

University of Texas Rio Grande Valley

ScholarWorks @ UTRGV

---

Electrical and Computer Engineering Faculty  
Publications and Presentations

College of Engineering and Computer Science

---

6-2021

## A Game-Theoretic Framework to Regulate Freeriding in Inter-Provider Spectrum Sharing

Mostafizur Rahman

*The University of Texas Rio Grande Valley*, [mostafizur.rahman@utrgv.edu](mailto:mostafizur.rahman@utrgv.edu)

Murat Yuksel

Thomas Quint

Follow this and additional works at: [https://scholarworks.utrgv.edu/ece\\_fac](https://scholarworks.utrgv.edu/ece_fac)



Part of the [Electrical and Computer Engineering Commons](#)

---

### Recommended Citation

M. Rahman, M. Yuksel and T. Quint, "A Game-Theoretic Framework to Regulate Freeriding in Inter-Provider Spectrum Sharing," in *IEEE Transactions on Wireless Communications*, vol. 20, no. 6, pp. 3941-3957, June 2021, doi: 10.1109/TWC.2021.3054558.

This Article is brought to you for free and open access by the College of Engineering and Computer Science at ScholarWorks @ UTRGV. It has been accepted for inclusion in Electrical and Computer Engineering Faculty Publications and Presentations by an authorized administrator of ScholarWorks @ UTRGV. For more information, please contact [justin.white@utrgv.edu](mailto:justin.white@utrgv.edu), [william.flores01@utrgv.edu](mailto:william.flores01@utrgv.edu).

# A Game-Theoretic Framework to Regulate Freeriding in Inter-Provider Spectrum Sharing

Mostafizur Rahman, *Student Member, IEEE*, Murat Yuksel, *Senior Member, IEEE*, and Thomas Quint

**Abstract**—Primary-secondary spectrum sharing is limited in terms of design space, and may not be sufficient to meet the ever-increasing demand of connectivity and high signal quality. The next step to increase spectrum sharing efficiency is to design markets where sharing takes place among primary providers rather than leaving it to the limited case where the primary licensee is idle. Attaining contractual spectrum sharing among primary providers, a.k.a. co-primary or inter-provider sharing, involves additional costs for the users, e.g., roaming fee. Co-primary spectrum sharing without additional charge to the users poses two major challenges: a) regulatory approaches must be introduced to incentivize providers to share spectrum resources, and b) small providers in co-primary spectrum sharing markets may freeride on large providers’ networks as the customers of the small providers may be using the spectrum and infrastructure resources of large providers. Such freeriding opportunities must be minimized to realize the benefits of primary-level sharing. We consider a subsidy-based spectrum sharing (SBSS) market to facilitate co-primary spectrum sharing where providers are explicitly incentivized to share spectrum resources. We focus on minimizing freeriding in SBSS markets and introduce a game-theoretic model to regulate the freeriding. We use the model to explore operational regimes with minimal freeriding.

**Index Terms**—Spectrum Sharing, Network Sharing, Spectrum Management, Game Theory, Subsidy Markets.

## I. INTRODUCTION

Spectrum utilization via sharing has seen several schemes over time, e.g., dynamic spectrum access (DSA) [2], device-to-device communications underlying a cellular network [3], and roaming [4]. However, most of these techniques focus on spectrum sharing at *primary-secondary* basis, i.e., a secondary user can use a band when the primary user is idle. Such sharing increases spectrum utilization, but the underlying techniques emphasize providers’ incentive rather than end users’ experience. Further, the potential of such secondary-level spectrum sharing is limited to utilizing whatever is left

Initial results of this work on equilibrium analysis of SBSS markets appeared in Proceedings of IEEE DySPAN Conference, 2018 [1]. This manuscript extends the conference paper by (i) exploring details of customers’ averseness to price, (ii) the effect of marginal signal improvement (i.e., a measure of how much room exists in a provider’s network for improvement) on market equilibrium, (iii) properties of the probability of customer switching (which models the likelihood of freeriding) and its effects on equilibrium, (iv) exploration of always-on full sharing mode of participating providers, (v) effect of sharing on different call types, i.e., home calls, foreign calls, rejected foreign calls, and (vii) game in a multi-provider market.

M. Rahman is with the Department of Electrical and Computer Engineering, University of Texas Rio Grande Valley, Edinburg, TX 78539 (E-mail: mostafizur.rahman@utrgv.edu).

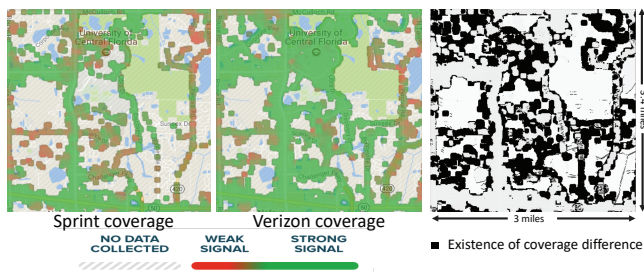
M. Yuksel is with the Department of Electrical and Computer Engineering, University of Central Florida, Orlando, FL 32816 (E-mail: murat.yuksel@ucf.edu).

T. Quint is with the Department of Mathematics and Statistics, University of Nevada, Reno, NV 89557 (E-mail: quint@unr.edu).

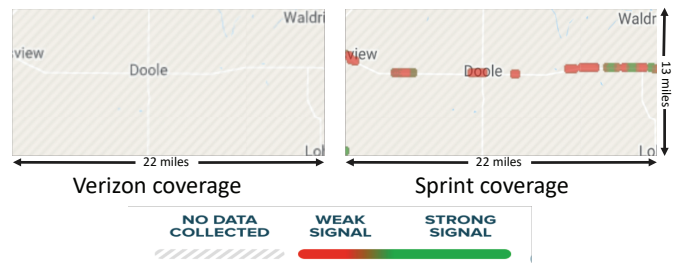
from the primary users. Recently, several attempts have aimed to take this sharing to *primary* level (e.g., co-primary [5], inter-operator [6], multi-operator [7] spectrum sharing) where sharing takes place even though the primary user may be busy. Spectrum sharing at *primary-level* seems necessary to maximize end users’ overall wireless experience and optimize “micro-opportunities” arising in radio propagation [8]. For example, in urban areas, a user might go into dark spots (e.g., no coverage or low signal quality) even though his provider has overall the largest coverage or the highest average signal quality. The user located in a dark spot could be served by other providers if there exists an extensive spectrum sharing among providers and users at the primary level.

Consider the real-world example shown in Fig. 1a. It illustrates an overlapping coverage map of two primary cellular providers Sprint and Verizon. If the providers have any difference in coverage signal strength within the operating region, it appears as dark black area in the coverage difference map. In some places, a Sprint customer may find himself out of coverage or get weak coverage while a Verizon customer enjoys seamless connectivity because of Verizon’s better coverage. On the other hand, a Verizon customer may find himself in a spot with a weak signal due complex radio propagation. We call these situations as “micro-opportunities” for sharing at the primary level. In another case, a Sprint customer will get better service than a Verizon customer as the existence of Verizon coverage was not reported in an area according to the map of Fig. 1b. Note that radio propagation may present many more micro-opportunities in overlapping coverage areas as Fig. 1a only shows the radio coverage difference. If providers (Verizon and Sprint in this example) are incentivized to serve each other’s customers, the end user’s experience could notably be improved. The only available economic framework for co-primary spectrum-shared market is cellular roaming [4]. An interesting feature of roaming is that it does not require any governmental regulation. However, it works well when participating providers have significant non-overlapping coverage. To reap the benefits of spectrum sharing in physically smaller markets, and utilize micro-opportunities for improving user experience, we need schemes to incentivize providers that may be competing with each other in overlapping regions of coverage.

Regulatory authorities around the world are aligning with the recent trends of spectrum sharing. The FCC’s vision of the U.S. National Broadband Plan [10] and the wireless community (i.e., industry and academic researchers) are exploring regimes where *sharing is pervasive and even the norm* [11]. However, due to the expenditures associated with the licensed bands, a provider is reluctant to share its spectrum resources



(a) Overlapping region [9]



(b) Non-overlapping region [9]

Fig. 1: Coverage Difference

with other providers unless the sharing brings financial benefit. Incentivizing providers to share their spectrum resources has picked up attention recently [12]. Such cooperation can also be done by injecting some governmental or semi-governmental regulation, although other motivations to improve the “larger good” (e.g., public safety) do exist. As providers get incentive to share their licensed bands, they get more freedom to improve infrastructure and can provide better service without charging more fees to end-users. This helps maximizing users’ welfare. Yet, such pervasive sharing introduces the problem of freeriding. If *strong providers* (i.e., large providers in terms of coverage areas, signal strengths and service rates) share their spectrum bands with the users of *weak providers*, the weak providers might freeride. In particular, if the incentive is given externally/explicitly (e.g., government subsidy in energy and power markets [13]), weak providers will have incentive to freeride in the shared spectrum markets. In this paper, we outline the emergence of freeriders in pervasively shared inter-provider spectrum markets where sharing takes place at the primary-level aiming to improve connectivity and wireless experience of end-users. We analyze this issue by introducing a game-theoretic model of head-to-head competitions between providers and exploring regimes where freeriding can be minimized.

In an open market, all participating providers prefer fewer government regulations. So, we try to involve the government as little as possible and focus on finding market equilibrium only involving activities of the providers. A government uses collected license fees to operate spectrum regulatory authority and a significant portion of surplus money goes to national reserve for serving other purposes [14]. A question now arises: why does not the government use this money to improve users’ wireless experiences? Incentivizing providers from this surplus money can motivate them to share spectrum resources and reduce additional fees, e.g., roaming fees, for the users. In our previous work [15], we have discussed *the government’s role* in giving a performance-based subsidy to the service providers with the goal of maximizing social welfare. In this paper, we focus on the issues arising with the *subsidy-based spectrum sharing* (SBSS) model and solving them, and involve the government only when providing a performance-based subsidy, determining the amount of subsidy to avoid freeriding and applying a penalty when required. Providers get subsidy on the basis of “proof-of-sharing”, which quantifies how much

a provider has shared its resources with another provider’s customers with a weaker signal/coverage at a certain time and location [16], [17]. This “proof-of-sharing” can ensure truthfulness of spectrum sharing and provide necessary information to the government. In our work, we assume truthfulness in the “proof-of-sharing” and use it to determine the subsidy amount. This subsidy goes for a certain time period. After each period, the provider’s sharing information gets assessed and the provider gets penalized based on its service record. Thus, effectively, the providers get a varying subsidy after each assessment period.

#### A. Contributions

Major contributions of this work on minimizing freeriding in a co-primary SBSS market are as follows:

- We develop a game-theoretic framework to minimize freeriding while maintaining spectrum sharing at the primary level.
- We formulate a two-player non-cooperative game of head-to-head competition in the market, with one large provider (a.k.a spectrum sharing provider) and one small provider (a.k.a probable freeriding provider), calculate revenue (i.e., payoff) of each provider, and find Nash Equilibrium (NE) of the game.
- Our model is able to determine the maximum possible sharing by the large provider while avoiding freeriding (considering other conditions remain unchanged).
- This model shows how to prevent or reduce the small providers’ freeriding opportunities, by adjusting the large provider’s subsidy amount.
- Our model sheds light on regulating providers’ subscription fees (i.e., service prices) to minimize freeriding, with respect to fair market fees.
- The game-theory analysis identifies operational regimes where the large provider earns at least the earnings of a non-shared market irrespective of its willingness to share (i.e., percentage of accepting incoming calls/service requests from the customers of the small provider) and the small provider’s strategies to steal customers and freeride.
- We show the existence of operational regimes without freeriding in a two-provider market with the exception where both providers enter into the freeriding Pure Strategy Nash Equilibrium (PSNE) region (explained later in Section VI) due to the excessive subsidy which was given

to the large provider. Before reaching this condition, the large provider gets at least the earnings of non-shared market by playing a mixed strategy or a freeriding-free pure strategy.

