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# Solving Uncertain Online Shopping Problem With Discounts Using Robust Counterpart Methodology

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## ABSTRACT

***Purpose:*** Online Shopping is a phenomenon that is growing rapidly at this time and consumers are an important element in the buying and selling competition in the market and consumers who make a diffference in determining the profifits of the sellers.

***Design/methodology/approach:*** This research discusses the problem of online shopping using the Robust Optimization method. Robust Optimization Method is a process to get optimal results with an uncertainty.

***Findings:*** Show your finding here Based on the demand model to optimize the buying price, an Integer Linear Programming model with discount functions is built which will be converted into Robust Optimization. In this study also used a tool that is the Maple application in the numerical calculation process.

***Originality/value:*** This Paper is Original.

***Paper type:*** Research paper

***Keyword:*** Discount function, Integer Linear Programming, Online Shopping, Robust Optimization

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## I. INTRODUCTION

Introduction Business is a profifit-oriented activity that produces goods or services for meeting community needs (Alma, 2014). Business conducted through the internet selling products, services, or advertisements is called an online business (Leyva, 2017). Transaction payment of buying and selling internet-based goods and services can be done quickly and easily, business people are competing to market their products and services through the internet in the hope that information about products that are owned more quickly spread among the people community along with these developments (Dewi and Kusumawati, 2018). Online Shopping optimization problem was fifirst put forward by Błazewicz *et al.*, (2010) with the aim of managing the list shopping from

Several stores is available at a minimum cost along with costs send. Then Błażewicz and Musiał, (2011) found the fifirst algorithm and introduced results computation. Furthermore, (Blazewicz *et al.*, 2014) redeveloped the Online Shopping optimization problem introduces new parameters that involve discounts and introduce some basic algorithms. Then (Musial *et al.*, 2016) issued a development Previous journalconsidering shipping costs with a discount policy based on world observation, that means the more money customers spend,

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the greater the discount they can get. With robust optimization, robust solutions obtained can help decision makers to avoid losses from uncertainty.

The role of internet technology in the framework of capital market activities, commodity futures trading, and other fifinancial market activities has a profound efect. The technological aspects of the internet have brought about changes related to the following. More eficient market activities are created by the role of internet technology in the framework. The acceleration of transactions in a larger scale and value is driven by the role of internet technology. Helping issuers, public companies, stock exchanges, to publish and distribute information in the context of protection for investors. The role of internet technology also shapes fraudulent practices carried out by market participants in order to achieve the intent and purpose of crime. Internet technology has caused a shift in the information disclosure paradigm. The development of a computer-based electronic trading system places more emphasis on understanding aspects of the underlying concepts, then the problem is analyzed and the computer-based system is designed to get a solution.

The issue of optimization of Online Shopping was fifirst raised by Blazewicz (2010) ( see (Błazewicz *et al.*, 2010) ) with the aim of managing shopping lists of several available stores with minimal costs accompanied by shipping costs. Then Błażewicz and Musiał, (2011) found the first algorithm and introduced the computation results. Furthermore, Blazewicz *et al.*, (2014) in (Blazewicz *et al.*, 2014) bredevelops this Online Shopping optimization problem by introducing new parameters that involve discounts and introducing some basic algorithms. Then Musial *et al.*, (2016) issued a journal development before taking into account the shipping cost along with a discount policy based on world observations, meaning that the more money the customer spends, the greater the discount he can get. In this journal, it is assumed that there is no difffference between the quality of goods sold at online stores in addition to the prices they charge for difffferent products and which stores can help consumers to buy products without spending large costs with the discount function available in each the case. Refers to Gorissen in Gorissen, Yanıkoğlu and den Hertog (2015), optimization problems in real life can involve uncertain data. The uncertain data is uncertain because of an error. Reasons for measuring errors in estimation errors stem from a lack of knowledge about mathematical model parameters (eg, uncertain requests in the supply model) or implementation errors can come from the impossibility of properly implementing computational solutions in real-life settings .

Based on the background that has been described, in this paper, the discussion about uncertain shipping costs to determine which stores will be selected in buying products to minimize the capital spent. In this problem it is assumed that shipping costs can vary due to the number of products purchased or the distance from the store to the destination. The goal is to pay as little as possible when we do an Online Shopping transaction. The following table state of the art for the problem of optimizing Online Shopping.

