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Bipedal Locomotion Structure with Passive Members

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BIPEDAL LOCOMOTION STRUCTURE WITH PASSIVE MEMBERS

BY

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This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirement for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Thesis Adviser　　　　　　Date

Head, Mechanical Engineering Department　　　　　　Date

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RDP

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NOMENCLATURE

a	One-half the length of lower section of extremities
α	Angle giving kinematic program of shifting of lower extremities, defined in (3-17)
A_i	Constants, defined in (3-16)
b	One-half the length of upper section of extremities
B_i	Functions, defined in (3-14) and (3-15)
β_1	Angular displacement of left passive member
β_2	Angular displacement of right passive member
c	Length from lower body mass center to compensating mass
d	One-half the distance between the extremities
e	One-half the length of the passive members
η	Torsional damping coefficient of passive members
$\{Fn\}$	Column matrix of final values
g	Gravitational constant
h	One-half the distance between passive members
$\{In\}$	Column matrix of initial values
J_a	Moment of inertia of lower section of extremities
J_e	Moment of inertia of passive members
J_o	Moment of inertia of lower body mass
m_a	Mass of lower section of extremities
m_b	Mass of upper section of extremities
m_c	Mass of compensating member
m_e	Mass of passive members
m_o	Mass of lower body

$\{M\}$	Moment matrix about the supporting point
ω	Cyclic frequency of gait
$\{\omega\}$	Angular velocity matrix
ω_p	Angular velocity of passive members, β_1 and β_2
P_j	Parameter varied in sensitivity function
ψ	Coordinate angle of compensating member (Fig. 2)
θ	Coordinate angle of compensating member (Fig. 2)
$[U]$	Sensitivity matrix defined in (4-4)
$[W]$	Matrix defined in (4-1)
$\{x\}_i$	Position vector of each member from the supporting point
$\{\ddot{x}\}_i$	Acceleration of mass center of each member
$\{x\}_p$	Position vector of the passive members from the passive member joints
ζ	Damping factor of passive members

CHAPTER I

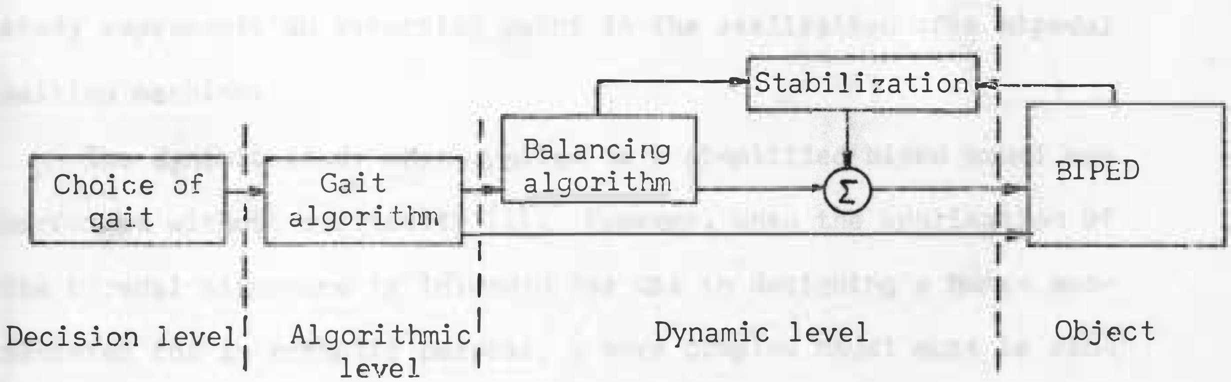
INTRODUCTION

In an early study of the dynamics and stability of bipedal locomotion [1,2], it was shown that a bipedal walking machine, being a large multivariable system, needed a hierarchial control structure in order to make its practical realization possible. The control levels suggested on the basis of minimum flow of information between different levels are

1) decision-making level
2) algorithmic level
3) dynamic level.

This multilevel control system is illustrated as shown in Fig. 1[3].

Fig. 1



The decision level represents the choice of the gait type out of a finite number of specific gaits stored in the control system memory. The algorithmic level controls the motion of kinematic members in such a manner as to perform the specified gait as well as controlling the motion of the balancing members that equilibrate the biped about the supporting point. An additional motion of balancing members is used at the same time to stabilize the biped gait. This additional motion also makes use of a feedback signal by sensing the instantaneous biped state.

The motion of the balancing members used to balance as well as stabilize the gait of the biped in motion results from the dynamics of the bipedal structure. Being inherently unstable, the structure needs constant dynamic equilibrium about the supporting point which is being achieved by an accelerated motion of the balancing members. However, this accelerated motion has the condition that results in a stable gait, that is, a gait that will be composed of repeated steps having a constant mean velocity. Thus, it appears that the dynamic study represents an essential point in the realization of a bipedal walking machine.

The dynamic study when applied to a simplified biped model was performed without difficulty [1]. However, when the application of the bipedal structure is intended for use in designing a human exo-skeleton for an orthotic purpose, a more complex model must be con-sidered. In addition to having more degrees of freedom to simulate an exoskeleton which supports the human body, different structures of the system can arise because of the different degrees of paralysis of the patient's members. Frequent is the case in which the patient's arms are incapacitated in addition to his legs. In such a case the arms swing freely in the form of multiple compound pendulums and thus become a form of passive members.

In treating such a large system with passive members [3], it was observed that settling of the solution obtained by numerical methods deteriorates as the step period approaches the period corresponding to the uncoupled natural frequency of the passive members. This effect seemed logical; consequently, to suppress this deterioration, a form of damping in the motion of the passive members was introduced. However, this did not help in all cases studied, and sometimes created an even worse effect. Thus, it became impossible to study the system at the above mentioned step periods which happened to be in the region of a normal walk speed.

The objective of the work presented in this thesis is to clarify this phenomenon of deterioration in settling of the solution when treating the system with passive members.

CHAPTER II

FORMULATION OF THE PROBLEM

The phenomenon of deterioration in settling of the solution obtained by numerical methods occurred with a complex system [3] that included both physical and mathematic algorithm properties. The physical sub-system properties included the following:

(1) The system was large, composed of many rigid bodies with many interacting degrees of freedom;

(2) The system was highly non-linear as a result of rigid body dynamics;

(3) The system was discontinuous as a result of discontinuous gait data;

(4) The system contained passive arms exposed to parametric excitation.

The mathematical sub-system properties in addition included the following:

(5) The system boundary conditions were satisfied by a successive approximation algorithm resulting from a form of Newton's method;

(6) Finite increment method was used for determination of sensitivity coefficients. These coefficients were used in the successive approximation algorithm.

(7) The system integration algorithm was Hamming's modified predictor corrector method with minimum accuracy of integration. Minimum accuracy was dictated by the system size and the

feasible computing time.

An analytic study of such a system for analysis of a particular phenomenon would be impossible due to the system size and the non-linearities involved. A systematic study of the full size system is also prohibitive. Thus to make a study at all possible, the system must be extensively simplified, retaining only what is thought to influence the phenomenon of interest.

Since the difficulty involved was observed definitely in connection with the addition of passive members, it thus followed that the natural sequence was to eliminate the maximum number of other complexities. In doing so, the number of possible influences was reduced to a reasonable number that could be handled with existing means. The possibility was also left open to add any of the complexities skipped if no difficulty of the type involved arose. Hence the following reduction of the above complex physical-mathematical system was decided:

(1) The system was reduced in size as much as possible by reducing the number of components and their degrees of freedom;

(2) The system non-linearities were retained;

(3) The discontinuous gait data of the system was smoothed;

(4) The passive members were retained as the main point of the study;

(5) The same algorithm for successive approximations was used;

(6) A true sensitivity analysis was used to replace the finite increment method eliminating its influence;

(7) The system integration algorithm used was the Runga-Kutta method with integration accuracy increased to eliminate its

influence.

The physical sub-system was chosen such that it satisfied the listed reduced specifications. The main part of the system was taken to be identical to the model treated by Juricic and Vukobratovic [1], and the passive hands were added in the simplest manner possible. Since the same model without the passive members using the same mathematical algorithm was free of the stated difficulties, it was thought to be the best model for studying the influence of the passive members. The final physical model chosen is given in Fig. 2.

The analysis was carried out with the following data mostly from the dimensions of the previous model [1] with added values taken for the passive members:

$a/c = .269$	$m_a/m_c = .200$
$b/c = .283$	$m_b/m_c = .292$
$d/c = .060$	$m_o/m_c = 1.115$
$g/c = 11.35\ sec^{-2}$	$m_e/m_c = .220$
$e/c = .300$	$J_a/m_c c^2 = .0048$
$h/c = .065$	$J_o/m_c c^2 = .0028$
	$J_e/m_c c^2 = .005$

The mathematical sub-system was modified as indicated by the introduction of true sensitivity coefficients through the solution of the sensitivity functions [4] as opposed to the finite increment method formerly used. The sensitivity equations were set up and solved simultaneously with the original system by use of the Runga-Kutta method.

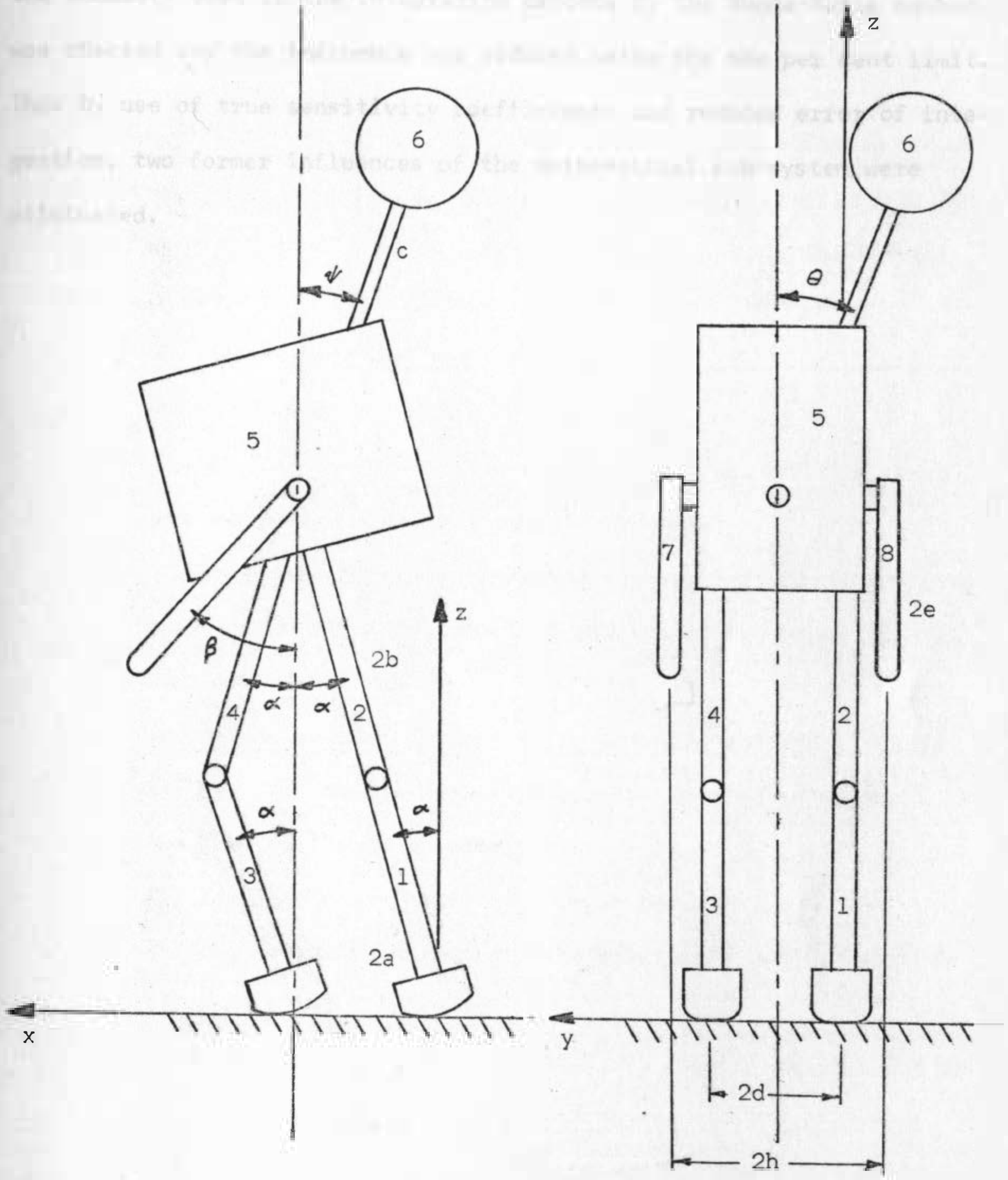


Fig. 2

The accuracy used in the integration process by the Runga-Kutta method was checked and the influence was reduced below the one per cent limit. Thus by use of true sensitivity coefficients and reduced error of integration, two former influences of the mathematical sub-system were eliminated.

CHAPTER III

MATHEMATICAL DESCRIPTION OF PHYSICAL MODEL

The model treated has been extensively simplified relative to the complex system [3] taking, for the main part, a model identical to the reduced system [1] with passive members added (Fig. 1). The large interacting degrees of freedom have been reduced in the following way:

(1) Two leg extremities perform a simple specified gait with a fixed kinematic program;

(2) The lower body mass represents the lower part of the torso and moves with a fixed kinematic program;

(3) The compensating mass represents the upper part of the torso and is used for achievement of balance of the biped as well as stability of the gait.

(4) Two passive members representing arms are added for study;

(5) All leg and passive member joints are treated as hinges.

The following are additional assumptions:

(1) All components are considered rigid bodies;

(2) All leg extremity components are considered uniform slender bars;

(3) The compensating mass is considered a point mass attached by a rod of negligible weight;

(4) The surface friction is sufficient to prevent slipping of extremity tips;

(5) The gait is specified in order that the moving extremity tip is in continuous frictionless contact with the ground. (This

assumption affects geometric relations only.)

The passive members are attached to the sides of the extended lower body mass with a form of damping added for the purpose of study. This damping is a function of the relative angular velocity of the passive members relative to the lower body mass. The criteria in choosing this specific point of attachment was the satisfaction of the system's simplification requirements. It was assumed that the passive members with such a simple joint position would equally well retain their characteristics.

The criteria essential to the dynamic description of a bipedal locomotion system is that it remain in dynamic equilibrium about the supporting point. This description is accomplished by use of D'Alembert's principle in determining the dynamic moment equations. The members of the system adopted are denoted in Fig. 2 with the inertia forces and moments as well as the position vectors shown in Fig. 3. The moments the inertia forces create are summed about the supporting point, this being the tip of the extremity which remains in contact with the ground while the other moves.

The step is divided into two half-steps; and as normal for a human gait, the second half-step is symmetric to the first one. Thus it is enough to study one half-step only. Since the supporting point changes during a half-step, each half-step is divided into two parts -- Part I when one extremity is advanced forward while the tip of the other extremity remains the supporting point; and Part II when the other extremity is brought forward while the tip of the first extremity remains the supporting point.