## II. RELATED WORKS

### B. Key Insights

We run a simulation based on our game-theoretic framework on an SBSS market where two providers (one large provider and one small provider) are competing in a region to attract customers from a common pool of 1000 participants who have high price elasticity. Each customer makes two types of calls: a) home calls and foreign calls. Initially the large provider enters with a high interest to share its spectrum resources and the small provider does not share (all detailed in Section VI). As our goal is to maintain better connectivity to improve end-users' wireless experience irrespective of their choice of provider, we only consider *call service* to evaluate our model. From this market, our findings are as follows:

- If the large provider's willingness to share is at most 69%, it can maintain this sharing for entire subsidy period by playing a pure strategy without incurring any revenue loss irrespective of the small provider's charging fee. If the willingness to share is more than 69%, the large provider plays a mixed strategy where it can only maintain this willingness to share for a certain duration of subsidy period to ensure no revenue loss from freeriding.
- If the ratio of the small provider's charging fee to the fair market fee is at most 41%, the large provider can operate with sharing its infrastructure at any level, i.e., willingness to share being 100%. If the fees ratio is greater than 41%, the game follows a mixed strategy.
- A small amount of subsidy can motivate a provider for more sharing if both providers get the chance to improve signal strengths (described in Section VI).
- If a participating provider is too large compared to the other provider, we observe a non-freeriding regime. It happens because the customers of the small provider perceive low quality signal compared the large provider. On the other hand, if the participating providers are similar in size, no freeriding takes place.
- A spectrum sharing provider is more interested to play mixed strategy due to customers' price elasticity.
- By considering all head-to-head competitions, a spectrum sharing provider gets necessary strategies from the game NEs to tackle each of its competitors' freeriding strategies in the market. Thus, our model provides effective solutions in a multi-provider environment.

The rest of the paper is organized as follows: In Section II, related works are discussed. Section III describes the freeriding issue in SBSS cellular markets and, in Section IV, we illustrate the SBSS market model. In Section V, we present a two-provider SBSS cellular market, formulate a two-provider game and finally, extend the game to a multi-provider environment. We outline the experimental evaluation of our model in Section VI. Finally, we conclude in Section VII.

Spectrum sharing via database-driven approaches can manage the shared co-existence of users in-case of heterogeneous access [18] and a full-duplex wireless technology can enable concurrent transmission, reception, higher spectrum efficiency and secondary throughput while minimizing hidden station problem [19]. A decentralized database-driven blockchain-based DSA was analyzed in [20]. It considers the benefits and limitations of blockchain solutions in general, and then examines their potential application on spectrum sharing. Geo-location-based spectrum sharing to priority users where higher priority users get spectrum allocation based on its traffic load and lower priority users complete to get spectrum from the unallocated portion can offer a stable and efficient spectrum allocation [21].

A cooperation among competing providers can be achieved with the "magnitude" of sharing which is counted by the number of favors each provider makes to other providers [22]. The virtualization of spectrum resources into fungible chunks can improve market liquidity by yielding viable market outcomes for secondary spectrum markets [23]. It explores the hinders of the emergence of liquid secondary markets for the radio spectrum, permits to develop an appropriate, tradeable, and spectrum-related commodity, and accounts the physical constraints inherent to the electromagnetic spectrum for the secondary market. A blind beamforming technique for secondary transmitters and a blind interference cancellation technique for secondary receivers was addressed in [24]. These two MIMO-based interference cancellation techniques use an underlay spectrum sharing scheme for cognitive radio networks where the primary users are oblivious to the secondary users. Deep reinforcement learning-based spectrum sharing using recurrent neural networks provides the ability in learning optimal strategies without any prior knowledge of any node's behavior to utilize multiple channels with the presence of multiple heterogeneous primary users was discussed in [25]. This work also demonstrates how a node copes with the multi-rate and multi-agents scenarios in dynamic spectrum sharing environments.

Spectrum sharing among micro-operators (MOs) to meet the demand for local service delivery in indoor small cell networks was discussed in [26]. It used a homogeneous Poisson point process to model the locations of the small cell base stations of the MOs that are going to share spectrum subbands and suggests which subbands a buyer MO should purchase from the regulator to satisfy QoS. Spectrum broker service for MOs and citizens-broadband-radio-service (CBRS) for priority-access-licenses (PAL) was considered in [27]. The spectrum broker allocates available spectrum to operators per demand and charges price. The concept of spectrum brokering is similar to our approach in aiming to attain a more effective and pervasive sharing of the spectrum at the primary level. A market model for the sub-leasing spectrum by PAL users with short-term license allocation in smaller areas with the coordination of a spectrum access system to maximizes the financial gain to the PAL holder was addressed in [28]. On-demand spectrum access from the CBRS to dynamically set-up application-

oriented virtual topologies to support user applications was discussed in [29]. It estimates the blocking probability of spectrum demands from PAL users, the network capacity, and the number of free spectrum bands available for lower priority general authorized access users. Utilizing mmWave spectrum bands using licensed spectrum access to coordinate sharing among cellular operators to increase the available capacity was addressed in [30]. It applies European and Italian regulatory conditions to simulate DSA in 5G scenarios.

The maximum total throughput of a coexisting spectrum-sharing Long Term Evolution (LTE) and WiFi network under two fairness constraints including throughput fairness and 3GPP fairness was addressed in [31]. It uses the Category 3 or Category 4 Listen-Before-Talk mechanism to improve the mean successful transmission time of the LTE network. End-users' provider selection based on the sum of congestion and price advertised by primary and secondary providers was addressed in [32]. They assume the congestion of the primary provider is caused by its subscribers and the congestion of secondary providers occurs with the presence of both subscribers. Although these efforts have a similar flavor to ours in tackling inter-provider issues, they only consider primary vs. secondary provider sharing cases.

Utility-based freeriding control in peer-to-peer (P2P) systems has been described in [33]. Users are penalized based on the total number of files shared, the total size of data shared and the popularity of the shared content. Payment-based control is another way to prevent freeriding in P2P systems. In [34], two decentralized payment methods were delineated: (1) the sender uploads the required payments and each intermediate node earns a fee when the packet traverses through that node, and (2) each node buys packet from previous node and sells to the next node, while finally the receiver pays the total cost. Although these freeriding control efforts have considered the game-theoretic issues, the freeriding problem in P2P systems is different from the freeriding in SBSS markets. Each participant in the P2P systems must be equal whereas the participant in our game must be different in size to see any freeriding. Further, the timescale of the game in P2P systems is much shorter than what is possible in the SBSS markets. To the best of our knowledge, detailed consideration and game-theoretic modeling of freeriding in shared spectrum markets has not been studied.

### III. FREERIDING IN SUBSIDY-BASED SPECTRUM SHARING (SBSS)

In SBSS markets [15], a subsidy from the government to providers can contribute to increasing user welfare. The end users will not need to pay more subscription fees for service when they are in suburban areas or out of their provider's coverage. Further, they will receive a better quality of experience due to micro-opportunities arising in urban settings. Thus, putting the end user's received quality of experience as the top priority is one of the main motivations for our work on SBSS markets.

An SBSS market operating within a region is visualized in Fig. 2. It is built on an existing cellular market with the inclusion of the government as an incentivizing organization.

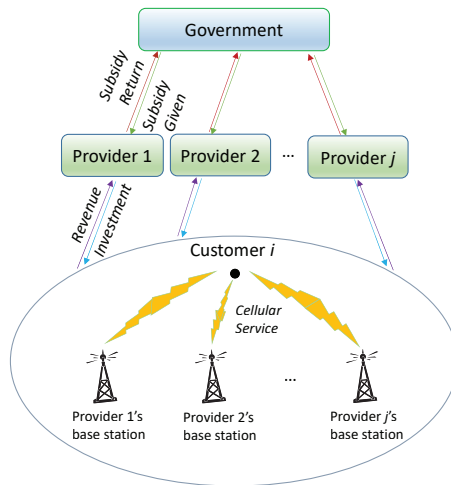


Fig. 2: An SBSS market

Initially, the customers choose providers based on the utility of the offered services. The government subsidizes the providers to share their spectrum with foreign customers (i.e., ones subscribed to another provider) and penalizes based on proof-of-sharing [16]. The providers try to maximize revenue earnings by serving as many foreign calls as they can without hurting their service quality.

In [15], we considered the benefits of an SBSS market. However, we did not analyze the potential risks of this market. It can result in a situation that provides an opportunity to a weak provider to freeride on a strong provider's infrastructure. We consider a weak provider as a small provider with weak signal strength, small coverage area, or low service rate compared to its competitor provider, which we call the strong provider or large provider. When a weak provider advertises relatively smaller subscription fees compared to its fair market fees and the strong provider's fees, it can unfairly attract customers who would normally go to the strong providers in a fair market. Since strong providers are motivated to serve the users of the weak providers because of the extra subsidy, the weak providers may exploit this situation and tend to freeride on the strong providers' network infrastructure. Further, strong providers will not be able to drop subscription fees below a certain level due to high maintenance costs, and may not retain their customers who eventually switch to other providers offering lower fees.

Assume an SBSS cellular market with Providers 1 and 2 as shown in Fig. 3. The dashed circles encapsulate each of the providers' coverage domination. Some regions are equally dominated by both providers while others are dominated by one of them. Provider 1 is, overall, stronger as it has more base station (BS) support than 2. Also, due to higher infrastructure costs, Provider 1's subscription fees are expected to be higher in an existing market. Consider a region where Provider 1 has better network coverage. If a customer wants to get a *strong* connection, he should subscribe to Provider 1 in an existing market. In fact, if there was no subsidy, Provider 1 would retain these customers. However, under the SBSS market, the weak one, Provider 2, can offer cheap subscription

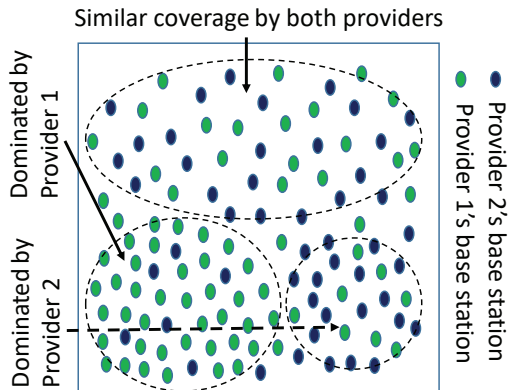


Fig. 3: Provider 1 and 2's coverage

fees due to its small infrastructure costs. Further, Provider 1 is incentivized to serve Provider 2's customers. Thus, if Provider 1 shares its BSeS with Provider 2's customers too much, the overall quality of the network service will appear to be similar for both Provider 1's and 2's subscribers. Then, for the customers, the only perceived difference between two providers will be the subscription fees. As the customers are highly sensitive to price, these fees differences will cause *unfair customer switching* (UCS) in favor of Provider 2. To make it fair, Provider 2 should increase its fees to fair market levels (i.e., similar to Provider 1's fees), but it has no motivation to do so in the SBSS market. Due to the potential revenue loss from freeriding, no strong providers will agree to join the SBSS market. To ensure their participation, we address freeriding and UCS issues in the SBSS markets, and minimize them using spectrum sharing strategies obtained from our proposed game-theoretic framework.

#### IV. A BASELINE MODEL FOR SBSS MARKET

Assume a set of  $J$  network providers, denoted by  $\mathcal{J}$ , that are competing within same region in an SBSS market. A customer  $i$  will choose  $j$  as "home provider" based on the overall signal strength/quality,  $\psi_j$ , and the subscription fee,  $f_j$ , of provider  $j$ , and treats any other provider  $k \in \mathcal{J} \setminus \{j\}$  as a "foreign provider". We assume a single subscription fee for all customers of a provider and do not consider different fees based on service quality within a provider. Since our focus is on freeriding among providers, differentiation of the fees across providers enables us to observe inter-provider dynamics in terms of regulating freeriding. The SBSS market may result in a customer being served by a foreign provider in addition to its subscribed home provider, depending on the received signal quality from these providers as well as the providers' willingness to share their infrastructure. A provider treats all customers as "foreign customer" if the customers are subscribed to other providers. Based on these selections (to be detailed next), the number of customers subscribing to provider  $j$  can be expressed as:

$$n_j = N(f_j, \psi_j) \quad (1)$$

where the demand function  $N(\cdot)$  is a decreasing convex function with respect to (w.r.t.)  $f_j$  and an increasing concave func-

tion w.r.t.  $\psi_j$ . The shape of a demand function w.r.t to price is usually a decreasing convex function [35], which we follow in our model. Further, the utility from a service diminishes with the increase of service consumption [36]. If we continuously increase the signal strength of a provider, after a certain limit, the marginal improvement of perceived signal strength will saturate and become insignificant. To model this, we shape the demand function as increasing concave form w.r.t. the provider's signal strength.

#### A. Customer $i$ 's Provider Selection

Assume each customer  $i$ , on average, makes  $\gamma$  calls during a fixed time period. Among them, he makes  $\beta_{i,j} = \sigma(X_j)$  home calls where  $0 \leq \beta_{i,j} \leq \gamma$ , and  $\sigma(\cdot)$  is an increasing concave function w.r.t. the number of BS,  $X_j$ , of his home provider  $j$ . The probability to find customer  $i$ 's home BS during making a call increases with the increase of home BS count. Since the marginal benefit of adding more BSeS diminishes in terms of covering calls from subscribed customers [37], we assume  $\sigma(\cdot)$  as a concave function. The customer  $i$  also makes  $\alpha_{i,j} = \gamma - \beta_{i,j}$  foreign calls.

