%A9r%C3%B4me_Kerviel
[101] http://zpravy.idnes.cz/zlodej-pul-miliardy-prochazka-vzal-penize-i-csob-f6f-/krimi.asp?c=A071210_215034_krimi_zra
[102] http://www.northernlife.ca/News/LocalNews/2007/06-18-07-bankstrike.asp?NLStory=06-18-07-bankstrike
http://www.thehindubusinessline.com/2004/02/07/stories/2004020701281701.htm
http://www.iht.com/articles/2008/03/04/business/4drachmafw.php

The other important feature of a strike is its extent – a strike can range from one branch to a country wide strike. A strike can also hit either one particular company or it can be an industry-wide. The reasons why employees decide to go on include a disagreement with changes in law or working conditions, pension funds, compensations or organizational changes etc. Several internal controls that may contribute to reduce the frequency of such event might be considered – e.g. a more professional human resource management, a competitive employee compensation scheme and an improved communication with trade unions.

For the purpose of this rigorous thesis it was assumed, that the employee from all regions would go on strike. Such a scenario has a very low probability, but if it occurred it would have significant negative impact on the bank. Optionally it can be assumed that the strike would hit just some regions – the frequency and severity estimates would have to be modified for such a limited extent scenario. The severity impact of the scenario depends on two factors – the extent and the duration of the strike. The extent was set to the whole country. The duration is assumed to range from one hour strike to five business days strike and the probability for each class was estimated according to the assumptions stated above.

A strike was assumed to cause four types of losses – the direct loss of lost revenue from branches was estimated based on the list of BANK branches and their revenues per day. The second source of loss are the costs connected with expenses on substitute employees that would be hired in order to maintain the bank critical operations. These costs increase with the duration of the strike and were estimated as a certain percentage of the direct loss of revenue (up to 40% for the most severe case). The third and the most severe type of loss is the loss of clients that was estimated as a proportion of yearly revenue from branches. While a 1 hour strike is not considered to have impact on customer satisfaction, in case of a whole week strike up to 5% of customers might decide to move to competitors. The last but not least type of the loss are the costs connected with commercial disputes. The losses were estimated based on interest costs from non-realized transactions and estimated amount of dispute penalties. The number of transactions is based on the number of transactions via internet banking and the proportion of customers that do not use internet banking. After taking into account all the assumed loss sources, the total loss was computed. The loss amounts and the probability distribution are listed in Table 5.2 – the loss amount grows as the duration of the strike increases.

Table 5.2: Strike duration probability distribution

Probability	***Duration***	***Estimated loss (EUR)***
70%	1 hour	138,515
25%	1 day	3,750,446
4%	2-4 days	9,056,450
1%	5 days	20,890,382

Source: Author

The worst case scenario is a strike that lasts five days. Under this case the loss amount reaches EUR 20M. Such strike is considered to cause significant harm to BANK – especially by the loss of 5% customers. Such scenario would also have very negative impact on the brand image and the banks reputation would be severely harmed. The average loss size is significantly lower though – EUR 1,6M.

Table 5.3: Custom hypothetical scenario details – loss amounts in EUR 1,000s

ID	Scenario name	Worst-case loss	Average loss
12	**Employee strike – whole state**	20,890	1,606

Source: Author

Table 5.3 provides an overview of the custom hypothetical scenario. This scenario will be used further during the stress testing and its potential effects in combination with other scenarios will be considered.

5.3 Tests – Scenario combinations and loss aggregation estimates

In total six tests were run. The aim was to analyze, whether BANK would be able to handle particular combinations of events defined in the scenarios employed for a particular test combination. The impact of such joint scenario was evaluated. Scenarios were denoted by the IDs assigned above. For the hypothetical scenarios (ID 1-8 and 12) two level of loss were considered – the worst-case level and the average level. For historical scenarios (ID9-11) only the worst-case loss amount is defined. The dates of event occurrence was set by a random number generator – this information is important for the BMM method.