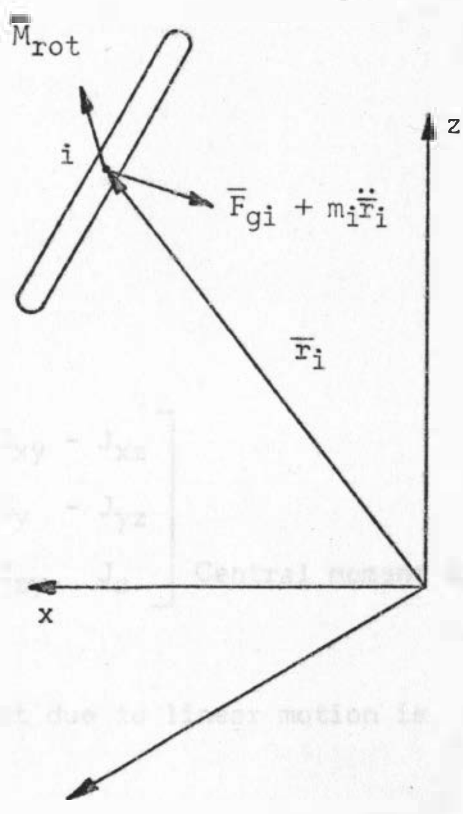


Fig. 3

The moment equations for the supporting point are formulated by use of a matrix notation. Referring to Fig. 3, the moments about the fixed x, y, z axes are divided into three parts for each member:

a) The inertia moment due to rotational motion is

$$\{M\}_{rot} = - d/dt([J]\{\omega\})$$

where

$$\{M\} = \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix}$$

$$\{\omega\} = \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix}$$

and

$$[J] = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{yx} & J_y & -J_{yz} \\ -J_{zx} & -J_{zy} & J_z \end{bmatrix}$$ Central moment of inertia

b) The inertia moment due to linear motion is

$$\{M\}_{lin} = -[x]\{F\}$$

where

$[x]$ is the skew symmetric matrix

$$[x] = \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$$

with x, y, z being components of the position vector $\bar{r}$, and

$$\{F\} = \begin{Bmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \end{Bmatrix} = m\{\ddot{x}\}$$

thus,

$$\{M\}_{lin} = - m[x]\{\ddot{x}\}$$

c) The moment due to gravity is

$$\{M\}_g = [x]\{F_{gr}\} = [x]\begin{Bmatrix} 0 \\ 0 \\ - mg \end{Bmatrix} = - m[x]\{g\}$$

For the physical model considered $\dot{\omega}_x$ and $\dot{\omega}_z$ are zero for all members concerned and $[J]$ reduces to a diagonal matrix $[J]$. The rotational moment when expanded becomes

$$\{M\}_{rot} = - (d/dt[J])\{\omega\} - [J]\{\dot{\omega}\})$$

and since J_y is constant but ω_x and ω_z are zero

$$d/dt[J] = 0$$

the equation reduces to

$$\{M\}_{rot} = -\begin{Bmatrix} 0 \\ J_y\,\dot{\omega}_y \\ 0 \end{Bmatrix} = -\{J\,\dot{\omega}\}$$

Combining the three parts of the moment, the resultant moment about the supporting point for each member will be

$$\{M\}_i = -\{J\,\dot{\omega}\}_i - m_i[x]_i\{\ddot{x}\}_i - m[x]_i\{g\}$$

For all the members of the bipedal structure, the following equation results:

$$\{M\} = \sum_{i=1}^{n}\{M\}_i =$$

$$\sum_{i=1}^{n}\left(-\{J\,\dot{\omega}\}_i - m_i[x]_i\{\ddot{x}\}_i - [x]_i\{g\}\right) = 0 \qquad (3\text{-}1)$$

Of the three moments, M_z need not be considered because sufficient friction to prevent such motion has been assumed.

A similar procedure for the moment equations for the passive member joints is used leading to the resultant moment about the joint p:

$$\{M\}_p = \sum_{i=1}^{m} \{M\}_{pi} =$$

$$\sum_{i=1}^{m} \left(-\{J\dot{\omega}\}_{pi} - m_{pi}[x]_{pi}\{\ddot{x}\}_i - m_{pi}[x]_{pi}\{g\} \right) = 0$$

where $\{x\}_p$ represents the position vector taken from the joint p to the mass center of the passive member and i goes only over the members influencing that moment.

For the physical model assumed only one member influences the moment; hence,

$$\{M\}_p = - \{J\dot{\omega}\}_p - m_p[x]_p\{\ddot{x}\} - m_p[x]_p\{g\} = 0$$

where p represents the left and right passive member joints. Only one moment component, M_y, needs to be used since the passive members are only allowed to swing about the y axis. The other components represent internal moments.

The damping considered must also be added to the moment equations about the passive member joints. Damping is a function of the difference between the angular velocities of passive members and the lower body mass. This difference will be denoted by $\Delta\omega$. The damping moment can be written in matrix form in the following manner:

$$\text{Damping moment} = \eta \begin{Bmatrix} 0 \\ \Delta\omega \\ 0 \end{Bmatrix}_p = \eta \{\Delta\omega\}_p$$

The resultant moment equations about the passive member joints are

$$\{M\}_p = - \{J\,\dot{\omega}\}_p - m_p[x]_p\{\ddot{x}\} - m_p[x]_p\{g\} - \eta\,\{\Delta\,\omega\}_p = 0 \tag{3-2}$$

Equations (3-1) and (3-2) were used to set up the four differential equations describing the system. The first two equations are obtained from equation (3-1) by substituting the following:

$$\left.\begin{aligned} &\omega_{y1} = \omega_{y2} = \omega_{y3} = -\omega_{y4} = \omega_{y5} = \dot{\alpha} \\ &\text{(according to the chosen gait)} \\ &\omega_{y7} = \beta_1 \\ &\omega_{y8} = \beta_2 \end{aligned}\right\} \tag{3-3}$$

From the scheme of denoting the members given in Fig. 2, the position vectors are evaluated for each component for Part I and Part II of the step.

<u>For Part I</u>

$$\{x\}_1 = \begin{Bmatrix} a \sin\alpha \\ 0 \\ a \cos\alpha \end{Bmatrix}$$

$$
\begin{gathered}
\{x\}_2 = \begin{Bmatrix} (2a + b)\sin\alpha \\ 0 \\ (2a + b)\cos\alpha \end{Bmatrix} \\[1em]
\{x\}_3 = \begin{Bmatrix} (a + 4b)\sin\alpha \\ 2d \\ a\cos\alpha \end{Bmatrix} \\[1em]
\{x\}_4 = \begin{Bmatrix} (3b + 2a)\sin\alpha \\ 2d \\ (2a + b)\cos\alpha \end{Bmatrix} \\[1em]
\{x\}_5 = \begin{Bmatrix} 2(a + b)\sin\alpha \\ d \\ 2(a + b)\cos\alpha \end{Bmatrix} \\[1em]
\{x\}_6 = \begin{Bmatrix} 2(a + b)\sin\alpha - c\cos\theta\sin\psi \\ c\sin\theta - d \\ 2(a + b)\cos\alpha + c\cos\theta\cos\psi \end{Bmatrix} \\[1em]
\{x\}_7 = \begin{Bmatrix} 2(a + b)\sin\alpha + e\sin\beta_1 \\ h + d \\ 2(a + b)\cos\alpha - e\cos\beta_1 \end{Bmatrix}
\end{gathered}
\tag{3-4}
$$

$$\{x\}_8 = \begin{Bmatrix} 2(a + b) \sin \alpha + e \sin \beta_2 \\ h-d \\ 2(a + b) \cos \alpha - e \cos \beta_2 \end{Bmatrix}$$

For Part II

$$\{x\}_1 = \begin{Bmatrix} (a - 4b) \sin \alpha \\ 2d \\ a \cos \alpha \end{Bmatrix}$$

$$\{x\}_2 = \begin{Bmatrix} (2a - 3b) \sin \alpha \\ 2d \\ (2a + b) \cos \alpha \end{Bmatrix}$$

$$\{x\}_3 = \begin{Bmatrix} a \sin \alpha \\ 0 \\ a \cos \alpha \end{Bmatrix}$$

$$\{x\}_4 = \begin{Bmatrix} (2a - b) \sin \alpha \\ 0 \\ (2a + b) \cos \alpha \end{Bmatrix} \tag{3-5}$$

$$\{x\}_5 = \begin{Bmatrix} 2(a - b) \sin \alpha \\ d \\ 2(a + b) \cos \alpha \end{Bmatrix}$$

$$\{x\}_6 = \begin{Bmatrix} 2(a - b)\sin\alpha - c\cos\theta\sin\psi \\ -c\sin\theta - d \\ 2(a + b)\cos\alpha + c\cos\theta\cos\psi \end{Bmatrix}$$

$$\{x\}_7 = \begin{Bmatrix} 2(a - b)\sin\alpha + e\sin\beta_1 \\ h + d \\ 2(a + b)\cos\alpha - e\cos\beta_1 \end{Bmatrix}$$

$$\{x\}_8 = \begin{Bmatrix} 2(a - b)\sin\alpha - e\sin\beta_2 \\ h - d \\ 2(a + b)\cos\alpha - e\cos\beta_2 \end{Bmatrix}$$

The second derivative with respect to time of these position vectors results in the acceleration of each mass center.

<u>For Part I</u>

$$\{\ddot{x}\}_1 = \begin{Bmatrix} a(\ddot{\alpha}\cos\alpha - \dot{\alpha}^2\sin\alpha) \\ 0 \\ -a(\ddot{\alpha}\sin\alpha + \dot{\alpha}^2\cos\alpha) \end{Bmatrix}$$

$$\{\ddot{x}\}_2 = \begin{Bmatrix} (2a + b)(\ddot{\alpha}\cos\alpha - \dot{\alpha}^2\sin\alpha) \\ 0 \\ -(2a + b)(\ddot{\alpha}\sin\alpha + \dot{\alpha}^2\cos\alpha) \end{Bmatrix}$$

$$\{\ddot{x}\}_3 = \begin{Bmatrix} (a + 4b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) \\ 0 \\ - a(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \end{Bmatrix}$$

$$\{\ddot{x}\}_4 = \begin{Bmatrix} (3b + 2a)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) \\ 0 \\ - (2a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \end{Bmatrix} \tag{3-6}$$

$$\{\ddot{x}\}_5 = \begin{Bmatrix} 2(a + b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) \\ 0 \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \end{Bmatrix}$$

$$\{\ddot{x}\}_6 = \begin{Bmatrix} 2(a + b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) + \\ \quad + c\,[- \cos \theta (\ddot{\psi} \cos \psi - \dot{\psi}^2 \sin \psi) + \\ \quad + 2\dot{\psi}\dot{\theta} \cos \psi \sin \theta + \sin \psi (\ddot{\theta} \sin \theta + \\ \quad + \dot{\theta}^2 \cos \theta)] \\ c(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) - \\ \quad - c\,[\cos \theta (\ddot{\psi} \sin \psi + \dot{\psi}^2 \cos \psi) - \\ \quad - 2\dot{\theta}\dot{\psi} \sin \psi \sin \theta + \cos \psi (\ddot{\theta} \sin \theta + \\ \quad + \dot{\theta}^2 \cos \theta)] \end{Bmatrix}$$

$$\{\ddot{x}\}_7 = \begin{Bmatrix} 2(a + b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) + \\ \quad + e(\ddot{\beta}_1 \cos \beta_1 - \dot{\beta}_1{}^2 \sin \beta_1) \\ 0 \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) + \\ \quad + e(\ddot{\beta}_1 \sin \beta_1 + \dot{\beta}_1{}^2 \cos \beta_1) \end{Bmatrix}$$

$$\{\ddot{x}\}_8 = \begin{Bmatrix} 2(a + b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) + \\ \quad + e(\ddot{\beta}_2 \cos \beta_2 - \dot{\beta}_2{}^2 \sin \beta_2) \\ 0 \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) + \\ \quad + e(\ddot{\beta}_2 \sin \beta_2 + \dot{\beta}_2{}^2 \cos \beta_2) \end{Bmatrix}$$

<u>For Part II</u>

$$\{\ddot{x}\}_1 = \begin{Bmatrix} (a - 4b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) \\ 0 \\ - a(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \end{Bmatrix}$$

$$\{\ddot{x}\}_2 = \begin{Bmatrix} (2a - 3b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) \\ 0 \\ - (2a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \end{Bmatrix}$$

$$\{\ddot{x}\}_3 = \begin{Bmatrix} a(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) \\ 0 \\ - a(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \end{Bmatrix}$$

$$\{\ddot{x}\}_4 = \left\{ \begin{matrix} (2a - b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) \\ 0 \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \end{matrix} \right\}$$

$$\{\ddot{x}\}_5 = \left\{ \begin{matrix} 2(a - b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) \\ 0 \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) \end{matrix} \right\} \qquad (3\text{-}7)$$

$$\{\ddot{x}\}_6 = \left\{ \begin{matrix} 2(a - b)(\ddot{\alpha} \cos \alpha - \ddot{\alpha}^2 \sin \alpha) + \\ \quad + c\ [- \cos \theta\,(\ddot{\psi} \cos \psi - \dot{\psi}^2 \sin \psi) + \\ \quad + 2\dot{\psi}\dot{\theta} \cos \psi \sin \theta + \sin \psi(\ddot{\theta} \sin \theta + \\ \quad + \dot{\theta}^2 \cos \theta)] \\ - c(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) - \\ \quad - c[\cos \theta\,(\ddot{\psi} \sin \psi + \dot{\psi}^2 \cos \psi) - \\ \quad - 2\dot{\theta}\dot{\psi} \sin \theta \sin \psi + \cos \psi(\ddot{\theta} \sin \theta + \\ \quad + \dot{\theta}^2 \cos \theta)] \end{matrix} \right\}$$

$$\{\ddot{x}\}_7 = \left\{ \begin{matrix} 2(a - b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) + e(\ddot{\beta}_1 \cos \beta_1 - \\ \quad - \dot{\beta}_1^2 \sin \beta_1) \\ 0 \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) + \\ \quad + e(\ddot{\beta}_1 \sin \beta_1 + \dot{\beta}_1^2 \cos \beta_1) \end{matrix} \right\}$$

$$\{\ddot{x}\}_8 = \begin{Bmatrix} 2(a + b)(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) + \\ + e(\ddot{\beta}_2 \cos \beta_2 - \dot{\beta}_2^2 \cos \beta_2) \\ 0 \\ - 2(a + b)(\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha) + \\ + e(\ddot{\beta}_2 \sin \beta_2 + \dot{\beta}_2^2 \cos \beta_2) \end{Bmatrix}$$

The last two equations in the system of four differential equations describing the system are obtained from equation (3-2) by the following substitutions:

<u>For member 7</u>

$$\left.\begin{aligned} \omega_p &= \dot{\beta}_1 \\ \Delta\omega &= (\dot{\beta}_1 + \dot{\alpha}) \\ \text{and} \quad \{x\}_p &= \begin{Bmatrix} e \sin \beta_1 \\ 0 \\ e \cos \beta_1 \end{Bmatrix} \end{aligned}\right\} \qquad (3\text{-}8)$$

The $\{\ddot{x}\}$ vector is identical to the $\{\ddot{x}\}_7$ used previously.

For member 8

$$\left.\begin{aligned} \omega_p &= \dot{\beta}_2 \\ \Delta\omega &= (\dot{\beta}_2 + \dot{\alpha}) \\ \text{and} \quad \{x\}_p &= \begin{Bmatrix} e \sin \beta_2 \\ 0 \\ e \cos \beta_2 \end{Bmatrix} \end{aligned}\right\} \qquad (3\text{-}9)$$

The $\{\ddot{x}\}$ vector is identical to the $\{\ddot{x}\}_8$ used previously.

Substituting equations (3-3), (3-4), and (3-6) into equation (3-1) results in two moment equations for Part I of the step. Substituting equations (3-3), (3-5), and (3-7) into equation (3-1) results in two similar moment equations for Part II of the step. The resulting four equations for the two parts of the step are similar except for a few terms which can be separated and defined for each part. These terms are denoted by B_i where $i = 0, 1, 2, \ldots 11$. The result is two equations in a concise form.