We model a customer's selection of a home provider via its utility of the service quality. Let a customer's utility function,  $u(\cdot)$ , be an increasing concave function of the signal intensity/quality of service available to that customer. Then, customer  $i$ 's overall utility from subscribing to provider  $j$  can be expressed as:

$$U(i, j) = \beta_{i,j}u(\psi_j) - f_j. \quad (2)$$

A provider's offered aggregate utility from service must be positive. Otherwise, in reality, no customer will be interested in getting a subscription. This positive difference is the gain a customer can expect from a subscription where  $\beta_{i,j}u(\psi_j) > f_j$ . If a provider cannot maintain this condition, the gained utility will not be attractive to the customers. In such a scenario, we set the offered aggregate utility to 0. As a result, the provider can not participate in the SBSS market because it will not have a customer base. We assume  $f_j \leq f_{max}$  where  $f_{max}$  is the maximum subscription fee above which no provider can run profitable cellular operations in a competitive market. Finding an optimal value of  $f_{max}$  involves exploration of several market factors as well as business strategies of providers. In this paper, we consider  $f_{max}$  as the maximum of all providers subscription fees for similar service in a non-shared market.

Based on (2), customer  $i$  will select provider  $j$  as his home provider with the following probability:

$$P_{i,j} = \frac{U(i, j)}{\sum_{j=1}^J U(i, j)}. \quad (3)$$

We get this probabilistic provider selection from Contest Theory, [38], where a customer will most likely choose a provider which offers best utility.

#### B. Provider $j$ 's Revenue Earnings

Provider  $j$  receives subsidy in addition to its regular subscription fees. In the SBSS market, a subsidy is determined by the total market value,  $T_{mv}$ . We denote the provider  $j$ 's

market value,  $V_j$ , as the number of customers,  $n_j$ , times subscription fees,  $f_j$ . Aggregating all providers' market values, we get the  $T_{mv}$ . In our model, we consider a percentage of the  $T_{mv}$  will be given to provider  $j$  as subsidy. Further, it can generate revenue by freeriding on other providers' networks. If a regular non-subsidized market consists of  $n$  customers among which  $n_j$  are subscribed to provider  $j$ , we can suppose that provider  $j$  earns  $n_j f_j$  subscription revenue. However, after joining in the SBSS market, provider  $j$  gets the subsidy  $\epsilon_j$ , some of which it can spend to improve its infrastructure, e.g., by increasing the number of base stations,  $X_j$ . It also has licensed bandwidth,  $b_j$ , divided equally among all of its BSEs, in order to run cellular operations. Available bandwidth and the number of BSEs determine the signal strength of provider  $j$ , which can be expressed as  $\psi_j = Q(X_j, b_j)$  where  $Q(\cdot)$  is an increasing concave function w.r.t.  $X_j$  and  $b_j$ , since marginal signal improvement diminishes with the increase of  $X_j$  and  $b_j$ .

In a multi-provider environment, the government can split subsidy  $\epsilon_1$  further in a way where the provider  $j$  earns  $\epsilon_j^k$ , a portion of  $\epsilon_j$  by serving the customers' of provider  $k$  only. However, such a split is beyond the scope of this work. We assume, the government will maintain a suitable policy to split  $\epsilon_1$  and penalize separately on the components where  $\epsilon_j = \sum_{k \in \mathcal{J} \setminus \{j\}} \epsilon_j^k$ . Assume that provider  $j$  serves  $F_j^k$  foreign calls generated by the customers of provider  $k$ , which is the proof of sharing of provider  $j$ 's BSEs with other provider's customers. A provider is penalized on its subsidy based on the number of foreign calls it serves. We call this a performance-based subsidy scheme. The higher  $F_j^k$ , the lower the penalty. We model the penalty function  $p(\cdot)$  which determines the percentage of the subsidy given to provider  $j$  for serving customers' of provider  $k$  needs to return to the government as:  $\mathcal{P}_j^k = p(F_j^k)$  where  $p(\cdot)$  is a linear function w.r.t.  $F_j^k$ . If provider  $j$  serves all foreign calls,  $\mathcal{P}_j^k$  becomes 0. Contrary,  $\mathcal{P}_j^k = 1$ , when provider  $j$  does not serve any foreign customer.

Based on the provider selection problem (3), a customer  $i$  of provider  $j$  is more likely to choose another provider  $k$  if  $U(i, k) > U(i, j)$ . Also, any change of subscription fees can change provider selection criteria along with the signal strength, which is the main driving factor behind  $U(\cdot)$ . In an SBSS market, the service utility received from providers  $j$  and  $k$  can be the same (or similar in a more general sense) due to the subsidy for sharing, i.e.,  $U(i, j) = U(i, k)$ . In such a case, if  $f_j < f_k$ , customer  $i$  of provider  $k$  will more likely subscribe to provider  $j$ . Assume, by advertising lower-than-fair subscription fees, i.e.,  $f_j < f_k$ , provider  $j$  can unfairly attract  $n_k \tau_{kj}$  subscribed customers of  $k$ , where  $\tau_{kj} = 1 - e^{-C f_k / f_j}$  is the probability of UCS from provider  $k$  to provider  $j$ . Here,  $C$  is customers' averseness to price. A high value of  $C$  indicates a high fees difference and a high customers' averseness to price. The value of  $C$  can be any number where  $C \geq 0$ . When  $f_k / f_j \approx \infty$ , we get  $\tau_{kj} \approx 1$  and if  $f_k / f_j \approx 0$ , we have  $\tau_{kj} \approx 0$ . Now, provider  $j$ 's revenue,  $R_j$ , becomes

$$\max_{f_j, b_j, X_j} \left( n_j f_j + \sum_{k \in \mathcal{J} \setminus \{j\}} (1 - \mathcal{P}_j^k) \epsilon_j^k + \sum_{k \in \mathcal{J} \setminus \{j\}} (n_k \tau_{kj} - n_j \tau_{jk}) f_j \right) \quad (4)$$

$$\text{where } \sum_{k \in \mathcal{J} \setminus \{j\}} \epsilon_j^k = \epsilon_j \quad (5)$$

$$0 \leq \beta_{i,j} \leq \gamma \quad (6)$$

$$\beta_{i,j} u(\psi_j) > f_j \quad (7)$$

$$0 < f_j \leq f_{max} \quad (8)$$

$$\text{and } b_j, X_j > 0, \forall j. \quad (9)$$

The first term of (4) denotes the earnings of provider  $j$  from its subscribed customers. The second term describes the leftover subsidy money. If provider  $j$  does not serve a satisfactory number of foreign calls, it has to return some or all of  $\epsilon_j$ . The third term delineates the earnings from net switching customers.

In an SBSS market, provider  $j$  may lose significant number of customers due to freeriding which eventually generates less revenue for  $j$  compared to a non-shared market. To make sure  $j$ 's participation in the SBSS market,  $j$ 's overall earnings must be at least as much as the revenue of the non-shared market. Thus, the following condition must be met to motivate  $j$  for sharing.

$$\sum_{k \in \mathcal{J} \setminus \{j\}} (1 - \mathcal{P}_j^k) \epsilon_j^k + \sum_{k \in \mathcal{J} \setminus \{j\}} (n_k \tau_{kj} - n_j \tau_{jk}) f_j \geq 0 \quad (10)$$

## V. GAME FORMATION

We mitigate freeriding issue in an SBSS market by developing a non-cooperative game to hold the condition of (10) by generate at least the earnings of a non-shared market. Initially, we consider a two-provider SBSS market, form a 2x2 non-cooperative game and determine the Nash equilibrium (NE) of the game. Later, we extend the game to a multi-provider market by applying the two-provider game among all providers considering head-to-head competitions.

### A. Two-Provider SBSS Market

Consider two providers, Provider I and Provider II, are competing in an SBSS cellular market with willingness to shares,  $\omega_1$  and  $\omega_2$ , respectively. The maximum number of foreign calls I and II can get from the market is  $\sum_{i=1}^{n_2} \alpha_{i,2}$  and  $\sum_{i=1}^{n_1} \alpha_{i,1}$  respectively, and are willing to serve  $F_1^2$  and  $F_2^1$  incoming foreign calls, respectively, where  $0 \leq F_1^2 \leq \sum_{i=1}^{n_2} \alpha_{i,2}$ ,  $0 \leq F_2^1 \leq \sum_{i=1}^{n_1} \alpha_{i,1}$ ,  $F_1^2 = \omega_1 \sum_{i=1}^{n_2} \alpha_{i,2}$ , and  $F_2^1 = \omega_2 \sum_{i=1}^{n_1} \alpha_{i,1}$ .

A customer's perceived signal strength of home provider depends on the distance between his location and the nearest home BS. So, perceived home signal strength can vary from customer to customer. Similarly, perceived signal strength for the foreign calls the customer makes can vary. Thus, the choice of BS selection to make a call varies from customer to customer which eventually generates different numbers of home and foreign calls for different customers. Suppose  $\theta_1$  denotes the probability that a customer of Provider I chooses Provider I's BS to make call (i.e. home call), and  $\theta_2$  denotes the probability that a customer of Provider II chooses Provider II's BS for a home call. Then  $\theta_1$  and  $\theta_2$  are given by the expressions

$$\begin{aligned} \theta_1 &= \text{Probability}[\psi_1 \geq \psi_2] + (1 - \text{Probability}[\psi_1 \geq \psi_2])(1 - \omega_2) \\ \theta_2 &= \text{Probability}[\psi_2 \geq \psi_1] + (1 - \text{Probability}[\psi_2 \geq \psi_1])(1 - \omega_1) \end{aligned} \quad (11)$$

where  $\psi_1$  and  $\psi_2$  are the signal strengths of Provider I and II, respectively. Now, the overall perceived signal strengths for customers of I and II will be

$$\psi_1^* = \theta_1 \psi_1 + (1 - \theta_1) \psi_2 \quad \text{and} \quad \psi_2^* = \theta_2 \psi_2 + (1 - \theta_2) \psi_1. \quad (12)$$

Consider Provider I has more BSes than II, i.e.,  $X_1 > X_2$ . According to the discussions of Section IV-A,  $\beta_{i,1}, \beta_{i,2}$  are increasing concave functions of the BS counts of the providers. Likewise, according to Section IV-B,  $\psi_1, \psi_2$  are increasing concave functions of the BS counts and bandwidths of the providers. As  $X_1 > X_2$ , comparing the first terms of (2),  $\beta_{i,1} * u(\psi_1) > \beta_{i,2} * u(\psi_2)$  holds. So, if  $f_1 \leq f_2$ , we can infer that  $U(i, 1) > U(i, 2)$ . If  $f_1 > f_2$ , we can write that  $U(i, 1) > U(i, 2)$  holds as long as the subscription fees are at their fair market values. We model an SBSS market which transforms from the existing market where both providers were charging fair market fees<sup>1</sup>  $f_1^*$  and  $f_2^*$ . Thus, considering different cases of both providers' fees, we can say, Provider I offers better utility than II before the effects of subsidy take place.

The SBSS cellular market makes a tunnel for the weak (i.e., small) provider to make extra earnings by freeriding on the strong (i.e., large) provider. Under the subsidized market, each provider's offered cellular utility,  $U(\cdot)$ , could become almost equal, and there may remain only a small perceived difference between cellular providers' service quality. Assuming no changes of previously subscribed customers' fees and considering the possibility of UCS, the revenues under the subsidized market becomes

$$R_1 = n_1 f_1 + (1 - \mathcal{P}_1^2) \epsilon_1 + f_1 (n_2 \tau_{21} - n_1 \tau_{12}) \quad (13)$$

$$\text{and} \quad R_2 = n_2 f_2 + (1 - \mathcal{P}_2^1) \epsilon_2 + f_2 (n_1 \tau_{12} - n_2 \tau_{21}). \quad (14)$$

In a two-provider SBSS market, we can say  $\epsilon_1^2 = \epsilon_1, \epsilon_2^1 = \epsilon_2, \mathcal{P}_2^1 = \mathcal{P}_2, \mathcal{P}_1^2 = \mathcal{P}_1$ , and will use these notations in the following sections. If  $f_1 \leq f_2$ , most of the potential customers will go for Provider I's service in those areas where I has better coverage. Because, Provider II may offer the same service utility, i.e.,  $U(i, 1) = U(i, 2)$ , it charges higher subscription fees. Thus, no UCS and freeriding take place, which is aligned with the existing market equilibrium. If Provider II aims to initiate UCS by lowering fees, i.e.,  $f_1 > f_2$ , then the market equilibrium changes. Now, Provider II may offer the same utility with lower fees, which will attract the customers of Provider I. Soon, Provider I will lose customers, even though it has a stronger signal and better coverage. If no subsidy agreement exists, Provider I could retain these customers. Depending on the government's offerings, the subsidy can be less than the revenue lost due to Provider II's freeriding, which demotivates Provider I to continue participating in the SBSS market.