Three statistical approaches were used to model the merged data sample – the g&h, the EVT – BMM Max Quarter and the EVT – POT 5% methods. The estimates provided by each of these methods were evaluated in the following sections. The goodness of fit statistics are not provided at this point, because they were already evaluated in chapter 3. Each of the scenario defines an extreme event that is expected to have significant impact on the capital estimates – and so the loss events belong to the tail of the data sample.

5.3.1 Test I – All scenarios

In the first test all scenarios were considered and all the defined losses were merged with the original empirical OR event database. The sample size grew by 12 more observations to the total of 669. All of the custom events belong to the top 5% of the data sample implying that all of these observations will be used for the POTM 5% method. For the BMM – Max quarter method not all custom events will be used, because some of them were generated to happen in the same quarter.

For the worst-case stress test, 4 out of 12 defined loss amounts are significantly higher than the rest of the data (all of them are higher than EUR 20M) – namely in the case of scenarios 5, 9, 11 and 12. So the tail of the data is much more heavier than in case of the original data sample – the highest loss amount is about 20 times higher than the highest loss amount from the original sample. The sum of all custom losses (EUR 278M) is 10 times higher than the sum of all 657 empirical losses. Because of these facts it can be rightly assumed that the capital estimates would be very high and that the bank might not be able to allocate such high level of capital.

Table 5.4: Test I – Worst-case scenarios capital estimates[103]

Distribution	Regulatory capital	Economic capital
G&H	91%	236%
POT 5%	207%	532%
BMM – Quarter	245%	443%

Source: Author

Table 5.4 clearly shows that the EVT based methods provide very high regulatory and economic capital estimates. The amount of the regulatory capital according to both EVT methods would reach five times higher value than the total equity of BANK, which would be about 40% of BANK total assets! Such huge loss would very likely mean that BANK would not be able to meet its liabilities and would be forced to default. The estimates provided by the g&h distribution are approximately two times lower than for the EVT methods. Even though this would mean quite a significant harm to BANK, under certain circumstances it can be considered that BANK would be able to handle these losses by additional financial sources provided by a public offer, external investors or SG.

In the average loss case,[104] the additional loss amounts are more comparable to the original loss amounts than in the previous case. The highest average loss is in fact lower than the highest empirical loss and the sum of all custom losses is EUR 10M, which is about 35% of the sum of all empirical loss

[103] The economic capital is measured as $CVaR_{0,99}$/Total Equity

[104] Only 9 average loss observations were considered - the historical scenarios do not define the average loss

amounts. But still all of the custom observations belong to the tail of the data sample. Thus it can be assumed that the capital estimates would be higher than in case of the original data sample, but the difference would be significantly lower than in the previous case.

Table 5.5: Test I – Average loss scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	11.7%	18.9%
POT 5%	4.3%	4.8%
BMM – Quarter	4.1%	3.4%

Source: Author

The capital estimates provided by both EVT methods are unreasonably low – even lower than for the original loss data sample. On the other hand, the g&h method provides reasonable results suggesting that BANK would be able to cover the OR losses from its own sources. The values of the capital estimates are approximately 3 times higher than in case of the original data sample.

The results indicate that the EVT methods are very sensitive to the number and magnitude of the added observations and if there is such extreme event as in case of the worst-case scenario combination, then the capital estimates are significantly overestimated. On the other hand, if the tail of the data is enriched by adding new observations with loss amounts that are comparable to each other as in case of the average Test I scenario, then the EVT results are underestimating capital needed to cover the risks. The conclusion is that the EVT approach is not consistent in modeling OR exposure of BANK and thus the EVT might be considered as an unsuitable method for OR measurement.

On the other hand, the g&h distribution provides reasonable results for both the worst-case and the average loss Test I methods. Even though the loss events defined for the worst-case scenario are so severe, that the estimated level of regulatory capital charge is unreasonably high for the bank to allocate. The only suitable solution would be to take the risk – especially, if the very low probability that all the events happen in the same 4 years period is considered.

5.3.2 Test II – All BANK scenario

For the Test II some scenarios defined by BANK (ID 1-8) were considered. The probability that this joint scenario would happen is very low. The probability that the worst-case joint scenario would happen is even much lower and limits to zero. Even though this case was considered. The eight worst-case event losses were added to the historical loss database which amplified the tail of the empirical distribution. The original loss events were merged with the additional ones.