From the moment about the x axis

$$\begin{aligned}
&[(B_0 \sin\theta - 1)\cos\psi - 2(A_1 + A_2)\cos\theta\cos\alpha]\ddot{\theta} + \\
&+ [(B_0 - \sin\theta)\cos\theta\sin\psi]\ddot{\psi} + \\
&+ [2B_0A_1M_1 + 2B_0M_2(2A_1 + A_2) + 2B_0(A_1 + A_2)M_3 - \\
&- 2(A_1 + A_2)\sin\theta + 2B_0(A_1 + A_2) - \\
&- 4B_0M_6(A_1 + A_2)]\ddot{\alpha}\sin\alpha + \\
&+ [M_6A_5(A_6 + B_0)\sin\beta_1]\ddot{\beta}_1 + \\
&+ [-M_6A_5(A_6 - B_0)\sin\beta_2]\ddot{\beta}_2 + \\
&+ [B_0\cos\theta\cos\psi + 2(A_1 + A_2)\sin\theta\cos\alpha]\dot{\theta}^2 + \\
&+ [(B_0 - \sin\theta)\cos\theta\cos\psi]\dot{\psi}^2 + \qquad\qquad (3\text{-}10) \\
&+ [2B_0(2A_1 + A_2)M_2 + 2B_0A_1M_1 + 2B_0(A_1 + A_2)M_3 + \\
&+ 2B_0(A_1 + A_2) - 2(A_1 + A_2)\sin\theta - \\
&- 4B_0M_6(A_1 + A_2)]\dot{\alpha}^2\cos\alpha + \\
&+ [M_6A_5(A_6 + B_0)\cos\beta_1]\dot{\beta}_1^2 + \\
&+ [-M_6A_5(A_6 - B_0)\cos\beta_2]\dot{\beta}_2^2 + \\
&+ [-2(B_0 - \sin\theta)\sin\theta\sin\psi]\dot{\theta}\dot{\psi} + \\
&+ - A_4(B_0 - \sin\theta) - B_0A_4(2M_1 + 2M_2 + M_3) + \\
&+ 2B_0M_6A_4 = 0
\end{aligned}$$

From the moment about the y axis

$$
\begin{aligned}
&[2B_2 \sin \Theta]\,\ddot{\Theta} + \\
&\quad + [-\,2B_3 \cos \Theta - \cos^2 \Theta]\,\ddot{\psi} + \\
&\quad + [4(A_1 + A_2)(A_1 + B_1) + 2B_4 \cos \Theta + 2A_1(A_1 + 2B_1)\,M_1 + \\
&\quad + 4(A_1 + B_1)(2A_1 + A_2)\,M_2 + 4(A_1 + A_2)(A_1 + B_1)\,M_3 + \\
&\quad + 2M_4 + M_5 + 2M_6A_5B_5 + 2M_6A_5B_8 - \\
&\quad - 8M_6(A_1 + A_2)(A_1 + A_3)]\,\ddot{\alpha} + \\
&\quad + [M_6A_5^2 - 2M_6A_5B_7 + M_7]\,\ddot{\beta}_1 + \\
&\quad + [M_6A_5^2 - 2M_6A_5B_{10} + M_7]\,\ddot{\beta}_2 + \\
&\quad + [2B_2 \cos \Theta]\,\dot{\Theta}^2 + \\
&\quad + [2B_2 \cos \Theta]\,\dot{\psi}^2 + \\
&\quad + [-2B_2 \cos \Theta - 2M_6A_5B_6 - 2M_6A_5B_9]\,\dot{\alpha}^2 + \\
&\quad + [2M_6A_5B_6]\,\dot{\beta}_1^2 + \\
&\quad + [2M_6A_5B_9]\,\dot{\beta}_2^2 + \\
&\quad + [4B_3 \sin \Theta + 2 \sin \Theta \cos \Theta]\,\dot{\Theta}\,\dot{\psi} + \\
&\quad + [-\,A_4\{2(A_1 + 2B_1)\,M_1 + 2(A_1 + B_1)(2M_2 + M_3) + \\
&\quad + 4M_6(A_1 + A_3)\} \sin \alpha + A_4A_5M_6 \sin \beta_1 + \\
&\quad + A_4A_5M_6 \sin \beta_2 + A_4 \cos \Theta \sin \psi - \\
&\quad - 2A_4(A_1 + 2B_1) \sin \alpha] + C(\dot{\beta}_1 + \dot{\beta}_2 + 2\dot{\alpha}) = 0
\end{aligned}
\tag{3-11}
$$

Similarly for the moment about the passive member joints, expanding equation (3-2) with the appropriate terms from equations (3-8) and (3-9) for each of the two parts of the step results in four equations combined into two equations with similar B terms separated and defined for each part of the step. From the moment about the left passive member joint

$$
\begin{aligned}
&[2M_6B_5A_5]\,\ddot{\alpha} + \\
&\quad + [M_6A_5^{\,2} + M_7]\,\ddot{\beta}_1 + \\
&\quad + [-\,2M_6A_5B_5]\,\dot{\alpha}^{\,2} + \\
&\quad + [A_4M_6A_5 \sin \beta_1] + \\
&\quad + C(\dot{\beta}_1 + \dot{\alpha}) = 0
\end{aligned} \tag{3-12}
$$

From the moment about the right passive member joint

$$
\begin{aligned}
&[2M_6B_8A_5]\,\ddot{\alpha} + \\
&\quad + [M_6A_5^{\,2} + M_7]\,\ddot{\beta}_2 + \\
&\quad + [-\,2M_6A_5B_9]\,\dot{\alpha}^{2} + \\
&\quad + [A_4M_6A_5 \sin \beta_2] + \\
&\quad + C(\dot{\beta}_2 + \dot{\alpha}) = 0
\end{aligned} \tag{3-13}
$$

The subscripted B values are

Part I

$$
\begin{aligned}
B_0 &= A_3 \\
B_1 &= A_2 \\
B_2 &= (A_1 + A_2) \sin (\psi + \alpha) \\
B_3 &= (A_1 + A_2) \cos (\psi + \alpha) \\
B_4 &= B_3
\end{aligned} \tag{3-14}
$$

$$B_5 = (A_1 + A_2) \cos (\beta_1 + \alpha)$$
$$B_6 = (A_1 + A_2) \sin (\beta_1 + \alpha)$$
$$B_7 = B_5$$
$$B_8 = (A_1 + A_2) \cos (\beta_2 + \alpha)$$
$$B_9 = (A_1 + A_2) \sin (\beta_2 + \alpha)$$
$$B_{10} = B_8$$

Part II

$$B_0 = - A_3$$
$$B_1 = - A_2$$
$$B_2 = A_1 \sin (\psi + \alpha) + A_2 \sin (\psi - \alpha)$$
$$B_3 = A_1 \cos (\psi + \alpha) + A_2 \cos (\psi - \alpha)$$
$$B_4 = A_1 \cos (\psi + \alpha) - A_2 \cos (\psi - \alpha)$$
$$B_5 = A_1 \cos (\beta_1 + \alpha) - A_2 \cos (\beta_1 - \alpha) \qquad (3\text{-}15)$$
$$B_6 = A_1 \sin (\beta_1 + \alpha) + A_2 \sin (\beta_1 - \alpha)$$
$$B_7 = A_1 \cos (\beta_1 + \alpha) + A_2 \cos (\beta_1 - \alpha)$$
$$B_8 = A_1 \cos (\beta_2 + \alpha) - A_2 \cos (\beta_2 - \alpha)$$
$$B_9 = A_1 \sin (\beta_2 + \alpha) + A_2 \sin (\beta_2 - \alpha)$$
$$B_{10} = A_1 \cos (\beta_2 + \alpha) + A_2 \cos (\beta_2 - \alpha)$$

Dimensionless constants resulting from division of complete set of equations by $m_c c^2$ are

$$A_1 = a/c \qquad M_1 = m_a/m_c$$
$$A_2 = b/c \qquad M_2 = m_b/m_c$$
$$A_3 = d/c \qquad M_3 = m_o/m_c$$
$$A_4 = g/c \qquad M_4 = J_a/m_c c^2 \tag{3-16}$$
$$A_5 = e/c \qquad M_5 = J_o/m_c c^2$$
$$A_6 = h/c \qquad M_6 = m_e/m_c$$
$$C = \eta\ /m_c c^2 \text{ sec}^{-1} \qquad M_7 = J_e/m_c c^2$$

where a, b, c, d, h, e are geometric lengths of the biped structure; m_a, m_b, m_c, m_o, m_e are the masses of appropriate components of the biped structure; J_a, J_o, J_e are moments of inertia of appropriate components; η is torsional damping coefficient of passive members; and g is the gravitational constant.

The resulting four dynamic equations of motion (3-10), (3-11), (3-12), and (3-13) are non-linear, non-homogeneous differential equations with time dependent coefficients. However, due to the specific kinematic program of the shifting of the lower extremities, the only output coordinates of the system are the angles ψ and θ representing the motion of the compensating mass and the angles β_1 and β_2 representing the angular motion of the left and right passive members, respectively.

The specific kinematic program of the driving system will be assumed to be

$$\alpha = (\alpha_m/2)(1 - \cos \omega t) \tag{3-17}$$

where α_m is the step size. This kinematic program was used in previous study [1] and was also suitable here because it satisfied the condition for smoothed gait data.

The boundary conditions resulting from the cyclic gait are as follows:

Part I

$$\begin{aligned} \omega t = 0 \quad & \psi = \psi_0;\ \dot{\psi} = \dot{\psi}_0;\ \theta = \theta_0;\ \dot{\theta} = \dot{\theta}_0; \\ & \beta_1 = \beta_{10};\ \dot{\beta}_1 = \dot{\beta}_{10};\ \beta_2 = \beta_{20};\ \dot{\beta}_2 = \dot{\beta}_{20} \\ \omega t = \pi \quad & \psi = \psi_1;\ \dot{\psi} = \dot{\psi}_1;\ \theta = \theta_1;\ \dot{\theta} = \dot{\theta}_1; \\ & \beta_1 = \beta_{11};\ \dot{\beta}_1 = \dot{\beta}_{11};\ \beta_2 = \beta_{21};\ \dot{\beta}_2 = \dot{\beta}_{21} \end{aligned} \tag{3-18}$$

Part II

$$\begin{aligned} \omega t = \pi \quad & \psi = \psi_1;\ \dot{\psi} = \dot{\psi}_1;\ \theta = \theta_1;\ \dot{\theta} = \dot{\theta}_1; \\ & \beta_1 = \beta_{11};\ \dot{\beta}_1 = \dot{\beta}_{11};\ \beta_2 = \beta_{21};\ \dot{\beta}_2 = \dot{\beta}_{21} \\ \omega t = 2\pi \quad & \psi = \psi_0;\ \dot{\psi} = \dot{\psi}_0;\ \theta = -\theta_0;\ \dot{\theta} = -\dot{\theta}_0; \\ & \beta_1 = \beta_{10};\ \dot{\beta}_1 = \dot{\beta}_{10};\ \beta_2 = \beta_{20};\ \dot{\beta}_2 = \dot{\beta}_{20} \end{aligned} \tag{3-19}$$

CHAPTER IV

SATISFACTION OF BOUNDARY CONDITIONS

The mathematical model describing the physical system is a system of four non-linear, non-homogenous ordinary differential equations (3-10), (3-11), (3-12), and (3-13) with cyclic boundary conditions (3-18) and (3-19). Because it represents a boundary value problem, the differential equations must be solved by successive approximations where neither the initial nor the final conditions are known -- only their relationships.

Basically the method of successive approximations used is to solve the system of differential equations with estimated initial conditions. This solution normally does not satisfy the final conditions, but the initial values are estimated to have this solution near to them. The initial conditions are then adjusted toward satisfaction of final conditions.

The improvement of the initial conditions to meet the specified boundary conditions was made in a systematic way by using a form of Newton's method for finding roots of equations. The system was linearized in the vicinity of the previous solution and the sensitivity of the final values relative to the initial values was evaluated. The boundary conditions, expressed in terms of the necessary changes in initial and final conditions, resulted in a set of algebraic equations. From this set of algebraic equations, the necessary improvement in initial conditions was determined. This solution was used in the next cycle and recalculated in successive cycles until the boundary conditions were satisfied to the required accuracy.

The following represents a set of estimated initial conditions in the form of a column matrix:

$$\{\theta_0\ \psi_0\ \beta_{10}\ \beta_{20}\ \dot{\theta}_0\ \dot{\psi}_0\ \dot{\beta}_{10}\ \dot{\beta}_{20}\} = \{In\}$$

and the following represents a set of final conditions in the form of a column matrix:

$$\{\theta_t\ \psi_t\ \beta_{1t}\ \beta_{2t}\ \dot{\theta}_t\ \dot{\psi}_t\ \dot{\beta}_{1t}\ \dot{\beta}_{2t}\} = \{Fn\}$$

where t represents T/2 or half of a step. Both sets together must satisfy the boundary conditions (3-18) and (3-19), that is,

$$\begin{aligned}
\theta_t &= -\theta_0 \\
\psi_t &= \psi_0 \\
\beta_{1t} &= \beta_{10} \\
\beta_{2t} &= \beta_{20} \\
\dot{\theta}_t &= -\dot{\theta}_0 \\
\dot{\psi}_t &= \dot{\psi}_0 \\
\dot{\beta}_{1t} &= \dot{\beta}_{10} \\
\dot{\beta}_{2t} &= \dot{\beta}_{20}
\end{aligned}$$

or in matrix form

$$\{Fn\} = [W]\{In\} \tag{4-1}$$

where

$$[W] = \begin{bmatrix} -1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & 1 & & & & \\ & & & & -1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{bmatrix}$$

With the estimated initial conditions the boundary conditions will normally not be satisfied, and a correction $\{\Delta In\}$ in $\{In\}$ will be necessary. Since the corrected initial conditions have to meet the corrected final conditions, the following from equation (4-1) must hold:

$$\{Fn\} + \{\Delta Fn\} = [W](\{In\} + \{\Delta In\}) \tag{4-2}$$

The changes in the final conditions $\{\Delta Fn\}$ are a function of changes in initial conditions and can be expanded in a series. The resulting equations for $\{\Delta Fn\}$ are

$$\{\Delta Fn\} = \left[\frac{\partial Fn_i}{\partial In_j} \right]_I \{\Delta In\} + \tag{4-3}$$

$$\frac{1}{2}\begin{bmatrix} \ddots & & \\ & \lfloor \Delta In \rfloor & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \left[\dfrac{\partial Fn}{\partial In_i \ \partial Fn_j}\right] \\ \vdots \end{bmatrix}_I \Delta In \ + \ldots$$

where

$$\left[\frac{\partial Fn_i}{\partial In_j}\right]_I$$

represents the matrix of the Jacobian's terms evaluated in the vicinity of the previous solution,

$$\begin{bmatrix} \ddots & & \\ & \lfloor \Delta In \rfloor & \\ & & \ddots \end{bmatrix}$$

represents a diagonal matrix composed of a series of identical row matrices $\lfloor \Delta In \rfloor$ on the diagonal and

$$\begin{bmatrix} \vdots \\ \left[\dfrac{\partial Fn}{\partial In_i \ \partial Fn_j}\right] \\ \vdots \end{bmatrix}_I$$

represents a column of submatrices composed of second derivatives [5]. Since the estimated initial values are near the solution and corrections are expected to be small, only the linear approximation was retained. Equation (4-3) takes the form

$$\{\Delta Fn\} = \left[\frac{\partial Fn_i}{\partial In_j}\right]_I \{\Delta In\} = [U]\{\Delta In\}$$

where [U] has been introduced as a short notation. Substituting this into equation (4-2)

$$\{Fn\} + [U]\{\Delta In\} = [W](\{In\} + \{\Delta In\})$$

and solving for $\{\Delta In\}$

$$\{\Delta In\} = ([U] - [W])^{-1}([W]\{In\} - \{Fn\}) \qquad (4\text{-}4)$$

The Jacobian's terms of matrix [U] were found by use of sensitivity analysis. The partial derivatives in [U] can be considered as sensitivity coefficients and found by solving for the sensitivity functions. These sensitivity coefficients will be dealt with in the next section.

CHAPTER V

SENSITIVITY EQUATIONS

As a requirement for the successive approximation algorithm for satisfaction of the system boundary conditions, sensitivity analysis [4] was used to determine the true sensitivity coefficients. In previous studies [1,2] a finite increment method for determination of sensitivity coefficients was used; however, due to the requirement of this particular study, true sensitivity coefficients were introduced.

The successive approximation algorithm required the sensitivity of the final conditions to a change in the initial conditions. This requirement was satisfied by using sensitivity analysis of the dynamic system when considering parameter variations. The dynamic system in this case was composed of the four moment equations which take the following form:

$$F_i(\ddot{\theta}, \dot{\theta}, \theta, \ddot{\psi}, \dot{\psi}, \psi, \ddot{\beta}_1, \dot{\beta}_1, \beta_1, \ddot{\beta}_2, \dot{\beta}_2, \beta_2, t) = 0$$

where i = 1, 2, 3, 4 and the parameters varied were the initial conditions

$$\{In\} = \{\theta_0 \ \ \psi_0 \ \ \beta_{10} \ \ \beta_{20} \ \ \dot{\theta}_0 \ \ \dot{\psi}_0 \ \ \dot{\beta}_{10} \ \ \dot{\beta}_{20}\}$$

To evaluate the sensitivity coefficients the preceding equations were differentiated with respect to a parameter to be varied.