**A. Uncertain Online Shopping Problem with Discounts**

E-commerce is a combination of technology, application, process, and strategic business that is a small portion of the facilities provided at Wild Internets can also bea means more specific for advertising, sales, shipping, services, and for utilizing the web show 24 hours a day for all customers. The existence of e-commerce is very important for increasing the business of companies that want to become international businesses (Sumijan and Santony, 2016).

**B. Online Shopping Problem Discount Models**

An optimization problem model has been formulated to minimize costs incurred the problem of online shopping that develops by considering shipping costs (Musial *et al.*, 2016) :

1. The cost of the product-$i$ at store-$j$ denoted by $pij$ , so you can consider the discounts obtained as well as shipping costs $dj$ from each part of the product-$i$ from the store-$j$.
2. The discount function that used in this Online Shopping problem is :

$$f(q)=\begin{cases} p_{ij},\ 0<q\leq 25 \\ 0.95p_{ij},\ 25<q\leq 50 \\ 0.90p_{ij},\ 50<q\leq 100 \\ 0.85p_{ij},\ 100<q\leq 200 \\ 0.80p_{ij},\ 200<q\leq 1000 \end{cases} \tag{1}$$

3. Online Shopping optimization problems involve several parameters, namely :
   pij : product price-i at store-j
   qi : number of product-i
   dj : delivery cost from store-j
4. In addition, the Online Shopping optimization problem consists of several variables:
   xij : indicator of product-i usage at store-j

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yj : indicator of delivery cost from store-j

The Online Shopping Optimization Model with Discounts can be formulated as Follows (Musial et al., 2016) :

$$\begin{aligned} &\min.\left(\sum_{i=1}^{m}\sum_{j=1}^{n}(p_{ij}q_i x_{ij})\right)+\sum_{j=1}^{n}(d_j y_j)\\ &s.t.\ \sum_{j=1}^{n} x_{ij}=1,\ i=1,\dots,m\\ &0\le x_{ij}\le y_j,\ i=1,\dots,m,\ j=1,\dots,n\\ &x_{ij}\in\{0,1\}, y_{ij}\in\{0,1\}, i=1,\dots,m, j=1,\dots,n\\ &q_i\in Z \end{aligned} \qquad (2)$$

## II. METHODOLOGY

Robust optimization is a method combined with computational tools for obtain an optimization problem where uncertainty exists in the data and is only known within form of set of uncertainties (Ben-Tal and Nemirovski, 2002). Robust Optimization assumes that data is uncertain in the set uncertainty. Robust Optimization is a way of looking at dealing with data uncertainty in optimization problems. The linear optimization problem is solved using Robust Optimization. The general form of formulation from indeterminate linear optimization is as follows (Gorissen, Yanıkoğlu and den Hertog, 2015).

$$\begin{aligned} &min.\ c^T x\\ &\text{s.t. } Ax \le b \\ &c, A, b \in u \end{aligned} \qquad (3)$$

where:

1. The function to be minimized cT x is an objective function.
2. Coeficient of c is the cost coeficient matrix.
3. The decision variables to be determined is x.
4. Inequality $Ax \le b$ is a constraint function. The coeficient A is called the coeficient matrix technology.
5. The constraint function of $(c, A, b) \in u$ states that A, b, c are in the set of uncertainty u.
6. Three assumptions underlying Robust Optimization (Ben-Tal, Ghaoui and Nemirovski, 2009) :
7. All decision variables $x \in Rn$ represent ”here and now” decisions, decisions $x \in Rn$ obtained from specifific numerical values as a result of problem solving before data the actual ” relevant itself”.
8. Decision makers are fully responsible for the decisions that must be made if and only if the data is actually specified in the set of uncertainties u.
9. The problem with the problem of indeterminate linear programming is ”hard”, meaning the maker decisions cannot tolerate violations of the slightest obstacle, when data is in the set of uncertainties u.