The results correspond with the assumptions for all methods. But again, the EVT method provides very high capital estimates. The g&h capital estimates are about 3 times lower than for the Test I. The results suggest that BANK would be able to undergo the events. However, the economic capital estimate reaches 80% of total equity!

Table 5.6: Test II – worst case scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	35.6%	77.5%
POT 5%	129.1%	377%
BMM – Quarter	135.7%	235.3%

Source: Author

For the average case scenario combination the loss amounts are more comparable to the historical losses. In fact they are very similar to the Test I which employs just one more observation (scenario ID12) and thus one might expect very similar results.

Table 5.7: Test II –average loss scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	10.1%	14.1%
POT 5%	5.2%	6.5%
BMM – Quarter	4%	3.3%

Source: Author

The results provided by the EVT methods are again unrealistically low suggesting that the EVT is not appropriate to model OR data with sufficient robustness to the specifics of the extreme observations. The g&h distribution provides results similar to Test I and thus similar conclusion can be made about BANK ability to handle the extreme losses. The economic capital level would reach 14% of total equity.

5.3.3 Test III – More frequent BANK scenarios

In order to increase the probability that all events will happen during 4 years period the BANK scenarios with higher frequency. This joint probability exceeds 1% and so the scenario combination should be assessed with higher importance than in the case of Test I and Test II. The probability of the worst case joint scenario would be, however, significantly lower.

The sum of the worst-case losses would still reaches EUR 100M. The capital estimates provided by the EVT methods are again unreasonably high. The g&h distribution provides very reasonable results which suggest that BANK would be able to undergo such events.

Table 5.8: Test III – worst case scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	20.4%	38.6%
POT 5%	144.5%	521.7%
BMM – Quarter	148%	295.6%

Source: Author

The EVT methods again provide unrealistically low estimates in case of average loss joint scenario compared to the results of the worst case joint scenarios. The regulatory capital charge estimate for average loss joint scenario given by the g&h distribution is about 2 times higher than in the case of the original data sample. This suggests that by adding a few extreme observations to the data sample the capital required to cover losses would increase significantly, but the new capital estimates would still be acceptable from the business continuity management point of view. The combination of the scenario analysis and the backward-looking LDA approach might thus provide reasonable estimates of OR exposure that would be acceptable by the bank management as well as by the regulators.

Table 5.9: Test III – average loss scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	8.8%	12%
POT 5%	6.6%	9.9%
BMM – Quarter	4.6%	4.2%

Source: Author

5.3.4 Test IV – All custom scenarios

Test IV consists of 4 custom scenarios (9-12). The average case scenario actually considers just the scenario ID12, because the historical scenarios were not assigned the average loss amounts. Also it is not possible to evaluate the frequency of such a joint scenario, because the historical events do not have frequency assigned. Since the historical scenario events are very severe, it can be assumed that the worst-case joint scenario capital estimates would be higher than in case of Test III. Especially the scenario ID9, which defines the loss amount of EUR 112M, is expected to significantly impact measures provided by the EVT.

In a correspondence with the assumptions, the capital estimated provides by the EVT methods are unreasonably high, especially for the POT 5% method that estimates the economic capital to the value of BANK total assets! If such capital should be set aside to cover losses, then BANK would not have any other funds to run the business. This is for sure not a possible solution. The g&h distribution provides reasonable results and suggests that BANK would be able to cover losses from its gross income, even though the amount of the funds required would be quite high.

Table 5.10: Test IV – Worst case scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	21.3%	38.6%
POT 5%	200%	1101%
BMM – Quarter	178%	488%

Source: Author

The average loss joint scenario actually considers just the loss defined in the custom hypothetical scenario ID12. So the results should be very similar to the original estimates because only one more observation was added to the data sample. The results provided by all methods follows the assumptions - but as for the other tests, the estimates provided by the EVT methods are actually lower than those for the original data sample, which further signals that the EVT methods are not ideal for modeling the OR data because of its high sensitivity to the number of extreme observations. The g&h distribution estimates are little higher because of the additional extreme observation that was added to the data sample.