The resulting equations are

$$\frac{\partial F_i}{\partial \ddot{\theta}}\frac{\partial \ddot{\theta}}{\partial P_j} + \frac{\partial F_i}{\partial \dot{\theta}}\frac{\partial \dot{\theta}}{\partial P_j} + \frac{\partial F_i}{\partial \theta}\frac{\partial \theta}{\partial P_j} +$$

$$\frac{\partial F_i}{\partial \ddot{\psi}}\frac{\partial \ddot{\psi}}{\partial P_j} + \frac{\partial F_i}{\partial \dot{\psi}}\frac{\partial \dot{\psi}}{\partial P_j} + \frac{\partial F_i}{\partial \psi}\frac{\partial \psi}{\partial P_j} + \tag{5-1}$$

$$\frac{\partial F_i}{\partial \ddot{\beta}_1}\frac{\partial \ddot{\beta}_1}{\partial P_j} + \frac{\partial F_i}{\partial \dot{\beta}_1}\frac{\partial \dot{\beta}_1}{\partial P_j} + \frac{\partial F_i}{\partial \beta_1}\frac{\partial \beta_1}{\partial P_j} +$$

$$\frac{\partial F_i}{\partial \ddot{\beta}_2}\frac{\partial \ddot{\beta}_2}{\partial P_j} + \frac{\partial F_i}{\partial \dot{\beta}_2}\frac{\partial \dot{\beta}_2}{\partial P_j} + \frac{\partial F_i}{\partial \beta_2}\frac{\partial \beta_2}{\partial P_j} + \frac{\partial F_i}{\partial P_j} = 0$$

where $i = 1, 2, 3, 4$ representing the four moment equations;

$j = 1, 2, 3, \ldots 8$ represents the eight parameters that are varied; or

$$\begin{aligned} P_1 &= \theta_0 & P_5 &= \dot{\theta}_0 \\ P_2 &= \psi_0 & P_6 &= \dot{\psi}_0 \\ P_3 &= \beta_{10} & P_7 &= \dot{\beta}_{10} \\ P_4 &= \beta_{20} & P_8 &= \dot{\beta}_{20} \end{aligned} \tag{5-2}$$

Also,

$$\frac{\partial F_i}{\partial P_j} = 0$$

since the functions are directly independent of P_j.

Equation (5-1) can further be reduced by introducing the following notation for the sensitivity functions:

$$\frac{\partial \theta}{\partial P_j} = U_{1j}$$

$$\frac{\partial \dot{\theta}}{\partial P_j} = \frac{d}{dt}\left(\frac{\partial \theta}{\partial P_j}\right) = \dot{U}_{1j}$$

$$\frac{\partial \ddot{\theta}}{\partial P_j} = \frac{d^2}{dt^2}\left(\frac{\partial \theta}{\partial P_j}\right) = \ddot{U}_{1j}$$

and similarly for nine analogous terms.

Rewriting equation (5-1) results in the following system:

$$\frac{\partial F_i}{\partial \ddot{\theta}} \ddot{U}_{1j} + \frac{\partial F_i}{\partial \dot{\theta}} \dot{U}_{1j} + \frac{\partial F_i}{\partial \theta} U_{1j} +$$

$$\frac{\partial F_i}{\partial \ddot{\psi}} \ddot{U}_{2j} + \frac{\partial F_i}{\partial \dot{\psi}} \dot{U}_{2j} + \frac{\partial F_i}{\partial \psi} U_{2j} +$$

$$\frac{\partial F_i}{\partial \ddot{\beta}_1} \ddot{U}_{3j} + \frac{\partial F_i}{\partial \dot{\beta}_1} \dot{U}_{3j} + \frac{\partial F_i}{\partial \beta_1} U_{3j} +$$

$$\frac{\partial F_i}{\partial \ddot{\beta}_2} \ddot{U}_{4j} + \frac{\partial F_i}{\partial \dot{\beta}_2} \dot{U}_{4j} + \frac{\partial F_i}{\partial \beta_2} U_{4j} = 0 \tag{5-3}$$

$i = 1, 2, 3, 4$ and $j = 1, 2, 3, \ldots 8$ where

$$\frac{\partial F_i}{\partial \ddot{\theta}}, \frac{\partial F_i}{\partial \dot{\theta}}, \ldots$$

are evaluated from the dynamic system equations. Expanding equation (5-3) for all j values results in 32 second order linear homogeneous ordinary differential equations grouped in eight systems of four simultaneous equations. These sensitivity equations are always linear because the coefficients

$$\frac{\partial F}{\partial \ddot{\theta}}, \frac{\partial F}{\partial \dot{\theta}}, \ldots$$

are not functions of U or of its derivatives.

Thus far the parameters to be varied do not appear in the sensitivity equations; however, as the parameters to be varied represent the initial conditions, they will appear when considering the initial conditions for the sensitivity equations.

For the first of the eight systems of four sensitivity equations that correspond to the parameter $P_1 = \theta_0$, the initial conditions will all be zero but $(U_{\theta\theta})_{t=0}$. That is, by varying θ at t = 0 no initial value will be influenced but θ itself. Thus,

$$(U_{\theta\theta})_{t=0} = \left(\frac{\partial \theta}{\partial \theta_0}\right)_{t=0} = \frac{\partial \theta_0}{\partial \theta_0} = 1$$

The solution of these first four sensitivity equations

$$(U_{\theta\theta})_{t=T/2}\ ,\quad (U_{\dot{\gamma}\theta})_{t=T/2}\ ,\ \cdots\ (U_{\dot{\beta}_2\theta})_{t=T/2}$$

will represent the sensitivity coefficients

$$U_{11}\ ,\ U_{21}\ ,\ \cdots\ U_{81}$$

in the first column of the matrix [U] in equation (4-4). Similarly the other seven systems of sensitivity equations will generate the remaining columns of the matrix [U].

The sensitivity functions, grouped in systems of four simultaneous equations, need the values of the original variables, as a function of time, obtained from the dynamic system equations. Instead of solving the dynamic equations, storing the numerical values obtained, and interpolating to obtain values for the sensitivity equations, it is more convenient to solve the system of sensitivity functions simultaneously with the dynamic system. Furthermore, this can be done for all eight sensitivity systems simultaneously, thus saving the computing time accumulated through repeated solution of the dynamic equations. Thus the total system of 36 second order differential equations was set up, consisting of four dynamic equations and eight sensitivity systems of four equations each. The total system was then reduced to a standard form of 72 first order differential equations to be solved simultaneously.

The coefficients for the sensitivity equations, starting with M_x, are as follows:

$$\frac{\partial F_1}{\partial \theta} = [B_0 \cos \theta \cos \psi + 2(A_1 + A_2) \sin \theta \cos \alpha] \ddot{\theta} + [- B_0 \sin \theta \sin \psi + \sin \psi (\sin^2 \theta - \cos^2 \theta)] \ddot{\psi} + [- 2(A_1 + A_2) \cos \theta \sin \alpha] \ddot{\alpha} + [- B_0 \sin \theta \cos \psi + 2(A_1 + A_2) \cos \theta \cos \alpha] \dot{\theta}^2 + [- B_0 \sin \theta \cos \psi + \cos \psi (\sin^2 \theta - \cos^2 \theta)] \dot{\psi}^2 + [- 2(A_1 + A_2) \cos \theta \cos \alpha] \dot{\alpha}^2 + [- 2 \cos \theta \sin \psi + 2 \sin \theta \cos \theta \sin \psi] \dot{\theta} \dot{\psi} + [+ A_4(\sin \theta)]$$

$$\frac{\partial F_1}{\partial \dot{\theta}} = [2B_0 \cos \theta \cos \psi + 4(A_1 + A_2) \sin \theta \cos \alpha] \dot{\theta} + [- 2(B_0 - \sin \theta) \sin \theta \sin \psi] \dot{\psi}$$

$$\frac{\partial F_1}{\partial \ddot{\theta}} = (B_0 \sin \theta - 1) \cos \psi - 2(A_1 + A_2) \cos \theta \cos \alpha$$

$$\frac{\partial F_1}{\partial \psi} = [- B_0 \sin\theta \sin\psi + \sin\psi]\ddot{\theta} +$$
$$+ [(B_0 - \sin\theta) \cos\theta \cos\psi]\ddot{\psi} +$$
$$+ [- B_0 \cos\theta \sin\psi]\dot{\theta}^2 +$$
$$+ [(\sin\theta - B_0) \cos\theta \sin\psi]\dot{\psi}^2 +$$
$$+ [- 2(B_0 = \sin\theta) \sin\theta \cos\psi]\dot{\theta}\dot{\psi}$$

$$\frac{\partial F_1}{\partial \dot{\psi}} = [2(B_0 - \sin\theta) \cos\theta \cos\psi]\dot{\psi} +$$
$$+ [- 2 (B_0 - \sin\theta) \sin\theta \sin\psi]\dot{\theta}$$

$$\frac{\partial F_1}{\partial \ddot{\psi}} = (B_0 - \sin\theta) \cos\theta \sin\psi$$

$$\frac{\partial F_1}{\partial \beta_1} = [M_6 A_5 (A_6 + B_0) \cos\beta_1]\ddot{\beta}_1 +$$
$$+ [- M_6 A_5 (A_6 + B_0) \sin\beta_1]\dot{\beta}_1^2$$

$$\frac{\partial F_1}{\partial \dot{\beta}_1} = [2 M_6 A_5 (A_6 + B_0) \cos\beta_1]\dot{\beta}_1$$

$$\frac{\partial F_1}{\partial \ddot{\beta}_1} = M_6 A_5 (A_6 + B_0) \sin\beta_1$$

$$\frac{\partial F_1}{\partial \beta_2} = [- M_6 A_5 (A_6 - B_0) \cos\beta_2]\ddot{\beta}_2$$
$$+ [M_6 A_5 (A_6 - B_0) \sin\beta_2]\dot{\beta}_2^2$$

$$\frac{\partial F_1}{\partial \dot{\beta}_2} = [-\ 2M_6A_5(A_6 - B_0)\cos\beta_2]\dot{\beta}_2$$

$$\frac{\partial F_1}{\partial \ddot{\beta}_2} = -\ M_6A_5(A_6 - B_0)\sin\beta_2$$

$$\begin{aligned}\frac{\partial F_2}{\partial \theta} = {} & [2B_2\cos\theta]\ \ddot{\theta} + \\ & + [2B_3\sin\theta + 2\cos\theta\sin\theta]\ \ddot{\psi} + \\ & + [-\ 2B_4\sin\theta]\ \ddot{\alpha} + \\ & + [-\ 2B_2\sin\theta]\ \dot{\theta}^2 + \\ & + [-\ 2B_2\sin\theta]\ \dot{\psi}^2 + \\ & + [+\ 2B_2\sin\theta]\ \dot{\alpha}^2 + \\ & + [4B_3\cos\theta + 2(\cos^2\theta - \sin^2\theta)]\ \dot{\theta}\ \dot{\psi} + \\ & + A_4\,[-\sin\theta\ \sin\psi]\end{aligned}$$

$$\begin{aligned}\frac{\partial F_2}{\partial \dot{\theta}} = {} & [4B_2\cos\theta]\ \dot{\theta} + [4B_3\sin\theta + \\ & + 2\sin\theta\cos\theta]\ \dot{\psi}\end{aligned}$$

$$\frac{\partial F_2}{\partial \ddot{\theta}} = 2B_2\sin\theta$$

$$\frac{\partial F_2}{\partial \psi} = [2DB_2 \sin\theta]\ddot{\theta} + $$
$$+ [-2DB_3 \cos\theta]\ddot{\psi} +$$
$$+ [2DB_4 \cos\theta]\ddot{\alpha} +$$
$$+ [2DB_2 \cos\theta]\dot{\theta}^2 +$$
$$+ [2DB_2 \cos\theta]\dot{\psi}^2 +$$
$$+ [2DB_2 \cos\theta]\dot{\alpha}^2 +$$
$$+ [4DB_3 \sin\theta]\dot{\theta}\ \dot{\psi} +$$
$$+ A_4 \cos\theta \ \cos\psi$$

$$\frac{\partial F_2}{\partial \dot{\psi}} = [4B_2 \cos\theta]\dot{\psi} +$$
$$+ [4B_3 \sin\theta + 2\sin\theta \cos\theta]\dot{\theta}$$

$$\frac{\partial F_2}{\partial \ddot{\psi}} = -2B_3 \cos\theta - \cos^2\theta$$

$$\frac{\partial F_2}{\partial \beta_1} = [2M_6A_5DB_5]\ddot{\alpha} +$$
$$+ [-2M_6A_5DB_7]\ddot{\beta}_1 +$$
$$+ [-2M_6A_5DB_6]\dot{\alpha}^2 +$$
$$+ [2M_6A_5DB_6]\dot{\beta}_1^2 +$$
$$+ [A_4A_5M_6 \cos\beta_1]$$

$$\frac{\partial F_2}{\partial \dot{\beta}_1} = [4M_6A_5B_6]\dot{\beta}_1 + \eta$$

$$\frac{\partial F_2}{\partial \ddot{\beta}_1} = M_6 A_5^2 - 2M_6 A_5 B_7 + M_7$$

$$\begin{aligned}\frac{\partial F_2}{\partial \beta_2} = {} & [2M_6 A_5 DB_8]\,\ddot{\alpha} + \\ & + [-\,2M_6 A_5 DB_{10}]\,\ddot{\beta}_2 + \\ & + [-\,2M_6 A_5 DB_9]\,\dot{\alpha}^2 + \\ & + [2M_6 A_5 DB_9]\,\dot{\beta}_2^2 + \\ & + [A_4 A_5 M_6 \cos \beta_2]\end{aligned}$$

$$\frac{\partial F_2}{\partial \dot{\beta}_2} = [4M_6 A_5 B_9]\,\dot{\beta}_2 + \eta$$

$$\frac{\partial F_2}{\partial \ddot{\beta}_2} = M_6 A_5^2 - 2M_6 A_5 B_{10} + M_7$$

From the left passive member moment

$$\begin{aligned}\frac{\partial F_3}{\partial \beta_1} = {} & [2M_6 DB_5 A_5]\,\ddot{\alpha} + \\ & + [-\,2\,M_5 A_5 DB_6]\,\dot{\alpha}^2 + \\ & + [A_4 M_6 A_5 \cos \beta_1]\end{aligned}$$

$$\frac{\partial F_3}{\partial \dot{\beta}_1} = \eta$$

$$\frac{\partial F_3}{\partial \ddot{\beta}_1} = M_6 A_5{}^2 + M_7$$

From the right passive member moment

$$\frac{\partial F_4}{\partial \beta_2} = [2M_6 DB_8 A_5]\,\ddot{\alpha} + + [-\,2M_6 A_5 DB_9]\,\dot{\alpha}^2 + + A_4 M_6 A_5 \cos \beta_2$$

$$\frac{\partial F_4}{\partial \dot{\beta}_2} = \eta$$

$$\frac{\partial F_4}{\partial \ddot{\beta}_2} = M_6 A_5{}^2 + M_7$$

The subscripted DB values are derivatives of B and are represented as follows:

Part I

$$DB_2 = (A_1 + A_2) \cos (\psi + \alpha)$$

$$DB_3 = -\,(A_1 + A_2) \sin (\psi + \alpha)$$

$$DB_4 = DB_3$$

$$DB_5 = -(A_1 + A_2)\sin(\beta_1 + \alpha)$$

$$DB_6 = (A_1 + A_2)\cos(\beta_1 + \alpha)$$

$$DB_7 = -(A_1 + A_2)\sin(\beta_1 + \alpha)$$

$$DB_8 = -(A_1 + A_2)\sin(\beta_2 + \alpha)$$

$$DB_9 = (A_1 + A_2)\cos(\beta_2 + \alpha)$$

$$DB_{10} = DB_8$$

Part II

$$DB_2 = A_1 \cos(\psi + \alpha) + A_2 \cos(\psi - \alpha)$$

$$DB_3 = -A_1 \sin(\psi + \alpha) - A_2 \sin(\psi - \alpha)$$

$$DB_4 = -A_1 \sin(\psi + \alpha) + A_2 \sin(\psi - \alpha)$$

$$DB_5 = -A_1 \sin(\beta_1 + \alpha) + A_2 \sin(\beta_1 - \alpha)$$

$$DB_6 = A_1 \cos(\beta_1 + \alpha) + A_2 \cos(\beta_1 - \alpha)$$

$$DB_7 = -A_1 \sin(\beta_1 + \alpha) - A_2 \sin(\beta_1 - \alpha)$$

$$DB_8 = -A_1 \sin(\beta_2 + \alpha) + A_2 \sin(\beta_2 - \alpha)$$

$$DB_9 = A_1 \cos(\beta_2 + \alpha) + A_2 \cos(\beta_2 - \alpha)$$

$$DB_{10} = -A_1 \sin(\beta_2 + \alpha) - A_2 \sin(\beta_2 - \alpha)$$

CHAPTER VI

COMPUTER ALGORITHMIZATION

The mathematical model describing the bipedal walking machine is, in a standard form, composed of eight dynamic equations originating from (3-10), (3-11), (3-12), and (3-13); 64 sensitivity equations originating from (5-3); and the equation of the successive approximation algorithm (4-4). The 72 first order ordinary differential equations, to be solved simultaneously, are solved by use of the digital computer through use of standard numerical methods. The specific method used here was the Runga-Kutta fourth order integration method.