### B. Game for a Two-Provider SBSS Market

To model the free-rider problem, we envision a simple 2x2 noncooperative strategic form game as follows. The two players are a large provider a.k.a Provider I and a small provider

		Provider II (Small provider)	
		Low fees	Fair market fees
Provider I (Large provider)	Cover calls	(a, b)	(c, d)
	Doesn't cover calls	(e, f)	(g, h)

Fig. 4: 2x2 game model to control freeriding

a.k.a Provider II. They are competing in an area where Provider I is able to offer much better service than Provider II. Now consider customers not subscribed to Provider I, who venture into this area and wish to make calls. Provider I then has a decision to make: it can either cover such calls or else not cover them. Covering them has the benefit of earning government subsidy benefits, but runs the risk of the freeriding problem described below. At the same time, Provider II decides whether to "undercut" Provider I, by offering low-fee service to these customers. If the Provider I is "covering" these customers calls, Provider II can freeride – it enrolls the customers by offering the low-fee services, collects the fees, but does not have to provide any service because Provider I is doing so.

The situation outlined above is modelled by the simple 2x2 noncooperative strategic form game as shown in Fig. 4.

The two strategies for Provider I are listed on the left, while those for Provider II are across the top. The four possible outcomes are represented by four cells. Of interest here is the free-rider outcome in the top left (if Provider I "cover calls" and Provider II charges "low fees"). Also note that there is another outcome which the government would like, namely for Provider I to cover foreign calls, but for Provider II to charge fair market fee and so no freeriding occurs. This is the top right cell of the bimatrix. The two entries in each cell represent the payoffs for Provider I and Provider II, respectively, if the providers play the corresponding strategies. For example, if Provider I does not cover calls while Provider II charges low fees, the payoff is  $e$  for Provider I and  $f$  for Provider II.

In our context, we may assume certain relationships among the quantities  $a, b, c, d, e, f, g$ , and  $h$ . First, we may expect Provider I to earn more with a non-covering strategy instead of a call-covering strategy against Provider II's undercutting low-fees strategy, so  $a < e$ . Next, if Provider I does not cover calls, its revenue will not be affected regardless of Provider II's strategies. Thus  $e = g$ . Also, if Provider II charges fair market fees, then Provider I earns more by covering than by non-covering – hence  $g < c$ . Next, if the large provider does NOT cover calls, Provider II will earn a lower revenue if it undercuts; so  $f < h$ . If Provider II charges fair market fees, then its earnings will not change, regardless of what Provider I does, as it is not losing any customers and gaining any subsidy because  $\omega_2 = 0$  – thus we get  $h = d$ . Finally, if the large provider is covering calls, it pays off for the small Provider II to free-ride; hence  $d < b$ . Putting all of this together, we claim that  $a < e = g < c$  and  $f < h = d < b$ .

<sup>1</sup>We show the calculation of the fair market fee from marginal signal utility in Section VI-A1.



We analyze our game using non-cooperative game theory. A mixed strategy for Provider I is a two-component vector  $(p, 1-p)$  in which  $p$  (between 0 and 1) represents the probability that it will cover calls and  $1-p$  is the probability it does not. Similarly, a mixed strategy for Provider II is given by  $(q, 1-q)$ , in which  $q$  is the probability that it charges low fees. Given mixed strategies for each provider, it is easy to calculate the expected payoff for each provider:  $pqa + p(1-q)c + (1-p)qe + (1-p)(1-q)g$  for Provider I, and  $pqb + p(1-q)d + (1-p)qf + (1-p)(1-q)h$  for Provider II.

A Nash Equilibrium (NE) is a pair of mixed strategies, one for each provider, in which both providers are maximizing their expected payoffs given what the other is doing. A stability concept, it is the most widely used solution concept in game theory [39]. In our game, under the assumptions  $a < e = g < c$  and  $f < h = d < b$ , the unique NE is when Provider I plays  $(p^*, 1-p^*)$  and Provider II plays  $(q^*, 1-q^*)$ , where  $p^* = (h-f)/(b-f)$  and  $q^* = (c-g)/(c-a)$ .

Now observe what happens if the government raises the subsidy to the Provider I for covering calls. This raises both  $a$  and  $c$  by the same amount, say  $x$ . For small  $x$ , the effect is to raise  $q^*$ , i.e., to make the Provider II more likely to undersell. If  $x = e - a$ , there is a continuum of NEs, all with  $q^* = 1$ , i.e., all with Provider II underselling. Similarly, if we find revenues  $a \geq e$  and  $d = b$ , we get a continuum of NEs where Provider I covers calls with  $p^* = 1$ . Provider II can play any strategy. Finally, if  $x > e - a$ , the unique NE outcome is the freeriding outcome of  $p^* = q^* = 1$ . Hence, we see how the government raising the subsidy triggers the freeriding problem. This additional subsidy forces the relations between the revenue earnings where  $a \geq e$ . In this state, covering calls is a dominating strategy the Provider I, i.e., it is in the best interest of the Provider I to cover calls, no matter what the Provider II does. Since  $d < b$ , the Provider II's best response to this is to charge low fees. Hence, the unique NE is the freeriding outcome.

So, how can the government encourage its desired outcome? It merely needs to raise the payoffs (to Provider II) for not underselling. In terms of our bimatrix, this would raise the quantities  $d$  and  $h$ . If done in conjunction with the subsidy idea above, it could force the NE to be the desired outcome, i.e.,  $p^* = 1$  and  $q^* = 0$ . In practice, this can be done by either imposing a price floor for the service subscription fee (so that charging too small a fee is a crime), or perhaps by allowing providers to keep more of their fees via lowering particular penalties on the subsidy benefits from the government.

### C. Observations

If we closely observe the game NEs, we see that Provider I's strategies ensure it to earn at least the earnings of a non-shared spectrum market which can be found at the bottom-right cell of the game matrix. In case of pure strategy NE (PSNE), it is obvious from the payoffs, e.g., the game NE at the top-left cell ensures Provider I to get as much as the earnings of a non-shared market. Similarly, Provider I gets guarantees from the freeriding-free outcome. In case of mixed strategy NEs, we can not directly observe such guarantees just looking into

the game matrix. However, if we compute the total revenue by applying NE strategies, the game guarantees Provider I no revenue loss. Let analyze this scenario with an example where the game follows mixed strategies. If  $a = 8, b = 6, c = 15, d = 5, e = 10, f = 3, g = 10$ , and  $h = 5$ , the game has an NE with the mixed strategies  $(0.67, 0.33)$  and  $(0.71, 0.29)$  for Provider I and II respectively. Provider I's expected payoff becomes 10.02, which is higher than the non-shared payoff 10. Similarly, we will get a higher payoff for all mixed strategy NEs. In this way, our proposed two-provider game holds the condition of (10).

### D. Game in a Multi-Provider SBSS Market

In a multi-provider SBSS market, we consider each of the competitor providers individually based on the head-to-head competitions in generating revenue. In this market, a provider's single willingness to share strategy against all providers may mitigate a freeriding provider's bad intentions, however, it can harm the cooperation, good intentions of other providers who tends to charge fair market fees. A spectrum sharing provider could increase its willingness to share to serve the customers of the providers who charge fair market fees. However, to minimize a freeriding provider's effects, the spectrum sharing provider is forced to reduce sharing. Eventually, such a strategy generates fewer subsidy revenue for it.

Regardless of the number of providers are present in an SBSS market, we always consider head-to-head business competitions among all providers in a multi-provider market and design the game model where a spectrum sharing provider plays a two-provider game separately with all of its competitors and finds an appropriate sharing strategy against each of the competitors. Thus, in a  $J$ -provider market, a spectrum sharing provider may end up with  $J-1$  willingness to share strategies and ensures none of the competitor's good/bad intentions get overlooked.

A provider  $j$  can apply strategic decision of sharing with the presence of a freeriding provider  $k$  by following the two-provider game settings described in earlier section, which holds the condition of (10), where  $(1 - \mathcal{P}_j^k)\epsilon_j^k - n_j\tau_{jk}f_j \geq 0$ . It ensures no revenue loss for provider  $j$ . Similarly, in a  $J$ -provider market, provider  $j$  can play the two-player game against each of the  $J-1$  providers and apply appropriate sharing strategies to overcome overall revenue loss where  $\sum_{k \in \mathcal{J} \setminus \{j\}} (1 - \mathcal{P}_j^k)\epsilon_j^k - \sum_{k \in \mathcal{J} \setminus \{j\}} n_j\tau_{jk}f_j \geq 0$ . Now, the revenue of provider  $j$  becomes:

$$R_j = n_j f_j + \sum_{k \in \mathcal{J} \setminus \{j\}} (1 - \mathcal{P}_j^k)\epsilon_j^k - \sum_{k \in \mathcal{J} \setminus \{j\}} n_j \tau_{jk} f_j \quad (15)$$

which is at least the earnings of a non-shared market. Thus, provider  $j$  can apply different strategic decisions of sharing against each of the competitor providers' freeriding strategies and continue participation in the SBSS market without losing revenue.

## VI. EXPERIMENTAL EVALUATION

In previous section, we observed a two-provider game NEs based on different scenarios. In this section, we will

TABLE I: Parameters &amp; Values

Parameter	Value	Parameter	Value
$n$	1000	$n_1, n_2$	$\frac{nX_1}{X_1+X_2}, \frac{nX_2}{X_1+X_2}$
$\omega_1, \omega_2$	(0.3, 0.8, 0.96), 0	$f_1^*, f_2^*$	$\frac{n_1}{X_1^\pi}, \frac{n_2}{X_2^\pi}$
$\gamma$	100	$V_1, V_2$	$n_1 f_1^*, n_2 f_2^*$
$\pi$	0.05, 0.5, 0.8, 0.95	$T_{mv}$	$V_1 + V_2$
$(X_1, X_2)$	(70, 30), (100, 1–100)	$\epsilon_1, \epsilon_2$	(4.49, (0–45))% of $T_{mv}$ , 0

perform quantitative analysis of these NEs using simulation. To analyze the providers' problem in an SBSS market, we consider two providers are competing within a single region where Providers I and II are the large and small providers, respectively. We use the parameters and initial settings outlined in Table I. If we look at the functions to determine the number of customers and fair market fees in Table I and Section VI-A1, we can see that similar sized providers will have a similar customer counts and subscription fees. If both providers are exactly the same, then both will have an equal customer count and charge equal fees.

On average, we find that a US customer is involved in 6 phone calls per day and 180 calls per month [40]. Thus the maximum limit of home calls can be 180 in a non-shared spectrum market. We consider approximately 55% of this number in our model, which is 100 as the total calls a customer will make monthly in the SBSS market. Limiting the maximum calls from real-world statistics helps us to overcome the diminishing effect of excessive home calls on subscription fees. We limit the maximum fee a provider can charge to 80 units based on the US's primary providers' maximum fees for similar service [41].

In our model, one provider shares spectrum resources and the other does not. Because of UCS, some customers may move from one provider to other. If both providers are similar in size and both are sharing, then the probability of UCS will be small and similar in both directions. If one of them takes new strategies on fees or willingness to share spectrum resources, the other provider will be forced to follow similar strategies to keep its subscribers. As fee difference is a key factor for freeriding, similar fees will overcome the freeriding issues. Our objective in this work is to identify potential freeriding possibilities and take countermeasures against them. Hence, we mostly focus on the harmful conditions where fees differences are significantly high. We can obtain such market conditions when the difference between the size of two providers is high. For this reason, we focus on a large and a small provider market to evaluate our model. We start with customer utility, demand and call quality. Later, we analyze customers' price averseness and UCS. Finally, we illustrate the strategies associated with each game NE.

#### A. Experimental Setup

We enlist the simulation parameters in Table I where  $n$  and  $\gamma$  denote the number of customers within the experimental region and the total number of calls each customer makes, respectively. The large Provider I has better area coverage than

II and is more willing to share its spectrum resources. These willingness to shares,  $\omega_1$  and  $\omega_2$ , represent the initial agreement between providers indicating the percentage of incoming foreign calls they will serve. Initially, we set Providers I and II's BS count to 70 and 30 respectively, a high value 0.8 for  $\omega_1$  to observe freeriding scenarios, and 0 for  $\omega_2$  as Provider II does not share spectrum resources. Except Section VI-B3, we use this count of BSEs throughout the experiments. We randomly set up each provider's BSEs in a  $35 \times 35$  square grid and calculate the number of customers,  $n_1, n_2$ , subscribed to the providers based on the BS counts as shown in Table I. Also, we position all customers in the experimental region randomly. We apply a linear penalty,  $\mathcal{P}_1 = 1 - F_1^2/F_t$ , to Provider I on given subsidy where  $F_t$  is the total incoming foreign calls to Provider I.

