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Method to find the Minimum 1D Linear Gradient Model for Seismic Tomography

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Abstract. The changes in the state of a geophysical medium before a strong earthquake can be found by studying of 3D seismic velocity images constructed for consecutive time windows. A preliminary step is to see changes with time in a minimum 1D model. In this paper we develop a method that finds the parameters of the minimum linear gradient model by applying a two-dimensional Taylor series of the observed data for the seismic ray and by performing least-square minimization for all seismic rays. This allows us to obtain the mean value of the discrete observed variable, close to zero value.

Keywords: local earthquake tomography, minimum one-dimensional model, linear gradient of velocity

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1. Introduction

Different from the human body, the Earth is not as assessable for studying internal structure. The most common way to investigate the Earth's interior is to process seismic observations. Our main objective is to examine if there are noticeable changes in the seismic velocity prior to strong earthquakes. In the case of a significant temporal change in a localized structure of the Earth, it is conceivable that it can be resolved in a 1D reliable approximation of the 3D structure. At the same time the main usefulness of such a 1D model to be an initial model for the 3D inversion. In this article the problem of determining the 1D minimum model with linear velocity gradient is solved. To make this study useful for other branches of computerized tomography we analyze fundamentals of computerized tomography and compare the problem formulations for different observational data, seismic and medical.

1.1. The fundament of computerized tomography and local earthquake tomography

The computerized tomography is based on the Radon Transform (RT) that was introduced by Johann Radon in 1917 [1]. There are several mathematical expressions that determine RT [2, 3]. We will use the following definition given in [4]. Let f be a function on R^n then RT is the function $\widehat{f}$ defined by:

$$\widehat{f}(\xi) = \int_{\xi} f(x)dm(x) \tag{1}$$

where the integral is taken over the hyperplane ξ. If f is a function on R^2, ξ are straight lines and the function $\widehat{f}$ is defined by the line integral of f along each line. If the complete set of $\widehat{f}$ (i.e. set of projections) is available the function f can be reconstructed uniquely. In medical imaging, e.g. the computerized axial tomography scan (CAT scan), X-rays are used to scan patients from all directions and the observational data ($\widehat{f}$) are attenuation of the X-rays along all ξ. Such use of the RT for medical purpose and the corresponding numerical technique to reconstruct the function f was proposed by Allan Cormack [5].

In order to properly define f the complete set of projections $\widehat{f}$ has to be measured from all angles. A similar approach is widely applied in the cross-borehole tomography, to investigate the rock properties between two boreholes. Seismic sources are distributed down in one borehole while receivers are located down the other borehole. Seismic rays are crossing the volume in various directions. The function of the acoustic slowness $f(x, y)$ is reconstructed in weakly heterogeneous structures by using the observed travel times of the seismic waves that are presented in equation (1) as $\widehat{f}$. The task in this formulation is analyzed in [6, 7] and the authors of [8] investigate ways to adapt the problem to incomplete sets of projections $\widehat{f}$.

When the observation data are travel times from local earthquakes, widely distributed within the imaging volume it is possible to reconstruct heterogeneous 3D velocity structure. However, in this case in the equation (1) there is a complex dependence of ray path ξ on the nonlinear velocity function $v(x, y, z)$, where z is a depth. Therefore the task to determine the slowness field $f(x, y, z)$ can be solved with linearized formulation that has been proposed and investigated by [9, 10]. Considering travel times t and t^0 along ray paths ξ and ξ^0 for the two media with velocities $v(x, y, z)$ and $v_0(z)$ (Figure 1) the problem of a search of $f(x, y, z)$ can be written in the following form:

$$t - t^0 \approx \int_{\xi^0} (f(x, y, z) - f(z))ds \tag{2}$$

where $f(x, y, z) = 1/v(x, y, z)$ and $f(z) = 1/v_0(z)$, s is a distance along the ray. Note that the integral (2) is taken along the curve $\xi^0(S, R)$ that corresponds to the source-receiver pair (S, R) and represents the seismic ray trajectory based on the a priori starting 1D model $v_0(z)$. This statement has been made in [9] under the assumption that the velocity $v(x, y, z)$ can be presented as $v(x, y, z) = v_0(z) + v(x, y, z)$, where $v_0 >> |v_1|$.

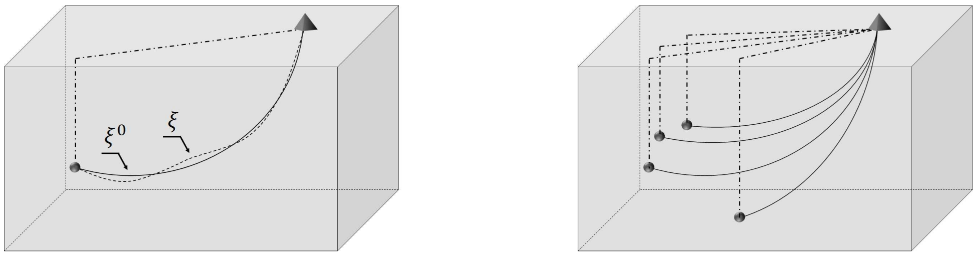

Figure 1. Ray paths ξ and ξ^0(on the left). Planes of rays that correspond to the medium with velocity $v_0(z)$ (on the right). The earthquake source and station are denoted by a small sphere and a tetrahedron, respectively.