The initial conditions for the eight dynamic equations are the estimated initial conditions. The initial conditions for the sensitivity equations are all zero except for the parameter varied which is equal to one in each of the system of equations. This simultaneous system with the initial conditions is illustrated in Fig. 5.

The estimated initial conditions are needed for the dynamic equations and must be selected so they will produce the final conditions near the initial conditions used. In order to satisfy this requirement initially, all initial conditions can be zero if the rate of step is fast. The fast rate of step produces the acceleration needed and results in small angular velocity and displacement; the final conditions will remain close to the zero estimated initial conditions. In this manner the system can achieve one step and then correct the zero initial conditions toward satisfaction of the final conditions. However, if the rate of step from zero is too slow, the coordinate angles on the coordi-

8 equations	Eight first order differential equations of the dynamic system originating from equations (3-10), (3-11), (3-12), and (3-13) with estimated initial conditions.
8 equations	Eight sensitivity equations originating from equation (5-3) with $P_1 = \Theta_0$ and with initial condition zero except $(U_{11})_0 = 1$
8 equations	Eight sensitivity equations originating from equation (5-3) with $P_2 = \psi_0$ and with initial conditions zero except $(U_{22})_0 = 1$
8 equations	Eight sensitivity equations originating from equation (5-3) with $P_3 = \beta_{10}$ and with initial conditions zero except $(U_{33})_0 = 1$
8 equations	Eight sensitivity equations originating from equation (5-3) with $P_4 = \beta_{20}$ and with initial conditions zero except $(U_{44})_0 = 1$
8 equations	Eight sensitivity equations originating from equation (5-3) with $P_5 = \dot{\theta}_0$ and with initial conditions zero except $(\dot{U}_{15})_0 = 1$
8 equations	Eight sensitivity equations originating from equation (5-3) with $P_6 = \dot{\psi}_0$ and with initial conditions zero except $(\dot{U}_{26})_0 = 1$

Fig. 5 (Continued)

8 equations	Eight sensitivity equations originating from equation (5-3) with $P_7 = \beta_{10}$ and with initial conditions zero except $(U_{37})_0 = 1$
8 equations	Eight sensitivity equations originating from equation (5-3) with $P_8 = \beta_{20}$ and with initial conditions zero except $(U_{48})_0 = 1$

Fig. 5

nate mass will become too large to achieve equilibrium (1.0 radian is maximum allowed coordinate angle on compensating mass because of physical reasons), and no final condition will be reached. Once the boundary conditions have been satisfied for a fast rate of step, the rate can then be slowed by incremental amounts taking initial conditions from the previous solution. This process is continued until the desired rate of step has been attained.

To each initial condition chosen in the way described above, a successive approximation algorithm was applied which led to the satisfaction of the boundary conditions. This applied successive approximation algorithm uses the final sensitivity coefficients from the sensitivity equations to correct the estimated initial conditions. This cycle is then repeated until accuracy of the boundary conditions has been achieved. The algorithm used can best be described by use of a basic flow chart (Fig. 6).

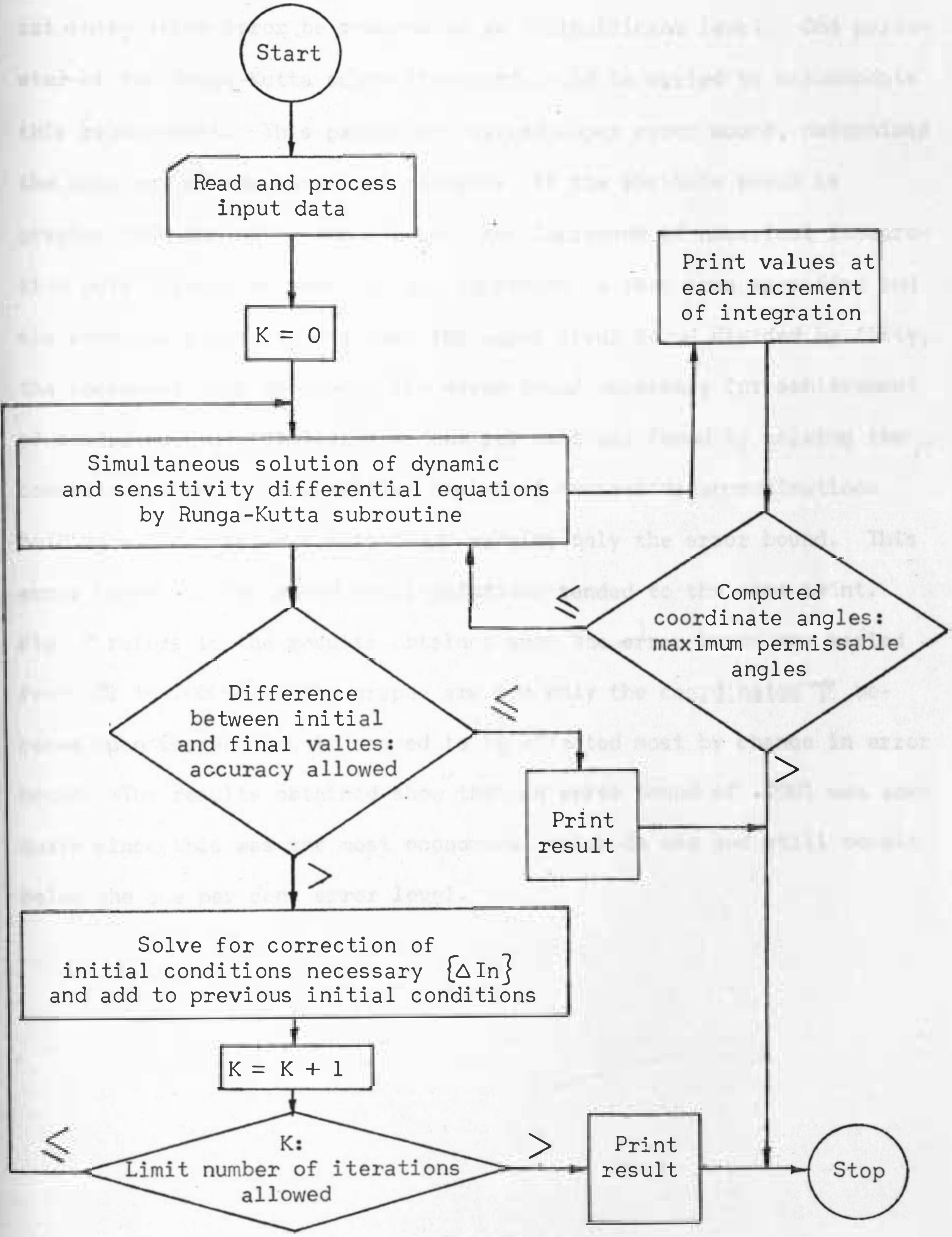

Start
Read and process input data
K = 0
Simultaneous solution of dynamic and sensitivity differential equations by Runga-Kutta subroutine
Print values at each increment of integration
Computed coordinate angles: maximum permissable angles
Difference between initial and final values: accuracy allowed
Print result
Solve for correction of initial conditions necessary {Δ In} and add to previous initial conditions
K = K + 1
K: Limit number of iterations allowed
Print result
Stop

Fig. 6

One of the stated requirements of this system was that the numerical integration error be reduced to an insignificant level. One parameter of the Runga-Kutta subroutine used could be varied to accommodate this requirement. This parameter, called upper error bound, determines the accuracy of the numerical process. If the absolute error is greater than the upper error bound, the increment of numerical integration gets halved; however, if the increment is less than specified and the absolute error is less than the upper error bound divided by fifty, the increment gets doubled. The error bound necessary for achievement of stated accuracy of less than one per cent was found by solving the complete system for a specified number of successive approximations holding all variables constant and varying only the error bound. This error bound was decreased until solutions tended to the same point. Fig. 7 refers to the results obtained when the error bound was varied from .01 to .000001. The graphs are for only the coordinates ψ because upon inspection, it seemed to be affected most by change in error bound. The results obtained show that an error bound of .0001 was adequate since this was the most economical value to use and still remain below the one per cent error level.

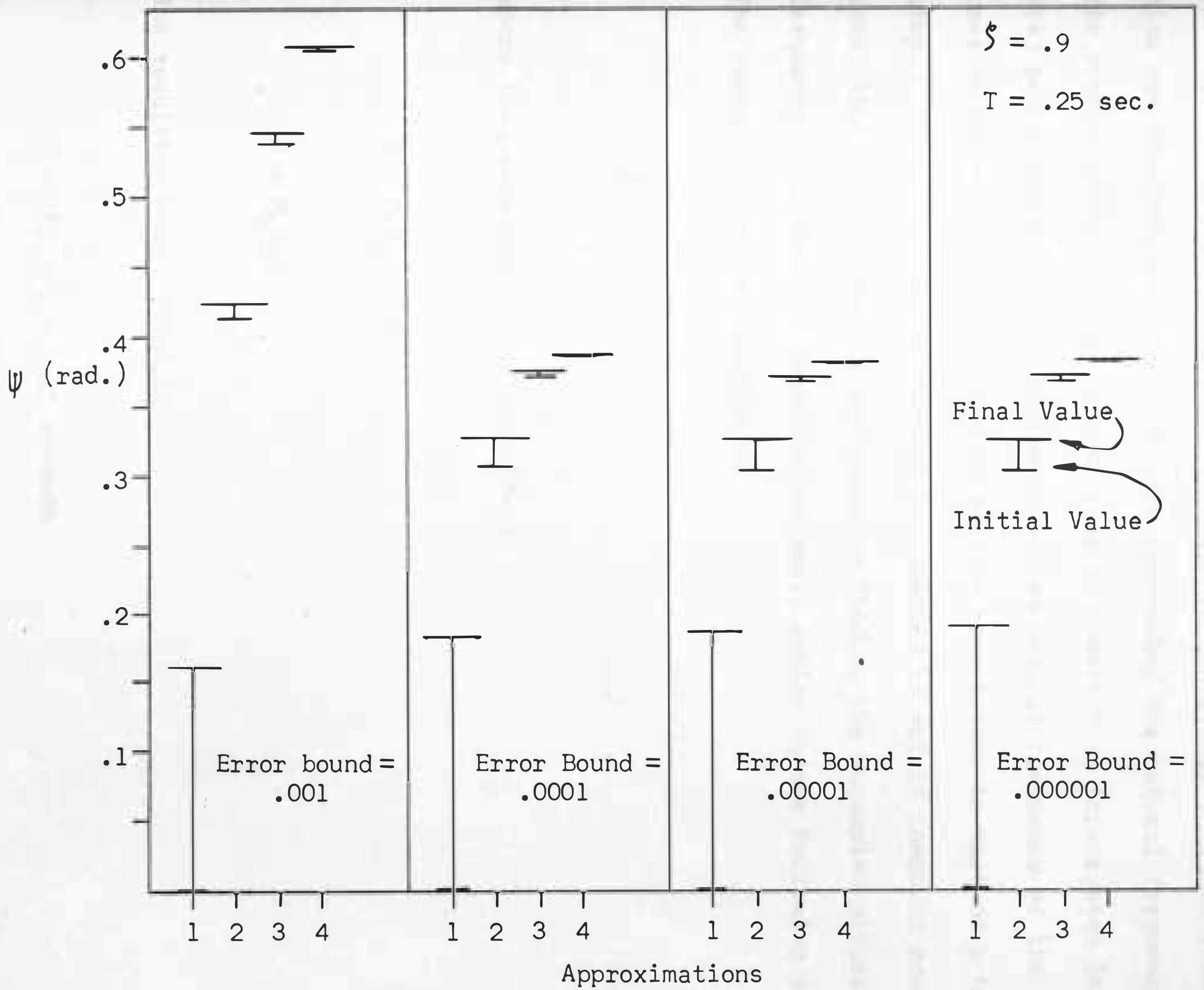


Fig. 7. Determination of Error Bound.

CHAPTER VII

ANALYSIS OF RESULTS

As stated previously, the deterioration in settling of the solution was observed when the step rate approached the natural frequency of the passive members. The rate of step in question in this system is the step period corresponding to the uncoupled natural frequency of the passive members. (Step period or rate of step refers to that of a half step.) Since the passive members are assumed to act as compound pendulums, this rate of step is estimated by finding the uncoupled natural frequency of a compound pendulum for small angles in the following way: The basic form of the equation is

$$m \ddot{\beta} + k\beta = 0$$

where from equations (3-12) and (3-13)

$$m = M_6 A_5^{\,2} + M_7$$

$$k = M_6 A_5 A_4$$

The resulting step period is

$$T = 2\pi\sqrt{m/k} = 1.15 \text{ seconds}$$

Fig. 8 gives the results of the first analysis of the system. Since physically the damping of passive members on a human is very small, analysis of the system was started with a very small damping factor. This damping factor was determined from the ratio of the damping coefficient to the critical damping coefficient in the following way: The basic form of the equation is

$$m\ddot{\beta} + c\dot{\beta} + k\beta = 0$$

where from equation (3-12) and (3-13)

$$m = M_6 A_5{}^2 + M_7$$

$$k = M_6 A_5 A_4$$

The resulting critical damping coefficient is

$$C_c = 2\sqrt{km}$$

and the resulting damping factor is

$$\zeta = c/2\sqrt{km} = c/.272$$

where c is the damping coefficient.

For this first analysis, $\zeta = .05$. As shown in Fig. 8, giving maximum amplitude of oscillation of passive members, the mathematical system functioned well achieving required accuracy after only five to eight corrections of the initial conditions for each of the step times used until a step time of 1.0 record was approached. (In all cases required accuracy was $\pm$ .001 between initial and final conditions.) Since step periods approaching 1.0 second are near the step period corresponding to the natural frequency of the passive members, it was first thought that the high angle of oscillation of the passive members at such step periods was disrupting the system.

To eliminate the high angle of oscillation, the damping factor was increased to near the point of critical damping. This did substantially decrease the angle of oscillation as is shown in Fig. 8. However, even with near critical damping, the problem of settling of the solution still remained.

Fig. 9 and Fig. 10 indicate the problem involved of the settling of the solution. These two graphs as well as following graphs show only the variables θ and $\dot{\theta}$ because they were the variables that were the greatest problem in settling. Since the resonance phenomenon was eliminated by introducing high damping, the attention was then turned toward the mathematical algorithm for successive approximations.

It was thought that this problem could be that of overcorrection of the initial conditions in an oscillating manner. If this were the case, it probably could be explained by the following: The hypersurface of the system whose tangents are used in Newton's method probably has an inflection at the step periods near the passive member resonant time. This problem can be explained by looking at the method of successive approximations used. This method of correcting the initial conditions was based on Newton's method. The form of Newton's method was that of using a linear means of obtaining the root of an equation by taking the tangent of the hypersurface of the function and determining when the function is zero. In Fig. 11 simplified to two dimensions, this possible problem in this system is illustrated. In this case the introduction of passive members cause an inflection near the axis, that is, the slope of the curve near the axis is very steep while the slope is much less steep away from the axis. When the successive approximation algorithm uses the tangent of the curve any distance from the solution, it either slowly approaches the solution or in some cases diverges from the solution (Fig. 12). To overcome this kind of problem, the initial conditions must be estimated very near the solution.

Another method to use in this case, and which was attempted here, is to eliminate the overcorrection oscillation by considering it analogous to an underdamped vibrating system. Since the corrected initial conditions seemed to oscillate about a mean value, a pseudo damping coefficient was impressed on the successive approximation algorithm. This pseudo damping reduced the oscillation by estimating the "steady state" which appeared to be near the mean value.

The reduction was made by dividing the correction in the initial conditions computed by the successive approximation algorithm by some constant value. Since the two variables θ and $\dot{\theta}$ were the problem, they were the ones reduced. A value of 4.0 was selected for the reduction because it appeared to produce the best results.

In other words, by knowing the oscillating values of the initial conditions, this reduction in the correction is just one attempt to estimate the initial conditions closer to the solution as is needed for Newton's method when there is an inflection. When this pseudo damping was added to the system algorithm, settling of the solution was greatly improved. This can be seen when comparing Fig. 9 and Fig. 10 with Fig. 13 and Fig. 14 for a step time of 1.0 second and more so with a step time of 1.2 second as shown in comparing Fig. 15 and Fig. 16 with Fig. 17 and Fig. 18. With the step time of 1.2 without the pseudo damping, the system oscillation in corrected initial conditions was so great that after 15 approximations the angles of the compensating mass became too large to achieve equilibrium.