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Assume is certain, formulation of the Robust Counterpart is as follows (Gorissen, Yanıkoğlu and den Hertog, 2015) :

$$\begin{aligned}&\min .\ c^T x\\&\text{s.t. } A(\zeta)x \leq b\\&\forall \zeta \in Z\\&\Leftrightarrow \min .\ c^T x\\&\text{s.t. } a^T(\zeta)x \leq b\\&\forall \zeta \in Z\end{aligned} \tag{4}$$

where $Z \subset R^L$ shows the use of a set of certain uncertainties. Solution $x \in R^n$ called robust feasible $[A(\zeta)x \leq b]$ if it meets all uncertain constraints for all realization of $\zeta \in Z$. Uncertainty parameters defined:

$$a(\zeta) = \overline{a} + P\zeta \tag{5}$$

where $\overline{a} \in R^n$, $P \in R^{n\times L}$, and $\overline{a}$ is a nominal value. Can be defined set of $u$:

$$u = \{a | a = \overline{a} + P\zeta, \zeta \in Z\} \tag{6}$$

A constraint obtained from (4) with the substitution of uncertainty parameters can be modeled as follows:

$$(\overline{a} + P\zeta)^T x \leq b, \forall \zeta \in Z \tag{7}$$

Based on the above conditions, the indeterminate problem (3) is changed by the Robust Optimization approach to become a single deterministic problem called the Robust Counterpart (RC) is:

$$\begin{aligned}&\pi^* = \min .\ c^T x\\&\text{s.t. } Ax \leq b\\&x \geq 0\\&\forall (c, A, b) \in u\end{aligned} \tag{8}$$

A vector $\chi^*$ mentioned the optimal Robuts solution if for all realization $\forall(c, A, b) \in u$ , $\chi^*$ feasible, and the value of objective function $\pi^*$ guaranteed the greatest value. According to Gorissen, Yanıkoğlu and den Hertog, (2015), equation (8)is equivalent to the problem of a linear objection function which is certain and only a function of an uncertain constraint :

$$\begin{aligned}&min.\ t\\&s.t.\ c^T x - t \leq 0\\&a_i^T x - b_i \leq 0;\ i = 1, ..., m\\&c, a, b \in u\end{aligned} \tag{9}$$

without reducing generality, it is assumed that the coefficient is certain, while the coefficient of $\{a_i,\ b_i\}_{i=1,...,m}$ is uncertaint, so equation (9) becomes:

$$\begin{aligned}&\min .\ \mathrm{t}\\&\text{s.t. } c^T x - t \leq 0\\&a_i^T x - b_i \leq 0\\&a, b \in u\end{aligned} \tag{10}$$

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Constraint at equation (10) can be changed by considering the semi-definite robust canonical constraints and stopping use $i$ being :

$$a^T x - b \leq 0, \forall (a,b) \in u \tag{11}$$

where $a$ is a vector inside $\mathbb{R}^n$ and $b$ is a scalar representing $a_i$. While $u$ representing $u_i$. Uncertaint parameter $a$ is a set of uncertainty $u$ can be transformed into a form of primitive factor $\zeta \in \mathbb{R}^L$, so it can be written as :

$$a = \bar{a} + P^T \zeta, u = \{(a = \bar{a} + P\zeta) | \zeta \in Z\} \tag{12}$$

where vector of $\bar{a}$ $in\mathbb{R}^n$ is called nominal, matrix of $P \in \mathbb{R}^{nxL}$ and vector $P \in \mathbb{R}^L$, and set of $Z \in R^L$ is a set of uncertainties for primitive factors. This representation of uncertainty by primitive factors is not obligatory, but only for convenience. To produce$u$ which is filled equation (12), the uncertainty in $a$ is represented by the following simple interval:

$$u = \{a | a^l \leq a \leq a^u\} \tag{13}$$

with defined,

$$\bar{a} = \frac{a^1 + a^u}{2}, P = diag\left(\frac{a^u - a^1}{2}\right) \text{ dan } Z = \{\zeta | -1 \leq \zeta_1 \leq 1, l = 1, ..., L\} \tag{14}$$

Furthermore, to analyze the Robust Counterpart that can be computationally tractable with show that the Robust Counterpart can be formed into contraints of Linear Programming, Conic Quadratic, or Semi-Defifinites. So the problem can be said to be linear Programming (LP) , Conic Quadratic Optimization (CQO) , or Semidefifinite Optimization (SDO) as stated in theorem 1 (Ben-Tal and Nemirovski, 2002) (Chaerani and Roos, 2013).

Proof. Proof has been given.