Table 5.11: Test IV – Average loss scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	5.3%	7.6%
POT 5%	8.5%	19.7%
BMM – Quarter	8.8%	18.9%

Source: Author

5.3.5 Test V – More frequent BANK scenarios + Custom scenarios

The fifth joint scenario combines Test III and Test IV by using BANK scenarios with higher assessment frequency and all the custom scenarios. The sum of the additional losses is again quite high, so the capital estimates should be similar to Test I, where all scenarios were considered. It might be rightly assumed that such combination of extreme events would be destructional for the financial institution, because the losses would be of such magnitude that BANK would not be able to cover them.

The capital estimates results for the worst case combination of events follow the assumption stated above. The POT 5% provides results similar to Test I or Test IV. Even though the results of the g&h distribution are high as well, it might be assumed that BANK would be able to survive the combination of losses but the recovering procedure would be very costly.

Table 5.12: Test V – Worst case scenario capital estimates

Distribution	*Regulatory capital*	*Economic capital*
G&H	70.3%	173.4%
POT 5%	320%	902%
BMM – Quarter	199%	466%

Source: Author

For the average loss case the estimates provided by the EVT are underestimated as for all other tests done. The g&h distribution suggests that even such a significant losses would not the move regulatory capital estimate over 10% gross income which is a reasonable level of capital for the bank to set aside.

Table 5.13: Test V – Average loss scenario capital estimates

Distribution	*Regulatory capital*	*Economic capital*
G&H	9%	13.3%
POT 5%	5.4%	7.2%
BMM – Quarter	4.8%	6.8%

Source: Author

5.3.6 Test VI – More frequent BANK scenarios + Custom hypothetical scenario

The last test combines Test III and the scenario ID12. The results should be quite similar to the results of Test III, because just one more extreme observation was added to the data sample. The worst case scenario results for the g&h distribution follow the assumptions – the estimated capital is about

30% higher than for Test III. The estimated capital level would be quite significant and it is not expected that the financial institution would need to reserve over 50% of its total equity to cover OR losses.

Table 5.14: Test VI – Worst case scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	29.7%	55.9%
POT 5%	123%	367%
BMM – Quarter	153%	470%

Source: Author

The results for the average loss joint scenario again show, that the EVT is not a consistent method. Since the number of extreme observations is very limited, each extreme observation significantly alters parameter estimates for the EVT methods and the aggregated distribution VaR measures. The g&h distribution estimates the regulatory capital to 10%. This would be a reasonable level for the financial institution to reserve in order to cover potential losses.[105]

Table 5.15: Test VI – Average loss scenario capital estimates

Distribution	Regulatory capital	Economic capital
G&H	9.3%	14.6%
POT 5%	5.4%	6.1%
BMM – Quarter	5.1%	9.1%

Source: Author

5.4 Comparison of the test results

This chapter combined 12 OR scenarios to 6 joint scenario combinations and analyzed the impact of the events, which are defined by the scenarios, on the regulatory and economic capital estimates. Two loss amounts for the additional events were used – the extreme worst-case and the average loss observations. The observations were merged with the original data sample and the aggregated loss distribution was constructed using the MC simulation.

[105] It should be noted that the results for the average loss joint scenario combination in Test VI should be exactly the same as for the average loss case in Test V. But since the historical average cases were actually included to the data sample and the loss size was set to EUR 0, then the parameter estimates are insignificantly different and so are the VaR measures.

All the tests suggest that the EVT method is not an appropriate one to model the OR data, because the results provided by both EVT methods (the BMM – Max quarter and the POTM 5%) were very sensitive to the number of the tail observations and to the length of the tail. If there is such extreme observation as the one defined by scenario the ID9, then the capital estimates given by the EVT method would be unreasonably high and in some cases reaching the amount of BANK total assets! On the other hand, if the less extreme average loss case events are added to the data sample, then the capital estimates provided by both EVT methods are unreasonably low. The EVT method is thus providing inconsistent results, and thus it cannot be considered as the best approach to model the OR data – even though the theory suggests that the EVT might be beneficial for the OR measurement. The application of the EVT methods to the empirical data provides overestimated results for the worst-case scenarios and underestimated results for the average loss scenarios.