To show that the hyperspace inflection is causing the delay in settling of the solution, the case for a step time of 1.0 second from Fig. 9 and Fig. 10 was run for more approximations. After 50 approximations (Fig. 19 and Fig. 20) the required accuracy was finally reached while the same accuracy was reached after 12 approximations with pseudo damping. Also, as can be seen from the figures, the solutions are identical.

By use of this method of reduced corrections, analysis was continued for step times from .25 second to 1.4 second with the system being varied as follows:

(1) For $\zeta = .9$ solve the system for specific step times from .25 second to 1.4 second taking estimated initial conditions as being the solution of the previous step time.

(2) Decrease damping to $\zeta = .5, .2, .1, .05$ taking the estimated initial conditions from the initial conditions of the previous solution of the particular step time.

The resulting graph (Fig. 21) gives the amplitude of oscillation of the passive members showing the high amplitude of oscillation at step times near that corresponding to the natural frequency of the passive members. These values were all obtained without difficulty when the pseudo damping was used even at high angles of oscillation of the passive members.

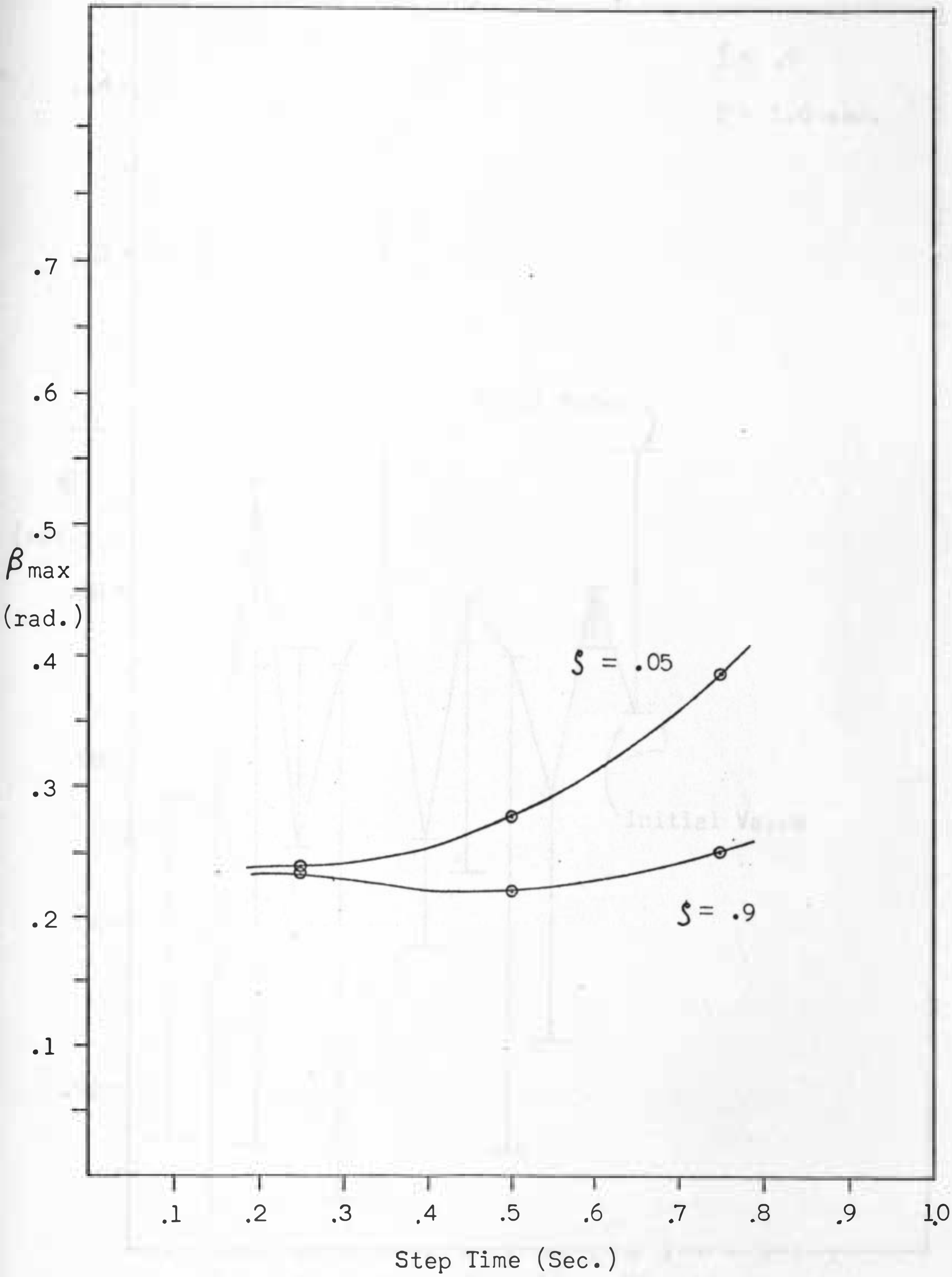


Fig. 8. Maximum amplitude of oscillation of passive hands before deterioration phenomenon.

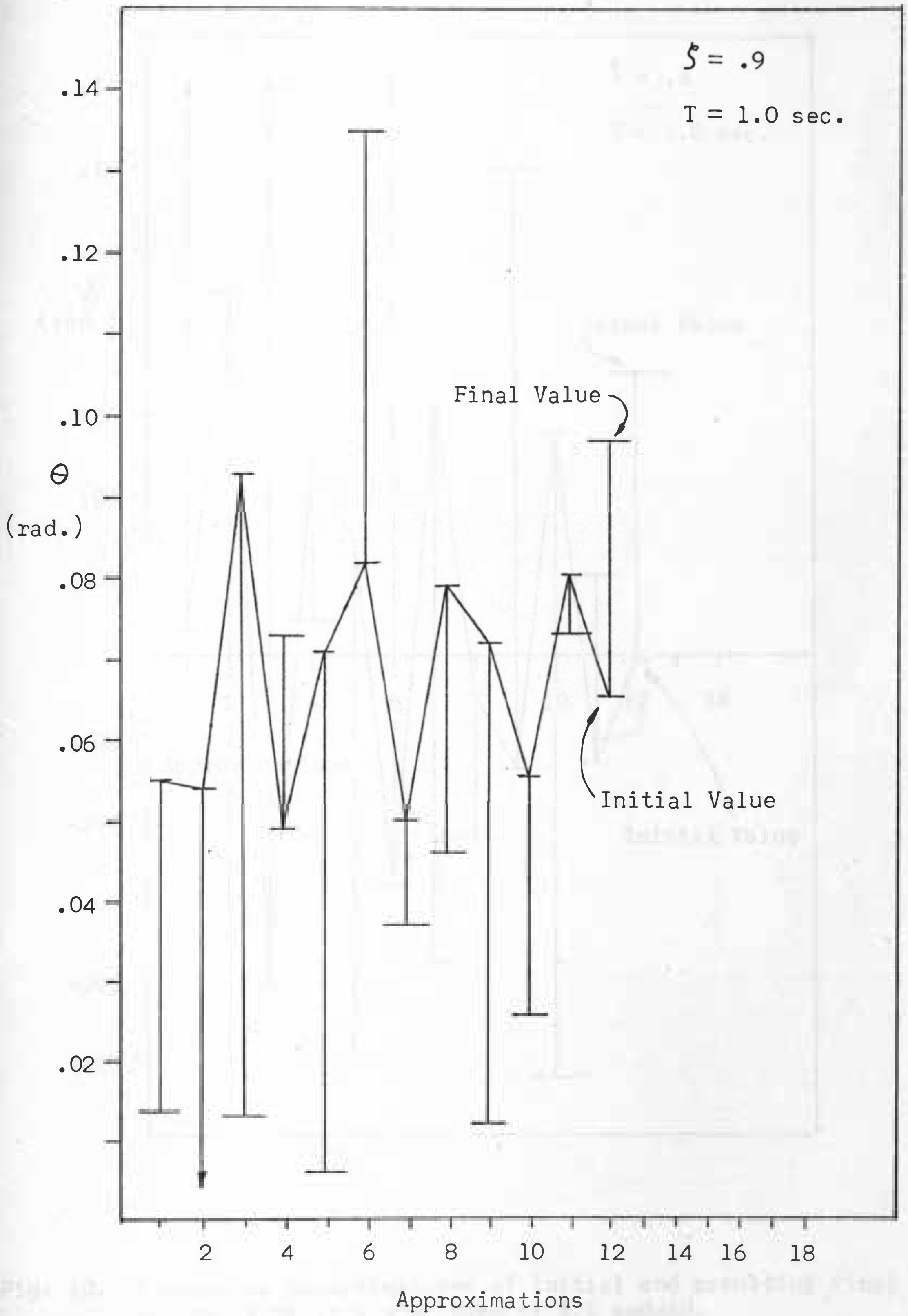


Fig. 9. Successive approximations of initial and resulting final values of θ at a step time of 1.0 second.

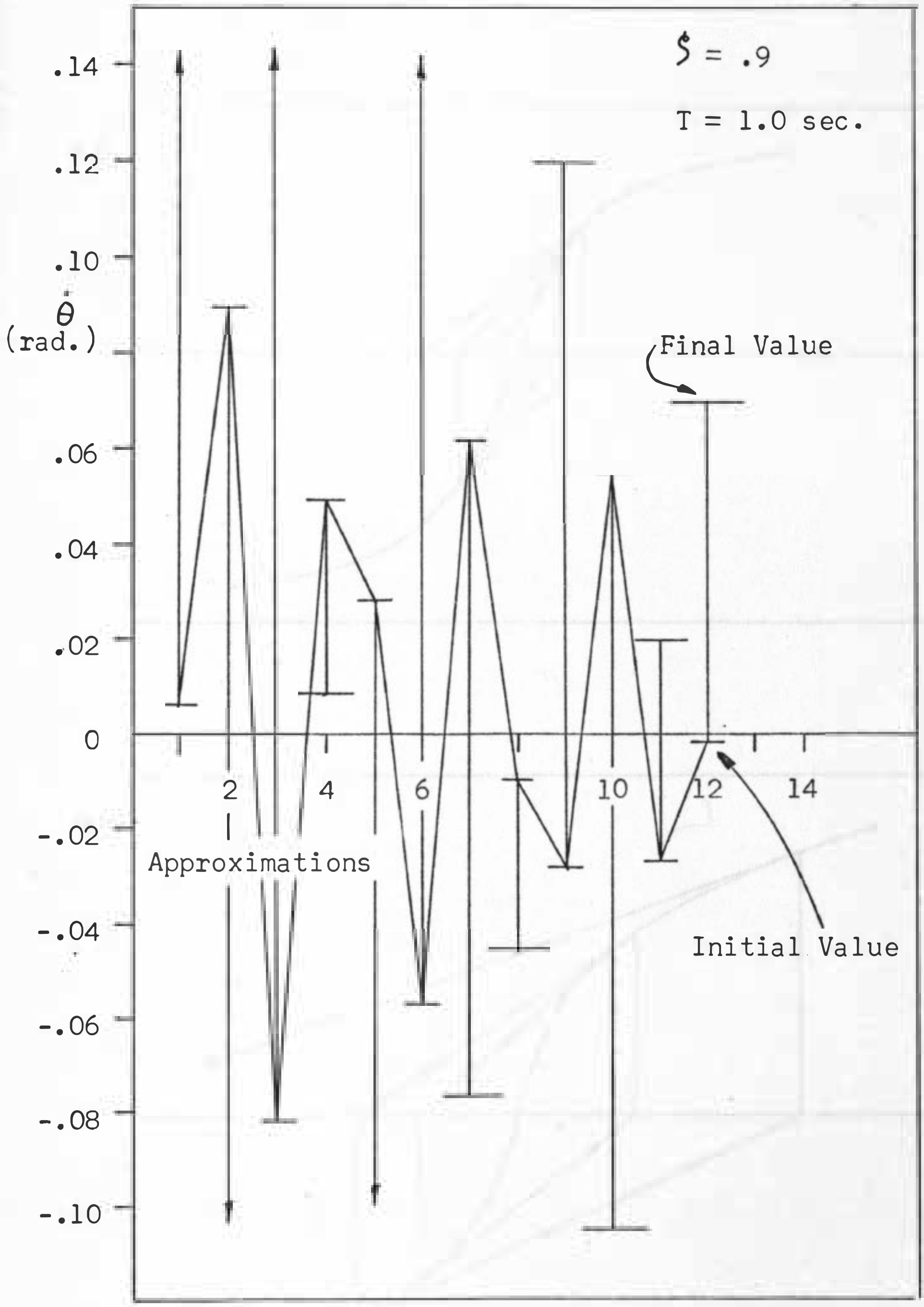


Fig. 10. Successive approximations of initial and resulting final values of $\dot{\theta}$ at a step time of 1.0 second.

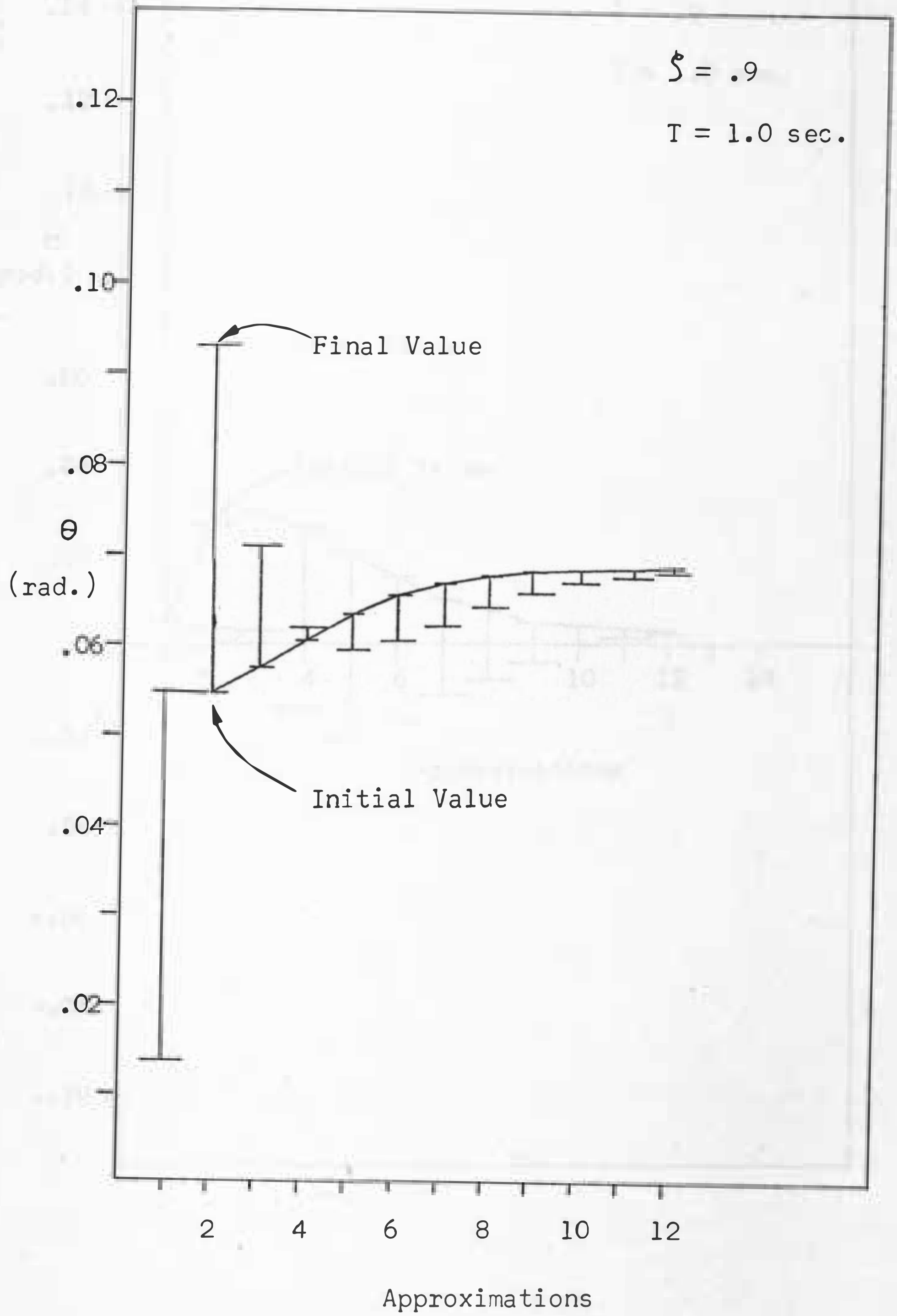


Fig. 13. Psuedo damped successive approximations of initial and resulting final values of θ at a step time of 1.0 second.