**A. Robust Counterpart in The Set of Box Uncertainty**

The Robust Counterpart formulation is “tractable”for the linear Robust Optimization problem with the Box Uncertanty area , which can be stated as follows (Gorissen, Yanıkoğlu and den Hertog, 2015) :

$$Z = \{\zeta : \zeta_\infty \leq 1\} \tag{15}$$

One of the assumptions discussed earlier is uncertainty is constraint-wise, so focus on one constraint (and omit index $i$):

$$(\overline{a} + P\zeta)^T x \leq b, \forall \zeta : \|\zeta\|_\infty \leq 1 \tag{16}$$

Equation constraints (19) is equivalent to:

$$\max_{\zeta : \|\zeta\|_\infty \leq 1} (\overline{a} + P\zeta)^T x \leq b \tag{17}$$

or

$$\overline{a}^T x + \max_{\zeta : \|\zeta\|_\infty \leq 1} (P^T x)^T \zeta \leq b \tag{18}$$

Norm - $\ell\infty$ is the search for the maximum value of each absolute value of the entry that (Kreyzig, 1978) :

$$x_\infty = max(|x_1|, |x_2|, \ldots, |x_n|) = max\,|x_i|$$

Then it can be changed to:

$$\max_{\zeta : \|\zeta\|_\infty \leq 1} (P^T x)^T \zeta = \max_{\zeta : \|\zeta\|_\infty \leq 1} \sum_i (P^T x)_i \zeta_i = \sum_i |(P^T x)_i| = \|P^T x\|_1 \tag{19}$$

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So, it can be concluded that $x$ fulfill equation (19) if and only if $x$ fulfill :

$$\bar{a}^T x + \|P^T x\|_1 \leq b \tag{20}$$

Constraint (23) does not have a semi-infinite structure as in equation (19). Then, this constraint is easily modeled as a set of linear constraints. Thus, although the number of variables and constraints increases, the final Robust Counterpart is linear. In the final Robust Counterpart form there is an additional "safety term" which depends on the value of the $x$ optimization variable.

Robust Counterpart in the set of Ellipsoidal Uncertainty The Robust Counterpart formulation which is stractable for linear robust optimization problems with ellipsoidal set areas of uncertainty can be stated as follows (Gorissen, Yanıkoğlu and den Hertog, 2015) :

$$Z = \{\|\zeta\|_2 \leq 1\} \tag{21}$$

defined a set of $u$:

$$u = \{(\bar{a} + P\zeta)\,|\forall\zeta : \|\zeta\|_2 \leq 1\} \tag{22}$$

In order to obtain the Robust Counterpart formulation from the Ellipsoidal Uncertainty set, the set (25) is applied in equation (7) so that it is obtained :

$$(\bar{a} + P\zeta)^T x \leq b, \forall\zeta : \|\zeta\|_2 \leq 1 \tag{23}$$

Equation constraints (26) is equivalent to:

$$\max_{\zeta:\|\zeta\|_2\leq 1} (\bar{a} + P\zeta)^T x = \bar{a}^T x + \max_{\zeta:\|\zeta\|_2\leq 1} \left(P^T x\right)^T \zeta \leq b \tag{24}$$

Choose $\zeta = \frac{(P^T x)}{\|P^T x\|_2}$ so $\max_{\zeta:\|\zeta\|_2\leq 1} (\bar{a} + P\zeta)^T x = \max_{\zeta:\zeta_2\leq 1} \left(P^T x\right)^T \zeta = \|P^T x\|_2$, then substitution in the formulation (27). Furthermore, the Robust Counterpart formulation is obtained from equation (27) which is equivalent to equation (26):

$$\bar{a}^T x + \|P^T x\|_2 \leq b \tag{25}$$

The final from of Robust Counterpart is guaranteed to be a problem that is computationally tractable with Conic Quadratic Constraints. If the Robust Counterpart formulation products another form, it-must re-determine the assumption of indeterminate parameters in the initial model of the journal (Ben-Tal and Nemirovski, 2002) and (Chaerani and Roos, 2013).

## III. RESULTS AND DISCUSSION

The Robust Counterpart with the uncertainty in delivery costs is done by assuming the uncertainty of the data in the Box Uncertainty and Ellipsoidal Uncertainty. Then a numerical simulation is performed on the Robust Counterpart Online Shopping Optimization model with uncertainty parameters by the Maple application. The data used is taken from (Błazewicz *et al.*, 2010). The following are the cases discussed

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**Table 1.** Request for Random Generator Results from the Maple Application for Case I ($0 < q \leq 25$)

| $q_i$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ |
|---|---|---|---|---|---|
| | 4 | 1 | 4 | 5 | 1 |

**Table 2.** Request for Random Generator Results from the Maple Application for Case II ($0 < q \leq 25$)

| $q_i$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ |
|---|---|---|---|---|---|
| | 10 | 7 | 10 | 7 | 8 |