The results of the EVT method are probably impacted also by the way the parameter estimates were computed. In this rigorous thesis the EasyFitXL software was used for this task. Teply, Chalupka (2008) suggest using other methods, which might be more feasible for the case, when the tail of the data does not contain enough observations. For the BMM method it is very crucial to question, when exactly the custom extreme losses happen. Since the number of custom losses as well as the length of observed period is quite low, the data generation algorithm might provide biased results. Also the VaR measures based on the aggregated loss distribution are very sensitive to the number of MC simulations. For all tests in this rigorous thesis we have used 50,000 MC trials. Due to technical limitations it was not possible to increase this number further – even though it might be beneficial to use e.g. one million MC trials, especially while working with the $VaR_{0.999}$ or the $CVaR_{0.99}$ measures. Such task is beyond the focus of this rigorous thesis and might be considered for further, more sophisticated, research.

However, it is rightly to assume that, even if more sophisticated methods would be used for the EVT parameter estimates and the loss aggregation process, the overall results would not probably change significantly and so both EVT methods would still overestimate the regulatory and economic capital estimates – in some cases the overestimation would be very significant. This conclusion follows the findings stated in the paper by Dutta, Perry (2007). On the other hand, Teply, Chalupka (2007) favor the EVT over g&h distribution – their conclusion might not consider the scenario analysis and the stress testing methods.

However, it might be expected that the results provided by the EVT method would improve the consistency, as the number of observations, both from the body and the tail of the empirical distribution, increases – but even though it might be assumed that the EVT results would still be less consistent than those provided by the g&h method.

The g&h distribution proved to be a very suitable one. Its results were consistent, as the extreme worst case and the average loss custom events were being added to the data sample – this conclusion corresponds with the findings of Degen (2007). The parameter estimates differ based on the number of the additional extreme events used for the scenario analysis – the more extreme losses were added to the data sample the higher the estimate for $\hat{h}$ and $\hat{g}$ was. So in case of Test I the $\hat{h}$ was seven times higher than in case of the original data sample – for more details see Table 5.16. The higher the estimated parameters, the higher the losses generated during the loss aggregation procedure.

Given the very low probability of the fact that a joint combination of all the worst-case custom losses would occur during the observed period, a financial institution should be relatively safe. Even though the potential losses assessed by using the g&h distributions are in most cases severe to the bank, but it is rightly to expect that even such high magnitude losses can be handled and that the financial institution would be able to cover the losses. The level of regulatory capital for all scenario analysis tests exceeded 20% of the gross income.

Table 5.16: Comparison of g&h distribution results- worst case scenarios

Test	Scenario IDs	$\hat{g}$	$\hat{h}$	Reg. capital	Ec. Capital
Original	n.a.	2.075	0.0509	4.43%	6.71%
Test we	ID1-12	2.210	0.3537	90.74%	236.33%
Test II	ID1-8	2.157	0.2379	35.66%	77.52%
Test III	ID3-5,7-8	2.136	0.1943	20.37%	38.62%
Test IV	ID9-12	2.124	0.1996	21.25%	38.64%
Test V	ID3-5,7-12	2.163	0.3163	70.32%	173.45%
Test VI	ID3-5,7-8,12	2.148	0.2222	29.75%	55.93%

Source: Author

The g&h distribution is, unlike the EVT, consistent even if less extreme but more frequent average loss cases are added to the data sample. In the average loss case the custom losses were of very similar magnitude as the most severe empirical losses. So the length of the tail remained the same – it was only made heavier. The parameter estimates are very similar to each other and so are the regulatory capital estimates. Even if all the scenarios were considered, the estimated regulatory capital would not exceed 12% of the gross income suggesting that BANK would be able to handle the losses of such high magnitude.