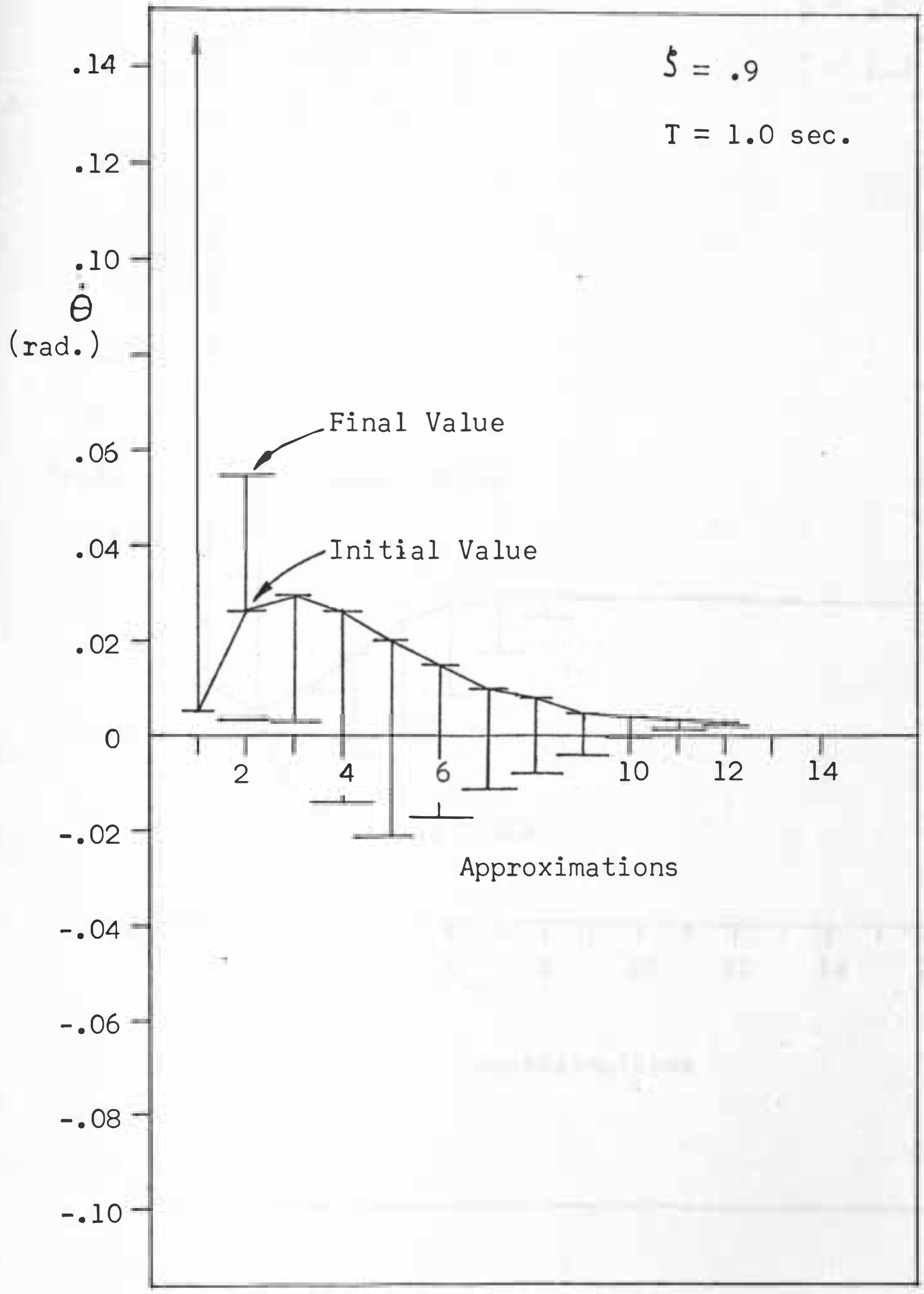


Fig. 14. Pseudo damped successive approximations of initial and resulting final values of $\dot{\theta}$ at a step time of 1.0 second.

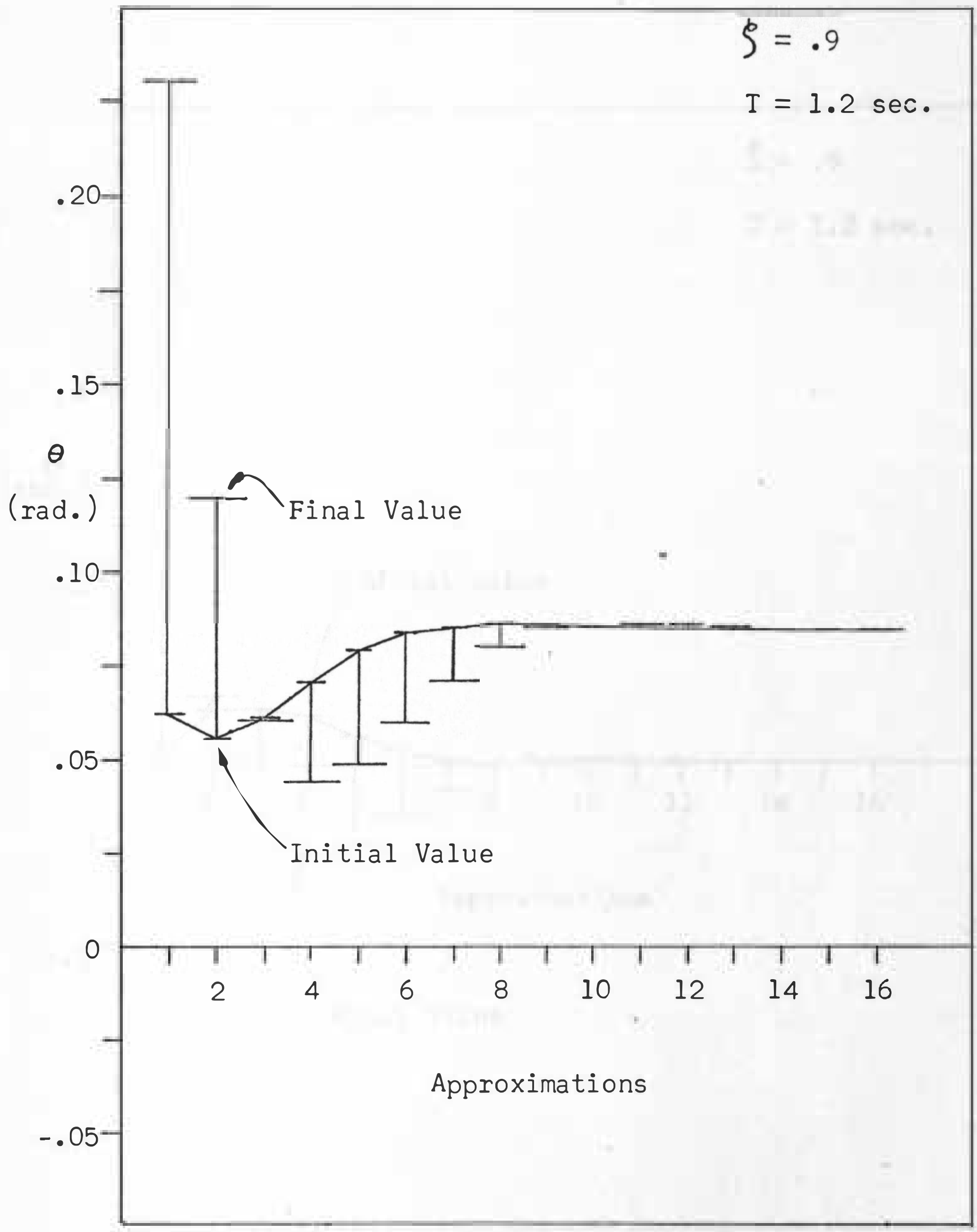


Fig. 15. Pseudo damped successive approximations of initial and resulting final values of θ at a step time of 1.2 second.

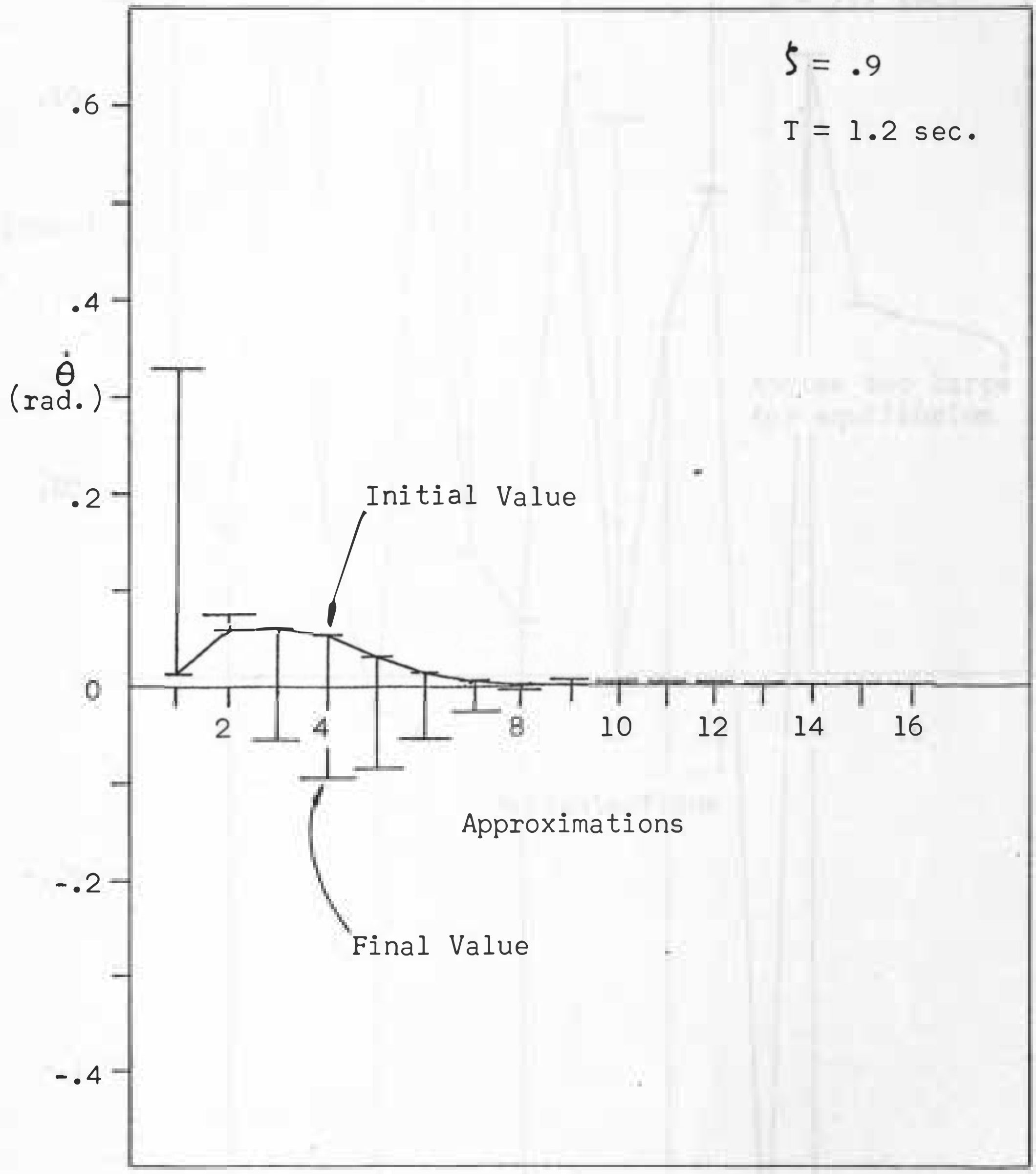


Fig. 16. Pseudo damped successive approximations of initial and resulting final values of $\dot{\theta}$ at a step time of 1.2 second.

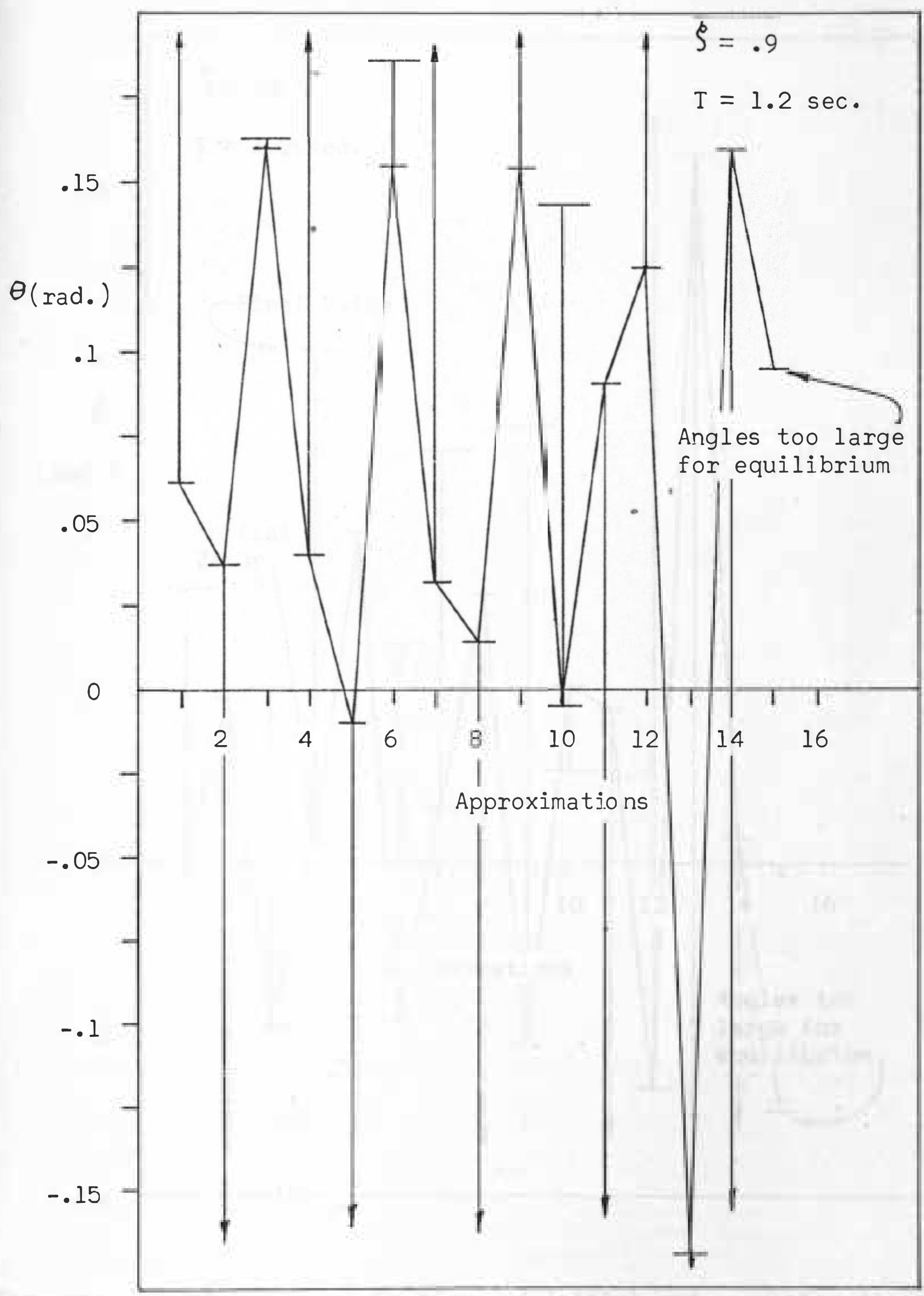


Fig. 17. Successive approximations of initial and resulting final values of θ at a step time of 1.2 second.