1) *Customer Utility and Demand*: We model a customer's perceived signal utility from Providers I and II as  $u_1 = u(\psi_1)$  and  $u_2 = u(\psi_2)$  where  $u(\psi_1) = \log(Q_1)$  and  $u(\psi_2) = \log(Q_2)$ , respectively, and formulate the signal quality offered by a provider as an increasing concave function of BS count, and decreasing convex function of the number of customers. In particular, we use  $Q_1 = X_1^\pi/n_1$  and  $Q_2 = X_2^\pi/n_2$  to represent the offered signal quality to the customers of Providers I and II where  $\pi$  is a constant expressing the benefit of having more BSEs to the signal strength. It describes, within a fixed size region, how the marginal improvement on the offered signal quality will diminish w.r.t. the BS count. To assure concavity,  $\pi$  must be in  $(0, 1)$ , and we chose  $\pi = 0.8$  for rest of the experiments except Section VI-B5. With same  $\pi$ , the small provider's offered signal quality and utility will improve more than the large providers' if we increase their base station counts by the same amount.

$\pi$  also helps to determine fair market fees for both providers. Using the marginal utilities,  $u'_1$  and  $u'_2$ , we define the fair market fees as:  $f_1^* = u'_1 = 1/Q_1 = n_1/X_1^\pi$  and  $f_2^* = u'_2 = 1/Q_2 = n_2/X_2^\pi$ . For  $\pi < 1$ , we get  $f_1^* > f_2^*$  which is expected in a normal market where market leader charges higher than the followers [42]. We determine both providers' market values,  $V_1, V_2$ , and the total market value,  $T_{mv}$ , using fair market fees as shown in Table I. Based on  $T_{mv}$ , we start with subsidies,  $\epsilon_1 = 4.49\%$  of  $T_{mv}$ , and  $\epsilon_2 = 0$ , as Provider II does not share spectrum resources. Later, we vary  $\epsilon_1$  between 0–45% of  $T_{mv}$  and observe market equilibrium. 45% of  $T_{mv}$  is already too much subsidy considering real markets. As a result, we do not consider higher subsidy amounts.

2) *Overall Perceived Quality*: To measure the signal strength of each provider, we consider the received power at free space. For simplicity, we only consider the distance from a caller to the BS for the main beam and ignore any multi-path beams. We define  $\psi_1 = 1/d_1^2$  and  $\psi_2 = 1/d_2^2$  where  $d_1$  and  $d_2$  represent the distance from a customer to the nearest BS of Provider I and Provider II, respectively. As we consider a simplified game model where only Provider I is willing to share (i.e.,  $\omega_1 > 0, \omega_2 = 0$ ), all calls of Provider I's customers are considered as home calls. However, for Provider II, it depends on the signal strength of both providers. If the nearest BS is one of Provider I's, then the decision to serve as a foreign call is made based on a random

## Algorithm 1 NE Strategies

---

**Input:**  $\omega_1, n_1, n_2, f_1, f_2, \gamma$   
**Output:** Both providers NE strategies

```

1: procedure NESTRATEGY( $\omega_1, n_1, n_2, f_1, f_2, \gamma$ )
2:   for  $i = 1$  to  $n_2$  do
3:     for  $j = 1$  to  $\gamma$  do
4:       Randomly determine customer  $i$ 's position
5:       if (Provider I's base station is the nearest) then
6:         if (A randomly generated number ranged  $[0:1] \leq \omega_1$ ) then
7:           Call is served by Provider I
8:         else
9:           Call is served by Provider II
10:        end if
11:      else
12:        Call is served by Provider II
13:      end if
14:    end for
15:  end for
16:  Compute  $\mathcal{P}_1$  and  $\tau_{12}$ 
17:  Compute all payoffs of the two-provider game
18:  Compute and return NE strategies:  $\{(p^*, 1 - p^*), (q^*, 1 - q^*)\}$ 
19: end procedure

```

---

number ranged between 0 to 1. If the number is at most  $\omega_1$ , then Provider I serves it as a foreign call. Thus, the number of home and foreign calls may vary for all customers of Provider II. Considering the randomness of the locations of BSes, customers, and determination of foreign vs. home calls, we have run our experiments 7 times and took the average of them to calculate the signal qualities  $\psi_1$  and  $\psi_2$ , and overall perceived qualities  $\psi_1^*$  and  $\psi_2^*$ .

3) *Price Averseness and UCS*: The probability of UCS is one of the key factors of freeriding. If  $\psi_2^* \geq \psi_1^*$ , and  $f_1^* > f_2$ , then the probability of the UCS initiated by Provider II becomes:  $\tau_{12} = 1 - e^{-Cf_1^*/f_2}$ . The probability of the UCS is high when the price averseness,  $C$ , is high. In a normal market, we believe customers are very averse with price. Hence, we use a moderate high value for  $C$  which is 0.9, except Section VI-B6. Due to UCS, Provider I's lost customers  $n_1\tau_{12}$ , can range between 0 to  $n_1$  depending on  $\tau_{12}$ .

4) *Algorithm to Find Nash Equilibrium*: To determine an NE of the game, we summarize the elaborate discussion of Section VI-A, VI-A1, VI-A2 and VI-A3 and transform to a simplified algorithm Algo. 1.

## B. Results

In this section, we illustrate observed NEs and both providers' strategies. We find the effects of different subsidies on game NEs, e.g., a large subsidy, study how the NE changes with the change of small provider's BS count, and the contribution of marginal signal improvement. We also analyze how an NE can shift from one state to another due to the small provider's UCS strategy as well as for the large provider's willingness to share.

1) *Strategies of Large and Small Providers at Equilibrium*: We have observed the NE strategies taken by Provider I,  $p^*$ , and Provider II,  $q^*$ , against Provider II's low fee,  $f_2$ , to fair market fee,  $f_2^*$ , ratio,  $r$ . We have taken 30, 50 and 70 different data points between the ranges  $0 \leq r \leq 0.199$ ,  $0.2 \leq r \leq 0.4$  and  $0.41 \leq r \leq 1$  respectively to evaluate. We consider Provider I's three reference willingness to shares ( $\omega_1 = 0.3, 0.8$  and  $0.96$ ) and draw the graphs, Fig. 5. If we

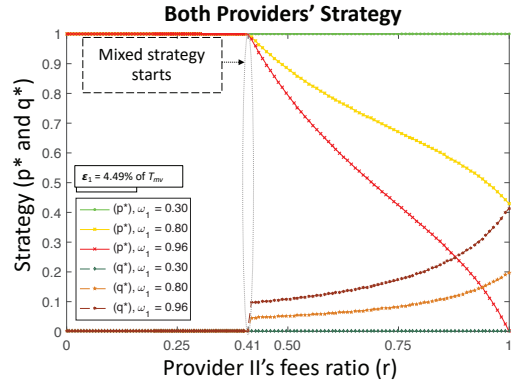


Fig. 5: NE strategies w.r.t.  $r$

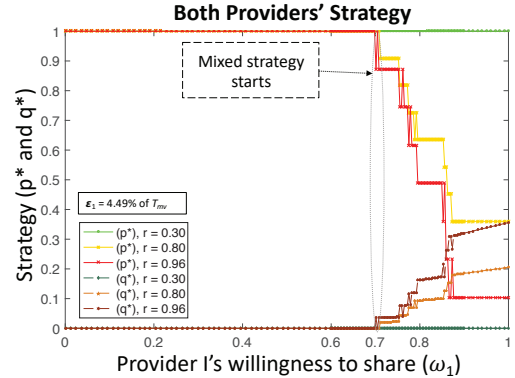


Fig. 6: NE strategies w.r.t.  $\omega_1$

continue increasing Provider II's fees towards the fair market fee, i.e.,  $r \rightarrow 1$ , Provider I plays a less sharing strategy (lowering  $p^*$ ), and Provider II undersells more (higher  $q^*$ ) at high  $\omega_1$ . It happens because of increasing fees nearer to the fair market fee causes less revenue loss for Provider II compared to the earnings from UCS, and high revenue loss for Provider I due to freeriding. We observe a transition of  $p^*$  from pure strategy,  $p^* = 1$ , to a mixed strategy between fees ratio,  $r = 0.41$  to  $r = 0.42$ . This,  $r = 0.41$ , is the terminal fees ratio up-to which Provider I can always maintain agreed sharing without facing any loss. At low willingness to share,  $\omega_1 = 0.3$ , we observe no freeriding and it takes place at high willingness to share,  $\omega_1 = 0.8$  or  $0.96$ . Once Provider I starts losing revenue due to UCS, it chooses mixed strategy to overcome the loss.

We have observed the NE strategies  $p^*$  and  $q^*$  for Provider I's all possible willingness to shares,  $\omega_1$ . We have considered 50, 90 and 10 different data points between the ranges  $0 \leq \omega_1 \leq 0.599$ ,  $0.6 \leq \omega_1 \leq 0.9$  and  $0.91 \leq \omega_1 \leq 1$  respectively, and plotted the graphs for three different  $r$  where the values are 0.3, 0.8 and 0.96, Fig. 6. At same  $\omega_1$ , Provider I is more interested for less sharing with the increase of Provider II's fees towards the fair market fee. We observe both providers play freeriding-free pure strategies,  $p^* = 1, q^* = 0$ , at low fees ratio,  $r = 0.3$ . In case of high fees ratio,  $r = 0.8$  or  $0.96$ , both starts playing mixed strategies above a certain  $\omega_1$ . We have seen a transition phase of  $p^*$  (from 1 to a proper fraction)

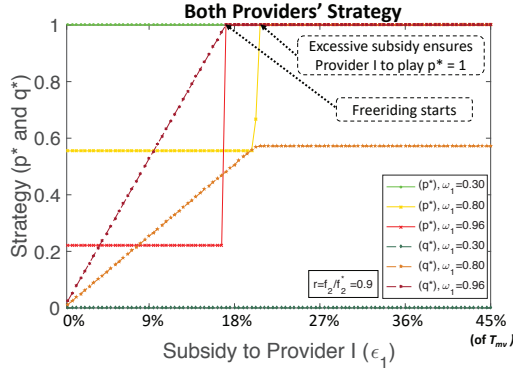


Fig. 7: NE for different  $\omega_1$  w.r.t.  $\epsilon_1$

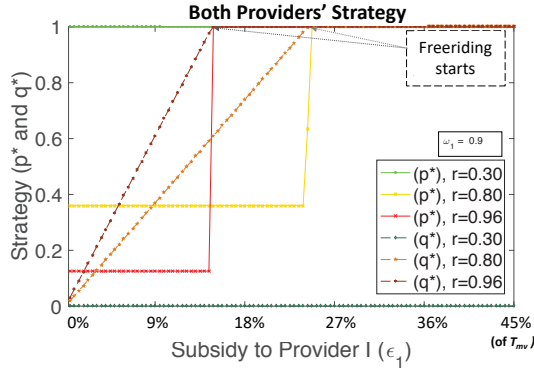


Fig. 8: NE for different  $r$  w.r.t.  $\epsilon_1$

when  $\omega_1$  is 0.69. It is Provider I's terminal willingness to share up-to which I can always maintain agreed willingness to share irrespective of Provider II's strategies. As we calculate  $p^*$  with the fraction of  $(h - f)$  to  $(b - f)$ , it is expected to reduce  $p^*$  to that level where the fraction is located in Fig. 6. We also observe that both providers maintain same strategy for certain period of  $\omega_1$ . It happens, because, between this period, Provider II cannot earn more from UCS.