Table 5.17: Comparison of g&h distribution results – average loss scenarios

Test	Scenario IDs	$\hat{g}$	$\hat{h}$	Reg. capital	Ec. capital
Original	n.a.	2.075	0.0509	4.43%	6.71%
Test we	ID1-12	2.163	0.112	11.76%	18.96%
Test II	ID1-8	2.157	0.10023	10.09%	14.07%
Test III	ID3-5,7-8	2.136	0.0884	8.75%	12.04%
Test IV	ID9-12	2.082	0.05802	5.29%	7.62%
Test V	ID3-5,7-12	2.141	0.1029	9.00%	13.33%
Test VI	ID3-5,7-8,12	2.148	0.09659	9.27%	14.60%

Source: Author

The combination of methods based on the empirical historical loss events as well as the events defined in the scenario made the capital estimates more robust to extreme plausible events. It was concluded that not the EVT but the g&h distribution is a suitable one for this approach.

The statistical fit of the EVT and the g&h distribution was not considered while running the scenario analysis tests. It is rightly to assume that the degree of the fit would be approximately the same for the average loss joint scenarios, while it can differ for the worst-case joint scenarios that add more extreme losses. It is also rightly to assume that the degree of the fit for the EVT methods would be generally higher than the degree of the fit of the g&h distribution – but it must be considered that the EVT is fitted just to the tail of the data while the g&h works with the whole sample.

The other crucial question is, how the tail and the body of the data sample are disbursed and to which part of the sample EVT methods are fitted. This rigorous thesis first considered original data sample and evaluated the EVT methods which provided the best results. Another methods can be used which could provide better results for the merged sample containing both the custom and the original OR events. However, this task is beyond the focus of this rigorous thesis.

5.4.1 Implications for the financial institution

As mentioned above, the scenario analysis added the custom hypothetical losses to the original loss database. Six tests were run in order to evaluate the effects of those plausible events on the financial distribution. Since all those events impose extreme losses, it was assumed that the estimates of the regulatory capital charge as well as of the economic capital would significantly increase. The statistical distribution that was finally considered to be the most suitable to measure the capital required to cover the OR losses – the g&h distribution – provided reasonable estimates for all the tests run.

In the cases where extreme worst-case losses were considered the final estimates for regulatory capital charge spiked up to 90% of the gross income. Such huge amount of capital cannot be set aside to cover risks, because it would make the financial institution noncompetitive - the cost of its capital would be much higher than the industry average. On the other hand, it is hardly to expect that all the worst case scenarios will ever happen in such short time period that was considered throughout this rigorous thesis – 4 years. But even if a longer time period - like 10 or 20 years – would be considered, the probability that the worst case joint scenario from Test I would occur limits to zero.

From this point of view it seems more reasonable to work with average loss joint scenario cases, which have higher probability of occurrence – in some cases over 2%. The tests that employed the average losses provided higher but still affordable level of capital estimates – up to 12% of the gross income for the capital charge and 19% of the total equity for the economic capital estimate defined as the $CVaR_{0.99}$ measure.

The conclusion of this rigorous thesis is that the combination of the scenario analysis and the LDA approach can improve applicability and soundness of the capital estimates over the methods, where just historical data are used. Since new internal and external OR data will be added to the loss databases in the future, the quantitative LDA techniques will be more important. But for now it is valuable to consider plausible events and evaluate, what would be the impact of these events. After all of the tests were run we can say that BANK would be able to survive losses imposed by the average joint scenario combination. The losses defined in the worst case scenarios are such extreme, that the bank would have to take the risks in order not to increase the cost of capital to an unacceptable level.

Conclusion

The main aim of this rigorous thesis was to evaluate the appropriateness of capital estimates based on historical loss events and to measure the impact of plausible OR events that were added to the empirical loss data sample provided by a Central European bank. The technique presented in this rigorous thesis claims to be consistent and applicable for other financial institutions. There were two main questions the rigorous thesis was aimed to answer:

- What is the appropriate statistical method to model the OR loss data distribution and to measure reasonable capital estimates for the institution?
- What is the impact of extreme events defined in extreme case scenarios on the capital estimates and on the financial institution?