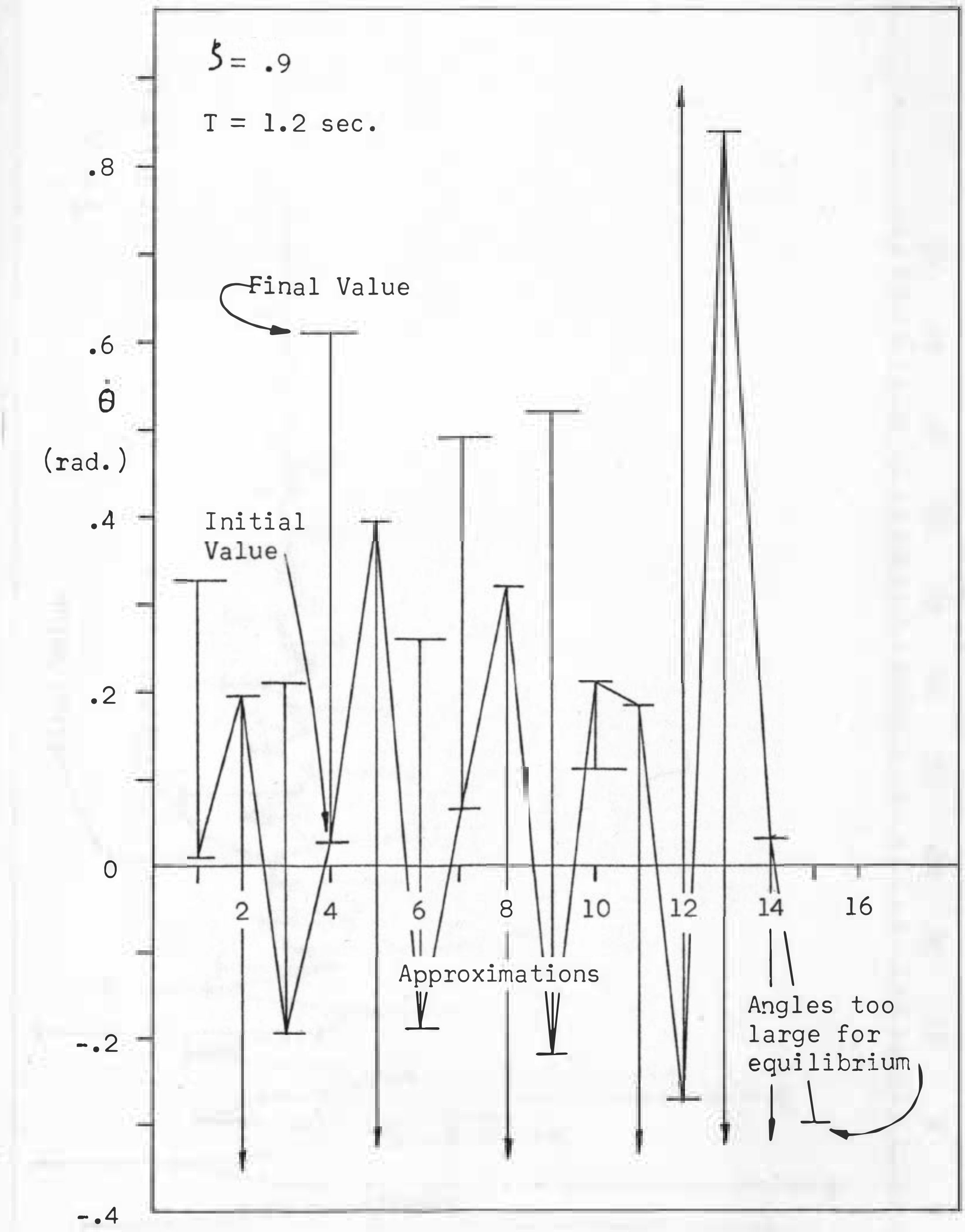


Fig. 18. Successive approximations of initial and resulting final values of $\dot{\theta}$ at a step time of 1.0 second.

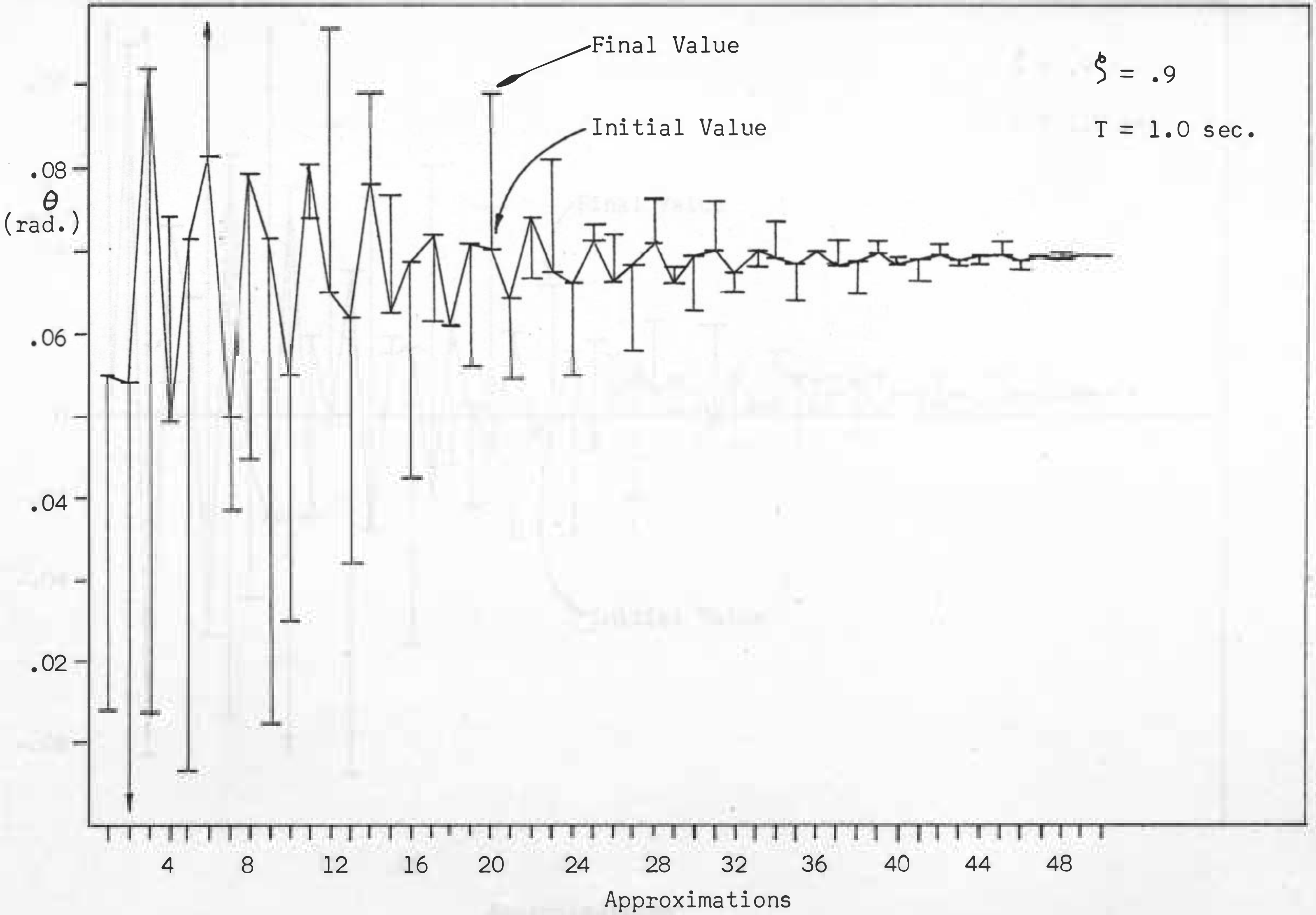


Fig. 19. Fifty successive approximations of initial and resulting final values of θ at a step time of 1.0 second.

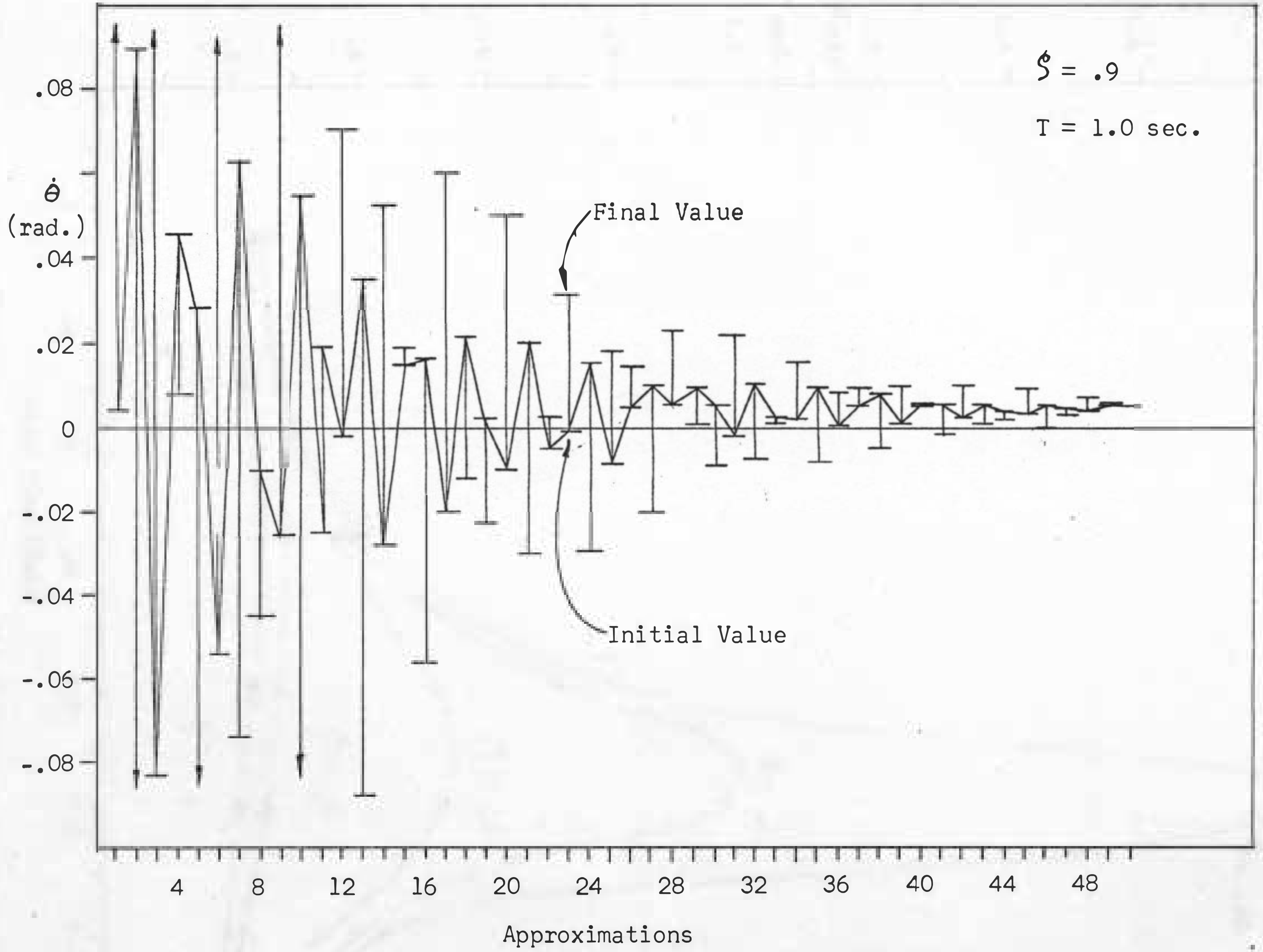


Fig. 20. Fifty successive approximations of initial and resulting final values of $\dot{\theta}$ at a step time of 1.0 second.

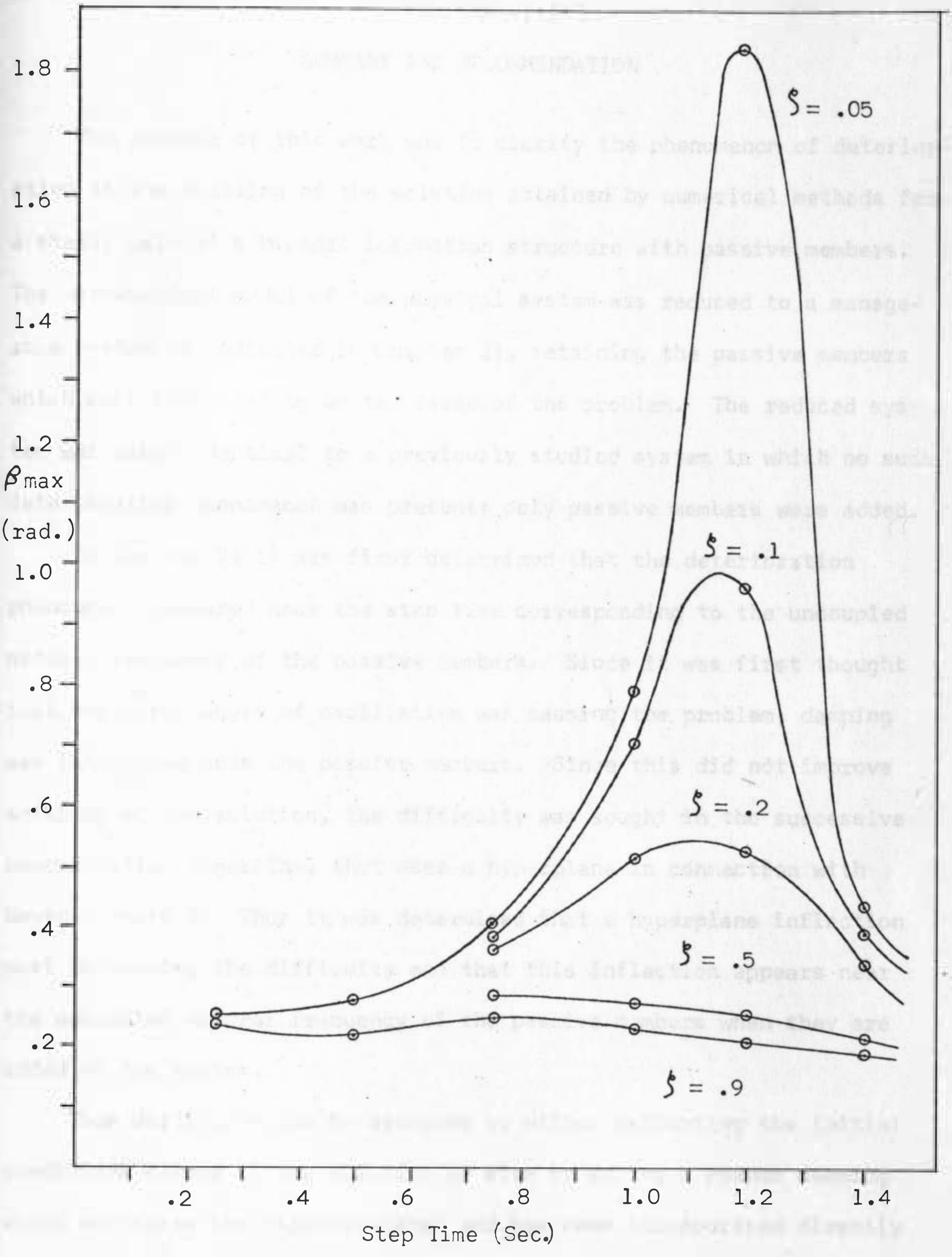


Fig. 21. Passive member angular response with different degrees of damping.

CHAPTER VIII

SUMMARY AND RECOMMENDATION

The purpose of this work was to clarify the phenomenon of deterioration in the settling of the solution obtained by numerical methods for a steady gait of a bipedal locomotion structure with passive members. The mathematical model of the physical system was reduced to a manageable system as indicated in Chapter II, retaining the passive members which were indicated to be the cause of the problem. The reduced system was taken identical to a previously studied system in which no such deterioration phenomenon was present; only passive members were added.

In Chapter VI it was first determined that the deterioration phenomenon appeared near the step time corresponding to the uncoupled natural frequency of the passive members. Since it was first thought that the large angle of oscillation was causing the problem, damping was introduced upon the passive members. Since this did not improve settling of the solution, the difficulty was sought in the successive approximation algorithm, that uses a hyperplane in connection with Newton's method. Thus it was determined that a hyperplane inflection must be causing the difficulty and that this inflection appears near the uncoupled natural frequency of the passive members when they are added to the system.

Thus difficulty can be overcome by either estimating the initial conditions closer to the solution or else by adding a pseudo damping which estimates the "steady state" and has been incorporated directly in the successive approximation algorithm.

A recommendation for further study would be to retain the second order terms of equation (4-3) and prove mathematically that the inflection is the cause of the problem.

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