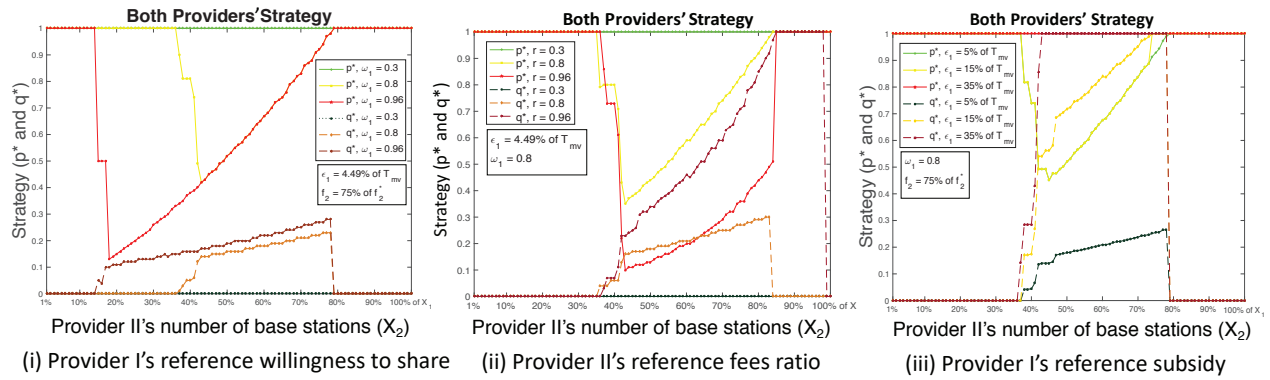
2) *Effect of Subsidy*: We run the experiment with Provider I's 100 different subsidies from 0% to 45% of  $T_{mv}$ , and observed the equilibrium behaviors. We have seen that Provider I is less interested in sharing with the increase of Provider II's fees closer to fair market fees. However, if we continue increasing  $\epsilon_1$ , Provider I is finally able to overcome its loss due to UCS. Additional subsidy can encourage it to play the most sharing strategy,  $p^* = 1$ , which may initiate freeriding. Fig. 7 describes the case where we plot graphs against Provider I's three different willingness to shares ( $\omega_1 = 0.3, 0.8, 0.96$ ). For same  $r$ , a high value of  $\omega_1$  encourages Provider II to undersell mostly, and this forces Provider I to play a less sharing strategy. In case of  $\omega_1 = 0.8$ , if we continue increasing  $\epsilon_1$ , Provider I is able to overcome revenue loss when  $\epsilon_1 = 0.202T_{mv}$ , which encourages Provider I to share more ( $p^* = 1$ ). However, Provider II continues its mixed strategy onwards as it cannot make more profits from UCS. When  $\omega_1 = 0.96$ , we find the start of freeriding,  $p^* = 1, q^* = 1$ , at  $\epsilon_1 = 0.175T_{mv}$ .

Fig. 8 tells the same story from a different perspective where NEs are observed against Provider II's three different fees ratios,  $r = 0.3, 0.8$  and  $0.96$ . When  $r = 0.8$  and  $r = 0.96$ , Provider I and Provider II play mixed strategies up to  $\epsilon_1 = 0.238T_{mv}$  and  $\epsilon_1 = 0.139T_{mv}$  respectively. After these  $\epsilon_1$ 's, freeriding takes place. We also observe, when  $r = 0.3$ , Provider I and Provider II play  $p^* = 1$  and  $q^* = 0$  respectively which remain unchanged. It happens because of Provider II's low charging fee. At  $r = 0.3$ , Provider II is charging 30% of its fair market fee. With this fee, it can attract a significant number of customers from Provider I, however, the aggregate revenue in this situation becomes lower than the revenue when it charges a fair market fee. Thus, Provider II's choice of not freeriding helps Provider I to play a maximum sharing strategy.

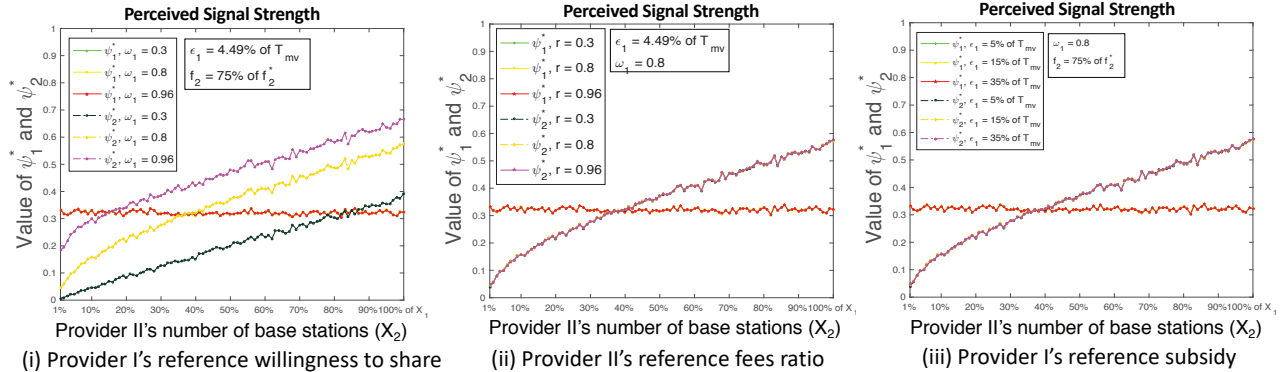
3) *Effect of Base Stations*: We evaluate game NEs with Provider I's fixed BS count to 100 and Provider II's varying BS count between 1-100% of Provider I's BS count. For all cases of Fig. 9, no freeriding takes place when the size of Provider II is too small. It happens because of Provider II's customers overall perceived signal strength. When Provider II's BS count is small, its customers will mostly find Provider I's BSeS as the nearest BS, and Provider I will serve these customers based on its strategic decision of sharing. As a result, Provider II's customers overall perceived signal strength cannot exceed that Provider I's customers enjoy. If Provider II's BS count is around half of Provider I's BS count, the perceived signal strengths by both providers customers changes and UCS occurs. It forces Provider I to reduce sharing and start playing mixed strategy. If Provider II's BS count is similar to the Provider I's BS count, no freeriding takes place. Though Provider II's customers receive better signal strength compared to Provider I's customers, the difference between both providers' charging fees remains low for which Provider II cannot initiate enough UCS to earn more by charging lower-than-fair fees. This low fees difference and low earning force Provider II to charge fair market fees. The freeriding and non-freeriding regions depend on parameters  $\epsilon_1, r$ , and  $\omega_1$ . From Fig. 9a(ii) and 9b(ii), when  $\omega_1 = 0.8, \epsilon_1 = 4.49\%$  of  $T_{mv}$  and  $r = 0.96$ , we observe a freeriding case ( $p^* = 1, q^* = 1$ ) if Provider II's BS count is between 85-98% of Provider I's BS count. This happens because of the high value of  $r$  (i.e.,  $f_2$  is closer to  $f_2^*$ ), Provider I loses few customers to Provider II and earns enough subsidy to overcome the loss. When both providers are almost equal in size (i.e.,  $X_2 = 99$  or  $100\%$  of  $X_1$ ), no freeriding takes place. From Fig. 9a(iii), we observe freeriding takes place if Provider II's BS count is between 74-78% and 43-78% of Provider I's BS count when  $\epsilon_1 = 15\%$  of  $T_{mv}$  and  $\epsilon_1 = 35\%$  of  $T_{mv}$  respectively.

In the SBSS market, a user receives better signal strength which enables better wireless experience. From Fig. 9b(i), a customer of Provider II enjoys higher signal strength if Provider I increases  $\omega_1$  when all other conditions remained unchanged. If both would share, then the perceived signal strength for both customers' would have improved. This delineates how spectrum sharing in the SBSS market can improve a user's perceived quality of service.

4) *Effect of Sharing on Call Types*: In an SBSS market, customers of Provider II can make calls similar to roaming.



(a) Both providers' strategies for different  $\omega_1$ ,  $r$  and  $\epsilon_1$



(b) Both providers' signal strength for different  $\omega_1$ ,  $r$  and  $\epsilon_1$

Fig. 9: NE strategies and perceived signal strengths for different  $X_2$  counts

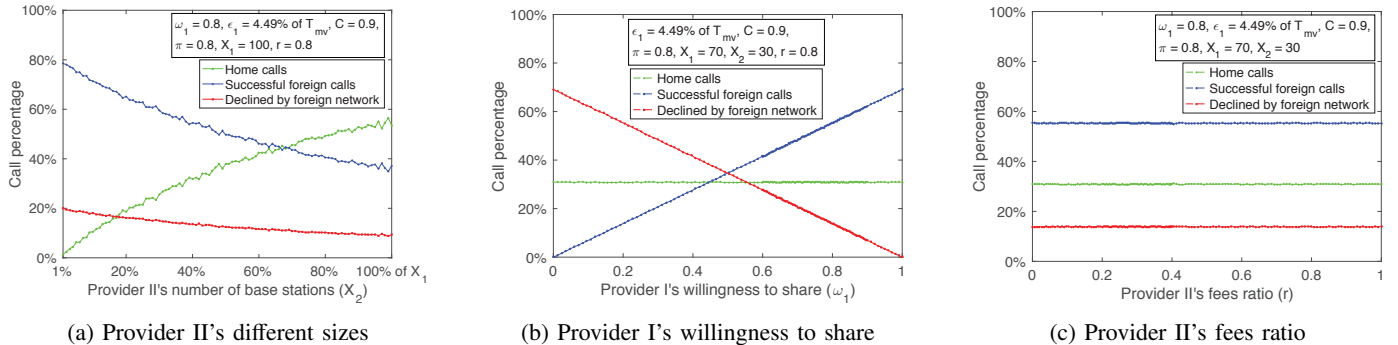


Fig. 10: Calls generated by the customers of Provider II

However, in roaming, the number of calls served by foreign providers are not high because of cellular providers geo-operational strategies. In the SBSS market, it can be significantly high because of pervasive sharing. If we vary Provider II's size from 1 to 100% of Provider I's size, customers of Provider II make fewer foreign calls using I's network with the increase of size, Fig. 10a. At the same time, the percentage of rejections by Provider I are also reduced and overall home calls made through II's network has been increased. For fixed-sized providers set, the percentage of foreign calls increases with the increase of  $\omega_1$ , Fig. 10b. If we only change Provider II's fees, the percentage of home calls, foreign calls or declined foreign calls remain unchanged. Because, these numbers depend  $\omega_1$  and  $X_2$ . From Fig. 10c, we observe

both  $X_2$ , and  $\omega_1$  remain unchanged. Now, if we only vary Provider II's fees,  $f_2$ , to observe the effect on the number of calls, it will not differ significantly. Lowering fees may attract customers from Provider I in the long run and change the stats, however, for a given condition the number of calls will remain unchanged as we are not considering customer switching effects for this particular experiment. Instead, we are interested to find the numbers of different types of calls on average Provider II's subscribed customers are making at an instance if Provider II charges any low-fee other than its fair market fee. If the market consists of more than two providers, declined customers may get served by other better providers instead of complete rejection or re-routed to home network.

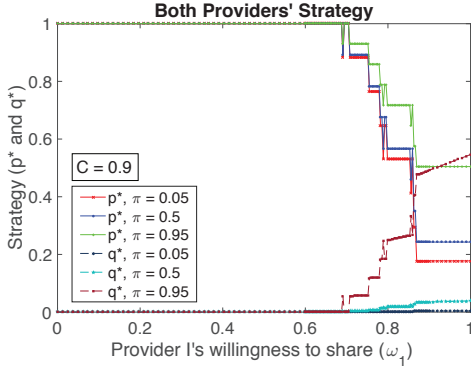


Fig. 11: NE for a set of  $\pi$  w.r.t.  $\omega_1$

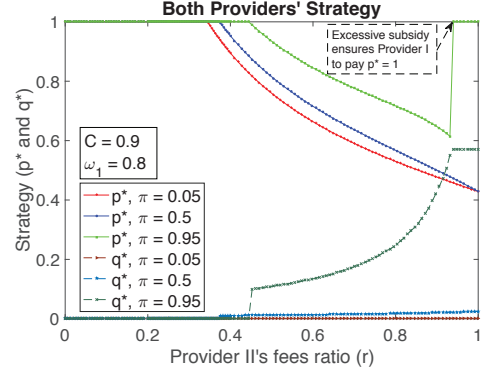
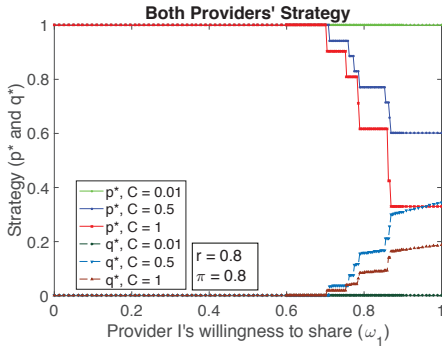
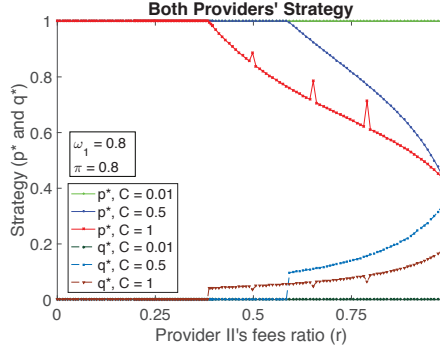


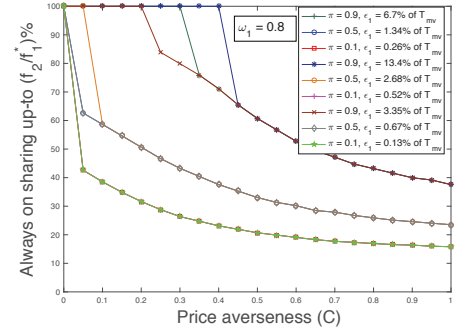
Fig. 12: NE for a set of  $\pi$  w.r.t.  $r$



(a) w.r.t.  $\omega_1$



(b) w.r.t. fees ratio ( $r$ )



(c) w.r.t. price averseness

Fig. 13: Equilibrium strategies and always on full sharing with a set of  $C$

5) *Effect of Marginal Signal Improvement*: Equilibrium strategies w.r.t. different  $\pi$  values are shown in Fig. 11 and 12. If we increase  $\pi$  by same amount for both providers, Provider I's higher sharing region increases. This happens due to two reasons, (1) the higher the  $\pi$  is, the closer each other's fees are, and (2) if the customers get equal improvement on signal strength from both providers, Provider I still remains ahead of Provider II. As a result, UCS probability remains low, which encourages Provider I for higher sharing. If  $\pi$  is high, and Provider II's charging fee stays near to fair market fee, Provider I can switch to play higher sharing pure strategy from a low sharing mixed strategy, Fig. 12. It happens, because, higher sharing brings more revenue from the subsidy than the subscription revenue lost due to UCS. In this case, a small amount of subsidy is enough to drive a provider for higher sharing, e.g., if  $\pi = 0.95$ , and  $r = 0.93$ ,  $\epsilon_1 = 0.0449T_{mv}$  is enough to motivate Provider I to switch from a mixed strategy to the higher sharing pure strategy.