The seismic data are collected from earthquakes non-uniformly distributed that illuminate the Earth from various directions. Combining all projections, which are differences between the observed travel times and travel times corresponding to the starting 1D model (travel time residuals) one can reconstruct a 3D velocity image in the given volume.

1.2. Conception of minimum 1D model

The function $v(z)$ can be found using travel data of seismic waves from explosions recorded by receivers located along the profile [11]. Figure 2 illustrates the arbitrary scheme of the profile experiment and the different layers with fixed average velocity.

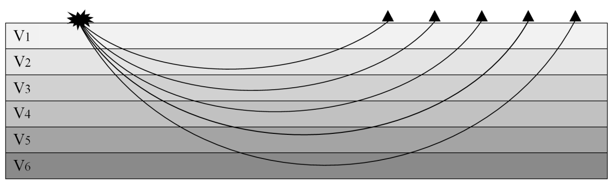

Figure 2. The profile experiment. Seismic rays are registered from the explosion (denoted by star) by geophones (denoted by triangles).

The concept of a minimum 1D model for local earthquakes was proposed in [12]. The minimum model by [12] was built by the minimization of the differences between the observed travel times and travel times calculated from the starting 1D layered model. The advantage such model for the 3D velocity imaging is described in [13]. In this paper, we propose another approach to find the minimum model. Seismic measurements show that velocity gradient models can be more appropriate. Therefore as a starting model, we consider the linear velocity gradient, that is, the function $v_0(z)$ is linearly dependent on the depth z, represented as $v_0(z) = a+bz$. By inverting travel times (minimizing travel time residuals) for all rays we find the corrections to the parameters a and b, which define the minimum model for a linear velocity gradient. We have tested this method on a small set of earthquakes in South Iceland recorded by the Iceland Meteorology Office (IMO).

2. The method to estimate parameters of minimum 1D linear gradient model

When velocity changes linearly with a depth: $v_0(z) = a + bz$, the travel time of seismic ray can be found by applying the following formula [14] :

$$t(a,b) = \frac{1}{b} \ln \frac{(a + bz^{source})(1 + \cos\theta)}{(a + bz^{receiver})(1 + \cos\theta_1)} \tag{3}$$

where θ_1 and θ are slopes of the ray to the vertical axis at the source depth z^{source} and the receiver depth $z^{receiver}$, respectively.

We use a two-dimensional Taylor series that is the expansion of $t(a, b)$ at point (a_0, b_0), which corresponds to the starting velocity model with linear gradient $v_0(z) = a_0 + b_0 z$. To our knowledge the geophysicist Keiiti Aki was the first who used the Taylor series expansion of observed seismic data [15]. The series by [15] are determined in terms of earthquake source and 3D velocity parameters. We perform the expansion in terms of other parameters, which are corrections Δa and Δb to the parameters a_0 and b_0 that characterize the initial 1D linear gradient model. In this case the observed travel time can be written in the following form:

$$t(a,b) = t(a_0, b_0) + \frac{\partial t}{\partial a}\Delta a + \frac{\partial t}{\partial b}\Delta b + e \tag{4}$$

where e are the higher order terms. The value of $t(a, b)$ corresponds to the observed travel time for a seismic ray while $t(a_0, b_0)$ corresponds to the travel time that was calculated from the initial model $v_0(z) = a_0 + b_0 z$. The travel time residual τ can be written as:

$$\tau = t(a,b) - t(a_0,b_0) = \vec{d}\binom{\Delta a}{\Delta b} + e \tag{5}$$

where $\vec{d} = (\partial t/\partial a, \partial t/\partial b)$.

Combining all rays and neglecting the higher order terms, the equation can be written in matrix form:

$$\widehat{\tau} = D\binom{\Delta a}{\Delta b} \tag{6}$$

where $\widehat{\tau}$ is a vector, whose components are travel time residuals, D is a matrix, whose rows are determined by vectors $\vec{d}$. By applying the least-square minimization to the system we can find corrections Δa and Δb that should be made for the initial linear gradient model. The parameters $a^{(1)} = a_0 + \Delta a$ and $b^{(1)} = b_0 + \Delta b$ describe the function $v^{(1)} = a^{(1)} + b^{(1)} z$ that is the first approximation to the minimum model. The process of search of corrections is repeated until the values of travel times $t(a, b)$ for all seismic rays have a mean value and a standard deviation value in the limits of pre-assigned accuracy. The pre-assigned accuracy is selected from conditions of the closeness of the mean value to the zero-value and the decrease of the standard deviation value in comparison with the initial value.