**Table 3.** Request for Random Generator Results from the Maple Application for Case III ($0 < q \leq 25$)

| $q_i$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ |
|---|---|---|---|---|---|
| | 17 | 20 | 16 | 12 | 14 |

**Table 4.** Request for Random Generator Results from the Maple Application for Case IV ($0 < q \leq 25$)

| $q_i$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ |
|---|---|---|---|---|---|
| | 35 | 33 | 26 | 22 | 36 |

**Table 5.** Request for Random Generator Results from the Maple Application for Case V ($0 < q \leq 25$)

| $q_i$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ |
|---|---|---|---|---|---|
| | 133 | 162 | 85 | 46 | 84 |

### *4.1. Robust Counterpart Optimization on Online Shopping Problems with Discounts in a Set of Box Uncertainty*

Next, assume that the uncertainty parameter in the Robust Counterpart formulation of the Online Shopping problem in the Set of Uncertainty set. The set of Box Uncertainty is defined as follows :

$$\begin{aligned} &Z = \{\zeta : \|\zeta\|_\infty \leq 1\} \\ &\Leftrightarrow Z = \{\zeta : |\zeta| \leq 1\} \end{aligned} \tag{26}$$

Then suppose that the set of $Z$ uncertainties is divided into the uncertainty set of parts $Z_1$ and $Z_2$ where :

$$\begin{aligned} Z_1 &= \{\zeta : \zeta = 0\} \\ Z_2 &= \{\zeta : 0 < \zeta \leq 1\} \end{aligned} \tag{27}$$

In obtaining the Robust Counterpart formulation, the Box Uncertainty set is applied to the first constraint function in the equation so that the following formulation is obtained:

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for $Z_1 = \{\zeta : \zeta = 0\}$, then :

$$\begin{aligned}
&Min.\ t\\
&s.t. \sum_{i=1}^{5}\sum_{j=1}^{6}(p_{ij}q_ix_{ij}) + \sum_{j=1}^{6}(\bar{d}_jy_j) - t + s_1 = 0\\
&\sum_{j=1}^{6} x_{ij} + a_1 = 1, i = 1, ..., 5\\
&0 \le x_{ij} \le y_j, i = 1, ..., 5, j = 1, ..., 6\\
&q_i \in Z\\
&t \ge 0\\
&x_{ij} \in \{0,1\}, y_j \in \{0,1\}, i = 1, ..., 5, j = 1, ..., 6
\end{aligned} \tag{28}$$

for $Z_2 = \{\zeta : 0 < \zeta \le 1\}$, then:

$$\begin{aligned}
&Min.t\\
&s.t. \sum_{i=1}^{m}\sum_{j=1}^{n}(p_{ij}q_ix_{ij}) + \sum_{j=1}^{n}(\bar{d}_jy_j) + \sum_{j}(P_jy_j) - t + s_1 = 0\\
&\sum_{j=1}^{n} x_{ij} + a_1 = 1, i = 1, ..., m\\
&0 \le x_{ij} \le y_j, i = 1, ..., m, j = 1, ..., n\\
&q_i \in Z\\
&t \ge 0\\
&x_{ij} \in \{0,1\}, y_j \in \{0,1\}, i = 1, ..., m, j = 1, ..., n
\end{aligned} \tag{29}$$

### *4.2. Robust Counterpart Optimization on Online Shopping Problems with Discounts in a Set of Ellipsoidal Uncertainty*

Assume that the uncertainty parameter in the Robust Counterpart formulation is the Online Shopping problem in the Ellipsoidal set of Uncertainty. The Ellipsoidal set of Uncertainty is defined as follows :

$$\begin{aligned}
&Z = \{\zeta : \|\zeta\|_2 \le 1\}\\
&\Leftrightarrow Z = \left\{\zeta : \sqrt{\sum_i \zeta_i^2} \le 1\right\}
\end{aligned} \tag{30}$$

Then suppose that the set of $Z$ uncertainties is divided into the uncertainty set of parts $Z_1$ and $Z_2$ where :

$$\begin{aligned}
Z_1 &= \left\{\zeta : \sqrt{\sum_i \zeta_i^2} = 0\right\}\\
Z_2 &= \left\{\zeta : 0 < \sqrt{\sum_i \zeta_i^2} \le 1\right\}
\end{aligned} \tag{31}$$