The evaluation of the OR exposure measurement employed different statistical methods and distributions – the most important ones were the EVT and the g&h distribution. For the original data sample the results for the EVT seemed consistent, statistically significant and economically reasonable. However, after the custom extreme events were added to the data sample, both EVT methods started to provide very inconsistent estimates – the inconsistency is most visible while comparing the estimates provided by tests, where very extreme worst-case events were considered to tests, where less extreme average case events were considered. While in the first case the estimates were unreasonably high, in the second case the estimates were even lower than in case of the original data sample. So the EVT method does not seem suitable to model the OR data even if it is widely favored by many researchers – it is main disadvantage is its sensitivity to the threshold choice. The appropriate threshold is very difficult to find given the limited historical data samples. Thus the EVT results were not robust to the data contamination and the outlier observations.

The alternative method to the EVT was the g&h distribution, which was evaluated as the most suitable from all the parametric distributions used. It proved itself very consistent to contamination and outlier observations and it provided very reasonable results even while very extreme worst-case losses were considered.

So the answer to the first question would be that the most suitable method to model the operational risk loss data distribution is to use the g&h distribution which is able to model the whole data sample “without trimming or truncating the data in an arbitrary or subjective manner”[106]. The null hypothesis stated in the introduction thus cannot be rejected, because the g&h proved consistent over all scenarios that were considered.

There might be other statistical distributions that are able to measure and model the tail structure of the OR data – we believe that a further research will be devoted to this issue and even more suitable measurement methods will be developed.

[106] Dutta, Perry (2007)

In order to answer the second question, the original data sample was enriched by adding events defined in 12 scenarios. The impact of these events was assessed. Given the fact that the original data sample was very limited and it consisted only of internal loss events, it is beneficial for the financial institution to measure the impact of such plausible event as an employee strike. In total six tests were run. The assumptions, that by adding an outlier event the capital estimate would increase, was fulfilled for all tests while using the g&h distribution. If the very low probability joint combination of the worst-case events was considered, the estimated level of the capital required to cover such losses would too high for the bank to set aside - over 90% of gross income for the 99.9% confidence level. It is not expected that such combination of extreme events occur in limited time period, so the only reasonable solution for the bank is to take this risk.

However, if a joint combination of extreme loss events with higher probability of occurrence – the average loss scenarios – were considered, the estimated regulatory and economic capital levels would be very reasonable capital estimates – 12% of the gross income for 99.9% confidence level. The financial institution should employ these OR events, while considering which level of capital to hold to cover the risk.

And so the answer to the second question is that, given the reasonable definition of the scenario analysis and the loss amounts defined under this scenario, the estimated regulatory charge has increased significantly but still to a level which is acceptable for the financial institution. The OR assessment method should be reasonable for the regulator as well and so this thesis provides a framework of how to combine the scenario analysis with the LDA approach. Using the scenario analysis can also help the financial institution to mitigate the OR and to decrease the impact of potential losses. This framework can be used for future application and the impact of other scenarios can be assessed.

Some further questions and tasks remain open. The external data could be merged with internal data in order to better capture the potential impact of events that have not happened to the financial institution yet. Statistical differences the between business lines and the event types should be analyzed. Robust methods or alpha stable distributions can be used as suggested by Chernobai (2007). Other EVT methods, particularly for the threshold estimation, could be used. The number of the Monte Carlo simulations can be further increased in order to achieve higher statistical relevance. However, this issue is beyond the scope of this rigorous thesis and is left for future consideration.

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[107] The software names are registered trademarks of the registered owners

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List of Abbreviations

A-D	Anderson-Darling Test
AMA	Advanced Measurement Approach
BIA	Basic Indicator Approach
BMM	Block Maxima Method
CVaR	Conditional Value-at-Risk
EC	Economic Capital
ESM	Empirical Sampling Method
EVT	Extreme Value Theory
GEV	Generalized Extreme Value
GOFT	Goodness of Fit Test
GPD	Generalized Pareto Distribution
K-S	Kolmogorov – Smirnoff Test
KRI	Key Risk Indicators
LDA	Loss Distribution Approach
MLE	Maximum Likelihood Estimators
POTM	Peak Over Threshold Method
PWM	Probability Weighted Method
QQ	Quantile – Quantile
RCSA	Risk Control Self Assessment
SA	Standardized approach
SCA	Scenario Analysis
SG	Société Générale
VaR	Value-at-Risk