6) *Effect of Price Averseness and UCS*: Customers are very averse with the price they pay for a service. In an SBSS market, a provider's willingness to share and low fees can cause UCS because of price averseness. If we increase  $C$ , the probability of UCS increases and the resource sharing provider loses customers. A resource sharing provider can maintain high sharing irrespective of customers' price averseness if a competitor provider always charges same fee. From Fig. 13a, if Provider II always charges 80% of its fair market fee, Provider I can maintain pure strategy ( $p^* = 1$ ) up-to  $\omega_1 = 0.69$

irrespective of  $C$ . However, if Provider II charges different fees, we can see in Fig. 13b that the price averseness affects Provider I's sharing and it chooses mixed strategy soon with the increase of  $C$ .

We observe the scenarios of how long Provider I can play pure strategy ( $p^* = 1$ ) or always maintain agreed sharing (i.e.,  $\omega_1 = 0.8$  in Fig. 13c) if we gradually increase  $C$  and take different  $\pi$ . It tells us the fees ratio ( $f_2/f_1^*$ ) up-to which Provider I will play higher sharing pure strategy whatever Provider II does. Above this fees ratio, Provider I starts playing mixed strategy. We observe that, with the increase of  $C$  and  $\pi$ , Provider I is more willing to share its spectrum resources. Because, it can tolerate a high price averseness at high marginal signal improvement. Also, if we increase  $\epsilon_1$ , the region of always on full sharing up-to agreed  $\omega_1$  increases. In this observation, the assumption of a high  $\omega_1$ , e.g., 0.8, comes from the experiment (described in Section VI-B7) where we found that for any  $\epsilon_1$ , Provider I always play higher sharing pure strategy up-to  $\omega_1 = 0.69$ .

7) *Large Provider's Willingness to Share and Weak Provider's Effort to Trigger UCS*: Higher sharing is expected in the SBSS market. However, higher sharing increases the chance of revenue loss for the provider who shares spectrum resources. Lowering fee is beneficial for a freeriding provider. However, if the fees are too low compared to fair market fee, lowering fees can be a bad move for it. Thus, there is always a willingness to share, and a fees ratio, below which any sharing can be ensured with PSNE.

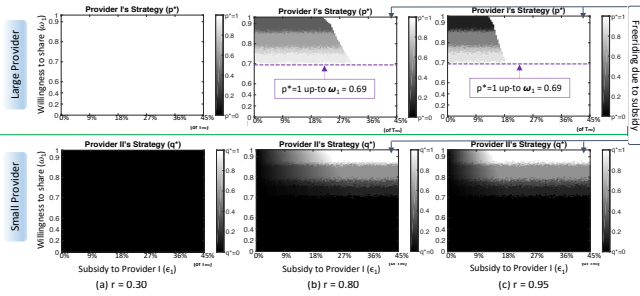


Fig. 14: Threshold of  $\omega_1$  w.r.t  $r$

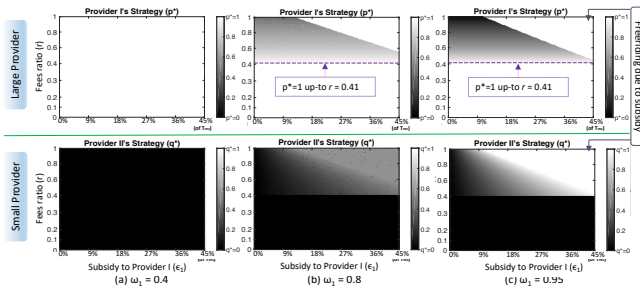


Fig. 15: Threshold of  $r$  w.r.t  $\omega_1$

We observe equilibrium strategies against given subsidy to Provider I and for a given set of Provider II's fees ratio as described in Fig. 14. When the fees ratio is 30%, we see both providers are interested to play freeriding-free pure strategies, i.e.,  $p^* = 1$  and  $q^* = 0$ , for any  $\omega_1$ , Fig. 14a. With the increase of fees ratio, we observe both pure and mixed strategy NEs, Fig. 14b and Fig. 14c. However, for any  $r$  and  $\epsilon_1$ , we observe a freeriding-free PSNE up-to  $\omega_1 = 0.69$ . We also observe NE strategies against given subsidy to Provider I and its a set of willingness to share in Fig. 15. When sharing is low, e.g.,  $\omega_1 = 0.4$  in Fig. 15a, both providers play the freeriding free PSNE. With the increase of sharing, we observe both pure and mixed strategy NEs, Fig. 15b and Fig. 15c. If the fees ratio  $r$  is high, the game exhibits a freeriding equilibrium with the increase of subsidy. The high  $\omega_1$ , the more we get the freeriding equilibrium when all game conditions remain unchanged. However, for all cases, if  $r \leq 0.41$ , we get freeriding-free PSNEs. Above of this ratio, we observe both players adjust their pure strategy to mixed strategy to ensure profits. In this case,  $r = 0.41$  is the threshold fees ratio, below which Provider II is not interested to play freeriding strategy,  $q^* = 1$ , as it can not make profit from UCS.

8) *Comparison with Cellular Roaming*: In general cellular roaming [4] is a “pay per use” service and highly dependent on the agreement among providers even though users may not know how much of their payment is for roaming. Being confidential data, the serving rate of a particular provider's roamed customers is not available. Thus, we needed to make an assumption. We assume 50% of Provider II's customers get paid roaming service from Provider I. This number is significantly high when compared with actual data as very few customers are interested in roaming because of the high charging rate. From the simulation setup in earlier sections,

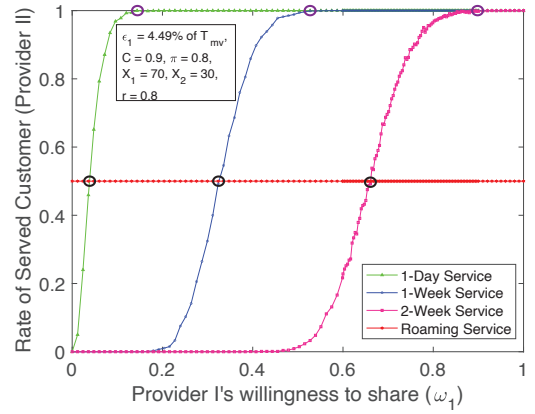


Fig. 16: Ratio of No. of customers get served

we already know that each customer makes 100 calls in a monthly subscription. Thus the number of calls a customer is expected to make in a day, a week, and bi-weekly are 3, 23, and 47 (all numbers are rounded) respectively. We consider these three cases to identify how many customers of Provider II actually get such services from Provider I in an SBSS market. We looked for  $\omega_1$  when the ratio of Provider II's number of beneficiary customers to II's total number of customers become 50%. We also observe the maximum number of customers that get served by Provider I at what  $\omega_1$ .

From Fig. 16, we find that 50% of Provider II's customers get roamed service from Provider I as per roaming agreement that includes all the three cases. In an SBSS market, if  $\omega_1$  is at least 0.03, 0.33 and 0.66, we find that 50% of Provider II's customers get 1-day, 1-week and 2-week equivalent services from Provider I. When  $\omega_1$  is 0.14, 0.53 and 0.9, Provider II's all customers get 1-day, 1-week, and 2-week equivalent services from Provider I respectively. By increasing  $\omega_1$ , Provider I can serve Provider II's same number of customers with long-duration services. However, in roaming, this number remains constant. Thus, by motivating Provider I for higher sharing, an SBSS market can improve Provider II's customers' overall wireless experience without paying any additional charges, and vice versa.

9) *Strategies in a Multi-provider SBSS Market*: We made our observation on a 3-provider SBSS market where Provider I, II, and III's base station counts are 50, 30, and 20 respectively. We compute all providers' customer counts and their fair market fees as proportionate to their base station counts as described in Table I.

We assume, Provider I gets subsidy  $\epsilon_1 = 0.05T_{mv}$  from the government. It will receive three-fifth of  $\epsilon_1$  for serving Provider II's customers and rest for Provider III's customers. In this market, we determine all providers' strategies by playing two-provider games where Provider I shares spectrum resources, and Provider II and III try to get additional benefits by charging 80% of their fair market fees, as shown in Fig. 17. We observe Provider I plays pure strategies to cover calls from the customers of Provider II until  $\omega_1 = 0.42$ . If Provider I makes an initial promise of  $\omega_1 > 0.42$ , it can not always

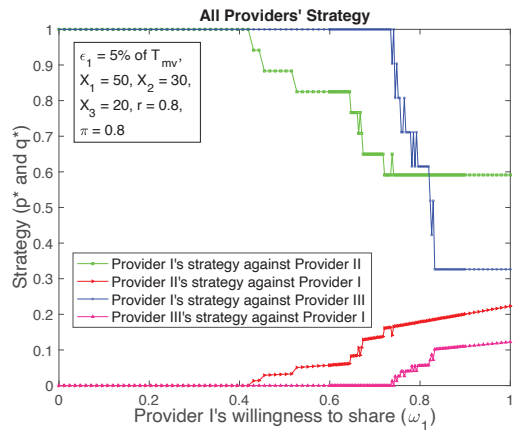


Fig. 17: 3-provider SBSS market strategies

maintain that agreement. Instead, it chooses to play a mixed strategy to maintain its non-SBSS revenue. In the case of serving Provider III's customers, Provider I can maintain high sharing up to  $\omega_1 = 0.74$ . Considering the market competition, Provider II is the immediate competitor for Provider I based on its size. Thus, any freeriding strategy from Provider II will hurt Provider I's revenue significantly. As a result, Provider I is more cautious while sharing resources with Provider II's customers. Here, we only consider a 3-provider SBSS market. It can be extended to a multi-provider market where the number of providers is greater than 3. For such a market, we will get Provider I's sharing strategies corresponding to all providers' freeriding strategies.

## VII. CONCLUSIONS AND FUTURE WORK

In this paper, we have considered a non-cooperative game-theoretic framework for an SBSS market, where the amount of customer switching triggered by a freeriding provider can be reduced and the earnings of the spectrum sharing provider remains at least as much as the earnings from the existing non-shared spectrum market. Our proposed game model can ensure the spectrum sharing provider to serve up to a certain sharing percentage of total incoming foreign calls at all times. Above this willingness to share, our model helps both providers to maintain a mixed strategy to control freeriding. Also, there exists a fees ratio (i.e., the potential disparity of the freeriding provider's fee to a fair market fee), below which a spectrum sharing provider can always share. We have considered customers' price averseness to determine customer switching probability arising from freeriding and shown the impact of subsidy in an SBSS cellular market. We also observed the NEs with different sizes of the providers. If the size difference between two providers is large or they are similar in size, we observed no freeriding takes place. We also discussed the application of the two-provider game in a multi-provider environment.

We leave exploration of NEs from the repeated games as future work. Exploration of marginal signal improvement is another worthy future work. Here, we have used same value for both providers. Different values may substantially increase/

decrease the size of the desired/undesired equilibrium regions. In a competitive market, along with different marginal signal improvement values, a small amount of subsidy can motivate a provider for higher sharing.