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In obtaining the Robust Counterpart formulation, the Ellipsoidal set of Uncertainty is applied to obtain the following formulation :

for $Z_1 = \{\zeta : \zeta = 0\}$, then :

$$\begin{aligned}
& Min.t \\
& s.t. \sum_{i=1}^{m} \sum_{j=1}^{n} (p_{ij} q_i x_{ij}) + \sum_{j=1}^{n} (\bar{d}_j y_j) - t + s_1 = 0 \\
& \sum_{j=1}^{n} x_{ij} + a_1 = 1, i = 1, ..., m \\
& 0 \leq x_{ij} \leq y_j, i = 1, ..., m, j = 1, ..., n \\
& q_i \in Z \\
& t \geq 0 \\
& x_{ij} \in \{0,1\}, y_j \in \{0,1\}, i = 1, ..., m, j = 1, ..., n
\end{aligned} \tag{32}$$

for $Z_2 = \{\zeta : 0 < \zeta \leq 1\}$, then:

$$\begin{aligned}
& Min.t \\
& s.t. \sum_{i=1}^{m} \sum_{j=1}^{n} (p_{ij} q_i x_{ij}) + \sum_{j=1}^{n} \left( \bar{d}_j y_j + \left( \sum_{j} P_j^2 \right)^{\frac{1}{2}} y_j \right) - t + s_1 = 0 \\
& \sum_{j=1}^{n} x_{ij} + a_1 = 1, i = 1, ..., m \\
& 0 \leq x_{ij} \leq y_j, i = 1, ..., m, j = 1, ..., n \\
& q_i \in Z \\
& t \geq 0 \\
& x_{ij} \in \{0,1\}, y_j \in \{0,1\}, i = 1, ..., m, j = 1, ..., n
\end{aligned} \tag{33}$$

The Robust Counterpart formulation on the Online Shopping problem that has been shown results in a form of linear constraint function, so that the formulation of the Robust Counterpart can be categorized as a Linear Programming problem. Therefore, it can be concluded that Robust Counterpart Optimization is guaranteed computationally tractable.

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**Table 6.** Numerical Calculation of Box Uncertainty and Ellipsoidal Uncertainty in Case I

| k | Box Uncertainty | Ellipsoidal Uncertainty |
|---|---|---|
| 1 | 485.999 | 485.999 |
| 2 | 489.999 | 498.999 |

**Table 7.** Numerical Calculation of Box Uncertainty and Ellipsoidal Uncertainty in Case II

| k | Box Uncertainty | Ellipsoidal Uncertainty |
|---|---|---|
| 1 | 1, 345.999 | 1, 345.999 |
| 2 | 1, 352.999 | 1, 364.734 |

**Table 8.** Numerical Calculation of Box Uncertainty and Ellipsoidal Uncertainty in Case III

| k | Box Uncertainty | Ellipsoidal Uncertainty |
|---|---|---|
| 1 | 2, 582.999 | 2, 582.999 |
| 2 | 2, 591.999 | 2, 603.734 |

**Table 9.** Numerical Calculation of Box Uncertainty and Ellipsoidal Uncertainty in Case IV

| k | Box Uncertainty | Ellipsoidal Uncertainty |
|---|---|---|
| 1 | 4, 584.999 | 4, 584.999 |
| 2 | 4, 854.999 | 4, 870.979 |

**Table 10.** Numerical Calculation of Box Uncertainty and Ellipsoidal Uncertainty in Case V

| k | Box Uncertainty | Ellipsoidal Uncertainty |
|---|---|---|
| 1 | 16, 059.999 | 16, 059.999 |
| 2 | 16, 068.999 | 16, 084.979 |

## IV. CONCLUSION

Conclussion This Robust Counterpart optimization on Online Shopping issues involves uncertainty on shipping costs which is an uncertainty parameter and is solved by the Box Uncertainty approach and the Ellipsoidal set of Uncertainty, resulting in the Online Shopping Robust Counterpart Optimization model. The Robust Counterpart Optimization Model also produces linear constraint functions so that it can be categorized as LinearProgramming. Therefore, the Robust Optimization Counterpart model is guaranteed to be computationally tractable. The numerical simulation results of the Robust Counterpart Optimization model on Online Shopping problems with uncertainty on shipping costs as an uncertainty parameter show which stores can help consumers to buy products without incurring large costs with discount functions available in each case.

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