2.1. Determination of the derivatives

The derivative of the function $t(a,b)$ with respect to a can be obtained as the derivative of the natural logarithm function that is the composite function, determined by the equation of (3). This derivative can be obtained in the process of corresponding transformations. Finally, we get the following formula:

$$\frac{\partial t}{\partial a} = -\frac{z^{source}}{a(a + bz^{source})} \tag{7}$$

The derivative of the function $t(a,b)$ with respect to b can be obtained as the derivative of the product of two functions. After corresponding transformations and substitutions we get the next formula:

$$\frac{\partial t}{\partial b} = -\frac{1}{b}t(a,b) + \frac{1}{b}\frac{z^{source}}{(a + bz^{source})} \tag{8}$$

3. Testing the method

In order to test the method we use travel time data from a small set of local earthquakes recorded in South Iceland in the time period since January, 2001 till October, 2001 by the South Iceland Lowland (SIL) seismic network. Minimum models have been constructed by applying the developed theory for two different starting models. The first starting model was found by using discrete values of seismic velocity V at different depths z that were estimated from previous tomographic research of the Tjornes Fracture Zone (in Northeast Iceland) [16]. These discrete values were approximated by a polynomial of degree 1 by using the MATLAB tool function polyfit, leading to $V(z) = 5.9895 + 0.0579z$. This general linear gradient model was suitable for the given observation data set. Ray tracing through this model for the test earthquakes with depths < 8 km (epicentral distances < 10 km) gives travel time residuals with the $mean = 0.238$ and $std = 0.058$. The second starting model is obtained by approximating the discrete values of seismic velocity that characterizes the seismic area within the SIL network [17]. Our adapted SIL velocity model is described as: $V(z) = 3.926 + 0.479z$. With this model, using the same set of earthquakes as before, the travel time residuals have the $mean = -0.0411$ and $std = 0.0377$.

The algorithm developed iteratively calculates the corrections Δa and Δb to find minimum models. For instance, using the first starting model we have calculated the following first approximation: $a^{(1)} = a_0 + \Delta a = 5.9895 + (-2.13466859) \approx 3.8548$ and $b^{(1)} = b_0 + \Delta b = 0.0579 + 0.00740185473 \approx 0.0653$. They describe the function $v^{(1)} = a^{(1)} + b^{(1)}z = 3.8548 + 0.0653z$. After 174 iterations we got $v^{(174)} = a^{(174)} + b^{(174)}z = 4.147 + 0.441z$. The corresponding travel time residuals have the $mean = -0.000134$ and $std = 0.0371$. Thus the minimum model can be characterized as: $V(z) = 4.147 + 0.441z$.

If we use the second starting model then the first approximation is the next: $a^{(1)} = a_0 + \Delta a = 3.926 + 0.413453907 \approx 4.3394$ and $b^{(1)} = b_0 + \Delta b = 0.479 + (-0.00460870098) \approx 0.4744$; they describe the function $v^{(1)} = a^{(1)} + b^{(1)}z = 4.3394 + 0.4744z$. The minimum model can be obtained after 18 iterations and it is nearly the same model as in the first case: $V(z) = 4.1468z + 0.4411z$. The corresponding travel time residuals have the $mean = -0.00013$ and $std = 0.0371$. A sample of data calculated from the two starting models and from the minimum model are given in Table 1.

A check of the proposed method as the approach, which helps to predict earthquakes can be made by using the extended data set collected in South Iceland for the sequence of time windows. The seismically active area is divided into subareas, one of which contains the main shock(s) of the strong earthquake.

Table 1. Travel time residuals calculated for the test earthquakes in South Iceland.

Epicentral distance	Travel time residual		
	SIL adapted model	General model	Minimum model
5,09082174	0,011568189	0,269914269	0,049516439
5,29005527	-0,080977321	0,184937119	-0,042094708
5,42234278	0,002081156	0,26837039	0,041070938
5,68836641	-0,087852716	0,206971407	-0,04546392
5,73525143	-0,052276135	0,310548842	-0,000537157
5,46838903	-0,085208535	0,191042662	-0,045081973
5,63388014	-0,11279726	0,206755996	-0,067219496
5,45617056	-0,009722829	0,263259649	0,030025601
6,30210209	0,03046608	0,397123337	0,082244277
5,62456656	-0,067387343	0,238366008	-0,02363348

By constructing minimum models for each subarea one can see the difference between models for the dangerous subarea and the surrounding subareas.

4. Conclusion

We have described the method to find a minimum 1D linear gradient velocity model for earthquake travel time data. This method is based on the approach of using a Taylor series expansion for the observed data (i.e. the tomographic projection) in terms of corrections to model parameters that are to be determined. By combining all projections we obtain a system of linear equations that can be solved in the least-squares sense to estimate the corrections. The method was tested on earthquake data in South Iceland. The method was not sensitive to the starting models and finds a minimum 1D velocity model with a good fit to the test data.

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