Our game-theoretic framework is a good way to identify good *willingness to share* values for a provider. But, a provider in SBSS will have to use its *willingness to share* to tune its strategy. If the provider wants to increase its subsidy revenue, it should increase its *willingness to share*. Yet, the provider will run the risk of exposing itself to potential freeriding providers. It is an open issue to develop strategies for a provider in the SBSS market. From a provider's perspective, the *willingness to share* parameter adds a new knob in addition to its subscription fee. By tuning both the *willingness to share* and subscription fee, the providers in SBSS can find the optimum operational regime for themselves. This is a dynamic optimization problem for SBSS, which we have not focused on in this paper. Also, we have not considered Mobile Virtual Operators (MVNOs) as part of the game-theoretic model. As such virtual operators without any infrastructure are being considered, looking at the effect of such operators on the SBSS markets will be an interesting direction to take.

## ACKNOWLEDGMENT

This work was supported in part by NSF awards 1663761, 1444077, and 1647189. The majority of this work was performed at the University of Central Florida.

## REFERENCES

- [1] M. Rahman, M. Yuksel, and T. Quint, "Regulating freeriding in inter-operator spectrum sharing," in *Proceedings of IEEE Int. Symp. on Dynamic Spectrum Access Networks (DySPAN)*, 2018.
- [2] E. Hossain, D. Niyato, and Z. Han, *Dynamic spectrum access and management in cognitive radio networks*. Cambridge university press, 2009.
- [3] R. Yin, C. Zhong, G. Yu, Z. Zhang, K. K. Wong, and X. Chen, "Joint spectrum and power allocation for D2D communications underlying cellular networks," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 4, 2015.
- [4] "International roaming explained," <https://www.gsma.com/latinamerica/wp-content/uploads/2012/08/GSMA-Mobile-roaming-web-English.pdf>, 2012.
- [5] Y. Teng, Y. Wang, and K. Horneman, "Co-primary spectrum sharing for denser networks in local area," in *Proceedings of IEEE Cognitive Radio Oriented Wireless Nets. and Comm. (CROWNCOM)*, 2014.
- [6] B. Cho, K. Koufos, R. Jäntti, and S.-L. Kim, "Co-primary spectrum sharing for inter-operator device-to-device communication," *IEEE Journal on Selected Areas in Comm.*, vol. 35, no. 1, pp. 91–105, 2017.
- [7] P. Luoto, M. Bennis, P. Pirinen, S. Samarakoon, and M. Latva-aho, "Enhanced co-primary spectrum sharing method for multi-operator networks," *IEEE Trans. on Mobile Computing*, 2017.
- [8] H. Liu, S. Hua, X. Zhuo, D. Chen, and X. Cheng, "Cooperative spectrum sharing of multiple primary users and multiple secondary users," *Digital Comm. and Nets.*, vol. 2, no. 4, pp. 191–195, 2016.
- [9] "Coverage maps you can trust," <https://opensignal.com>.
- [10] "Connecting america: The national broadband plan," <http://download.broadband.gov/plan/national-broadband-plan.pdf>, 2010.
- [11] M. Yuksel, I. Guvenc, W. Saad, and N. Kapucu, "Pervasive spectrum sharing for public safety communications," *IEEE Comm. Mag.*, vol. 54, no. 3, pp. 22–29, 2016.
- [12] S. M. Yu and S.-L. Kim, "Guaranteeing user welfare in network service: Comparison of two subsidy schemes," *SIGMETRICS Perform. Eval. Rev.*, vol. 40, no. 2, pp. 22–25, 2012.
- [13] Z. Fan, Y. Mao, and T. Horgor, "What smart grid means to an iso/rto?" in *Proceedings of IEEE Transmission and Distribution Conference and Exposition*, 2010, pp. 1–8.



- [14] "Spectrum pricing: Gsma public policy position," [https://www.gsma.com/spectrum/wp-content/uploads/2017/09/spectrum\\_pricing\\_positioning\\_2017.pdf](https://www.gsma.com/spectrum/wp-content/uploads/2017/09/spectrum_pricing_positioning_2017.pdf), 2017.
- [15] A. Merwaday, M. Yuksel, T. Quint, I. Güvenç, W. Saad, and N. Kapucu, "Incentivizing spectrum sharing via subsidy regulations for future wireless networks," *Computer Networks*, vol. 135, pp. 132–146, 2018.
- [16] M. Rahman, S. H. Ahmed, and M. Yuksel, "Proof of sharing in inter-operator spectrum sharing markets," in *Proceedings of IEEE Int. Symp. on Dyn. Spectrum Access Nets.(DySPAN)*, 2018.
- [17] M. Rahman and M. Yuksel, "Mobile proof-of-sharing architectures for inter-operator spectrum sharing markets," in *Proceedings of IEEE Int. Symp. on Dyn. Spectrum Access Nets.(DySPAN)*, 2019.
- [18] S. Anand, V. P. Kumar, and R. Chandramouli, "Spectrum management over dynamic spectrum access based hetnets," in *2019 IEEE Int. Symp. on Dynamic Spectrum Access Networks (DySPAN)*, 2019.
- [19] S. K. Sharma, T. E. Bogale, L. B. Le, S. Chatzinotas, X. Wang, and B. Ottersten, "Dynamic spectrum sharing in 5g wireless networks with full-duplex technology: Recent advances and research challenges," *IEEE Communications Surveys Tutorials*, vol. 20, no. 1, pp. 674–707, 2018.
- [20] M. B. Weiss, K. Werbach, D. C. Sicker, and C. E. C. Bastidas, "On the application of blockchains to spectrum management," *IEEE Transactions on Cognitive Communications and Networking*, vol. 5, no. 2, 2019.
- [21] C. Xin, P. Paul, M. Song, and Q. Gu, "On dynamic spectrum allocation in geo-location spectrum sharing systems," *IEEE Transactions on Mobile Computing*, 2018.
- [22] B. Singh, K. Koufos, O. Tirkkonen, and R. Berry, "Co-primary inter-operator spectrum sharing over a limited spectrum pool using repeated games," in *Proceedings of IEEE Int. Conf. on Comm. (ICC)*, 2015.
- [23] M. Gomez, M. Weiss, and P. Krishnamurthy, "Improving liquidity in secondary spectrum markets: Virtualizing spectrum for fungibility," *IEEE Transactions on Cognitive Communications and Networking*, vol. 5, no. 2, 2019.
- [24] P. K. Sangdeh, H. Pirayesh, H. Zeng, and H. Li, "A practical underlay spectrum sharing scheme for cognitive radio networks," in *IEEE Int. Conference on Computer Communications (INFOCOM)*, 2019.
- [25] Y. Xu, J. Yu, and R. M. Buehrer, "The application of deep reinforcement learning to distributed spectrum access in dynamic heterogeneous environments with partial observations," *IEEE Transactions on Wireless Comms*, vol. 19, no. 7, 2020.
- [26] T. Sanguanpuak, S. Guruacharya, E. Hossain, N. Rajatheva, and M. Latva-aho, "On spectrum sharing among micro-operators in 5G," in *2017 European Conference on Networks and Communications (EuCNC)*, 2017, pp. 1–6.
- [27] T. Kokkinen, H. Kokkinen, and S. Yrjölä, "Spectrum broker service for micro-operator and cbrs priority access licenses," in *Proceedings of Springer International Conference on Cognitive Radio Oriented Wireless Networks*, 2017.
- [28] D. Stojadinovic and M. Buddhikot, "Design of a secondary market for fractional spectrum sub-leasing in three-tier spectrum sharing," in *IEEE Int. Symp. on Dynamic Spectrum Access Networks (DySPAN)*, 2019, pp. 1–8.
- [29] C. Xin and M. Song, "Analysis of the on-demand spectrum access architecture for CBRS cognitive radio networks," *IEEE Transactions on Wireless Comms*, vol. 19, no. 2, 2020.
- [30] M. Rebato and M. Zorzi, "A spectrum sharing solution for the efficient use of mmWave bands in 5G cellular scenarios," in *IEEE Int. Symp. on Dynamic Spectrum Access Networks (DySPAN)*, 2018, pp. 1–5.
- [31] X. Sun and L. Dai, "Towards fair and efficient spectrum sharing between LTE and WiFi in unlicensed bands: Fairness-constrained throughput maximization," *IEEE Transactions on Wireless Comms*, vol. 19, no. 4, 2020.
- [32] C. Liu and R. A. Berry, "Competition with shared spectrum," in *Proceedings of IEEE Int. Symp. on Dynamic Spectrum Access Nets. (DySPAN)*, 2014.
- [33] L. Ramaswamy and L. Liu, "Free riding: A new challenge to peer-to-peer file sharing systems," in *Proceedings of IEEE Hawaii Int. Conf. on System Sciences*, 2003.
- [34] M. Feldman and J. Chuang, "Overcoming free-riding behavior in peer-to-peer systems," *ACM SIGecom Exch.*, vol. 5, no. 4, pp. 41–50, 2005.
- [35] S. Flynn, A. Hollis, and M. Palmedo, "An economic justification for open access to essential medicine patents in developing countries," *The Journal of Law, Medicine & Ethics*, vol. 37, no. 2, pp. 184–208, 2009.
- [36] L. Kaplow, "Concavity of utility, concavity of welfare, and redistribution of income," *International Tax and Public Finance*, vol. 17, no. 1, pp. 25–42, 2010.
- [37] L. Tan and Y. Zhang, "Optimal resource allocation with principle of equality and diminishing marginal utility in wireless networks," *Wireless Personal Communications*, vol. 84, no. 1, pp. 671–693, 2015.
- [38] E. Dechenaux, D. Kovenock, and R. M. Sheremeta, "A survey of experimental research on contests, all-pay auctions and tournaments," *Experimental Economics*, vol. 18, no. 4, pp. 609–669, 2015.
- [39] G. Bacci, S. Lasaulce, W. Saad, and L. Sanguinetti, "Game theory for signal processing in networks," *IEEE Signal Processing Magazine*, vol. 33, no. 1, pp. 94–119, 2016.
- [40] "No time to talk: Americans sending/receiving five times as many texts compared to phone calls each day, according to new report," <https://www.prnewswire.com/news-releases/no-time-to-talk-americans-sendingreceiving-five-times-as-many-texts-compared-to-phone-calls-each-day-according-to-new-report-300056023.html>, 2015.
- [41] "Best cell phone plans of 2020," <https://www.reviews.org/mobile/best-cell-phone-plans/>, 2020.
- [42] "Here's how the 'unlimited' plans from verizon, at&t, sprint, and t-mobile compare," <http://www.businessinsider.com/unlimited-plans-comparison-verizon-att-sprint-tmobile-2018-2>, 2018.



**Mostafizur Rahman** is an Assistant Professor in the Department of Electrical and Computer Engineering at the University of Texas Rio Grande Valley (UTRGV), Edinburg, TX. He obtained a Ph.D. and an MS in computer engineering from the University of Central Florida (UCF), Orlando, Florida, USA in 2020 and 2018, respectively. He received a BS degree in computer science and engineering from Bangladesh University of Engineering and Technology (BUET), Bangladesh in 2012. He worked as a senior software engineer at Samsung R&D Institute Bangladesh (SRBD) Ltd., Dhaka, Bangladesh from 2012 to 2015. His research interest includes spectrum sharing, cellular networks, mobile computing and machine learning.



**Murat Yuksel** is a Professor at the ECE Dept. of the University of Central Florida (UCF), Orlando, FL. Prior to UCF, he was with the CSE Dept. of the University of Nevada - Reno (UNR), Reno, NV as a faculty member until 2016. He was with the ECSE Dept. of Rensselaer Polytechnic Institute (RPI), Troy, NY as a Postdoctoral Associate and a member of Adjunct Faculty until 2006. He received a B.S. degree in computer engineering of Ege University, Izmir, Turkey in 1996. He received M.S. and Ph.D. degrees in computer science of RPI in 1999 and 2002 respectively. He worked as a software engineer at Pepperdata, Sunnyvale, CA and a visiting researcher at AT&T Labs and Los Alamos National Lab. His research interests are in the area of networked, wireless, and computer systems with a recent focus on big-data networking, UAV networks, optical wireless, public safety communications, device-to-device protocols, economics of cyber-security and cyber-sharing, routing economics, network management, and network architectures. He has been on the editorial board of Computer Networks, and published more than 150 papers at peer-reviewed journals and conferences and is a co-recipient of the IEEE LANMAN 2008 Best Paper Award. He is a senior member of IEEE, senior and life member of ACM, and was a member of Sigma Xi and ASEE.



**Thomas Quint** is a Professor in the Department of Mathematics and Statistics at the University of Nevada Reno (UNR). He obtained his PhD in operations research from Stanford in 1987. In addition to UNR, he has taught at UCLA, the US Naval Academy, Yale, and the Universidad Nacional de San Luis in Argentina. His research interests lie in cooperative and noncooperative game theory, mathematical economics, and graph theory. His publications have appeared in journals such as the International Journal of Game Theory, the Annals of Finance, Mathematics of Operations Research, and Linear Algebra and its Applications. He also coauthored a book published by the Yale University Press in 2014.